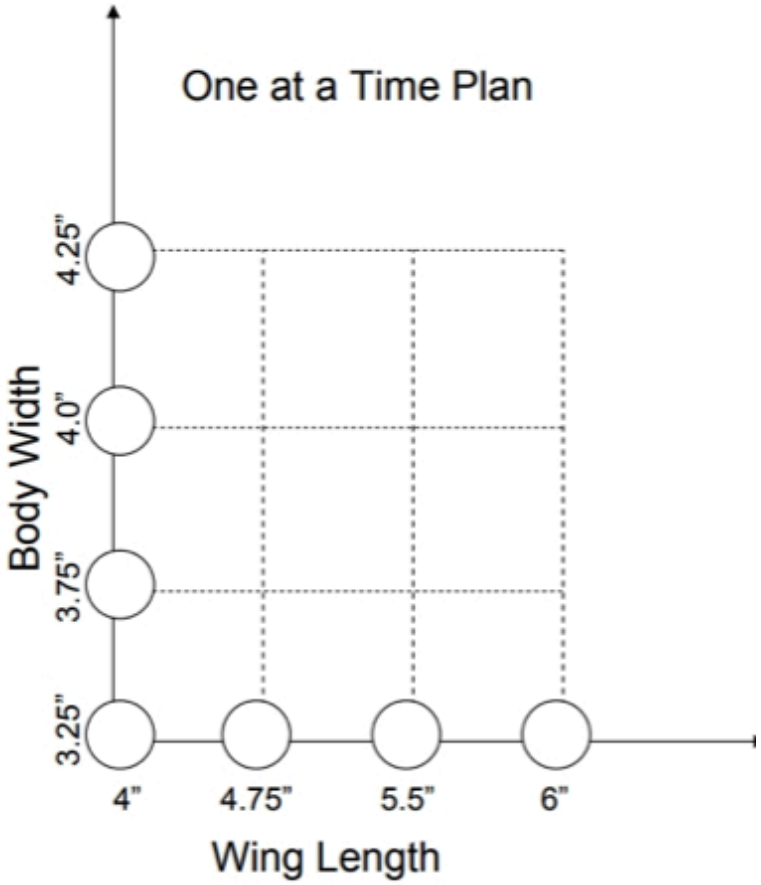
Factorial Designs

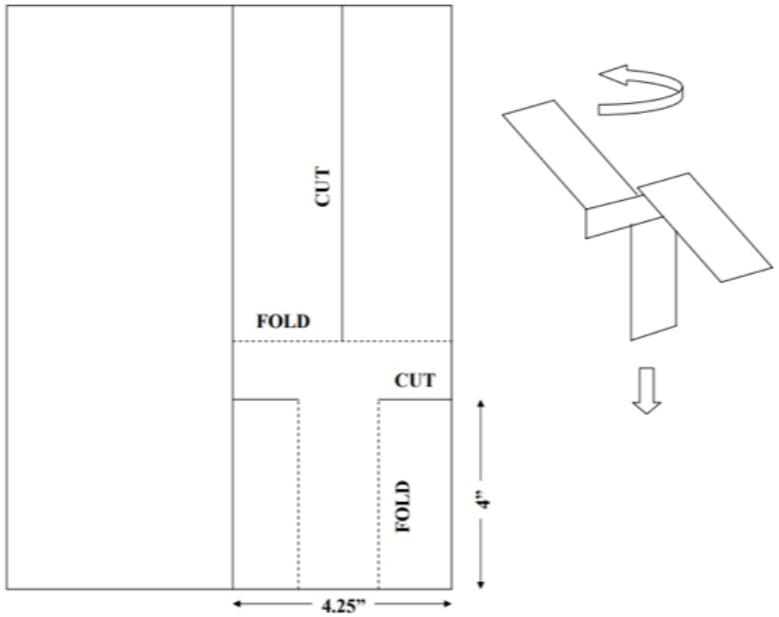
Why do we use factorial designs?

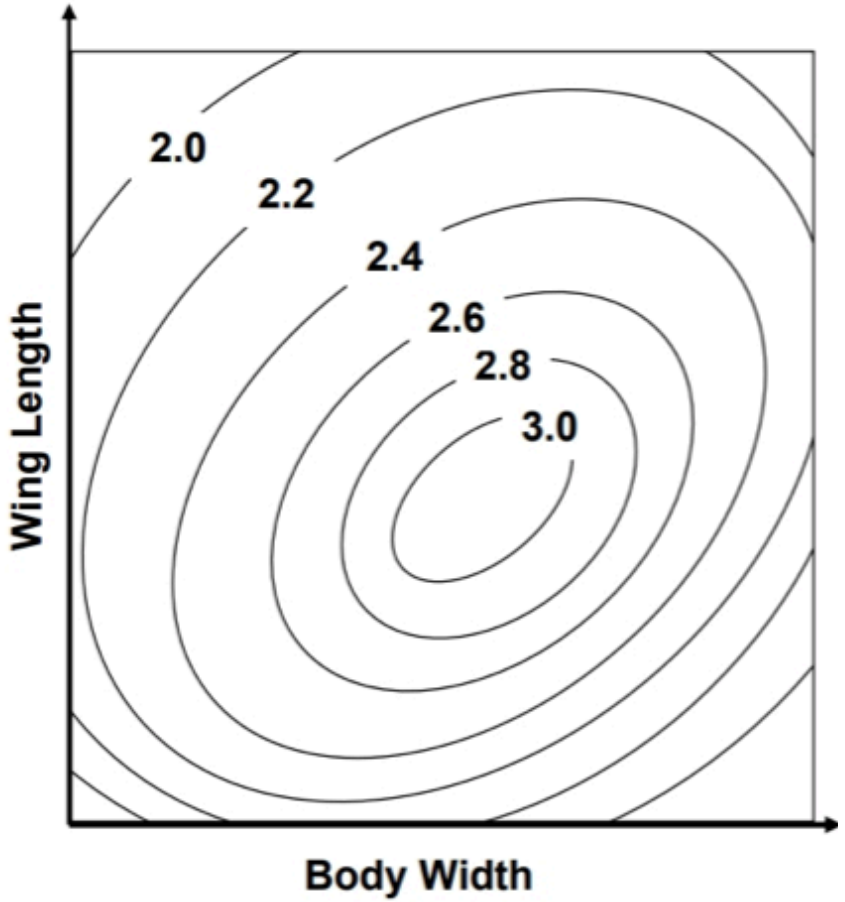
- Factorial designs find better optima.
- Factorial designs are more efficient.
- Factorial designs make better estimates of effect sizes.

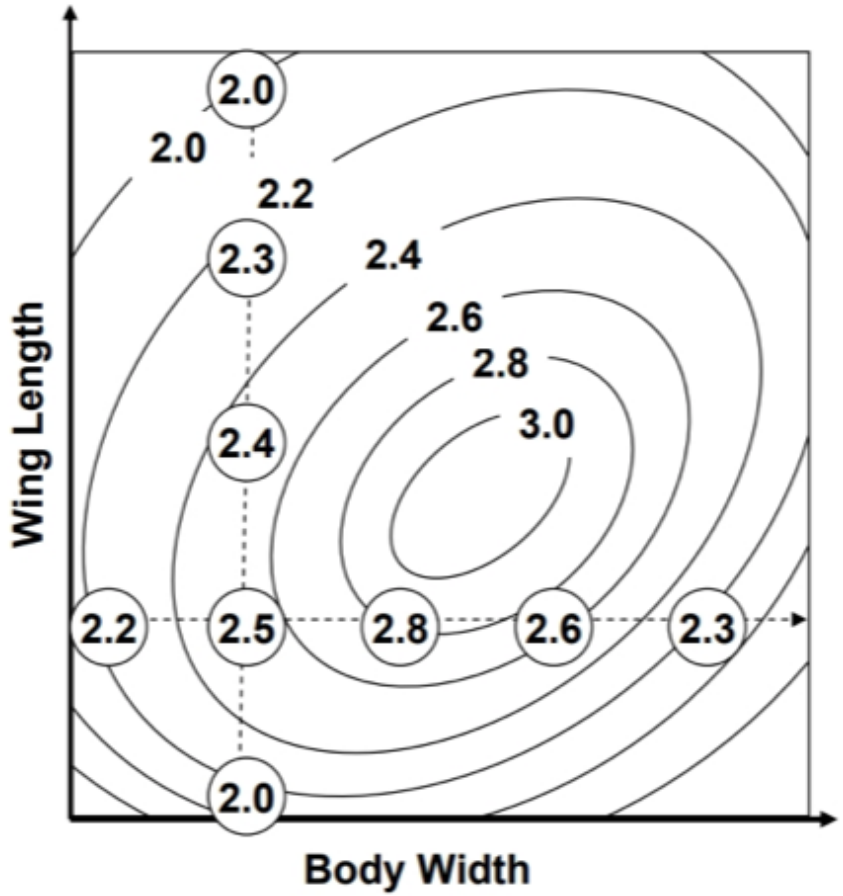
What is a factorial design?



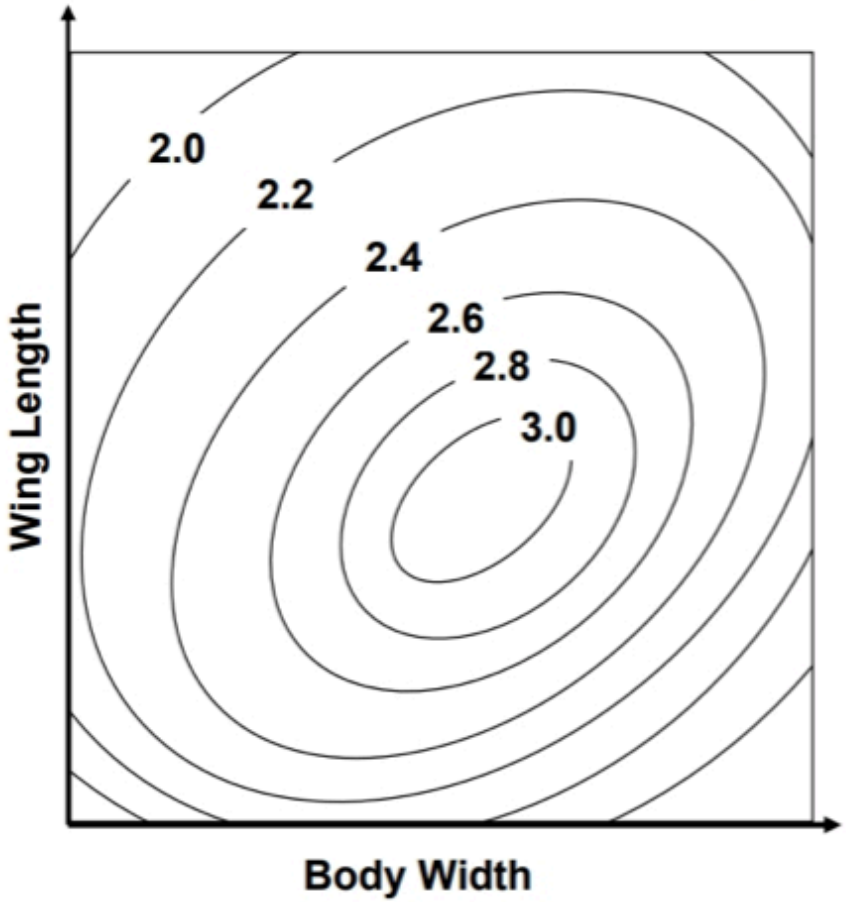
Factorial designs find better optima.

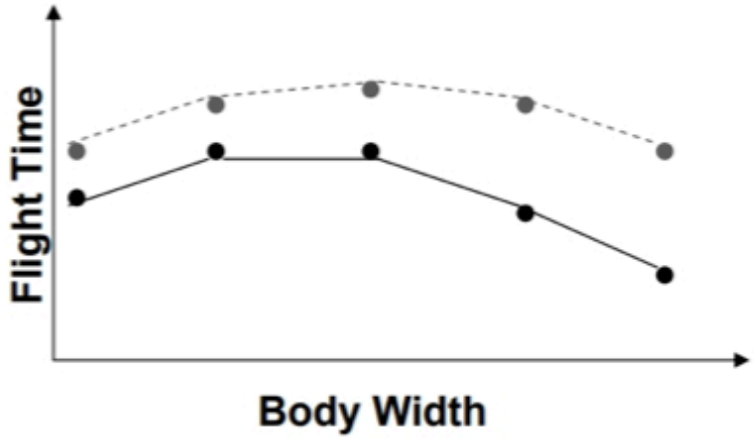






Interactions are the problem.



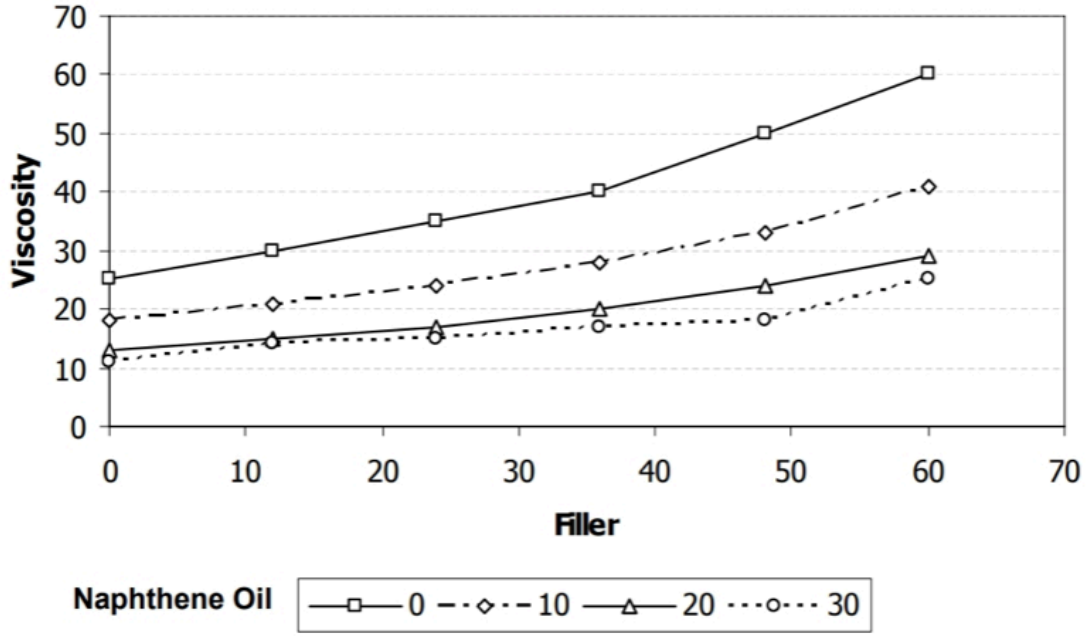


Interaction plots for diagnosis.

Table 3.1 *Mooney Viscosity of Silica B at 100° C*

Naphthene Oil (phr)	Filler (phr)					
	0	12	24	36	48	60
0	25	30	35	40	50	60
10	18	21	24	28	33	41
20	13	15	17	20	24	29
30	11	14	15	17	18	25

Figure 3.4 *Interaction Plot of Filler and Naphthene Oil*



Why do we use factorial designs?

- Factorial designs find better optima.
- **Factorial designs are more efficient.**
- Factorial designs make better estimates of effect sizes.

Factorial designs seem less efficient.

- Imagine an experiment with four variables, each with two levels (0, +). We want three replicates for each level.

- One-at-a-time Design
  3 runs at level 0
  4 variables x 3 runs at level +
  15 total experiments
- Factorial design
  2^4=16 levels x 3 runs
  48 total experiments?

One-at-a-time design

+	0	0	0
+	0	0	0
+	0	0	0
0	+	0	0
0	+	0	0
0	+	0	0
0	0	+	0
0	0	+	0
0	0	+	0
0	0	0	+
0	0	0	+
0	0	0	+
0	0	0	0
0	0	0	0
0	0	0	0

One-at-a-time design

+	0	0	0
+	0	0	0
+	0	0	0
0	+	0	0
0	+	0	0
0	+	0	0
0	0	+	0
0	0	+	0
0	0	+	0
0	0	0	+
0	0	0	+
0	0	0	+

Factorial Design

0	0	0	0
+	0	0	0
0	+	0	0
+	+	0	0
0	0	+	0
+	0	+	0
0	+	+	0
+	+	+	0
0	0	0	+
+	0	0	+
0	+	0	+
+	+	0	+
0	0	+	+
+	0	+	+
0	+	+	+
+	+	+	+

Factorial designs give more replicates per additional run.

- Imagine a design with p variables with k levels.
- After the initial design, adding another replicate requires

  pk runs for a one-at-a-time design

  k runs for a factorial design

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One-at-a-time design

+	0	0	0
+	0	0	0
+	0	0	0
0	+	0	0
0	+	0	0
0	+	0	0
0	0	+	0
0	0	+	0
0	0	+	0
0	0	0	+
0	0	0	+
0	0	0	+

Factorial Design

0	0	0	0
+	0	0	0
0	+	0	0
+	+	0	0
0	0	+	0
+	0	+	0
0	+	+	0
+	+	+	0
0	0	0	+
+	0	0	+
0	+	0	+
+	+	0	+
0	0	+	+
+	0	+	+
0	+	+	+
+	+	+	+

What do the effect sizes quantify?

- In one-at-a-time designs, β_i is the effect of moving x_i from 0 to + **while all other variables held at 0**.

- In factorial designs, β_i is the average effect of moving x_i from 0 to + **across the entire design space**.

Factorial Design

0	0	0	0
+	0	0	0
0	+	0	0
+	+	0	0
0	0	+	0
+	0	+	0
0	+	+	0
+	+	+	0
0	0	0	+
+	0	0	+
0	+	0	+
+	+	0	+
0	0	+	+
+	0	+	+
0	+	+	+
+	+	+	+

when $x_3 = 0$:

0	0	0
+	0	0
0	+	0
+	+	0
0	0	+
+	0	+
0	+	+
+	+	+

when $x_3 = +$:

0	0	0
+	0	0
0	+	0
+	+	0
0	0	+
+	0	+
0	+	+
+	+	+