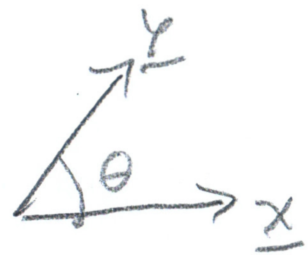
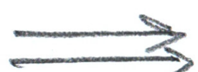
Scalar Mult.

$$k\begin{pmatrix}x_1\\ \vdots \\ x_n\end{pmatrix} = \begin{pmatrix}kx_1\\ \vdots \\ kx_n\end{pmatrix}$$

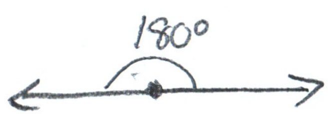
Dot Product

$$\underline{x}\cdot\underline{y} = \|\underline{x}\|\,\|\underline{y}\|\cos\theta$$



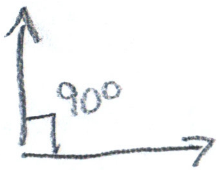
 $\theta = 0^\circ$ $\quad \cos\theta = 1$

$$\underline{x}\cdot\underline{y} = \|\underline{x}\|\,\|\underline{y}\|$$



$\cos\theta = -1$

$\theta = 0^\circ$



$$\cos 90^\circ = 0$$

$$\underline{x} \cdot \underline{y} = \|\underline{x}\| \, \|\underline{y}\| \cos\theta$$

$$= 0 \text{ if } \underline{x} \perp \underline{y}$$

orthogonal

Unit Cartesian vectors

In 3D:

$$\hat{\underline{e}}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \hat{\underline{e}}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad \hat{\underline{e}}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad \cancel{\hat{i}, \hat{j}, \hat{k}}$$

$$\begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix} = -2 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + 1 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + 3 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 3 \\ 4 \end{pmatrix} = 3 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + 4 \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

In 3D

$$\underline{x} \cdot \underline{y}$$

$$= \underline{x} \cdot \left(y_1 \hat{\underline{e}}_1 + y_2 \hat{\underline{e}}_2 + y_3 \hat{\underline{e}}_3 \right)$$

$$= y_1 \underbrace{\underline{x} \cdot \hat{\underline{e}}_1} + y_2 \underline{x} \cdot \hat{\underline{e}}_2 + y_3 \underline{x} \cdot \hat{\underline{e}}_3$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = x_1$$

$$= y_1 x_1 + y_2 x_2 + y_3 x_3$$

$$= \sum_{i=1}^{n} x_i y_i$$

$$\underline{x} \cdot \underline{y} = \|\underline{x}\| \|\underline{y}\| \cos\theta$$

$$= \sum_{i=1}^{n} x_i y_i$$

$$\Rightarrow \cos\theta = \frac{\underline{x} \cdot \underline{y}}{\|\underline{x}\| \|\underline{y}\|}$$

$$\begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 8 \\ 0 \end{pmatrix} = -2 \times 4 + 1 \times 8 + 3 \times 0 = \|\underline{x}\| \|\underline{y}\| \cos\theta$$

$$= 0$$

$\Rightarrow$ orthogonal

$$\underline{x} \xrightarrow{\text{mult.}} \underline{y}$$

$\underline{x} \in \mathbb{R}^n$, $\underline{y} \in \mathbb{R}^n$

$$y_1 = a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n$$

$$y_2 = a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n$$

$$\vdots$$

$$y_n = a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n$$

Matrix Multiplication

$$\begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix} = \underbrace{\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & \ddots & & \\ \vdots & & \ddots & \\ a_{n1} & & & a_{nn} \end{pmatrix}}_{\underline{A}} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$$

$y_i = \underline{A}(i,:) \cdot \underline{x}$

$\underline{C} = \underline{A}\,\underline{B} \iff c_{ij} = \underline{A}(i,:) \cdot B(:,j)$

$y_{i1} = \underline{A}(i,:) \cdot \underline{x}$

$\underline{A}$ + $\underline{B}$ must be <u>conformable</u> for matrix mult.

$$c_{23} = \begin{pmatrix} 2^{nd} \text{ Row} \\ \underline{A} \end{pmatrix} \cdot \begin{pmatrix} 3^{rd} \text{ col} \\ \underline{B} \end{pmatrix}$$

$$\dim(\text{Row } \underline{A}) = \dim(\text{col } \underline{B})$$

$$= \#\text{col } \underline{A} = \#\text{ Rows } \underline{B}$$

$\underline{A} \in \mathbb{R}^{m \times n}$, $\underline{B} \in \mathbb{R}^{p \times q}$

$(m \times n)(p \times q) \quad n = p$

$\underline{C} = \underline{A}\,\underline{B}$ $\quad$ $\underline{A}$ is $m \times n$

$\underline{B}$ is $n \times q$

$\underline{C}$ is $m \times q$

What is 1?

$$\underline{\underline{I}} = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & & \vdots \\ 0 & 0 & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \; 1 \end{pmatrix}$$

$\underline{I}\,\underline{x} = \underline{x}$

$\underline{A}\,\underline{I} = \underline{A}$

Matrix mult. <u>does not</u> commute.

$\underline{A}\,\underline{B} \neq \underline{B}\,\underline{A}$

$\underline{A}\,\underline{B}\,\underline{C} = (\underline{A}\,\underline{B})\underline{C} = \underline{A}(\underline{B}\,\underline{C})$ ✓ $\quad$ $\underline{A}(\underline{B} + \underline{C}) = \underline{A}\,\underline{B} + \underline{A}\,\underline{C}$