BIOE 210, Spring 2021

Homework 2

Due Friday, 2/19/2021 by 5:00pm.
Upload your answers to Gradescope. If submitting a single PDF, you must mark the location of all answers.

Part I (10 points)

1. Solve the following linear system using elementary row operations to bring the augmented matrix into **row-echelon form**, followed by **backsubstitution**. Show all of your work (by hand).

$$\begin{pmatrix} 3 & 4 & -2 \\ 2 & 0 & 1 \\ 1 & 2 & 4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -14 \\ 10 \\ 22 \end{pmatrix}$$

2. Solve the following linear system using elementary row operations to bring the augmented matrix into **reduced row-echelon form (RREF)**. Show all of your work (by hand).

$$\begin{pmatrix} 1 & -1 & 1 \\ 2 & 10 & -3 \\ 4 & 4 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 31 \\ 28 \end{pmatrix}$$

3. Find the inverse of the following matrix (by hand) using the side-by-side method.

$$\begin{pmatrix} 0 & 2 \\ 3 & -4 \end{pmatrix}$$

   Use this inverse to solve the linear systems

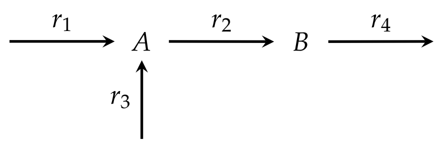
$$\begin{aligned} 3x_1 - 4x_2 &= 1 \\ 2x_2 &= 10 \end{aligned}$$

   and

$$\begin{aligned} x_2 &= 4 \\ 3x_1 - 4x_2 &= 6 \end{aligned}$$

Part II (5 points)

In this problem you will use a technique called Flux Balance Analysis to analyze chemical reaction networks. You will find fluxes (or *rates*) for multiple reactions that satisfy the conservation of mass. Consider the four reaction network below:



The metabolites A and B are produced and consumed by four reactions. The rates of the reactions are the unknowns $r_1, \ldots, r_4$. It is convenient to think of the four individual rates as entries in a four-dimensional flux vector $\mathbf{r}$. The connectivity of a network is captured by a *stoichiometric matrix* $\mathbf{S}$.

$$\mathbf{S} = \begin{pmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & 1 & -1 \\ 0 & -1 & 0 & 1 \end{pmatrix}$$

Conservation of mass requires that $\mathbf{Sr} = \mathbf{0}$. This homogeneous system has the trivial solution $\mathbf{r} = \mathbf{0}$, but we want to find the more interesting nontrivial solutions.

1. Using elementary row operations, calculate the rank of $\mathbf{S}$.
2. Find a parameterized solution for the system $\mathbf{Sr} = \mathbf{0}$.
3. Find a specific solution for the system by choosing values for the parameters.
4. Explain why you would expect the system to have the rank it does. Your answer should discuss the reaction network and the conservation of mass, not the linear dependence of the rows or column in $\mathbf{S}$. We want you to focus on what the rank means in terms of the dependence of the reaction rates.

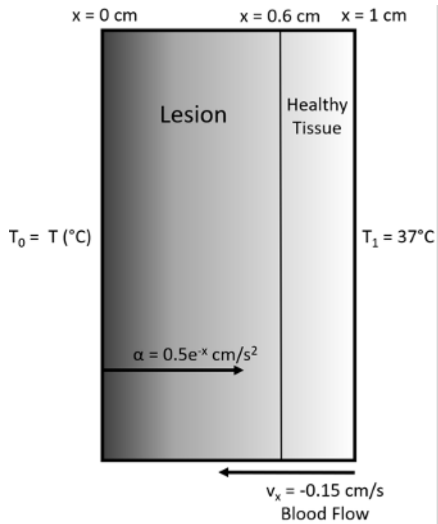
Part III: Machine Problem (15 points)

Cryotherapy is a common form of medical intervention for the removal of targeted tissue without surgical excision. During the procedure, liquid nitrogen is sprayed directly onto the skin to destroy lesions such as those caused by cancer. The dead tissue will form a blister and fall off in the following weeks. However, there is a balancing act to this therapy. Overexposure to the freezing temperatures will rapidly begin to freeze healthy tissue, and underexposure will result in diseased tissue surviving and continuing to grow. Is it possible to overcome this shortcoming by substituting another cryogenic at a less extreme temperature?

Your goal is to analyze a one-dimensional temperature profile of the lesion using cryotherapy at differing temperatures. At steady state, the temperature T of the lesion and healthy tissue, treated as a slab for the sake of simplicity, is subject to the heat transfer by conduction and convection by the governing differential equation

$$\alpha \frac{d^2T}{dx^2} - v_x \frac{dT}{dx} = 0.$$

The parameter α is the heat diffusivity constant and v_x is the x component of the velocity of blood in the capillaries feeding the lesion.



For our problem, we set $v_x = -0.15$ cm/s. Assume that the heat diffusivity of the lesion exponentially decays as the tissue extends further into healthy skin, so $\alpha = 0.5e^{-x}$ cm/s^2 , and the temperature 1 cm into the tissue (which is too far to be damaged) is held constant at $T(1) = 37$ °C. Use a finite difference approximation to solve for the temperature profile, T, at 11 points spanning the domain $[0, 1]$ cm under three conditions:

a.) $T(0) = 0$ °C

b.) $T(0) = -75$ °C

c.) $T(0) = -196$ °C

Questions

1. Make a table listing the index k, position x, and heat diffusivity α at each of the eleven nodes.
2. Use the finite difference approximation to discretize the differential equation for an **interior** node k.
3. Write equations representing the boundary conditions at the first and last nodes.
4. Assemble your equations into matrix form and solve for the temperature profile in MATLAB. Repeat this procedure for all three temperatures $T(0)$. Include MATLAB code in your submission.
5. Plot all three conditions on the same plot with labels. Title the plot and both axes. Include your MATLAB code in your submission.
6. Which of these three therapeutic scenarios results in adequate therapeutic effect with minimal damage to healthy tissue (assuming biological tissue needs to reach 0 °C to be considered "treated")? How do you know?
7. Conceptually, why is v_x negative and what effect would changing the direction have on the temperature profiles?
8. Discuss the practicality of using a less extreme temperature to reach a steady state system as a treatment method for this type of therapy. Is it better or worse?