Root finding

$$g(x) = 0$$

$$\underline{g}(\underline{x}) = \underline{0}$$

Newton's Method

1. Pick a guess x_0
2. Iterations

$$x_1 = x_0 - \frac{g(x_0)}{g'(x_0)}$$

3. Go back

$x_0 \longrightarrow g(x_0), g'(x_0)$

$x_1 \longrightarrow g(x_1), g'(x_1)$

x_2

$$x_{k+1} = x_k - [g'(x_k)]^{-1} g(x_k)$$

$$\underline{x}_{k+1} = \underline{x}_k - \underline{\underline{J}}^{-1}(\underline{x}_k)\, \underline{g}(\underline{x}_k)$$

$$\underline{x}_{k+1} = \underline{x}_k - \underline{J}^{-1}(\underline{x}_k)\,\underline{g}(\underline{x}_k)$$

$$\underline{J}(\underline{x}_k)\,\underline{x}_{k+1} = \underline{J}(\underline{x}_k)\,\underline{x}_k - \cancel{\underline{J}(\underline{x}_k)\underline{J}^{-1}(\underline{x}_k)}\,\underline{g}(\underline{x}_k)$$

$$\underline{J}(\underline{x}_k)\left(\underline{x}_{k+1} - \underline{x}_k\right) = -\underline{g}(\underline{x}_k)$$

$$\underline{J}(\underline{x}_k)\,\underline{\xi} = -\underline{g}(\underline{x}_k)$$

$\Rightarrow \underline{\xi}$ by Gauss Elim.

$$\underline{\xi} = \underline{x}_{k+1} - \underline{x}_k \Rightarrow \underline{x}_{k+1} = \underline{\xi} + \underline{x}_k$$

Optimization

$$\min_{\underline{x}} f(\underline{x})$$

$$\Updownarrow$$

$$\max_{\underline{x}} -f(\underline{x})$$

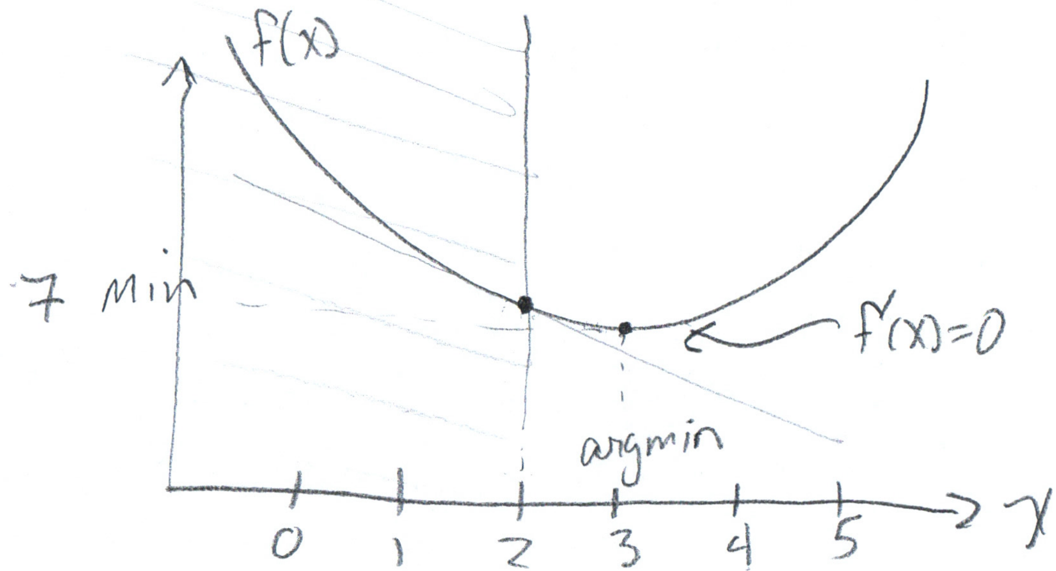
Root Finding

$$\underline{g}(\underline{x}) = \underline{0}$$

$$f(\underline{x}) = \|\underline{g}(\underline{x})\|$$

$$\min_{\underline{x}} f(\underline{x}) = \|\underline{g}(\underline{x})\|$$

Un-constrained vs. constrained opt.



$$\min_{x} f(x)$$
s.t. $x \leq 2$

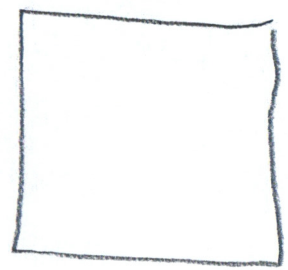
Unconst. $\rightarrow$ $f'(x) = 0$

Const. $\rightarrow$ Either $f'(x) = 0$ (x feasible)

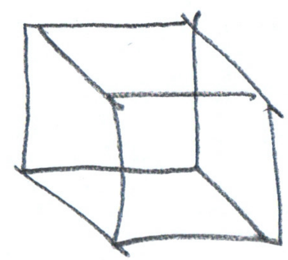
OR x at boundary

1D

2D

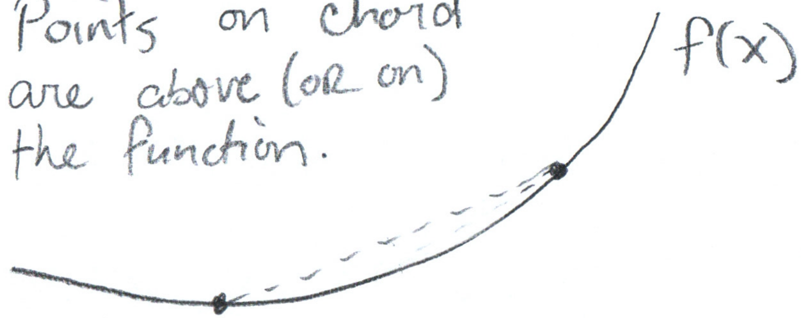


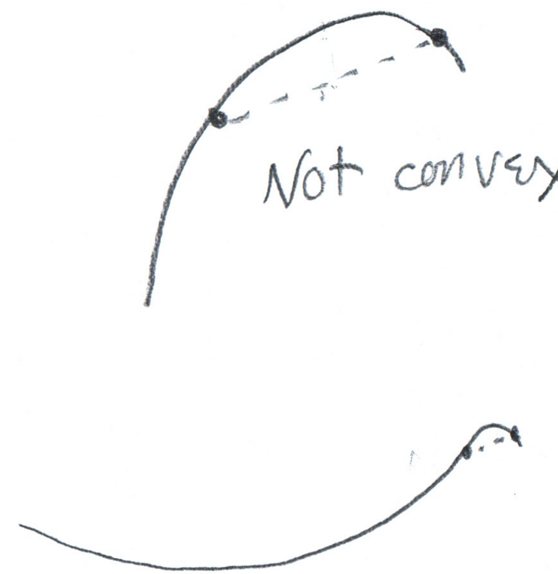
3D



Convex functions

Points on chord are above (or on) the function.





When minimizing a convex function over a convex set, all local minima are global minima.

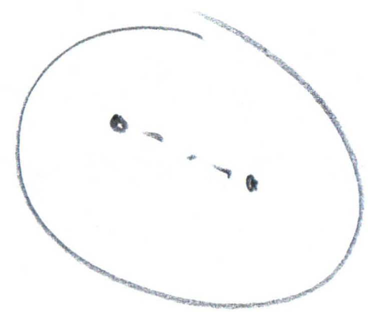
Solution: walk downhill

1. Reach a valley
2. Hit the boundary

$\Big\}$ local minima

Convexity

1. Convex Sets
2. Convex functions.

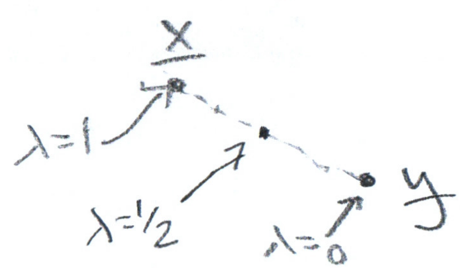


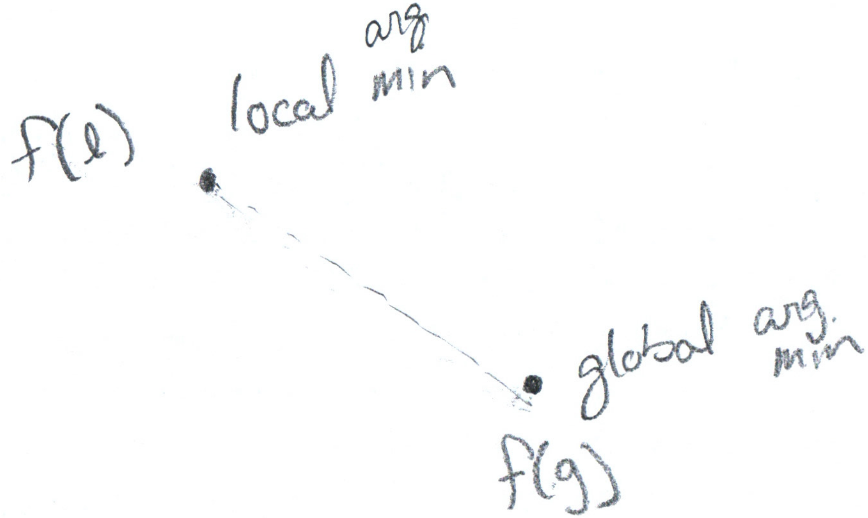
Convex combination

$\underline{x} \;\; \underline{y}$

$$\lambda \underline{x} + (1-\lambda)\underline{y}, \quad 0 \le \lambda \le 1$$

$$\frac{1}{2}\underline{x} + \frac{1}{2}\underline{y}$$





$$f(g) < f(l)$$

Linear systems are convex.

$$\left.\begin{aligned} &\min_{\underline{x}} \; f(\underline{x}) \\ &\text{s.t.} \quad \underline{A}\,\underline{x} = \underline{b} \end{aligned}\right\} \text{convex set}$$