BIOE 210, Spring 2022

Homework 5

Due Monday, 2/21/2022 by 5:00pm.
You must upload your answers to Compass and assign each question.

1. Find the inverse of the following matrix using the side-by-side method.

$$\begin{pmatrix} -2 & 3 \\ 1 & 3 \end{pmatrix}$$

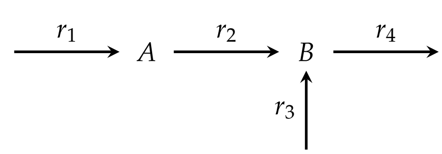
Verify that your solution is a true inverse using matrix multiplication.

2. In lecture we used elementary matrices to prove the existence of the matrix inverse. Here we will use elementary matrices to construct an inverse for the matrix

$$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ -2 & 3 \end{pmatrix}$$

   (a) Write out the elementary row operations needed to transform **A** into the identity matrix. (You should need no more than three row operations.)
   (b) Write the elementary matrices corresponding to each elementary row operation.
   (c) Write the inverse as a product of the elementary matrices. Compute the product.
   (d) Verify that your inverse is correct by multiplication with **A**.

3. In this problem you will use a technique called Flux Balance Analysis to analyze chemical reaction networks. You will find fluxes (or *rates*) for multiple reactions that satisfy the conservation of mass. Consider the four reaction network below:



The metabolites A and B are produced and consumed by four reactions. The rates of the reactions are the unknowns $r_1, \ldots, r_4$. It is convenient to think of the four individual rates as entries in a four-dimensional flux vector $\mathbf{r}$. The connectivity of a network is captured by a *stoichiometric matrix* $\mathbf{S}$.

$$\mathbf{S} = \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & 1 & -1 \\ 1 & 0 & 1 & -1 \\ 0 & -1 & -1 & 1 \end{pmatrix}$$

Conservation of mass requires that $\mathbf{Sr} = \mathbf{0}$. This homogeneous system has the trivial solution $\mathbf{r} = \mathbf{0}$, but we want to find the more interesting nontrivial solutions.

(a) Using elementary row operations, calculate the rank of $\mathbf{S}$.

(b) Find a parameterized solution for the system $\mathbf{Sr} = \mathbf{0}$.

(c) Find a specific solution for the system by choosing values for the parameters.

(d) Explain why you would expect the system to have the rank it does. Your answer should discuss the reaction network and the conservation of mass, not the linear dependence of the rows or column in $\mathbf{S}$. We want you to focus on what the rank means in terms of the dependence of the reaction rates.