Nominal-the-Best Optimization

BIOE 498/598 PJ

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Location vs. Dispersion

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- **Location** describes the central tendency of a response
  - Mean, median, mode
  - All of our models so far use response = location

Location vs. Dispersion

- Sometimes we want to study the variation in the response, not the response itself.
- **Location** describes the central tendency of a response
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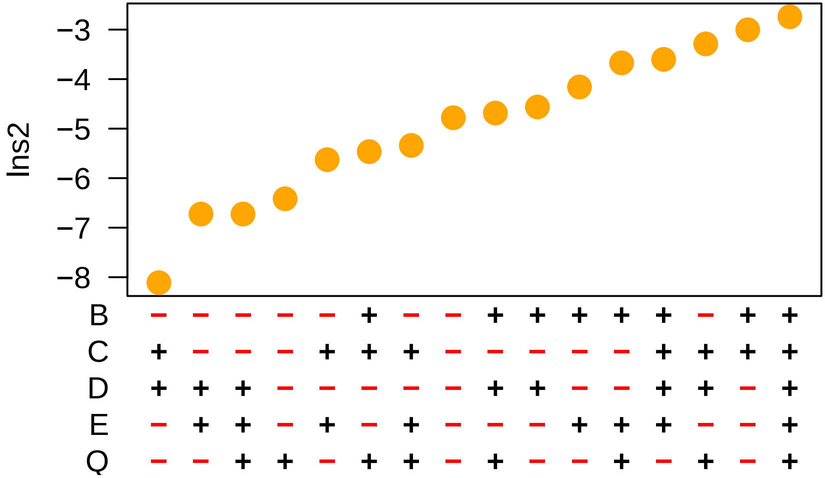
  - All of our models so far use response = location
- **Dispersion** describes the spread of a response
  - Range, inter-quartile range (IQR), variance, standard deviation
- Location can be studied with unreplicated or replicated designs
- Studying dispersion always requires replicates

Studying dispersion

- The variance σ^2 is the natural statistic for studying dispersion with linear models fit by least-squares
- However, the sample variance s^2 is not a good response for studying σ^2
  - s^2 is left-censored ($s^2 \geq 0$)
  - s^2 follows a χ^2 distribution, not a normal distribution
- Both problems are fixed by modeling $\ln s^2$ instead of s^2
- Moreover, maximizing $-\ln s^2$ minimizes the variance, so we can keep the same maximization-based framework used for location models

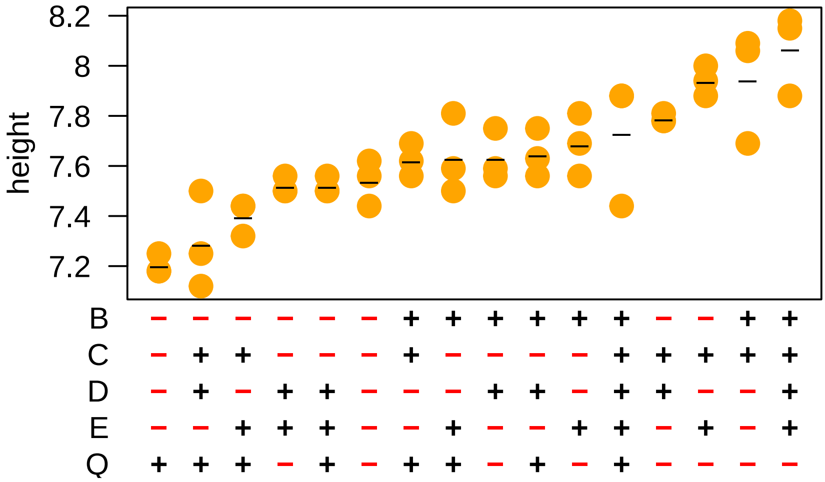
Visualizing the **dispersion**

```
farplot(disp, factors=c("B","C","D","E","Q"), response="lns2")
```



Visualizing the **data**

```
farplot(data, factors=c("B","C","D","E","Q"), response="height")
```



Building the model

```r
disp_model <- lm(-lns2 ~ B+C+D+E+Q +
                         B:Q + C:Q + D:Q + E:Q + B:C + B:D + B:E,
                 data=disp)
```

Confounding in the 2^{5-1} design with I=BCDE:

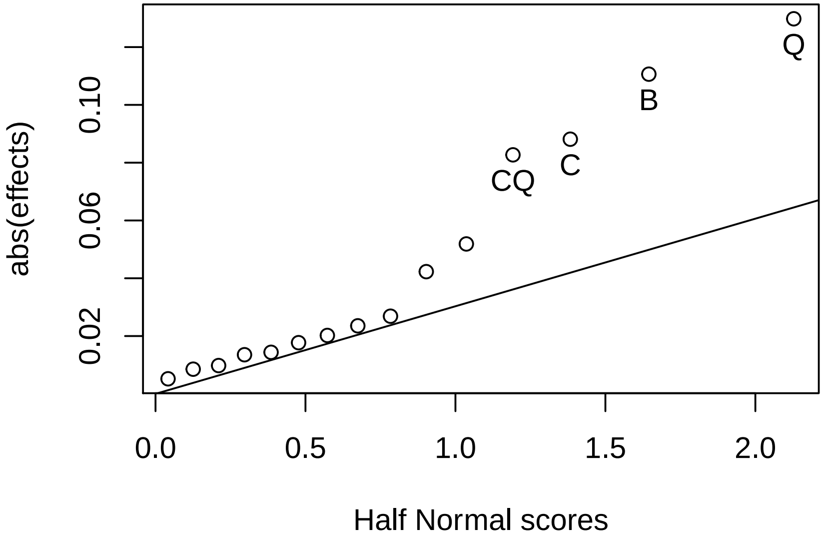
- main effects clear
- BQ
- CQ
- DQ
- EQ
- BC=DE
- BD=CE
- BE=CD

```r
show_effects(disp_model, ordered="abs")
```

```
## (Intercept)    4.93131
##           B    -.94543
##         D:Q    -.55538
##         B:E    -.33523
##         C:Q    -.2989
##         B:Q     .29437
##           C    -.28434
##         B:D    -.21234
##           Q    -.13976
##           D     .12375
##           E    -.10777
##         E:Q    -.06457
##         B:C     .00079
```

Factors affecting **location** (spring height)

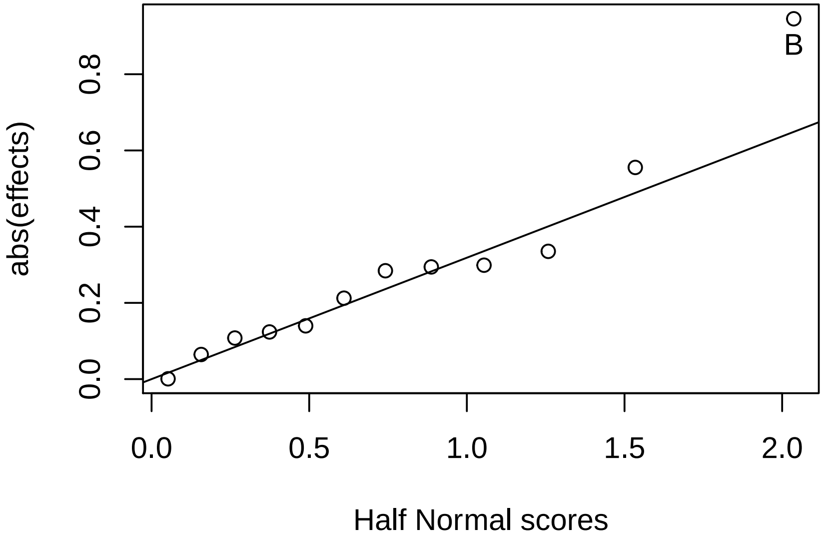
```
daewr::halfnorm(na.omit(get_effects(model)))
```




```
## zscore= 0.0417893 0.1256613 0.2104284 0.2967378 0.3853205 0.4770404
```

Factors affecting **dispersion** ($\ln s^2$)

```
daewr::halfnorm(get_effects(disp_model))
```




```
## zscore= 0.05224518 0.1573107 0.264147 0.3740954 0.4887764 0.6102946
```

Final Models

$$
\begin{aligned}
\text{height} &= 7.64 + 0.11B + 0.09C - 0.13Q - 0.08CQ \\
-\ln s^2 &= 4.93 - 0.95B
\end{aligned}
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- Often we have a *nominal value* for a response and need to balance reducing variance with keeping the response near the nominal value.

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- We want to minimize process variance (maximize $-\ln s^2$), but what about height?
- Often we have a *nominal value* for a response and need to balance reducing variance with keeping the response near the nominal value.

- In this case, we set $B = -$ and adjust the nominal value with C and Q.
- C and Q are called *adjustment factors* since the appear in the model for location but not dispersion.

Nominal-the-Best Optimization

1. Use the dispersion model to reduce variation.
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3. If the location is too far off, repeat but reduce the variation less than before.

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Why it works

Given a nominal value t for a response y, our goal (using quadratic loss) is

$$\begin{aligned}\min \mathbb{E}(y-t)^2 &= \mathbb{E}[(y-\mathbb{E}(y)) + (\mathbb{E}(y)-t)]^2 \\ &= \mathrm{Var}(y) + (\mathbb{E}(y)-t)^2\end{aligned}$$

Robust Parameter Design

- Factors can be split into two groups
  - **Control factors** can be changed easily
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- Factors can be split into two groups
  - **Control factors** can be changed easily
  - **Noise factors** are difficult or impossible to change
- Robust Parameter Design finds settings for control factors that mitigate variation from noise factors.
- Mitigation is achieved through *noise* $\times$ *control* interactions.

Example: Manufacturing boxed cake mixes

- Control factors: Ingredients in the box $A, \ldots, D$.
- Noise factors: Controlled by the customer
  - E: Egg (small or large)
  - M: Milk (skim or 2%)
  - T: Oven temperature (340°–360°F)

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Controlling for egg size variation (E) using baking soda (B)

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Controlling for egg size variation (E) using baking soda (B)

Recall the definition of $\mathrm{Int}(EB)$:

$$\mathrm{Int}(EB) = \frac{\mathrm{ME}(E|B+) - \mathrm{ME}(E|B-)}{2}$$

Mitigating noise trades optimality for robustness

Partial model for egg size E and baking soda B:

$$\text{taste} = \ldots + 0.2B - 0.7E + 0.4EB + \ldots$$

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When $B = +$

- $E+ \Rightarrow \text{taste} = -0.1$
- $E- \Rightarrow \text{taste} = 0.5$

When $B = -$

- $E+ \Rightarrow \text{taste} = -1.3$
- $E- \Rightarrow \text{taste} = 0.9$

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When $B = -$

- $E+ \Rightarrow \text{taste} = -1.3$
- $E- \Rightarrow \text{taste} = 0.9$

What should we do?

- The optimal cake has low baking soda and instructions to use a small egg (taste=0.9).
- If the customer uses a large egg, the taste drops a lot (-1.3).
- Using high baking soda gives a suboptimal taste (0.5 with small egg).
- Customers incorrectly using a large egg will not change taste as much (-0.1).

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- Typically, a Resolution V design is needed.
- But, the noise$\times$control interaction are most important, so a clever choice of generator can work with Resolution IV.

In the leaf spring experiment, the generator $E = BCD$ produced a 2^{5-1}_{IV} design with:

- main effects clear
- BQ
- CQ
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All the interactions with the noise factor Q (quench oil temperature) are clear.