Questions about partial derivative and total derivative in Frank's model.

The optimum probability of induction, y^*, is obtained by maximization of w with respect to small variants in y, from the solution of

$$\frac{\mathrm{d}w}{\mathrm{d}y} = \frac{\partial w}{\partial y} + r\frac{\partial w}{\partial z} = 0, \qquad 1$$

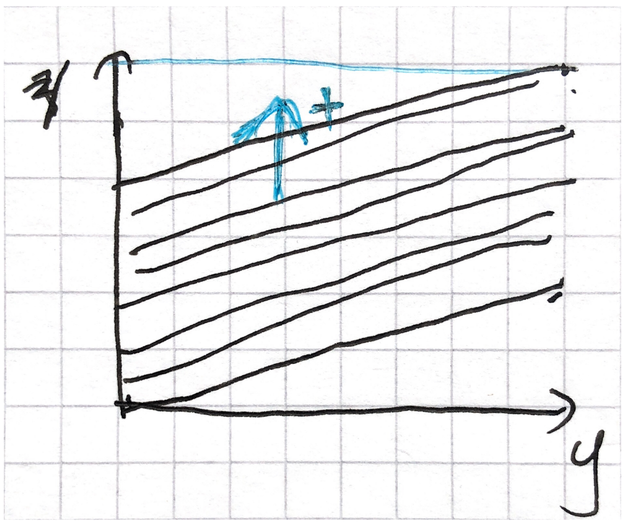
evaluated at $y = z = y^*$. The partial derivatives arise from standard application of the chain rule for differentiation, and $r = \mathrm{d}z/\mathrm{d}y$ is the coefficient of relatedness from kin-selection theory (Taylor & Frank 1996). Note that this coefficient is simply the slope of group genotype on individual genotype or, equivalently, the slope of the average social partner's genotype on the actor's genotype.

The first term of the derivative is

$$\frac{\partial w}{\partial y} = a(1 - cy^*)f(y^*) - c[1 - a(1 - y^*)f(y^*)], \qquad 2$$

which is the direct, marginal effect of induction on the individual's own fitness. The second term is

$$r\frac{\partial w}{\partial z} = -ra(1 - cy^*)(1 - y^*)f'(y^*), \qquad 3$$



Here in equation 1 is the total derivative regarding y. This should include r. Assuming r is a constant means that y and z are linearly related. So this is more like a directional derivative, like this dW[z(y)]/dy. To solve this, we can just use the chain rule. But what is the geometry meaning of the solution? But to make a directional derivative, a definite relation between y and z is needed. We don't know the real relation between y and z, there could be a constant, like y=rz+b. And the constant must be there, otherwise how could y=z?

So what are we doing geometrically? Finding the best y for the scalar field w, given that y=x, despite b?

And then if it does not matter, how could we guarantee that y and z both exists at the point of solution?

In 2 and 3, why he made the partial derivative of y and z? We know that W is always an increasing function of z (mathematically and biologically). So is that we are calculating the point with maximum W along the upper limit of the shaded area I drew?

I know that with this assumption that y and z are related linearly, z changes with y in a known rate, if given y. It might mathematically work, but in reality there must be more limits. Which means, not every point the shaded area can be reached.

The following is my apprehension of the solution:

On the line y=z, we want to find where we got dW=0 in the r slope direction. But evolution does not necessarily happen along y=z or y=rz+b.

Is this right?