Project report (MAN-300) on

Factorization of polynomials using Rational Root Test

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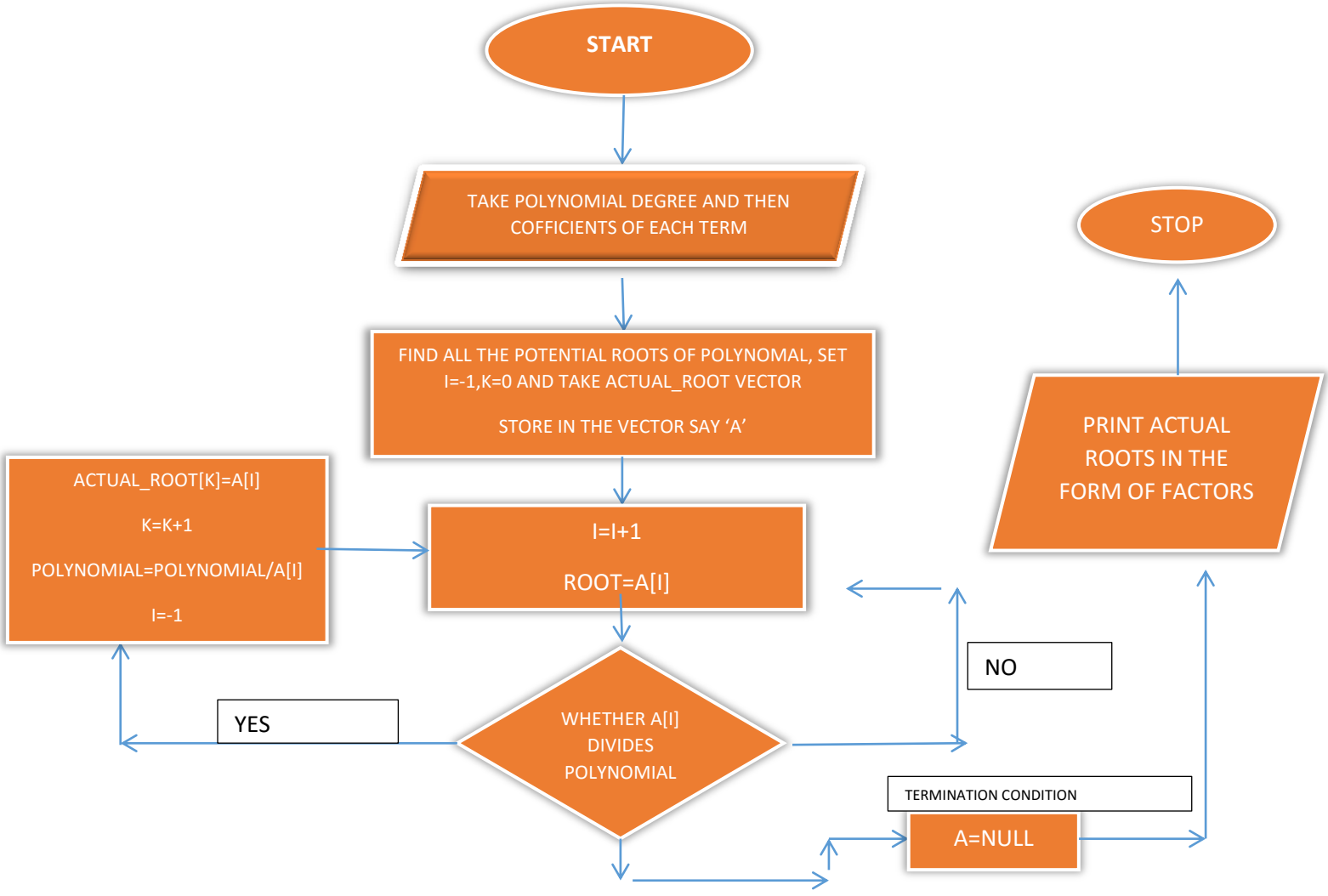
Abstract:

There are various applications like MATLAB, Maple etc. These applications are capable of factoring polynomial in real time.
The problem is that they are not open source; they are not available for free use.
I, under the proper support and guidance of Dr. Maheshanand, am trying to create an open source algorithm which can factorize a polynomial with only real rational roots with any multiplicity.
Algorithm test the potential roots to satisfy the given polynomial. If it is satisfied then the test potential root is actual root of that polynomial.

Algorithm flowchart:



Time complexity:

Synthetic division for division of polynomial by rational root costs: O(n); n→ no. of terms in polynomial.

Factor function have complexity in worst case as O(m); m→ no. m.

Rational_root function has complexity of O(xy); x→no. of factors in constant term, y→no. of factors in leading coefficient.

Algorithm complexity→O(max{m,n,xyk}); k→ sum of all multiplicities

Future Expectations:

Since the algorithm is open and free to use, modify, copy and reuse. It will help many students and researchers to develop this algorithm as per their needs and use to solve other polynomial problems.

Codes:

```
#include<iostream>

#include<vector>  // used for vector<>

#include<math.h> // used for abs()


using namespace std;


//structure to hold the division result

struct division_result{

        vector<float> quotient;

        float remainder;

};
```

```
//synthetic division function  complexity O(n=degree of poly)
division_result synthetic_division(vector<float>cofficient, float root){
		float rem=0;
		vector<float>poly_coff;

		for(std::vector<float>::iterator i=cofficient.begin();i!=cofficient.end();i++){
			rem=rem*root+(*i);
			poly_coff.push_back(rem);
			//poly_coff.pop_back();
			cout<<rem<<"  ";
		}
		division_result result;
		result.remainder=rem;
		poly_coff.pop_back();
		result.quotient=poly_coff;

		return result;
}
//end of synthetic division function

//calculation of factors
vector<float> factors(int f){
  vector<float>fact;
  for(int i=1;i<=abs(f);++i)
   {
      if(f%i==0)
        fact.push_back(i);
```

```
        }
        return fact;
}
//end of factors

//calculation of all rational roots start
vector<float> rational_roots(vector<float> poly){

        int leading_coff = poly.front();
        int constant_term = poly.back();

        cout<<"Factors of leading coff: ";
        vector<float> leading_coff_factors= factors(leading_coff);
        for(std::vector<float>::iterator i=leading_coff_factors.begin();i!=leading_coff_factors.end();i++)
        cout<<*i<<"   ";

        cout<<"\nFactors of constant term: ";
        vector<float> constant_term_factors= factors(constant_term);
        for(std::vector<float>::iterator j=constant_term_factors.begin();j!=constant_term_factors.end();j++)
        cout<<*j<<"   ";

        vector<float> rational_roots;
        for(std::vector<float>::iterator i=leading_coff_factors.begin();i!=leading_coff_factors.end();i++){
                for(std::vector<float>::iterator j=constant_term_factors.begin();j!=constant_term_factors.end();j++){
                        rational_roots.push_back((*j)/(*i));
                        rational_roots.push_back(-(*j)/(*i));
                }
```

```
        }
        return rational_roots;
}
//end of rational_roots


//testing
int main(){
        vector<float>factor_of_polynomial;
        cout<<"Testing Synthetic division:\n";
        vector<float> fop;
        division_result res;
        cout<<"Degree of Polynomial:";
        int deg;
        cin>>deg;
        vector<float>poly;
        float temp;
        cout<<"cofficient of polynomial (n+1) terms,leading cofficient term first:";
        while(deg+1){
                cin>>temp;
                poly.push_back(temp);
                deg--;
        }



        fop= rational_roots(poly);
        cout<<"\nRational Roots:";
        for(std::vector<float>::iterator i=fop.begin();i!=fop.end();i++)
```

```
                cout<<*i<<"  ";
        for(std::vector<float>::iterator k=fop.begin();k!=fop.end();k++){
                cout<<"\ngoing with "<<*k<<"\n.....\n";
        res= synthetic_division(poly,*k);
        cout<<"\nremainder:  "<<res.remainder<<"\nQuotient: ";

        if(res.remainder==0){
                factor_of_polynomial.push_back(*k);
                poly=res.quotient; //// solved:: multiplicity condition is checked
                k=fop.begin();
        }
        for(std::vector<float>::iterator i=res.quotient.begin();i!=res.quotient.end();i++){
                cout<<*i<<"  ";
        }
        }
        cout<<"\n\n**********\nfactorization: ";
        for(std::vector<float>::iterator g=factor_of_polynomial.begin();g!=factor_of_polynomial.end();g++)
        cout<<"(x"<<showpos<<-*g<<")";
}
```

References:

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NO