1. $\begin{pmatrix}3\\2\end{pmatrix}$ *decomposed onto* $\left\{\begin{pmatrix}1\\0\end{pmatrix},\begin{pmatrix}1\\1\end{pmatrix}\right\}$

$$\begin{pmatrix}3\\2\end{pmatrix} = a_1\begin{pmatrix}1\\0\end{pmatrix} + a_2\begin{pmatrix}1\\1\end{pmatrix} \rightarrow$$

$$3 = a_1 + a_2$$

$$2 = (0)(a_1) + a_2 \rightarrow a_2 = 2 \; and \; a_1 = 1$$

$$\begin{pmatrix}3\\2\end{pmatrix} = 1\begin{pmatrix}1\\0\end{pmatrix} + 2\begin{pmatrix}1\\1\end{pmatrix}$$

2. $\begin{pmatrix}3\\2\end{pmatrix} = a_1\begin{pmatrix}1\\-2\end{pmatrix} + a_2\begin{pmatrix}1\\1\end{pmatrix} + a_3\begin{pmatrix}-1\\1\end{pmatrix} \rightarrow$

$$3 = a_1 + a_2 - a_3$$

$$2 = -2a_1 + a_2 + a_3 \rightarrow a_1 = 0, a_2 = 2.5, a_3 = -0.5$$

$$\begin{pmatrix}3\\2\end{pmatrix} = 0\begin{pmatrix}1\\-2\end{pmatrix} + 2.5\begin{pmatrix}1\\1\end{pmatrix} - 0.5\begin{pmatrix}-1\\1\end{pmatrix}$$

   a. No, the number of vectors (3) does not match the dimensions (2).
   b. Yes.
   c. $a_1 = 1$(Arbitrary) →

$$2 = a_2 - a_3$$

$$4 = a_2 + a_3 \rightarrow 4 = 2 + 2a_3 \rightarrow$$

$$a_2 = 3, a_3 = 1$$

$$\begin{pmatrix}3\\2\end{pmatrix} = 1\begin{pmatrix}1\\-2\end{pmatrix} + 3\begin{pmatrix}1\\1\end{pmatrix} + 1\begin{pmatrix}-1\\1\end{pmatrix}$$

3. Testing if vectors form a basis:
   a. Number of vectors match number of dimensions: TRUE (2-2)
   b. Vectors span V: TRUE (other 2 are true)
   c. Vectors are linearly independent: TRUE
      i. $\begin{pmatrix}1 & 0\\1 & -1\end{pmatrix} \rightarrow -1(R_2 - R_1) \rightarrow \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$
   d. These vectors form a basis.

4.
   a. Orthonormal basis: $\left\{\begin{pmatrix}1\\0\\0\end{pmatrix},\begin{pmatrix}0\\\frac{1}{\sqrt{5}}\\\frac{2}{\sqrt{5}}\end{pmatrix},\begin{pmatrix}0\\\frac{2}{\sqrt{5}}\\-\frac{1}{\sqrt{5}}\end{pmatrix}\right\}$
   b. Decomposition

      $a_1 = \begin{pmatrix}-2\\1\\3\end{pmatrix}\cdot\begin{pmatrix}1\\0\\0\end{pmatrix} = -2$

$$a_2 = \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix} \cdot \frac{1}{\sqrt{5}} \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} = \frac{7}{\sqrt{5}}$$

$$a_3 = \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix} \cdot \frac{1}{\sqrt{5}} \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} = -\frac{1}{\sqrt{5}}$$

$$\rightarrow -2 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \frac{7}{\sqrt{5}} \begin{pmatrix} 0 \\ \frac{1}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} \end{pmatrix} - \frac{1}{\sqrt{5}} \begin{pmatrix} 0 \\ \frac{2}{\sqrt{5}} \\ -\frac{1}{\sqrt{5}} \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix}$$

5. $\boldsymbol{A} = \begin{pmatrix} 5 & 3 & 1 \\ 3 & 2 & 1 \\ 1 & 1 & 1 \end{pmatrix}$
    a. Eigenvectors: $\left\{ \begin{pmatrix} 0.4082 \\ -0.8165 \\ 0.4082 \end{pmatrix}, \begin{pmatrix} 0.3913 \\ -0.2475 \\ -0.8863 \end{pmatrix}, \begin{pmatrix} 0.8247 \\ 0.5216 \\ 0.2185 \end{pmatrix} \right\}$
    Eigenvalues: $\{0, 0.8377, 7.1623\}$
    b. $0 \begin{pmatrix} 0.4082 \\ -0.8165 \\ 0.4082 \end{pmatrix} - 2.7627 \begin{pmatrix} 0.3913 \\ -0.2475 \\ -0.8863 \end{pmatrix} + 2.5234 \begin{pmatrix} 0.8247 \\ 0.5216 \\ 0.2185 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$
    c. $\boldsymbol{Ax} = a_1 \lambda_1 \boldsymbol{v_1} + a_2 \lambda_2 \boldsymbol{v_2} + a_3 \lambda_3 \boldsymbol{v_3} = \begin{pmatrix} 14 \\ 10 \\ 6 \end{pmatrix}$