BIOE 210, Spring 2021

Homework 1

Due Friday, 2/5/2021 by 5:00pm.
Upload your answers to Gradescope. If submitting a single PDF, you must mark the location of all answers.

Part I (30 points)

1. For each statement, 1.) write the statement in words, and 2.) write if the statement is true or false.
   (a) $\exists\, p < 3$ and $q < 3$ s.t. $pq > 9$
   (b) $a + b \in \mathbb{Z} \Leftrightarrow a \in \mathbb{Z}$ and $b \in \mathbb{Z}$
   (c) $m \in \mathbb{Q}$ and $n \in \mathbb{Q} \Rightarrow m/n \in \mathbb{Q}$
   (d) $x^2 > |x| \quad \forall x \geq 1$
2. Compute the following expressions.
   (a) $\begin{pmatrix} 2 & -1 \\ 4 & 6 \end{pmatrix} \begin{pmatrix} 7 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ 1 \end{pmatrix}$
   (b) $\begin{pmatrix} 1 & -a \\ 0 & 2 \end{pmatrix} \begin{pmatrix} 3 & 1 \\ b & -1 \\ 0 & 2 \end{pmatrix}^{\mathsf{T}}$
3. Let $\mathbf{y} = \begin{pmatrix} 3 \\ \theta \\ -2 \end{pmatrix}$. Compute
   (a) $\mathbf{y} \cdot \mathbf{y}$
   (b) $\mathbf{y}^{\mathsf{T}}\mathbf{y}$
4. Let $\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ and $\mathbf{Q} = \begin{pmatrix} 1 & -2 \\ -2 & 3 \end{pmatrix}$. What is $\mathbf{x}^{\mathsf{T}}\mathbf{Q}\mathbf{x}$?
5. Prove that the following functions are either linear or nonlinear.
   (a) $f(\beta) = \beta x^2$
   (b) $f(x) = \frac{dx}{dt}$
   (c) $f(x) = \int x dt$
6. Let $\mathbf{a} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}$. Compute
   (a) $\|\mathbf{a}\|$
   (b) $\|\mathbf{b}\|$
   (c) $\|\mathbf{a} + \mathbf{b}\|$

(d) $\|3\mathbf{a}\|$

7. What is the angle between the following vectors?

   (a) $\begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

   (b) $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix}$

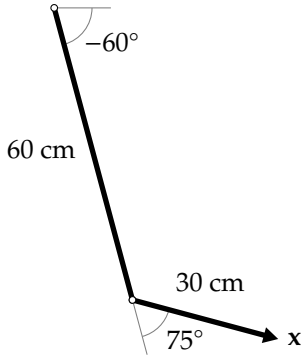
8. Let $\mathbf{a} = \begin{pmatrix} 1 \\ k \\ 2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$. What is

   (a) $\mathbf{a}^\mathsf{T}\mathbf{b}$

   (b) $\mathbf{a}\mathbf{b}^\mathsf{T}$

   (c) $\mathbf{b}\mathbf{a}^\mathsf{T}$ *Hint*: *There is a shortcut using* (*b*).

9. Using rotation and translation matrices, find the position ($\mathbf{x}$) of the end of the following multi-bar linkage arm. Write out the matrices, but you may use MATLAB to compute the final answer.



Part II: Machine Problem (5 points)

Define the following matrix in MATLAB.

$$\mathbf{A} = \begin{pmatrix} 4 & 8 & -12 & 44 \\ 3 & 6 & -8 & 32 \\ -2 & -1 & 0 & -7 \end{pmatrix}$$

Apply the following elementary row operations to $\mathbf{A}$.

1. Multiply the 1st row by 1/4
2. Subtract the 3 times the 1st row from the 2nd row
3. Add 2 times the 1st row to the 3rd row
4. Swap the 2nd and 3rd rows
5. Multiply the 2nd row by 1/3
6. Subtract 2 times the 2nd row from the 1st row

7. Subtract the 3rd row from the 1st row
8. Add 2 times the 3rd row to the 2nd row

Hint: The following syntax can be used for indexing in matrices:

- $\mathbf{A}(i,:)$ refers to the ith row of $\mathbf{A}$.
- $\mathbf{A}(:,j)$ refers to the jth column of $\mathbf{A}$.
- In order to alter a row, set it equal to the desired quantity. For example, if you want to set the 1st row equal to the sum of the 2nd and 3rd rows, use:

$$\mathbf{A}(1,:) = \mathbf{A}(2,:) + \mathbf{A}(3,:)$$

For each operation (1-8), turn in the command you used and the resulting matrix.

Part III: Machine Problem (5 points)

Download the files `bioe210_test_suite.m` and `lasso_data.mat` from the course website. Both files need to be placed in the same directory, as `bioe210_test_suite.m` loads data from `lasso_data.mat`. Check the extensions on the files; some browsers change the names upon download (to .exe, for example). If so, try another browser or adjust your browser's MIME settings.

Run the file `bioe210_test_suite.m` and turn in the output.

For full credit, there should be no errors when you run the script. If the script completes without errors, you have installed all the Matlab toolboxes you will need for the course. (Note that there may be warnings that some functions "will be removed in a future release". This is not a problem. We are using the old names for functions to allow compatibility with previous Matlab releases.)