Part I

1. $\begin{pmatrix}3 & 4 & -2\\2 & 0 & 1\\1 & 2 & 4\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\begin{pmatrix}-14\\10\\22\end{pmatrix}\rightarrow\begin{pmatrix}3 & 4 & -2 & -14\\2 & 0 & 1 & 10\\1 & 2 & 4 & 22\end{pmatrix}\rightarrow\frac{R_1}{3}\rightarrow\begin{pmatrix}1 & \frac{4}{3} & -\frac{2}{3} & -\frac{14}{3}\\2 & 0 & 1 & 10\\1 & 2 & 4 & 22\end{pmatrix}\rightarrow$

   $R_2-2R_1\rightarrow\begin{pmatrix}1 & \frac{4}{3} & -\frac{2}{3} & -\frac{14}{3}\\0 & -\frac{8}{3} & \frac{7}{3} & \frac{58}{3}\\1 & 2 & 4 & 22\end{pmatrix}\rightarrow R_3-R_1\rightarrow\begin{pmatrix}1 & \frac{4}{3} & -\frac{2}{3} & -\frac{14}{3}\\0 & -\frac{8}{3} & \frac{7}{3} & \frac{58}{3}\\0 & \frac{2}{3} & \frac{14}{3} & \frac{80}{3}\end{pmatrix}\rightarrow-\frac{3}{8}R_2\rightarrow$

   $\begin{pmatrix}1 & \frac{4}{3} & -\frac{2}{3} & -\frac{14}{3}\\0 & 1 & -\frac{7}{8} & -\frac{29}{4}\\0 & \frac{2}{3} & \frac{14}{3} & \frac{80}{3}\end{pmatrix}\rightarrow R_3-\frac{2}{3}R_2\rightarrow\begin{pmatrix}1 & \frac{4}{3} & -\frac{2}{3} & -\frac{14}{3}\\0 & 1 & -\frac{7}{8} & -\frac{29}{4}\\0 & 0 & 21 & \frac{80}{3}\end{pmatrix}\begin{pmatrix}1 & \frac{4}{3} & -\frac{2}{3} & -\frac{14}{3}\\0 & 1 & -\frac{7}{8} & -\frac{29}{4}\\0 & \frac{2}{3} & \frac{14}{3} & \frac{80}{3}\end{pmatrix}\rightarrow$

   $R_3-\frac{2}{3}R_2\rightarrow\begin{pmatrix}1 & \frac{4}{3} & -\frac{2}{3} & -\frac{14}{3}\\0 & 1 & -\frac{7}{8} & -\frac{29}{4}\\0 & 0 & \frac{21}{4} & \frac{63}{2}\end{pmatrix}\rightarrow\frac{4}{21}R_3\rightarrow\begin{pmatrix}1 & \frac{4}{3} & -\frac{2}{3} & -\frac{14}{3}\\0 & 1 & -\frac{7}{8} & -\frac{29}{4}\\0 & 0 & 1 & 6\end{pmatrix}$

   a. $x_1+\frac{4}{3}x_2-\frac{2}{3}x_3=-\frac{14}{3}$
   b. $x_2-\frac{7}{8}x_3=-\frac{29}{4}$
   c. $\boldsymbol{x_3=6}$
   d. Solve x2: $x_2-\frac{7}{8}(6)=-\frac{29}{4}$
      i. $\boldsymbol{x_2=-2}$
   e. Solve x3: $x_1+\frac{4}{3}(-2)-\frac{2}{3}(6)=-\frac{14}{3}$
      i. $\boldsymbol{x_3=2}$

2. $\begin{pmatrix}1 & -1 & 1\\2 & 10 & -3\\4 & 4 & 0\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\begin{pmatrix}4\\31\\28\end{pmatrix}\rightarrow\begin{pmatrix}1 & -1 & 1 & 4\\2 & 10 & -3 & 31\\4 & 4 & 0 & 28\end{pmatrix}\rightarrow R_2-2R_1\rightarrow$

   $\begin{pmatrix}1 & -1 & 1 & 4\\0 & 12 & -5 & 23\\4 & 4 & 0 & 28\end{pmatrix}\rightarrow R_3-4R_1\rightarrow\begin{pmatrix}1 & -1 & 1 & 4\\0 & 12 & -5 & 23\\0 & 8 & -4 & 12\end{pmatrix}\rightarrow\frac{1}{12}R_2\rightarrow$

   $\begin{pmatrix}1 & -1 & 1 & 4\\0 & 1 & -\frac{5}{12} & \frac{23}{12}\\0 & 8 & -4 & 12\end{pmatrix}\rightarrow R_3-8R_2\rightarrow\begin{pmatrix}1 & -1 & 1 & 4\\0 & 1 & -\frac{5}{12} & \frac{23}{12}\\0 & 0 & -\frac{2}{3} & -\frac{10}{3}\end{pmatrix}\rightarrow-\frac{3}{2}R_3\rightarrow$

   $\begin{pmatrix}1 & -1 & 1 & 4\\0 & 1 & -\frac{5}{12} & \frac{23}{12}\\0 & 0 & 1 & 5\end{pmatrix}\rightarrow R_2+\frac{5}{12}R_3\rightarrow\begin{pmatrix}1 & -1 & 1 & 4\\0 & 1 & 0 & 4\\0 & 0 & 1 & 5\end{pmatrix}\rightarrow R_1-R_3+R_2\rightarrow$

   $\begin{pmatrix}1 & 0 & 0 & 3\\0 & 1 & 0 & 4\\0 & 0 & 1 & 5\end{pmatrix}$

   a. $\boldsymbol{(x_1,\ x_2,\ x_3)=(3,4,5)}$

3. $\begin{pmatrix} 0 & 2 & 1 & 0 \\ 3 & -4 & 0 & 1 \end{pmatrix} \rightarrow \frac{1}{2}R_1 \leftrightarrow \frac{1}{3}R_2 \rightarrow \begin{pmatrix} 1 & -\frac{4}{3} & 0 & \frac{1}{3} \\ 0 & 1 & \frac{1}{2} & 0 \end{pmatrix} \rightarrow R_1 + \frac{4}{3}R_2 \rightarrow \begin{pmatrix} 1 & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & 1 & \frac{1}{2} & 0 \end{pmatrix}$

   a. $\boldsymbol{A^{-1}} = \begin{pmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{2} & \mathbf{0} \end{pmatrix}$

   b. $\begin{pmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{2} & 0 \end{pmatrix} \begin{pmatrix} 10 \\ 1 \end{pmatrix} = \begin{pmatrix} 7 \\ 5 \end{pmatrix} \rightarrow (\boldsymbol{x_1}, \boldsymbol{x_2}) = (\mathbf{7}, \mathbf{5})$

   c. $\begin{pmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{2} & 0 \end{pmatrix} \begin{pmatrix} 8 \\ 6 \end{pmatrix} = \begin{pmatrix} \frac{22}{3} \\ 4 \end{pmatrix} \rightarrow (\boldsymbol{x_1}, \boldsymbol{x_2}) = \left(\frac{\mathbf{22}}{\mathbf{3}}, \mathbf{4}\right)$

Part II

1. $\begin{pmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & 1 & -1 \\ 0 & -1 & 0 & 1 \end{pmatrix} \rightarrow R_4 + R_2 \rightarrow \begin{pmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \rightarrow R_3 - R_1 - R_2 \rightarrow$

   $\begin{pmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \rightarrow R_1 + R_2 \rightarrow \begin{pmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

   a. **Rank = 2**

2. Let $r_4 = \alpha$ and $r_3 = \beta$

$$r_2 = r_4$$
$$= \alpha$$
$$r_1 = r_2 - r_3$$
$$= \alpha - \beta$$

3. Let $\alpha = 5$ and $\beta = 3$

$$r_1 = 2$$
$$r_2 = 5$$
$$r_3 = 3$$
$$r_4 = 5$$

4. The structure of this metabolic pathway necessitates that all the mass must flow through two nodes, and every rate is connected to at least one of these nodes. So, by specifying any two nodes other than r_2 and r_4 (which are the same) we know the value of every reaction rate. Therefore, the system only has two independent pieces of information making the rank equal two.

Part III

1. ***(Tables may vary!)***

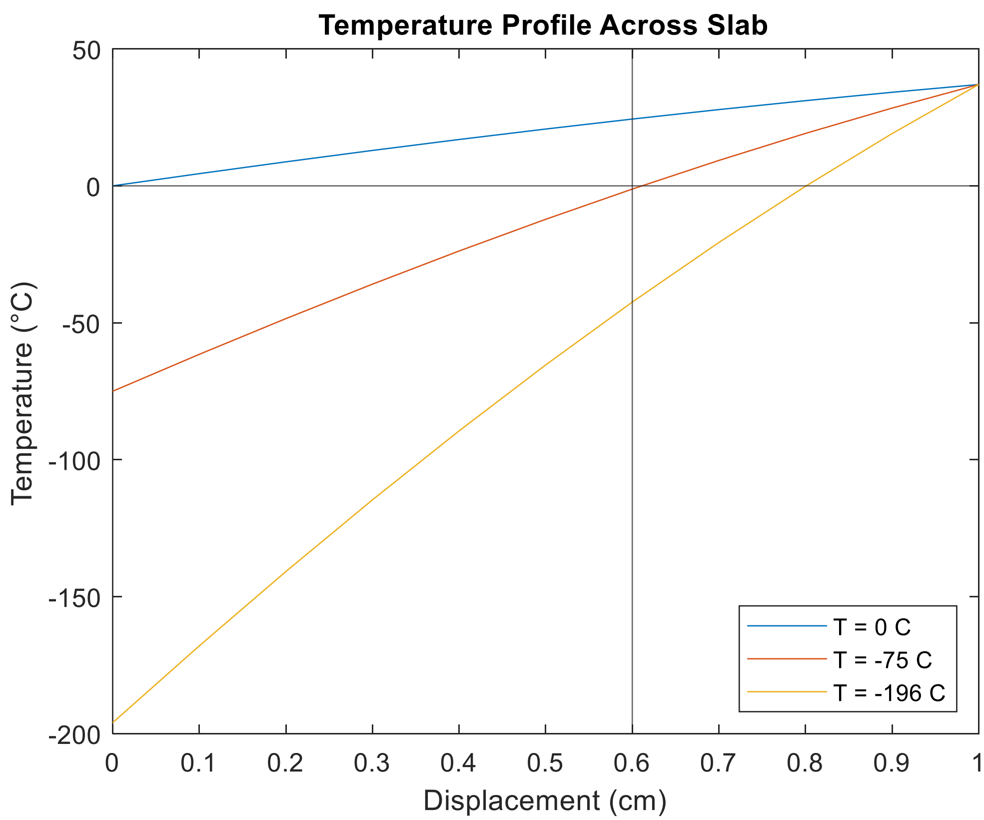
Index k	0	1	2	3	4	5	6	7	8	9	10
x (cm)	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
α	0.50	0.45	0.41	0.37	0.34	0.30	0.27	0.25	0.22	0.20	0.18

2. $\alpha \frac{d^2T}{dx} - v_x \frac{dT}{dx} = 0 \rightarrow \alpha \frac{u^{k+1} - 2u^k + u^{k-1}}{\Delta x^2} - v_x \frac{u^{k+1} - u^k}{\Delta x} = 0$
3. $u^0 = T(x = 0), u^{10} = 37°\text{C}$
4. Code:

```
clc
clear
close all

T11 = 37; %degrees Celsius
dx = 0.1; %displacement across nodes
x = 0:dx:1; %initiates the 11 points spanning [0,1]
n = 11; %# of nodes
vx = -0.15; %convection in cm/s

y = zeros(n,1); %Temp at node n-1
Tp1 = zeros(n, n); %coefficients
Tp1(1,1)= 1;
Tp1(n,n) = 1;
y(1) = 0;
y(11)=T11;
for i=2:length(x)-1 %iteration over i to find change in T over [0,1]
    A = 0.5.*exp(-x(i));%heat diffusivity
    Tp1(i, i+1)=A/(dx.^2)-vx./dx;
    Tp1(i,i)=A.*(-2)/(dx.^2)+vx./dx;
    Tp1(i,i-1)=A/(dx.^2);
end

plot(x,Tp1\y)
hold on %allows for addition of new graphics to figure

%(2)----------------------------------------------------------------------

y = zeros(n,1); %Temp at node n-1
Tp2 = zeros(n, n);
Tp2(1,1)= 1;
Tp2(n,n) = 1;
y(1) = -75; %temp = -75C
y(11)=T11;
for i=2:length(x)-1 %iteration over i to find change in T over [0,1]
    A = 0.5.*exp(-x(i));
    Tp2(i, i+1)=A/(dx.^2)-vx./dx;
    Tp2(i,i)=A.*(-2)/(dx.^2)+vx./dx;
    Tp2(i,i-1)=A/(dx.^2);
end

plot(x,Tp2\y)

%(3)----------------------------------------------------------------------

y = zeros(n,1); %Temp at node n-1
Tp3 = zeros(n, n);
Tp3(1,1)= 1;
Tp3(n,n) = 1;
y(1)= -196;%temp = -196C
y(11)=T11;
```

```
for i=2:length(x)-1 %iteration over i to find change in T over [0,1]
    A = 0.5.*exp(-x(i));
    Tp3(i, i+1)=A/(dx.^2)-vx./dx;
    Tp3(i,i)=A.*(-2)/(dx.^2)+vx./dx;
    Tp3(i,i-1)=A/(dx.^2);
end

plot(x,Tp3\y)
yline(0)
xline(0.6)
title('Temperature Profile Across Slab');
xlabel('Displacement (cm)');
ylabel('Temperature (°C)');
legend('T = 0 C', 'T = -75 C', 'T = -196 C', 'Location', 'southeast');
```

5. Output:



6. The scenario that uses -75°C cryogenic produces a sufficient therapeutic effect to bring the temperature of the lesion down to below 0°C while the healthy tissue is nearly entirely above this threshold. This implies the entirety of the lesion will die while only a fraction of a millimeter of healthy tissue will be lost. Compared to 0°C where none of the lesion is killed and -196°C where almost 2 millimeters of healthy tissue is lost.

7. v_x is negative because it is the velocity of the blood flow which is moving in the negative x direction based on how our system is defined.

8. ***(Answers may vary!)*** A practicality of using a less extreme treatment compared to liquid nitrogen is that, depending on the depth required to reach a therapeutic effect, we can remove timing as a factor and allow the system to reach steady state where we know we have achieved maximal therapeutic effect with minimal damage. There are several drawbacks including this procedure taking longer than it would if a lower temperature cryogenic was applied which means the patient must endure the pain for a longer period, and the temperature of the cryogenic must be determined on an individual basic decided by the geometry of the lesion and dynamics of the tissue.