Factorial Designs

BIOE 498/598 PJ

Spring 2021

What is a factorial design?

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A factorial design includes runs with every combination of factor at every level.

Sample factorial designs

2^2 Factorial design

x_1	x_2
$-$	$-$
$+$	$-$
$-$	$+$
$+$	$+$

2^3 Factorial design

x_1	x_2	x_3
$-$	$-$	$-$
$+$	$-$	$-$
$-$	$+$	$-$
$+$	$+$	$-$
$-$	$-$	$+$
$+$	$-$	$+$
$-$	$+$	$+$
$+$	$+$	$+$

3^2 Factorial design

x_1	x_2
$-$	$-$
0	$-$
$+$	$-$
$-$	0
0	0
$+$	0
$-$	$+$
0	$+$
$+$	$+$

Why do we use factorial designs?

- Factorial designs find better optima.
- Factorial designs are more efficient.
- Factorial designs make better estimates of effect sizes.

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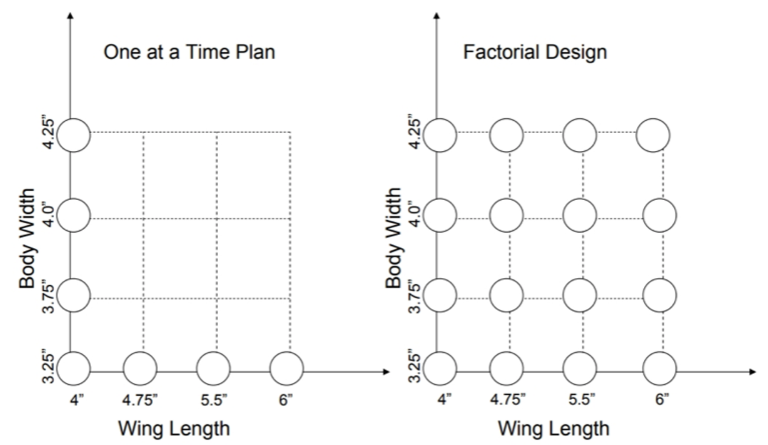
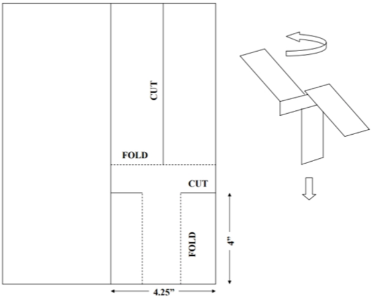
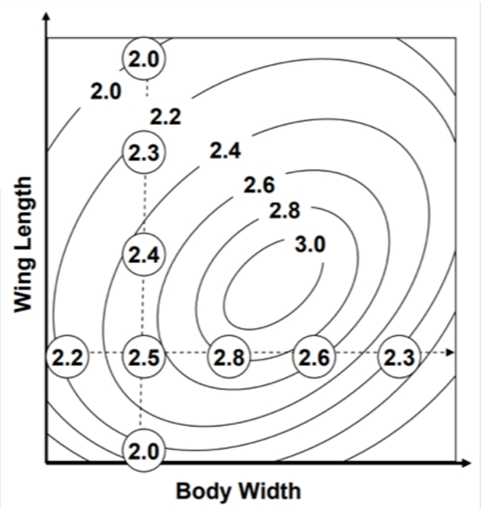
What is a factorial design?

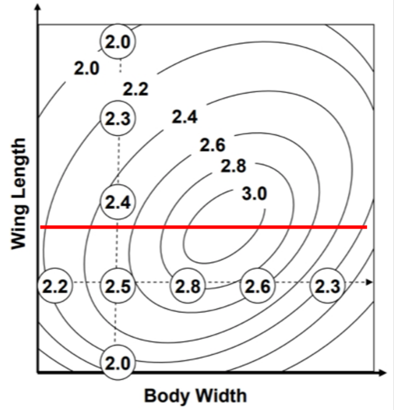


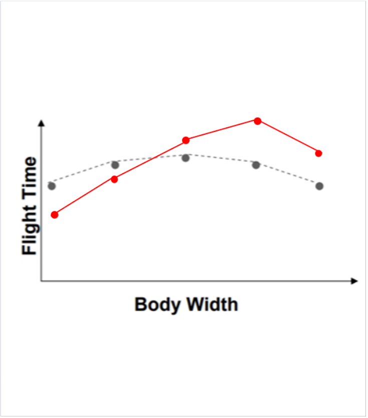
Factorial designs find better optima





The problem with OFAT: Interactions



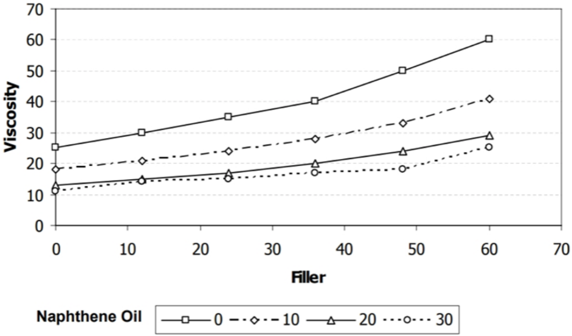


Using interaction plots for diagnosis

Table 3.1 *Mooney Viscosity of Silica B at 100° C*

Naphthene Oil (phr)	Filler (phr)					
	0	12	24	36	48	60
0	25	30	35	40	50	60
10	18	21	24	28	33	41
20	13	15	17	20	24	29
30	11	14	15	17	18	25

Figure 3.4 *Interaction Plot of Filler and Naphthene Oil*



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Factorial designs seem *less* efficient...

Imagine an experiment with four factors, each with two levels $(-, +)$. We want three replicates for each level.

One Factor at a Time Design

- 3 runs at level $(-)$
- 4 factors $\times$ 3 runs at $(+) = 12$ runs
- **15 total runs**

Factorial Design

- $2^4 =$ **16 total runs**

...until you look at the designs

OFAT design

x_1	x_2	x_3	x_4
−	−	−	−
−	−	−	−
−	−	−	−
+	−	−	−
+	−	−	−
+	−	−	−
−	+	−	−
−	+	−	−
−	+	−	−
−	−	+	−
−	−	+	−
−	−	+	−
−	−	−	+
−	−	−	+
−	−	−	+

Factorial design

x_1	x_2	x_3	x_4
−	−	−	−
+	−	−	−
−	+	−	−
+	+	−	−
−	−	+	−
+	−	+	−
−	+	+	−
+	+	+	−
−	−	−	+
+	−	−	+
−	+	−	+
+	+	−	+
−	−	+	+
+	−	+	+
−	+	+	+
+	+	+	+

Factorial designs give more replicates per run

A factorial design in n variables has 2^n runs, but 2^{n-1} replicates at each level $(-, +)$.

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A factorial design in n variables has 2^n runs, but 2^{n-1} replicates at each level $(-, +)$.

Imagine a design with n variables at k levels.
After the initial design, adding another replicate requires

- nk runs for a OFAT design
- $\sim k$ runs for a factorial design

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What are the other factors doing when x_3 is high?

OFAT design

x_1	x_2	x_3	x_4
−	−	−	−
−	−	−	−
−	−	−	−
+	−	−	−
+	−	−	−
+	−	−	−
−	+	−	−
−	+	−	−
−	+	−	−
−	−	+	−
−	−	+	−
−	−	+	−
−	−	−	+
−	−	−	+
−	−	−	+

Factorial design

x_1	x_2	x_3	x_4
−	−	−	−
+	−	−	−
−	+	−	−
+	+	−	−
−	−	+	−
+	−	+	−
−	+	+	−
+	+	+	−
−	−	−	+
+	−	−	+
−	+	−	+
+	+	−	+
−	−	+	+
+	−	+	+
−	+	+	+
+	+	+	+

What do the effect sizes estimate?

For OFAT designs:

β_i is the effect of moving x_i from $-$ to $+$
while all other factors stay at $-$.

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For OFAT designs:

β_i is the effect of moving x_i from $-$ to $+$
while all other factors stay at $-$.

For factorial designs:

β_i is the effect of moving x_i from $-$ to $+$
averaged over all other factors at all levels.

Factorial designs are nested

Factorial design

x_1	x_2	x_3	x_4
$-$	$-$	$-$	$-$
$+$	$-$	$-$	$-$
$-$	$+$	$-$	$-$
$+$	$+$	$-$	$-$
$-$	$-$	$+$	$-$
$+$	$-$	$+$	$-$
$-$	$+$	$+$	$-$
$+$	$+$	$+$	$-$
$-$	$-$	$-$	$+$
$+$	$-$	$-$	$+$
$-$	$+$	$-$	$+$
$+$	$+$	$-$	$+$
$-$	$-$	$+$	$+$
$+$	$-$	$+$	$+$
$-$	$+$	$+$	$+$
$+$	$+$	$+$	$+$

When $x_3 = -$

x_1	x_2	x_4
$-$	$-$	$-$
$+$	$-$	$-$
$-$	$+$	$-$
$+$	$+$	$-$
$-$	$-$	$+$
$+$	$-$	$+$
$-$	$+$	$+$
$+$	$+$	$+$

When $x_3 = +$

x_1	x_2	x_4
$-$	$-$	$-$
$+$	$-$	$-$
$-$	$+$	$-$
$+$	$+$	$-$
$-$	$-$	$+$
$+$	$-$	$+$
$-$	$+$	$+$
$+$	$+$	$+$