

# BIOE 210, Spring 2022

## Homework 4

Due Monday, 2/14/2022 by 5:00pm.

**You must upload your answers to Gradescope and assign each question.**

1. Classify the following differential equations as *linear* or *nonlinear*.

(a)

$$\frac{dv}{dt} - tv = 0$$

(b)

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = xy$$

(c)

$$(1 - u)\frac{du}{dt} = u + 1$$

(d)

$$\frac{d\mathbf{x}}{dt} = \mathbf{A}\mathbf{x}$$

2. Find the inverse of the following matrix using the side-by-side method.

$$\begin{pmatrix} -2 & 3 \\ 1 & 3 \end{pmatrix}$$

Verify that your solution is a true inverse using matrix multiplication.

3. Use the inverse from the previous question to solve the following linear systems:

(a)

$$\begin{aligned} -2x_1 + 3x_2 &= 5 \\ x_1 + 3x_2 &= -4 \end{aligned}$$

(b)

$$\begin{aligned} x_1 + 3x_2 &= 2 \\ -2x_1 + 3x_2 &= 5 \end{aligned}$$

(c)

$$\begin{aligned} 4x_1 - 6x_2 &= 8 \\ x_1 + 3x_2 &= 2 \end{aligned}$$

4. In lecture we used elementary matrices to prove the existence of the matrix inverse. Here we will use elementary matrices to construct an inverse for the matrix

$$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ -2 & 3 \end{pmatrix}$$

- (a) Write out the elementary row operations needed to transform  $\mathbf{A}$  into the identity matrix. (You should need no more than three row operations.)
  - (b) Write the elementary matrices corresponding to each elementary row operation.
  - (c) Write the inverse as a product of the elementary matrices. Compute the product.
  - (d) Verify that your inverse is correct by multiplication with  $\mathbf{A}$ .
5. Find a solution to the linear system

$$\begin{pmatrix} 1 & -1 & 2 \\ 1 & 1 & 0 \\ 3 & 1 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ 10 \end{pmatrix}$$