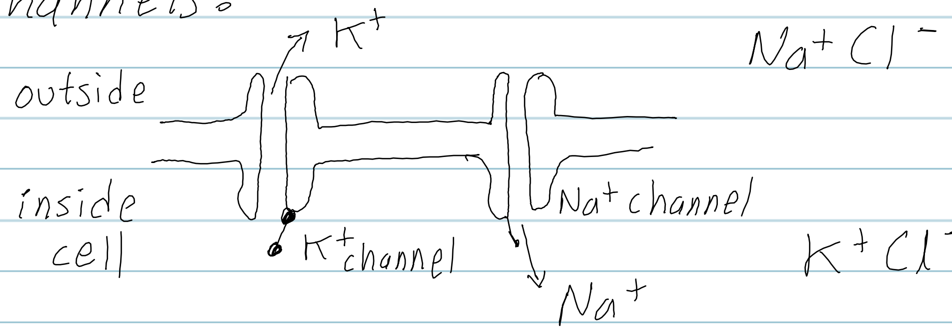
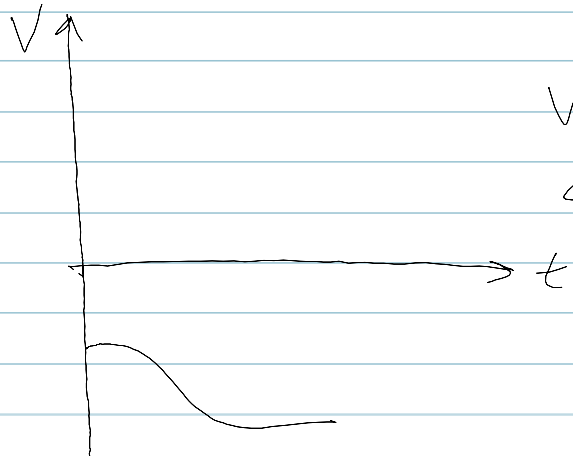
Hodgkin-Huxley Model

Paper in J. Physiology, 1952

Ion channels:

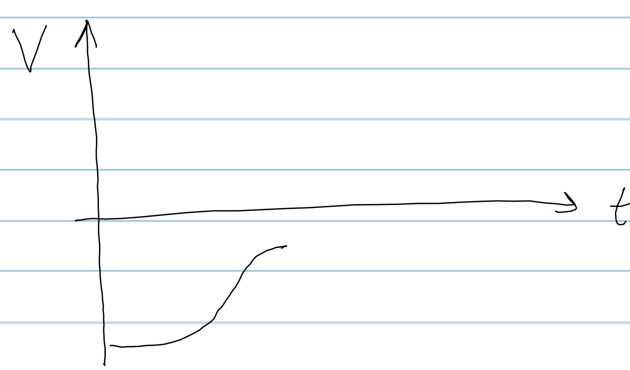


K^+ flows through K^+ channel. There is an outward current, defined as positive current.



V decreases via the quantity I_K

Na^+ flows through Na^+ channels giving an inward current, i.e. a negative current.



V increases via the quantity I_{Na}

The flux of ions through an open channel is the <u>single-channel conductance</u>

Conductance in a $1\ cm^2$ patch of membrane is

$$\bar{g}_K = (\text{single channel conductance})(\#\ \text{channels in patch})$$

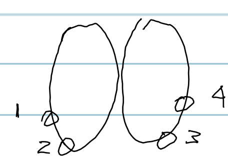
This assumes that every K^+ channel is open.

The actual conductance at some particular time when some channels may not be open is g_K (without bar)

$$g_K = \bar{g}_K \cdot \text{Prob}[K^+ \text{ channel open}]$$

Similar relation for Na^+.

In a K^+ channel, 4 "gates" must open in order for the channel to open:



→ 4 activation gates

$$\Rightarrow \text{Prob}[K^+ \text{ channel open}] = n^4$$

where n is the activation variable, $n \in [0,1]$

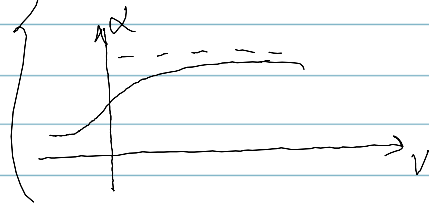
This transition can be represented as:

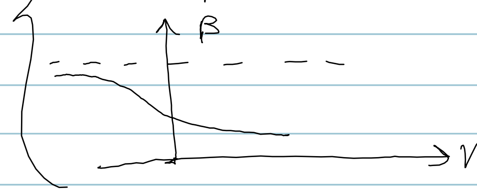
$$\underset{\underset{1-n}{\shortparallel}}{(\text{not active})} \underset{\beta}{\overset{\alpha}{\rightleftharpoons}} \underset{\underset{n}{\shortparallel}}{(\text{activated})}$$

Use the law of mass action to write down an equation for n:

(1) $$\frac{dn}{dt} = \alpha_n (1-n) - \beta_n n$$

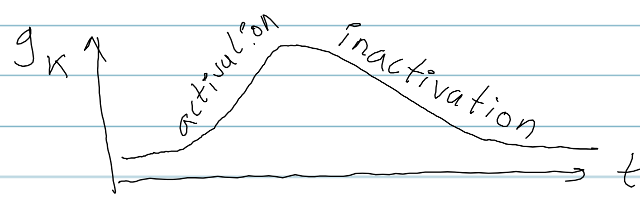
where $\alpha_n = \alpha_n(V)$ and $\beta_n = \beta_n(V)$





So, $g_K = \bar{g}_K n^4$ where n is given by (1)

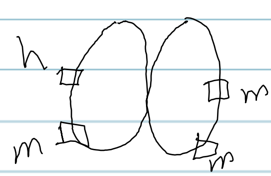
and:



Na^+ channels activate rapidly and ~~extend~~ inactivate slowly. It has two types of gates:

(i) activation gate m = Prob[gate activated]

(ii) inactivation gate h = Prob[gate not activated]



When m are activated and h is inactivated, there is flow.

$\Rightarrow$ Prob [Na^+ channel opens] $= m^3 h$,

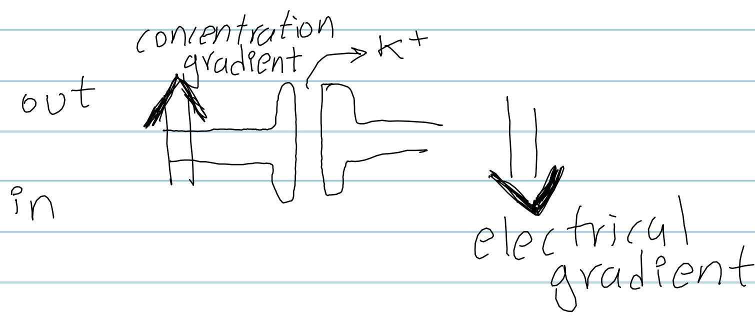
so, $$g_{Na} = \overline{g}_{Na} \, m^3 h$$

where
$$(2) \quad \frac{dm}{dt} = \alpha_m (1-m) - \beta_m m$$
$$(3) \quad \frac{dh}{dt} = \alpha_h (1-h) - \beta_h h$$

and $\alpha_m = \alpha_m(V)$, $\beta_m = \beta_m(V)$... and so on.

By ohm's law: $$V = IR = I\left(\frac{1}{g}\right)$$
$$\Rightarrow I = gV$$

It is crucial to note that there is an equilibrium potential at which there is not current flow. This potential is called Nerst potential and occurs at about -70mV.



$$V_K = \frac{RT}{F} \ln \frac{[K]_{outside}}{[K]_{inside}}$$
$$\approx -70mV$$

where R = gas constant
T = temperature
F = Faraday's constant.

For sodium: $I_{Na} = g_{Na}(V - V_{Na})$

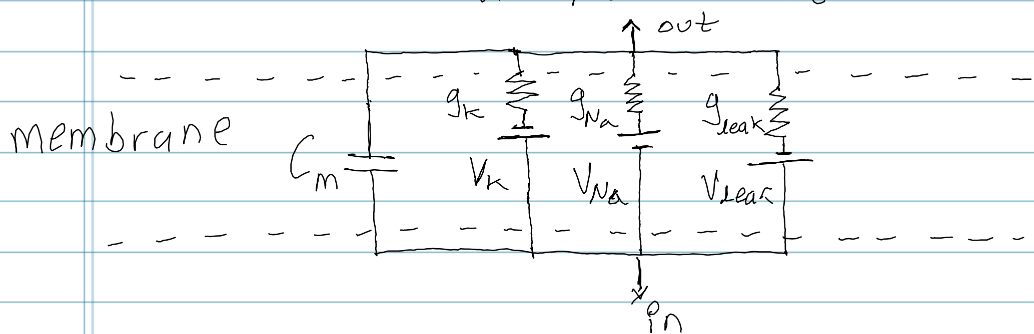
where $V_{Na} \approx 50mV$

There is also a leakage current:

$$I_{leak} = g_{leak}(V - V_{leak})$$

where g_{leak} is constant and $V_{leak} \approx -40mV$

The electrical model of the membrane is:



where C_m = membrane's capacitance (constant)
I_{cap} = capacitance current, assuming membrane = capacit

By Kirchoff's current law: $I_{cap} + I_{Na} + I_K + I_{leak} = 0$

$$\Rightarrow I_{cap} = -(I_{Na} + I_K + I_{leak})$$

(4) $$\Rightarrow C_m \frac{dV}{dt} = -\left[\bar{g}_{Na} m^3 h (V - V_{Na}) + \bar{g}_K n^4 (V - V_K) + g_{leak}(V - V_{leak})\right]$$

Equations (1) through (4) are the H-H model.