1.2: Transform the expression $3x + 5y - 5z$ into the expression $5(y - z) + 3x$, stating the axiom you used for each step.

$$3x + 5y - 5z = 3x + 5(y - z) \quad \text{Distribution}$$

$$= 5(y - z) + 3x \quad \text{Commutation of addition}$$

1.3: Let $\mathbf{a} = \begin{pmatrix} 1 \\ 5 \\ 3 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} -5 \\ 2 \\ 6 \end{pmatrix}$, and $\mathbf{c} = \begin{pmatrix} 8 \\ -7 \\ -5 \end{pmatrix}$. What is $\mathbf{a} + \mathbf{b} - \mathbf{c}$?

$$\underline{a} + \underline{b} - c = \begin{pmatrix} 1 \\ 5 \\ 3 \end{pmatrix} + \begin{pmatrix} -5 \\ 2 \\ 6 \end{pmatrix} - \begin{pmatrix} 8 \\ -7 \\ -5 \end{pmatrix}$$

$$= \begin{pmatrix} 1-5-8 \\ 5+2+7 \\ 3+6+5 \end{pmatrix}$$

$$= \begin{pmatrix} -12 \\ 14 \\ 14 \end{pmatrix}$$

1.5: Is the function $f(x) = x^2 - x$ linear?

$$f(kx) = (kx)^2 - kx$$

$$= k^2x^2 - kx$$

$$= k(kx^2 - x) \neq kf(x)$$

No.

1.6: Find the magnitude of the vector $\mathbf{x} = \begin{pmatrix} 6 \\ -2 \\ 3 \end{pmatrix}$. Use the magnitude to convert $\mathbf{x}$ into a unit vector.

$$\|\underline{x}\| = \left(6^2 + (-2)^2 + 3^2\right)^{1/2}$$

$$= \sqrt{49} = 7$$

$$\hat{\underline{x}} = \begin{pmatrix} 6/7 \\ -2/7 \\ 3/7 \end{pmatrix}$$

1.7: Let $\mathbf{x} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$ and $\mathbf{y} = \begin{pmatrix} 6 \\ 1 \\ 4 \end{pmatrix}$. What is $\mathbf{x} \cdot \mathbf{y}$? What is the angle between the vectors?

$$\underline{x} \cdot \underline{y} = 2 \times 6 - 3 \times 1 + 1 \times 4$$
$$= 13$$

$$\underline{x} \cdot \underline{y} = \|\underline{x}\| \, \|\underline{y}\| \cos \theta$$

$$\|\underline{x}\| = \left(2^2 + (-3)^2 + 1^2\right)^{1/2} = \sqrt{14}$$
$$\|\underline{y}\| = \left(6^2 + 1^2 + 4^2\right)^{1/2} = \sqrt{53}$$

$$\theta = \cos^{-1}\left(\frac{\underline{x} \cdot \underline{y}}{\|\underline{x}\| \, \|\underline{y}\|}\right)$$

$$= \cos^{-1}\left(\frac{13}{\sqrt{14}\,\sqrt{53}}\right) = 61.5^\circ$$

2.1a: Which of the following matrix products are compatible for multiplication?

$$\mathbf{A} = \mathbb{R}^{2\times3}$$
$$\mathbf{B} = \mathbb{R}^{5\times5}$$
$$\mathbf{C} = \mathbb{R}^{3\times5}$$

Write the dimension of the product whenever the matrices are compatible.

1. **AB** No
2. **BA** No
3. **BC** No
4. **CA** No
5. **CB** Yes: 3x5

2.1b: Compute the product

$$\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 5 & 6 & 3 \\ 1 & 5 & 4 \end{pmatrix}.$$

$$= \begin{pmatrix} 7 & 16 & 11 \\ 11 & 17 & 10 \end{pmatrix}$$

2.3: Simplify the expression $(\mathbf{A}\mathbf{A}^{-1})^{-1}\mathbf{A}$.

$$
\begin{aligned}
(\underline{A}\,\underline{A}^{-1})^{-1}\underline{A} &= (\underline{I})^{-1}\underline{A} \\
&= \underline{I}\,\underline{A} \\
&= \underline{A}
\end{aligned}
$$

2.6: Which is the most efficient way to compute the product **ABC** when dim(**A**) = 100×400, dim(**B**) = 400 × 600, and dim(**C**) = 600 × 200?

a. (**AB**)**C**

b. **A**(**BC**)

Using the guidelines in §2.6

$(\underline{A}\,\underline{B})\underline{C}$: $100 \times 400 \times 600 + 100 \times 600 \times 200 = 36\,M$

$\underline{A}(\underline{B}\,\underline{C})$: $400 \times 600 \times 200 + 100 \times 400 \times 200 = 56\,M$

Choose $(\underline{A}\,\underline{B})\underline{C}$.

3.1: Rotate the point $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$ counter-clockwise by 87°.

$$\underline{R}(87^\circ)\begin{pmatrix} 3 \\ -4 \end{pmatrix} = \begin{pmatrix} \cos 87^\circ & -\sin 87^\circ \\ \sin 87^\circ & \cos 87^\circ \end{pmatrix}\begin{pmatrix} 3 \\ -4 \end{pmatrix}$$

$$\approx \begin{pmatrix} 0.0523 & -0.999 \\ 0.999 & 0.0523 \end{pmatrix}\begin{pmatrix} 3 \\ -4 \end{pmatrix}$$

$$= \begin{pmatrix} 4.15 \\ 2.79 \end{pmatrix}$$

3.3: Point $\mathbf{p}$ is located at $\begin{pmatrix}1\\2\end{pmatrix}$. Shift $\mathbf{p}$ by 7 horizontally and 6 vertically, and rotate the shifted point 90° counter-clockwise about the origin. What is the final coordinates of the point?

$$\underline{p}' = \underline{R}(90^\circ)\,\underline{T}(7,6)\,\underline{p}$$

$$= \begin{pmatrix} \cos 90^\circ & -\sin 90^\circ & 0 \\ \sin 90^\circ & \cos 90^\circ & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 7 \\ 0 & 1 & 6 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 8 \\ 8 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} -8 \\ 8 \\ 1 \end{pmatrix}$$

The new coordinates are $\begin{pmatrix} -8 \\ 8 \end{pmatrix}$.

4.0: Solve the system of equations

$$-3x + 5y = 45$$
$$8x + 6y = 112$$

$$\begin{pmatrix} -3 & 5 & 45 \\ 8 & 6 & 112 \end{pmatrix}$$

$$\xrightarrow{-\frac{1}{3}R_1} \begin{pmatrix} 1 & -5/3 & -15 \\ 8 & 6 & 112 \end{pmatrix}$$

$$\xrightarrow{-8R_1 \to R_2} \begin{pmatrix} 1 & -5/3 & -15 \\ 0 & 58/3 & 232 \end{pmatrix}$$

$$\xrightarrow{\frac{3}{58} R_2} \begin{pmatrix} 1 & -5/3 & -15 \\ 0 & 1 & 12 \end{pmatrix}$$

$$\Rightarrow y = 12$$

$$x - \frac{5}{3} y = -15$$

$$x = -15 + \frac{5}{3} y$$

$$= -15 + \frac{5}{3} 12$$

$$= 5$$

5a: Write a finite difference approximation for the following differential equation using five nodes spanning the interval $x \in [-2, 2]$.

$$e^x \frac{du}{dx} - \sin(x) = 4, \quad u(-2) = 1$$

Write your equations in matrix form and solve for u at each node.

node (k):	0	1	2	3	4
x	-2	-1	0	1	2

$$\Delta x = \frac{2-(-2)}{4} = 1$$

Interior Nodes:

$$k=1: \quad e^{-1}\left(\frac{u^{(1)}-u^{(0)}}{1}\right) - \sin(-1) = 4$$

$$k=2: \quad e^{0}\left(\frac{u^{(2)}-u^{(1)}}{1}\right) - \sin 0 = 4$$

$$k=3: \quad e^{1}\left(\frac{u^{(3)}-u^{(2)}}{1}\right) - \sin 1 = 4$$

$$k=4: \quad e^{2}\left(\frac{u^{(4)}-u^{(3)}}{1}\right) - \sin 2 = 4$$

Boundary Node:

$$k=0: \quad u^{(0)} = 1$$

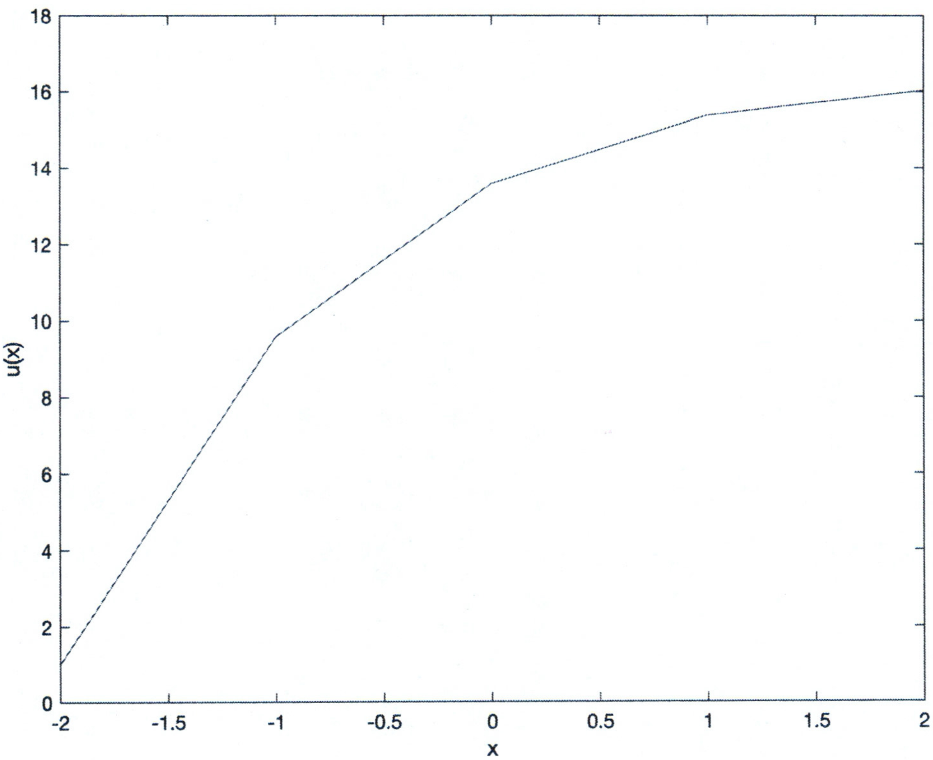
```
n = 5;
x = linspace(-2,2,n);
dx = x(2) - x(1);

A = zeros(n);
b = zeros(n,1);

for i = 2:n
    A(i,i) = exp(x(i))/dx;
    A(i,i-1) = -exp(x(i))/dx;
    b(i) = 4 + sin(x(i));
end

A(1,1) = 1;
b(1) = 1;

[A b]
```

```
ans = 5×6
    1.0000         0         0         0         0    1.0000
   -0.3679    0.3679         0         0         0    3.1585
         0   -1.0000    1.0000         0         0    4.0000
         0         0   -2.7183    2.7183         0    4.8415
         0         0         0   -7.3891    7.3891    4.9093
```

```
A \ b
```

```
ans = 5×1
    1.0000
    9.5858
   13.5858
   15.3668
   16.0313
```

```
plot(x, A\b)
xlabel('x')
ylabel('u(x)')
```


u(x)
x
18
16
14
12
10
8
6
4
2
0
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2

5b: Write a finite difference approximation for the following differential equation using four nodes spanning the interval $t \in [0,1]$.

$$\frac{d^2y}{dt^2} - t\frac{dy}{dt} = 0, \quad y(0) = 0, \quad y(1) = 4$$

node: 0 1 2 3

t: 0 1/3 2/3 1

$\Delta t = \frac{1-0}{3} = 1/3$

Interior Nodes (1 + 2)

$k=1$: $\frac{u^{(2)} - 2u^{(1)} + u^{(0)}}{1/3} - \frac{1}{3}\frac{u^{(2)} - u^{(1)}}{1/3} = 0$

$k=2$: $\frac{u^{(3)} - 2u^{(2)} + u^{(1)}}{1/3} - \frac{2}{3}\frac{u^{(3)} - u^{(2)}}{1/3} = 0$

Boundary Nodes:

$k=0$: $u^{(0)} = 0$

$k=3$: $u^{(3)} = 4$

6.2: Create a 2×2 elementary matrix for the row operation $-3R_2 \to R_1$. Use this matrix to perform the row operation on the matrix

$$\begin{pmatrix} 2 & 6 \\ -1 & 3 \end{pmatrix}.$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \xrightarrow{-3R_2 \to R_1} \begin{pmatrix} 1 & -3 \\ 0 & 1 \end{pmatrix} = \underline{E}_{-3R_2 \to R_1}$$

$$\begin{pmatrix} 1 & -3 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 2 & 6 \\ -1 & 3 \end{pmatrix} = \begin{pmatrix} 5 & -3 \\ -1 & 3 \end{pmatrix}$$

6.4: Use the side-by-side method to compute the inverse of the matrix

$$\begin{pmatrix} 1 & 2 \\ 3 & -2 \end{pmatrix}.$$

$$\left(\begin{array}{cccc} 1 & 2 & 1 & 0 \\ 3 & -2 & 0 & 1 \end{array}\right) \xrightarrow{-3R_1 \to R_2} \left(\begin{array}{cccc} 1 & 2 & 1 & 0 \\ 0 & -8 & -3 & 1 \end{array}\right)$$

$$\xrightarrow{-\frac{1}{8}R_2} \left(\begin{array}{cccc} 1 & 2 & 1 & 0 \\ 0 & 1 & 3/8 & -1/8 \end{array}\right)$$

$$\xrightarrow{-2R_2 \to R_1} \left(\begin{array}{cccc} 1 & 0 & 1/4 & 1/4 \\ 0 & 1 & 3/8 & -1/8 \end{array}\right)$$

$$\Rightarrow \begin{pmatrix} 1 & 2 \\ 3 & -2 \end{pmatrix}^{-1} = \begin{pmatrix} 1/4 & 1/4 \\ 3/8 & -1/8 \end{pmatrix}$$

7.1a: Use Gaussian elimination to compute the rank of the matrix

$$\begin{pmatrix} 1 & 2 & -3 \\ -2 & -4 & 6 \\ 0 & 4 & 1 \end{pmatrix}.$$

$$\begin{pmatrix} 1 & 2 & -3 \\ -2 & -4 & 6 \\ 0 & 4 & 1 \end{pmatrix} \xrightarrow{2R_1 \to R_2} \begin{pmatrix} 1 & 2 & -3 \\ 0 & 0 & 0 \\ 0 & 4 & 1 \end{pmatrix}$$

$$\xrightarrow{R_2 \leftrightarrow R_3} \begin{pmatrix} 1 & 2 & -3 \\ 0 & 4 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

The rank is 2.

7.1b: Find a solution to the system of equations

$$\begin{aligned} x_1 + 2x_2 - 3x_3 &= 4 \\ -2x_1 - 4x_2 + 6x_3 &= -6 \\ 4x_2 + x_3 &= 3 \end{aligned}$$

$$\begin{pmatrix} 1 & 2 & -3 & 4 \\ -2 & -4 & 6 & -6 \\ 0 & 4 & 1 & 3 \end{pmatrix} \xrightarrow{2R_1 \to R_2} \begin{pmatrix} 1 & 2 & -3 & 4 \\ 0 & 0 & 0 & 2 \\ 0 & 4 & 1 & 3 \end{pmatrix}$$

$$\xrightarrow{R_2 \leftrightarrow R_3} \begin{pmatrix} 1 & 2 & -3 & 4 \\ 0 & 4 & 1 & 3 \\ 0 & 0 & 0 & 2 \end{pmatrix}$$

The last row is not consistent. There is no solution.