A few Bayesian lectures for the Uninitiated

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Why Bayes?

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- (Fine-tune shrinking against collinearity, for model selection.)
- (Get interpretable "credible intervals".)

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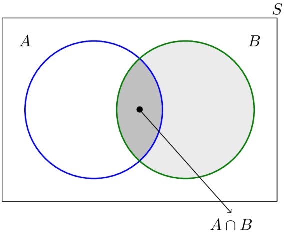
These *cannot* be valid arguments, otherwise we'll never see progress in science!

Deriving Bayes Theorem



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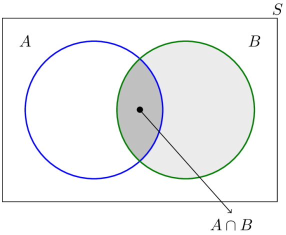

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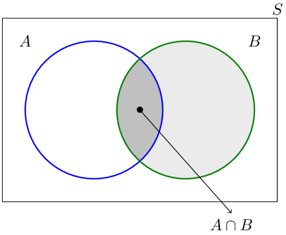

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Re-arrange:

$$\boxed{P(A|B) = \frac{P(B|A)P(A)}{P(B)}} \tag{1}$$

Final slide: Prosecutor's Fallacy

In court, a forensic scientist states that the probability that an innocent has a DNA-match to the sample found at a crime scene is 1:3 million:

$$P(\text{DNA-match}|\text{innocent}) = 1/3\text{mio}.$$

The accused has a DNA-match: a clear case! – Or is it?

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Bring in the Bayes Theorem!

$$\boxed{P(A|B) = \frac{P(B|A)P(A)}{P(B)}} \tag{2}$$

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$$P(\text{innocent}|\text{DNA-match}) = \frac{P(\text{DNA-match}|\text{innocent})P(\text{innocent})}{P(\text{DNA-match})} \tag{3}$$

If there are 10 million people in the DNA database, we'll get:

$$P(\text{innocent}|\text{DNA-match}) = \frac{(1/3 \cdot 10^6) \cdot (10^7 - 1)}{(1/3 \cdot 10^6 \cdot 10^7) + 1}$$

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Bayes Theorem shows us how to do the maths.

Next time:
Turning a little GLM-style regression into something weirdly complicated, just to call it Bayesian.