$$y = \beta_0 + \beta_1 x_1 + \cdots \beta_n x_n + \varepsilon$$

$$\left(y = \beta_0 + \beta_1 x + \beta_2 x^2\right) + \beta_3 x^3 + \cdots \beta_n x^n$$

y	x
8	3
7	-2
6	1

$$\begin{array}{c} y \\ \hline \begin{pmatrix} 8 \\ 7 \\ 6 \end{pmatrix} \end{array} = \begin{pmatrix} 1 & 3 & 9 \\ 1 & -2 & 4 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{pmatrix}$$

Curvilinear Models

Exponential Cell Growth

$$N(t) = N_0 e^{\mu t}$$

exponential growth rate

$$\log N(t) = \log\left(N_0 e^{\mu t}\right)$$

$$= \log N_0 + \log e^{\mu t}$$

$$\underbrace{\log N(t)}_{y} = \underbrace{\log N_0}_{\beta_0} + \mu t$$

$$y = \beta_0 + \beta_1 t$$

$$\beta_0 = \log N_0$$

$$e^{\beta_0} = e^{\log N_0} = N_0$$

Linearizing

Nonlinear Systems:

- Don't know # of solutions
  how to get them.

Two Problems:

1.) Root-finding

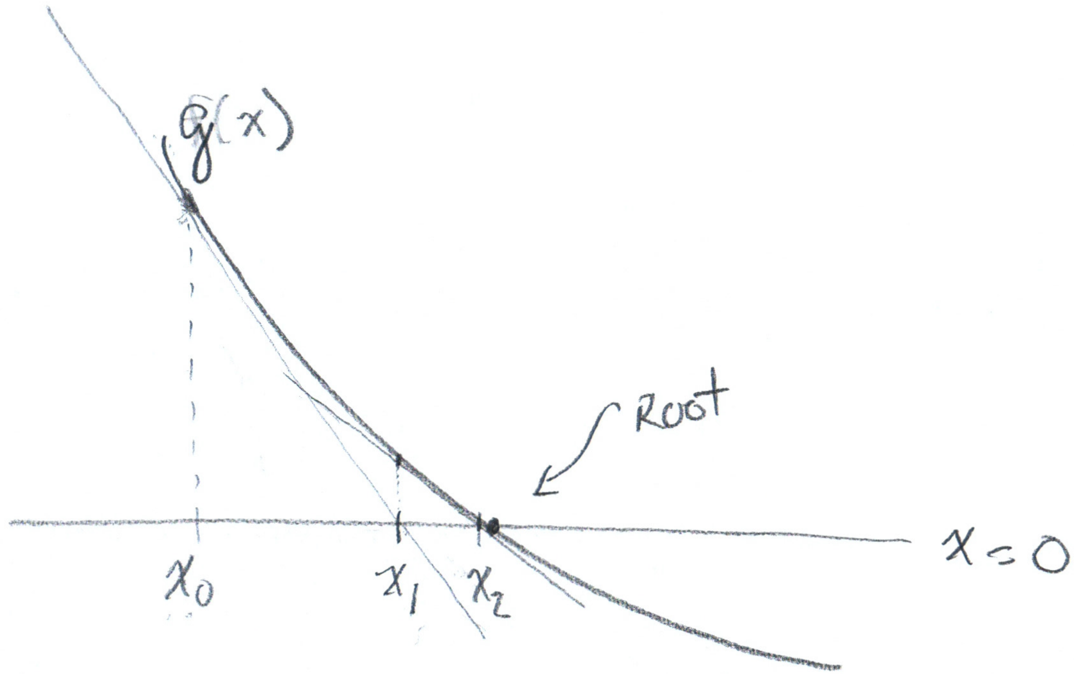
$g(x) = 0$ $\qquad$ $h(x) = 17$

$\underline{g}(\underline{x}) = \underline{0}$ $\qquad$ $g(x) = h(x) - 17$

$g(x) = 0 \Leftrightarrow h(x) = 17$

2.) optimization

$\min_{\underline{x}} f(\underline{x})$ $\qquad$ $\min_{\beta} L(\beta)$



$$g'(x_0) = \frac{\text{RISE}}{\text{Run}} = \frac{g(x_0) - 0}{x_0 - x_1}$$

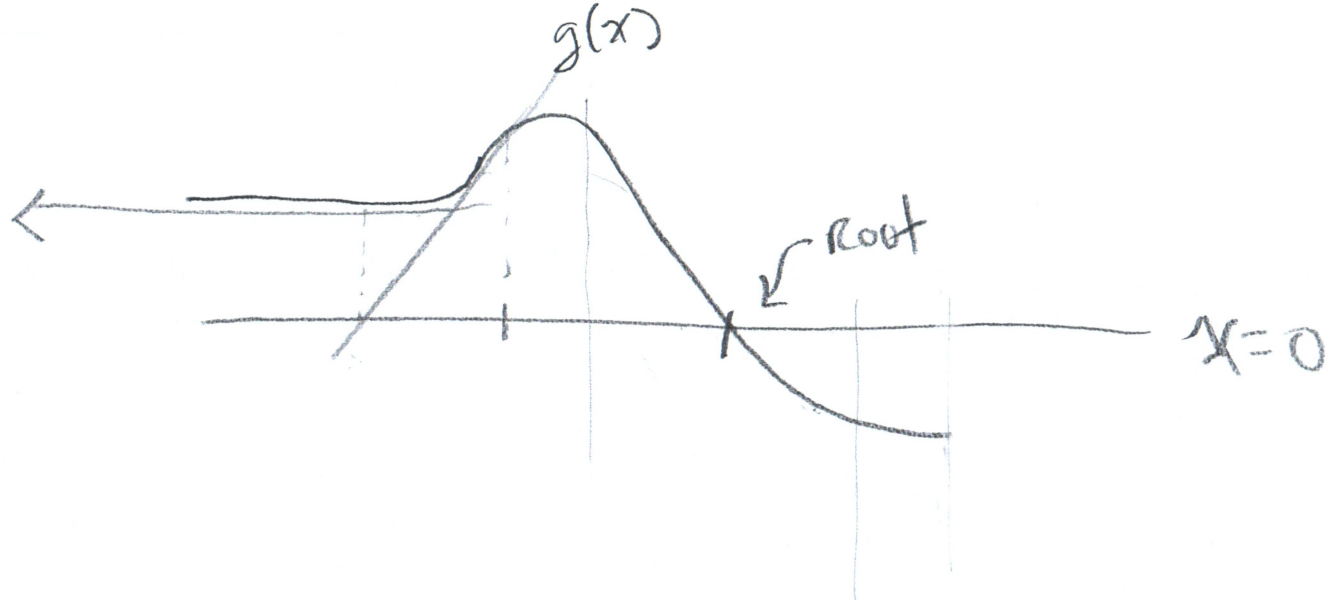
$$g'(x_0)\left(x_0 - x_1\right) = g(x_0)$$

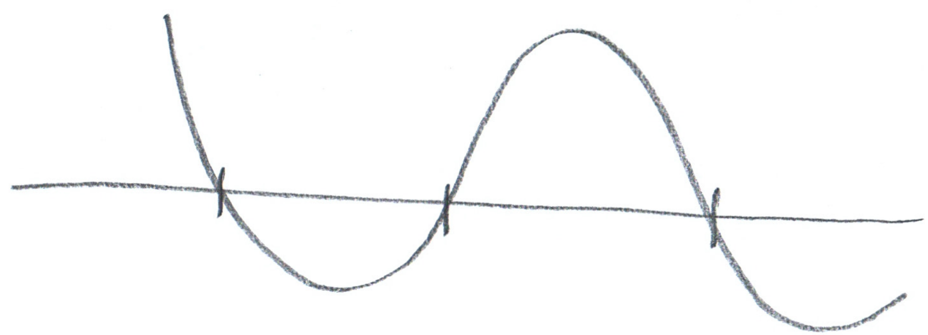
Newton's Method.

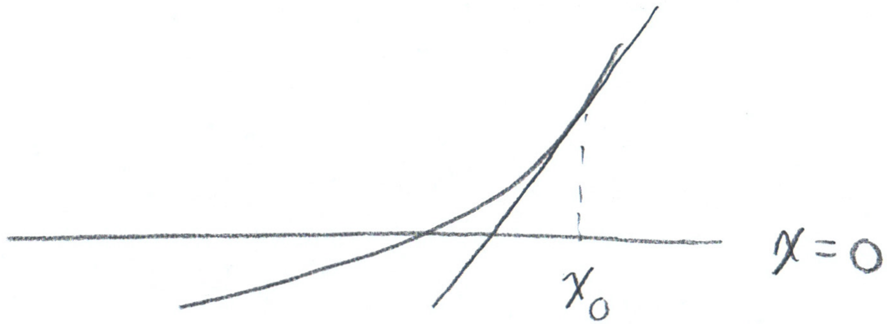
$$x_0 - x_1 = \frac{g(x_0)}{g'(x_0)}$$

$$\boxed{x_1 = x_0 - \frac{g(x_0)}{g'(x_0)}}$$

$$x_{k+1} = x_k + \frac{g(x_k)}{g'(x_k)}$$


g(x)
Root
x=0





Taylor Series:

$$g(x) = \left(g(x_0) + \frac{g'(x_0)}{1!}(x - x_0) \right) + \overbrace{\frac{g''(x_0)}{2!}(x-x_0)^2 + \ldots}^{\text{bounded by a quadratic}}$$

Multivariate Newton's Method

$$\underline{g}(\underline{x}) = 0$$

$$\underline{g}(\underline{x}) = \begin{pmatrix} x_1 - x_3 \\ x_3^2 + 2x_2 \\ \cos x_1 \end{pmatrix} \qquad \underline{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

3 inputs, 3 outputs

$$\frac{\partial g_1}{\partial x_1} = 1 + 0$$

$$\underline{g}(\underline{x}) = \begin{pmatrix} g_1(\underline{x}) \\ g_2(\underline{x}) \\ g_3(\underline{x}) \end{pmatrix}$$

JACOBIAN

$$\underline{J}(\underline{x}) = \begin{pmatrix} \frac{\partial g_1}{\partial x_1} & \frac{\partial g_1}{\partial x_2} & \frac{\partial g_1}{\partial x_3} \\ \frac{\partial g_2}{\partial x_1} & \frac{\partial g_2}{\partial x_2} & \frac{\partial g_2}{\partial x_3} \\ \frac{\partial g_3}{\partial x_1} & \frac{\partial g_3}{\partial x_2} & \frac{\partial g_3}{\partial x_3} \end{pmatrix}$$

$$x_{n+1} = x_n - \frac{g(x_n)}{g'(x_n)}$$

Newton's Method

$$x_{n+1} = x_n - \frac{1}{g'(x_n)} g(x_n)$$

Multivariate N.M.

$$\underline{x}_{n+1} = \underline{x}_n - \underline{J}^{-1}(\underline{x}_n)\, \underline{g}(\underline{x}_n)$$