

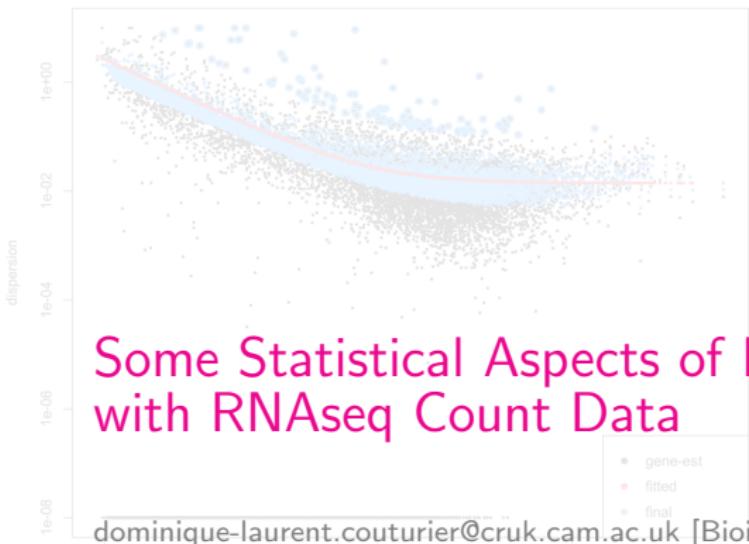


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## Some Statistical Aspects of DE Analysis with RNAseq Count Data

dominique-laurent.couturier@cruk.cam.ac.uk [Bioinformatics core]

(Source: O. Rueda, MRC-BSU; G. Marot, INRIA)

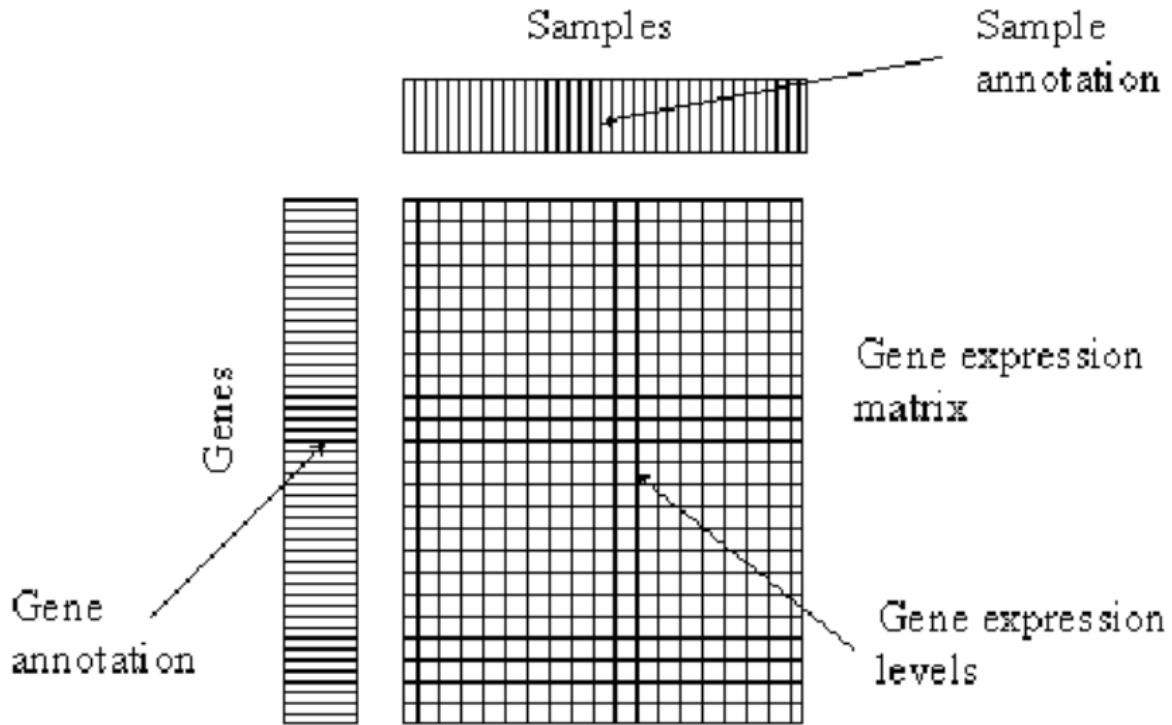
raw count for gene i, sample j

The mean is taken as "normalized counts" scaled by a normalization factor

$$K_{ij} \sim NB(s_{ij}q_{ij}, \alpha_i)$$

one dispersion per gene

# Introduction



# Introduction

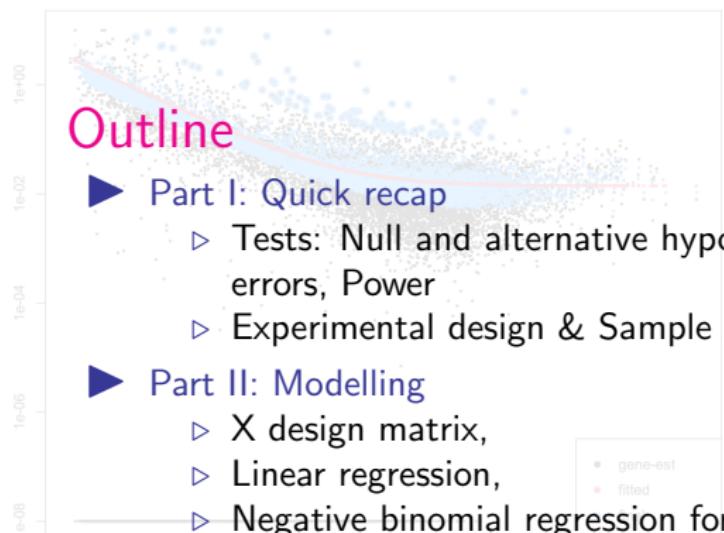
```
> set.seed(777)
> cnts <- matrix(rnbinom(n=20000, mu=100, size=1/.25), ncol=20)
> cond <- factor(rep(1:2, each=10))

> dds <- DESeqDataSetFromMatrix(cnts, DataFrame(cond), ~ cond)
> dds <- DESeq(dds)
> results(dds)

log2 fold change (MLE): cond 2 vs 1
Wald test p-value: cond 2 vs 1
DataFrame with 1000 rows and 6 columns
  baseMean log2FoldChange      lfcSE      stat     pvalue     padj
  <numeric>      <numeric>      <numeric>      <numeric>      <numeric>
1    97.3140     -0.682067    0.344525   -1.979730  0.0477339  0.745842
2   109.9860     -0.228819    0.450720   -0.507676  0.6116808  0.944354
3    98.8111      0.104291    0.462113    0.225683  0.8214483  0.978382
4   103.2615      0.306400    0.297682    1.029284  0.3033460  0.944354
5    97.9406      0.316338    0.357242    0.885501  0.3758864  0.944354
...
996   86.8057      0.0467703   0.287042    0.162939  0.8705668  0.980044
997  101.4437     -0.2070806   0.339886   -0.609264  0.5423495  0.944354
998   78.1356     -0.6372790   0.369515   -1.724637  0.0845930  0.824310
999   89.2920      0.7554725   0.306192    2.467314  0.0136131  0.614613
1000  103.5569     -0.0728875   0.348655   -0.209053  0.8344065  0.978382
```

# Outline

- ▶ Part I: Quick recap
  - ▷ Tests: Null and alternative hypotheses, Type I and type II errors, Power
  - ▷ Experimental design & Sample size calculation.
- ▶ Part II: Modelling
  - ▷ X design matrix,
  - ▷ Linear regression,
  - ▷ Negative binomial regression for counts.
- ▶ Part III: Multiplicity correction
  - ▷ Familywise error rate (FWER)
  - ▷ False discovery rate (FDR)



$$K_{ij} \sim NB(s_{ij}q_{ij}, \alpha_i)$$

The mean is taken as "normalized counts" scaled by a normalization factor  
one dispersion per gene

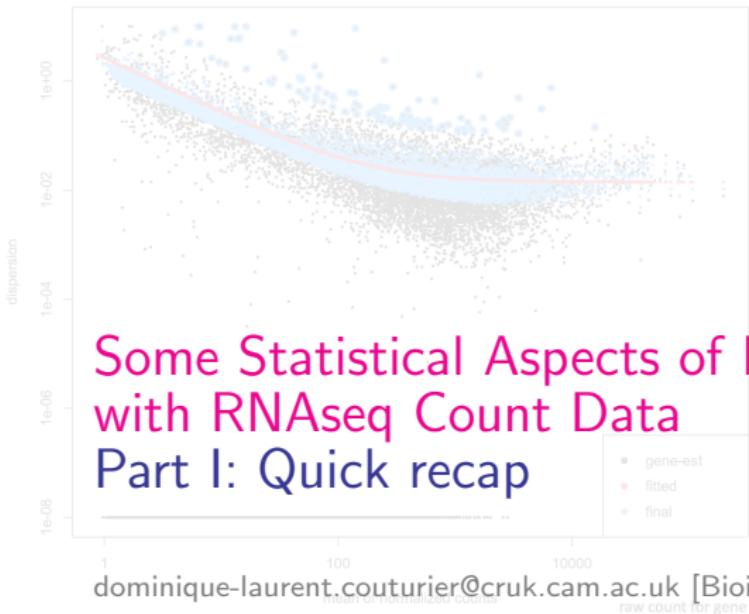


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## Some Statistical Aspects of DE Analysis with RNAseq Count Data

### Part I: Quick recap

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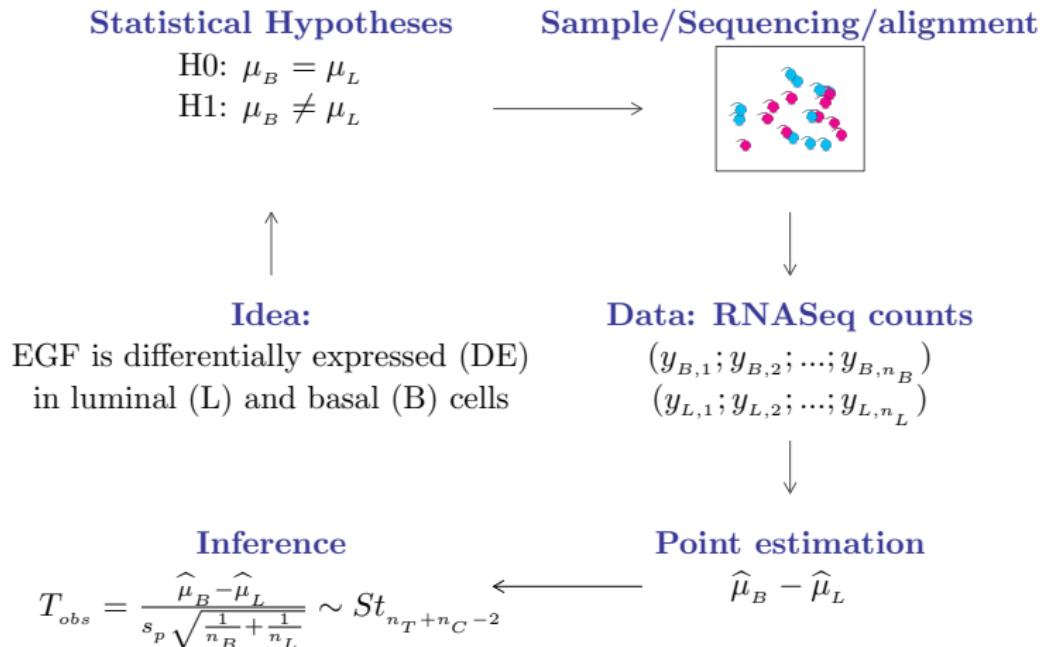
raw count for gene i, sample j

The mean is taken as "normalized count" divided by a normalization factor

$$K_{ij} \sim NB(s_{ij}q_{ij}, \alpha_i)$$

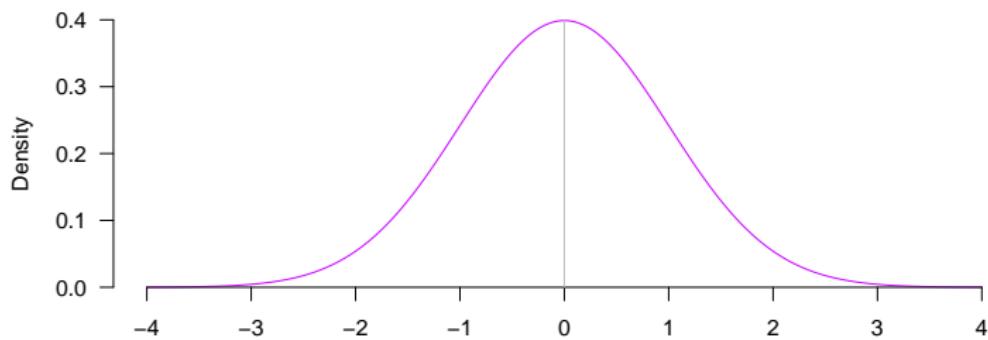
one dispersion per gene

# Grand Picture of Statistics



# Statistical tests

Assess how likely the observed test statistics is compared to the test statistics distribution under H0:



P-value for a two-sided test:  $p\text{-value} = P(|T| > T_{obs})$

i.e. the probability of getting a test statistic as extreme or more extreme than the calculated test statistic if H0 is true

# Statistical tests

## 4 possible outcomes

Conclude:

- ▶ if  $p\text{-value} > \alpha \rightarrow$  do not reject  $H_0$ .
- ▶ if  $p\text{-value} < \alpha \rightarrow$  reject  $H_0$  in favour of  $H_1$ .

		Test Outcome	
		$H_0$ not rejected	$H_1$ accepted
Unknown Truth	$H_0$ true	$1 - \alpha$ [TN]	$\alpha$ [FP]
	$H_1$ true	$\beta$ [FN]	$1 - \beta$ [TP]

where

- ▶  $\alpha$  is the type I error,
- ▶  $\beta$  is the type II error.

Want to minimise FP and FN through design

# Experimental design

## 3 fundamental aspects of sounds experiments (Fisher 1935)

- ▶ **Replication**

Try to capture all sources of variability  
(Biological versus technical variability)

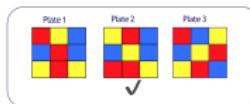
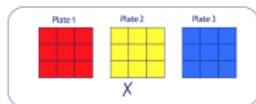
- ▶ **Blocking**

Try to remove technical biases/confounding  
(Lane and batch effects)



- ▶ **Randomisation**

Try to remove confounding due to other factors



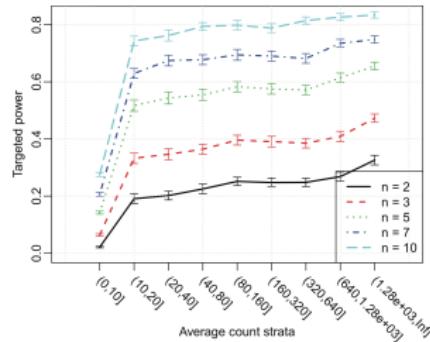
# Experimental design

## Sample size per condition

### Sample size calculation:

Aim is to define the sample size allowing to detect an effect of a given size at the  $\alpha$  level with a given probability (power):

- ▶  $\delta$ , the effect size: function of  $\mu_L$  and  $\mu_B$  (log fold change, standardised difference),
- ▶  $1 - \beta$ , the power,
- ▶  $\alpha$ , the type I error.
- ▶  $\phi$ , nuisance parameters (variability, sequencing depth, multiplicity correction)



(Wu, Wang and Wu (2015))

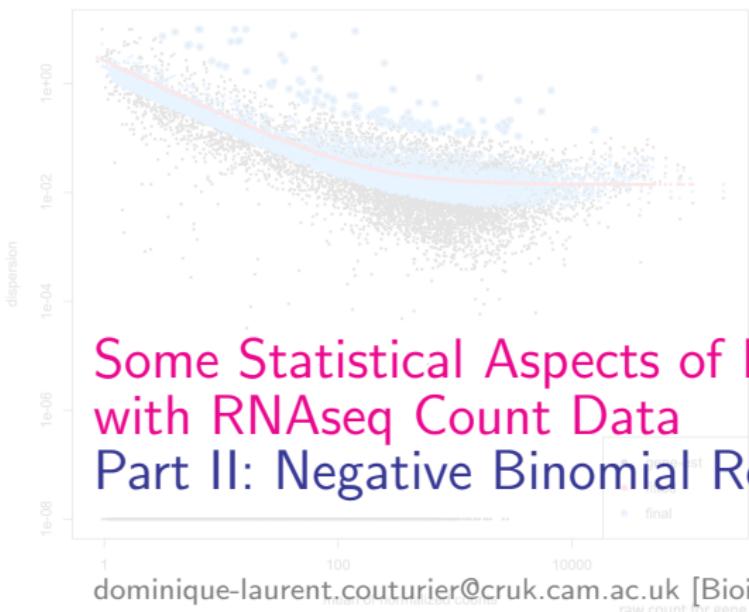


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## Some Statistical Aspects of DE Analysis with RNAseq Count Data

### Part II: Negative Binomial Regression

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(Source: O. Rueda, MRC-BSU)

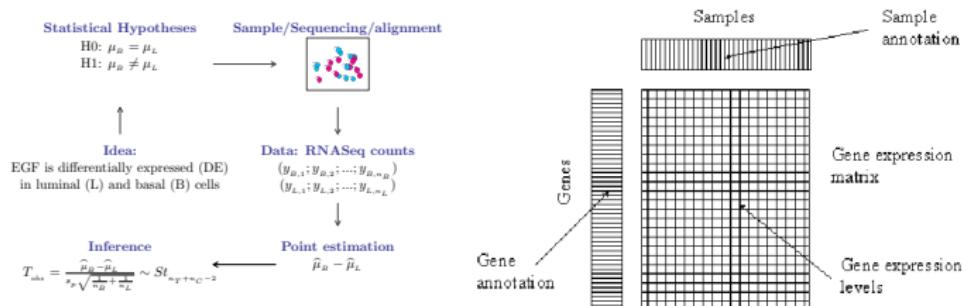
raw count for gene  $i$ , sample  $j$

The mean is taken as "normalized counts" divided by a normalization factor

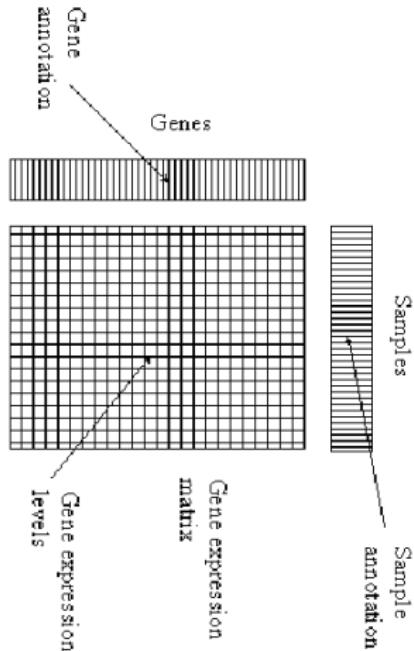
$$K_{ij} \sim NB(s_{ij}q_{ij}, \alpha_i)$$

one dispersion per gene

# Statistical modelling



# Statistical modelling

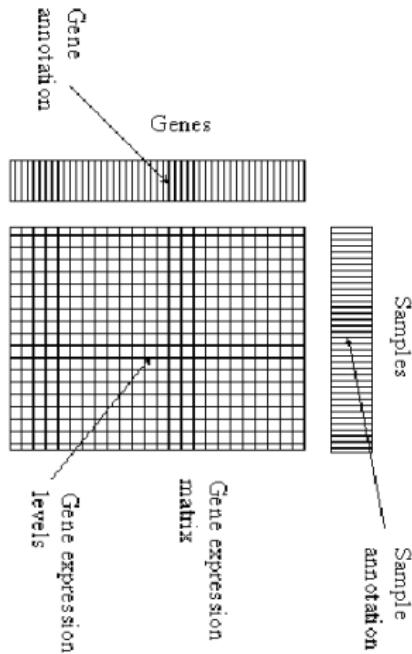


$$\mathbf{y} = f(\mathbf{X}) + \epsilon$$
$$E[\mathbf{y}] = f(\mathbf{X})$$

where

- ▶  $\mathbf{y}$  denotes the  $(n \times 1)$  vector of expression intensities of a given gene,
- ▶  $\mathbf{X}$  denotes the  $(n \times p)$  design/predictor matrix,
- ▶  $\epsilon$  denotes the  $(n \times 1)$  stochastic error vector,
- ▶  $E[\mathbf{y}]$  denotes the expectation of  $\mathbf{y}$

# Statistical modelling : Linear regression

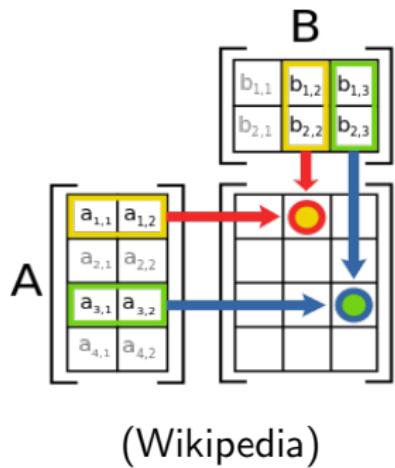


$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}$$
$$E[\mathbf{y}] = \mathbf{X}\boldsymbol{\beta}$$

where

- ▶  $\mathbf{y}$  denotes the  $(n \times 1)$  vector of expression intensities of a given gene,
- ▶  $\mathbf{X}$  denotes the  $(n \times p)$  design/predictor matrix,
- ▶  $\boldsymbol{\beta}$  denotes the  $(p \times 1)$  parameter vector,
- ▶  $\boldsymbol{\epsilon} \sim N(0, \sigma^2)$  denotes the  $(n \times 1)$  stochastic error vector,
- ▶  $E[\mathbf{y}]$  denotes the expectation of  $\mathbf{y}$

# Statistical modelling : Linear regression



$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \epsilon$$

$$E[\mathbf{y}] = \mathbf{X}\boldsymbol{\beta}$$

where

- ▶  $\mathbf{y}$  denotes the  $(n \times 1)$  vector of expression intensities of a given gene,
- ▶  $\mathbf{X}$  denotes the  $(n \times p)$  design/predictor matrix,
- ▶  $\boldsymbol{\beta}$  denotes the  $(p \times 1)$  parameter vector,
- ▶  $\epsilon \sim N(0, \sigma^2)$  denotes the  $(n \times 1)$  stochastic error vector,
- ▶  $E[\mathbf{y}]$  denotes the expectation of  $\mathbf{y}$

## Statistical modelling : Strategy

- ▶ Collect the information related to each sample for the predictors of interest,
- ▶ define  $\beta$ , the sets of parameters we are interested in,
- ▶ build the  $X$  matrix that relates the sample information with the  $\beta$ ,
- ▶ estimate the  $\beta$ ,
- ▶ use statistical inference to assess significance ( $p$ -values).

## Statistical modelling : X contrast matrix

- ▶ Linear regression:  
 $E[y] = \mathbf{X}\boldsymbol{\beta}$ ,
- ▶ Cox regression:  
 $h(t) = h_0(t)e^{\mathbf{X}\boldsymbol{\beta}}$ ,
- ▶ Logistic regression:  
 $\pi = \frac{e^{\mathbf{X}\boldsymbol{\beta}}}{1+e^{\mathbf{X}\boldsymbol{\beta}}}$ ,
- ▶ Mean expression level for a given gene in DESeq2:  
 $E[y] = 2^{\mathbf{X}\boldsymbol{\beta}}$ ,

# Statistical modelling : X contrast matrix

Contrast matrices for models with

- ▶ one factor / categorical predictor,
  - ▷ two experimental conditions (dichotomous predictor),  
[t-test](#)
  - ▷ several experimental conditions,  
[ANOVA](#)
- ▶ two factors / categorical predictors,
  - ▷ without interaction,
  - ▷ with interaction,
- ▶ [Two-way ANOVA](#)
- ▶ categorical and continuous factors.

## Design matrix for models with a two-level factor

Sample	Treatment
Sample1	Treatment A
Sample 2	Control
Sample 3	Treatment A
Sample 4	Control
Sample 5	Treatment A
Sample 6	Control

Number of samples: 6

Number of factors: 1 with 2 levels (Control and Treatment A)

Possible parameters (What differences are important)?

- Effect of Treatment A
- Effect of Control

## Design matrix for models with a two-level factor

Sample	Treatment
Sample 1	Treatment A
Sample 2	Control
Sample 3	Treatment A
Sample 4	Control
Sample 5	Treatment A
Sample 6	Control

$$\begin{array}{l} \text{Sample 1} \\ \text{Sample 2} \\ \text{Sample 3} \\ \text{Sample 4} \\ \text{Sample 5} \\ \text{Sample 6} \end{array} \left[ \begin{array}{l} S1 \\ S2 \\ S3 \\ S4 \\ S5 \\ S6 \end{array} \right] = \left( \begin{array}{c} \text{Treat. A} \\ \text{Control} \end{array} \right) \quad \left[ \begin{array}{l} T \\ C \end{array} \right] \xrightarrow{\beta \text{ Parameter vector}}$$

X design Matrix

$C$  is the mean expression of the control  
 $T$  is the mean expression of the treatment

# Design matrix for models with a two-level factor

Different parameterisation: using intercept

Sample	Treatment
Sample1	Treatment A
Sample 2	Control
Sample 3	Treatment A
Sample 4	Control
Sample 5	Treatment A
Sample 6	Control

Let's now consider this parameterization:

$C$ = Baseline expression

$T_A$ = Baseline expression + effect of treatment

So the set of parameters are:

$C$  = Control (mean expression of the control)

$a = T_A - C$  = Control (mean change in expression under treatment)

## Design matrix for models with a two-level factor

Different parameterization:  
using an intercept

$$\begin{array}{l} \text{Sample 1} \\ \text{Sample 2} \\ \text{Sample 3} \\ \text{Sample 4} \\ \text{Sample 5} \\ \text{Sample 6} \end{array} \left[ \begin{array}{c} S1 \\ S2 \\ S3 \\ S4 \\ S5 \\ S6 \end{array} \right] = \left( \begin{array}{c} \text{Intercept} \\ \text{Treatment A} \end{array} \right) \left[ \begin{array}{c} \beta_0 \\ a \end{array} \right]$$

$\beta$  Parameter vector

X design Matrix

The Intercept measures the baseline expression and  $a$  measures now the differential expression between Treatment A and Control

## Design matrix for models with a three-level factor

Sample	Treatment
Sample1	Treatment A
Sample 2	Treatment B
Sample 3	Control
Sample 4	Treatment A
Sample 5	Treatment B
Sample 6	Control

Number of samples: 6

Number of factors: 1 with 3 levels (Control, Treatment A, Treatment B)

Possible parameters (What differences are important)?

- Effect of Treatment A
- Effect of Treatment B
- Effect of Control
- Differences between treatments?

## Design matrix for models with a three-level factor

Sample	Treatment
Sample1	Treatment A
Sample 2	Treatment B
Sample 3	Control
Sample 4	Treatment A
Sample 5	Treatment B
Sample 6	Control

Control = Baseline

$T_A = \text{Baseline} + a$

$T_B = \text{Baseline} + b$



$$\begin{bmatrix} S1 \\ S2 \\ S3 \\ S4 \\ S5 \\ S6 \end{bmatrix} = \begin{pmatrix} & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \end{pmatrix} \begin{bmatrix} T_A \\ T_B \\ C \end{bmatrix}$$

$$\begin{bmatrix} S1 \\ S2 \\ S3 \\ S4 \\ S5 \\ S6 \end{bmatrix} = \begin{pmatrix} & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \end{pmatrix} \begin{bmatrix} \beta_0 \\ a \\ b \end{bmatrix}$$

```
> dds <- DESeqDataSetFromMatrix(cnts, DataFrame(cond), ~ cond)
> results(DESeq(dds))
```

## Design matrix for models with a three-level factor: R code

```
> one3levelfactor = data.frame(condition =
  rep(c("TreatmentA", "TreatmentB", "Control"), 2))

# model without intercept and default levels:
> X1 = model.matrix(~ condition - 1, data = one3levelfactor)

# model with intercept and default levels
> X2 = model.matrix(~ condition, data = one3levelfactor)

# model with intercept and self-defined levels
> levels(one3levelfactor$condition)
> levels(one3levelfactor$condition) = c("TreatmentB", "TreatmentA", "Control")
> X3 = model.matrix(~ condition, data = one3levelfactor)
```

## Models with 2 factors

Sample	Treatment	ER status
Sample1	Treatment A	+
Sample 2	No Treatment	+
Sample 3	Treatment A	+
Sample 4	No Treatment	+
Sample 5	Treatment A	-
Sample 6	No Treatment	-
Sample 7	Treatment A	-
Sample 8	No Treatment	-

Number of samples: 8

Number of factors: 2 two-level factors

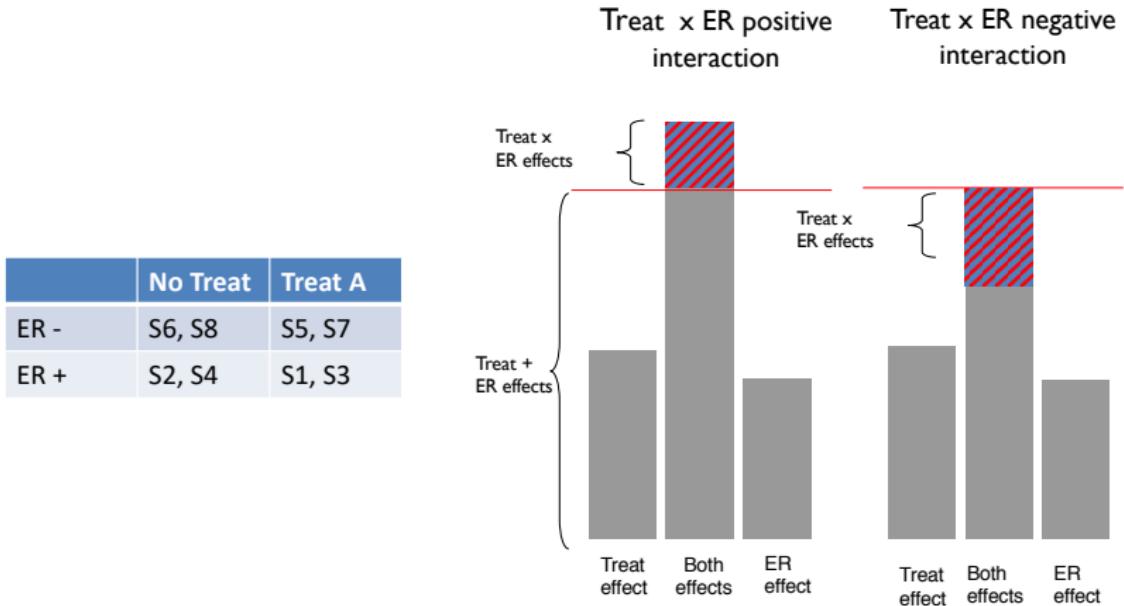
## Models with 2 factors: no interaction

```
x1 = model.matrix(~ treatment + er, data=two2levelfactor)
```

$$\begin{bmatrix} S1 \\ S2 \\ S3 \\ S4 \\ S5 \\ S6 \\ S7 \\ S8 \end{bmatrix} = \begin{pmatrix} & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \end{pmatrix} \begin{bmatrix} \beta_0 \\ a \\ er + \end{bmatrix}$$

	No Treat	Treat A
ER -	S6, S8	S5, S7
ER +	S2, S4	S1, S3

## Models with 2 factors: interactions



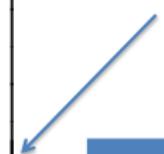
(Adapted from Natalie Thorne, Nuno L. Barbosa Morais)

## Models with 2 factors: with interaction

```
> X2 = model.matrix(~ treatment * er, data=two2levelfactor)
> X3 = model.matrix(~ treatment + er + treatment:er, data=two2levelfactor)
```

$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \\ Y_5 \\ Y_6 \\ Y_7 \\ Y_8 \end{bmatrix} = \begin{pmatrix} & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \end{pmatrix} \begin{bmatrix} \beta_0 \\ a \\ er + \\ a.er + \end{bmatrix}$$

Interaction effect of Treatment A on ER+ samples

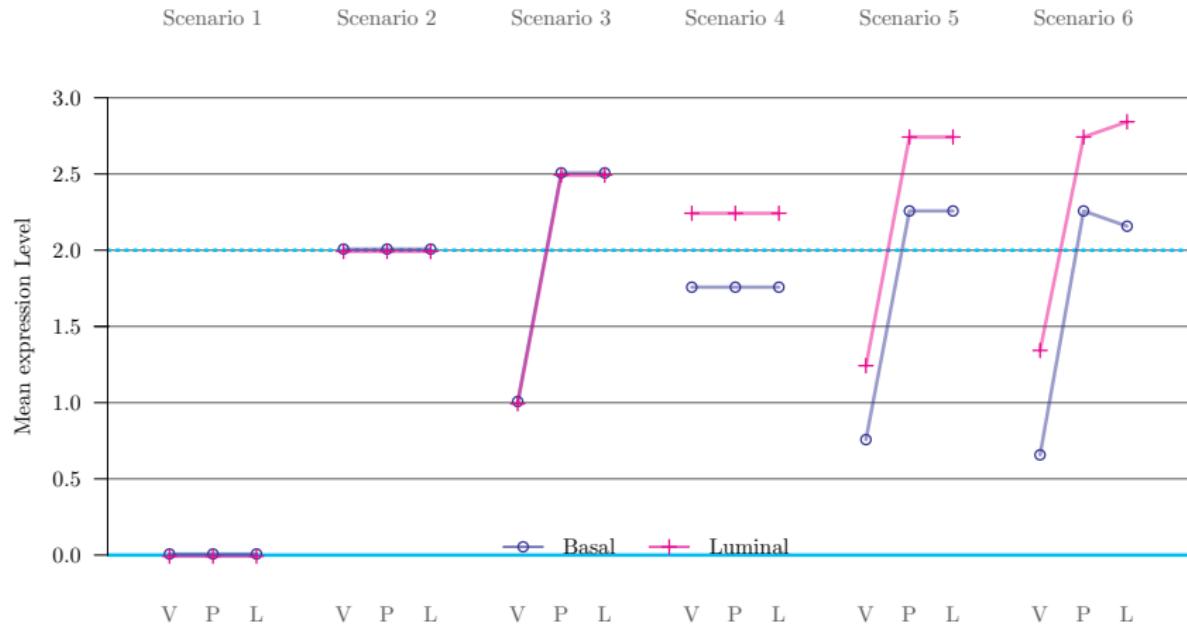


	No Treat	Treat A
ER -	S6, S8	S5, S7
ER +	S2, S4	S1, S3

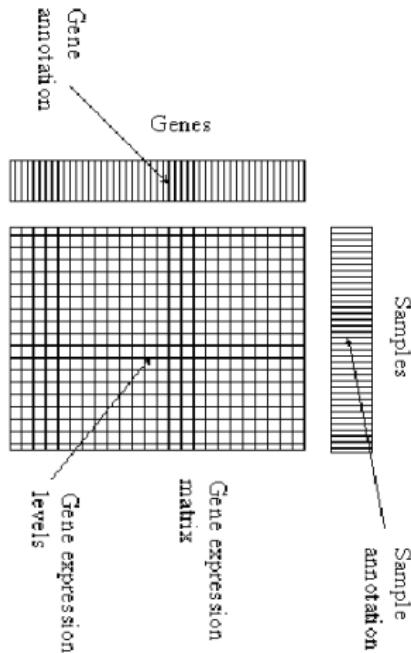
# Models with 2 factors: possible scenarios

2 factors:

- ▶ cell type (2 levels): luminal versus basal
- ▶ mouse type (3 levels): virgin, pregnant, lactating



# Negative binomial regression: Model



$$\mathbf{y} \sim \text{NB}(\mu, \phi)$$

$$E[\mathbf{y}] = \mu = s 2^{\mathbf{X}\beta}$$

where

- ▶  $\mathbf{y}$  denotes the  $(n \times 1)$  count vector of expression intensities of a given gene,
- ▶  $\mathbf{X}$  denotes the  $(n \times p)$  design/predictor matrix,
- ▶  $\beta$  denotes the  $(p \times 1)$  parameter vector,
- ▶  $\phi$  denotes the dispersion parameter,
- ▶  $s$  denotes the scaling factor vector (library size),
- ▶  $E[\mathbf{y}] = \mu$  denotes the expectation of  $\mathbf{y}$

## Negative binomial regression: Probability mass function

$$\mathbf{y} \sim \text{NB}(\boldsymbol{\mu}, \phi)$$

$$f(\mathbf{y}|\boldsymbol{\mu}, \phi) = \frac{\Gamma(\mathbf{y} + \frac{1}{\phi})}{\Gamma(\frac{1}{\phi})\Gamma(\mathbf{y} + 1)} \left( \frac{\phi\boldsymbol{\mu}}{1 + \phi\boldsymbol{\mu}} \right)^{\mathbf{y}} \left( \frac{1}{1 + \phi\boldsymbol{\mu}} \right)^{\frac{1}{\phi}}$$

with expectation and variance given by

- ▶  $E[\mathbf{y}] = \boldsymbol{\mu} = s^2 \mathbf{X}\boldsymbol{\beta}$
- ▶  $\text{Var}[\mathbf{y}] = \boldsymbol{\mu}(1 + \phi\boldsymbol{\mu})$

# Negative binomial regression: Log2 FC

```
log2 fold change (MLE): cond 2 vs 1
Wald test p-value: cond 2 vs 1
DataFrame with 1000 rows and 6 columns
  baseMean log2FoldChange    lfcSE      stat     pvalue     padj
  <numeric>      <numeric> <numeric> <numeric> <numeric> <numeric>
1     97.3140     -0.682067  0.344525 -1.979730  0.0477339  0.745842
2    109.9860     -0.228819  0.450720 -0.507676  0.6116808  0.944354
...
999   89.2920     0.7554725  0.306192  2.467314  0.0136131  0.614613
1000  103.5569     -0.0728875  0.348655 -0.209053  0.8344065  0.978382
```

- ▶  $E[y|'cond 1'] = 2^{\hat{\beta}_0}$
- ▶  $E[y|'cond 2'] = 2^{\hat{\beta}_0 + \hat{\beta}_1} = 2^{\hat{\beta}_0} 2^{\hat{\beta}_1}$

▷ If not DE,  $\hat{\beta}_1 = 0$  so that  $E[y|'cond 2'] = 2^{\hat{\beta}_0} 2^0 = 2^{\hat{\beta}_0}$ ,

▷ If DE,  $\hat{\beta}_1 \neq 0$  so that  $E[y|'cond 2'] = 2^{\hat{\beta}_0} 2^{\hat{\beta}_1}$

Interpretation: *Multiplicative change in observed gene*

*expression level of  $2^{\hat{\beta}_1} = 2^{-0.682067} = 0.6232717$  compared to the condition 1*

# Negative binomial regression: Significativity

```
log2 fold change (MLE): cond 2 vs 1
Wald test p-value: cond 2 vs 1
DataFrame with 1000 rows and 6 columns
  baseMean log2FoldChange      lfcSE      stat     pvalue     padj
  <numeric>      <numeric> <numeric> <numeric> <numeric> <numeric>
1     97.3140      -0.682067  0.344525 -1.979730  0.0477339  0.745842
2    109.9860      -0.228819  0.450720 -0.507676  0.6116808  0.944354
...
999   89.2920      0.7554725  0.306192  2.467314  0.0136131  0.614613
1000 103.5569      -0.0728875  0.348655 -0.209053  0.8344065  0.978382
```

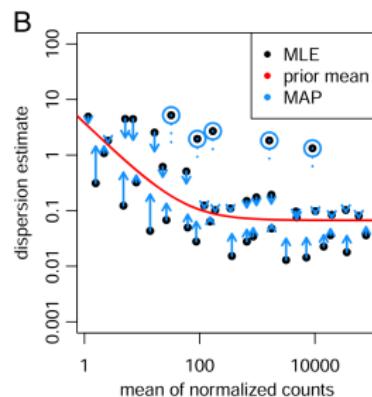
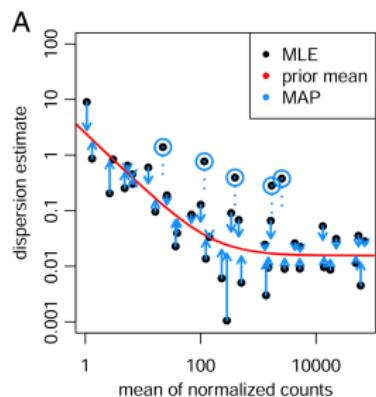
Wald T-test to assess if a Log2 FC is significantly different from 0:

- ▶ **H0:**  $\beta_1 = 0$  versus **H1:**  $\beta_1 \neq 0$
- ▶ T-statistic =  $\frac{\hat{\beta}_1}{\hat{\sigma}_{\hat{\beta}_1}} = \frac{-0.682067}{0.344525} = -1.979730$
- ▶ P-value =  $P(|T| > \text{T-statistic})$  where  $T \sim N(0, 1)$  under **H0**  
 $> 2*(1-\text{pnorm}(\text{abs}(-1.979730))))$

```
[1] 0.04773388
```

# Negative binomial regression: Assumed Distribution

- ▶ The **assumed distribution of counts per condition for a given gene** depends on
  - ▷  $\hat{\beta}$ , the estimate of the parameter vector,
  - ▷  $\hat{\phi}$ , the estimate of the dispersion parameter for that gene.
- ▶ There are **3 ways to estimate  $\phi$  in DESeq2:**
  - ▷ **gene-wise** dispersion estimates via ML (black dots) [no efficient],
  - ▷ **smooth curve** (red line) [strong assumption],
  - ▷ Bayesian **combination of both** [mid-way optimal solution].

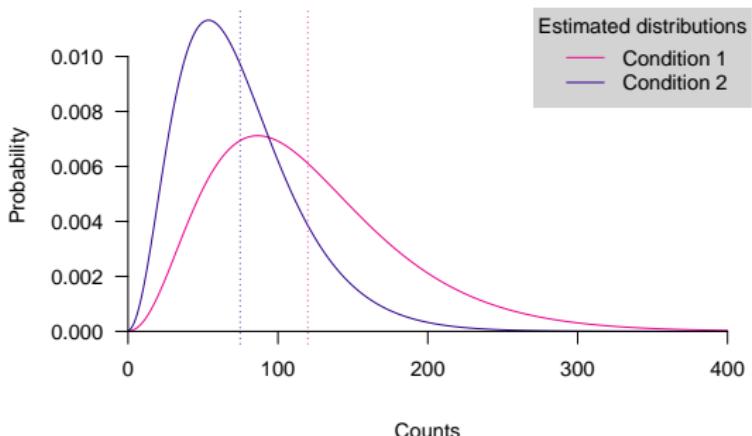


(Love et al (2015))

# Negative binomial regression: Assumed Distribution

```
-> mcols(dds)[,c("Intercept","cond_2_vs_1","dispGeneEst","dispFit","dispersion")]
Data Frame with 1000 rows and 5 columns
  Intercept cond_2_vs_1 dispGeneEst dispFit dispersion
  <numeric>   <numeric>    <numeric> <numeric>   <numeric>
1     6.90565 -0.682067   0.294082  0.234624   0.274708
2     6.89102 -0.228819   0.479231  0.230525   0.479231
...
999    6.05380  0.7554725   0.206644  0.229562   0.213730
1000   6.73029 -0.0728875   0.304930  0.235483   0.282745
```

- ▶ For gene 1 and condition 1, we have  
 $y \sim NB(\hat{\mu} = 2^{6.90565} = 119.8969, \hat{\phi} = 0.274708)$
- ▶ For gene 1 and condition 2, we have  
 $y \sim NB(\hat{\mu} = 2^{6.90565} 2^{-0.682067} = 74.72831, \hat{\phi} = 0.274708)$



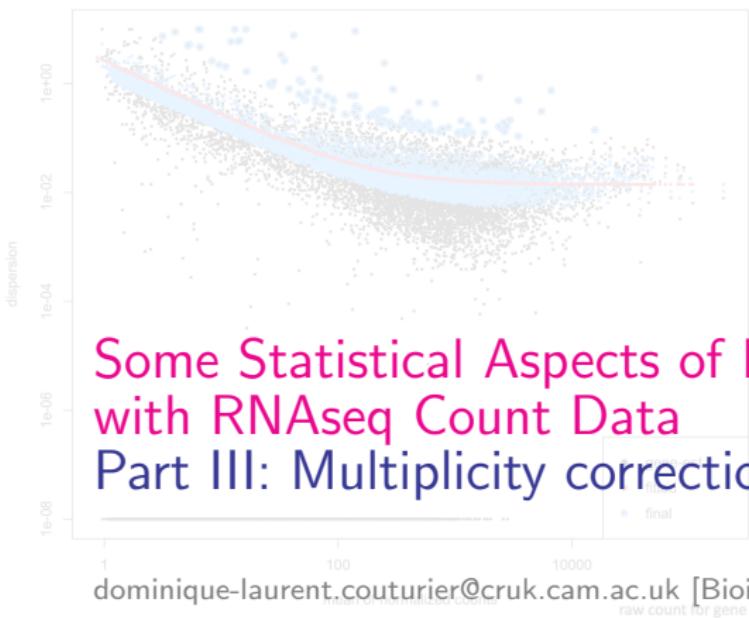


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## Some Statistical Aspects of DE Analysis with RNAseq Count Data Part III: Multiplicity correction

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(Source: G. Marot, INRIA)

raw count for gene  $i$ , sample  $j$

The mean is taken as "normalized counts" divided by a normalization factor

$$K_{ij} \sim NB(s_{ij}q_{ij}, \alpha_i)$$

one dispersion per gene

# Multiplicity correction

Experimental design

Exploration

Normalization

Differential analysis

Multiple testing

## The Family Wise Error Rate (FWER)

### Definition

Probability of having at least one Type I error (false positive), of declaring DE at least one non DE gene.

$$FWER = \mathbb{P}(FP \leq 1)$$

### The Bonferroni procedure

Either each test is realized at  $\alpha = \alpha^*/G$  level

or use of adjusted pvalue  $pBonf_i = \min(1, p_i * G)$  and  $FWER \leq \alpha^*$ .

For  $G = 2000$ ,  $\leq \alpha^* = 0.05$ ,  $\alpha = 2.510^{-5}$ .

**Easy but conservative and not powerful.**

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## The False Discovery Rate (FDR)

Idea : Do not control the error rate but the proportion of error  
⇒ less conservative than control of the FWER.

### Definition

The false discovery rate of [Benjamini and Hochberg, 1995] is the expected proportion of Type I errors among the rejected hypotheses

$$\text{FDR} = \mathbb{E}(FP/P) \text{ if } P > 0 \text{ and } 0 \text{ if } P = 0$$

### Prop

$$\text{FDR} \leq \text{FWER}$$

# Multiplicity correction

```
> set.seed(777)
> cnts <- matrix(rnbinom(n=20000, mu=100, size=1/.25), ncol=20)
> cond <- factor(rep(1:2, each=10))

> dds <- DESeqDataSetFromMatrix(cnts, DataFrame(cond), ~ cond)
> dds <- DESeq(dds)
> results(dds)

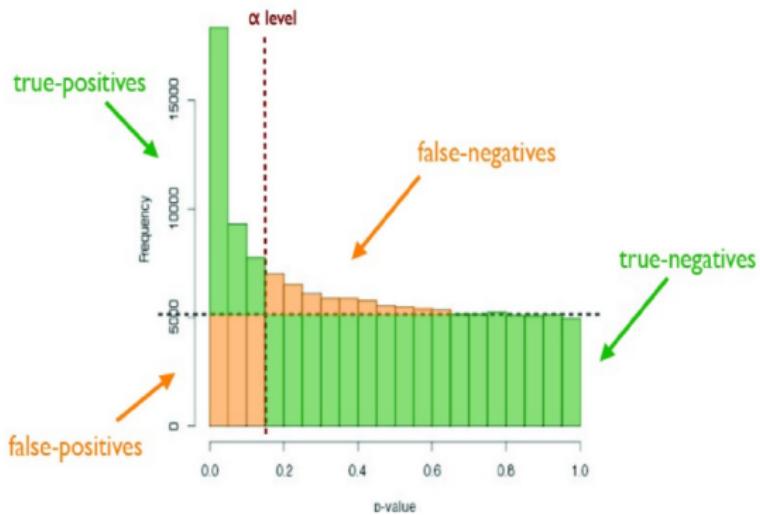
log2 fold change (MLE): cond 2 vs 1
Wald test p-value: cond 2 vs 1
DataFrame with 1000 rows and 6 columns
  baseMean log2FoldChange      lfcSE      stat     pvalue     padj
  <numeric>      <numeric>  <numeric>  <numeric>  <numeric>  <numeric>
1    97.3140     -0.682067  0.344525 -1.979730  0.0477339  0.745842
2   109.9860     -0.228819  0.450720 -0.507676  0.6116808  0.944354
3    98.8111      0.104291  0.462113  0.225683  0.8214483  0.978382
4   103.2615      0.306400  0.297682  1.029284  0.3033460  0.944354
5    97.9406      0.316338  0.357242  0.885501  0.3758864  0.944354
...
996   86.8057      0.0467703  0.287042  0.162939  0.8705668  0.980044
997  101.4437     -0.2070806  0.339886 -0.609264  0.5423495  0.944354
998   78.1356     -0.6372790  0.369515 -1.724637  0.0845930  0.824310
999   89.2920      0.7554725  0.306192  2.467314  0.0136131  0.614613
1000  103.5569     -0.0728875  0.348655 -0.209053  0.8344065  0.978382

> p.adjust(results(dds)[, "pvalue"], method="BH") [c(1:5, 996:1000)]

[1] 0.7458417 0.9443538 0.9783822 0.9443538 0.9443538 0.9800445 0.9443538 0.8243099
[9] 0.6146133 0.9783822
```

# Multiplicity correction

## Standard assumption for p-value distribution



Source : M. Guedj, Pharnext

# Multiplicity correction

Experimental design

Exploration

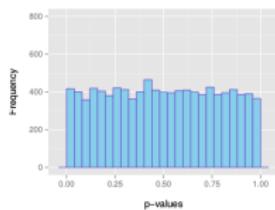
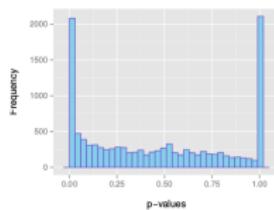
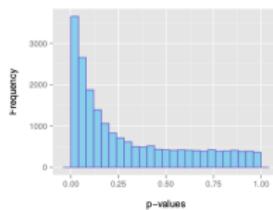
Normalization

Differential analysis

Multiple testing

## p-values histograms for diagnosis

Examples of expected overall distribution



(a) : the most desirable shape

(b) : very low counts genes usually have large p-values

(c) : do not expect positive tests after correction

# Multiplicity correction

Experimental design

Exploration

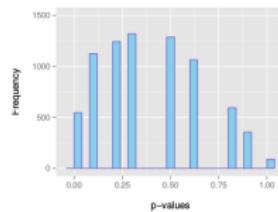
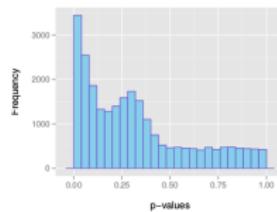
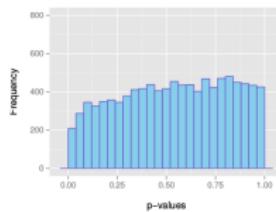
Normalization

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## p-values histograms for diagnosis

Examples of **not expected** overall distribution



- (a) : indicates a batch effect (confounding hidden variables)
- (b) : the test statistics may be inappropriate (due to strong correlation structure for instance)
- (c) : discrete distribution of p-values : unexpected

# CONCLUSION

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```