Let us consider the Stokes problem defined in the unit square $\Omega := (0,1)^2$

$$\begin{cases} -\Delta \mathbf{u} + \nabla p = 0, & \text{in } \Omega, \\ \text{div } \mathbf{u} = 0, & \text{in } \Omega, \\ \mathbf{u} = 1\mathbf{i} + 0\mathbf{j}, & \text{on } \Gamma^{\text{up}} = \{0 \leq x \leq 1, \, y = 1\}, \\ \mathbf{u} = \mathbf{0}, & \text{on } \partial\Omega \backslash \Gamma^{\text{up}}. \end{cases}$$

1 Assemble the block matrix associated to the problem using the stable pair of spaces $\mathbb{P}^2/\mathbb{P}^1$. Solve the system using the GMRES method.

2 Using a value of 10^{-6} for the tolerance, does the number of iterations needed to reach the convergence increase with the number of the elements? (Use a uniformly refined grid with $n = 10, 20, 40$ subdivisions). Use the `Firedrake` function `LinearVariationalSolver` with the following parameters:

```
{'ksp_type': 'gmres', 'pc_type': 'none',
 'ksp_rtol': 1.e-6, 'ksp_max_it': 10000}
```

3 Now let us introduce the block diagonal preconditioner

$$P = \begin{bmatrix} A & 0 \\ 0 & M_p \end{bmatrix},$$

where M_p is the mass matrix for the pressure unknown. Does the number of iterations needed to reach the convergence increase with the number of the elements? (Use a uniformly refined grid with $n = 10, 20, 40$ subdivisions and for the tolerance consider a value of 10^{-6}).

4 Study the eigenvalues for both the unpreconditioned and preconditioned system.