Power Analysis

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When is an effect size significant?

Linear models compute estimates of the true value of the parameters β.

The uncertainty in our estimate is quantified by the *standard error*.

Let's say we estimate β using n samples from a population with standard deviation σ. The standard error of our estimate is

$$\text{s.e.} = \sigma/\sqrt{n}$$

The 95% confidence interval for a parameter is 1.96 standard errors* on each side of the estimate:

$$95\%\ \text{C.I. of}\ \beta = [\beta - 1.96\text{s.e.},\ \beta + 1.96\text{s.e.}]$$

* We often use 2 standard errors as a slightly conservative (and more convenient) estimate of the 95% confidence interval.

The 95% Confidence Interval

If a parameter estimate has a 95% C.I. that includes zero, we cannot be certain that the true value of the parameter is nonzero.

A parameter estimate is *significant* if and only if the 95% C.I. excludes zero.

The 95% C.I. depends on the number of samples (s.e. $= \sigma/\sqrt{n}$). We can narrow the 95% C.I. and improve our estimate of β by increasing n.

Example

Let's say we fit a model and found an estimate of $\beta = 3.1$ with s.e. $= 1.9$ using $n = 4$ samples. How many samples would we need before our estimate of β is significant?

We always assume that the population standard deviation (σ) is independent of n. In our example, $\sigma = \sqrt{n}\,\text{s.e.} = 3.8$. For our estimate to be significant, the lower end of the 95% C.I. must exclude zero, so

$$\begin{aligned} \beta - 1.96\sigma/\sqrt{n} &> 0 \\ n &> (1.96\sigma/\beta)^2 \\ n &> (1.96 \times 3.8/3.1)^2 \\ n &> 5.77 \end{aligned}$$

Two more samples ($n = 6$) would have been sufficient for our estimate of β to be significantly nonzero.

Power Analysis

The previous example makes two assumptions:

- The standard deviation (σ) will not change in subsequent experiments.
- The parameter estimate (β) will not change when new samples are added.

The first assumption is valid since σ is a property of the underlying population. Assuming our samples are drawn from the same population, they will have the same variation.

Our assumption about β is not valid. Remember that β is only an estimate of the true parameter value. If we re-sample the population we will get a new estimate. If the new estimate of β is any lower, the confidence interval in the previous example will again include zero.

Power Analysis (continued)

We need to be more conservative in our estimate of n to account for differences in the new estimates of β. Adding another 0.84 s.e. to our bound will ensure the 95% C.I. for β excludes zero **for 80% of the new estimates of** β. Our new estimate for the sample size is

$$\beta - (1.96\text{s.e.} + 0.84\text{s.e.}) > 0$$
$$\Rightarrow n > (2.80\sigma/\beta)^2$$

Even with this conservative estimate, there is still a 20% chance that our estimate of β will not be significant, although this level of uncertainty seems to be acceptable to most experimenters.