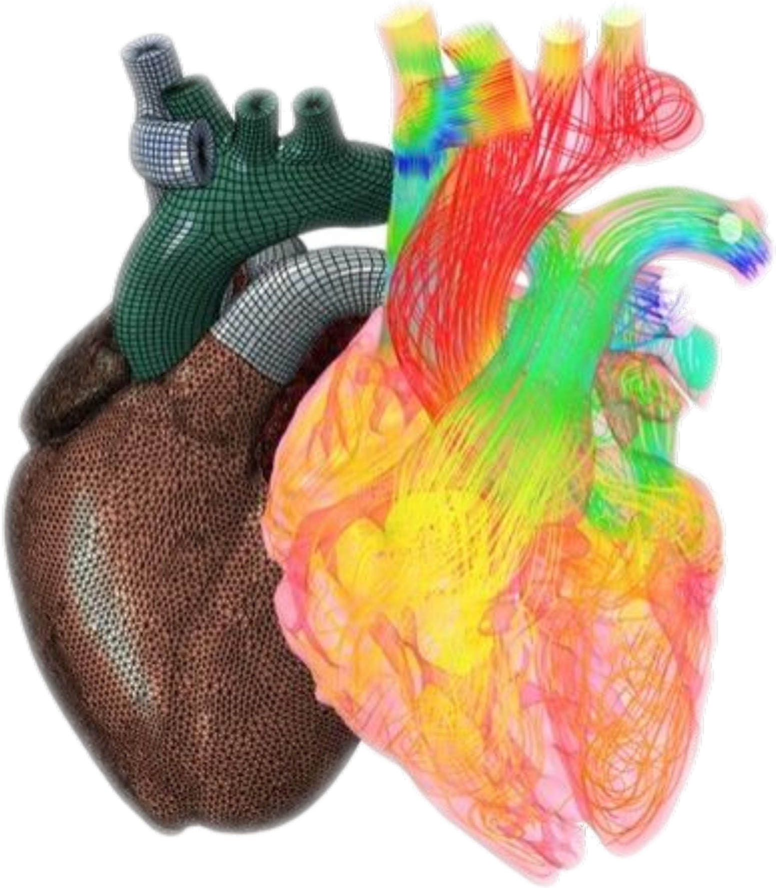
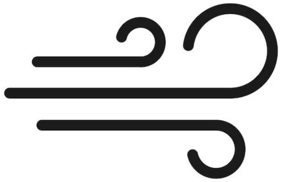
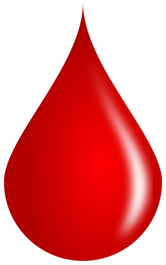
Cardiovascular Fluid Mechanics



National Biomechanics Day

What is a fluid?





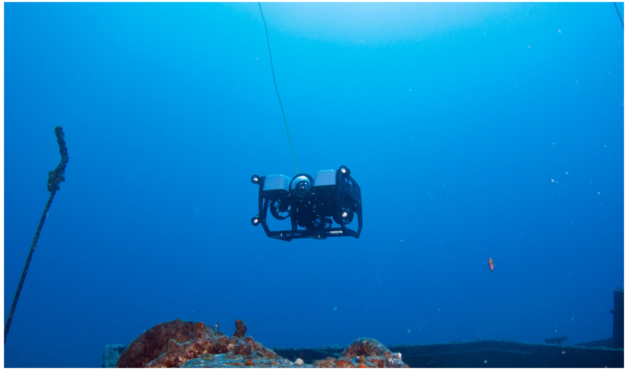
Fluid:

A material that continuously deforms under applied shear stress

Fluid mechanics is the study of fluids in…

motion (*fluid dynamics*)

at rest (*fluid statics*)

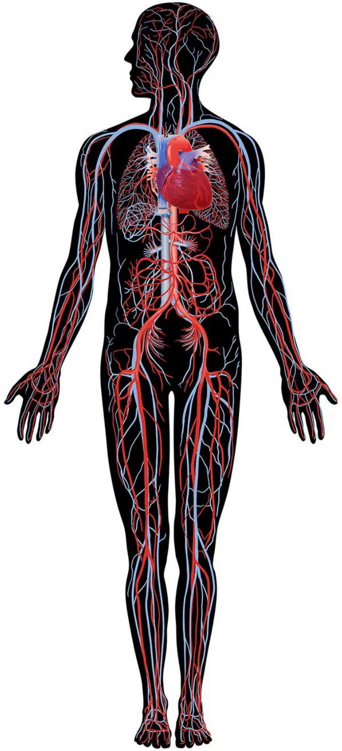


Why do we study fluid dynamics?

1. Understand fluid flow (velocity, pressure)

2. Understand how influences objects (forces, energy)

Why do we care about fluids in our body?



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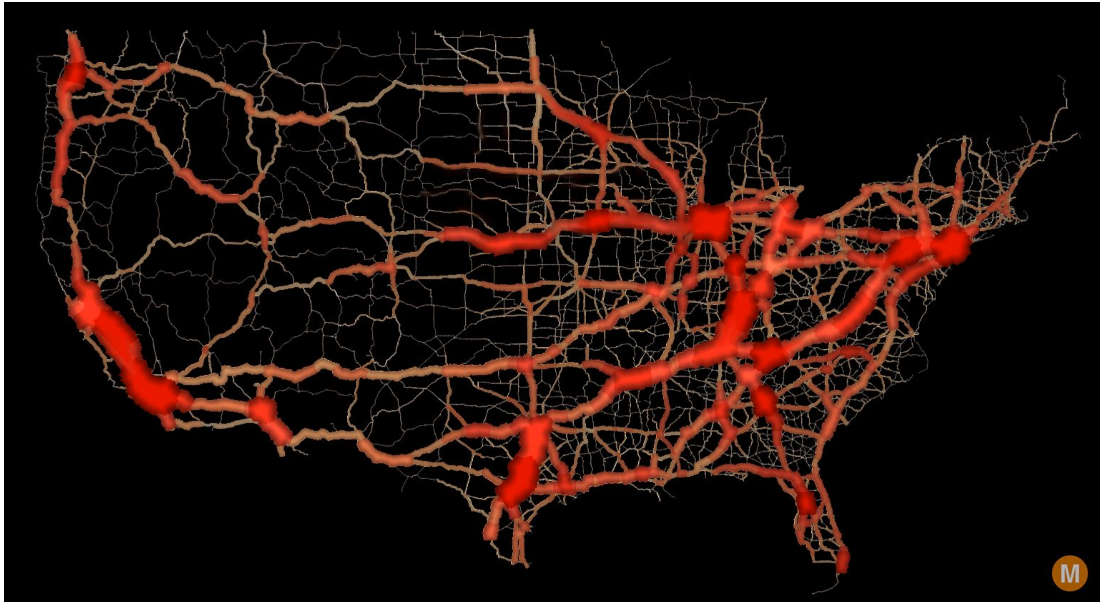

We use the ***Navier-Stokes Equations**** to describe the behavior of any fluid

$$\nabla \cdot \overline{u} = 0$$

$$\rho \frac{D\overline{u}}{Dt} = -\nabla p + \mu \nabla^2 \overline{u} + \rho \overline{F}$$

But… these equations are very difficult to solve for complex problems!

*One of the seven Millennium Prize Problems:
https://www.claymath.org/millennium-problems/navier–stokes-equation

However, we can solve these equations for some simple problems!

Simple example 1: Poiseuille flow

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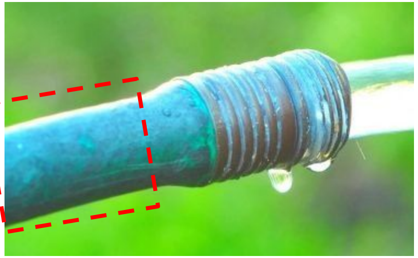
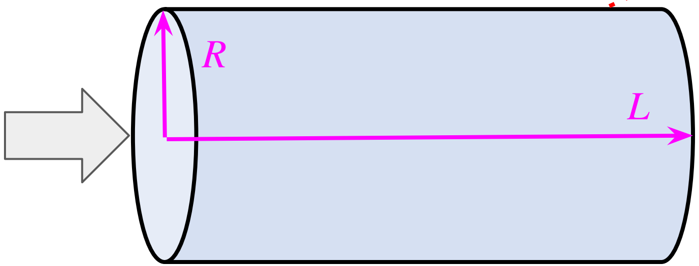


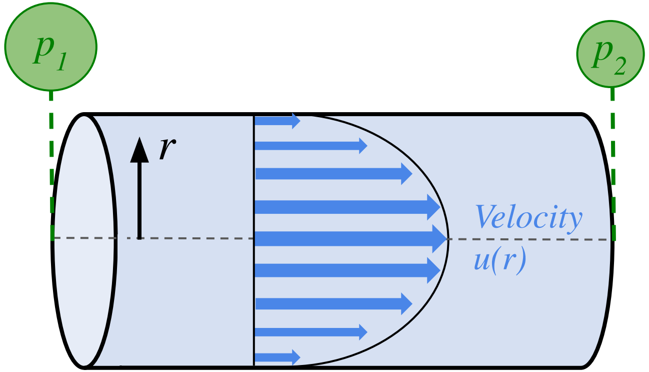
Q: Flow rate [ml/s]



What does the velocity change throughout the pipe

How much pressure do we need to push this water?

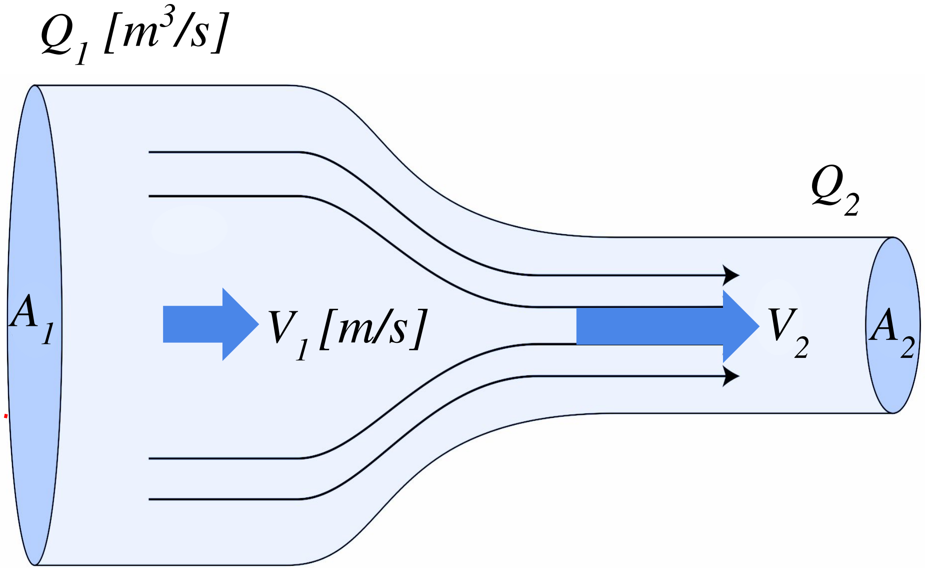
Using Navier-Stokes, we can predict that…



$$P_1 - P_2 = \Delta P = \frac{8\mu L Q}{\pi R^4} \quad [Pa]$$

$$u = \frac{2Q}{\pi R^2}(R^2 - r) \quad [m/s]$$

Simple example 2: Mass conservation

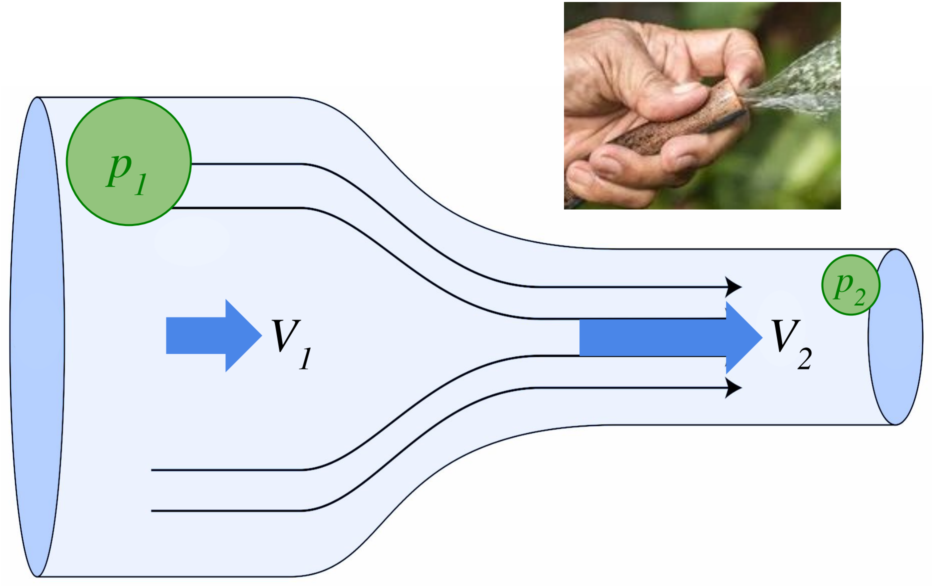


$$Q_1 = Q_2$$

$$A_1 V_1 = A_2 V_2$$

The flow rate does not change, but velocity does!

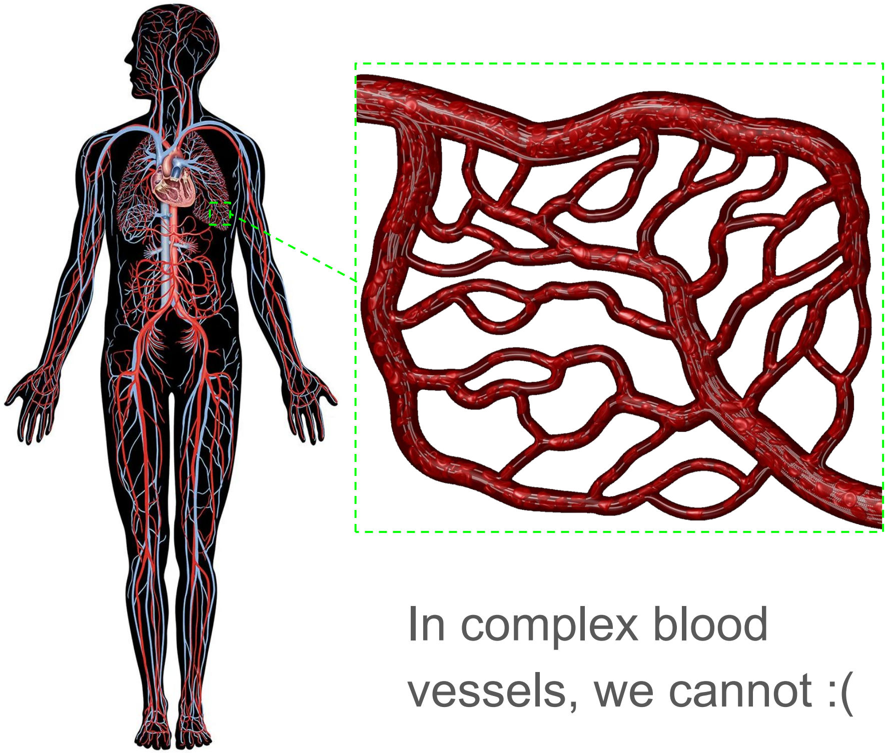
Simple example 3: Bernoulli's Principle



$$\overbrace{P_1 + \frac{\rho}{2} V_1^2}^{\text{Region 1}} = \overbrace{P_1 + \frac{\rho}{2} V_2^2}^{\text{Region 2}}$$

Bernoulli's Principle tells us how *energy* (velocity, pressure) *changes throughout* the fluid

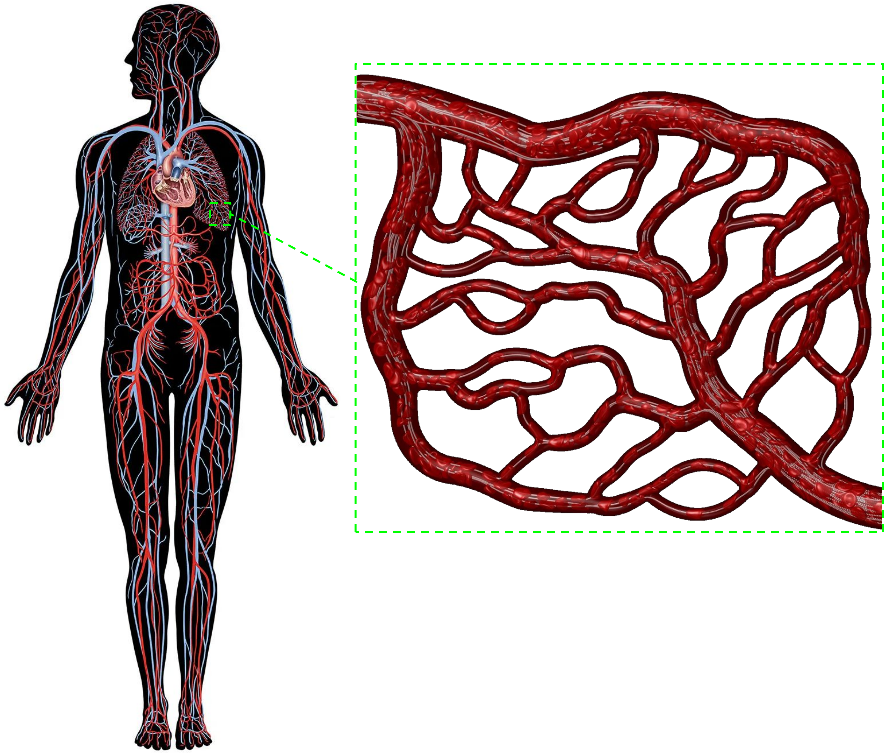
Going from simple fluid dynamics problems to complex ones



In a straight pipe, we can find a simple equation!

In complex blood vessels, we cannot :(

What do we do for complex problems, like the human body?



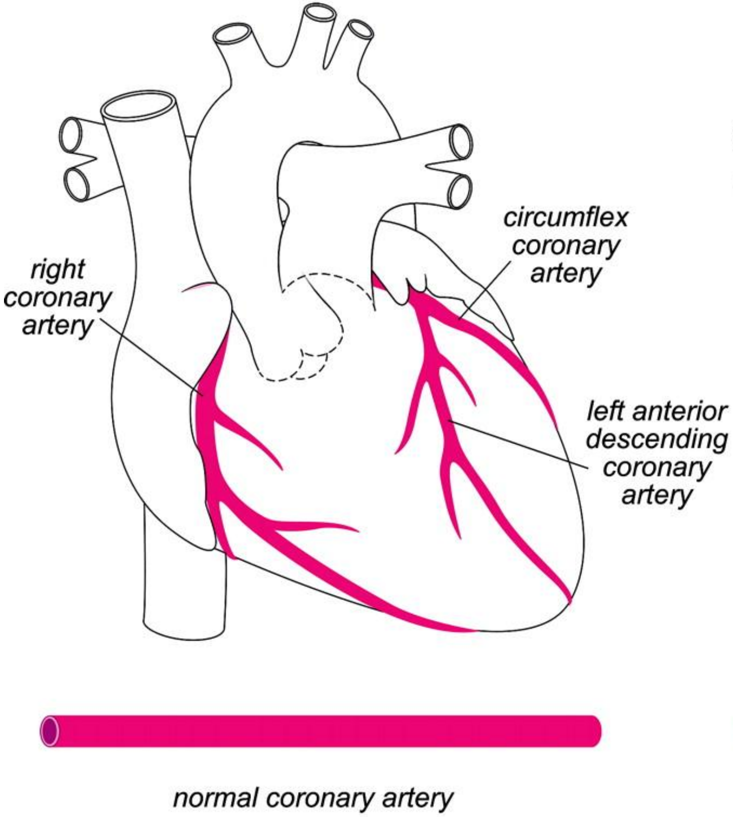
We have to solve these equations with *computational fluid dynamics*!

$$\nabla \cdot \overline{u} = 0$$

$$\rho \frac{D\overline{u}}{Dt} = -\nabla p + \mu \nabla^2 \overline{u} + \rho \overline{F}$$

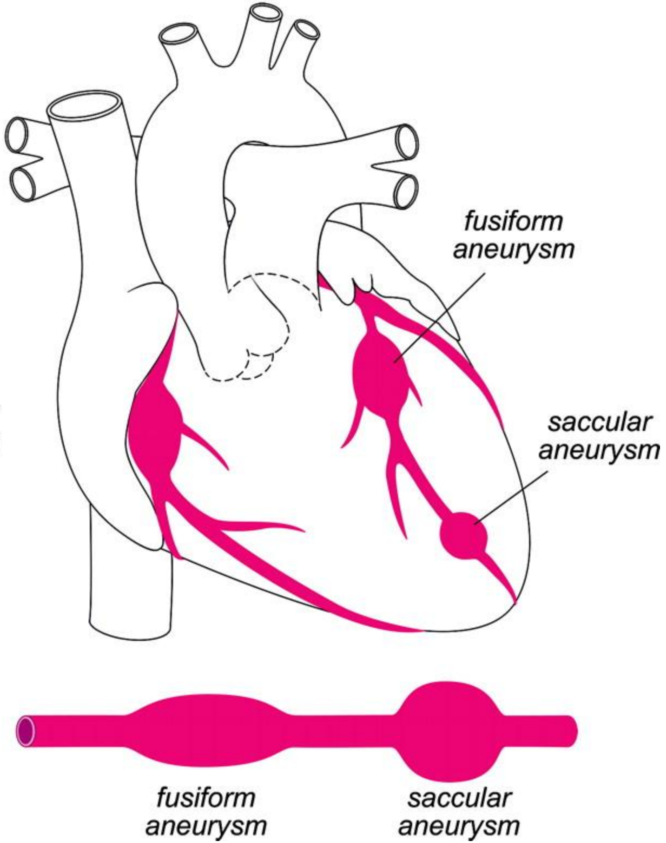
Heart with Normal Coronary Arteries

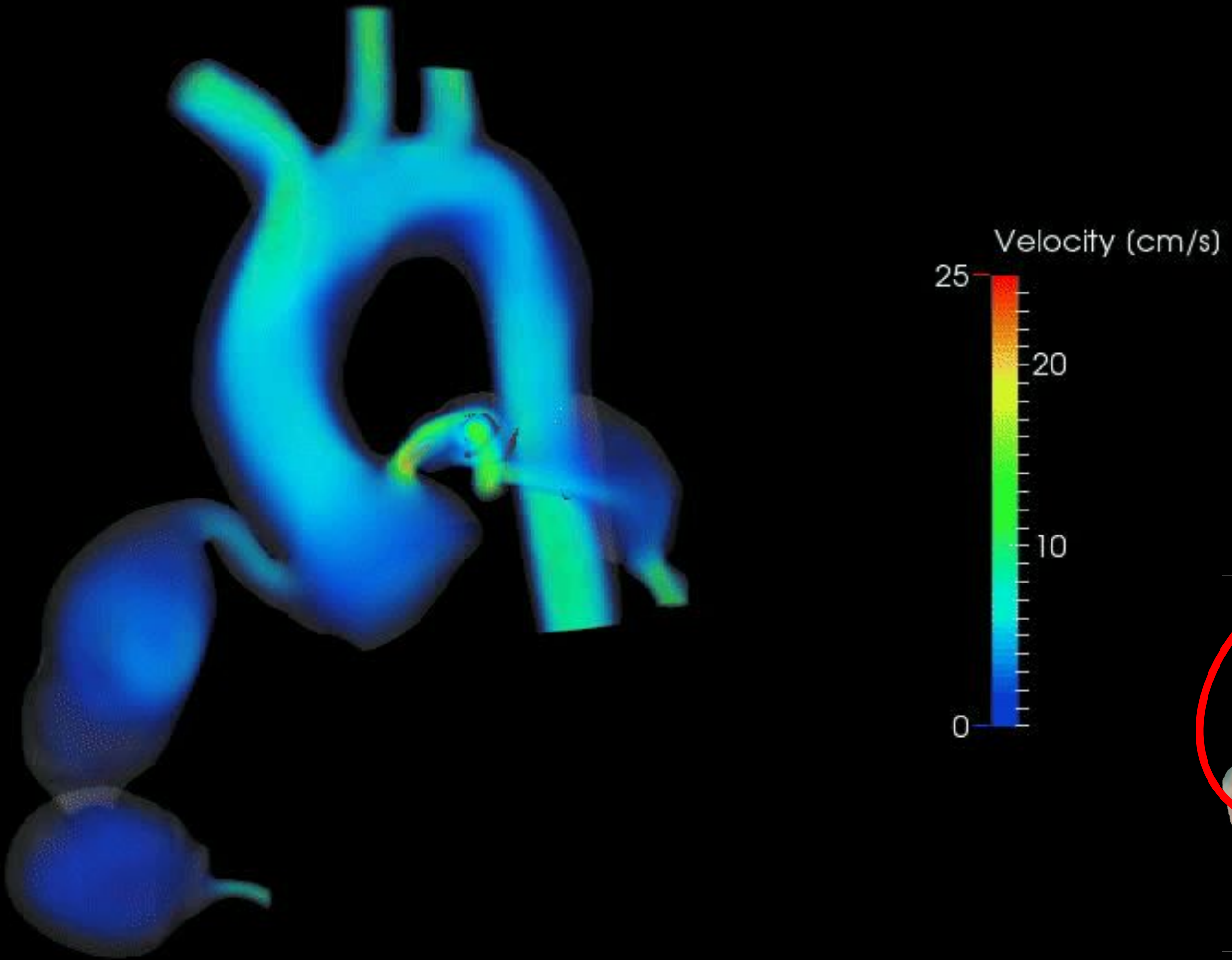
(in red)

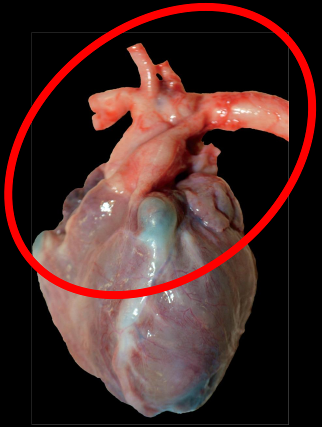


Heart with Coronary Artery Aneurysms

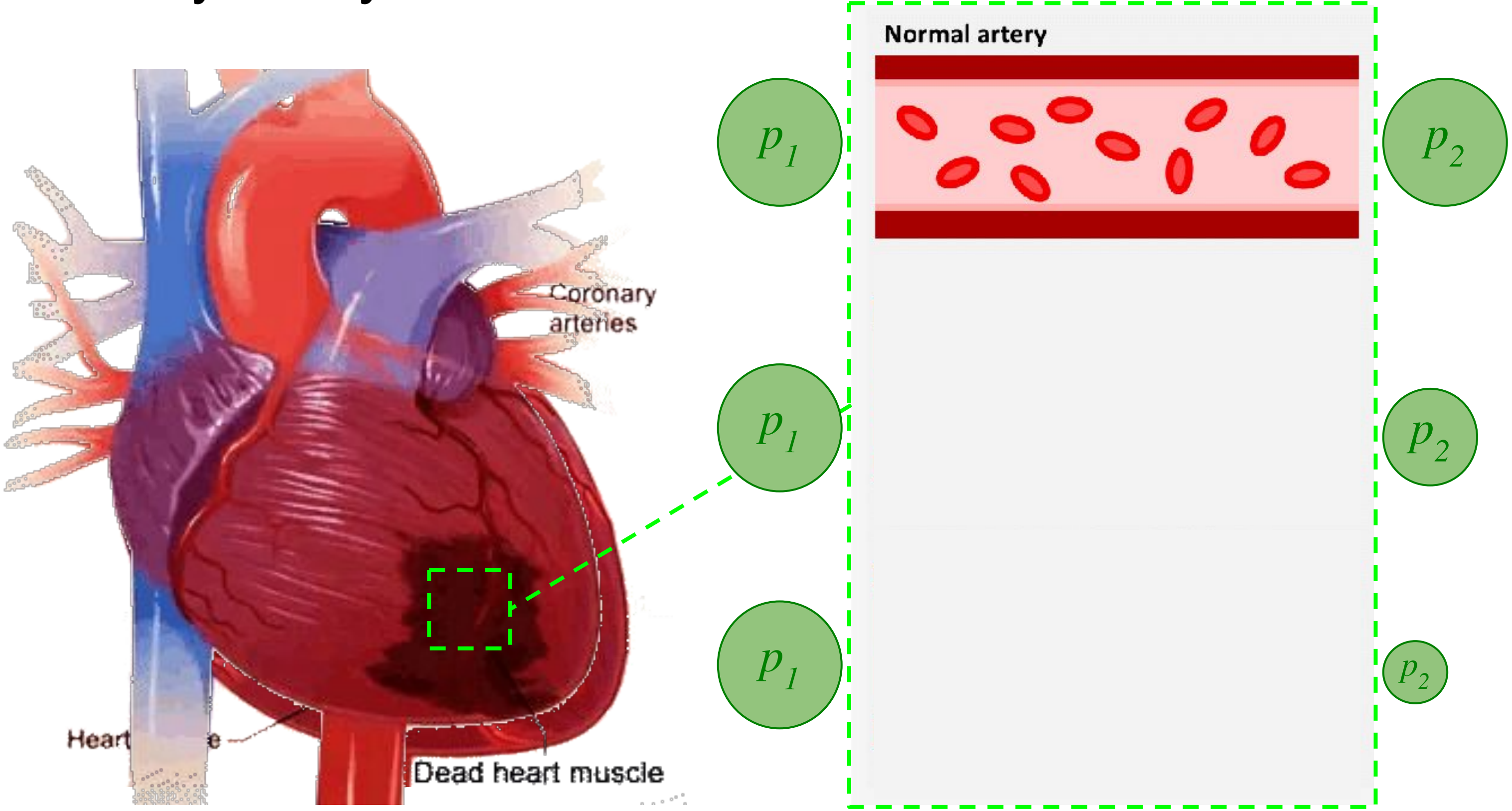
(in red)




Velocity (cm/s)
25
20
10
0

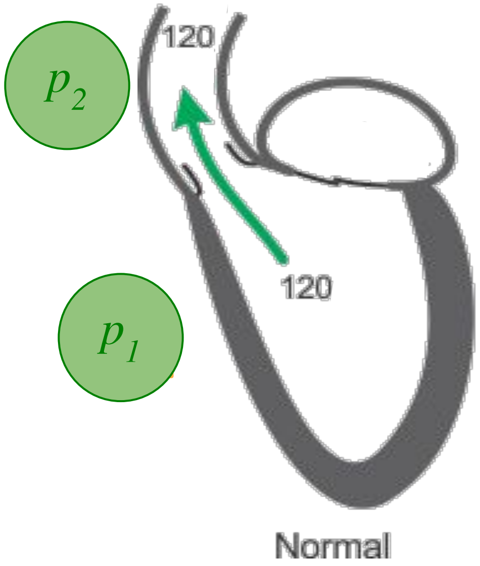


Coronary artery disease

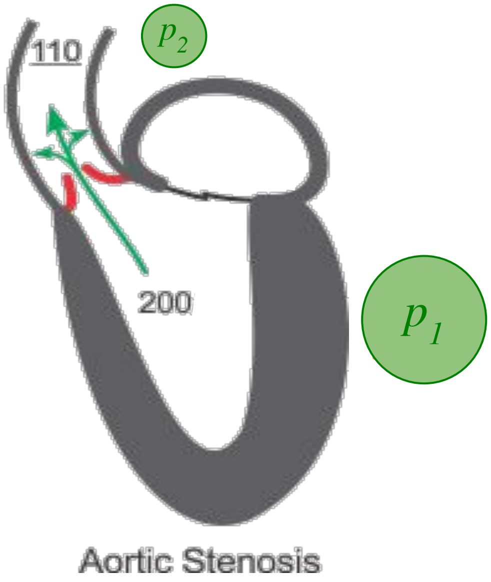


Aortic valve stenosis

Aortic valve stenosis



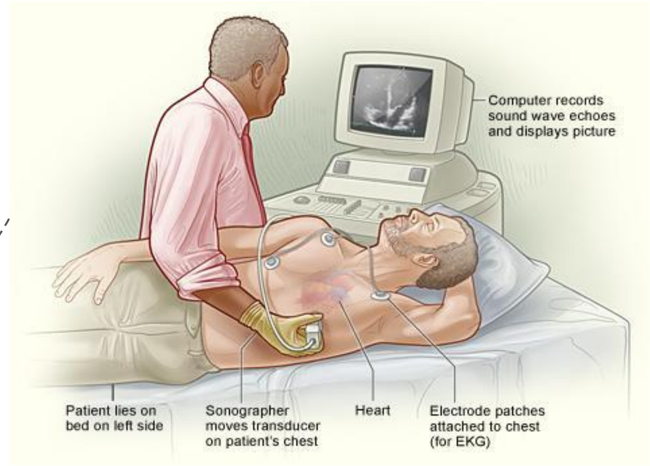
$$p_1 - p_2 = 120 - 120 = 0\ mmHg$$

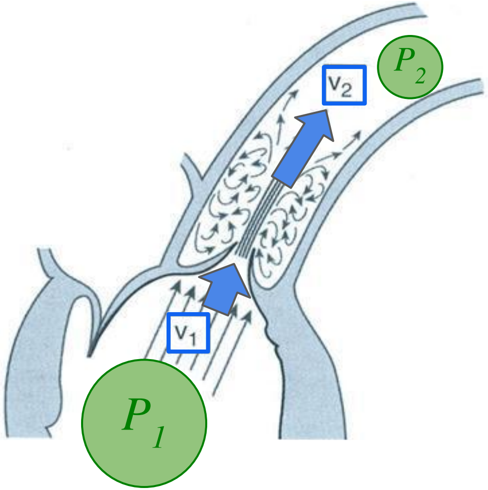


$$p_1 - p_2 = 200 - 110 = 90\ mmHg$$

Measuring pressure inside someone's heart is **hard!**

Application of Bernoulli's Principle





Echocardiography uses sound waves to create images of our heart and to measure blood velocity (**Doppler effect**)

We can measure v_1,v_2 !

$$p_1 - p_2 = \frac{1}{2}\rho(v_2^2 - v_1^2)$$