## Part 1

1.

a. There exists a value "p" that is less than 3 and "q" that is less than 3 such that "p" multiplied by "q" is greater than 9. **True.** Ex: (-1x-10)

b. "a" added to "b" is an integer if and only if "a" is an integer and "b" is an integer. **False.** Ex: (1.25 + 1.75 = 3)

c. "m" is rational and "n" is rational implies "m" divided by "n" is rational. False. (n=0)

d. "x" squared is greater than the absolute value of "x" for all "x" greater than or equal to 1. **False.** Ex:  $1^2 = |1|$ 

2.

a. 
$$\begin{pmatrix} 2 & -1 \\ 4 & 6 \end{pmatrix} \begin{pmatrix} 7 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ 1 \end{pmatrix} = \begin{pmatrix} 11 \\ 46 \end{pmatrix} - \begin{pmatrix} 4 \\ 1 \end{pmatrix} = \begin{pmatrix} 7 \\ 45 \end{pmatrix}$$

b. 
$$\begin{pmatrix} -1 & -a \\ 0 & 2 \end{pmatrix} \begin{pmatrix} 3 & 1 \\ b & -1 \\ 0 & 2 \end{pmatrix}^T = \begin{pmatrix} -1 & -a \\ 0 & 2 \end{pmatrix} \begin{pmatrix} 3 & b & 0 \\ 1 & -1 & 2 \end{pmatrix} = \begin{pmatrix} -3 - a & -b + a & -2a \\ 2 & -2 & 4 \end{pmatrix}$$

3. 
$$y = \begin{pmatrix} 3 \\ \theta \\ -2 \end{pmatrix}$$

a. 
$$\mathbf{y} \cdot \mathbf{y} = \begin{pmatrix} 3 \\ \theta \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ \theta \\ -2 \end{pmatrix} = 9 + \theta^2 + 4 = \mathbf{13} + \boldsymbol{\theta}^2$$

b. 
$$\mathbf{y}^T \mathbf{y} = \mathbf{y} \cdot \mathbf{y} = \mathbf{13} + \boldsymbol{\theta}^2$$

4. 
$$\mathbf{x}^{T}\mathbf{Q}\mathbf{x} = (x_{1} \quad x_{2})\begin{pmatrix} 1 & -2 \\ -2 & 3 \end{pmatrix}\begin{pmatrix} x_{1} \\ x_{2} \end{pmatrix} = (x_{1} - 2x_{2} \quad -2x_{1} + 3x_{2})\begin{pmatrix} x_{1} \\ x_{2} \end{pmatrix} = (x_{1}(x_{1} - 2x_{2}) + x_{2}(-2x_{1} + 3x_{2})) = x_{1}^{2} - 2x_{1}x_{2} - 2x_{1}x_{2} + 3x_{2}^{2} = \mathbf{x}_{1}^{2} - \mathbf{4}\mathbf{x}_{1}\mathbf{x}_{2} + 3\mathbf{x}_{2}^{2}$$

5. 
$$f(k_1x_1 + k_2x_2) = k_1f(x_1) + k_2f(x_2)$$

a. 
$$f(\beta) = \beta x^2 \rightarrow (k_1 x_1 + k_2 x_2) x^2 = k_1 x_1 x^2 + k_2 x_2 x^2 \rightarrow True$$
: **Linear**

b. 
$$f(x) = \frac{dx}{dt} \to \frac{d}{dt}(k_1x_1 + k_2x_2) = k_1\frac{d}{dt}(x_1) + k_2\frac{d}{dt}(x_2) = k_1f(x_1) + k_2f(x_2) \to True: Linear$$

c. 
$$f(x) = \int x \, dx = \int (k_1 x_1 + k_2 x_2) \, dx = k_1 \int x_1 \, dx + k_2 \int x_2 \, dx \to True$$
: **Linear**

6. 
$$\boldsymbol{a} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}, \boldsymbol{b} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}$$

a. 
$$\|a\| = \sqrt{(-2)^2 + 1^2} = \sqrt{5}$$

b. 
$$\|\boldsymbol{b}\| = \sqrt{(0)^2 + 4^2} = \mathbf{4}$$

c. 
$$\|\boldsymbol{a} + \boldsymbol{b}\| = \left\| {\binom{-2}{5}} \right\| = \sqrt{(-2)^2 + 5^2} = \sqrt{29}$$

d. 
$$||3a|| = \left\| {\binom{-6}{3}} \right\| = \sqrt{(-6)^2 + 3^2} = \sqrt{45} = 3\sqrt{5}$$

7. 
$$x \cdot y = ||x|| ||y|| \cos \theta$$

a. 
$$\rightarrow 2 = \sqrt{14}\sqrt{3}\cos\theta \rightarrow \theta = \cos^{-1}\left(\frac{2}{\sqrt{14}\sqrt{3}}\right) = 72.02^{\circ}$$

b. 
$$\rightarrow dot \ product = 0 \rightarrow 90^{\circ}$$

8. 
$$\boldsymbol{a} = \begin{pmatrix} 1 \\ k \\ 2 \end{pmatrix}, \boldsymbol{b} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$$

a. 
$$\boldsymbol{a}^T \boldsymbol{b} = \begin{pmatrix} 1 & k & 2 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} = (\boldsymbol{k} + \boldsymbol{4})$$

b. 
$$ab^{T} = \begin{pmatrix} 1 \\ k \\ 2 \end{pmatrix} (0 \quad 1 \quad 2) = \begin{pmatrix} 0 & 1 & 2 \\ 0 & k & 2k \\ 0 & 2 & 4 \end{pmatrix}$$
  
c.  $ba^{T} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} (1 \quad k \quad 2) = \begin{pmatrix} 0 & 0 & 0 \\ 1 & k & 2 \\ 2 & 2k & 4 \end{pmatrix}$  or  $ba^{T} = (ab^{T})^{T} = \begin{pmatrix} 0 & 0 & 0 \\ 1 & k & 2 \\ 2 & 2k & 4 \end{pmatrix}$ 

```
1 - d = [0;0;1];
2 - Short_arm = [1, 0, 30; 0, 1, 0; 0, 0, 1];
3 - Long_arm = [1, 0, 60; 0, 1, 0; 0, 0, 1];
4 - R_75 = [cosd(75), -sind(75), 0; sind(75), cosd(75), 0; 0 0 1];
5 - R_minus60 = [cosd(-60), -sind(-60), 0; sind(-60), cosd(-60), 0; 0 0 1];
6 
7 - Location = R_minus60 * Long_arm * R_75 * Short_arm * d;

>> Location

Location =

58.9778
-44.1970
1.0000
```

The final location is  $\begin{pmatrix} 58.98 \text{ cm} \\ -44.20 \text{ cm} \end{pmatrix}$ 

9.

Command Window

>> HW1\_Part2

## Part 2

```
A =
  1
     2 -3 11
     6 -8 32
  3
  -2
     -1
        0 -7
A =
     2 -3
           11
  1
       1
     0
           -1
  0
     -1
A =
  1 2 -3 11
    0 1 -1
A =
           11
     2 -3
  1
        -6
  0
      3
      0
         1
A =
     2 -3
           11
A =
  1
     0
        1
     1
        1
           -1
  0
A =
  1 0 0 2
     1 -2 5
      0
        1 -1
  0
A =
         0
        0
     1
            3
       1 -1
     0
```

```
A = [4 \ 8 \ -12 \ 44; \ 3 \ 6 \ -8 \ 32; \ -2 \ -1 \ 0 \ -7];
 2
       % 1
       A(1,:) = 0.25.*A(1,:)
3 -
       § 2
 4
       A(2,:) = A(2,:) - 3.*A(1,:)
5 -
6
7 -
       A(3,:) = 2.*A(1,:) + A(3,:)
       % 4
8
9 -
       A([2 3],:) = A([3 2],:)
10
       % 5
       A(2,:) = A(2,:)./3
11 -
12
       % 6
       A(1,:) = A(1,:) - 2.*A(2,:)
13 -
14
       % 7
15 -
       A(1,:) = A(1,:) - A(3,:)
16
       % 8
       A(2,:) = A(2,:) + 2.*A(3,:)
17 -
```

## Part 3

```
Command Window
 Optimal solution found.
 ans =
    45.1546
    -0.0082
    0.1969
  ans =
    1.4534
    -2.3954
         0
  linx =
    0.6667
     1.3333
 quadx =
    0.6667
    1.3333
  species =
   2×1 <u>cell</u> array
     {'virginica' }
     {'versicolor'}
  ans =
    36.8218 29.6073 -12.9818 23.7147 -0.5532
 ans =
    -0.0170 -0.0161 -0.0171 -0.0191 -0.0200
```