4 Exercises for the fourth lecture day of the course on quantum field theory

4.1 Exercise 1

Carefully (re)read chapters 6 and 7 in Mandl and Shaw. Most of it was explicitly discussed during the lecture. Pay particular attention to section 7.3. If you have any questions left on the Feynman rules, ask me about them at the start of the next lecture (or during one of the breaks)!

4.2 Exercise 2

In the lecture (before lunch break) we calculated one term of the matrix element for Compton scattering in leading order in perturbation theory completely from scratch. As an exercise do it for another process (a particularly useful choice might be Compton scattering for positrons where you will learn a few more things). Compare your result with what you get from directly applying the Feynman rules (well you should get the same answer...).

4.3 Exercise 3

Use the Feynman rules to show that the expansion in number of loops is an expansion in powers of $\hbar$. Hint: obviously you will have to reinstate the factors $\hbar$ into the expressions for the interaction vertex and the Feynman propagators! You will also need the rule we derived which states that the number of loops (which is the same as the number of undetermined momenta) is equal to the number of propagators in a diagram minus the number of interaction vertices plus one.

4.4 Exercise 4

Given a scalar field ϕ with Lagrange density,

$$\mathcal{L} = \frac{1}{2}\,\partial_\mu\phi\,\partial^\mu\phi - \frac{m^2}{2}\,\phi^2 - \frac{\lambda_3}{3!}\,\phi^3 - \frac{\lambda_4}{4!}\,\phi^4. \tag{1}$$

1. What are the interaction vertices?

2. Determine the Feynman rules for the vertices (warning: the combinatorics will be non-trivial!).

3. Determine in terms of Feynman diagrams the leading terms in the perturbative expansion for the process 2 scalars $\to$ 2 scalars. Are you able to write down the corresponding Feynman amplitudes as well?

4.5 Exercise 5

In the afternoon of last lecture we calculated the cross section for pair production. Carefully study sections 8.1 through 8.4 of Mandl and Shaw. Read sections 8.5 through 8.8 as well.