5a: Write a finite difference approximation for the following differential equation using five nodes spanning the interval $x \in [-2, 2]$.

$$e^x \frac{du}{dx} - \sin(x) = 4, \quad u(-2) = 1$$

Write your equations in matrix form and solve for u at each node.

5b: Write a finite difference approximation for the following differential equation using four nodes spanning the interval $t \in [0, 1]$.

$$\frac{d^2y}{dt^2} - t\frac{dy}{dt} = 0, \quad y(0) = 0, \quad y(1) = 4$$

5a: Write a finite difference approximation for the following differential equation using five nodes spanning the interval $x \in [-2, 2]$.

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Write your equations in matrix form and solve for u at each node.

node (k): 0 1 2 3 4

x -2 -1 0 1 2

$\Delta x = \frac{2-(-2)}{4} = 1$

Interior Nodes:

$k=1$: $e^{-1}\left(\frac{u^{(1)}-u^{(0)}}{1}\right) - \sin(-1) = 4$

$k=2$: $e^{0}\left(\frac{u^{(2)}-u^{(1)}}{1}\right) - \sin 0 = 4$

$k=3$: $e^{1}\left(\frac{u^{(3)}-u^{(2)}}{1}\right) - \sin 1 = 4$

$k=4$: $e^{2}\left(\frac{u^{(4)}-u^{(3)}}{1}\right) - \sin 2 = 4$

Boundary Node:

$k=0$: $u^{(0)} = 1$

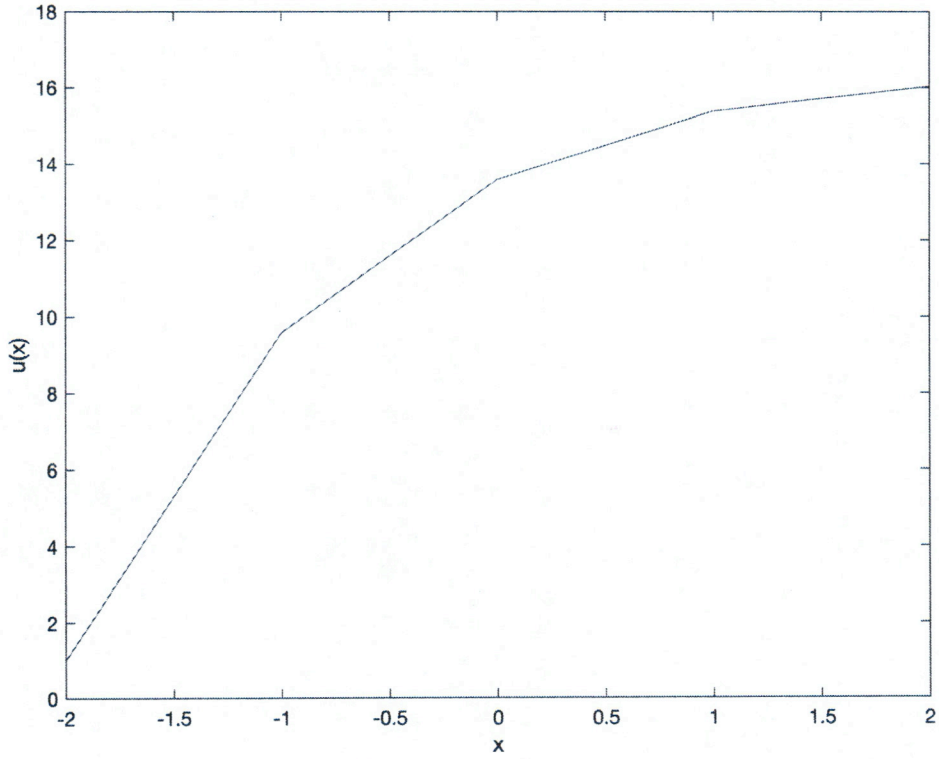
```
n = 5;
x = linspace(-2,2,n);
dx = x(2) - x(1);

A = zeros(n);
b = zeros(n,1);

for i = 2:n
    A(i,i) = exp(x(i))/dx;
    A(i,i-1) = -exp(x(i))/dx;
    b(i) = 4 + sin(x(i));
end

A(1,1) = 1;
b(1) = 1;

[A b]
```

```
ans = 5×6
    1.0000         0         0         0         0    1.0000
   -0.3679    0.3679         0         0         0    3.1585
         0   -1.0000    1.0000         0         0    4.0000
         0         0   -2.7183    2.7183         0    4.8415
         0         0         0   -7.3891    7.3891    4.9093
```

```
A \ b
```

```
ans = 5×1
    1.0000
    9.5858
   13.5858
   15.3668
   16.0313
```

```
plot(x, A\b)
xlabel('x')
ylabel('u(x)')
```


18
16
14
12
10
8
6
4
2
0
u(x)
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
x

5b: Write a finite difference approximation for the following differential equation using four nodes spanning the interval $t \in [0, 1]$.

$$\frac{d^2y}{dt^2} - t\frac{dy}{dt} = 0, \quad y(0) = 0, \quad y(1) = 4$$

node:	0	1	2	3
t	0	$1/3$	$2/3$	1

$$\Delta t = \frac{1-0}{3} = 1/3$$

Interior Nodes (1 & 2)

$$k=1: \quad \frac{u^{(2)} - 2u^{(1)} + u^{(0)}}{(1/3)^2} - \frac{1}{3}\frac{u^{(2)} - u^{(1)}}{1/3} = 0$$

$$k=2: \quad \frac{u^{(3)} - 2u^{(2)} - u^{(1)}}{(1/3)^2} - \frac{1}{3}\frac{u^{(3)} - u^{(2)}}{1/3} = 0$$

Boundary Nodes:

$$k=0: \quad u^{(0)} = 0$$

$$k=3: \quad u^{(3)} = 4$$