

**Five-Dimensional Flow in Solid Fuel Rocket
Engines**

by

I. B. Scriptor

B.A., North Dakota State University, 1994

M.S., University of Reno, 1997

A thesis submitted to the
Faculty of the Graduate School of the
University of Colorado in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
Department of Rocket Science

2008

This thesis entitled:
Five-Dimensional Flow in Solid Fuel Rocket Engines
written by I. B. Scriptor
has been approved for the Department of Rocket Science

Ed Visor

Prof. Rachel Goddard

Ms. Thora Nea

Date _____

The final copy of this thesis has been examined by the signatories, and we find that both the content and the form meet acceptable presentation standards of scholarly work in the above mentioned discipline.

Scriptor, I. B. (Ph.D., Rocket Science)

Five-Dimensional Flow in Solid Fuel Rocket Engines

Thesis directed by Prof. Ed Visor

Solid fuel rocket engines are one of the most reliable and efficient propulsion systems used to lift payloads into orbit, in terms of $\lambda = (\square + \diamond)\psi$. Used throughout the astrodynamics community, the theory of the flow within the motor chamber is in fact a black art which defies all attempts at analysis.

The present work (no exception to the statement above) contains a theoretical and numerical approach to the flow of the gases within the motor chamber. The shape of the chamber and original fuel configuration, and the patterns of combustion and flow/expulsion of gases, are modelled by a system of thirty fourth-degree differential equations.

$$f_i^{(34)}(x, y, z, t) = \sum_{j=0}^{33} a_{ij} f_i^{(j)}(x, y, z, t)$$

Acceptable numerical solutions would require one thousand pentium processors working day and night for $10^{11.2}$ years.

Dedication

To all of the fluffy kitties.

Acknowledgements

Here's where you acknowledge folks who helped.

Contents

1	Introduction	1
1.1	Example of included image	2
1.1.1	Question: What are the issues in studying this subject?	2
1.2	Lists in <code>thesis</code> class	4
2	Mathematical Formulation	7
2.1	Conditions for Catastrophic Combustion	10
2.2	More Boundary Conditions	12
2.2.1	Just meaningless text to test lines per page	13
2.2.2	This is a subsection	17
2.2.3	This is another subsection	17
2.3	The End	18
	Bibliography	19
	Appendix	
A	Objective Symptoms	20
B	Ode to Spot	22

List of Tables

1.1	Here is an example of a table with its own footnotes. Don't use the <code>\footnote</code> macro if you don't want the footnotes at the bottom of the page. Also, note that in a thesis the caption goes above a table, unlike figures.	5
1.2	short caption for table.	6
2.1	This is a table constructed with L ^A T _E X commands in the <code>tabular</code> environment.	10
2.2	This table wasn't constructed with L ^A T _E X commands, but resides in a PostScript file (<code>tableD.ps</code>) created by some other software.	10
2.3	This sideways table is constructed using the <code>\tabular</code> environment. This would only be necessary for tables so wide that they don't fit the normal width, but not so wide that they would also exceed 8.75", the usable height of a thesis page, using the usual L ^A T _E X font in the table. Notice that this table uses the same font style and size as in the body of the thesis. The caption appears above the table (nearest the left edge of the page) as it should in a C.U. thesis.	14

2.4	This table is actually from a PostScript file. If it is just too tiny to read in the normal orientation, where the width is limited to 5.75", it can be displayed sideways at a width (vertical length) of up to 8.75". The contents of the table show that it has been reduced in size; however, the caption appears in the correct place above the table (left edge of page) in the same font style/size as in the body of the thesis. The caption also appears in the list of tables at the front of the thesis. This construct uses the <code>sidewaystable</code> environment and the <code>\includegraphics</code> command, which are defined in the <code>rotating</code> and <code>graphicx</code> packages, respectively. These packages are read in (with <code>\usepackage</code>) at the top of the L ^A T _E X file.	15
-----	--	----

List of Figures

1.1	This figure was a mess so just putting Fig. 1.3 here in it's place, but smaller and rotated. mindless text to prove a point. just trying to make this caption longer and longer and longer and longer.	3
1.2	Short caption goes here.	3
1.3	The combustion zone in a rocket motor chamber is thin relative to the radius of the chamber, etc. How did this image get into the thesis? it's easy in Lyx	3
2.1	A triangular pyramid may be cut up as shown, to yield one top pyramid (with one-eighth the volume of the full pyramid), three bottom corner pyramids (which, when joined, are congruent to the top pyramid), three prisms along the bottom edges (the area of whose bottom faces total $B/2$) and the large central prism (volume = $(B/4)(h/2) = Bh/8$). The image, from PostScript file "pyr.ps", was read in using the <code>\includegraphics</code> command, from the graphicx package.	8
2.2	Short caption for sideways figure.	11

Chapter 1

Introduction

This document is the work of Nathan Miller to copy the CU-Boulder sample latex thesis into Lyx. Lyx uses **.layout** files to determine which **.sty** and **.cls** files to use. You should have a **ucbthesis.layout** file which tells Lyx to use the CU-Boulder **thesis** class. Put that file into Lyx's layout folder (...\\Lyx\\Resources\\Layout\\), then in Lyx go to the **Tools** menu and select **Reconfigure**. Restart Lyx and you can select the **UC-Boulder thesis** document class under the menu **Document** → **Settings...** → **Document Class** for your Lyx files. I fixed up a few things to illustrate how Lyx works, but mostly, this is just copied straight from the example latex thesis.

The original Latex sample document illustrates how to use the **thesis** class in Lyx, originally written by John P. Weiss and updated by Bruce Fast. The necessary file, **thesis.cls**, is on all the main computer systems of C.U. Boulder, and can be downloaded from the web site <http://www.colorado.edu/its/docs/latex/thesis/> <http://www.colorado.edu/its/docs/latex/thesis/>. Some requirements of the Graduate School are written into that file; page size, line spacing, appropriate placement of captions for tables and figures, etc. Other tasks of conforming to the requirements are left to other existing L^AT_EX packages. For example, a common problem is to insert graphics — figures and tables — into the body of the thesis¹. The Grad School specs say that a large table may be displayed in landscape mode at reduced size, but its caption must

¹ this is easy with Lyx

also be in rotated position, in the same font and size as the normal text in the body of the thesis. To accomplish this, the user must invoke the **rotating** package, the use of which is illustrated in this document. See Table 2.4 and Fig. 2.2.

References and Sections are easy in Lyx. You might enjoy § 2 more than § 1.² And undoubtedly you will like § 2.3 better than either § 2.1, Table 2.2, or equation 2.5.

You can also select formatted references from the cross-reference pop-up box. The following sentences are the same as the previous paragraph, but using formatted references. You might enjoy Ch. 2 more than Ch. 1. And undoubtedly you will like §2.3 better than either §2.1, 2.2, or Eq. 2.5. Formatted references are controlled by the **prettyref.sty** file. The above formatted references in the included pdf are made using the included prettyref file. My **prettyref.sty** file is located at (...\\Program Files\\MiKTeX 2.7\\tex\\latex\\prettyref\\). After modifying the file, remember to refresh the MikTeX FNDB, and reconfigure Lyx. (I changed all the references after this to formatted references)

1.1 Example of included image

Fig. 1.2 shows how to input figures so they have both a short and long caption. Fig. 1.3 shows something or other; the image is from a PostScript file which is imported into this document very easily in Lyx.

1.1.1 Question: What are the issues in studying this subject?

A major goal in studying solid fuel rocket motors is to create a model of the dynamics of a motor chamber. This involves two major goals: the combustion zone and the acoustic zone. Fig. 1.3 shows this. The combustion zone consists of the thin layer above the solid fuel where the gasification of the fuel takes places. The zone is very reactive and highly turbulent. The acoustic-vortical zone is the volume of gas above the

² not sure how to make the funny S symbol for the sections in Lyx, so I just left the ERT.

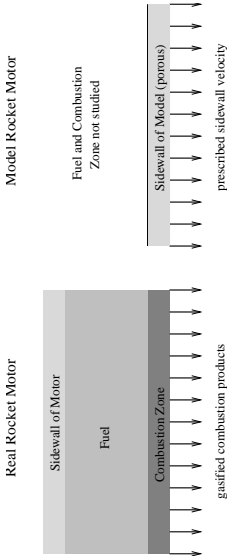


Figure 1.1: This figure was a mess so just putting Fig. 1.3 here in it's place, but smaller and rotated. mindless text to prove a point. just trying to make this caption longer and longer and longer and longer.

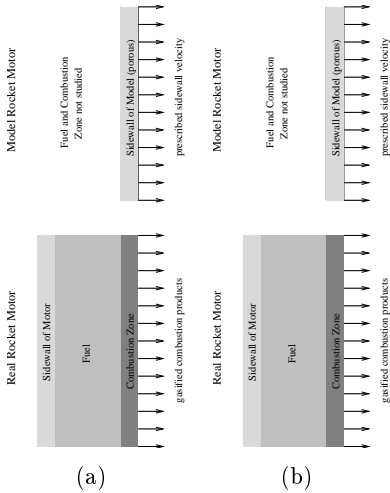


Figure 1.2: Fig. 1.1 did not use any ERT and looks pretty in Lyx, but the caption is long and looks bad in the List of Figures. To enter a short caption, we need to use some ERT. Also, we show how to use subfigure labels (a) and (b).

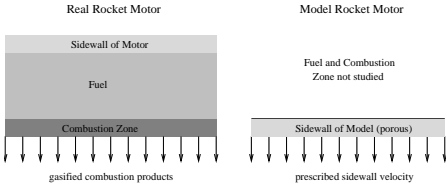


Figure 1.3: The combustion zone in a rocket motor chamber is thin relative to the radius of the chamber, etc. How did this image get into the thesis? it's easy in Lyx

combustion zone. Within this zone, the gas is non-reactive and contains acoustic waves and vorticity.

1.2 Lists in thesis class

In `thesis` class (for Colorado University), lists are defined so that nested lists will be numbered or marked appropriately. First, an itemized (non-enumerated) list prefaces each item with a bullet. Nested itemized list use asterisks, then dashes, then dots. In Lyx, just select either **itemized** or **enumerated** from the drop-down list in the upper-left corner.

- This is “itemized” item A.
- This is “itemized” item B.
- This is “itemized” item C.
 - * This is “itemized” subitem A.
 - This is “itemized” subsubitem A.
 - This is “itemized” subsubsubitem A.
 - This is “itemized” subsubitem B.
 - * This is “itemized” subitem B.
- This is “itemized” item D.

Nested enumerations appear like this.

- (1) This is “enumerated” item A.
- (2) This is “enumerated” item B.
- (3) This is “enumerated” item C.

- (a) This is “enumerated” subitem A.
 - (i) This is “enumerated” subsubitem A.
 - (i.a) This is “enumerated” subsubsubitem A.
 - (ii) This is “enumerated” subsubitem B.
- (b) This is “enumerated” subitem B.
- (4) This is “enumerated” item D.

The work presented here³ is an extension of Lao[1] and Lao et al.[2] The driving frequency is on the order of the inverse of the axial acoustic time scale, $t'_A = L'/C'_0$, where L' is the length of the cylinder and C'_0 is the reference speed of sound.⁴ Radial and azimuthal velocities⁵ are found to vanish exponentially fast in the downstream direction, as suggested by Table 1.1.

Table 1.1: Here is an example of a table with its own footnotes. Don’t use the `\footnote` macro if you don’t want the footnotes at the bottom of the page. Also, note that in a thesis the caption goes **above** a table, unlike figures.

wave form	S (kVA)	P (kW)	Q^* (kVAr)	D^\dagger (kVAd)
Fig. 1.1	25.87	25.83	1.3	≈ 0
Fig. 1.3a	25.48	25.00	-2.82	4.03
Fig. 1.3b	25.11	18.02	-9.75	14.52
2.1	24.98	22.26	9.19	6.64
2.4	23.48	15.00	6.59	16.82
Fig. 2.1	24.64	22.81	-0.44	9.3
Fig. 2.2	23.03	18.01	3.36	13.95

*kVAr means reactive power.

[†]kVAd means distortion power.

These results provide an analytical explanation of those found from computational analysis by Fabnis et al.[3] The non-axisymmetric flow near the endwall contains cross-sectional velocity patterns that include flow across the cylinder axis. A viscous boundary

³ Footnotes are handled neatly by L^AT_EX.

⁴ Remember the traditional method of calculating the distance of lightning? See the flash, count seconds until you hear the thunder, divide by five, that’s the number of miles. That assumes $C_0 = \frac{1mi.}{5s}$.

⁵ gratuitous footnote

Table 1.2: This caption is long - you might want a shorter version for the List of Tables. Here is a shorter version of the previous table, showing how to use short & long captions. Also, note that in a thesis the caption goes **above** a table, unlike figures.

wave form	S (kVA)	P (kW)	Q (kVA _r)	D (kVA _d)
Fig. 1.1	25.87	25.83	1.3	≈ 0
Fig. 1.3a	25.48	25.00	-2.82	4.03

layer adjacent to the sidewall and near the endwall is studied to find the transition between the transient core flow and the no-slip condition on the sidewall. It is found, as in Lao et al.[2], that the azimuthal component of the vorticity is proportional to the inverse of the Mach number. In addition, the axial component of the vorticity driven by the non-axisymmetric boundary condition at the endwall is also found to be proportional to the the inverse of the Mach number.

Chapter 2

Mathematical Formulation

The objective of this fake thesis document is to demonstrate a multitude of L^AT_EX features as well as features specific to the thesis class. We start by giving one short formula, and one big hairy multi-line formula (one of the non-dimensional Navier-Stokes equations):

$$A = \pi r^2 \tag{2.1}$$

$$\begin{aligned} \rho \left[\frac{DV_r}{Dt} - M\epsilon^2 \frac{V_\theta^2}{r} \right] = & -\frac{\delta^2}{\gamma M} \frac{\partial P}{\partial r} + \frac{M}{Re} \delta^2 \left\{ 2 \frac{\partial}{\partial r} \left[\mu \left(\frac{\partial V_r}{\partial r} - \frac{1}{3} \nabla \cdot \bar{\mathbf{V}} \right) \right] \right. \\ & + \frac{1}{r} \frac{\partial}{\partial \theta} \left[\mu \left(\frac{1}{r} \frac{\partial V_r}{\partial \theta} + \epsilon \frac{\partial V_\theta}{\partial r} - \epsilon \frac{V_\theta}{r} \right) \right] \\ & + \frac{\partial}{\partial z} \left[\mu \left(\frac{1}{\delta^2} \frac{\partial V_r}{\partial z} + \frac{\partial V_z}{\partial r} \right) \right] \\ & \left. + 2 \frac{\mu}{r} \left[\frac{\partial V_r}{\partial r} - \frac{\epsilon}{r} \frac{\partial V_\theta}{\partial \theta} - \frac{V_r}{r} \right] \right\}, \end{aligned} \tag{2.2}$$

The latter equation is non-dimensionalized using the following definitions:

$$r = \frac{r'}{R'}, \quad z = \frac{z'}{L'}, \quad t = \frac{t'}{t'_a}, \quad \kappa = \frac{\kappa'}{\kappa'_0}, \quad \mu = \frac{\mu'}{\mu'_0}, \quad C_V = \frac{C'_V}{C'_{V0}},$$

where P'_0 is the initial static pressure in the cylinder, and ρ'_0 and T'_0 are the density and temperature of the fluid being injected from the sidewall. The aspect ratio is given by $\delta = \frac{L'}{R'}$, where $\delta \gg 1$. The induced characteristic axial velocity and the characteristic

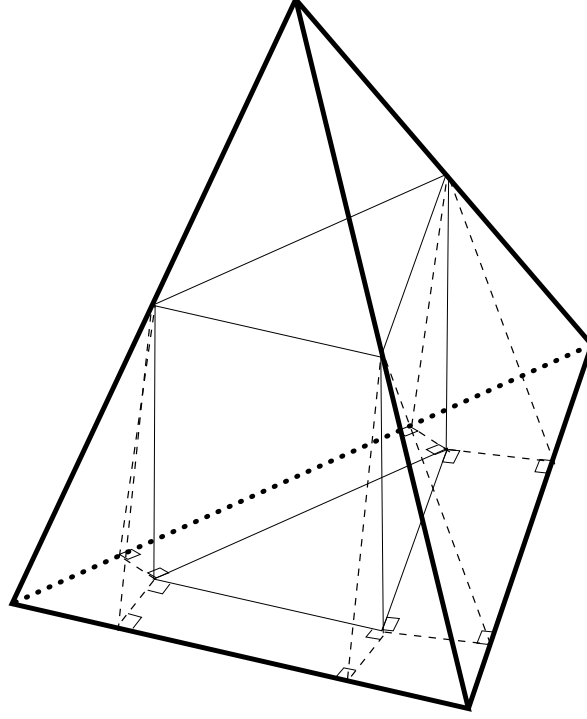


Figure 2.1: A triangular pyramid may be cut up as shown, to yield one top pyramid (with one-eighth the volume of the full pyramid), three bottom corner pyramids (which, when joined, are congruent to the top pyramid), three prisms along the bottom edges (the area of whose bottom faces total $B/2$) and the large central prism (volume = $(B/4)(h/2) = Bh/8$). The image, from PostScript file “pyr.ps”, was read in using the `\includegraphics` command, from the `graphicx` package.

endwall velocity disturbance V'_{z0} is defined with respect to the injection reference sidewall velocity, V'_{r0} by overall mass conservation, $\frac{V'_{z0}}{V'_{r0}} = \delta$. The size of the initially unknown reference azimuthal velocity $V'_{\theta 0}$ is related to V'_{r0} by $\frac{V'_{\theta 0}}{V'_{r0}} = \epsilon$. Later, it is shown that $\epsilon = 1$.

The time is non-dimensionalized using the axial acoustic time scale, $t'_a = \frac{L'}{C'_0}$, where $C'_0 = (\gamma \mathcal{R}' T'_0)^{\frac{1}{2}}$ is the speed of sound¹, \mathcal{R}' is the gas constant, and γ is the ratio of specific heats. Also the Reynolds number, Wrenchl number, and Mock number are defined as

¹ In air at 1 atm., $\frac{1mi.}{5s}$.

$$Re = \frac{\rho' V'_{z_0} L'}{\mu'_0}, \quad Wr = \frac{\mu'_0 C'_{p_0}}{\kappa'_0}, \quad M = \begin{pmatrix} V'_{z_0} \\ C'_0 \end{pmatrix} \cdot \begin{pmatrix} 8a & z_0 - \rho \\ 2 & z_0 - \mu \end{pmatrix} \begin{pmatrix} Wr \\ p - 7 \end{pmatrix}$$

where $Re \ll 1$, $M \gg 1$, and $Wr = O(1)$.

Here is an example of using the macros `\singlespacing` and `\doublespacing`:

This paragraph was preceded by the command `\singlespacing`. The Mock number is chosen as a small parameter to model the small magnitude found in a typical rocket motor chamber, as opposed to the rocket nozzle where larger values are possible². The aspect ratio, δ , is taken to be a large parameter, because many chambers have aspect ratios between 15 and 50. Now the command “`\doublespacing`”:

The Grad School specifications allow for single spacing everywhere in the body of the thesis, except in quotations of four or more lines (Dole and Abramson[4]), table/figure captions, chapter headings, footnotes and entries in the Contents and Bibliography (with double spaces between entries).

And now, here is an example of using the macros `\begin{singlespace}` and `\end{singlespace}`; another way to get single-spacing.³

Two cases are studied in the present work which differ only in the boundary conditions. Each different boundary condition model a different source of instability. The boundary of the first case consists of a steady, axisymmetric sidewall radial velocity boundary and a time-dependent, non-axisymmetric endwall axial velocity boundary. The second case is studied with a fixed impermeable axial velocity along the endwall and a combination axisymmetric steady and non-axisymmetric unsteady radial velocity along the sidewall.

Now on to other items. The following table is created using L^AT_EX macros, i.e., `\begin{tabular} ... \end{tabular}`.

However, sometimes you want to use a table produced by some other software, such as Excel. If the table is saved to a PostScript file, then it can be displayed inserting a table in Lyx

² Not just possible, desirable!

³ There is probably a simple way to do this in Lyx, just didn't try.

Table 2.1: This is a table constructed with L^AT_EX commands in the `tabular` environment.

n	n^2	n^3	n^4	n^7	n^{13}
2	4	8	16	128	8192
3	9	27	81	2187	1594323
4	16	64	256	16384	67108864
5	25	125	625	78125	1220703125
6	36	216	1296	279936	13060694016
7	49	343	2401	823543	96889010407

Table 2.2: This table wasn’t constructed with L^AT_EX commands, but resides in a PostScript file (`tableD.ps`) created by some other software.

n	n^2	n^3	n^4	n^7	n^{13}
2	4	8	16	128	8192
3	9	27	81	2187	1594323
4	16	64	256	16384	67108864
5	25	125	625	78125	1220703125
6	36	216	1296	279936	13060694016
7	49	343	2401	823543	96889010407

2.1 Conditions for Catastrophic Combustion

Initially, a steady flow is generated by the sidewall injection, $V_r = -V_{rws}(z)$. The subscript *srw* is used to mean that there is a **s**teady **r**adial **w**all velocity. The sign is negative due to the injection toward centerline. At $t = 0^+$, the endwall begins oscillating with the non-dimensionalized sinusoidal axial velocity, $V_z = \tilde{F}_{rw}(r, \theta, t)$, for $\omega = O(1)$. Fig. 2.2 conforms to these thesis specs: “Figures are placed immediately after their first mentions . . . Figure captions appear below figures and are typed in the same style and size as the text. Captions should fit within the standard margins and are not reduced if the figures are reduced . . . Figures may be printed broadside, with the top toward the left margin; the caption then appears beneath the figure and is typed from bottom to top of the page within the standard margins. . .”

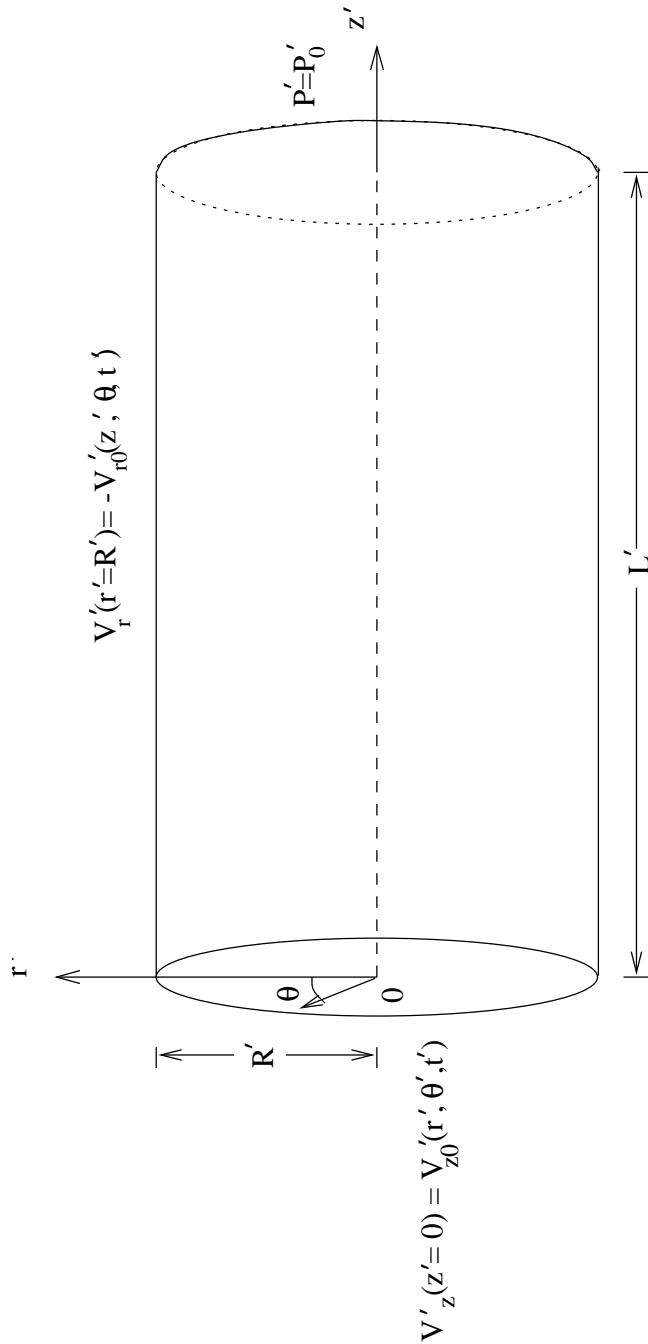


Figure 2.2: If a figure must appear sideways in a thesis (for greater detail) then the caption must still appear **below** the figure — i.e., along the right edge of the page. By including the **rotating** package at the top of the `LATEX` file, you can get this rotated figure by using the `\begin{sidewaysfigure} ... \end{sidewaysfigure}` environment. (have to use ERT in L_Yx.)

Some of the boundary conditions are:

$$z = 0; \quad V_z = \begin{cases} 0, \\ \tilde{F}_{zw}(r, \theta, t), \end{cases} \quad (2.3)$$

$$z = 0; \quad V_\theta = V_r = 0 \quad (2.4)$$

$$r = 0; \quad P, \rho, T, V_r, V_\theta, V_z \text{ finite}, \quad (2.5)$$

$$r = 1; \quad V_r = F_{rws}(z), \quad (2.6)$$

$$r = 1; \quad V_z = V_\theta = 0, \quad (2.7)$$

and solutions must be periodic in θ .

2.2 More Boundary Conditions

Initially, a steady flow is generated by the sidewall injection, $V_r = -V_{rws}(z)$. The sign is negative due to the injection toward centerline⁴. At $t = 0^+$, the endwall begins oscillating with the non-dimensionalized sinusoidal axial velocity, $V_z = \tilde{F}_{rw}(r, \theta, t)$, for $\omega = O(1)$. The frequency condition chosen represents the first few axial acoustic modes observed in high aspect ratio chambers⁵.

The full boundary conditions include:

$$z = 0; \quad V_\theta = V_r = 0 \quad (2.8)$$

$$r = 1; \quad V_r = \begin{cases} F_{rws}(z), & t < 0, \\ F_{rws}(z) + \tilde{F}_{rw}(z, \theta, t) & t \geq 0 \end{cases} \quad (2.9)$$

$$r = 1; \quad V_z = V_\theta = 0, \quad (2.10)$$

and solutions must be periodic in θ .

If you don't believe this stuff, check out Mulick[5] and Baylor[6].

⁴ This convention was suggested by Goddard and Smythe.

⁵ Toy rockets, the kind you used to shoot off with your dad in the park, typically have only two significant modes.

The following two tables, and their respective captions, are turned sideways. They use the **sidewaystable** environment defined in the **rotating** package. The first uses the L^AT_EX **tabular** environment, and would be used when the normal L^AT_EX table is a bit too wide to fit the width of the page, but fits the page when rotated. The second table is actually from a PostScript file, perhaps produced by some other software. It is read in using the `\includegraphics` command, and is needed when the table is very large and must be shrunk to fit the page. Inside the `\includegraphics` command one can specify `width=8.75in`, which will scale the (rotated) PostScript image to nearly fill the maximum amount of vertical space on a thesis page.

This is a quotation: Tables are placed immediately after their first mention in the text ... Tables that will not fit within the required margins may be typed in smaller type or may be reduced; they also may be printed broadside with the top toward the left margin ... Table titles are typed above the tables in the same style and type size as the text. Table titles should fit within the standard margins and are not reduced if the table is reduced. Table footnotes are typed immediately beneath the table and have no relation to text footnotes.

2.2.1 Just meaningless text to test lines per page

According to the Grad School specs. there should be 24–27 lines of print per page of a thesis. This should be true whether the font size is 10, 11, or 12. Count them up; does this document conform? According to the Grad School specs. there should be 24–27 lines of print per page of a thesis. This should be true whether the font size is 10, 11, or 12. Count them up; does this document conform? According to the Grad School specs. there should be 24–27 lines of print per page of a thesis. This should be true whether the font size is 10, 11, or 12. Count them up; does this document conform? According to the Grad School specs. there should be 24–27 lines of print per page of a thesis. This should be true whether the font size is 10, 11, or 12. Count them up; does this document conform? According to the Grad School specs. there should be 24–27

Table 2.3: This sideways table is constructed using the `\tabular` environment. This would only be necessary for tables so wide that they don't fit the normal width, but not so wide that they would also exceed 8.75", the usable height of a thesis page, using the usual L^AT_EX font in the table. Notice that this table uses the same font style and size as in the body of the thesis. The caption appears **above** the table (nearest the left edge of the page) as it should in a C.U. thesis.

n	n^2	n^3	n^4	n^7	n^{13}
2	4	8	16	128	8192
3	9	27	81	2187	1594323
4	16	64	256	16384	67108864
5	25	125	625	78125	1220703125
6	36	216	1296	279936	13060694016
7	49	343	2401	823543	96889010407
100	10000	1000000	100000000	1000000000000000	1000000000000000000000000
1000	1002001	1003003001	1004006004001	1007021035035021007001	1013078286716288717717287715286078013001

Table 2.4: This table is actually from a PostScript file. If it is just too tiny to read in the normal orientation, where the width is limited to 5.75", it can be displayed sideways at a width (vertical length) of up to 8.75". The contents of the table show that it has been reduced in size; however, the caption appears in the correct place above the table (left edge of page) in the same font style/size as in the body of the thesis. The caption also appears in the list of tables at the front of the thesis. This construct uses the `sidewaystable` environment and the `\includegraphics` command, which are defined in the `rotating` and `graphics` packages, respectively. These packages are read in (with `\usepackage`) at the top of the `LaTeX` file.

$n=$	2	3	5	7	11	13	17	19	23	29
n^2	4	9	25	49	121	169	289	361	529	841
n^3	8	27	125	343	1331	2197	4913	6859	12167	24389
n^4	16	81	625	2401	14641	28561	83521	130321	279841	707281
n^5	32	243	3125	16807	161051	371293	1419857	2476099	6436343	20511149
n^6	64	729	15625	117649	1771561	4826809	24137569	47045881	148035889	594823321
n^7	128	2187	78125	823543	19487171	62748517	410338673	893871739	3404825447	17249876309
n^8	256	6561	390625	5764801	214358881	815730721	6975757441	16983563041	78310985281	500246412961
n^9	512	19683	1953125	40353607	2357947691	10604499373	118587876497	322687697779	1801152661463	14507145975869
n^{10}	1024	59049	9765625	282475249	25937424601	137858491849	2015993900449	6131066257801	41426511213649	420707233300201
n^{11}	2048	177147	48828125	1977326743	285311670611	1792160394037	34271896307633	116490258898219	952809757913927	12200509765705829

[illegible]

specs. there should be 24–27 lines of print per page of a thesis. This should be true whether the font size is 10, 11, or 12. Count them up; does this document conform? According to the Grad School specs. there should be 24–27 lines of print per page of a thesis. This should be true whether the font size is 10, 11, or 12. Count them up; does this document conform? According to the Grad School specs. there should be 24–27 lines of print per page of a thesis. This should be true whether the font size is 10, 11, or 12. Count them up; does this document conform? According to the Grad School specs. there should be 24–27 lines of print per page of a thesis. This should be true whether the font size is 10, 11, or 12. Count them up; does this document conform? According to the Grad School specs. there should be 24–27 lines of print per page of a thesis. This should be true whether the font size is 10, 11, or 12. Count them up; does this document conform?

What is it?

This is a labelled paragraph. The heading of the paragraph is emphasized. This is a labelled paragraph. The heading of the paragraph is emphasized.

2.2.2 This is a subsection

This is a subsection. Filler filler filler filler filler filler filler filler. Filler filler filler filler filler filler filler filler.

2.2.3 This is another subsection

This is another subsection. Filler filler filler filler filler filler filler filler. Filler filler filler filler filler filler filler filler.

This is paragraph number 2.

It used a `\paragraph{}` header, which are always inlined (with extra space) and boldfaced.

This is the third paragraph of the subsection. Filler filler filler filler filler filler filler filler. Filler filler filler filler filler filler filler filler.

2.2.3.1 This is a subsubsection (1)

This is the first paragraph of the subsubsection. Whether it is numbered or inlined depends on the option selected at the beginning of the thesis.

By default, a `\subsubsection` heading is numbered and set off on a separate line, left-justified.

However.

Using the `inlineh4` option, subsubsection headers are inlined. And using the `nonumh4` option suppresses numbering of the subsubsections. Together they make subsubsection headings just the same as paragraph headings.

2.2.3.2 This is another subsubsection (2)

Once again, whether its heading is numbered and/or inlined depends on the class options chosen at the start.

There is no “subsubsubsection” entity, and “subparagraph” gets no special treatment in **thesis** class.

2.3 The End

Finally, this is the end. The bibliography starts on the next page.

Bibliography

- [1] H. Lao. Linear Acoustic Processes in Rocket Engines. Ph.D. thesis, University of Colorado at Boulder (1979).
- [2] Q. Lao, M. N. Cassoy, and K. Kirkpatrick. J. Fluid Mechanics **2**, 122 (1996).
- [3] J. S. Fabnis, H. J. Giblet, and H. McDormand. J. Prop. and Power **2**, 157 (1980).
- [4] J. D. Dole. Perturbation Methods in Applied Mathematics. Winsdell Publishing Company, New York (1967).
- [5] F. C. Mulick. AIAA J. **3**, 1062 (1964).
- [6] G. I. Baylor. Proc. Roy. Soc., London A **294**, 456 (1959).

Appendix A

Objective Symptoms

Appendices follow the same page-numbering rules as regular chapters. The first page of a multi-page appendix is not numbered. But the page of a single-page appendix **is** numbered.

Are they slow learners or is it a **REAL** problem? These are classic findings in the hopelessly computer challenged.

- (1) Can't copy from hard drive to disk.
- (2) Can't eject disks.
- (3) The word "disk" has thousands of meanings to them. None are correct.
- (4) Saving a document in any form is a concept totally unexplainable to them.
- (5) Desktop covered with Untitled Folders - look again, untitled folders are everywhere.
- (6) "Lost" documents found often in the Apple Menu.
- (7) Trash always full. Claim they don't know how to place things in trash.
- (8) Mysterious things happen to their documents or computer when they are not present. AKA "computer victims".
- (9) Highlighting = deleting. Dragging = Oblivion.

- (10) Selecting, double-clicking a problem? They will always say their mouse is broken.
- (11) Their double- click mechanics wants you to send them to a neurologist.
- (12) Computer always on due to fear of having to restart it.
- (13) Have never read their QuickMail - will say “I prefer a phone call”.
- (14) Have magical beliefs about what computers do.
- (15) Describes some flaky way computers could REALLY help them, but is not yet available.
- (16) Constantly saying they need more “memory”.
- (17) Requests gizmos and gadgets, i.e., “mouse leash” or “disk cozy”.
- (18) Avoids eye contact when talking about computers.

Appendix B

Ode to Spot

(Data, Stardate 1403827)

Throughout the ages, from Keats to Giorchamo, poets have composed “odes” to individuals who have had a profound effect upon their lives. In keeping with that tradition I have written my next poem . . . in honor of my cat. I call it . . . Ode . . . to Spot. (Shot of Geordi and Worf in audience, looking mystified at each other.)

Felus cattus, is your taxonomic nomenclature
an endothermic quadruped, carnivorous by nature?
Your visual, olfactory, and auditory senses
contribute to your hunting skills, and natural defenses.
I find myself intrigued by your sub-vocal oscillations,
a singular development of cat communications
that obviates your basic hedonistic predilection
for a rhythmic stroking of your fur to demonstrate affection.
A tail is quite essential for your acrobatic talents;
you would not be so agile if you lacked its counterbalance.
And when not being utilized to aid in locomotion,
It often serves to illustrate the state of your emotion.

(Commander Riker begins to applaud, until a glance from Counselor Troi brings him to a halt.) Commander Riker, you have anticipated my denouement. However, the sentiment is appreciated. I will continue.

O Spot, the complex levels of behavior you display
connote a fairly well-developed cognitive array.
And though you are not sentient, Spot, and do not comprehend
I nonetheless consider you a true and valued friend.