1.
$$J(x) = \begin{pmatrix} -x_2 \sin(x_1) & \cos(x_1) \\ 0 & 4x_2 \end{pmatrix}$$
2.
$$g(x^{(0)}) = \begin{pmatrix} \cos(1) \\ 2-1 \end{pmatrix} = \begin{pmatrix} 0.54 \\ 1 \end{pmatrix} \neq \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$
3.
$$x^{(1)} = x^{(0)} - J^{-1}(x^{(0)})g(x^{(0)}) = \begin{pmatrix} 1 \\ 1 \end{pmatrix} - \begin{pmatrix} -1.1884 & 0.1605 \\ 0 & 0.25 \end{pmatrix}^{-1} \begin{pmatrix} \cos(1) \\ 1 \end{pmatrix} = \begin{pmatrix} 1.48 \\ 0.75 \end{pmatrix}$$
a.
$$g(x^{(1)}) = \begin{pmatrix} 0.0668 \\ 0.1250 \end{pmatrix}$$
4.

a.
$$x^{(2)} = \begin{pmatrix} 1.48 \\ 0.75 \end{pmatrix} - \begin{pmatrix} -1.3387 & 0.0398 \\ 0 & 0.3333 \end{pmatrix}^{-1} \begin{pmatrix} 0.0668 \\ 0.1250 \end{pmatrix} = \begin{pmatrix} 1.5661 \\ 0.7083 \end{pmatrix}$$
i.
$$g(x^{(2)}) = \begin{pmatrix} 0.0034 \\ 0.0035 \end{pmatrix}$$
b.
$$x^{(3)} = \begin{pmatrix} 1.5661 \\ 0.7083 \end{pmatrix} - \begin{pmatrix} -1.4118 & 0.0024 \\ 0 & 0.3529 \end{pmatrix}^{-1} \begin{pmatrix} 0.0034 \\ 0.0035 \end{pmatrix} = \begin{pmatrix} 1.5708 \\ 0.7071 \end{pmatrix}$$
i.
$$g(x^{(3)}) = \begin{pmatrix} 0.5765 \\ 0.3004 \end{pmatrix} \times 10^{-5}$$

Code:

```
clc; clear; close all
        syms x1 x2
        g = [x2.*cos(x1); 2.*x2.^2 - 1];
                                                                                                                                           Solution to question 1:
        fprintf('Solution to question 1:')
4
        J = jacobian(g)
                                                                                                                                            -x_2 \sin(x_1) \cos(x_1)
        fprintf('Solution to question 2:')
                                                                                                                                           Solution to question 2:
        g = Q(x) [x(2).*cos(x(1)); 2.*x(2).^2 - 1];
                                                                                                                                           g at x nought = 2 \times 1
        x_0 = [1 1];
        g_at_x_nought = g(x_0)
                                                                                                                                           Solution to question 3:
        fprintf('Solution to question 3:')
                                                                                                                                               1.4816 0.7500
        g_at_x_1 = 2 \times 1
                                                                                                                                                0.0668
        g_at_x_1 = g(x_1)
13
                                                                                                                                           Solution to question 4:
        fprintf('Solution to guestion 4:')
14
         \begin{array}{l} \mbox{Jinv} = @(x) \ [-1/(x(2)^* \sin(x(1))) \ \cos(x(1))/(4^*x(2)^2 * \sin(x(1))); \ \theta \ 1/(4^*x(2))]; \\ x\_2 = (x\_1' \ - \mbox{Jinv}(x\_1)^* g(x\_1))'; \\ \end{array} 
                                                                                                                                           g_at_x_2 = 2 \times 1
15
                                                                                                                                                0.0034
        g_{at_x_2} = g(x_2)

x_3 = (x_2' - Jinv(x_2)*g(x_2))';
                                                                                                                                           g_at_x_3 = 2×1
        g_at_x_3 = g(x_3)
                                                                                                                                                0.5765
clc; clear; close all
syms x1 x2
g = [x2.*cos(x1); 2.*x2.^2 - 1];
```

```
fprintf('Solution to question 1:')
J = jacobian(g)
```

```
fprintf('Solution to question 2:')
g = @(x) [x(2).*cos(x(1)); 2.*x(2).^2 - 1];
x_0 = [1 1];
g_at_x_nought = g(x_0)
```

```
 \begin{array}{lll} & \text{fprintf('Solution to question 3:')} \\ & \text{Jinv} = @(x) \left[ -1/(x(2)*\sin(x(1))) \; \cos(x(1))/(4*x(2)^2*\sin(x(1))); \; 0 \; 1/(4*x(2))]; \\ & \text{x\_1} = (x\_0' \; - \; \text{Jinv}(x\_0)*g(x\_0))' \\ & \text{g\_at\_x\_1} = g(x\_1) \\ \end{array}
```

```
fprintf('Solution to question 4:')
Jinv = @(x) [-1/(x(2)*sin(x(1))) cos(x(1))/(4*x(2)^2*sin(x(1))); 0 1/(4*x(2))];
x_2 = (x_1' - Jinv(x_1)*g(x_1))';
g_at_x_2 = g(x_2)
x_3 = (x_2' - Jinv(x_2)*g(x_2))';
g_at_x_3 = g(x_3)
```

Part II

Code:

```
clear; close all; clc;
a = 0.01;
x = zeros(1000,3);
y = zeros(1000, 1);
f = Q(x) (x(1)-2)^2 + (x(2)+3)^2 + (x(3)-x(1))^2;
g = 0(x) [2*(x(1)-2) - 2*(x(3)-x(1)); 2*(x(2)+3); 2*(x(3)-x(1))];
 for k = 1:1000
     x(k+1,1:3) = (x(k,1:3)' - a*g(x(k,1:3)))';
     y(k) = f(x(k, 1:3));
 end
figure(1)
plot(1:1000,y)
title('a = 0.01')
xlabel('k')
ylabel("Value of y")
figure(2)
plot(x)
title('a = 0.01')
xlabel('k')
ylabel("Value of x")
clear;
a = 0.1;
x = zeros(1000,3);
y = zeros(1000,1);
f = @(x) (x(1)-2)^2 + (x(2)+3)^2 + (x(3)-x(1))^2;
q = 0(x) [2*(x(1)-2) - 2*(x(3)-x(1)); 2*(x(2)+3); 2*(x(3)-x(1))];
 for k = 1:1000
     x(k+1,1:3) = (x(k,1:3)' - a*g(x(k,1:3)))';
     y(k) = f(x(k, 1:3));
 end
figure(3)
plot(x)
title('a = 0.1')
xlabel('k')
```

```
ylabel("Value of x")
clear;
a = 0.001;
x = zeros(1000,3);
  = zeros(1000,1);
  = @(x) (x(1)-2)^2 + (x(2)+3)^2 + (x(3)-x(1))^2;
g = @(x) [2*(x(1)-2) - 2*(x(3)-x(1)); 2*(x(2)+3); 2*(x(3)-x(1))];
 for k = 1:1000
       x(k+1,1:3) = (x(k,1:3)' - a*g(x(k,1:3)))';
       y(k) = f(x(k, 1:3));
 end
figure(4)
plot(x)
title('a = 0.001')
xlabel('k')
ylabel("Value of x")
                         a = 0.01
                                                                               a = 0.01
     14
                                                           1.5
     12
     10
                                                          0.5
   Value of y
                                                         Value of x
                                                          -0.5
                                                          -1.5
                                                          -2.5
                  300
                          500
                              600
                                  700 800 900 1000
                                                                   200
                                                                          400
                                                                                 600
                                                                                              1000
                                                                                                     1200
                         a = 0.1
                                                                               a = 0.001
                                                           1.5
    1.5
                                                          0.5
    0.5
     0
                                                        Value of x
  Value of x
    -0.5
                                                          -1.5
    -1.5
                                                           -2
     -2
    -2.5
                                                          -2.5
                                                           -3 <sup>r</sup>
            200
                   400
                          600
                                 800
                                        1000
                                               1200
                                                                   200
                                                                                 600
                                                                                        800
                                                                                              1000
                                                                                                     1200
```

Changing alpha modifies the size of the step in the gradient descent. Therefore, a larger alpha will result in a steeper step downhill and a more rapid approach of the variables to their optimal values.