For a physical system of n *distinguishable* particles,

$$X=Y(n,3)\,.$$

For a physical system of n *indistinguishable* particles,

$$X=Y(n,3)/S_n\,.$$

The quotient space X is the set of equivalence classes of points in $Y(n,3)$ under permutations belonging to the symmetric group S_n, and is given the identification topology under the natural projection $p\colon Y(n,3)\rightarrow X$. We observe that S_n acts effectively on $Y(n,3)$; that is to say, given any point $y\in Y$ and any element $\alpha\in S_n$ except the identity, then $\alpha(y)\neq y$ (this is true because we have excluded points of coincidence). Because $Y(n,3)$ is simply connected and S_n act effectively, $(Y(n,3),p)$ is a universal covering space for X and the fundamental group of X is isomorphic to S_n.[8,9] There are only two scalar unitary representations of the symmetric group.

$D^1(\alpha)=+1$ for all α,

$D^2(\alpha)=\pm 1$ according as α is an even or odd permutation,

$$K^{\text{Bose}}=\sum_\alpha D^1(\alpha)K^\alpha\,,$$

$$K^{\text{Fermi}}=\sum_\alpha D^2(\alpha)K^\alpha\,.$$

[8] Peter Hilton, *Algebraic Topology—An Introductory Course* (Courant Institute of Mathematical Sciences, New York University, New York, 1969), p. 67.

[9] Edwin H. Spanier, *Algebraic Topology* (McGraw-Hill, New York, 1966), pp. 87–89.

ACKNOWLEDGMENTS

We are much indebted to Raoul Bott, who patiently helped us make sense out of intuitively formulated and incomplete ideas, and to Yvonne Choquet-Bruhat, who did not give up the physical meaning when the mathematics could not provide any guidance. We also wish to thank R. P. Walker for helpful conversations on homotopy theory. Credit is due to the Battelle Rencontres for the interactions from which this work has evolved.

PHYSICAL REVIEW D VOLUME 3, NUMBER 6 15 MARCH 1971

The Photon as a Composite State of a Neutrino-Antineutrino Pair

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(Received 22 September 1970)

The possibility that the photon may be a composite state of a neutrino-antineutrino pair has been examined from field-theoretic considerations on the basis of the photon-neutrino weak-coupling theory. It is shown that the difficulties which generally crop up in quantum electrodynamics to describe a photon as a composite state of an electron-positron pair using the $Z_3=0$ condition are avoided in neutrino dynamics (photon-neutrino weak interaction). In view of this, we conclude that the photon may be taken as a composite state of a neutrino-antineutrino pair when the composite character is described by the vanishing of the wave-function renormalization constant and the nonvanishing of a certain composite coupling constant.

I. INTRODUCTION

THE possibility that the photon may be a composite state of a neutrino-antineutrino pair was first suggested by de Broglie.[1] However, this simple picture of the photon was found not to obey Bose statistics because of the underlying Fermi statistics of its components. In fact, if two such photons were in the state with momentum **p**, two-component neutrinos (and antineutrinos) of these photons would be in the state with the same momentum **k**. In view of this, Jordan[2] suggested a model of the photon, composed of two neutrinos each being a superposition of states with different momenta. This assumption, with the proper choice of the superposition coefficients, provided the correct statistics for the theory. After this, Kronig[3] succeeded in constructing the photon field out of that of neutrinos. However, Pryce[4] has shown that the Kronig theory is not invariant under the group of spatial rotations about the direction of the photon momentum as an axis.

In recent times, several authors revived the discussions on the neutrino theory of photons. Barbour, Bietty, and Touschek[5] argued that a photon in neutrino theory is always longitudinally polarized. Ferretti[6] suggested that the photon be considered as a limiting state of a bound system of two nonzero mass particles with a given angular momentum when the binding energy as well as the mass tends to zero. Perkins[7] considered the usual four-component solutions with

[1] L. de Broglie, Compt. Rend. **195**, 862 (1932); **199**, 813 (1934).

[2] P. Jordan, Z. Physik **93**, 464 (1935).

[3] R. Kronig, Physica **3**, 1120 (1936).

[4] M. H. L. Pryce, Proc. Roy. Soc. (London) **165**, 247 (1938).

[5] I. M. Barbour, A. Bietti, and B. F. Touschek, Nuovo Cimento **28**, 453 (1963).

[6] B. Ferretti, Nuovo Cimento **33**, 265 (1964).

[7] W. A. Perkins, Phys. Rev. **137**, B1291 (1965).

definite momentum and helicity of the Dirac equation with a zero-mass term. He constructed in some special way the electromagnetic field tensor $F_{\mu\nu}$ of the two neutrino operators in the configuration space. In this formalism, the photon appears as composed of the pair $(\nu_1\bar{\nu}_2)$ or $(\nu_2\bar{\nu}_1)$, where ν_1 (ν_2) denotes the neutrino with spin parallel (antiparallel) to its momentum. However, the impossibility of constructing linearly polarized photons seems to be a very serious defect of this theory. In fact, judging from all these attempts, we can say that the construction of the photon as a composite state of a neutrino-antineutrino pair has remained a problem until now.

In a recent paper, Bandyopadhyay[8] suggested that photons can interact weakly with neutrinos. In view of this, it is possible to study the composite character of photons from field-theoretic considerations. Here we study this problem, and show that the photon can indeed be taken as a composite state of a neutrino-antineutrino pair.

II. PHOTON AS COMPOSITE PARTICLE IN QUANTUM FIELD THEORY

There have been attempts in recent years to describe composite particles by a local Lagrangian quantum field theory. The general feature of composite particles has been developed by different authors[9] on the vanishing of the wave-function renormalization constant and the nonvanishing of a certain composite coupling constant. Broido[10] has argued that a composite particle in a local Lagrangian quantum field theory is obtained by taking an elementary particle and letting its wave-function renormalization constant tend to zero,

$$Z_3=0\,, \tag{1}$$

in such a way that

$$\lim_{Z_3\to 0}\frac{g'Z_1}{Z_3\delta m^2} \tag{2}$$

is finite and nonzero, where g' is the renormalized coupling constant and δm^2 is the mass shift describing the composite.

It is to be emphasized that in quantum electrodynamics within the framework of the perturbation theory (QEDP), Z_3^{-1} diverges to all orders.[11] The divergence of Z_3^{-1} is an ultraviolet divergence which will have to be removed by cutoffs. Thus the $Z_3\to 0$ transition process is accomplished by the transition $\Lambda^2\to\infty$, where Λ is the cutoff factor. Thus in QEDP the divergence of Z_3^{-1} has nothing to do with the zero mass of the photon and would persist also for small nonzero values of the renormalized photon mass. Hence zero is not an isolated point of the spectrum of p_μ^2 for the photon field, and if this is regarded as a composite, there is no particle interpretion for it. Pointing out these drawbacks, Broido[10] has argued that in QEDP, the photon cannot be considered as a composite state of electron-positron pairs. Also, from an analysis outside the framework of the perturbation theory, Broido reached the same conclusion.

Considering that a photon can interact weakly with neutrinos, we here attempt to describe the photon as a composite state of a neutrino-antineutrino pair, and see whether the above drawbacks, as observed by Broido, are removed in the case of neutrino dynamics (photon-neutrino weak interaction). It is to be remarked here that photons can interact weakly only with massless two-component neutrinos[8] having an interaction Lagrangian of the form

$$\begin{aligned} L_I&=ig\bar{\varphi}\gamma_\mu\phi A_\mu\\ &=ig\bar{\psi}\gamma_\mu(1+\gamma_5)\psi A_\mu\,. \end{aligned} \tag{3}$$

Here ϕ is the two-component neutrino wave function and ψ is the four-component function defined as

$$\phi=\tfrac{1}{2}(1+\gamma_5)\psi\,, \tag{4}$$

where g is the photon-neutrino weak-coupling constant. Evidently, the theory is renormalizable. Analyzing in a similar way as in QEDP, we can use the following spectral representation to calculate Z_3^{-1} for neutrino dynamics:

$$Z_3^{-1}=\int_0^\infty d\sigma^2\rho_R(\sigma^2)\,, \tag{5}$$

where

$$\rho_R(\sigma^2)=\delta(\sigma^2)+(g^2/12\pi^2)\theta(\sigma^2)(1/\sigma^2) \tag{6}$$

with

$$\begin{aligned} \theta(\sigma^2)&=1\,,\quad \sigma^2>0\\ &=0\,,\quad \sigma^2<0\,. \end{aligned}$$

Here g is the renormalized photon-neutrino weak-coupling constant.

It is noted that we come across here the same divergence difficulties as in QEDP and this has to be removed by cutoffs. That is, apart from the dependence of Z_3 on the mass and coupling constant, it is also dependent on the cutoff factor, and we can write in a generalized form

$$Z_3^{-1}=1+\int_0^{\Lambda^2} f(M^2,g^2,m_c)dM^2/M^2\,, \tag{7}$$

where m_c is the renormalized photon mass.

However, it is to be observed that for two-component neutrinos, the renormalized mass as well as the bare mass is zero. The vanishing of the physical and bare mass of neutrinos is ensured by the fact that the total Lagrangian for the photon-neutrino interaction, taking

[8] P. Bandyopadhyay, Phys. Rev. **173**, 1481 (1968); Nuovo Cimento **55A**, 367 (1968).

[9] B. Jouvet, Nuovo Cimento **5**, 1 (1957); M. M. Broido and J. G. Taylor, Phys. Rev. **147**, 993 (1966).

[10] M. M. Broido, Phys. Rev. **157**, 1444 (1967).

[11] M. Gell-Mann and F. E. Low, Phys. Rev. **95**, 1300 (1954).

into account the interaction Lagrangian (3), is invariant under the transformation

$$\psi \rightarrow e^{i\alpha\gamma_5}\psi \,, \tag{8}$$

which shows that the interaction term does not contribute to any mass effect.[12] Considering that the physical mass of a neutrino is zero, we note that the renormalized photon mass m_c must be *identically* zero. This follows from the fact that if a photon composed of a pair consisting of a massless neutrino and antineutrino attains a certain mass due to interaction, it will be unstable and should decay spontaneously into a neutrino-antineutrino pair. However, this is in contradiction to the fact that a real photon having a rest mass, however small, cannot interact weakly with neutrinos.[8] Indeed, this is because if a weak interaction of massive photons is allowed, we cannot forbid *a priori* gauge-noninvariant interactions such as $ig\bar{\psi}_e\gamma_\mu(1+\gamma_5) \times\psi_e A_\mu$ depicting the weak interaction of photons with electrons. But this leads to a contradiction, for in that case, photons in an external field should create electron-positron pairs both electromagnetically and weakly, which is absurd. So the weak interaction of a photon behaving as a particle of nonzero rest mass must be forbidden. Thus zero is essentially an isolated point of the spectrum of $p_\mu{}^2$ for the photon field. This shows that in neutrino dynamics, the vanishing of Z_3 can arise from its functional dependence on the external parameters. Thus, the main difficulty which crops up in using the $Z_3=0$ condition in QEDP, is removed in neutrino dynamics.

In view of the above conclusions, we have to consider condition (2) seriously. To this end, we here write explicitly the renormalized constants:

$$\begin{aligned}
&\phi = Z_2^{-1/2}\phi^{(u)} \,, \\
&j_\mu = Z_1\bar{\phi}_\nu\gamma_\mu\phi_\nu \\
&\quad = Z_1Z_2^{-1}\bar{\phi}_\nu{}^{(u)}\gamma_\mu\phi_\nu{}^{(u)} \,, \\
&A_\mu = Z_3^{-1/2}A_\mu{}^{(u)} \,, \quad g = Z_1^{-1}Z_2Z_3^{1/2}g^{(u)} \,.
\end{aligned} \tag{9}$$

Here ϕ is the two-component neutrino wave function. The superscript (u) stands for "unrenormalized."

It is noted that the two-component neutrino current $\bar{\phi}_\nu\gamma_\mu\phi_\nu$ is a conserved quantity and the constant of motion here is the "lepton number." We may remark here that a two-component spinor is equivalent to a four-component Majorana spinor[13] and, in view of this, the coupling constant g cannot behave as "electric charge."[8] Because of the conservation of the two-component neutrino current, the Ward-Takahasi identity holds in neutrino dynamics too, and as a result, we have $Z_1=Z_2$. Maris *et al.*[14] have studied QED, taking the vanishing bare spinor mass and $Z_3=0$. They have shown that in this special case $Z_1=Z_2=0$ in all physical gauges. Evidently, this result is valid in our present case too and thus in neutrino dynamics we get the interesting result that all the renormalization constants become zero.

To find out the value of $Z_3\delta m^2$, we here consider the analysis of Kang and Land.[15] In an analysis to show the equivalence between the composite particle in S-matrix theory and the elementary particle in field theory, they have pointed out that for the condition $Z_1=0$, the mass renormalization is always finite and thus the conditions $Z_3=0$ and $Z_1=0$ necessarily ensure $Z_3\delta m^2=0$.[16] However, to show that condition (2) is satisfied, we should analyze the asymptotic behavior of the different field-theoretic quantities.

Following Kang and Land,[15] we assume the Lehmann representation for the propagator with no subtractions:

$$\Delta'(s) = \frac{1}{m^2-s} + \alpha + \frac{1}{\pi}\int_{s_t}^{\infty} ds' \frac{\rho(s')\,|\Lambda(s')|^2}{s'-s} \,, \tag{10}$$

or, if the integral diverges, with one subtraction,

$$\Delta'(s) = \frac{1}{m^2-s} + \alpha + \frac{s-m^2}{\pi}\int_{s_t}^{\infty} ds' \frac{\rho(s')\,|\Lambda(s')|^2}{(s'-m^2)(s'-s)} \,. \tag{11}$$

In these expressions, α is a constant and s_t is the threshold energy. The quantity $\Lambda(s)$ is related to the form factor $F(s)$ by

$$F(s) = (m^2-s)\Lambda(s) \,. \tag{12}$$

The proper vertex function $\Gamma(s)$ is related to the propagator by

$$\Lambda(s) = \Gamma(s)\Delta'(s) \,. \tag{13}$$

For both (10) and (11), the inverse propagator has the solution

$$\Delta^{-1}(s) = (m^2-s)\left[1 + \frac{s-m^2}{\pi}\int_{s_t}^{\infty} \frac{ds'\rho(s')\,|\Gamma(s')|^2}{(s'-m^2)^2(s'-s)} + \sum_i \frac{R_i(s-m^2)}{(s_i-s)(s_i-m^2)^2}\right] , \tag{14}$$

where R_i's denote the residual terms represented in a series.

For both (10) and (11), we have[15]

$$\int_{s_t}^{\infty} \frac{\rho(s)\,|\Gamma(s)|^2}{s^2} ds < \infty \,. \tag{15}$$

Moreover, for Eq. (11), we have the further restriction

$$\int_{s_t}^{\infty} \frac{\rho(s)\,|\Gamma(s)|^2}{s} ds < \infty \,. \tag{16}$$

[12] B. Touschek, Nuovo Cimento **5**, 1281 (1957).

[13] C. Ryan and S. Okubo, Nuovo Cimento Suppl. **2**, 234 (1964).

[14] Th. A. J. Maris, D. Dillenburg, and S. Jacob, Nucl. Phys. **B18**, 366 (1970).

[15] K. Kang and D. J. Land, Nuovo Cimento **63A**, 1053 (1969).

[16] The fact that the condition $Z_1=Z_2=0$ may be essential for the finiteness of the vacuum polarization has also been emphasized by Dillenburg and Maris. See D. Dillenburg and Th. A. J. Maris, Nucl. Phys. **B18**, 390 (1970).

Defining $Z(s)$ by

$$Z^{-1}(s)=(m^2-s)\Delta'(s)\,, \tag{17}$$

we get, by taking the limit $s\to\infty$, the wave-function renormalization constant

$$Z_3=1-\frac{1}{\pi}\int_{s_t}^{\infty}ds\frac{\rho(s)|\Gamma(s)|^2}{(s-m^2)^2}-\sum_i\frac{R_i}{(s_i-m^2)^2}. \tag{18}$$

Also, we have

$$Z(s)-Z_3=\frac{1}{\pi}\int_{s_t}^{\infty}ds'\frac{\rho(s')|\Gamma(s')|^2}{(s'-m^2)(s'-s)}+\sum_i\frac{R_i}{(s_i-m^2)(s_i-s)}. \tag{19}$$

This exists for both (10) and (11). In particular, for Eq. (11), the expression

$$\lim_{s\to\infty}s[Z(s)-Z_3]=-\frac{1}{\pi}\int_{s_t}^{\infty}ds\frac{\rho(s)|\Gamma(s)|^2}{s-m^2}-\sum_i\frac{R_i}{s_i-m^2} \tag{20}$$

also exists. For $Z_3=0$, we obtain from Eq. (18) the sum rule

$$1=\frac{1}{\pi}\int_{s_t}^{\infty}ds\frac{\rho(s)|\Gamma(s)|^2}{(s-m^2)^2}+\sum_i\frac{R_i}{(s_i-m^2)^2}. \tag{21}$$

Also from Eq. (20), we have

$$-\lim_{s\to\infty}sZ(s)=\frac{1}{\pi}\int_{s_t}^{\infty}ds\frac{\rho(s)|\Gamma(s)|^2}{s-m^2}+\sum_i\frac{R_i}{s_i-m^2}. \tag{22}$$

The mass renormalization $\delta m^2=m^2-m_0{}^2$ is given by

$$\delta m^2=\frac{1}{Z_3}\lim_{s\to\infty}(s-m^2)[Z(s)-Z_3] \tag{23}$$

and thus

$$-Z_3\delta m^2=\frac{1}{\pi}\int_{s_t}^{\infty}ds\frac{\rho(s)|\Gamma(s)|^2}{s-m^2}+\sum_i\frac{R_i}{s_i-m^2}. \tag{24}$$

It may be mentioned that $Z_3\delta m^2$ always exists for the subtracted propagator of Eq. (11), while it may not for the unsubtracted propagator.

From Eqs. (22) and (24), we note that for $Z_3=0$,

$$Z_3\delta m^2=\lim_{s\to\infty}sZ(s)\,. \tag{25}$$

Again, for Z_1, we have the conventional relation

$$Z_1=\lim_{s\to\infty}\Gamma(s)\,. \tag{26}$$

Now, for $Z_3=0$ and $Z_1=0$, the asymptotic behaviors for the field-theoretic quantities in the approximation of elastic unitarity are as follows[15]:

$$\begin{array}{cc|c} \Gamma(s) & \Delta'(s) & F(s) \\ \hline 0 & \infty & \\ 1/\ln s & \ln s & s \end{array} \tag{27}$$

From the asymptotic relations (25)–(27), we see that the quantity $\lim_{Z_3\to 0}(gZ_1/Z_3\delta m^2)$ is nonzero and finite. Thus condition (2) is also satisfied in neutrino dynamics.

III. DISCUSSION

We have shown above that within the framework of perturbation theory in a quantum field-theoretic formalism, the photon appears as a composite particle when the basic spinors are neutrinos, and the photon is taken to interact weakly with the neutrinos. Also, one may study the compositeness criterion outside perturbation theory. However, there are serious difficulties in this procedure, as has been pointed out by Broido.[10]

We may remark here that from a phenomenological point of view, the compositeness of strongly interacting particles is generally considered on the basis of bootstrap and self-consistency notions. However, these techniques cannot be used in electrodynamics to discuss the compositeness of photons. For we do not yet have any satisfactory S-matrix formalism for electrodynamics. Moreover, the calculational techniques used for bootstraps are not suitable to deal with processes involving the exchange of more than one particle, and this is unsatisfactory in electrodynamics because of the difficulties associated with large numbers of low-energy photons leading to infrared divergences.[17] Evidently, these arguments are valid for neutrino dynamics too, as considered here. In view of these considerations, the composite character of photons described by the vanishing of the wave-function renormalization constant and the nonvanishing of a certain composite coupling constant seems to be worthwhile.

ACKNOWLEDGMENT

The authors are thankful to S. Dey for helpful discussions.

[17] G. Källén, in *Handbuch der Physik*, edited by S. Flügge (Springer, Berlin, 1958), Vol. 5, Chap. 1.