Part 1

1.
   a. There exists a value "p" that is less than 3 and "q" that is less than 3 such that "p" multiplied by "q" is greater than 9. **True.** Ex: (-1x-10)
   b. "a" added to "b" is an integer if and only if "a" is an integer and "b" is an integer. **False.** Ex: (1.25 + 1.75 = 3)
   c. "m" is rational and "n" is rational implies "m" divided by "n" is rational. **False.** Ex: 1/0
   d. "x" squared is greater than the absolute value of "x" for all "x" greater than or equal to 1. **False.** Ex: $1^2 = |1|$
2.
   a. $\begin{pmatrix} 2 & -1 \\ 4 & 6 \end{pmatrix}\begin{pmatrix} 7 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ 1 \end{pmatrix} = \begin{pmatrix} 11 \\ 46 \end{pmatrix} - \begin{pmatrix} 4 \\ 1 \end{pmatrix} = \begin{pmatrix} \mathbf{7} \\ \mathbf{45} \end{pmatrix}$
   b. $\begin{pmatrix} 1 & -a \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ b & -1 \\ 0 & 2 \end{pmatrix}^T = \begin{pmatrix} 1 & -a \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 3 & b & 0 \\ 1 & -1 & 2 \end{pmatrix} = \begin{pmatrix} \mathbf{3-a} & \mathbf{b+a} & \mathbf{-2a} \\ \mathbf{2} & \mathbf{-2} & \mathbf{4} \end{pmatrix}$
3. $\boldsymbol{y} = \begin{pmatrix} 3 \\ \theta \\ -2 \end{pmatrix}$
   a. $\boldsymbol{y} \cdot \boldsymbol{y} = \begin{pmatrix} 3 \\ \theta \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ \theta \\ -2 \end{pmatrix} = 9 + \theta^2 + 4 = \mathbf{13} + \boldsymbol{\theta}^\mathbf{2}$
   b. $\boldsymbol{y}^T\boldsymbol{y} = (3 \quad \theta \quad -2)\begin{pmatrix} 3 \\ \theta \\ -2 \end{pmatrix} = 9 + \theta^2 + 4 = \mathbf{13} + \boldsymbol{\theta}^\mathbf{2}$
4. $\boldsymbol{x}^T\boldsymbol{Q}\boldsymbol{x} = (x_1 \quad x_2)\begin{pmatrix} 1 & -2 \\ -2 & 3 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = (x_1 - 2x_2 \quad -2x_1 + 3x_2)\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \big(x_1(x_1 - 2x_2) + x_2(-2x_1 + 3x_2)\big) = x_1^2 - 2x_1x_2 - 2x_1x_2 + 3x_2^2 = \boldsymbol{x}_\mathbf{1}^\mathbf{2} - \mathbf{4}\boldsymbol{x}_\mathbf{1}\boldsymbol{x}_\mathbf{2} + \mathbf{3}\boldsymbol{x}_\mathbf{2}^\mathbf{2}$
5. $f(k_1x_1 + k_2x_2) = k_1f(x_1) + k_2f(x_2)$
   a. $f(\beta) = \beta x^2 \to (k_1x_1 + k_2x_2)x^2 = k_1x_1x^2 + k_2x_2x^2 \to True\colon \boldsymbol{Linear}$
   b. $f(x) = \frac{dx}{dt} \to \frac{d}{dt}(k_1x_1 + k_2x_2) = k_1\frac{d}{dt}(x_1) + k_2\frac{d}{dt}(x_2) = k_1f(x_1) + k_2f(x_2) \to True\colon \boldsymbol{Linear}$
   c. $f(x) = \int x\,dx = \int (k_1x_1 + k_2x_2)\,dx = k_1\int x_1\,dx + k_2\int x_2\,dx \to True\colon \boldsymbol{Linear}$
6. $\boldsymbol{a} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}, \boldsymbol{b} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}$
   a. $\|\boldsymbol{a}\| = \sqrt{(-2)^2 + 1^2} = \sqrt{\mathbf{5}}$
   b. $\|\boldsymbol{b}\| = \sqrt{(0)^2 + 4^2} = \mathbf{4}$
   c. $\|\boldsymbol{a} + \boldsymbol{b}\| = \left\|\begin{pmatrix} -2 \\ 5 \end{pmatrix}\right\| = \sqrt{(-2)^2 + 5^2} = \sqrt{\mathbf{29}}$
   d. $\|3\boldsymbol{a}\| = \left\|\begin{pmatrix} -6 \\ 3 \end{pmatrix}\right\| = \sqrt{(-6)^2 + 3^2} = \sqrt{45} = \mathbf{3}\sqrt{\mathbf{5}}$
7. $\boldsymbol{x} \cdot \boldsymbol{y} = \|\boldsymbol{x}\|\|\boldsymbol{y}\|\cos\theta$
   a. $\to 2 = \sqrt{14}\sqrt{3}\cos\theta \to \theta = \cos^{-1}\left(\frac{2}{\sqrt{14}\sqrt{3}}\right) = \mathbf{72.02}°$
   b. $\to dot\ product = 0 \to \mathbf{90}°$
8. $\boldsymbol{a} = \begin{pmatrix} 1 \\ k \\ 2 \end{pmatrix}, \boldsymbol{b} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$

a. $\boldsymbol{a}^T\boldsymbol{b} = (1 \quad k \quad 2)\begin{pmatrix}0\\1\\2\end{pmatrix} = (\boldsymbol{k+4})$

b. $\boldsymbol{a}\boldsymbol{b}^T = \begin{pmatrix}1\\k\\2\end{pmatrix}(0 \quad 1 \quad 2) = \begin{pmatrix}\boldsymbol{0} & \boldsymbol{1} & \boldsymbol{2}\\ \boldsymbol{0} & \boldsymbol{k} & \boldsymbol{2k}\\ \boldsymbol{0} & \boldsymbol{2} & \boldsymbol{4}\end{pmatrix}$

c. $\boldsymbol{b}\boldsymbol{a}^T = \begin{pmatrix}0\\1\\2\end{pmatrix}(1 \quad k \quad 2) = \begin{pmatrix}\boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\\ \boldsymbol{1} & \boldsymbol{k} & \boldsymbol{2}\\ \boldsymbol{2} & \boldsymbol{2k} & \boldsymbol{4}\end{pmatrix}$ **or** $\boldsymbol{b}\boldsymbol{a}^T = (\boldsymbol{a}\boldsymbol{b}^T)^T = \begin{pmatrix}\boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\\ \boldsymbol{1} & \boldsymbol{k} & \boldsymbol{2}\\ \boldsymbol{2} & \boldsymbol{2k} & \boldsymbol{4}\end{pmatrix}$

9.

```
d = [0;0;1];
Short_arm = [1, 0, 30; 0, 1, 0; 0, 0, 1];
Long_arm = [1, 0, 60; 0, 1, 0; 0, 0, 1];
R_75 = [cosd(75), -sind(75), 0; sind(75), cosd(75), 0; 0 0 1];
R_minus60 = [cosd(-60), -sind(-60), 0; sind(-60), cosd(-60), 0; 0 0 1];

Location = R_minus60 * Long_arm * R_75 * Short_arm * d;
```

```
>> Location

Location =

   58.9778
  -44.1970
    1.0000
```

The final location is $\begin{pmatrix}\mathbf{58.98\ cm}\\ \mathbf{-44.20\ cm}\end{pmatrix}$

Part 2

```
A = [4 8 -12 44; 3 6 -8 32; -2 -1 0 -7];
% 1
A(1,:) = 0.25.*A(1,:)
% 2
A(2,:) = A(2,:) - 3.*A(1,:)
% 3
A(3,:) = 2.*A(1,:) + A(3,:)
% 4
A([2 3],:) = A([3 2],:)
% 5
A(2,:) = A(2,:)./3
% 6
A(1,:) = A(1,:) - 2.*A(2,:)
% 7
A(1,:) = A(1,:) - A(3,:)
% 8
A(2,:) = A(2,:) + 2.*A(3,:)
```

Command Window

```
>> HW1_Part2

A =

     1     2    -3    11
     3     6    -8    32
    -2    -1     0    -7


A =

     1     2    -3    11
     0     0     1    -1
    -2    -1     0    -7


A =

     1     2    -3    11
     0     0     1    -1
     0     3    -6    15


A =

     1     2    -3    11
     0     3    -6    15
     0     0     1    -1


A =

     1     2    -3    11
     0     1    -2     5
     0     0     1    -1


A =

     1     0     1     1
     0     1    -2     5
     0     0     1    -1


A =

     1     0     0     2
     0     1    -2     5
     0     0     1    -1


A =

     1     0     0     2
     0     1     0     3
     0     0     1    -1

fx >>
```

Part 3

Command Window

```
Optimal solution found.


ans =

   45.1546
   -0.0082
    0.1969


ans =

         0
    1.4534
         0
   -2.3954
         0


linx =

    0.6667
    1.3333


quadx =

    0.6667
    1.3333


species =

  2×1 cell array

    {'virginica' }
    {'versicolor'}


ans =

   36.8218   29.6073  -12.9818   23.7147   -0.5532


ans =

   -0.0170   -0.0161   -0.0171   -0.0191   -0.0200
```