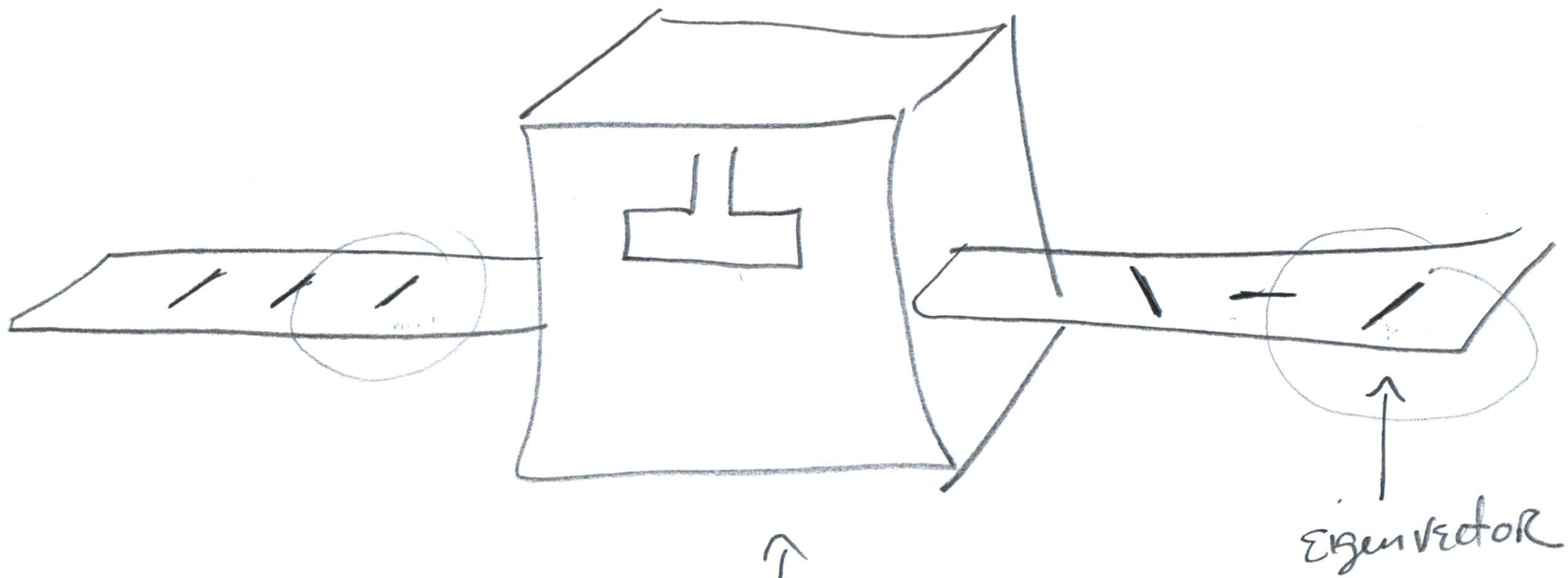

$$\underline{A}\,\underline{v} = \lambda \underline{v}$$

$$\underline{A}\,\underline{0} = \lambda \underline{0} \quad \forall \lambda$$

$$\underline{0} = \underline{0}$$

$\underline{0}$ is not an eigenvector

Stability of linear syst. §18.3.2

$\frac{d\underline{x}}{dt} = \underline{A}\,\underline{x}$ is stable iff all eigenvalues of $\underline{A}$ are negative, or at least not positive.

$c\underline{v}e^{\lambda t}$

Positive Def §18.3.3.

$\underline{x}^T\underline{A}\,\underline{x} > 0$	P.D	All. eigenvalues	> 0
≥ 0	P.S.D.	" "	≥ 0

Determinant

$\det(A)$ = product of eigenvalues of $\underline{A}$.

Pg 165: Properties of determinant.

$\underline{v}_1, \underline{v}_2, \cdots, \underline{v}_n$

Vectors **span** a space if for any $\underline{x} \in$ space,

$$\underline{x} = a_1 \underline{v}_1 + a_2 \underline{v}_2 + \cdots + a_n \underline{v}_n$$

Vectors are a **basis** if the decomposition is unique.

1. Span the space.
2. Linearly independent
3. # of vectors = dimension of space.

$$c_1 \underline{v}_1 + c_2 \underline{v}_2 + \cdots + c_n \underline{v}_n = \underline{0} \iff c_1 = c_2 = \cdots = c_n = 0$$

Lin ind.

$$\underline{V} = (\underline{v}_1\ \underline{v}_2 \cdots \underline{v}_n), \quad \text{Rank}(\underline{V}) = n$$

orthonormal basis: basis mutually orthogonal + all vectors are unit vectors.

Eigendecomp.

$$\underline{A} = \underline{V}\,\underline{\Lambda}\,\underline{V}^{-1}$$

columns equal to eigenvectors

$$\underline{\Lambda} = \begin{bmatrix} \lambda_1 & & & \underline{0} \\ & \lambda_2 & & \\ & & \ddots & \\ \underline{0} & & & \lambda_n \end{bmatrix}$$

SVD

$$\underline{A} = \underline{U}\,\underline{\Sigma}\,\underline{V}^{T}$$

$\underline{V}$ is orthogonal : $\underline{V}^{-1} = \underline{V}^{T}$

(orthonormal columns)

~~Singular~~

All eigenvectors for a matrix are unique.

But, eigenvalues are not necessarily unique.

Basis $\underline{v}_1, \cdots, \underline{v}_n$

To decompose $\underline{x}$ onto it.

$$\underline{V} = (\underline{v}_1\ \underline{v}_2 \cdots \underline{v}_n)$$

Coefficients $\underline{a}$ satisfy

$$\underline{V}\,\underline{a} = \underline{x}$$

$$\underline{a} = \underline{V}^{-1}\underline{x}$$

Shortcut for orthonormal basis

$$a_i = \underline{x} \cdot \underline{v}_i$$

$$\underline{y} = \underline{\sigma}\left(\underline{W}_d \cdots \underline{\sigma}\left(\underline{W}_2\left(\underline{\sigma}\left(\underline{W}_1 \underline{x}\right)\right)\right)\right)$$

Multi Layer Perceptron

$$\underline{W}_{i+1}\left(\underline{\sigma}\left(\underline{W}_i \cdots\right)\right)$$

without $\underline{\sigma}$,

$$\underbrace{\underline{W}_{i+1}\,\underline{W}_i}_{\underline{W}}$$