# BMEN 619.24

# HW01

Sayed Thaqafi

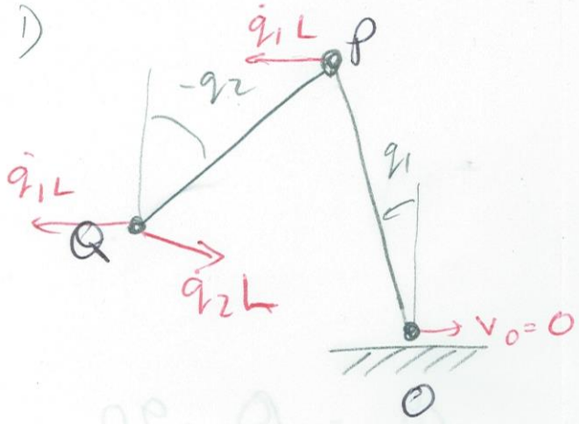


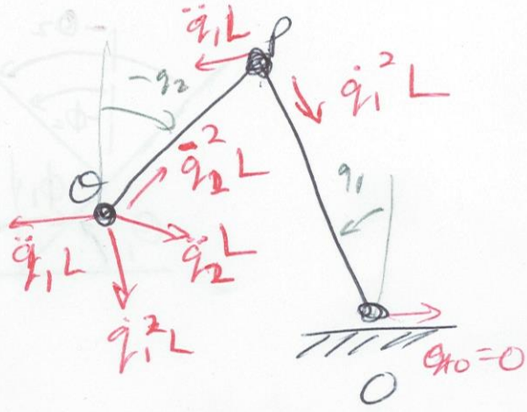
## Velocities

## Acceleration

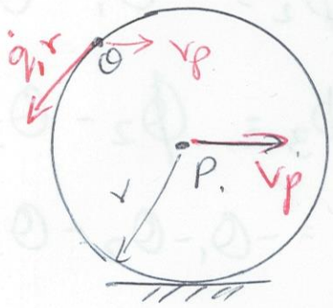
A1)

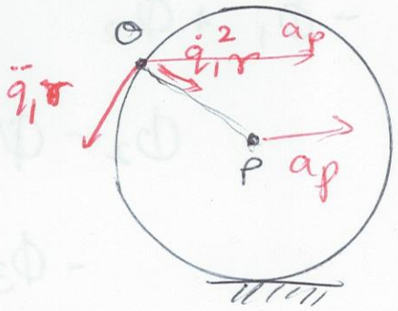
1)



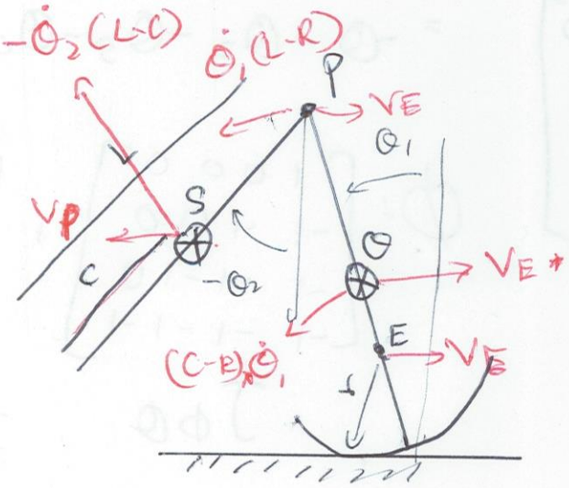



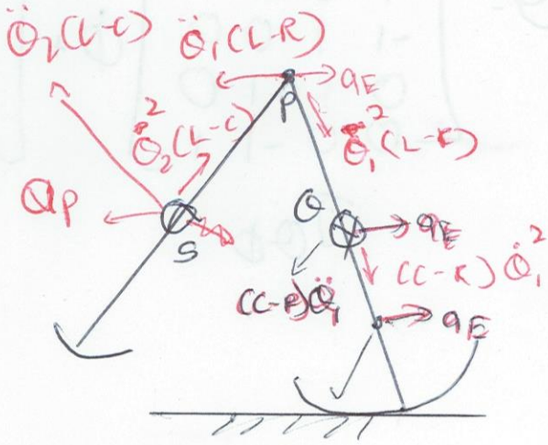

2)



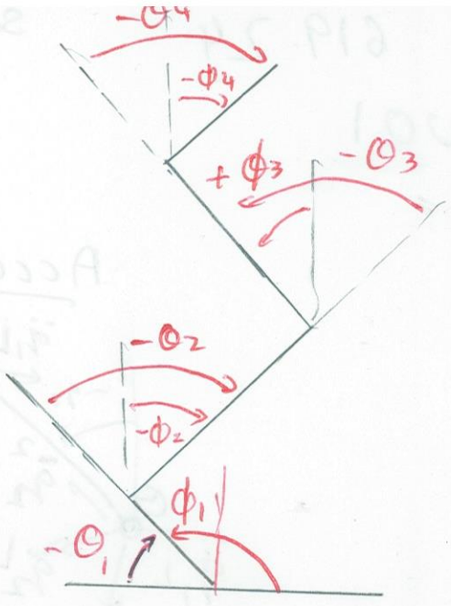




3)



A2A)




$$\theta_1 = \phi_1 - 90$$

$$\theta_2 = -\phi_1 + \phi_2$$

$$\theta_3 = \phi_2 - \phi_3$$

$$\theta_4 = -\phi_3 + \phi_4$$

$$\theta = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix} \phi + \begin{bmatrix} -90 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$J_{\theta\phi}$

$$\phi_1 = \theta_1 + 90$$

$$\phi_2 = -\theta_1 - \theta_2 + 90$$

$$\phi_3 = \phi_2 - \theta_3$$

$$= -\theta_1 - \theta_2 - \theta_3 + 90$$

$$\phi_4 = \phi_3 - \theta_4$$

$$= -\theta_1 - \theta_2 - \theta_3 - \theta_4 + 90$$

$$\phi = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 \\ -1 & -1 & -1 & 0 \\ -1 & -1 & -1 & -1 \end{bmatrix} + \begin{bmatrix} 90 \\ 90 \\ 90 \\ 90 \end{bmatrix}$$

$J_{\phi\theta}$

A2A) No offset ∴ Jacobian is a transformation, initial conditions are left out Like in integration.

A2B) $$\dot{X} = J_{x\phi}\,\dot{\phi}$$

$$\underset{12\times 1}{\begin{bmatrix} \dot{x}_{c1} \\ \dot{y}_{c1} \\ \dot{\phi}_{c2} \\ \vdots \end{bmatrix}} = \underset{12\times 4}{\begin{bmatrix} -l_1\cos & 0 & 0 & 0 \\ -l_1\sin & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & +l_2\cos & 0 & 0 \\ 0 & -l_2\sin & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix}} \underset{4\times 1}{\begin{bmatrix} \dot{\phi}_1 \\ \dot{\phi}_2 \\ \dot{\phi}_3 \\ \dot{\phi}_4 \end{bmatrix}}$$

A2C)

$$\dot{X} = J_{x\phi}\,\dot{\phi} \quad -①$$

$$\theta = J_{\theta\phi}\,\phi$$

$$\dot{\theta} = J_{\theta\phi}\,\dot{\phi}$$

$$J_{\theta\phi}^{-1}\,\dot{\theta} = \dot{\phi} \quad -②$$

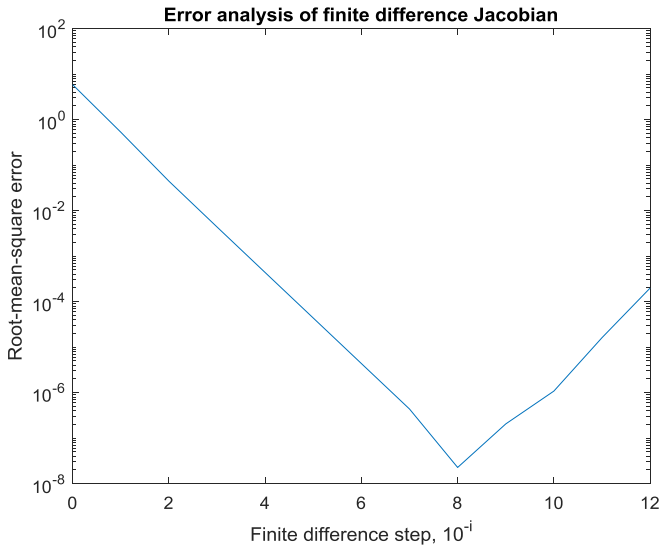
Sub ② into ①

$$\dot{X} = \left[J_{x\phi}\,J_{\theta\phi}^{-1}\right]\dot{\theta}$$

$$\downarrow$$

$$J_{x\theta}$$

3B)




I expected the numerical methods to get more and more accurate as step size increases, but with diminishing returns. I cannot explain why the error grows again after a point.

A5a)

The energy is conserved and remains constant.

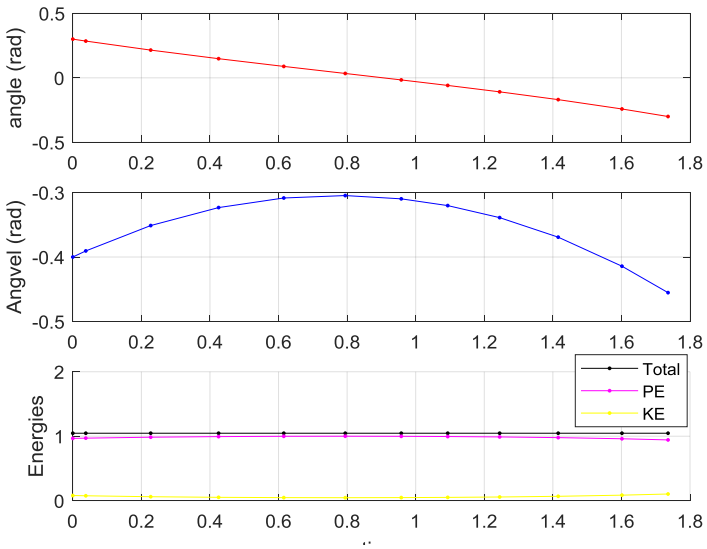



Figure 1: the angle, angular velocity and energies of the rimless wheel over 1 stance phase.

5B

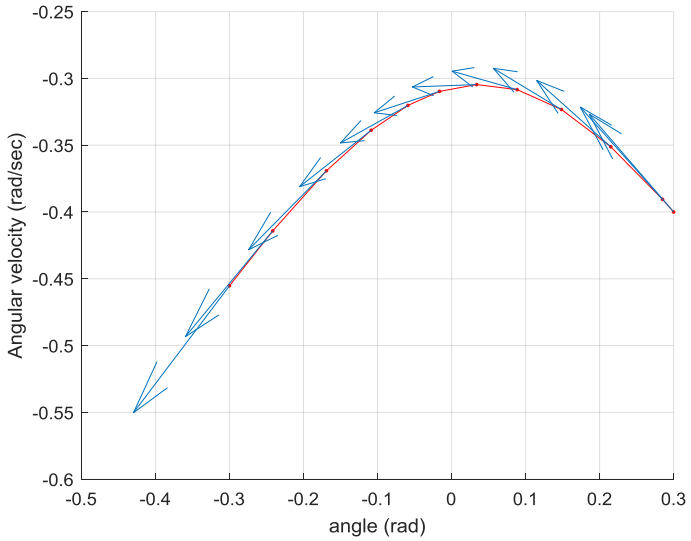



Figure 2 The phase plot for the rimless wheel over 1 stance phase. The arrows indicate state derivatives.

A5c. When allowed to run for 20 steps, the wheel slowly settles into a periodic limit cycle.