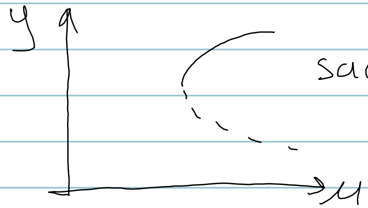
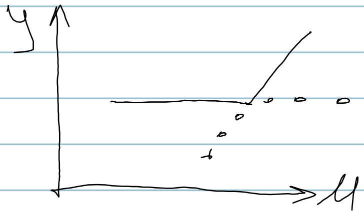
# Additional notes on bifurcations

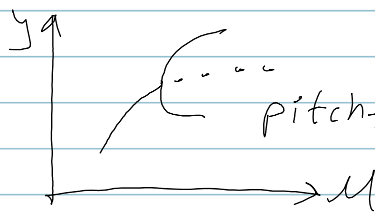
Recall the different type of bifurcations:



saddle node



transcritical



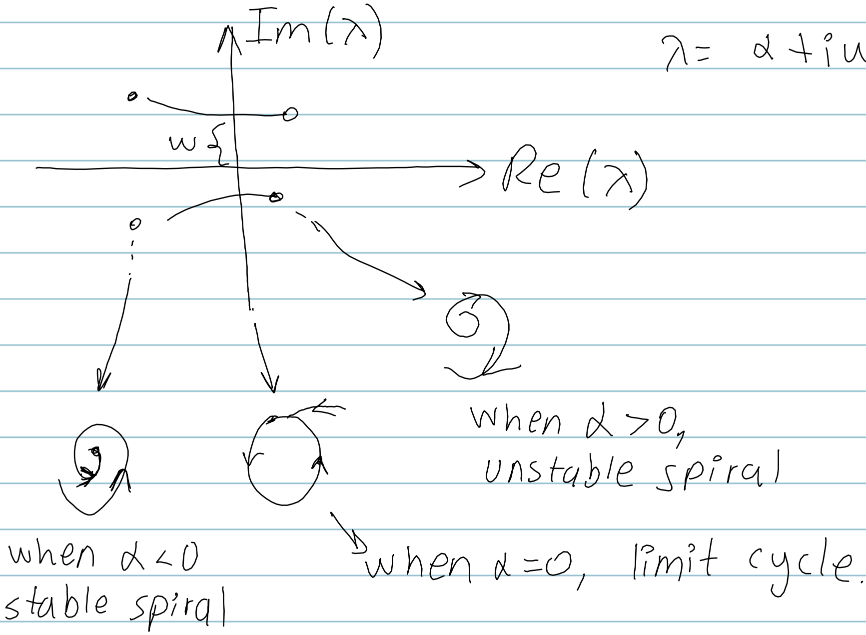
pitchfork

In each case, there is a zero eigenvalue at the bifurcation point.

How are periodic solutions (limit cycles) born?

# Hopf Bifurcation:

$\lambda = \alpha + i\omega$

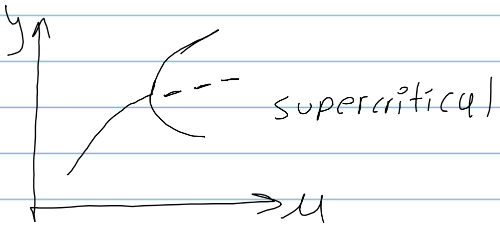


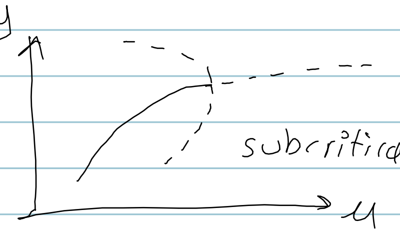

$\omega$ is angular frequency

$\omega = 2\pi f$, where $f$ is frequency.

when $\alpha = 0$, the period of oscillation is $T = \frac{2\pi}{\omega}$, since period is the reciprocal of frequency ($f$)

The Hopf bifurcation can be supercritical if the limit cycle is stable, or subcritical if the limit cycle is unstable:



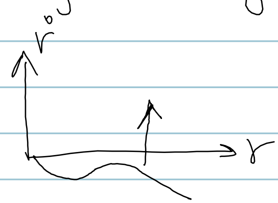



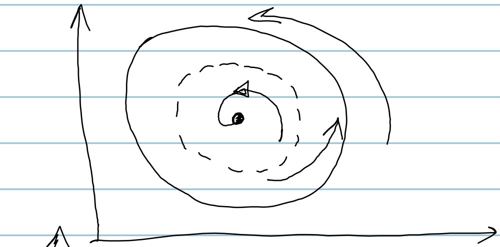

<u>Global bifurcations:</u>

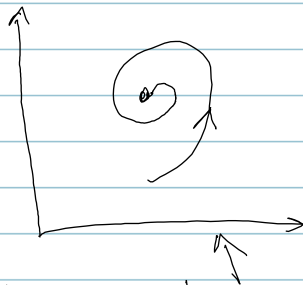
So far all bifurcations have been local involving linearization about S.S. Some bifurcations are off of limit cycles and steady states

- Saddle node of periodics

Study the system $\dot{r} = \mu r + r^3 - r^5$
$\dot{\theta} = \omega + b r^2$





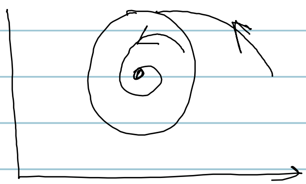


1 outer stable limit cycle

⊥ inner unstable L.C.

⊥ inner stable S.S.

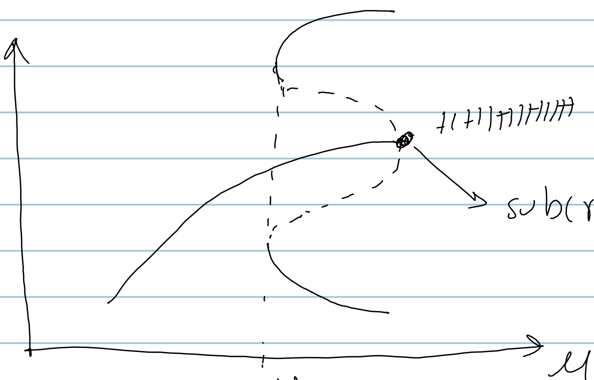
⊥ stable state



⊥ inner S.S.

⊥ half-stable L.C

Bifurcation diagram:



The unstable branch of periodics is created at subcritical Hopf bifurcation.

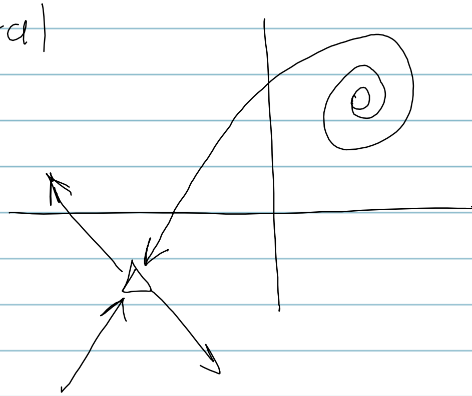
Homoclinic Bifurcations:

Needs a saddle point.

Δ = saddle

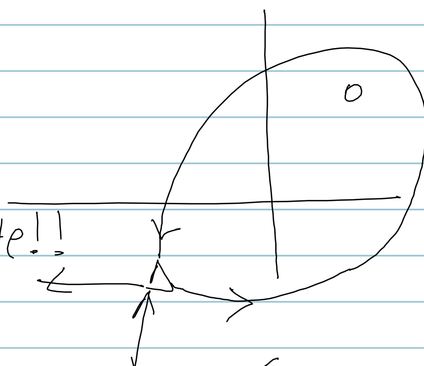
o = unstable spiral

$\mu < \mu_{HM}$

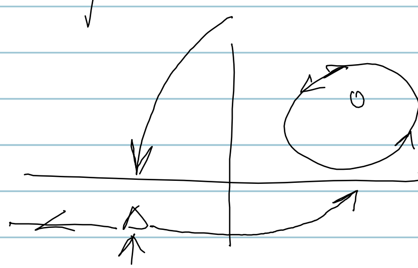


$\mu = \mu_{HM}$

period is infinite!!



$\mu > \mu_{HM}$



Limit cycle born at homoclinic bifurcation. It has large but finite period.