1. Show that $f(0) = 0$ for any linear system.

2. What is the angle between the vectors $\begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$?

3. Is $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ a unit vector? If not, normalize it.

4. Is the function $f(x) = \frac{dx}{dt}$ linear?

5. What is the angle between the vectors
$$\begin{pmatrix} 1 \\ 0 \\ 3 \\ 0 \\ 4 \\ 0 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 0 \\ 8 \\ 0 \\ -6 \\ 0 \\ 12 \end{pmatrix}$$

6. What is $\begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 2 \\ 4 \end{pmatrix}$?

7. Given matrices
$$\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 0 & -3 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} 2 & 0 \\ -1 & 1 \end{pmatrix}$$
Show that $(\mathbf{AB})\mathbf{C} = \mathbf{A}(\mathbf{BC})$.

8. Given $\mathbf{A} \in \mathbb{R}^{3\times 4}$, $\mathbf{B} \in \mathbb{R}^{4\times 3}$, and $\mathbf{C} \in \mathbb{R}^{3\times 3}$, what permutations of $\mathbf{ABC}$ are conformable? What are the resulting dimensions?

9. *Show that $(\mathbf{AB})^\mathsf{T} = \mathbf{B}^\mathsf{T}\mathbf{A}^\mathsf{T}$.

10. Is $\mathbf{A}^\mathsf{T}\mathbf{A}$ always conformable? How about $\mathbf{A}\mathbf{A}^\mathsf{T}$? If so, what are the dimensions?

11. Given
$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \quad \mathbf{Q} = \begin{pmatrix} q_{11} & q_{12} \\ 0 & q_{22} \end{pmatrix}$$
What is $\mathbf{x}^\mathsf{T}\mathbf{Q}\mathbf{x}$?

12. Solve the linear system

$$x_1 + x_2 + x_3 = 3 \tag{1}$$
$$4x_1 + 3x_2 + 4x_3 = 8 \tag{2}$$
$$9x_1 + 3x_2 + 4x_3 = 7 \tag{3}$$

13. Which of the following differential equations are linear?

$$\frac{du}{dx} + \sin(3x)u = 0$$

$$\frac{\partial T}{\partial x} = \alpha\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial x \partial t} + \sin(t)$$

$$y\frac{d^2 y}{dx^2} = 4$$

14. Use the finite difference approximation to write algebraic equation that solve

$$\frac{du}{dx} + 3u = 0, \quad u'(0) = 1$$

using four nodes on the interval $[0, 1]$.

15. Given

$$\mathbf{A} = \begin{pmatrix} 1 & -2 \\ -1 & 3 \end{pmatrix}$$

find $\mathbf{A}^{-1}$.

16. Given

$$\mathbf{A} = \begin{pmatrix} -2 & 4 \\ 0 & 1 \end{pmatrix}$$

find $\mathbf{A}^{-1}$ using a product of elementary matrices.