

THE ACTIVE TRANSPORT OF WATER UNDER TEMPERATURE GRADIENTS

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I. INTRODUCTION

In presenting this review on a particular aspect of active transport of water I want first of all to discuss the thermodynamics of transport processes in a general way to try to bring out more clearly the distinction between active and passive mechanisms, and to show just where the phenomenon discussed in this paper fits into the general picture. The procedure I want to adopt follows that of the relatively new Thermodynamics of Irreversible Processes which is destined, no doubt, to play an increasingly important part in biological theory (see, for instance, de Groot, 1951).

The meaning of 'active transport'

In a general way the meaning of 'active transport' is fairly clear. Whenever movement of matter occurs in a direction which we cannot explain as helping towards the attainment of equilibrium we recognize a case of active transport. Very often it is an extremely easy matter to decide whether we have such a case or not, but sometimes it is not at all simple, and our usual definitions land us in anomalies or fail us altogether. Consider one or two practical examples. Electro-osmosis can be either positive or negative, i.e. it can promote the movement of water in a direction either helping, or hindering, the normal *osmotic* flow. Now in both cases there is reason to believe that the mechanism is essentially similar; consequently if negative electro-osmosis is regarded as active (as it must be), so must positive electro-osmosis. In other words, an active movement can take place *down* a gradient of chemical potential or activity, as well as *up* one. Thus the very common definition of 'active transport' as being transport against a gradient of activity fails us—it expresses only half of the truth. Then consider the case to be discussed in this paper, where the temperature is not uniform. Here no special significance attaches to activity gradients at all; in fact, the difference in activity is only unambiguously known when the temperature is uniform. This makes the usual criterion of active movement really meaningless—a most unsatisfactory position. A knowledge of the real distinction between active and passive movements is obviously desirable,

and I think it can be found by following the general approach introduced, I believe, by the Norwegian physicist Onsager in 1931.

Consider a very simple type of system (Fig. 1) divided into two equal halves by some sort of membrane, and containing water and a number of other substances. Imagine it at first to be in absolute equilibrium, so that the temperature, pressure and the concentrations of all its constituents are uniform everywhere. Further, let it be imagined to be entirely isolated, so that neither matter nor energy can enter or leave it.

Suppose we proceed now to disturb its equilibrium in a number of different ways. First, each one of the constituents can be redistributed between the two halves so that instead of being equal in amount on both sides it is now present in unequal amounts. Thus if glucose is a constituent there may be a mass m_g^I on one side of the membrane and a mass m_g^{II} on the

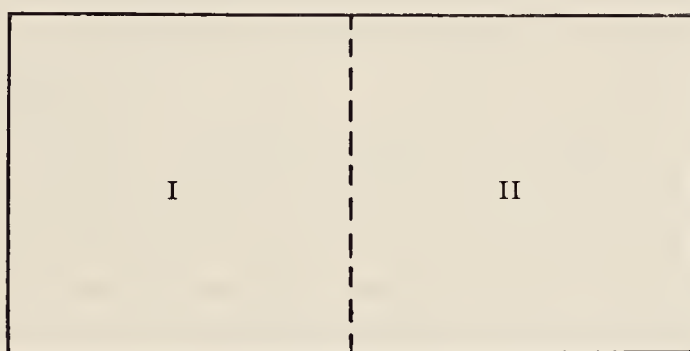


Fig. 1.

other, and the extent to which we have upset the equilibrium can be specified by the difference $m_g^{II} - m_g^I = \Delta m_g$, which of course was originally zero. For each chemical substance present we shall have one degree of freedom in disturbing the equilibrium, and when we have finished, in imagination, displacing matter from one side to the other the state of our system can be specified by the variables $\Delta m_1, \Delta m_2, \Delta m_3, \dots, \Delta m_k$, of which there will be one for each distinct sort of substance present. We can go further than this, however, and imagine that heat energy is made to flow from one side to the other, setting up a temperature difference ΔT between them. As a matter of fact, there will be many other ways still of disturbing the equilibrium (such as producing an electrical potential difference between the two sides, or promoting chemical changes), but for the purpose of illustration the matter need be taken no further.

We now have a system not in equilibrium whose condition is completely specified by a number of variables $\Delta m_1, \Delta m_2, \dots, \Delta m_k$ and ΔT , and these, moreover, are all independent. In general, of course, our manipulations will have produced a pressure difference ΔP between the sides as well, but

it will not be necessary to specify it, for it is already implied. In other words, it is not a further *independent* variable. We can if we so desire include it in the specification, but then not all our variables will be independent; one of them, say Δm_k , can be left out.

‘Fluxes’ and ‘forces’

The foregoing example illustrates the point that when a system is not in equilibrium its momentary condition can be specified by means of a certain minimum number of variables $\alpha_1, \alpha_2, \dots, \alpha_n$, all of which are independent. There is a certain latitude in choosing these, but their number is always the same, however they are chosen.

Consider now how the system returns to equilibrium. The one thing that can be said about it from the thermodynamic point of view is that its entropy continuously increases; it manifests a continuous tendency to increase its entropy, and when it can do so no more it comes to rest. It would seem reasonable therefore to regard the derivative $\partial S/\partial \alpha$ as measuring the tendency of α to change, i.e. as a measure of the ‘force’ changing it. Further, the time-derivative $d\alpha/dt$ can obviously be spoken of as the ‘flux’ of α . To make this clearer, however, let us refer again to the previous example. If $\alpha = \Delta m_g$ measures the extent to which the glucose distribution is ‘out of balance’, then $\partial S/\partial \alpha$ can be regarded as the force promoting transport of glucose and $d\alpha/dt$ as the rate of glucose transport.

Now the rate at which the entropy of the whole system changes can be written

$$\frac{dS}{dt} = \frac{\partial S}{\partial \alpha_1} \frac{d\alpha_1}{dt} + \frac{\partial S}{\partial \alpha_2} \frac{d\alpha_2}{dt} + \frac{\partial S}{\partial \alpha_3} \frac{d\alpha_3}{dt} + \dots + \frac{\partial S}{\partial \alpha_n} \frac{d\alpha_n}{dt}, \quad (1)$$

by the basic property of partial differential coefficients, the partial derivative $\partial S/\partial \alpha_1$, for instance, being taken with all the other variables $\alpha_2, \alpha_3, \dots, \alpha_n$ constant. The quantity dS/dt can thus be split up into the sum of a number of products of ‘forces’ such as $X_1 = \partial S/\partial \alpha_1$ and ‘fluxes’ such as $J_1 = d\alpha_1/dt$; furthermore, it is bound by the second law of thermodynamics to be positive. Calling $dS/dt = \sigma$, equation (1) can therefore be written in the more concise form

$$\sigma = X_1 J_1 + X_2 J_2 + X_3 J_3 + \dots + X_n J_n, \quad (2)$$

a relation which is rather analogous to the electrical one

$$\text{watts} = \text{volts} \times \text{amperes}. \quad (3)$$

Equation (2) expresses how rapidly the system is increasing its entropy in terms of the instantaneous values of the forces and fluxes. However, these quantities are not independent; they can be related together in a way analogous to that in which the volts and amperes of equation (3) are related by Ohm’s law:

$$\text{amperes} = \text{volts} \times \text{conductance coefficient}. \quad (4)$$

The 'Ohm's law' relations have, however, to be much more general, for it is a common observation that the movement of any one kind of matter promotes to *some* extent movement of all the other kinds present. Thus the flux J_1 will in general depend not only on the force X_1 but also on all the other forces X_2 , X_3 and so on. Thus in general we must write

$$J_1 = L_{11}X_1 + L_{12}X_2 + L_{13}X_3 + \dots + L_{1n}X_n, \quad (5)$$

and similarly for the other fluxes, the coefficients L_{11} , L_{12} , ... being 'conductance' coefficients whose magnitude will depend on the size and geometry of the system and on the nature of the mechanism of transport across the dividing membrane. By means of equations such as (5) (which are purely descriptive like Ohm's law), we can express the contribution of, say, the flux J_1 to the general process of increase of entropy. Calling this contribution σ_1 we have from (2) and (5)

$$\sigma_1 = X_1J_1 = L_{11}X_1^2 + L_{12}X_1X_2 + L_{13}X_1X_3 + \dots + L_{1n}X_1X_n, \quad (6)$$

which is analogous to

$$\text{watts} = (\text{conductance of circuit}) \times \text{volts}^2. \quad (7)$$

Now the interesting thing about equation (6) is that the only term on the right which is *necessarily* positive is the first one; the others may clearly be either positive or negative. In other words, that part of the flux $J_1 = d\alpha_1/dt$ which can be attributed to the *conjugate* force $X_1 = \partial S/\partial\alpha_1$ necessarily contributes to an increase in the entropy of the system; that part which can be attributed to the *non-conjugate* forces, $\partial S/\partial\alpha_2$, $\partial S/\partial\alpha_3$ and so on, may contribute towards either an increase or a decrease in the entropy. This distinction would therefore seem to be just what is wanted to mark the division between passive and active movements; the passive component is due to the conjugate force, the active to the non-conjugate.

As a matter of fact this division is not entirely unambiguous, on account of the fact mentioned earlier that the variables α_1 , α_2 , α_3 , ..., α_n can be chosen in a variety of ways. However, the ambiguity can be overcome if the α 's are so chosen that, in the simple example given, the fluxes $d\alpha/dt$ represent the rates of transfer of the individual chemical substances present together with the rate of flow of heat. Suppose this is done in a system containing water and one other chemical substance, say glucose. Then the *passive* movement of water on the present analysis proves to be associated with the force $\left\{ v_w \Delta P + \left(\frac{\partial \mu_w}{\partial c_g} \right)_{T,P} \Delta c_g \right\}$, where v_w is the partial volume of water, ΔP the pressure difference, μ_w the chemical potential of water and c_g the glucose concentration per unit mass of mixture. Needless to say it represents no departure to attribute the nature of passive agents to the pressure and

concentration gradients,† though if v_w should happen to be negative instead of positive the pressure gradient will operate in the reverse direction. The *active* movement of the water likewise proves to be due to the forces ΔT , the temperature differential, and $\left\{v_g \Delta P + \left(\frac{\partial \mu_g}{\partial c_g}\right)_{T,P} \Delta c_g\right\}$, where v_g is the partial volume of glucose and c_g is again the glucose concentration per unit mass. This emphasizes that any movement of water caused by a gradient in the chemical potential of another substance is necessarily an active movement; and that a pressure gradient has an active effect as well as a passive one, the active effect being dependent on $v_g \Delta P$. Further, and this will be the subject of the rest of this paper, any movement of water brought about by a temperature difference must be regarded as an active one, the criterion being, as mentioned earlier, whether it contributes *necessarily* or not to an increase in the entropy of the system.

II. THE IMPORTANCE OF TEMPERATURE

It is usually considered that as far as makes no difference living cells are isothermal systems. This is probably because the process of thermal conduction over minute distances is so extremely rapid that it is difficult to conceive of two points close together—say on either side of the plasma membrane—possessing temperatures measurably different. It seems to me that this view is in some respects inadequate, and elsewhere (Spanner, 1953) I have put forward the conception that where metabolism is active any given point in the cell possesses not *one* temperature, but many; in fact, one to each different species of molecule present. What makes this view at least a possible one is the fact that though thermal equilibration over molecular distances may be an extremely rapid process, yet there are other processes, such as chemical reaction and diffusion, which are comparable in rapidity. Thus these processes may be able to ‘take advantage’ of microscopic temperature fluctuations before thermal conduction has evened them out.

An important fact in connexion with the active transport of water under temperature gradients lies in the quite unexpectedly great effect of small temperature differences. Consider a very simple model. Two open Petri dishes I and II (Fig. 2) contain an aqueous solution of osmotic pressure π . Initially both are at the same temperature, and we can suppose them enclosed in a larger sealed vessel. If now dish I be raised in temperature from T to $T + \Delta T$, the vapour pressure above it will be increased and water will evaporate from it and condense in dish II, which will become more dilute

† These two gradients can be included together as a gradient of chemical potential.

in consequence. This process will go on till the dilute and cooler solution in dish II has the same vapour pressure as the stronger and warmer solution in dish I, when the system will come in a sense to a standstill. There will then be a difference in osmotic pressure $\Delta\pi$ between the two solutions, and this difference can broadly be said to balance the temperature difference ΔT . The relation between $\Delta\pi$ and ΔT can easily be shown to be (Spanner, 1952)

$$\frac{\Delta\pi}{\Delta T} = \frac{L}{VT}, \quad (8)$$

where L is the latent heat of vaporization and V is the molar volume of water, i.e. about 18.0 c.c. A simple calculation from this shows that a temperature difference of only $\frac{1}{100}^\circ \text{C}$. can cause a pressure difference of over four-fifths of an atmosphere.

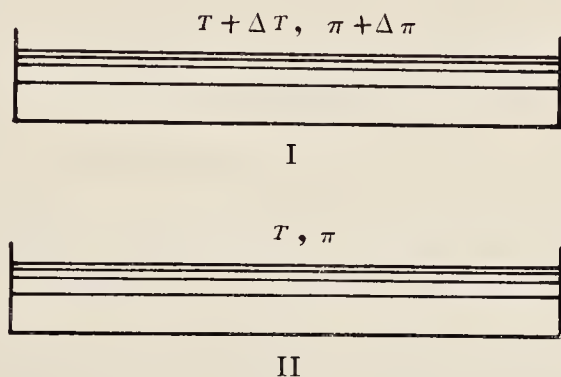


Fig. 2.

A general thermodynamic relation

The simple example just given is a particular case of a general relation governing transport under a temperature gradient. The system we considered consisted of two homogeneous parts I and II separated by a divisional wall represented by the air space. In general this type of system can be represented by such a diagram as Fig. 3. It is called a discontinuous system because its properties change abruptly on crossing the membrane from one side to the other, though within each section all properties are uniform.

Suppose that in such a system (containing for simplicity, water only) a temperature difference ΔT is imposed between the two sides. In general this will cause a movement of water from one side to the other, and provided the walls are solid this movement will build up a pressure difference ΔP which will ultimately bring the flow to a standstill. When this occurs the system is said to be in a 'steady state', and the relation between ΔP and ΔT will be given by the perfectly general equation

$$\frac{\Delta P}{\Delta T} = - \frac{Q^*}{VT}, \quad (9)$$

where V is the volume of unit mass of water and Q^* is a quantity called the 'heat of transfer'. In a sense therefore equation (9) gives the equivalence of temperature and pressure differences in promoting flow across the membrane; if the right-hand side is large then a very small temperature difference can cause the same rate of flow as a much larger pressure difference. The key to the situation obviously lies in the interpretation of the quantity Q^* , which must now be discussed.

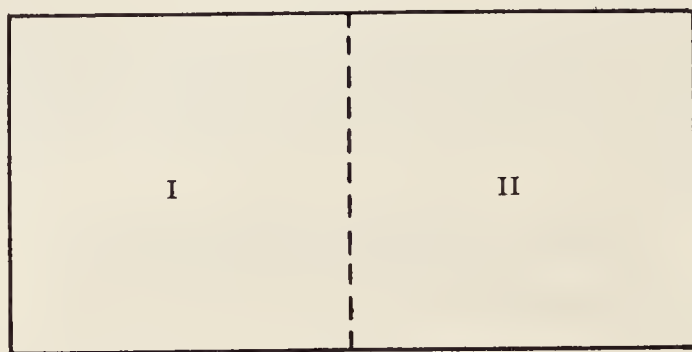


Fig. 3.

III. THE 'HEAT OF TRANSFER' Q^*

Imagine that the whole system represented by Fig. 3 is at a uniform temperature, but that owing to a small pressure difference (ideally an infinitesimal one) water is flowing across the membrane. In general, the process by which the water traverses the membrane will act differentially on the faster and slower molecules so that the water which crosses is not usually a representative sample but contains either a larger, or a smaller, proportion of 'hot' or 'cold' molecules than the bulk. This is very obviously the case in the example previously given, where the membrane is an air space and the process of crossing it involves, first, evaporation and, secondly, condensation. Thus, as the water moves across, it carries a quantity of heat with it, and while compartment I is left cooler, compartment II grows warmer—again a result which is very evident in the evaporation example. Suppose now that as the water flows across the membrane heat is continuously abstracted from compartment II to maintain its temperature constant. Then the quantity of heat so abstracted per unit mass of water flowing is the 'heat of transfer', Q^* . In the earlier example (Fig. 2) it is obviously very nearly the same as the latent heat of evaporation, showing that equation (8) is a particular case of equation (9).

Conditions for large 'heat of transfer'

It can be seen at once that the condition which must be fulfilled if there is to be a definite heat of transfer is that the membrane should act in such a way as to distinguish between 'hot' and 'cold' water molecules. In general, there are two ways in which this can come about, and these two incidentally correspond to the two classical theories of cell permeability. In the first, the membrane may act as a sieve. If the pores of this are coarse, no selective effect is apparent; water flowing down a main does not tend to get hotter as it goes. On the other hand, if the pores are really minute, in fact comparable to the intermolecular distance of water, then the faster molecules penetrate in relatively greater numbers than the slower ones, and a 'heat of transfer' appears. This is particularly easy to show with gases and a fine-grained porous pot. The magnitude of Q^* , however, is small, and even in the most favourable cases it never numerically exceeds $\frac{1}{2}RT$ per mole.

In the second mechanism, the membrane acts as a potential energy barrier. This means that it does not merely act less favourably towards low-energy molecules (as the sieve does); it positively turns them back. Thus in the example of Fig. 2 only those water molecules with energies sufficient to overcome the latent heat forces can escape into the 'membrane'; all with lower energies are held back. From this it can at once be seen that the 'heat of transfer' can be very large in such cases; in fact it is very nearly the same as the height of the potential energy barrier, i.e. as the 'activation energy' required to cross the membrane.

This second type of membrane may take many different forms. It may be simply an air space with transport occurring in the state of vapour; or it may impose chemical reactions as a condition of crossing, as when some gases diffuse across metallic diaphragms. A very important type of membrane involves what can be called 'solution in an uncongenial medium', and it is here that we can probably place the case of water molecules penetrating the lipoid membrane of the protoplast. Still another type of potential energy barrier is the electrical one. This is exemplified by a charged membrane exposed to ions, and illustrates the fact that the barrier can be either positive or negative, i.e. it can either oppose or facilitate crossing. In the latter case the 'heat of transfer' may be negative, since the barrier (which might better be called a 'ditch' in this case) acts to promote the crossing of disproportionate numbers of *low*-speed molecules.

Whatever the mechanism of crossing, however, equation (9) holds, exemplifying in this respect the typical nature of a thermodynamic result. However, before any practical use can be made of this relation it is obviously necessary to know the order of magnitude of Q^* . Where the mechanism of

'permeability' is known it may be possible to arrive at this from consideration of existing and well-known data, as in the case of evaporation, where it is equal roughly to the latent heat. As a matter of fact, as mentioned earlier, in all cases Q^* is equal very nearly to the height of the potential energy barrier (if this is large); but as this is very rarely known, some other way of evaluating it must be found.

IV. MEASUREMENT OF THE 'HEAT OF TRANSFER'

Fortunately, Q^* can be calculated very simply from a knowledge of the temperature dependence of the permeability. As a semi-empirical but fairly exact relation Arrhenius showed that the equation

$$\frac{\partial}{\partial T} \ln(\text{rate}) = \frac{E}{RT^2} \quad (10)$$

can be used to describe the variation of rate processes with temperature, E being the 'activation energy' of the process. Originally this equation was applied to chemical reactions, but it can be used for transport and other processes as well. It has been regarded till fairly recently† as only having a partial justification in theory, but it appears (Appendix) that it can be treated as quite exact if E , the 'activation energy', is replaced with Q^* , the heat of transfer. When this is done and the ordinary permeability μ is introduced into the equation we get the exact result

$$\frac{\partial}{\partial T} \ln \left(\frac{\mu T}{V} \right) = \frac{Q^*}{RT^2}, \quad (11)$$

where V is the partial molar volume of water. Since V is nearly constant it can be omitted, and assuming that Q^* does not vary greatly with temperature equation (11) becomes on integration

$$\ln \left(\frac{\mu_2 T_2}{\mu_1 T_1} \right) = \frac{Q^*}{R} \left(\frac{1}{T_1} - \frac{1}{T_2} \right). \quad (12)$$

Writing $T_1 \approx T_2 = T$, $T_2 - T_1 = 10$ and introducing the temperature coefficient Q_{10} of permeability we get the final result

$$\frac{10Q^*}{RT^2} \approx \ln(1.034 Q_{10}) \quad (13)$$

at ordinary temperatures. This result gives us a simple way of evaluating the heat of transfer across the cell membrane.

† The theory of absolute reaction rates developed by Eyring and others since 1935 gives, of course, a very satisfactory basis for this equation.

V. ACTIVE TRANSPORT OF WATER ACROSS THE CELL MEMBRANE

Broadly speaking, the cell can be regarded as a discontinuous system of the type described, with an internal watery phase separated from an external one by a thin lipid layer constituting a potential energy barrier. The height of this barrier probably varies considerably, and sometimes the membrane may even act as a molecular sieve. However, assuming a typical value for the temperature coefficient of permeability it is possible to make an estimate of the 'thermomolecular pressure effect', as $\Delta P/\Delta T$ in the steady state is called. For plant cells a fairly typical value for the Q_{10} for water would seem to be about 2.6 (Davson & Danielli, 1943), and using equation (13) this gives the 'heat of transfer' as

$$Q^* = \frac{1.986 \times 293^2}{10} \ln(1.034 \times 2.6) \\ = 16,900 \text{ calories per mole.}$$

Substituting this value in equation (9) the pressure effect is seen to be

$$\frac{\Delta P}{\Delta T} = - \frac{16,900 \times 41.3}{18.0 \times 293}, \\ = -132 \text{ atmospheres per degree C.,}$$

the negative sign indicating that the pressure develops on the low-temperature side. This is an astonishingly large value; it means that a temperature difference of only $\frac{1}{100}^\circ \text{C.}$ can cause movement of water at the same rate as a pressure difference of well over an atmosphere. Before passing to the question, however, of whether a thermomolecular mechanism can be of any practical importance in the life of the organism, it is necessary to consider two objections which raise rather serious difficulties.

Temperature gradients in the cell

The first difficulty arises from the extreme thinness of the plasma membrane. There is a considerable amount of evidence (see, for instance, Davson & Danielli, 1943) that the plasma membrane is normally of the order of 10^{-6} cm. in thickness. Now a temperature difference of $\frac{1}{100}^\circ \text{C.}$ over this distance means a temperature gradient of $10^{-2}/10^{-6} = 10,000^\circ \text{C./cm.}$, which is hardly of the order of magnitude to be expected in living cells; about one-thousandth of this value would seem to be more within the realm of possibility. However, this objection can be partially met in three ways. The thermal conductivity of a typical oil such as olive oil is about one-third of that of water. Further, when molecules are definitely oriented,

as in crystals, the conductivity may be widely different in different directions, a factor of even 5 or 6 being sometimes reached. Now the lipid molecules in the membrane are arranged more or less parallel, and it is not inconceivable that this arrangement might considerably influence the heat conductivity across the layer. If this is so the conductivity of the membrane might be less than that of water by a factor as low as one-tenth or even less. This, however, only goes a small way towards removing the objection. A second possibility lies in the fact that over a potential energy barrier the flow of heat is impeded as well as the flow of matter, and in a diffuse gaseous system the resistance to heat flow can easily be augmented by such enormous factors as 10^{10} or more. This arises from the fact that in diffuse gases heat flow is by diffusion of molecules, whereas in solids collision is the mechanism and no diffusion at all need take place. Liquids come in between, and heat conduction is partly by one means and partly by the other. To what extent therefore thermal flow is hindered by the fact that the plasma membrane constitutes an energy barrier is not apparent, but at least it is a factor to be taken into account, and it may possibly be very important. Where a whole tissue or several layers of cells is concerned of course the necessary temperature differential will be divided between all the plasma membranes, and this will naturally reduce the *gradients* over them; the same is true where the cell seems to possess multiple membranes, as reported recently by Sjöstrand (1953) for mouse pancreas and kidney cells.

Finally, it may be observed that where the temperature gradient is excessively high the ordinary law of thermal conduction will break down, just as the analogous laws for diffusion and electrical conduction do.

It is suggested that these three considerations, taken together, may conceivably raise the resistance to heat flow across the plasma membrane to several hundred, and perhaps even a thousand, times its value for a comparable thickness of bulk aqueous phase, and this will of course correspondingly sharpen the temperature differential over it.

The heat flow accompanying transport

The second objection is an equally serious one. It will readily be appreciated that while transport is actually proceeding (as opposed to the steady state in which it has ceased) very large quantities of heat energy will be passing across the barrier, since all the molecules crossing will be high-velocity ones. As a matter of fact, to a first approximation, the amount of heat flowing across per unit amount of water will be the heat of transfer, Q^* . At this rate a man would have to consume a remarkable weight of potatoes merely to keep his kidneys functioning! In the course of an hour between 3 and 4 l. of water are actively reabsorbed by the kidneys, and if the

heat of transfer was only 6900 cal./mole (corresponding to a Q_{10} of 1.5) the heat flow would be 1,340,000 cal., requiring the complete oxidation of no less than 360 g. of glucose per hour, or 19 lb. per day! Fortunately, the body functions more efficiently than this, but it looks on the surface as if any thermomolecular mechanism is ruled out at once. The difficulty arises from the fact that this mechanism implies that the cell membrane functions as a heat engine, and as one working over the extremely small temperature range ΔT . This at once limits the thermodynamic efficiency of the process, as Carnot showed, to the excessively small value $\Delta T/T$, and consequently any direct provision of heat by chemical reactions is inconceivable as a significant contribution. However, an interesting possibility remains. If a heat engine is very inefficient because it works over a small temperature range, it is correspondingly highly efficient when it works in reverse as a heat pump over the same range. In other words, the amount of heat it transfers can be an enormous multiple of the free energy it consumes. Such a reversed mechanism can be imagined in the cell or organ. A very simple case is ordinary osmosis. When a *Paramoecium*, for instance, draws in water osmotically from its environment this entering water will carry a large amount of heat in with it, an amount measured by Q^* for the plasma membrane. There is energetically no reason why this heat should not be sufficient to expel water by a thermomolecular process into the contractile vacuole, whose membrane, conceivably, has a lower value of Q^* . Ultimately, of course, some source of free energy is required, and this may operate by maintaining the required osmotic pressure difference at the points of water entry and exit. In the case of the kidney the possibilities are somewhat similar. Pressure filtration in the Bowman's capsule will hardly imply any considerable flow of heat, as the membrane almost certainly acts as a fairly coarse sieve; but in the proximal convoluted tubules where apparently glucose is reabsorbed it is not at all impossible to conceive that the development of osmotic activity might draw in water from the capillaries across a high energy barrier, the heat provided being later used, perhaps in the loop of Henle, to sustain a reversed flow into the capillaries through an energy barrier substantially lower. Such a process, involving first a heat pump and then a heat-engine activity, might possess quite an appreciable thermodynamic efficiency. At least it appears to be a possibility deserving of attention.

VI. CONCLUSION

The theory outlined in this paper has probably its more important potential application to water movement, though it can also be applied to the active transport of solutes such as sugars and ions. However, in the case of these there seem several much more promising suggestions

now being debated, and only in the case of water does it seem likely that none of the existing mechanisms is adequate to meet the requirements. What, in effect, the present theory does is to show that under certain conditions a temperature differential can bring about results hitherto associated only with a pressure difference. Thus the cells of a green alga spending its whole life in fresh water maintain themselves in equilibrium so far as water content is concerned only because they possess a firm wall which is capable of sustaining a considerable internal pressure. This possibility is normally denied to the single-celled organisms of the animal kingdom, and they are consequently faced with the problem of actively excreting water to counterbalance that entering by osmosis. It might, of course, prove to be the case that they excrete actively not water, but ions or other solutes, the water being merely drawn out after them passively by ordinary osmosis. This can only be decided by analysis of the fluid excreted, and it certainly remains an attractive theory, especially in the light of the suggestions put forward a year or two ago by Goldacre (1952) on protein contractility as a basis for osmotic work. However, living systems are amazingly complex, and it is hardly a flight of fancy to suppose that the ultimate explanation of the phenomena of active water movement will be found not in one, but in a combination of several physical processes.

APPENDIX

On the thermodynamic theory

The following is a brief account of the derivation of the fundamental equation. It follows the treatment given by de Groot (1951) and illustrates the method of the Thermodynamics of Irreversible Processes.

Consider the simple system discussed earlier (Fig. 3), water being the only component present and all intensive properties, such as temperature and pressure, being uniform throughout each sub-section. Let superscripts I, II denote the two sections. Further, let subscripts i , e denote increments of an extensive quantity gained internally (i.e. from the other subsection) or externally (from the surroundings). Then it is required to find an expression giving the rate of increase (σ) of the entropy of the system supposing it to be held completely isolated from its surroundings. To do this it is convenient to consider the system as merely *closed*, but not *isolated*; and to separate the expression for its rate of entropy increase into two parts, one representing the entropy gained from the surroundings, the other the entropy produced internally. The latter will be the value of σ , the quantity required. The further development of the theory will then require the use of Onsager's Theorem.

If m^I , m^{II} are the masses of water on the two sides of the membrane, the Law of Conservation of Mass gives

$$m^I + m^{II} = \text{constant},$$

$$\text{or} \quad dm^I + dm^{II} = 0. \quad (1)$$

Further, if U is the internal energy we have the analogous relation

$$d_i U^I + d_i U^{II} = 0. \quad (2)$$

Finally, if the whole system is changing slowly enough each of the sides can be regarded as an open system in internal equilibrium, and we can apply to it the equation of Gibbs:

$$dU = T dS - P dV + \mu dm. \quad (3)$$

The validity of this procedure will naturally depend on the transport processes in operation not being of too rapid a nature; in practical cases, however, equation (3) will hold to a very good degree of approximation. Applying it to each side in turn we get

$$dU^I = T^I dS^I - P^I dV^I + \mu^I dm^I, \quad (4)$$

$$dU^{II} = T^{II} dS^{II} - P^{II} dV^{II} + \mu^{II} dm^{II}. \quad (5)$$

Rearranging these and adding,

$$dS = dS^I + dS^{II} = [(d_e U^I + P^I dV^I)/T^I + (d_e U^{II} + P^{II} dV^{II})/T^{II}] + \{d_i U^I/T^I + d_i U^{II}/T^{II} - \mu^I dm^I/T^I - \mu^{II} dm^{II}/T^{II}\}, \quad (6)$$

where dU has been split up into $d_i U + d_e U$. The quantity in square brackets represents the entropy gained from the surroundings† ($d_e S$); that in braces, the entropy ($d_i S$) generated within the system. Had the system been isolated $d_e S$ would of course have been zero, while $d_i S$ would have been unaffected. It is this latter quantity therefore which is relevant to the Onsager theory.

Eliminating dm^{II} and $d_i U^{II}$ from (6) by means of (1) and (2) and dividing by dt , we get for the rate of entropy production

$$\begin{aligned} \sigma &= \frac{d_i S}{dt} = \frac{d_i U^I}{dt} \left(\frac{1}{T^I} - \frac{1}{T^{II}} \right) - \frac{dm^I}{dt} \left(\frac{\mu^I}{T^I} - \frac{\mu^{II}}{T^{II}} \right) \\ &= \frac{d_i U^I}{dt} \frac{\Delta T}{T^2} + \frac{dm^I}{dt} \Delta \left(\frac{\mu}{T} \right), \end{aligned} \quad (7)$$

where the symbol ΔT , for instance, stands for $T_2 - T_1$, and $T_1 \approx T_2 = T$. Equation (7) has the form mentioned earlier; the 'fluxes' are $J_u = -d_i U^I/dt$,

† The expression $(d_e U^I + P^I dV^I)$, for instance, is equal to q_e^I , the heat absorbed from the surroundings by subsection I. This follows from the first law, $P^I dV^I$ being the work done by subsection I in expanding against the surroundings.

the flow of energy from I to II, and $J_w = -dm^I/dt$, the flow of water. The appropriate 'forces' are $X_v = -\Delta T/T_2$ for the flow of energy, and $X_u = -\Delta(\mu/T)$ for the flow of water.

The theory now relates the J 's and X 's by means of rather general equations analogous to 'Ohm's Law'. This leads to the expressions

$$J_u = L_{uu}X_u + L_{uw}X_w, \quad (8)$$

$$J_w = L_{wu}X_u + L_{ww}X_w, \quad (9)$$

where the L 's are conductance coefficients which will depend on the size and geometry of the system and on the nature of the divisional membrane. However, Onsager's theorem states that provided forces and fluxes are measured in the way described the general result always holds that

$$L_{uw} = L_{wu}. \quad (10)$$

This is a result of the Principle of Microscopic Reversibility, according to which reversal of the motions of all the particles in an isolated system would simply cause the system to retrace its former history.

Before proceeding with the theory it is convenient to replace the flow of energy J_u with the flow of heat, J_Q . In a rough way it can be seen that the energy carried from I to II per unit mass of water will be made up of, first, a term depending on the internal energy u per unit mass of water; secondly, an amount of work represented by the product of the volume of water per unit mass (v) and the pressure under which it flows; and thirdly, a quantity of heat. The first two terms make up the heat function per unit mass (h), since

$$h = u + Pv. \quad (11)$$

This leads to the suggestion that we define the heat flow as

$$J_Q = J_u - hJ_w. \quad (12)$$

The quantity h will differ for the two sides of the system, but since the difference is small this will be of no account. It simply underlines the fact that the theory in any case is strictly exact only for infinitesimal departures from equilibrium.

If we introduce J_Q from (12) into (7) we find that the force X_w is altered. This illustrates the ambiguity mentioned earlier when we were discussing the distinction between active and passive movements; the force X_w which is the passive agent for water movement depends on whether we take J_u or J_Q as the other flux. However, it seems logical to take J_Q instead of J_w since not only is the flow of heat independent of the flow of matter in a sense in which the flow J_u is not, but the choice of J_Q leads to values of the forces which are uniquely defined; that is, they have no unknown additive constant as h or μ , for instance, have.

Substituting for J_u from (12) into (7) the new forces become

$$X_Q = -\frac{\Delta T}{T^2}, \quad (13)$$

$$\begin{aligned} X_w &= -h \frac{\Delta T}{T^2} - \Delta \left(\frac{\mu}{T} \right) \\ &= -\frac{v \Delta P}{T}. \end{aligned} \quad (14)$$

The latter result follows from the fact that $\mu = h - Ts$, where s is the entropy per unit mass; also, since we are dealing with a single substance μ is a function of P and T only. Thus, considering a fixed amount of water we have the Gibbs equation

$$dG = V dP - S dT, \quad (15)$$

which, on dividing by the total mass and writing Δ instead of d , gives

$$\Delta \mu = v \Delta P - s \Delta T. \quad (16)$$

The 'Ohm's Law' equations can now be written in the form

$$J_Q = L_{QQ} \left(-\frac{\Delta T}{T^2} \right) + L_{Qw} \left(-\frac{v \Delta P}{T} \right), \quad (17)$$

$$J_w = L_{wQ} \left(-\frac{\Delta T}{T^2} \right) + L_{ww} \left(-\frac{v \Delta P}{T} \right), \quad (18)$$

with
$$L_{wQ} = L_{Qw}. \quad (19)$$

The condition for the steady state is found by writing $J_w = 0$. This gives, on rearrangement,

$$\frac{\Delta P}{\Delta T} = -\frac{L_{wQ}}{L_{ww}} \frac{1}{vT}. \quad (20)$$

But by definition, if ΔT is put equal to zero the heat flow per unit of mass (J_Q/J_w) is the heat of transfer Q^* . This gives

$$Q^* = \frac{L_{Qw}}{L_{ww}}. \quad (21)$$

Introducing (21) with (19) into (20) it follows that

$$\frac{\Delta P}{\Delta T} = -\frac{Q^*}{vT}. \quad (22)$$

The temperature dependence of the permeability

Let ν^I , ν^{II} be the rates at which molecules of water are passing across the membrane from the two sides respectively. At equilibrium ν^I and ν^{II} will be equal, and this will also be true for the steady state.

Starting with the whole system in equilibrium, imagine the temperature and pressure of section I to be raised by ΔT and ΔP respectively. Then the condition that must be fulfilled if the net flow of water is to remain zero is

$$\Delta \nu^I = 0. \quad (23)$$

Treating ν as a function of T and P this becomes

$$\frac{\partial \nu}{\partial T} \Delta T + \frac{\partial \nu}{\partial P} \Delta P = 0, \quad (24)$$

or, more explicitly,
$$\frac{\Delta P}{\Delta T} = - \frac{\partial \nu}{\partial T} \bigg/ \frac{\partial \nu}{\partial P} \quad (25)$$

$$= - \frac{\partial (\ln \nu)}{\partial T} \bigg/ \frac{\partial (\ln \nu)}{\partial P}. \quad (26)$$

Now it can be shown that ν is proportional to the vapour pressure p . Hence

$$\frac{\partial (\ln \nu)}{\partial P} = \frac{\partial (\ln p)}{\partial P} = \frac{v}{RT}, \quad (27)$$

by a well-known thermodynamic relation, v being the volume of unit mass of water.

But from (26) we have

$$\frac{\partial (\ln \nu)}{\partial T} = - \frac{\Delta P}{\Delta T} \frac{\partial (\ln \nu)}{\partial P}, \quad (28)$$

or introducing (22) and (27),

$$\begin{aligned} \frac{\partial (\ln \nu)}{\partial T} &= \frac{Q^*}{vT} \frac{v}{RT} \\ &= \frac{Q^*}{RT^2}, \end{aligned} \quad (29)$$

which is the familiar Arrhenius type of equation for the temperature variation of a rate.

The ordinary permeability μ is the net rate of flow of water under unit pressure difference and zero temperature difference, i.e.

$$\mu = \left(\frac{\partial \nu}{\partial P} \right)_T. \quad (30)$$

Using (27) it is possible to write

$$\mu = \frac{\nu v}{RT},$$

or

$$\nu = \frac{\mu RT}{v}. \quad (31)$$

Thus from (29)
$$\frac{\partial}{\partial T} \ln \left(\frac{\mu T}{v} \right) = \frac{Q^*}{RT^2}, \quad (32)$$

or since v will be nearly constant,

$$\frac{\partial}{\partial T} \ln (\mu T) = \frac{Q^*}{RT^2}, \quad (33)$$

which is the result sought.

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