Reinforcement Learning: Discounting, TD-learning, and Q-factors

BIOE 498/598 PJ

Spring 2021

Review

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- A single pass with a random base policy provides good, but not necessarily optimal, behavior.
- Iteration and exploration are required to find optimal policies.

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- A single pass with a random base policy provides good, but not necessarily optimal, behavior.
- Iteration and exploration are required to find optimal policies.
- **Today:** Model-free learning with discounted rewards and Q-factors.

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The *discount factor* $\gamma \in [0, 1]$ determines the length of the horizon.

- $\gamma = 0$ makes the algorithms greedy; only the immediate reward r_i influences the agent.
- $\gamma = 1$ equally weights all rewards to the end of the trajectory.

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Penalized stepping with $r_i = -1$, $r_T = 0$:

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\text{reward} &= r_0 + r_1 + \cdots + r_{T-1} + r_T \\
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Discounting with $r_i = 0$, $r_T = 1$, $\gamma < 1$:

$$\begin{aligned}\text{reward} &= r_0 + \gamma(r_1 + \gamma(r_2 + \gamma(\cdots\gamma(r_{T-1} + \gamma(r_T))))) \\ &= r_0 + \gamma r_1 + \gamma^2 r_2 + \cdots \gamma^{T-1} r_{T-1} + \gamma^T r_T \\ &= \gamma^T\end{aligned}$$

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In both cases, the maximum reward is achieved by minimizing the number of steps T.

When to discount?

Almost all algorithms are written with a discount factor

$$\max_a \mathbb{E}\left\{r_i + \gamma V(s_{i+1})\right\}.$$

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Discounting is also the key to solving non-episodic (infinite horizon) problems. While the MDP never terminates, the discounted rewards become so small that the agent stops caring after a finite number of steps.

Model-free learning

- Monte Carlo methods like rollout require a *model* to simulate ahead when estimating value functions.
- *Model-free* algorithms learn directly from experience. Their only method of sampling is to interact with the environment.
- Model-free algorithms try to maximize the information that can be extracted from every trajectory.

Temporal difference learning

- Model-free algorithms learn directly from experience.
- Each trajectory is "expensive" relative to a simulated trajectory.
- Ideally, we would update our estimates of the value function from every trajectory; however, a single trajectory is a noisy estimate of value.
- **Temporal difference (TD) learning** balances new experiences with previous results when updating $V(s)$.

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3. For each state s_i in the trajectory, calculate the *TD target*

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TD-learning is a *bootstrap* method since $V(s)$ is updated using $V(s_i)$ and $V(s_{i+1})$ from the previous iteration. New information only enters through r_i when estimating the TD target $\hat{V}(s_i)$.

Q-factors

Learning $V(s)$ is not the end. We still need to find a policy that solves

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This requires knowing s_{i+1} given s_i and a, or at least the probability distribution for ending up in each state.

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For many problems it is easier to learn the value of each state/action pair, called a Q-factor or $Q(s,a)$.

Learning Q-factors

Using Q-factors, the policy problem at state s

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We can learn Q-factors using a TD approach given a trajectory s_0, a_0, r_0, s_1, a_1, r_1 …, s_T, r_T:

$$\hat{Q}(s_i, a_i) = r_i + \gamma Q(s_{i+1}, a_{i+1}) \qquad \text{target}$$

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This approach is also called *SARSA*.

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- **Next time:** Tic-Tac-Go!