# Sequential Experiments: Introduction

BIOE 498/598

# Sequential Experimentation

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- Design experiments
- Collect data
- ► Fit and analyze a model
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The exception was RSM, which allowed two blocks of experiments and included an optimization that presumably led to additional experiments.

RSM is one example of strategies for increasing knowledge by **sequential experimentation**.

# Why sequential experimentation?

There is an inherent tradeoff between the breath and width of our experimental designs. This tradeoff is described by the *knowledge line*.

Figure 13.1 The State of Knowledge Line

| State of<br>Knowledge | 0% 100%                                |                                  |                                                                            |                                                                                    |                                                                                                                |
|-----------------------|----------------------------------------|----------------------------------|----------------------------------------------------------------------------|------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------|
| Stage                 | Preliminary<br>Exploration             | Screening<br>Factors             | Effect<br>Estimation                                                       | Optimization                                                                       | Mechanistic<br>Modeling                                                                                        |
| Purpose               | Determine<br>Sources of<br>Variability | Identify<br>Important<br>Factors | Estimate Main<br>Effects, Interactions<br>and Optimum<br>Conditions Tested | Fit Empirical<br>Models and<br>Interpolate to<br>Find Optimum in<br>Bounded Region | Estimate Parameters<br>of Theoretical Model<br>and Extrapolate<br>Predictions Outside<br>Region of Experiments |
| Useful<br>Designs     | RSE<br>NSE<br>SNSE                     | CRFF<br>PB<br>OA<br>SPFF         | CRFD<br>RCBF<br>CCBF<br>PCBF<br>CRSP<br>RBSP                               | CRRS D-optimal Designs BRS RSSP EESPRS                                             | D-optimal<br>Designs                                                                                           |

## Stage 1: Preliminary Exploration

- ▶ What factors could possibly matter for my system?
- ▶ Overlaps with needs assessment from design.
- We did not cover this stage.

# Stage 2: Screening Factors

- What factors actually matter?
- ► Resolution III fractional factorial designs.
- Estimate main effects only; interactions are usually confounded.

# Stage 3: Effect Estimation

- How much do the factors affect the response?
- ► Full factorial, resolution IV and V fractional designs.
- Often the last stage for science, the beginning stage for engineering.

# Stage 4: Optimization

- What are the optimal operating conditions?
- ► RSM
- ▶ All factors must be important since designs are expensive.

# Stage 5: Mechanistic modeling

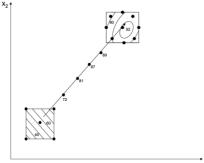
- ► A mechanistic model contains only important factors and allows effect estimation and optimization.
- Experiments are only needed to fit parameter values.
- Often the best at extrapolating beyond the region of experimentation.

# Early sequential experiments: Box & Wilson (1951)

- A full/fractional design identified the direction of steepest ascent for process improvement.
- Experiments were run one-at-a-time in this direction until improvement stopped.
- ► An RSM-like procedure was run at the endpoint to assess the curvature and declare a maximum.

Figure 13.2 The Method of Steepest Ascent





X<sub>1</sub>

#### RSM with a CCD allows sequential experimentation

Fractional Factorial + Central Factorial center points Composite State of Knowledge 0% 100% Screening Effect Estimation Optimization

Figure 13.3 Blocked Response Surface

### Online learning

Process improvement is often at odds with the goals of production.

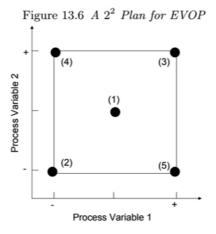
The goals of production are:

- Quality products
- High output
- Low costs
- Low variability

Our experimental designs require runs with multiple changes and large perturbations. This strategy cannot be implemented online.

### How do we balance learning with production?

In 1957 Box invented Evolutionary Operation (EVOP) to learn from an active production line.



# EVOP example: 2<sup>2</sup> full factorial

- Rotate the production through cycles of five runs:
  - $\triangleright$   $y_{00}$  is the current operating conditions
  - **▶** *y*<sub>−−</sub>, *y*<sub>++</sub>, *y*<sub>−+</sub>, *y*<sub>+−</sub>
- After every cycle, estimate the effects:
  - $\beta_1 = \mathsf{mean}(y_+) \mathsf{mean}(y_-)$
  - $\beta_2 = \mathsf{mean}(y_{\cdot,+}) \mathsf{mean}(y_{\cdot,-})$
  - $\beta_{12} = [\text{mean}(y_{++}) \text{mean}(y_{--})] [\text{mean}(y_{+-}) \text{mean}(y_{-+})]$
- ► Test for significant effects using s.e.  $= 2\sigma/\sqrt{4r}$  where r is the cycle number.

Since r increases over time, any truly nonzero effect will become significant over time *regardless of the effect size*. These data can be used to modify the process.

### Next steps: Reinforcement Learning

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#### Examples of RL systems:

- Computer chess
- Chatbots
- Self-driving cars
- Humans