# Factorial Designs

BIOE 498/598 PJ

Spring 2021

# Why do we use factorial designs?

- Factorial designs find better optima.
- Factorial designs are more efficient.
- Factorial designs make better estimates of effect sizes.

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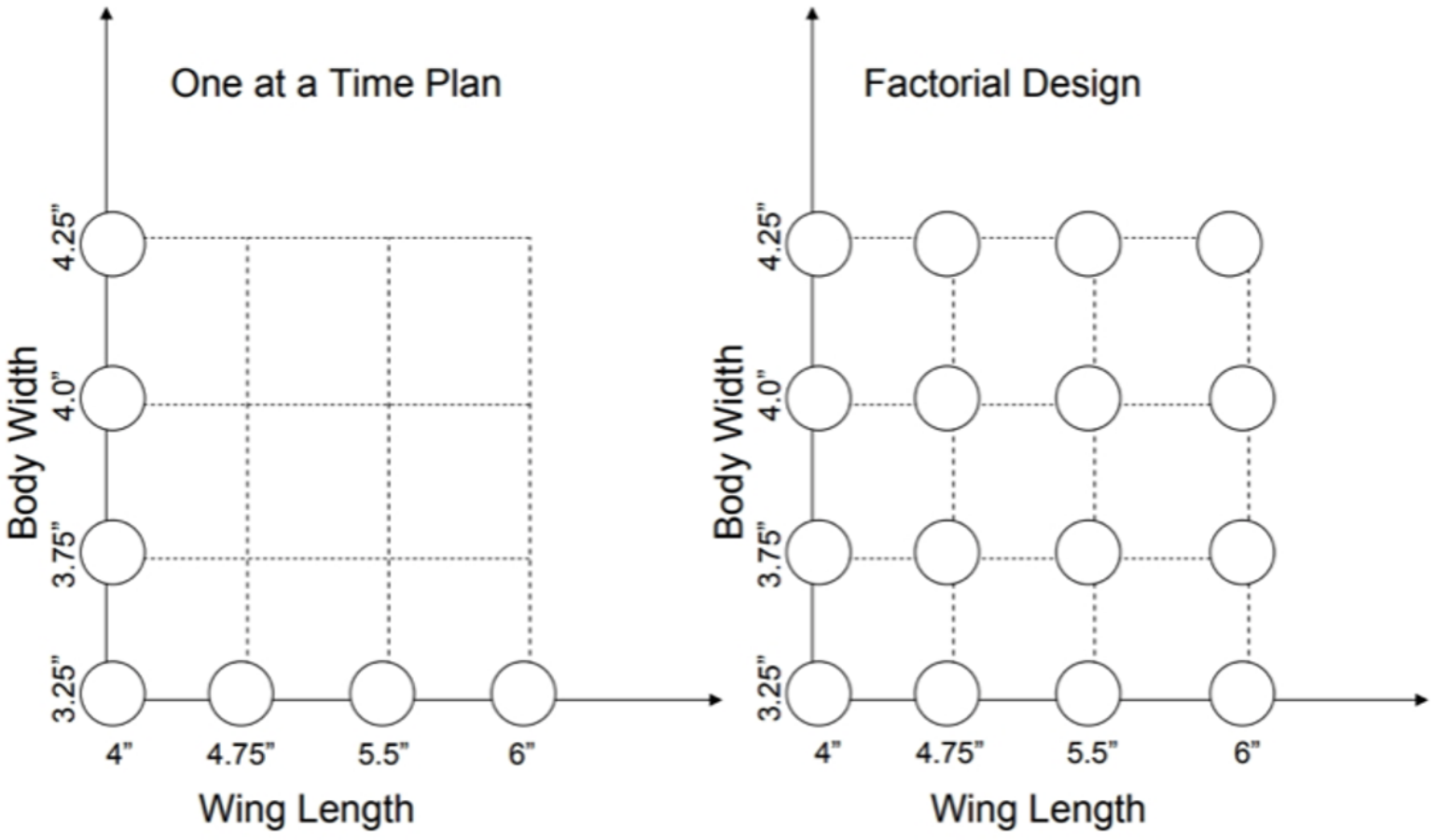
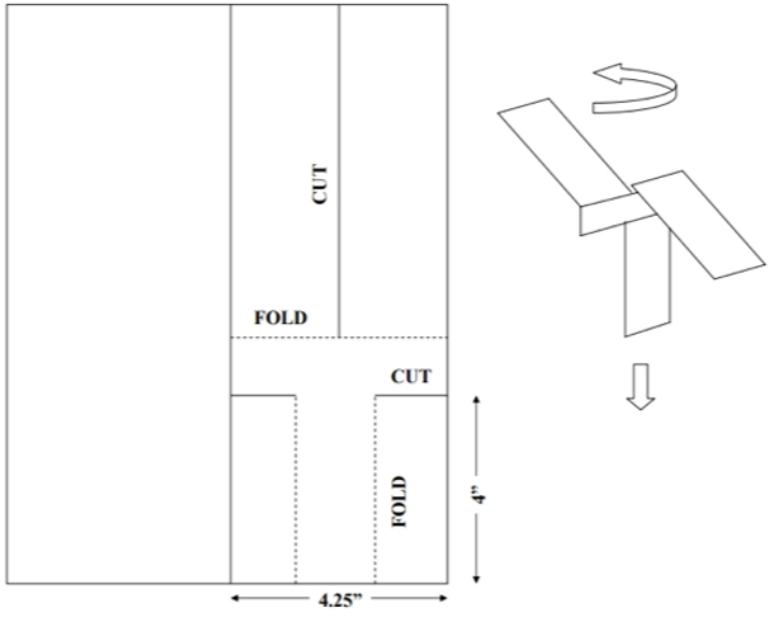
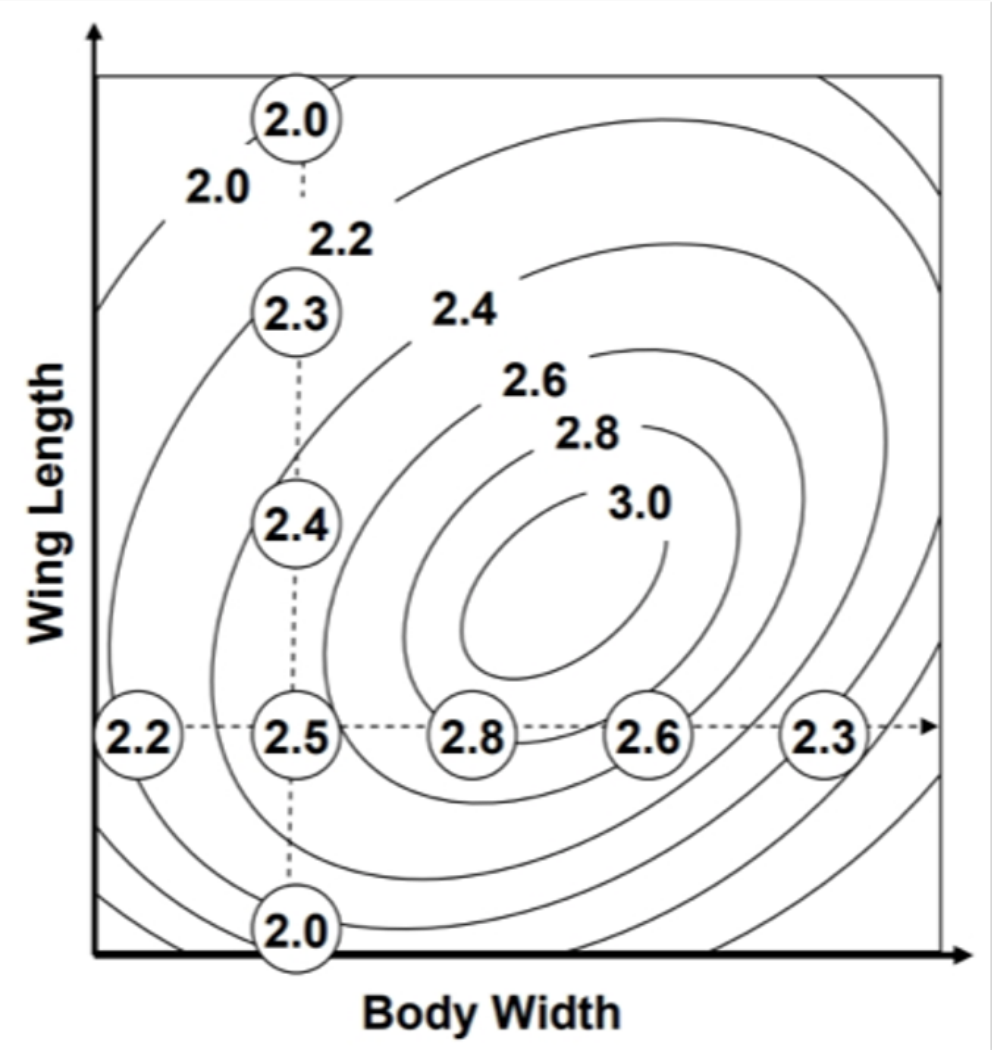
# What is a factorial design?

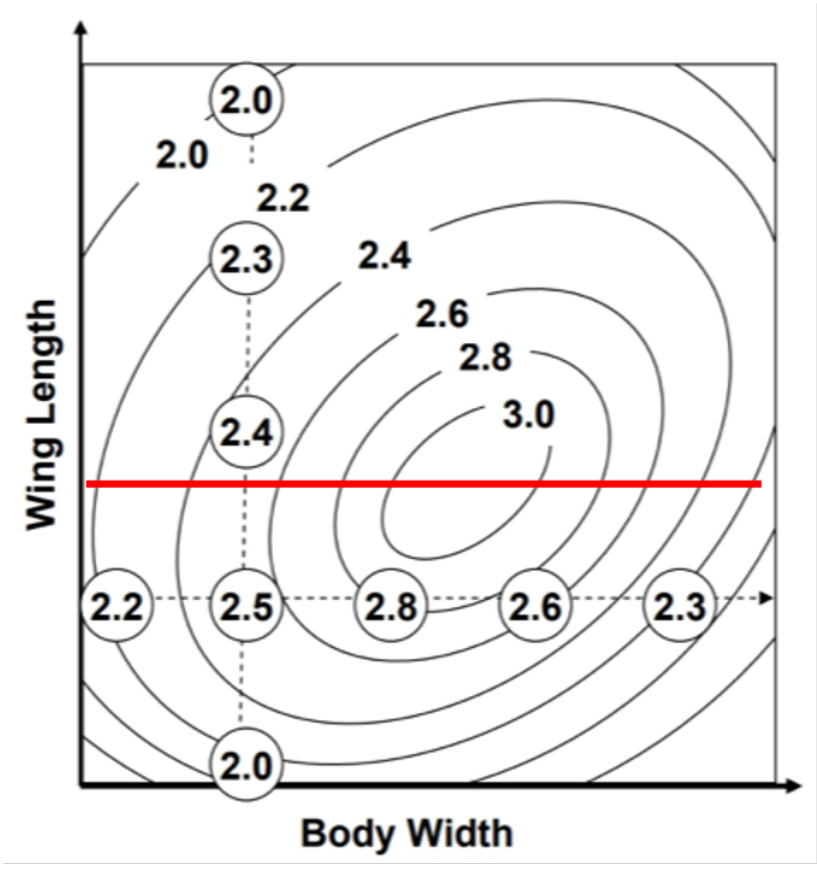


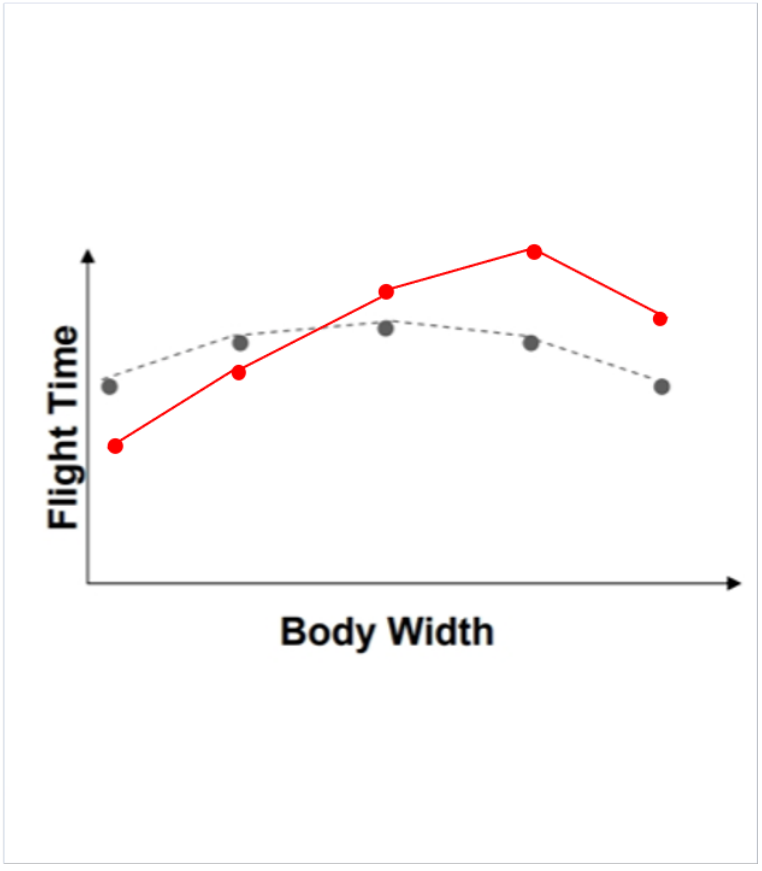
# Factorial designs find better optima



# The problem with OFAT: Interactions



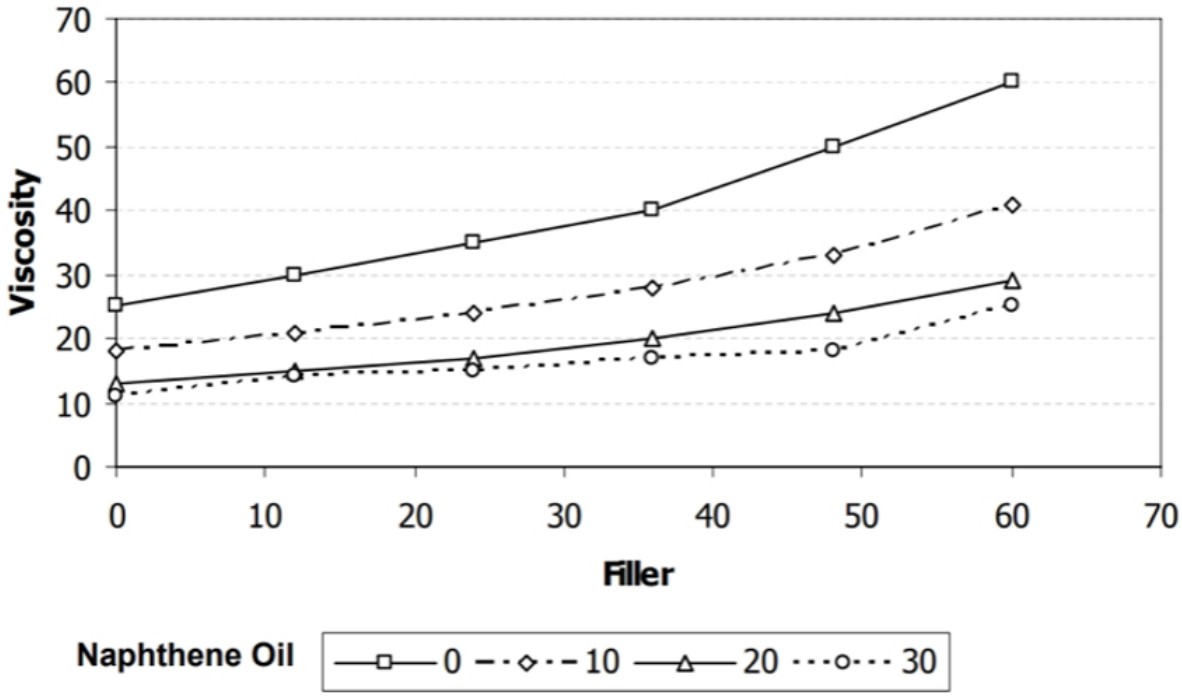



# Using interaction plots for diagnosis

Table 3.1 *Mooney Viscosity of Silica B at 100° C*

| Naphthene Oil (phr) | Filler (phr) | | | | | |
|---|---|---|---|---|---|---|
| | 0 | 12 | 24 | 36 | 48 | 60 |
| 0 | 25 | 30 | 35 | 40 | 50 | 60 |
| 10 | 18 | 21 | 24 | 28 | 33 | 41 |
| 20 | 13 | 15 | 17 | 20 | 24 | 29 |
| 30 | 11 | 14 | 15 | 17 | 18 | 25 |

Figure 3.4 *Interaction Plot of Filler and Naphthene Oil*



# Why do we use factorial designs?

- Factorial designs find better optima.
- **Factorial designs are more efficient.**
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# Factorial designs seem *less* efficient. . .

Imagine an experiment with four factors, each with two levels $(-, +)$.
We want three replicates for each level.

One Factor at a Time Design

- 3 runs at level $(-)$
- 4 factors $\times$ 3 runs at $(+) = 12$ runs
- **15 total runs**

Factorial Design

- $2^4 =$ **16 total runs**

# . . . until you look at the designs

OFAT design

| $x_1$ | $x_2$ | $x_3$ | $x_4$ |
|---|---|---|---|
| − | − | − | − |
| − | − | − | − |
| − | − | − | − |
| + | − | − | − |
| + | − | − | − |
| + | − | − | − |
| − | + | − | − |
| − | + | − | − |
| − | + | − | − |
| − | − | + | − |
| − | − | + | − |
| − | − | + | − |
| − | − | − | + |
| − | − | − | + |
| − | − | − | + |

Factorial design

| $x_1$ | $x_2$ | $x_3$ | $x_4$ |
|---|---|---|---|
| − | − | − | − |
| + | − | − | − |
| − | + | − | − |
| + | + | − | − |
| − | − | + | − |
| + | − | + | − |
| − | + | + | − |
| + | + | + | − |
| − | − | − | + |
| + | − | − | + |
| − | + | − | + |
| + | + | − | + |
| − | − | + | + |
| + | − | + | + |
| − | + | + | + |
| + | + | + | + |

# Factorial designs give more replicates per run

A factorial design in $n$ variables has $2^n$ runs, but $2^{n-1}$ replicates at each level $(-, +)$.

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A factorial design in $n$ variables has $2^n$ runs, but $2^{n-1}$ replicates at each level $(-, +)$.

Imagine a design with $n$ variables at $k$ levels.
After the initial design, adding another replicate requires

- $nk$ runs for a OFAT design
- $\sim k$ runs for a factorial design

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# What are the other factors doing when $x_3$ is high?

OFAT design

| $x_1$ | $x_2$ | $x_3$ | $x_4$ |
|---|---|---|---|
| − | − | − | − |
| − | − | − | − |
| − | − | − | − |
| + | − | − | − |
| + | − | − | − |
| + | − | − | − |
| − | + | − | − |
| − | + | − | − |
| − | + | − | − |
| − | − | + | − |
| − | − | + | − |
| − | − | + | − |
| − | − | − | + |
| − | − | − | + |
| − | − | − | + |

Factorial design

| $x_1$ | $x_2$ | $x_3$ | $x_4$ |
|---|---|---|---|
| − | − | − | − |
| + | − | − | − |
| − | + | − | − |
| + | + | − | − |
| − | − | + | − |
| + | − | + | − |
| − | + | + | − |
| + | + | + | − |
| − | − | − | + |
| + | − | − | + |
| − | + | − | + |
| + | + | − | + |
| − | − | + | + |
| + | − | + | + |
| − | + | + | + |
| + | + | + | + |

# What do the effect sizes estimate?

For OFAT designs:

$\beta_i$ is the effect of moving $x_i$ from $-$ to $+$
**while all other factors stay at $-$.**

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For OFAT designs:

$\beta_i$ is the effect of moving $x_i$ from $-$ to $+$
**while all other factors stay at $-$.**

For factorial designs:

$\beta_i$ is the effect of moving $x_i$ from $-$ to $+$
**averaged over all other factors at all levels.**

# Factorial designs are nested

Factorial design

| $x_1$ | $x_2$ | $x_3$ | $x_4$ |
|---|---|---|---|
| − | − | − | − |
| + | − | − | − |
| − | + | − | − |
| + | + | − | − |
| − | − | + | − |
| + | − | + | − |
| − | + | + | − |
| + | + | + | − |
| − | − | − | + |
| + | − | − | + |
| − | + | − | + |
| + | + | − | + |
| − | − | + | + |
| + | − | + | + |
| − | + | + | + |
| + | + | + | + |

When $x_3 = -$

| $x_1$ | $x_2$ | $x_4$ |
|---|---|---|
| − | − | − |
| + | − | − |
| − | + | − |
| + | + | − |
| − | − | + |
| + | − | + |
| − | + | + |
| + | + | + |

When $x_3 = +$

| $x_1$ | $x_2$ | $x_4$ |
|---|---|---|
| − | − | − |
| + | − | − |
| − | + | − |
| + | + | − |
| − | − | + |
| + | − | + |
| − | + | + |
| + | + | + |