Linear Models: Main Effects

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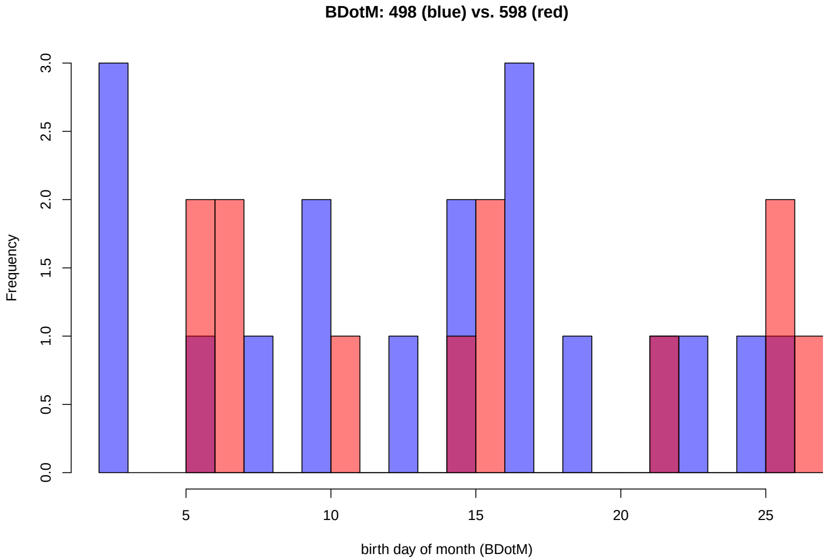
Does BDotM differ for the BIOE 498 and BIOE 598 students?



Shortcut method: the *t*-test

```r
t.test(days498, days598, alternative="two.sided")
```

```
##
##  Welch Two Sample t-test
##
## data:  days498 and days598
## t = -0.4488, df = 22.194, p-value = 0.6579
## alternative hypothesis: true difference in means is not equal
## 95 percent confidence interval:
##  -7.647553  4.925331
## sample estimates:
## mean of x mean of y
##  13.88889  15.25000
```

New technique: Linear Models

Previous approach: Split data into two groups; test for a difference in BDotM

New appraoch: Build a model that predicts BDotM; ask if 498/598 knowledge helps

Linear Models

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_n x_n + \epsilon$$

- y is the **response**
- x_i is a **predictor** or **factor**
- β_0 is the **intercept**
- $\beta_i, i > 0$ is a **coefficient**, **effect size**, or **parameter**
- ϵ is a **residual**

Re-organizing our data into a **data frame**

```r
head(days_data)
```

```
##   day undergrad
## 1  23      TRUE
## 2  10      TRUE
## 3  10      TRUE
## 4  17      TRUE
## 5   7     FALSE
## 6  27     FALSE
```

Working with data frames

Checking the size (# rows and # columns)

```r
dim(days_data)
```

```
## [1] 30  2
```

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```
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```
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Extracting a single entry

```
days_data[12, ]
```

```
##    day undergrad
## 12  22     FALSE
```

Working with data frames

Checking the size (# rows and # columns)

```
dim(days_data)
```

```
## [1] 30  2
```

Extracting a single entry

```
days_data[12, ]
```

```
##    day undergrad
## 12  22     FALSE
```

Or just the day

```
days_data[12,"day"]
```

```
## [1] 22
```

Our data frame contains two vectors

```
days_data$day
```

```
##  [1] 23 10 10 17  7 27  2  7 25 22 26 22  3 16 17 26  5  5 17
## [24]  8 15 15 13 16 15 11
```

```
days_data$undergrad
```

```
##  [1]  TRUE  TRUE  TRUE  TRUE FALSE FALSE  TRUE FALSE  TRUE  T
## [12] FALSE  TRUE FALSE  TRUE FALSE FALSE FALSE  TRUE  TRUE  T
## [23]  TRUE  TRUE  TRUE  TRUE  TRUE FALSE FALSE FALSE
```

Building a linear model with `lm`: $y = \beta_0 + \epsilon$

```r
lm( days_data$day ~ 1 )
```

```
##
## Call:
## lm(formula = days_data$day ~ 1)
##
## Coefficients:
## (Intercept)
##       14.43
```

Building a linear model with `lm`: $y = \beta_0 + \epsilon$

```r
lm( days_data$day ~ 1 )
```

```
##
## Call:
## lm(formula = days_data$day ~ 1)
##
## Coefficients:
## (Intercept)
##       14.43
```

```r
mean(days_data$day)
```

```
## [1] 14.43333
```

Modeling with a predictor: $y = \beta_0 + \beta_1 x + \epsilon$

```r
lm( days_data$day ~ 1 + days_data$undergrad )
```

```
##
## Call:
## lm(formula = days_data$day ~ 1 + days_data$undergrad)
##
## Coefficients:
##            (Intercept)  days_data$undergradTRUE
##                 15.250                   -1.361
```

Modeling with a predictor: $y = \beta_0 + \beta_1 x + \epsilon$

```r
lm( days_data$day ~ 1 + days_data$undergrad )
```

```
##
## Call:
## lm(formula = days_data$day ~ 1 + days_data$undergrad)
##
## Coefficients:
##           (Intercept)  days_data$undergradTRUE
##                15.250                   -1.361
```

```r
mean(days498) - mean(days598)
```

```
## [1] -1.361111
```

What does this model mean?

We fit the parameters β_0 and β_1 in the model

$$\text{day} = \beta_0 + \beta_1 \times \text{undergrad} + \epsilon$$

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$$\text{day} = 15.3 - 1.4 \times \text{undergrad} + \epsilon$$

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For undergrads: $\text{day} = 15.3 - 1.4 \times 1 = 13.9$

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For undergrads: $\text{day} = 15.3 - 1.4 \times 1 = 13.9$

For grad students: $\text{day} = 15.3 - 1.4 \times 0 = 15.3$

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$$\text{day} = \beta_0 + \beta_1 \times \text{undergrad} + \epsilon$$

Substituting the fitted values:

$$\text{day} = 15.3 - 1.4 \times \text{undergrad} + \epsilon$$

For undergrads: $\text{day} = 15.3 - 1.4 \times 1 = 13.9$

For grad students: $\text{day} = 15.3 - 1.4 \times 0 = 15.3$

These are the means for each group.

Let's clean up our calls to `lm`: Intercepts

An intercept is always assumed.

```
lm( y ~ 1 + x )
```

is equivalent to

```
lm( y ~ x )
```

If you don't want an intercept, use a 0

```
lm( y ~ 0 + x )
```

Let's clean up our calls to `lm`: Naming a data frame

We can give `lm` a data frame where it can find our response and predictor variables.

```
lm( days_data$day ~ days_data$undergrad )
```

is equivalent to

```
lm( day ~ undergrad, data=days_data )
```

Storing our model for further analysis

We can assign the output of a model to a variable.

```r
model <- lm( day ~ undergrad, data=days_data )
summary(model)
```

```
##
## Call:
## lm(formula = day ~ undergrad, data = days_data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -11.889  -7.389   0.750   6.340  12.111
##
## Coefficients:
##                Estimate Std. Error t value Pr(>|t|)
## (Intercept)      15.250      2.306   6.614 3.56e-07 ***
## undergradTRUE    -1.361      2.976  -0.457    0.651
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' '
##
## Residual standard error: 7.987 on 28 degrees of freedom
## Multiple R-squared:  0.007413,	Adjusted R-squared:  0.0280
```

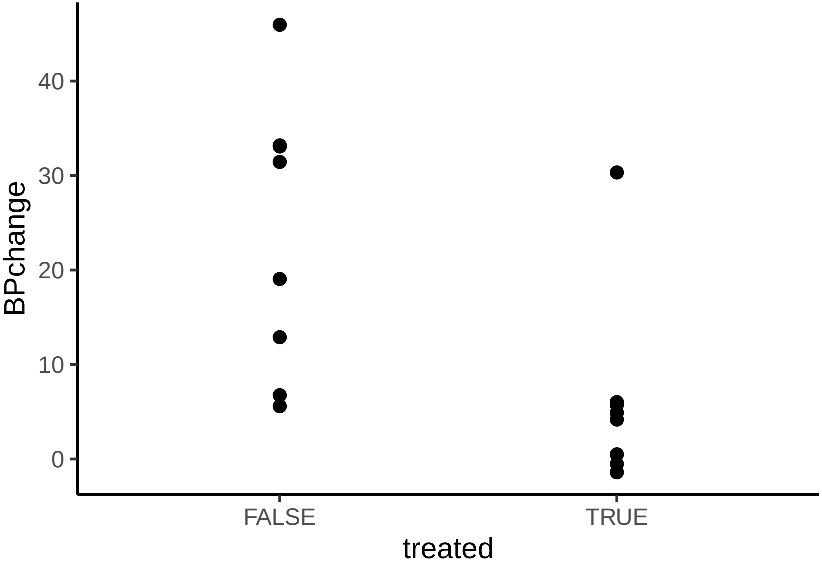
Testing a new blood presure medication

```r
head(bp_data)
```

```
## # A tibble: 6 x 3
##   BPchange treated male
##      <dbl> <lgl>   <lgl>
## 1   -0.525 TRUE    FALSE
## 2    4.17  TRUE    FALSE
## 3    6.03  TRUE    TRUE
## 4   -1.40  TRUE    FALSE
## 5    0.493 TRUE    FALSE
## 6   12.9   FALSE   TRUE
```

Does our BP treatment work?

```
qplot(data=bp_data, x=treated, y=BPchange, size=I(5))
```



Hypothesis testing the treatment effect

```r
t.test(BPchange ~ treated, data=bp_data)
```

```
##
##  Welch Two Sample t-test
##
## data:  BPchange by treated
## t = 2.7499, df = 12.51, p-value = 0.01704
## alternative hypothesis: true difference in means is not equal
## 95 percent confidence interval:
##   3.649731 30.903566
## sample estimates:
## mean in group FALSE  mean in group TRUE
##           23.492472            6.215823
```

A linear model with effect of treatment

```r
model <- lm(BPchange ~ treated, bp_data)
summary(model)
```

```
##
## Call:
## lm(formula = BPchange ~ treated, data = bp_data)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -17.905  -6.960  -1.678   8.357  24.110
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   23.492      4.442   5.288 0.000115 ***
## treatedTRUE  -17.277      6.283  -2.750 0.015647 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' '
##
## Residual standard error: 12.57 on 14 degrees of freedom
## Multiple R-squared:  0.3507, Adjusted R-squared:  0.3043
## F-statistic: 7.562 on 1 and 14 DF,  p-value: 0.01565
```

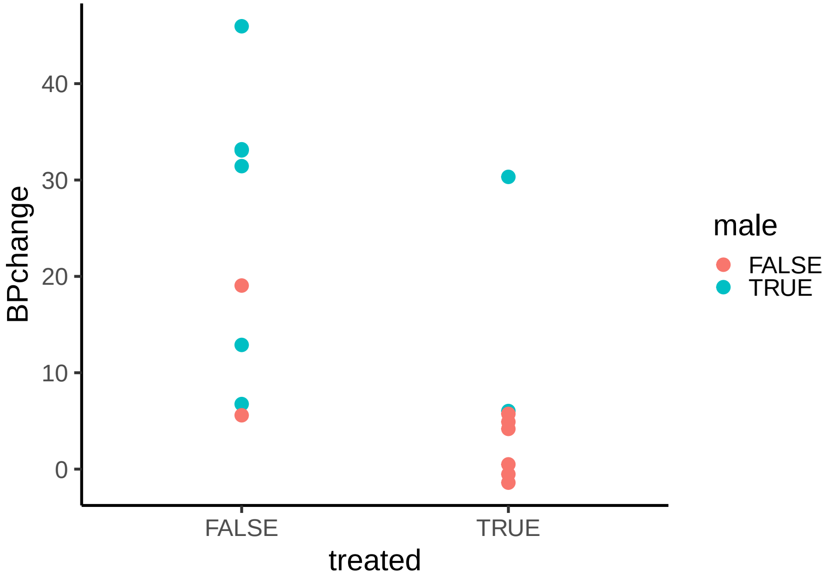
What else could explain the effect?

```r
summary(lm(BPchange ~ treated + male, bp_data))
```

```
##
## Call:
## lm(formula = BPchange ~ treated + male, data = bp_data)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -20.596  -4.407   2.178   5.756  18.607
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   11.926      5.998   1.988   0.0683 .
## treatedTRUE   -9.566      6.195  -1.544   0.1466
## maleTRUE      15.422      6.195   2.489   0.0271 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' '
##
## Residual standard error: 10.73 on 13 degrees of freedom
## Multiple R-squared:  0.5603, Adjusted R-squared:  0.4927
## F-statistic: 8.283 on 2 and 13 DF,  p-value: 0.004791
```

Does our BP treatment work?

```
qplot(data=bp_data, x=treated, y=BPchange, color=male, size=I(5))
```



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- Multivariate linear models consider how **all** factors affect the response. This is a form of conditioning.
- Next time: What if the factors interact?