

Interactions in Linear Models

BIOE 210

What is an interaction?

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What if there is another effect that depends on both x_1 and x_2 ?
This is an **interaction** between x_1 and x_2 .

How do we model interactions?

We model the interaction of x_1 and x_2 using the product of these variables.

$$y = \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + \epsilon$$

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Why do we multiply x_1 and x_2 ? There are at least two ways to interpret this term.

The coded factor interpretation

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on/off \rightarrow interaction when both "on"

| x_1 | x_2 | x_1x_2 |
|-------|-------|----------|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

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|-------|-------|----------|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

high/low \rightarrow interaction when both "high" or both "low"

| x_1 | x_2 | x_1x_2 |
|-------|-------|----------|
| -1 | -1 | +1 |
| -1 | +1 | -1 |
| +1 | -1 | -1 |
| +1 | +1 | +1 |

The augmented slope interpretation

We can also interpret the interaction as one variable changing the effect of the other variable.

$$\begin{aligned}y &= \beta_1 x_1 + \beta_2 (x_1) x_2 + \epsilon \\&= \beta_1 x_1 + (\beta_2 + \beta_{12} x_1) x_2 + \epsilon \\&= \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + \epsilon\end{aligned}$$

Things to remember about interactions

- ▶ Interaction are modeled as the product of variables.
- ▶ The interaction effect is “above and beyond” the independent effects (synergy/super-additivity, antagonism/sub-additivity).
- ▶ Higher-order interactions are possible (e.g. $x_1x_2x_3$), but these are rare.