

1.

- a. Linear
- b. Linear
- c. Non-Linear
- d. Linear

$$2. \begin{pmatrix} -2 & 3 & 1 & 0 \\ 1 & 3 & 0 & 1 \end{pmatrix} \rightarrow R_1 + 2R_2 \rightarrow \begin{pmatrix} 0 & 9 & 1 & 2 \\ 1 & 3 & 0 & 1 \end{pmatrix} \rightarrow R_2 - \frac{R_1}{3} \rightarrow \begin{pmatrix} 0 & 9 & 1 & 2 \\ 1 & 0 & -\frac{1}{3} & \frac{1}{3} \end{pmatrix} \rightarrow$$

$$R_2 \leftrightarrow \frac{R_1}{9} \rightarrow \begin{pmatrix} 1 & 0 & -\frac{1}{3} & \frac{1}{3} \\ 0 & 1 & \frac{1}{9} & \frac{2}{9} \end{pmatrix}$$

$$a. \begin{pmatrix} -2 & 3 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} -\frac{1}{3} & \frac{1}{3} \\ \frac{1}{9} & \frac{2}{9} \end{pmatrix} = \begin{pmatrix} \frac{2}{3} + \frac{3}{9} & -\frac{2}{3} + \frac{6}{9} \\ -\frac{1}{3} + \frac{3}{9} & \frac{1}{3} + \frac{6}{9} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

3.

$$a. \begin{pmatrix} -2 & 3 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} 5 \\ -4 \end{pmatrix} \rightarrow \begin{pmatrix} -\frac{1}{3} & \frac{1}{3} \\ \frac{1}{9} & \frac{2}{9} \end{pmatrix} \begin{pmatrix} 5 \\ -4 \end{pmatrix} = \begin{pmatrix} -3 \\ -\frac{1}{3} \end{pmatrix}$$

$$b. \begin{pmatrix} 1 & 3 \\ -2 & 3 \end{pmatrix} \begin{pmatrix} 2 \\ 5 \end{pmatrix} \rightarrow \begin{pmatrix} -\frac{1}{3} & \frac{1}{3} \\ \frac{1}{9} & \frac{2}{9} \end{pmatrix} \begin{pmatrix} 5 \\ 2 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$c. \begin{pmatrix} 4 & -6 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} 8 \\ 2 \end{pmatrix} \rightarrow \begin{pmatrix} -\frac{1}{3} & \frac{1}{3} \\ \frac{1}{9} & \frac{2}{9} \end{pmatrix} \begin{pmatrix} -4 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

4.

$$a. E_{R_1 \leftrightarrow R_2} \rightarrow E_{R_1 - 3R_2} \rightarrow E_{\frac{R_1}{-2}}$$

$$b. \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & -3 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} -\frac{1}{2} & 0 \\ 0 & 1 \end{pmatrix}$$

$$c. \begin{pmatrix} -\frac{1}{2} & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -3 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1.5 & -0.5 \\ 1 & 0 \end{pmatrix}$$

$$d. \begin{pmatrix} 1.5 & -0.5 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -2 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

5. Two possible solutions:

- a.  $x_1 = 3$   $x_2 = 1$   $x_3 = 0$
- b.  $x_1 = 2$   $x_2 = 2$   $x_3 = 1$
- c. etc.