

Lower Fractional Designs

BIOE 498/598 PJ

Spring 2022

Review

- ▶ A full factorial design with k factors requires 2^k runs.
- ▶ A half factorial design uses only 2^{k-1} runs.
 - ▶ Begin with a base design.
 - ▶ Set the remaining factor equal to an interaction (generator, $E = AB$)
 - ▶ Compute the defining relation ($I = \dots$) and confounding/alias structure.

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- ▶ Half-fractional factorials in action
- ▶ Today we define quarter- or eighth-factorial designs!

A large (2^5) unreplicated study

```
tumor <- read.csv("TumorInhibition.csv")  
head(tumor)
```

##		A	B	C	D	E	inhibition	run	half	pb
##	1	-1	-1	-1	-1	-1	61	1	0	1
##	2	1	-1	-1	-1	-1	53	2	1	0
##	3	-1	1	-1	-1	-1	63	3	1	1
##	4	1	1	-1	-1	-1	61	4	0	0
##	5	-1	-1	1	-1	-1	53	5	1	0
##	6	1	-1	1	-1	-1	56	6	0	1

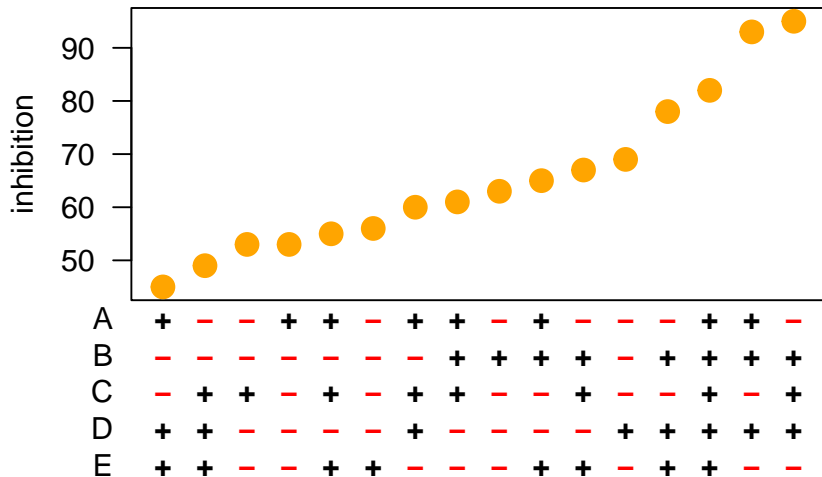
Let's pretend it was a 2^{5-1} half-fractional

```
half_tumor <- tumor[tumor$half==1, ]  
half_tumor
```

##		A	B	C	D	E	inhibition	run	half	pb
## 2	1	-1	-1	-1	-1		53	2	1	0
## 3	-1	1	-1	-1	-1		63	3	1	1
## 5	-1	-1	1	-1	-1		53	5	1	0
## 8	1	1	1	-1	-1		61	8	1	0
## 9	-1	-1	-1	1	-1		69	9	1	0
## 12	1	1	-1	1	-1		93	12	1	1
## 14	1	-1	1	1	-1		60	14	1	1
## 15	-1	1	1	1	-1		95	15	1	1
## 17	-1	-1	-1	-1	1		56	17	1	0
## 20	1	1	-1	-1	1		65	20	1	0
## 22	1	-1	1	-1	1		55	22	1	0
## 23	-1	1	1	-1	1		67	23	1	1
## 26	1	-1	-1	1	1		45	26	1	0
## 27	-1	1	-1	1	1		78	27	1	0
## 29	-1	-1	1	1	1		49	29	1	1
## 32	1	1	1	1	1		82	32	1	0

Step 2: Look at the data

```
farplot(half_tumor, response="inhibition", factors=c("A","B","C","D","E"))
```



Step 3: Fit a linear model

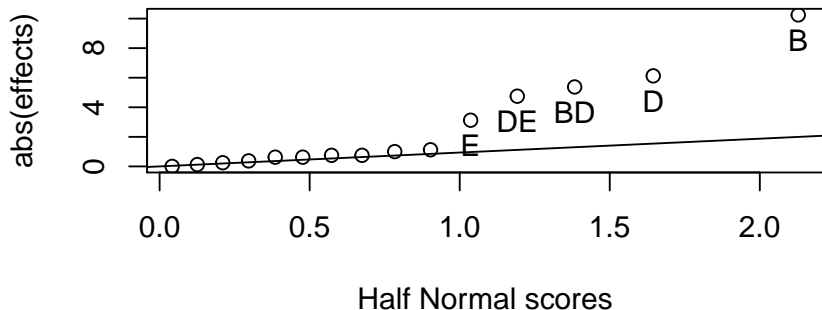
```
half_model <- lm(inhibition ~ A*B*C*D*E, data=half_tumor)
```

```
show_effects(half_model, scaling=2, show_effects(model, scaling=2, in18r
```

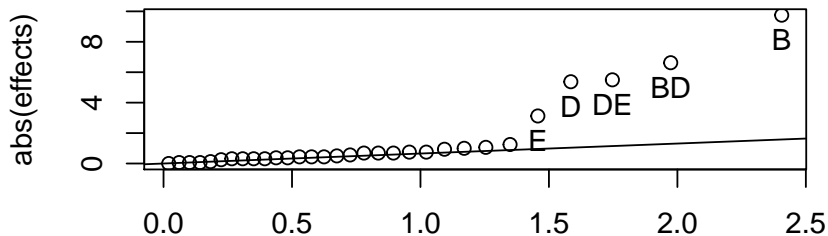
##	B	20.5	##	B	19.5
##	D	12.25	##	B:D	13.25
##	B:D	10.75	##	D:E	-11.
##	D:E	-9.5	##	D	10.75
##	E	-6.25	##	E	-6.25
##	C:E	2.25	##	A:C:E	-2.5
##	A	-2.	##	C:D	2.125
##	A:B	1.5	##	B:E	2.
##	B:C	1.5	##	A:B:E	-1.875
##	A:E	1.25	##	A:B:C	1.5
##	B:E	1.25	##	A:B:C:E	1.5
##	A:D	-.75	##	A:B:D	1.375
##	A:C	.5	##	A:B	1.375
##	C:D	.25	##	A	-1.375
##	C	.	##	B:C:D	1.125
##	A:B:C	NA.	##	A:C:D:E	1.
##	A:B:D	NA.	##	A:D	-.875
##	A:C:D	NA.	##	B:C	.875

Visualizing with half-normal plots

```
daewr::halfnorm(na.omit(get_effects(half_model)))
```



```
daewr::halfnorm(get_effects(model))
```



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Also, $I^2 = I = (ABD)(ACE) = BCDE$.

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$$A + BD + CE + ABCDE$$

$$B + AD + ABCE + CDE$$

$$C + ABCD + AE + BDE$$

$$D + AB + ACDE + BCE$$

$$E + ABDE + AC + BCD$$

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$$D + AB + ACDE + BCE$$

$$E + ABDE + AC + BCD$$

$$BC + ACD + ABE + DE$$

$$BE + ADE + ABC + CD$$

Eighth fractional design: 2^{6-3}

Factors $A, B, C, D = AB, E = AC, F = BC$

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Factors $A, B, C, D = AB, E = AC, F = BC$

$$I = ABD = ACE = BCF$$

Also, all combinations:

$$I^2 = I = (ABD)(ACE) = BCDE$$

$$I^2 = I = (ABD)(BCF) = ACDF$$

$$I^2 = I = (ACE)(BCF) = ABEF$$

$$I^3 = I = (ABD)(ACE)(BCF) = DEF$$

Defining relation:

$$I = ABD = ACE = BCF = BCDE = ACDF = ABEF = DEF$$

Which generator should I choose?

A generator's optimality is assessed with three criteria:

- ▶ **Resolution:** difference in the level of confounding.
- ▶ **Aberration:** the multiplicity of the worst confounding.
- ▶ **Clarity:** # of confounded main effects or two-way interactions.

Criterion #1: Design Resolution

The resolution of a fractional design is the length of the shortest word in the defining relation.

For the 2^{5-2} design generated by $D=AB$ and $E=AC$, the defining relation is

$$I = ABD = ACE = BCDE$$

This is a Resolution III design. (Resolution is written with Roman numerals.)

Resolution measures the degree of confounding

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- ▶ Main effects ($i = 1$) are confounded with secondary ($3 - 1 = 2$) interactions.

Resolution IV

- ▶ Main effects ($i = 1$) are confounded with tertiary ($4 - 1 = 3$) interactions.
- ▶ TWIs ($i = 2$) are confounded with other TWIs ($4 - 2 = 2$).

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A design with resolution R contains a full factorial design for any subset of $k = R - 1$ factors.

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If after the fractional experiments you drop to k factors you can re-analyze the data for all the interactions.

Criterion #2: Design Aberration

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- ▶ **Aberration:** Number of words with length equal to the resolution.

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$I = ABCDF = ABCEG = DEFG$ resolution IV, aberration 1

$I = ABCF = ADEG = BCDEFG$ resolution IV, aberration 2

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- ▶ **Resolution:** Length of the shortest word in the defining relation.
- ▶ **Aberration:** Number of words with length equal to the resolution.

$$I = ABCDF = ABCEG = DEFG \quad \text{resolution IV, aberration 1}$$

$$I = ABCF = ADEG = BCDEFG \quad \text{resolution IV, aberration 2}$$

We favor the design with the lower aberration. It will have fewer main effects confounded with low-order interactions.

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A main effect or two-way interaction effect is **clear** if it is only confounded with higher order terms (three-way or higher).

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Clear effects always lead to tradeoffs. For a 2^{6-2} design:

$$I = ABCE = ABDF = CDEF$$

6 main effects clear

$$I = ABE = ACDF = BCDEF$$

3 main effects + 6 TWIs clear

Overall design guidelines

1. Choose the highest **resolution** that fits your budget.
2. For that resolution, choose the **minimum aberration** design.
3. If you have particular effects that you know are significant, try to choose a factor or generator that clears them.