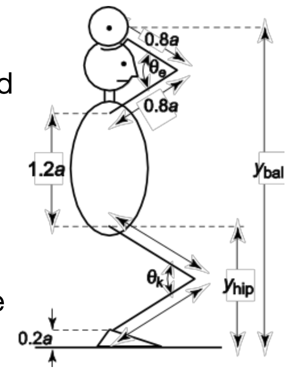
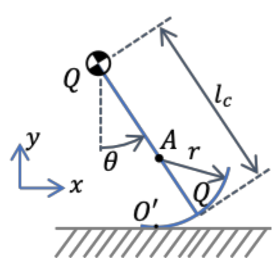
Homework 07

1. The Matlab file `runshotput.m` runs a simulation of the shotput using Alexander\'s model. It is designed to study the observation that shotputters tend to activate their lower extremity muscles first, followed by upper extremity muscles, with delay parameter `tdelay`. Examine the m-file, and try to understand how the simulation is performed.



   A. Using the provided intermediate-delay simulation, find the ball's speed and energy at the time of release, in the default dimensionless units (or, equivalent to speed in units of $\sqrt{a/g}$ and energy in units of Mga. Then express these same quantities in SI units, for a typical adult. How much more energy would the ball be expected to gain if thrown by a person twice as heavy? (Hint: Try it in simulation, or look at how M enters into the equations.)
   B. Notice that the simulations performed in (a.) all end due to maximum torque-velocity limits. Find the parameter that makes this a limiting factor, and try changing it to see if it results in faster throws. Briefly explain the result.
   C. What happens to ball energy and release time when small and larger values of 0.2 and 1.0 are used? Examine the time over which the legs are extending, and the time for the arm. Try to explain why intermediate delays are better, by examining these times as well as the angle and angular velocity trajectories over time.
   D. Plot the ball's relative energy (normalizing for ball mass) as a function of dimensionless delay ranging from 0 to 1, and repeat this for ball masses of 0.04, 0.10, and 0.25 *M* body mass). How does the optimum delay time change with increasing ball mass? How does the relative work performed on the ball change with ball mass?
   E. The provided forward kinematics function shows the velocities computed explicitly. However, transformation may also be written in Jacobian form, $\dot{y} = J\dot{\theta}$. Find J.
   F. Note that the simulation uses states $x = \left[y_{\text{hip}}, y_{\text{ball}}, \dot{y}_{\text{hip}}, \dot{y}_{\text{ball}}\right]^T$, but the dynamics are largely described in terms of θ_{knee}, θ_{elbow}, etc. Briefly describe how you would formulate the simulation using joint angle states. Note that the state-derivative equations yield $\ddot{y}_{\text{hip}}$ and $\ddot{y}_{\text{ball}}$.

2. Find constraint Jacobians for a rolling arc foot. The arc foot has radius r, and is attached to a leg with angle θ measured counter-clockwise from vertical. Each of the points O', A, Q has Cartesian location such as $\left(x_Q, y_Q\right) = (-r\theta - (l_c - r)\sin\theta, ?)$, where the origin is located at where the foot-ground contact point would be when $\theta = 0$.



   A. Find the pose Jacobian to describe the center-of-mass and segment pose, $V_Q = \left[\dot{x}_Q, \dot{y}_Q, \dot{\theta}\right]^T$. First find the full position $\left(x_Q, y_Q\right)$, and take its time-derivative to find the Jacobian J_p, where $V_Q = J_p\dot{\theta}$.

B. Use graphical kinematics to find the pose Jacobian separately. Use $\mathbf{v}_Q = \mathbf{v}_A + \omega \times \mathbf{r}_{Q/A}$, where for example $\mathbf{v}_A = [?,\ 0]^T$. Notice that no time-differentiation is needed, because the kinematics equation already includes velocities.
C. Find the corresponding constraint Jacobian, such that $J_c \cdot V_Q = 0$. The same Cartesian location $\left(x_Q, y_Q\right)$, constraint used in part (a.) should be used here, except use its time-derivative to find the constraint matrix. For example, $\dot{y}_Q = ? \cdot \dot{\theta}$ translates into a matrix row $[0 \quad 1 \quad -?]$.
D. Use graphical kinematics to find the constraint Jacobian separately, again using $\mathbf{v}_Q = \mathbf{v}_A + \omega \times \mathbf{r}_{Q/A}$.
E. Verify that $J_c J_p = 0$.

3. The provided `runanthrowalk2` demonstrates the "anthropomorphic" walking model with human-like inertial properties (continuing the previous assignment). Briefly explain the steps taken in the file to compute ground reaction forces following a simulation.
   A. Using the provided parameter studies as example, perform your own study of any other parameter, and describe how different values affect walking performance (e.g., speed, collision losses, etc.)
   B. Examine the step-to-step eigenvalues and eigenvectors, which describe stability of the system. Does the nominal simulation have local asymptotic stability?
   C. Note that one of the eigenvalues is zero. The corresponding eigenvector (say, v_{zero}) represents a combination of state perturbations $c \cdot v_{\text{zero}}$ (where c is small) that result in no error. Apply such a perturbation to the initial condition, and demonstrate that there is no effect on the next step's states. How is it possible for the states to be perturbed and have no effect on the states? (Hint: There is an effect on time.)
   D. The simulation runs forward in time (1 dimension) in state space, and its trajectory intersects the Poincaré section (ground contact condition). How many dimensions does the ground contact condition have, for the rimless wheel, and for the anthropomorphic model? (Thus in principle the fixed point can be described in less than N dimensional state space, related to (c.) above.)

4. Suppose a rigid body trunk is to be added to the anthropomorphic model, with a hinge joint attached to the hip. The trunk is to be modeled as an inverted pendulum of length l_t, with a point mass m_t at the top, and an additional moment of inertia described a radius of gyration r_{gyrt} about the hip. (The total moment of inertia about the hip is therefore $m_t\left(l_t^2 + r_{\text{gyrt}}^2\right)$.)
   A. Describe how the equations of motion could be implemented, using the pose Jacobian method. Hint: One additional body segment, three additional rows to pose Jacobian.
   B. Describe how the constraint Jacobian would be modified. Hint: Two additional constraints.
   C. Optional (advanced): Implement the anthropomorphic model with trunk, and find and demonstrate a limit cycle. Hint: The search for a limit cycle is made easier by starting with the model without trunk, and then making an initial guess with the trunk starting at vertical position. Use an artificially large r_{gyrt}, which need not be physically meaningful, so that the trunk is very slow to fall over. Once a limit cycle is found, continuous methods may be used to reduce the radius of gyration to zero.