Decomposition

1. Network Analysis
2. Image compression

3. Recommender System.

Vector Decomposition

$$\underline{u} = a_1 \underline{v}_1 + a_2 \underline{v}_2 + \cdots + a_m \underline{v}_m$$

BIOE ≈ MATH + CHEM + BIO + PHYSICS

$$\begin{pmatrix} -2 \\ 4 \\ 5 \end{pmatrix} = -2\underset{\underline{\hat{e}}_1}{\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}} + 4\underset{\underline{\hat{e}}_2}{\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}} + 5\underset{\underline{\hat{e}}_2}{\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}}$$

$$\underline{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = a_1\begin{pmatrix} 1 \\ 1 \end{pmatrix} + a_2\begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$a_1 - a_2 = u_1 \qquad \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$
$$a_1 + a_2 = u_2$$

$$a_1 = \frac{u_1 + u_2}{2}, \quad a_2 = \frac{u_2 - u_1}{2}$$

$$\underline{u} = \begin{pmatrix} -2 \\ 4 \end{pmatrix} \Rightarrow a_1 = 1 \ \& \ a_2 = 3$$

$$\begin{pmatrix} -2 \\ 4 \end{pmatrix} = 1\begin{pmatrix} 1 \\ 1 \end{pmatrix} + 3\begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

In 2D: $a_1\begin{pmatrix}1\\0\end{pmatrix} + a_2\begin{pmatrix}0\\1\end{pmatrix}$

$$a_1\begin{pmatrix}1\\1\end{pmatrix} + a_2\begin{pmatrix}-1\\1\end{pmatrix}$$

Vectors <u>span</u> a space if any vector $\underline{u}$ in the space can be decomposed onto the vectors.

<u>Linear Independence</u>

$$a_1 \underline{v}_1 + a_2 \underline{v}_2 + \cdots + a_n \underline{v}_n = \underline{0}$$

$$\iff a_1 = a_2 = \cdots = a_n = 0$$

Basis: A set of vectors that

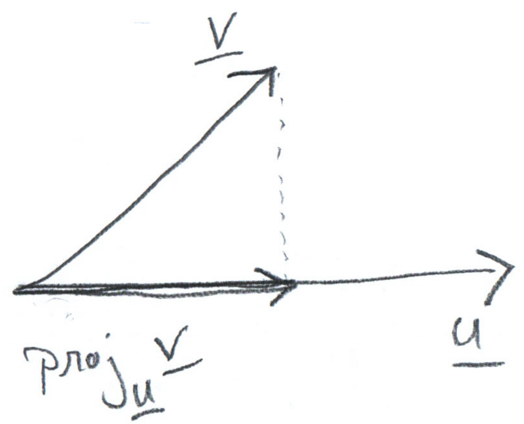
- span a space
- linearly independent.

A set of vectors is a basis for space V.
iff:

1. The # of vectors (n) matches the dimension of V
2. The vectors span V.
3. The vectors are linearly independent.

$$\begin{pmatrix} -3 \\ 2 \\ 1 \end{pmatrix} \qquad \underline{v}_1, \underline{v}_2, \cdots \underline{v}_n$$

$$\underline{u} = a_1 \underline{v}_1 + \cdots a_n \underline{v}_n \iff \begin{pmatrix} | & | & & | \\ \underline{v}_1 & \underline{v}_2 & \cdots & \underline{v}_n \\ | & | & & | \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix} = \underline{\mathbf{V}}\,\underline{a} = \underline{u}$$



Given $\underline{v}$ & $\underline{u}$, the vector $\underline{v} - \text{proj}_{\underline{u}}\underline{v}$ is orthogonal to $\underline{u}$.