$$\min_{\underline{x}} f(\underline{x}) \leftrightarrows \underline{g}(\underline{x}) = \underline{0}$$

$$\underline{x}^{(0)} \to \underline{x}^{(1)} \to \underline{x}^{(2)} \to \cdots \to \underline{x}^{(k)}$$

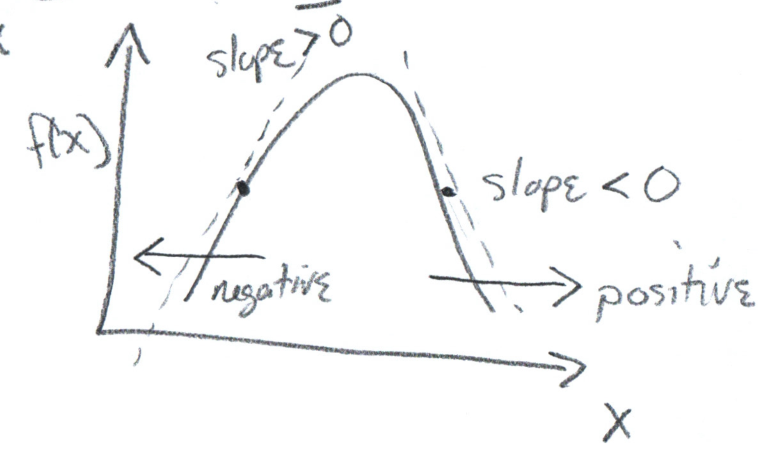
$$f(\underline{x}^{(0)}) > f(\underline{x}^{(1)}) > f(\underline{x}^{(2)}) > \cdots > f(\underline{x}^{(k)})$$

$$\underline{x}^{(k+1)} = \underline{x}^{(k)} + (\text{downhill step})$$

$$= \underline{x}^{(k)} + \underbrace{(\text{step size})}_{\alpha}\underset{-\underline{g}(\underline{x})}{(\text{downhill direction})}$$

downhill direction

− gradient



$$\boxed{\underline{x}^{(k+1)} = \underline{x}^{(k)} - \alpha\, \underline{g}(\underline{x}^{(k)})}$$

$$\underline{g}(\underline{x}^{(k)}) = \begin{pmatrix} \partial f / \partial x_1 \\ \dfrac{\partial f}{\partial x_2} \\ \vdots \\ \partial f / \partial x_n \end{pmatrix}$$

$\cancel{\nabla f}$

$$f(x) = x^4 - 2x^3 - 23x^2 + 24x + 147$$

$$g(x) = \frac{df}{dx} = 4x^3 - 6x^2 - 46x + 24$$

$\alpha = 0.01$

$x^{(0)} = 2$

$$x^{(1)} = x^{(0)} - \alpha\, g(x^{(0)})$$
$$= 2 - (0.01)(-60)$$
$$= 2.6$$

$g(2) = -60$

$f(x^{(0)}) = 103$

$f(x^{(1)}) = 64.5$

$$\underline{x}^{(k+1)} = \underline{x}^{(k)} - \alpha \, \underline{g}(\underline{x}^{(k)})$$

1. Iterate convergence.

$$\| \underline{x}^{(k+1)} - \underline{x}^{(k)} \| < \varepsilon$$

2. Objective convergence

$$| f(\underline{x}^{(k+1)}) - f(\underline{x}^{(k)}) | < \delta$$

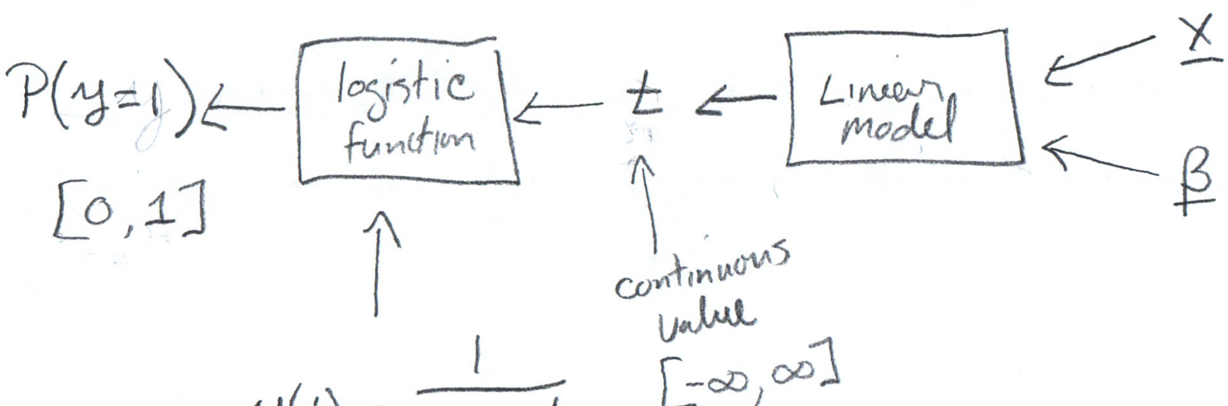
§12.4 *optional.

Predicting Binary Outcomes

$\{0, 1\}$

$\underline{y} = \underline{X}\underline{\beta}$ $\quad \log(\text{CFUs}) = f(c_1, c_2 \ldots)$

Logistic Regression



$$y(t) = \frac{1}{1+e^{-t}}$$

$$\text{odds}(y) = \frac{P(y=1)}{P(y=0)}$$

$$\text{odds}(y) = 1 \Rightarrow P(y=1) = P(y=0)$$

$$\log(\text{odds}(y)) = \beta_0 + \beta_1 x_1 + \cdots + \beta_n x_n$$