A few Bayesian lectures for the Uninitiated

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Part 4: priors

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Bayes?!

Remember Bayes' Theorem in terms of fitting a model:

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Now: how to choose the *prior*, $P(\theta)$?

Priors galore!

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improper
conjugate
Jeffreys (scale invariant)
shrinkage
savvy (AIC-like)
hyperprior

See here for more:

https://jrnold.github.io/bayesian_notes/priors.html

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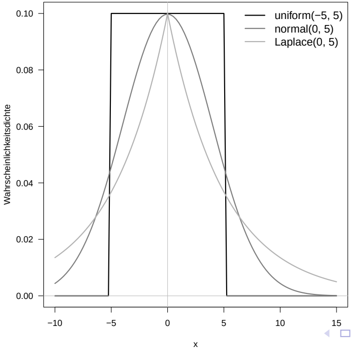

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hyperprior	prior for a hyper-parameter

Which prior distribution to choose?

1. Informative or not? $\longrightarrow$ informative if possible![1]

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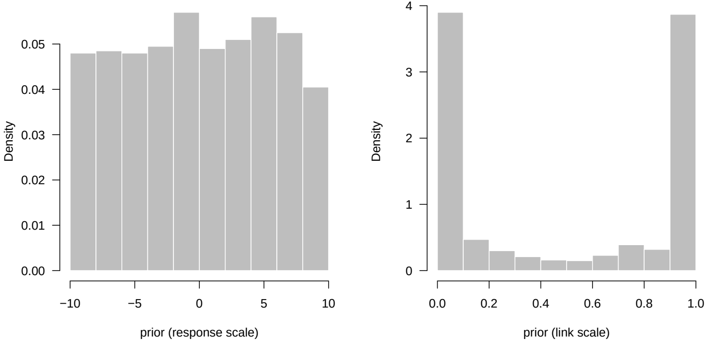

Uniform$(-10, 10)$ as prior for binomial model's intercept: it is no longer uninformative at the response scale!
(Jeffreys priors *remain* uninformative!)

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4. Are parameters correlated? $\longrightarrow$ multivariate priors (e.g. inverse (!) Wishart for MVN)

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- Priors are our friends: we can use them to our advantage!
- We may not like priors, but they are inevitable in Bayesian analysis.

Next time:
Mixed effect models from a Bayesian perspective (with R)