Partially Ionized Plasmas

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Astronomy 253: Plasma Astrophysics

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These lecture notes are based off of Leake et al. (2014, preprint), Meier & Shumlak (2012), Zweibel (1989), Zweibel et al. (2011), and other sources.

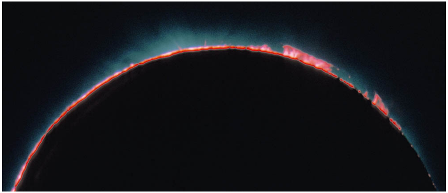
Outline

- Where are partial ionization effects important?
- Governing equations
- Ambipolar diffusion
- Non-equilibrium ionization (NEI)

Introduction

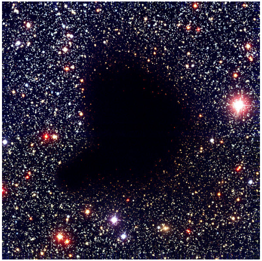
- MHD assumes that plasmas are fully ionized
- Plasmas below 10^4 K are partially ionized (assuming: collisional)
- There exist many partially/weakly ionized plasmas in astrophysics and elsewhere
  - Stellar chromospheres
  - Cold neutral medium and warm neutral medium in ISM
  - Molecular clouds
  - Protoplanetary disks
  - Earth's ionosphere
  - Some laboratory plasma experiments
- Need to modify equations to include neutrals

The solar chromosphere



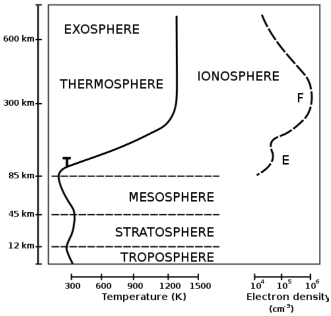
- Ionization fraction ranges from $\sim$0.5% to $\sim$50%
- As we go outward from the photosphere to corona, there are transitions from
  - Pressure dominated ($\beta \gg 1$) to magnetically dominated ($\beta \ll 1$)
  - Weakly ionized to fully ionized
  - Optically thick to optically thin
- Modeling the chromosphere requires partial ionization effects and NLTE radiative transfer
- Very dynamic region with reconnection, instabilities, etc.

Molecular clouds



- Temperatures of $\sim$10–20 K; densities of $\sim 10^2$–10^6 cm^{-3}
- Ionization due to cosmic rays even for very cold temperatures
- Partial ionization effects important because of very long length scales
    - Coupling between ions and neutrals
- Problem: how is mass transported across field lines so stars can form?
- In protoplanetary disks, the Hall effect may be important

Earth's thermosphere and ionosphere



- Transition region between atmosphere and magnetosphere
  - Similarities to chromosphere (Leake et al. 2014)
- Driven from above and below
- Ionized by EUV and X-ray solar radiation
  - Variation over days, seasons, solar cycle
- Affects radio propagation

The continuity equations

- There are separate continuity equations for ions, neutrals, and electrons

$$
\begin{aligned}
\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \mathbf{V}_i) &= +\Gamma^{ion} - \Gamma^{rec} \quad (1) \\
\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \mathbf{V}_e) &= +\Gamma^{ion} - \Gamma^{rec} \quad (2) \\
\frac{\partial n_n}{\partial t} + \nabla \cdot (n_n \mathbf{V}_n) &= -\Gamma^{ion} + \Gamma^{rec} \quad (3)
\end{aligned}
$$

  where we are considering only neutrals and singly charged ions
- If we assume $n_e = n_i$ then the ion and electron equations become identical
- The ionization and recombination rates are Γ^{ion} and Γ^{rec}

Let's look at two forms of the momentum equations

- Full form
  - What physics effects are important?
  - Used in many recent simulation efforts (with a few approximations)
- Simplified form
  - Make lots of approximations
  - Provides more physical insight
  - Useful for deriving ambipolar diffusion

The momentum equations

- The momentum equations are

$$\frac{\partial}{\partial t}\left(\rho_i \mathbf{V}_i\right) + \nabla \cdot \left(\rho_i \mathbf{V}_i \mathbf{V}_i\right) = -\nabla \cdot \mathbf{P}_i + q_i n_i \left(\mathbf{E} + \frac{\mathbf{V}_i \times \mathbf{B}}{c}\right) + \mathbf{R}_i^{ie} + \mathbf{R}_i^{in} + \Gamma^{ion} m_i \mathbf{V}_n - \Gamma^{rec} m_i \mathbf{V}_i \quad (4)$$

$$\frac{\partial}{\partial t}\left(\rho_e \mathbf{V}_e\right) + \nabla \cdot \left(\rho_e \mathbf{V}_e \mathbf{V}_e\right) = -\nabla \cdot \mathbf{P}_e - q_e n_e \left(\mathbf{E} + \frac{\mathbf{V}_e \times \mathbf{B}}{c}\right) + \mathbf{R}_i^{ie} + \mathbf{R}_e^{en} + \Gamma^{ion} m_e \mathbf{V}_n - \Gamma^{rec} m_e \mathbf{V}_e \quad (5)$$

$$\frac{\partial}{\partial t}\left(\rho_n \mathbf{V}_n\right) + \nabla \cdot \left(\rho_n \mathbf{V}_n \mathbf{V}_n\right) = -\nabla \cdot \boldsymbol{P}_n - \mathbf{R}_i^{in} - \mathbf{R}_e^{en} + \Gamma_n^{rec}\left(m_i \mathbf{V}_i + m_e \mathbf{V}_e\right) \quad (6)$$

- What does this all mean?

The full form of the momentum equations

- Pressure is a tensor (not a scalar)
- Lorentz forces on charged components (not neutrals)
- Momentum transfer due to collisions acting as drag forces
  - $\mathbf{R}_i^{in}$: momentum transfer due to ion-neutral collisions
- Momentum transfer due to ionization and recombination (e.g., $\Gamma^{ion} m_i \mathbf{V}_n$)
- Momentum transfer due to charge exchange not included
- This form assumes only three components: ions, electrons, neutrals
- The electron equation of motion can be used to derive Ohm's law

A simpler form of the momentum equations

- Represent momentum transfer as a drag force

$$\rho_n \left(\frac{\partial}{\partial t} + \mathbf{V}_n \cdot \nabla \right) \mathbf{V}_n = -\nabla p_n - \rho_n \nu_{ni} \left(\mathbf{V}_n - \mathbf{V}_i \right) \tag{7}$$

$$\rho_i \left(\frac{\partial}{\partial t} + \mathbf{V}_i \cdot \nabla \right) \mathbf{V}_i = -\nabla p_i + \frac{\mathbf{J} \times \mathbf{B}}{c} - \rho_i \nu_{in} \left(\mathbf{V}_i - \mathbf{V}_n \right) \tag{8}$$

with

$$\rho_n \nu_{ni} = \rho_i \nu_{in} = \frac{\rho_i \rho_n \langle \sigma V \rangle_{in}}{m_i + m_n} \tag{9}$$

  - $\langle \sigma V \rangle_{in}$ is the rate coefficient (averaging over velocity distribution)
  - σ is the cross section
  - V is the relative velocity in the center of mass frame

For a weakly ionized plasma, we can neglect ion inertia and the ion pressure gradient

- The ion momentum equation becomes

$$\frac{\mathbf{J} \times \mathbf{B}}{c} = \rho_i \nu_{in} (\mathbf{V}_i - \mathbf{V}_n) \tag{10}$$

- The induction equation in the ion frame is

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V}_i \times \mathbf{B}) \tag{11}$$

- Use $\mathbf{V}_i \approx \mathbf{V} + (\mathbf{V}_i - \mathbf{V}_n)$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B}) + \nabla \times \Big(\underbrace{\frac{\mathbf{J} \times \mathbf{B}}{\rho_i \nu_{in} c}}_{\text{ambipolar drift}} \times \mathbf{B} \Big) \tag{12}$$

Ambipolar drift: simple geometry

- Suppose $\mathbf{B} = \hat{\mathbf{z}} B_z(x, t)$. Then

$$\frac{\partial B_z}{\partial t} = \frac{\partial}{\partial x}\left(D_{AD}\frac{\partial B_z}{\partial x}\right) \tag{13}$$

where

$$D_{AD} \equiv \frac{V_A^2}{\nu_{ni}} \tag{14}$$

Ambipolar drift acts like nonlinear diffusion in slab geometry

- The magnetic field decouples from the bulk flow when

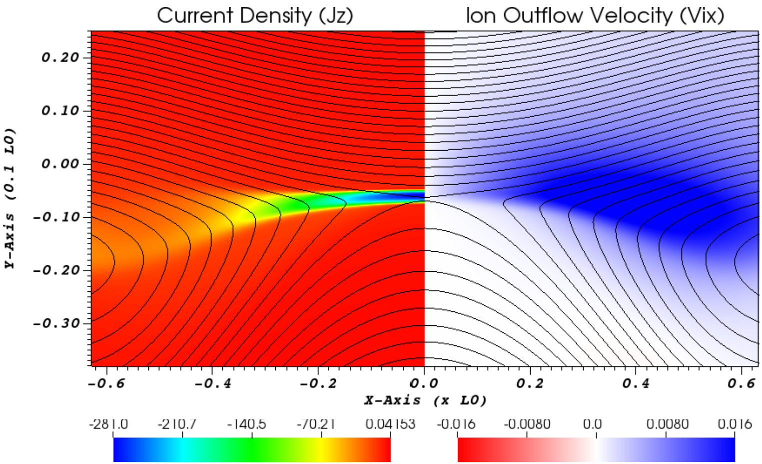
$$R_{AD} \equiv \frac{LV}{D_{AD}} \lesssim 1 \tag{15}$$

- Ambipolar diffusion can facilitate formation of singularities

There are separate energy equations for different species

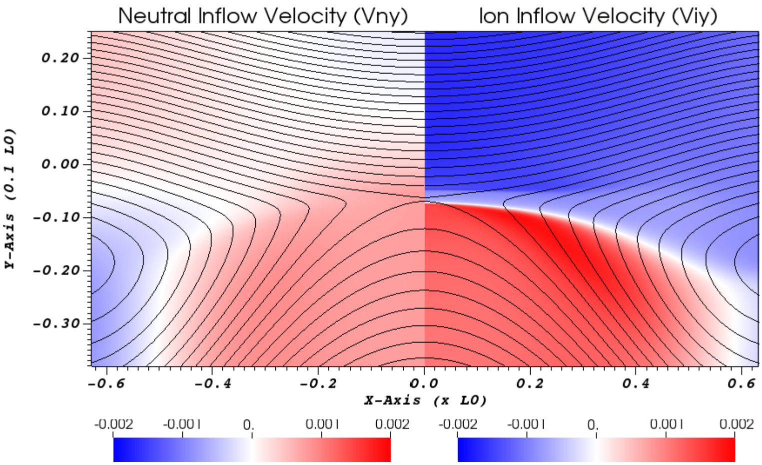
- Heating due to ion/neutral friction
- Isotropic thermal conduction for neutrals
- Anisotropic thermal conduction for ions & electrons
- Heat transfer between species due to collisions, charge exchange
- Ionization energy
- For details see Meier & Shumlak (2012)

Application: asymmetric reconnection in the chromosphere



- $B_{top} = 10$ G; $B_{bot} = 5$ G
- $T_{top} = 8750$ K; $T_{bot} = 9250$ K

Application: asymmetric reconnection in the chromosphere



- Asymmetric decoupling of neutrals & ions in inflow regions
- Outflow is well-coupled (not shown)

Non-equilibrium ionization (NEI)

- Ionization equilibrium assumes that the ionization and recombination time scales $\ll$ thermodynamical time scales
  - This assumption is not met in many diffuse, quickly evolving plasmas!
- Examples of NEI plasma:
  - Solar wind & CMEs (outside of a few $R_\odot$)
  - Supernova remnants
- Relatively simple to model hydrogen, but we also care about heavier elements

Building up intuition for NEI processes

- Suppose you rapidly heat plasma
  - Ionization takes time to catch up to temperature changes
  - The charge state distribution will imply that the plasma is cooler than it actually is
  - The plasma is *underionized*
- Suppose plasma expands rapidly $\Rightarrow$ quick adiabatic cooling
  - Recombination takes time to catch up to temperature increase
  - The charge state distribution will imply that the plasma is hotter than it actually is
  - The plasma is *overionized*

How do you model non-equilibrium ionization plasmas?

- Following a parcel of plasma, evolve the equation for every charge state of each element of interest

$$\begin{aligned}\frac{\mathrm{d}n_z}{\mathrm{d}t} &= n_e n_{z-1} q_i(Z, z-1, T) \\ &\quad - n_e n_z \left[q_i(Z, z, T) + \alpha_r(Z, z, T)\right] \\ &\quad + n_e n_{z+1} \alpha_r(Z, z+1, T) \qquad (16)\end{aligned}$$

  where z is the charge, Z is the atomic number, q_i is the ionization rate, and α_r is the recombination rate. Assumes collisionally dominated.
- Beware: atomic data have errors!
  - $\sim$10–20% errors for best data
  - Higher errors for less well known data & theoretical calculations
  - Energetic particles can increase ionization rates

Summary

- Examples of partially ionized plasmas include stellar chromospheres, molecular clouds, and Earth's ionosphere
- Partially ionized plasmas are described using separate equations for the neutrals, ions, and electons
- These equations include momentum transport and energy transfer between species
- Ambipolar diffusion arises when the induction equation is written using the bulk velocity instead of the ion velocity
  - Can lead to formation of singularities
- Partial ionization effects modify dispersion relations for waves & instabilities
- Non-equilibrium ionization is important in diffuse plasmas when temperature changes occur more quickly than ionization and recombination can keep up

Final Thoughts

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Plasma astrophysics is a rich and emerging field

- Plasma physics is necessary to describe many astrophysical phenomena:
  - Structure of galaxies, star formation, stellar atmospheres, jets, turbulence, evolution of shocks, cosmic rays/particle acceleration, accretion disks, interaction of planetary magnetospheres with stellar winds, compact objects
- Old astrophysical adage: *If you don't understand it, invoke magnetic fields.*
  - Well, now you can!

Plasma processes are often coupled to other plasma processes

- Examples include:
  - Particle acceleration is coupled to shocks & instabilities
  - Dynamos are coupled to turbulence & reconnection
  - Turbulence is coupled to reconnection, waves, & headaches
- Important research topics for next few decades:
  - How are plasma processes connected with each other?
  - What are the nonlinear dynamics of these processes?

To understand astrophysical plasmas, we must connect to laboratory and heliospheric plasmas

- Astrophysical plasmas
  - Extreme regions of parameter space
  - Very limited observations of small-scale processes
- Solar atmosphere
  - Global consequences of small-scale plasma processes
  - Observations possible at very high spatial resolution
- Space plasmas
  - Direct investigation of collisionless plasma processes
  - *In situ* measurements for details of small-scale processes
- Laboratory experiments
  - Test models under controlled settings
  - Abundant diagnostics but relatively modest plasma parameters

Professional organizations working on plasma astrophysics

- APS: Topical Group in Plasma Astrophysics (GPAP)
- APS: Division of Plasma Physics (DPP)
  - Laboratory focus but with a plasma astrophysics contingent
- AAS: Laboratory Astrophysics Division (LAD)
  - Focus goes beyond plasma experiments
- New England Space Science Consortium (NESSC)
  - Mostly heliophysics, but there are regional cross-disciplinary meetings that focus on plasma processes

Thoughts?

- This is a new course, and the first time we're teaching it
- Suggestions?
  - Topics
  - Homeworks
  - Projects
  - Level of difficulty