Practice Exam 1, MATH 4780/6780

Spring 2013

WARNING: TIME YOURSELF FOR ONE HOUR. The test will be different, but comparable. Please be careful with your sketches and show your work clearly. Be sure to read directions and to include the things that are requested.

1. For the system $\dot{x} = x(e^x - r)$, with $r > 0$, sketch all the qualitatively distinct phase portraits that occur as r is varied. Be sure to indicate the stability of all steady states. Determine all bifurcation points and draw a bifurcation diagram. Finally, classify the type of bifurcation or bifurcations that occur.
2. Consider the damped harmonic oscillator, $\ddot{x} + \alpha\dot{x} + x = 0$. Convert this into a system of two first-order differential equations. Then determine for all values of the friction parameter $\alpha > 0$ whether the steady state at the origin is a node or a spiral, and whether it is stable or unstable.
3. Consider the linear system $\dot{\vec{x}} = A\vec{x}$, and let $\Delta =$Det(A) and $\tau =$Trace(A). Classify (type and stability) the fixed point at the origin if:

- $\Delta = 1$, $\tau = 3$
- $\Delta = -1$, $\tau = 2$
- $\Delta = 1$, $\tau = 0$
- $\Delta = 0$, $\tau = 1$

4. Sketch a phase portrait for the system $\dot{x} = x^2 - y$, $\dot{y} = x - y$. Include nullclines and direction arrows, and perform a linear stability analysis for all steady states. Sketch an approximate stable manifold for the saddle point.

5. Answer the following questions about planar systems:

(a) Suppose that the eigenvalues of a linear system are equal. How can you tell whether $\vec{x}^* = \vec{0}$ is a degenerate node or a star node?
(b) Is hysteresis possible in the vicinity of a transcritical bifurcation? Why or why not?
(c) Suppose that a planar nonlinear system has two stable fixed points and a saddle point. What curve separates the basins of attraction of the stable fixed points? That is, what curve forms the separatrix?
(d) For what type of steady state is the linearization technique for determining stability not valid?