1.2: Transform the expression $3x + 5y - 5z$ into the expression $5(y - z) + 3x$, stating the axiom you used for each step.

1.3: Let $\mathbf{a} = \begin{pmatrix} 1 \\ 5 \\ 3 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} -5 \\ 2 \\ 6 \end{pmatrix}$, and $\mathbf{c} = \begin{pmatrix} 8 \\ -7 \\ -5 \end{pmatrix}$. What is $\mathbf{a} + \mathbf{b} - \mathbf{c}$?

1.5: Is the function $f(x) = x^2 - x$ linear?

1.6: Find the magnitude of the vector $\mathbf{x} = \begin{pmatrix} 6 \\ -2 \\ 3 \end{pmatrix}$. Use the magnitude to convert $\mathbf{x}$ into a unit vector.

1.7: Let $\mathbf{x} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$ and $\mathbf{y} = \begin{pmatrix} 6 \\ 1 \\ 4 \end{pmatrix}$. What is $\mathbf{x} \cdot \mathbf{y}$? What is the angle between the vectors?

2.1a: Which of the following matrix products are compatible for multiplication?

$$\mathbf{A} = \mathbb{R}^{2\times 3}$$
$$\mathbf{B} = \mathbb{R}^{5\times 5}$$
$$\mathbf{C} = \mathbb{R}^{3\times 5}$$

Write the dimension of the product whenever the matrices are compatible.

1. **AB**
2. **BA**
3. **BC**
4. **CA**
5. **CB**

2.1b: Compute the product

$$\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 5 & 6 & 3 \\ 1 & 5 & 4 \end{pmatrix}.$$

2.3: Simplify the expression $(\mathbf{A}\mathbf{A}^{-1})^{-1}\mathbf{A}$.

2.6: Which is the most efficient way to compute the product **ABC** when $\dim(\mathbf{A}) = 100 \times 400$, $\dim(\mathbf{B}) = 400 \times 600$, and $\dim(\mathbf{C}) = 600 \times 200$?

a. $(\mathbf{AB})\mathbf{C}$

b. $\mathbf{A}(\mathbf{BC})$

3.1: Rotate the point $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$ counter-clockwise by 87°.

3.3: Point $\mathbf{p}$ is located at $\begin{pmatrix}1\\2\end{pmatrix}$. Shift $\mathbf{p}$ by 7 horizontally and 6 vertically, and rotate the shifted point 90° counter-clockwise about the origin. What is the final coordinates of the point?

4.0: Solve the system of equations

$$\begin{aligned} -3x + 5y &= 45 \\ 8x + 6y &= 112 \end{aligned}$$

5a: Write a finite difference approximation for the following differential equation using five nodes spanning the interval $x \in [-2, 2]$.

$$e^x \frac{du}{dx} - \sin(x) = 4, \quad u(-2) = 1$$

Write your equations in matrix form and solve for u at each node.

5b: Write a finite difference approximation for the following differential equation using four nodes spanning the interval $t \in [0, 1]$.

$$\frac{d^2y}{dt^2} - t\frac{dy}{dt} = 0, \quad y(0) = 0, \quad y(1) = 4$$

6.2: Create a 2×2 elementary matrix for the row operation $-3R_2 \to R_1$. Use this matrix to perform the row operation on the matrix

$$\begin{pmatrix} 2 & 6 \\ -1 & 3 \end{pmatrix}.$$

6.4: Use the side-by-side method to compute the inverse of the matrix

$$\begin{pmatrix} 1 & 2 \\ 3 & -2 \end{pmatrix}.$$

7.1a: Use Gaussian elimination to compute the rank of the matrix

$$\begin{pmatrix} 1 & 2 & -3 \\ -2 & -4 & 6 \\ 0 & 4 & 1 \end{pmatrix}.$$

7.1b: Find a solution to the system of equations

$$\begin{aligned} x_1 + 2x_2 - 3x_3 &= 4 \\ -2x_1 - 4x_2 + 6x_3 &= -6 \\ 4x_2 + x_3 &= 3 \end{aligned}$$