This is a description of my modification to the Frank(1998) model.

In this model, the result of attack is calculated for either induce a defence or not. The pattern of attack in this model follows poisson distribution, individuals with and without defence has difference maximum tolerance to the attack. P1 and P2 are the cumulative function of a poisson distribution, counting the sum of the probability that the total number of attack does not exceed the maximum tolerance. The variable y is the probability of an individual inducing a defence. Different from Frans's model, f(z) here is an increasing function that increases the survival probability.

How dw/dx changes into dw/dy and r referrers to the Taylor and Frank paper in 1996.

$$(1-cy)\left[P_2 \cdot y + P_1(1-y)\right]$$

$$w=(1-cy)\left[P_2\, y\, f(z) + P_1(1-y)\, f(z)\right]$$

$$\frac{dw}{dx} = \frac{\partial w}{\partial y}\cdot\frac{dy}{dx} + \frac{\partial w}{\partial z}\cdot\frac{dz}{dx}$$

$$\frac{dw}{dy} = \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z}\cdot r$$

$$\frac{\partial w}{\partial y}\ \left\{(1-cy)P_2\cdot y\cdot f(z) + (1-cy)P_1(1-y)f(z)\right\}'$$

$$=\left[P_2\cdot f(z)\,(y-cy^2) + P_1 f(z)\,(cy^2-y-cy+1)\right]'$$

$$= P_2\cdot f(z)\,(1-2cy) + P_1 f(z)\,(2cy-1-c)$$

$$r\cdot\frac{\partial w}{\partial z} = \left[(1-cy)P_2\cdot y\cdot f'(z) + P_1(1-y)f'(z)\right] r$$

$$\frac{\partial w}{\partial y} = P_2\cdot f(z)(1-2cy) - P_1 f(z)(1-2cy) - f(z)\cdot c$$

$$=(P_2-P_1)f(z)(1-2cy) - f(z)\cdot c$$

$$r\,\frac{\partial w}{\partial z} = y\cdot P_2 f'(z) - cy^2 P_2 f'(z) + P_1 f'(z) - y P_1 f'(z)$$

$$=(P_2-P_1)\, y\, f'(z) + (P_1 - cy^2 P_2)\, f'(z)$$

Here is my result.

What is not stated in the graphs is f(z)=0.1z+0.9. This function is set purely based on my personal favour, and it is good enough to demonstrate this model. When the whole group grow stronger, individuals are more likely to survive, but no matter how hard they try, z<=1, f(z)<=1. Which means they still need to rely on themselves, the P1 and P2.

Both graph shows the derivative of W or dW/dy. When dW/dy reaches 0, the equilibrium reaches, changing y either way will be pushed back by its decreasing fitness. The graph on the left shows that **when relatedness increase, the optimal y increase**, since one need to fight for its “family”. The graph on the right shows the selection power of different frequency of attack. Two extreme cases worth to be noticed. When the attack is so frequent, W becomes a constantly increasing function of y, which means, **the harder one fight, the more it gains**. When the attack ratio becomes very low, **the more one pay, the more one loose**. In between this to situations, the individual always has a optimal spot for paying.