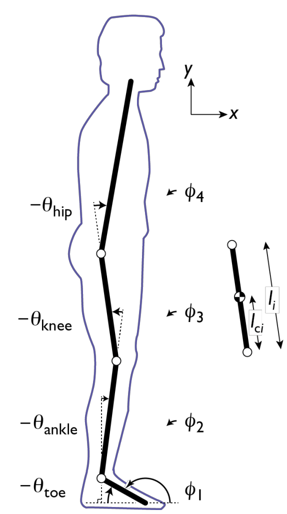
Homework 03

1. The file `legextension` demonstrates a Hill model of maximum knee moment during leg extension with two legs. It assumes a quasi-static situation and models the force-length curves of muscle. There is no force-velocity relationship, nor tendon.
   A. Plot the maximum available moment that can be exerted by the two legs for three cases: moment arm as given, and moment arm ± 10%. In a few sentences, explain how the maximum moment is calculated. (Try to understand and interpret the code.)
   B. Examine the m-file, and modify it to plot the active and passive force-length curves. Try a range of 50% to 150% of optimal fiber length. Briefly explain what the curves represent.
   C. Suppose the extension moment was not quasi-static, that is, when force-velocity matters. What would you expect would happen to the maximum moment curve?
   D. The model also does not include the effect of elastic tendon. What would (qualitatively) be expected to happen to the maximum moment vs. angle curve if elastic tendon were included? Explain in a sentence or two.

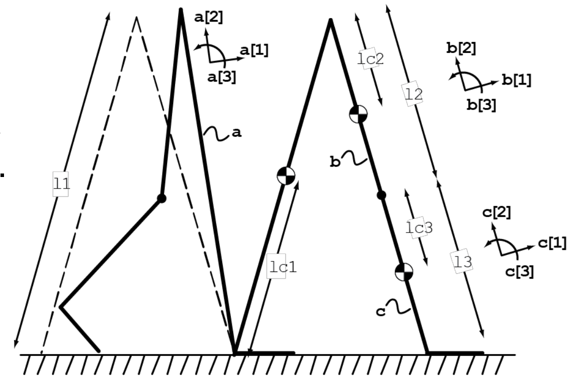
2. We previously found the pose Jacobian $J_{\mathbf{X}\phi}$ for a sagittal plane representation of the human body. For the same system, $m_1 - m_4$ are the masses of the segments, $l_1 - l_4$ are the lengths, $l_{c1} - l_{c4}$ are the distances from the bottom of the segment to its center of mass. Define x and y directions as positive to the right and upward, respectively.



   A. Find a Jacobian transforming segment angle displacements into whole body COM translational displacements. Assume that $J_{\mathbf{X}\phi}$ is known.
   B. Sketch a configuration of interest to you (e.g. squatting, sitting), and sketch arrows indicating the motion of each segment's center of mass that would result from a unit displacement in each of the joint angles. Based on that, also try to predict the overall COM motion that would result from each joint motion. It is not necessary to make any calculations, and the masses and other parameters may be estimated qualitatively.
   C. Find an expression for a mathematical constraint that must be satisfied if the body center of mass is required to move only vertically, that is, with zero horizontal motion.
   D. Now that you have a relationship between joint angular velocities and COM velocity, what does the corresponding force/torque relationship represent, using the transformer rule?
   E. In your own words, describe what the columns of the Jacobian (for whole body center of mass) represent.
   F. Describe what the rows of the Jacobian represent.

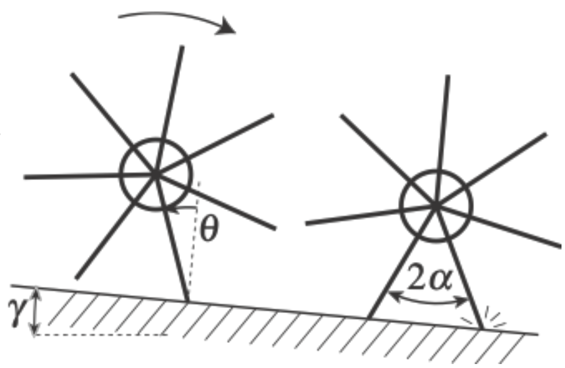
3. In model simulations, the initial and final conditions must often be determined computationally. This typically entails defining a constraint function, such as the event function used for detection of final conditions. Here we will use a constraint function to perform inverse kinematic assembly, for the ballistic walking model. It is desired that the initial configuration for the simulation be such that the swing toe is on the ground, and is separated from the stance toe by a given step length `SL`.

A. Starting with `assembleBWroot` , assemble the initial configuration for the model. The configuration is actually under-determined. Treat `q1` as fixed, and find the `q2` and `q3` that satisfy the conditions. The `drawballwalk` function may be helpful for understanding the kinematics.



B. The initial velocities have yet to be determined. Suppose the desired final condition is that the heel strikes the ground with step length `SL` , simultaneous with the knee reaching full extension, in a desired step period `ST` . Explain how this could be used to determine the initial velocities to correspond with the initial configuration, including solving for `q1` . This is an example of a two-point boundary value problem. (No need to implement this in Matlab.)

4. The file `testrimlesswheel2` demonstrates a periodic gait (limit cycle) found by root-finding, as well as a parameter study.



A. Examine the parameter study of the effect of ground slope `gamma` on walking speed. Briefly explain how slope affects walking speed, and why there appears to be a minimum slope and speed for this system.

B. Add your own parameter study examining how half inter-leg angle `alpha` affects speed. Briefly explain what happens, and why there appears to be a maximum `alpha` that yields a period gait.

C. Notice that the parameter studies work best when a parameter is varied by small increments, and the initial guess for root-finding uses the previous, nearby solution. Explain why this works better than providing a single initial guess for all cases.