Appendix A

Algebraic Formulas

Algebraic Manipulation

$x + y = y + x$	Commutative Rule (addition)	(A.1)
$xy = yx$	Commutative Rule (addition)	(A.2)
$(x + y) + z = z + (y + z)$	Associative Rule (addition)	(A.3)
$(xy)z = x(yz)$	Associative Rule (multiplication)	(A.4)
$x(y + z) = xy + xz$	Distributive Rule	(A.5)

Fractions

$\dfrac{x+y}{z} = \dfrac{x}{z} + \dfrac{y}{z}$	(A.6)	$\dfrac{x}{p} + \dfrac{y}{q} = \dfrac{xp + yq}{pq}$	(A.7)
$\dfrac{x}{y} \cdot \dfrac{p}{q} = \dfrac{xp}{yq}$	(A.8)	$\dfrac{x/y}{p/q} = \dfrac{xq}{py}$	(A.9)

Differences of Powers

$$x^1 - y^1 = (x - y) \tag{A.10}$$

$$x^2 - y^2 = (x - y)(x + y) \tag{A.11}$$

$$x^3 - y^3 = (x - y)(x^2 + xy + y^2) \tag{A.12}$$

$$x^4 - y^4 = (x - y)(x^3 + x^2y + xy^2 + y^3) \tag{A.13}$$

$$\begin{aligned} x^n - y^n = (x - y)(&x^{n-1} + x^{n-2}y + x^{n-3}y^2 + \cdots \\ &+x^2y^{n-3} + xy^{n-2} + y^{n-1}) \end{aligned} \tag{A.14}$$

Completing the Squares

$$ax^2 + bx + c = a\left(x^2 + \frac{b}{a}x\right) + c = a\left(x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2\right) + c \tag{A.15}$$

$$= a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a} \tag{A.16}$$

Quadratic Formula

The roots of $ax^2 + bx + c = 0$ are given by $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Binomial Theorem

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k} = \sum_{k=0}^{n} \frac{n!}{k!(n-k)!} x^k y^{n-k} \tag{A.17}$$

$$(x+y)^2 = x^2 + 2xy + y^2 \tag{A.18}$$

$$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^2 \tag{A.19}$$

$$(x+y)^4 = x^4 + 4x^3y + 6^2y^2 + 4xy^3 + y^4 \tag{A.20}$$

$$(x+y)^n = \binom{n}{0} x^n y^0 + \binom{n}{1} x^{n-1} y^1 + \binom{n}{2} x^{n-2} y^2 + \cdots + \binom{n}{n-1} x^1 y^{n-1} + \binom{n}{n} x^0 y^n \tag{A.21}$$

Pascals Triangle

The coefficients in the sum of $(x+y)^m$ are given by the m^{th} row of Pascal's Triangle. Each number on the inside is the sum of the two numbers above it. The equation is given by the **Binomial Theorem**.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

Equation of a Line

Let m be the slope of a line, a be its x intercept, b its y intercept, and (x_1, y_1) and (x_2, y_2) be any distinct points on the line, with $x_1 \neq x_2$.

$\frac{x}{a} + \frac{y}{b} = 1$	intercept-intercept form	(A.22)
$m = \frac{y_2 - y_1}{x_2 - x_1}$	slope of the line	(A.23)
$y = mx + b$	slope - intercept form	(A.24)
$y = y_1 + m(x - x_1)$	point-slope form	(A.25)

Roots and Exponents

$x^n = \overbrace{x \cdot x \cdot x \cdots x}^{\text{repeated } n \text{ times}}$	(A.26)	$y = x^n \iff x = y^{1/n}$	(A.27)
$x^{1/n} = \sqrt[n]{x}$	(A.28)	$x^{-n} = \dfrac{1}{x^n},\ (x \neq 0)$	(A.29)
$\sqrt{\dfrac{x}{y}} = \dfrac{\sqrt{x}}{\sqrt{y}}$	(A.30)	$\sqrt{xy} = \sqrt{x}\sqrt{y}$	(A.31)
$x^{p-q} = \dfrac{x^p}{x^q}$	(A.32)	$x^{p+q} = x^p \cdot x^q$	(A.33)
$x^{p/q} = \sqrt[q]{x^p}$	(A.34)	$(x^p)^q = x^{pq}$	(A.35)
$(xy)^p = x^p y^p$	(A.36)	$\left(\dfrac{x}{y}\right)^n = \dfrac{x^n}{y^n},\ (y \neq 0)$	(A.37)

Sums of Powers of Integers

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2} \tag{A.38}$$

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6} \tag{A.39}$$

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4} \tag{A.40}$$

Exponentials and Logarithms

$y = e^x \iff x = \ln y$	(A.41)	$\ln(xy) = \ln x + \ln y$	(A.42)
$e^{\ln x} = \ln e^x = x$	(A.43)	$\ln(x/y) = \ln x - \ln y$	(A.44)
$e^{x+y} = e^x e^y$	(A.45)	$\ln y^x = x \ln y$	(A.46)
$e^{x-y} = \dfrac{e^x}{e^y}$	(A.47)	$\log_b x = \dfrac{\ln x}{\ln b}$	(A.48)

Hyperbolic Functions and Identities

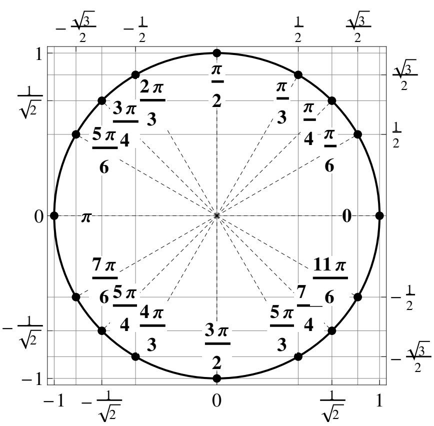
$\cosh x = \frac{1}{2}(e^x + e^{-x})$	(A.49)	$\sinh x = \frac{1}{2}(e^x - e^{-x})$	(A.50)
$e^x = \cosh x + \sinh x$	(A.51)	$e^{-x} = \cosh x - \sinh x$	(A.52)
$\cosh^2 x - \sinh^2 x = 1$	(A.53)	$1 - \tanh^2 x = \operatorname{sech}^2 x$	(A.54)
$\cosh(2x) = \cosh^2 x + \sinh^2 x$	(A.55)	$\sinh 2x = 2 \sinh x \cosh x$	(A.56)
$\operatorname{arcsinh}(x) = \ln\left(x + \sqrt{x^2+1}\right)$	(A.57)	$\operatorname{arccosh}(x) = \ln\left(x + \sqrt{x^2-1}\right)$	(A.58)

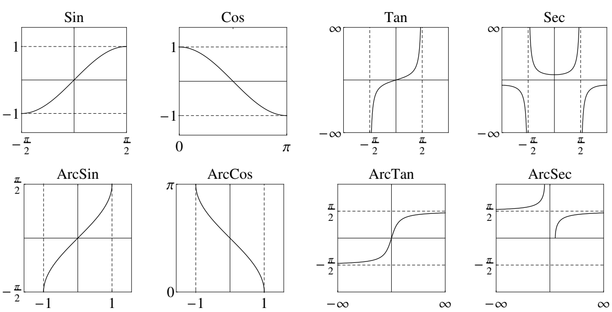
Trigonometry

$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$ (A.59)	$\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$ (A.60)
$\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{\text{opposite}}{\text{adjacent}}$ (A.61)	$\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{\cos\theta}{\sin\theta} = \dfrac{\text{adjacent}}{\text{opposite}}$ (A.62)
$\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{\text{hypotenuse}}{\text{adjacent}}$ (A.63)	$\csc\theta = \dfrac{1}{\sin\theta} = \dfrac{\text{hpotenuse}}{\text{opposite}}$ (A.64)

Trigonometric Identities

$\sin^2 x + \cos^2 x = 1$ (A.65)	$\tan^2 x + 1 = \sec^2 x$ (A.66)
$1 + \cot^2 x = \csc^2 x$ (A.67)	$\cos(2x) = \cos^2 x - \sin^2 x$ (A.68)
$\sin(2x) = 2\sin x \cos x$ (A.69)	$\tan(2x) = \dfrac{2\tan x}{1-\tan^2 x}$ (A.70)
$\sin^2 x = \dfrac{1}{2}(1-\cos(2x))$ (A.71)	$\cos^2 x = \dfrac{1}{2}(1+\cos(2x))$ (A.72)
$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$ (A.73)	$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$ (A.74)





θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞	-2	$-\sqrt{2}$	$-\frac{2}{\sqrt{3}}$	-1
$\csc\theta$	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞
$\cot\theta$	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	$-\frac{1}{\sqrt{3}}$	-1	$-\sqrt{3}$	∞

Appendix B

Table of Derivatives

$$\frac{d}{dx}(Cf(x)) = C\frac{d}{dx}f(x) \tag{B.1}$$

$$\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}f(x) \pm \frac{d}{dx}g(x) \tag{B.2}$$

$$\frac{d}{dx}\left(f(x) \cdot g(x)\right) = f(x) \cdot g'(x) + f'(x) \cdot g(x) \tag{B.3}$$

$$\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2} \tag{B.4}$$

$$\frac{d}{dx}f(g(x)) = f'(g(x))g'(x) \tag{B.5}$$

$$\frac{d}{dx}x^n = nx^{n-1} \tag{B.6}$$

$$\frac{d}{dx}f(x)^n = nf(x)^{n-1}f'(x) \tag{B.7}$$

$$\frac{d}{dx}\ln x = \frac{1}{x} \tag{B.8}$$

$$\frac{d}{dx}\ln f(x) = \frac{f'(x)}{f(x)} \tag{B.9}$$

$$\frac{d}{dx}e^x = e^x \tag{B.10}$$

$$\frac{d}{dx}e^{f(x)} = e^{f(x)}f'(x) \tag{B.11}$$

$$\frac{d}{dx}a^x = a^x \ln a \tag{B.12}$$

$$\frac{d}{dx}a^{f(x)} = a^{f(x)}f'(x)\ln a \tag{B.13}$$

$$\frac{d}{dx}x^x = x^x(1 + \ln x) \tag{B.14}$$

$$\frac{d}{dx}x^{f(x)} = x^{f(x)}\left(\frac{f(x)}{x} + f'(x)\ln x\right) \tag{B.15}$$

$$\frac{d}{dx}\sin x = \cos x \tag{B.16}$$

$$\frac{d}{dx}\cos x = -\sin x \tag{B.17}$$

$$\frac{d}{dx}\tan x = \sec^2 x \tag{B.18}$$

$$\frac{d}{dx}\cot x = -\csc^2 x \tag{B.19}$$

$$\frac{d}{dx}\sec x = \sec x \tan x \tag{B.20}$$

$$\frac{d}{dx}\csc x = -\csc x \cot x \tag{B.21}$$

$$\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1 - x^2}} \tag{B.22}$$

$$\frac{d}{dx}\cos^{-1} x = -\frac{1}{\sqrt{1 - x^2}} \tag{B.23}$$

$$\frac{d}{dx}\tan^{-1} x = \frac{1}{x^2 + 1} \tag{B.24}$$

$$\frac{d}{dx}\cot^{-1} x = -\frac{1}{x^2 + 1} \tag{B.25}$$

$$\frac{d}{dx}\sec^{-1} x = \frac{1}{x\sqrt{x^2 - 1}} \tag{B.26}$$

$$\frac{d}{dx}\csc^{-1} x = -\frac{1}{x\sqrt{x^2 - 1}} \tag{B.27}$$

$$\frac{d}{dx}\sinh x = \cosh x \tag{B.28}$$

$$\frac{d}{dx}\cosh x = \sinh x \tag{B.29}$$

$$\frac{d}{dx}\tanh x = \operatorname{sech}^2 x \tag{B.30}$$

$$\frac{d}{dx}\operatorname{coth}\ x = -\operatorname{csch}^2\ x \tag{B.31}$$

$$\frac{d}{dx}\operatorname{sech}\ x = -\tanh x\ \operatorname{sech} \tag{B.32}$$

$$\frac{d}{dx}\operatorname{csch}\ x = -\coth x\ \operatorname{csch}\ x \tag{B.33}$$

$$\frac{d}{dx}\sinh^{-1} x = \frac{1}{\sqrt{x^2+1}} \tag{B.34}$$

$$\frac{d}{dx}\cosh^{-1} x = \frac{1}{\sqrt{x^2-1}} \tag{B.35}$$

$$\frac{d}{dx}\tanh^{-1} x = \frac{1}{1-x^2} \tag{B.36}$$

$$\frac{d}{dx}\coth^{-1} x = -\frac{1}{1-x^2} \tag{B.37}$$

$$\frac{d}{dx}\operatorname{sech}^{-1} x = -\frac{1}{x\sqrt{1-x^2}} \tag{B.38}$$

$$\frac{d}{dx}\operatorname{csch}^{-1} x = -\frac{1}{|x|\sqrt{1+x^2}} \tag{B.39}$$

Appendix C

Table of Integrals

$$\int Cf(x)dx = C\int f(x)dx \tag{C.1}$$

$$\int (f \pm g)dx = \int f dx \pm \int g dx \tag{C.2}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1}, n \neq -1 \tag{C.3}$$

$$\int \frac{1}{x} dx = \ln|x| \tag{C.4}$$

$$\int \ln x \; dx = x\ln x - x \tag{C.5}$$

$$\int x\ln x = \frac{1}{2}x^2\ln x - \frac{1}{4}x^2 \tag{C.6}$$

$$\int e^x dx = e^x \tag{C.7}$$

$$\int xe^x \; dx = xe^x - e^x \tag{C.8}$$

$$\int x^2 e^x \; dx = e^x(x^2 - 2x + 2) \tag{C.9}$$

$$\int a^x dx = \frac{a^x}{\ln a} \tag{C.10}$$

$$\int xa^x dx = \frac{a^x(x\ln a - 1)}{(\ln a)^2} \tag{C.11}$$

$$\int \sin x dx = -\cos x \tag{C.12}$$

$$\int \cos x dx = \sin x \tag{C.13}$$

$$\int \tan x dx = \ln|\sec x| \tag{C.14}$$

$$\int \cot x dx = \ln|\sin x| \tag{C.15}$$

$$\int \sec x dx = \ln|\sec x + \tan x| \tag{C.16}$$

$$\int \csc x dx = -\ln|\csc x + \cot x| \tag{C.17}$$

$$\int \sec^2 x dx = \tan x \tag{C.18}$$

$$\int \csc^2 x dx = -\cot x \tag{C.19}$$

$$\int \sec x\tan x dx = \sec x \tag{C.20}$$

$$\int \csc x\cot x dx = -cscx \tag{C.21}$$

$$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x \tag{C.22}$$

$$\int \frac{dx}{1+x^2} = \tan^{-1} x \tag{C.23}$$

$$\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1} x \tag{C.24}$$

$$\int \frac{dx}{\sqrt{x^2+1}} = \sinh^{-1} x \tag{C.25}$$

$$\int \frac{dx}{\sqrt{x^2-1}} = \cosh^{-1} x \tag{C.26}$$

$$\int \frac{dx}{1-x^2} = \tanh^{-1} x \tag{C.27}$$

$$\int \sinh x \ dx = \cosh x \tag{C.28}$$

$$\int \cosh x \ dx = \sinh x \tag{C.29}$$

$$\int \tanh x \ dx = \ln|\cosh x| \tag{C.30}$$

$$\int \coth x \ dx = \ln|\sinh x| \tag{C.31}$$

$$\int \operatorname{sech} x \, dx = 2\tan^{-1} \ \tanh\frac{x}{2} \tag{C.32}$$

$$\int \operatorname{csch} x \, dx = \ln\tanh\frac{x}{2} \tag{C.33}$$

$$\int \operatorname{sech}^2 x \ dx = \tanh x \tag{C.34}$$

$$\int \operatorname{csch}^2 x \ dx = -\coth \ x \tag{C.35}$$

$$\int \operatorname{sech} \ x \tanh x \ dx = -\operatorname{sech} \ x \tag{C.36}$$

$$\int \operatorname{csch} \ x \ \coth \ x \ dx = -\operatorname{csch} \ x \tag{C.37}$$

$$\int x\sin x \ dx = \sin x - x\cos x \tag{C.38}$$

$$\int x\cos x \ dx = \cos x + x\sin x \tag{C.39}$$

$$\int e^x \sin x \ dx = \frac{e^x}{2}\left[\sin x - \cos x\right] \tag{C.40}$$

$$\int e^x \cos x \ dx = \frac{e^x}{2}\left[\sin x + \cos x\right] \tag{C.41}$$

