BIOE 210, Spring 2022

Homework 12

Due Monday, 4/18/2022 by 5:00pm.
Upload your answers to Gradescope. If submitting a single PDF, you must mark the location of all answers.

Solve the following problems by hand.

1. Decompose the vector $\begin{pmatrix}3\\2\end{pmatrix}$ onto the vectors $\left\{\begin{pmatrix}1\\0\end{pmatrix}, \begin{pmatrix}1\\1\end{pmatrix}\right\}$.
2. Decompose the vector $\begin{pmatrix}3\\2\end{pmatrix}$ onto the vectors $\left\{\begin{pmatrix}1\\-2\end{pmatrix}, \begin{pmatrix}1\\1\end{pmatrix}, \begin{pmatrix}-1\\1\end{pmatrix}\right\}$.
   (a) Does the set of vectors in the previous problem constitute a basis? Why or why not?
   (b) Does the set of vectors span the space $\mathbb{R}^2$?
   (c) Find a second decomposition of $\begin{pmatrix}3\\2\end{pmatrix}$ onto the same vectors.
3. Show that the vectors $\left\{\begin{pmatrix}1\\1\end{pmatrix}, \begin{pmatrix}0\\-1\end{pmatrix}\right\}$ are a basis.
4. Consider the vectors $\left\{\begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\2\end{pmatrix}, \begin{pmatrix}0\\1\\0\end{pmatrix}\right\}$.
   (a) Construct an orthonormal set of basis vectors from these vectors.
   (b) Decompose the vector $\begin{pmatrix}-2\\1\\3\end{pmatrix}$ onto your orthonormal basis.

You can use Matlab or a calculator to answer the following questions.

5. Consider the matrix
$$\mathbf{A} = \begin{pmatrix}5 & 3 & 1\\3 & 2 & 1\\1 & 1 & 1\end{pmatrix}$$
   (a) Find the eigenvectors and the corresponding eigenvalues for the matrix.
   (b) Decompose the vector $\mathbf{x} = \begin{pmatrix}1\\2\\3\end{pmatrix}$ on the eigenvectors.
   (c) Using your eigenvectors and eigenvalues instead of matrix multiplication, what is $\mathbf{Ax}$?