

1.

$$\begin{aligned} \text{a. } \begin{pmatrix} 4 & 0 & -3 \\ 0 & 1 & 5 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} &= \begin{pmatrix} 5 \\ 8 \\ 6 \end{pmatrix} \rightarrow \begin{pmatrix} 4 & 0 & -3 & 5 \\ 0 & 1 & 5 & 8 \\ 1 & 1 & 1 & 6 \end{pmatrix} \rightarrow R_1 \leftrightarrow R_3 \rightarrow \\ \begin{pmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 5 & 8 \\ 4 & 0 & -3 & 5 \end{pmatrix} &\rightarrow R_3 - 4R_1 \rightarrow \begin{pmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 5 & 8 \\ 0 & -4 & -7 & -19 \end{pmatrix} \rightarrow R_3 + 4R_2 \rightarrow \\ \begin{pmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 5 & 8 \\ 0 & 0 & 13 & 13 \end{pmatrix} &\rightarrow \frac{R_3}{13} \rightarrow \begin{pmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 5 & 8 \\ 0 & 0 & 1 & 1 \end{pmatrix} \end{aligned}$$

i. $x_3 = 1$

ii. $x_2 = 3$

iii. $x_1 = 2$

$$\text{b. } \begin{pmatrix} 1 & a & -2 \\ 2 & -2 & 3 \end{pmatrix} \rightarrow R_2 - 2R_1 \rightarrow \begin{pmatrix} 1 & a & -2 \\ 0 & -2 - 2a & 7 \end{pmatrix}$$

i. $x_2 = -\frac{7}{2(a+1)}$

ii. $x_1 = -2 + \frac{7a}{2(a+1)}$

$$\begin{aligned} 2. \begin{pmatrix} 4a + 2b + o = 2.5 \\ a + b + 0o = 0.75 \\ 2b + 2o = 0.5 \end{pmatrix} &\rightarrow \begin{pmatrix} 4 & 2 & 1 & 2.5 \\ 1 & 1 & 0 & 0.75 \\ 0 & 2 & 2 & 0.5 \end{pmatrix} \rightarrow R_1 \leftrightarrow R_2 \rightarrow \begin{pmatrix} 1 & 1 & 0 & 0.75 \\ 4 & 2 & 1 & 2.5 \\ 0 & 2 & 2 & 0.5 \end{pmatrix} \rightarrow R_2 - \\ 4R_1 &\rightarrow \begin{pmatrix} 1 & 1 & 0 & 0.75 \\ 0 & -2 & 1 & -0.5 \\ 0 & 2 & 2 & 0.5 \end{pmatrix} \rightarrow (R_2 + R_3)/3 \rightarrow \begin{pmatrix} 1 & 1 & 0 & 0.75 \\ 0 & 0 & 1 & 0 \\ 0 & 2 & 2 & 0.5 \end{pmatrix} \rightarrow R_2 \leftrightarrow \frac{R_3}{2} \rightarrow \\ \begin{pmatrix} 1 & 1 & 0 & 0.75 \\ 0 & 1 & 1 & 0.25 \\ 0 & 0 & 1 & 0 \end{pmatrix} \end{aligned}$$

a. Oranges are free (\$0)

b. Bananas are \$0.25

c. Apples are \$0.50

$$\begin{aligned} 3. \begin{pmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} &= \begin{pmatrix} 0.63 \\ 1.25 \\ 2.78 \\ 1.44 \end{pmatrix} \rightarrow R_2 - R_1 \rightarrow R_3 - R_1 \rightarrow R_4 - R_1 \rightarrow R_3 - R_2 \rightarrow R_1 - \\ R_3 &\rightarrow R_2 + R_3 \rightarrow R_4 + R_3 \rightarrow \frac{R_4}{2} \rightarrow R_1 + R_4 \rightarrow R_2 - R_4 \rightarrow R_3 - R_4 \rightarrow \\ \begin{pmatrix} 1 & 0 & 0 & 0 & 0.27 \\ 0 & 1 & 0 & 0 & 0.98 \\ 0 & 0 & 1 & 0 & 0.36 \\ 0 & 0 & 0 & 1 & 1.17 \end{pmatrix} \end{aligned}$$

a. $x_1 = 0.27$

b. $x_2 = 0.98$

c. $x_3 = 0.36$

d. $x_4 = 1.17$