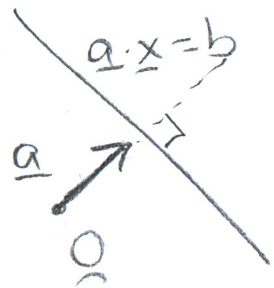
$$\underline{a} \cdot \underline{x} = b \qquad \underline{A}\,\underline{x} = \underline{b}$$

$\underline{a}$ is normal to the H.P.

$$\underline{A}(1,:) \cdot \underline{x} = b_1$$

Every row in a linear system is a hyperplane. (HP)



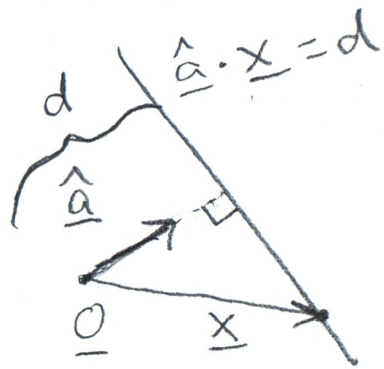
$$\underline{x} = \underline{A}^{-1}\underline{b}$$

$$\frac{\underline{a}}{\|\underline{a}\|} \cdot \underline{x} = b/\|\underline{a}\|$$

$\Rightarrow$ $\underline{x}$ is a point on all of the hyperplanes in $\underline{A}$, $\underline{b}$.

$$\underline{\hat{a}} \cdot \underline{x} = d$$

Hesse Normal Form



How far is the closest point on the line $3x_1 + 4x_2 = 7$ from the origin?

$$\underline{a} \cdot \underline{x} = b$$

$$\begin{pmatrix} 3 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 7$$

$$\hat{\underline{a}} \cdot \underline{x} = d$$

$$\|\underline{a}\| = \sqrt{3^2 + 4^2} = 5$$

$$\hat{\underline{a}} = \begin{pmatrix} 3/5 \\ 4/5 \end{pmatrix}$$

$$\begin{pmatrix} 3/5 \\ 4/5 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 7/5$$

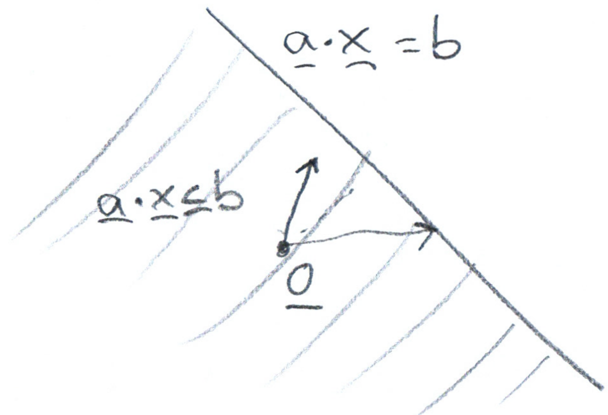
$$d = 7/5$$

Solving $\underline{A}\,\underline{x} = \underline{b}$ is akin to finding intersection of all the rows.

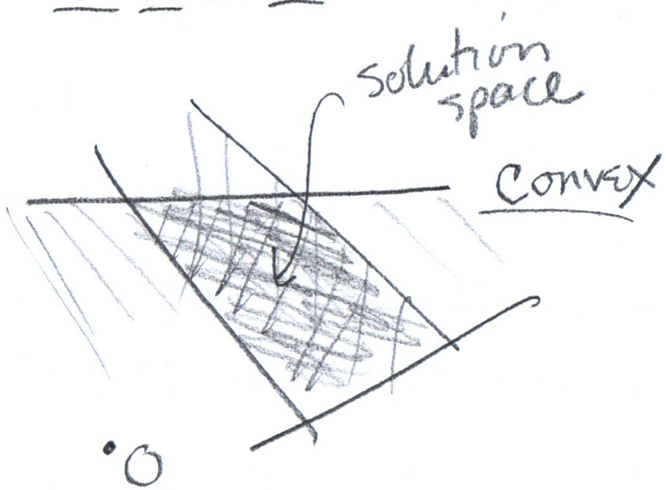
Inequalities

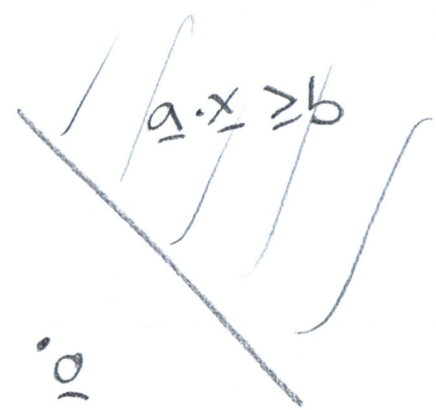
$$\underline{a} \cdot \underline{x} \leq b$$

$$\underline{\hat{a}} \cdot \underline{x} \leq d$$

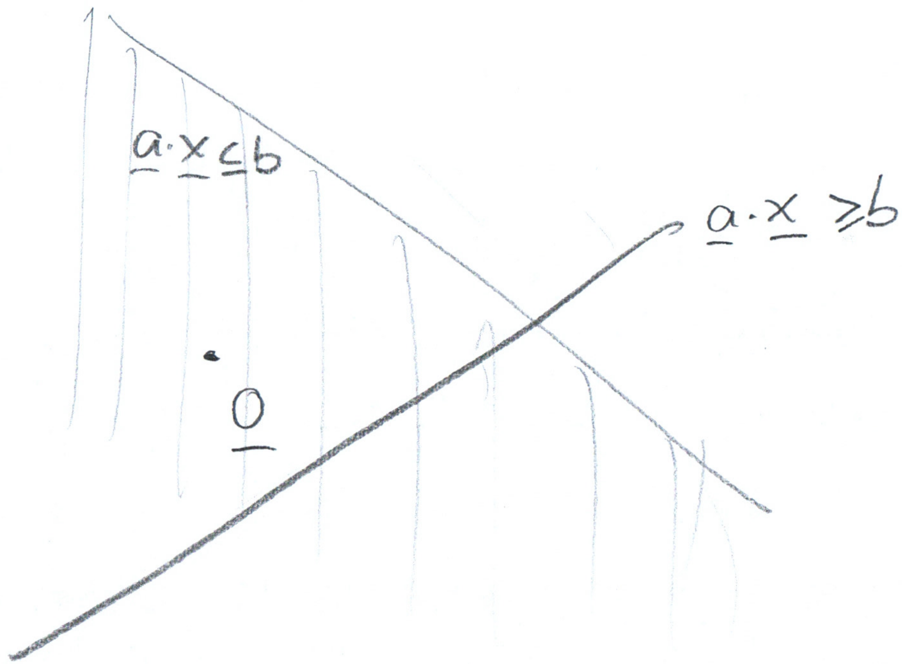


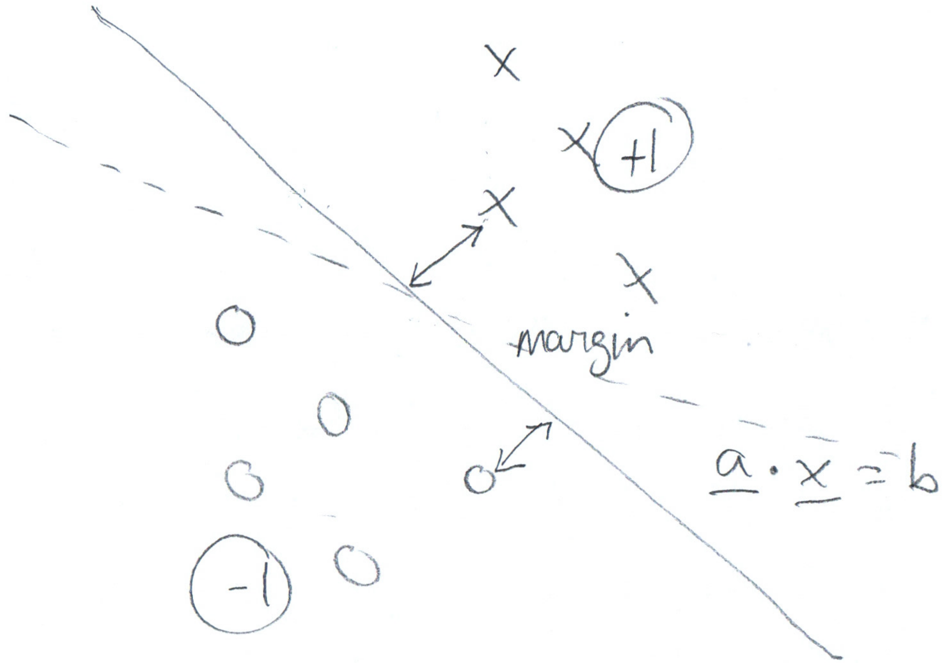
$$\underline{A}\,\underline{x} \leq \underline{b}$$





$\underline{A}\,\underline{x} \le \underline{b}$





Maximum margin separating hyperplane.

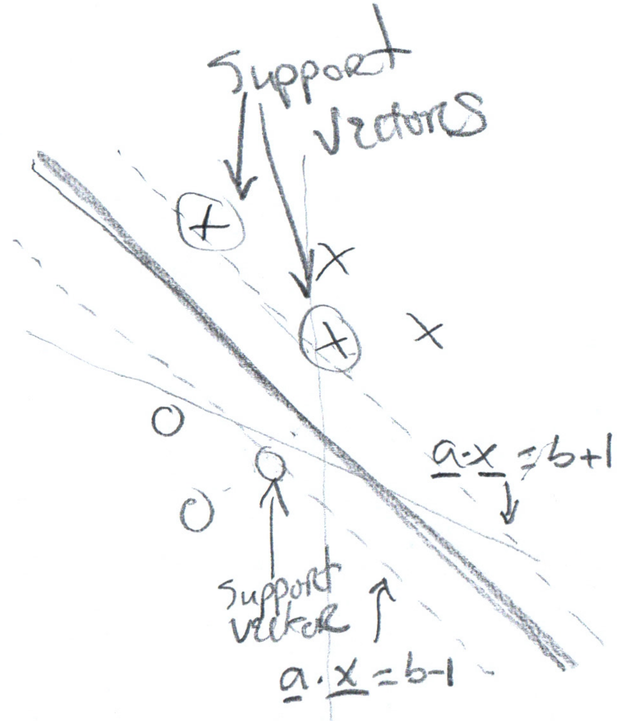
find $\underline{a}$, b such that

X above $\underline{a} \cdot \underline{x} = b$

O below $\underline{a} \cdot \underline{x} = b$

$\underline{a} \cdot \underline{x} \geq b$ for +1

$\underline{a} \cdot \underline{x} \leq b$ for +1



Two Separating HP.

$$\underline{a} \cdot \underline{x} = b+1$$

$$\underline{a} \cdot \underline{x} = b-1$$

max distance b/w HP's

s.t.

$\underline{a} \cdot \underline{x} \geq b+1$ for +1 points

$\underline{a} \cdot \underline{x} \leq b-1$ for -1 points distance b/w $\underline{a}\cdot\underline{x} = b+1$ + $\underline{a}\cdot\underline{x} = b-1$

$$\Downarrow \qquad\qquad \Downarrow$$

$$\hat{\underline{a}}\cdot\underline{x} = \frac{b+1}{\|\underline{a}\|} \qquad \hat{\underline{a}}\cdot\underline{x} = \frac{b-1}{\|\underline{a}\|}$$

$$= \frac{b+1}{\|\underline{a}\|} - \frac{b-1}{\|\underline{a}\|} = \frac{2}{\|\underline{a}\|}$$

$$\text{max distance} = \max \frac{2}{\|\underline{a}\|} = \min \|\underline{a}\|$$

$$= \min \sqrt{a_1^2 + a_2^2 + \cdots a_n^2}$$

$$= \min a_1^2 + a_2^2 + \cdots a_n^2$$

SVM

$$\min_{\underline{a},b} a_1^2 + a_2^2 + \cdots a_n^2$$

$\underline{a}\cdot\underline{x} \geq b+1$ for +1 points

$\underline{a}\cdot\underline{x} \leq b-1$ for −1 points