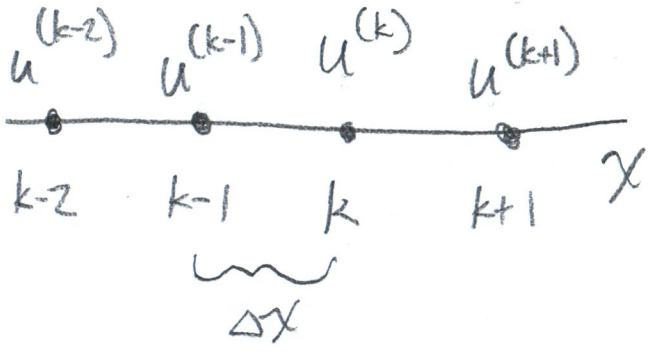
$$\frac{du}{dx} = \lim_{a \to b} \frac{u(b) - u(a)}{b - a}$$

$$= \lim_{\Delta x \to 0} \frac{u(x + \Delta x) - u(x)}{\Delta x}$$

$$\frac{du}{dx} \approx \frac{u^{(k+1)} - u^{(k)}}{\Delta x}$$



$$\frac{d^2u}{dx^2} \approx \frac{\left(\frac{du}{dx}\right)^{(k+1)} - \left(\frac{du}{dx}\right)^{(k)}}{\Delta x}$$

$$= \frac{\left(\frac{u^{(k+1)} - u^{(k)}}{\Delta x}\right) - \left(\frac{u^{(k)} - u^{(k-1)}}{\Delta x}\right)}{\Delta x} = \frac{u^{(k+1)} - 2u^{(k)} + u^{(k-1)}}{\Delta x^2}$$

Linear Differential Equations.

$$t^2 \frac{\partial u}{\partial t} = c_1 \frac{\partial^2 u}{\partial x^2} + c_2 \sin x \frac{\partial u}{\partial x} + c_3 e^{tx}$$

$\Rightarrow$ Linear in u

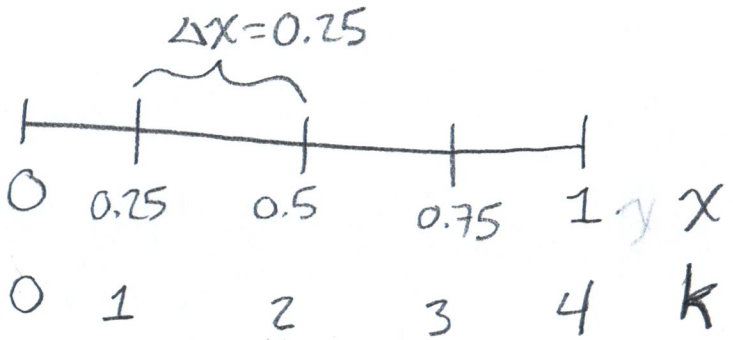
"Linear" for ODE/PDE's $\rightarrow$ "affine"

Deriv. of u are linear

$$k_1 u = k_2 u + k_3 u + k_4 \quad \checkmark$$

$$u \frac{du}{dx} = 7$$

1. Discretize



5 nodes, 0 ... n=4

$\Delta x = \ell/n = 1/4 = 0.25$

$$\frac{d^2u}{dx^2} + \frac{du}{dx} - 6u = 0$$

at Node 1: $x = 0.25$, $k = 1$

$$\frac{u^{(2)} - 2u^{(1)} + u^{(0)}}{(0.25)^2} + \frac{u^{(2)} - u^{(1)}}{0.25} - 6u^{(1)} = 0$$

at Node 2: $x = 0.5$, $k = 2$

$$\frac{u^{(3)} - 2u^{(2)} + u^{(1)}}{(0.25)^2} + \frac{u^{(3)} - u^{(2)}}{0.25} - 6u^{(2)} = 0$$

at Node 3: $x = 0.75$, $k = 3$

$$\frac{u^{(4)} - 2u^{(3)} + u^{(2)}}{(0.25)^2} + \frac{u^{(4)} - u^{(3)}}{0.25} - 6u^{(3)} = 0$$

at Node 0:

~~$u^{(1)} - 2u^{(0)} - u^{(1)}$~~

@ Node 0: $x=0$, $k=0$

$u^{(0)} = 0$ by B.C.

@ Node 4: $x=1$, $k=4$

$u^{(4)} = 3$ by B.C.