Mapping

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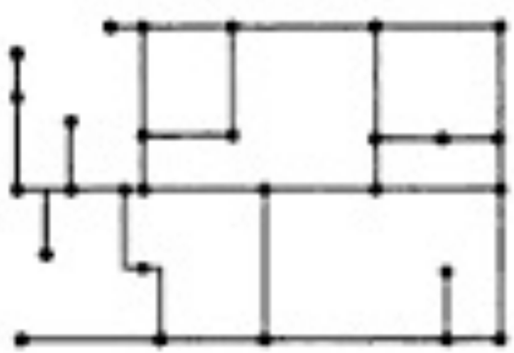
Mapping

- A **map** is a model of the environment used for robot localization and to compute paths. A map is impacted by robot pose representation.
- **Mapping** is the task of generating models of robot environments from sensor data.
- **Map precision** must match robot and application. The higher the map precision the higher the computational complexity.

Mapping

Mapping

1. High level features (e.g. landmarks for topological maps, etc.): Low volume, filters out lot of the information
2. Low level features (e.g. lines, etc.): Medium volume, filters out some information
3. Raw sensor data: Large volume, uses all acquired information



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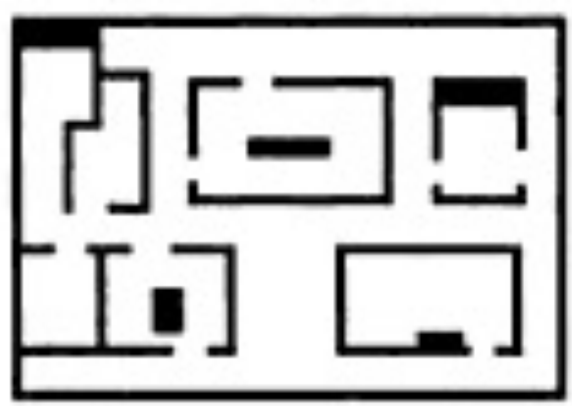

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Mapping

- **Odometric map**: distances between locations (no landmarks)
- **Landmark-based map**: distances and orientations in relation to external landmarks
- **Topological map**: similar to landmark-based map with nodes and edges representing particular locations (no odometry)
- **Metric map**: combines all previous types of maps with precise measurements between map locations and landmarks

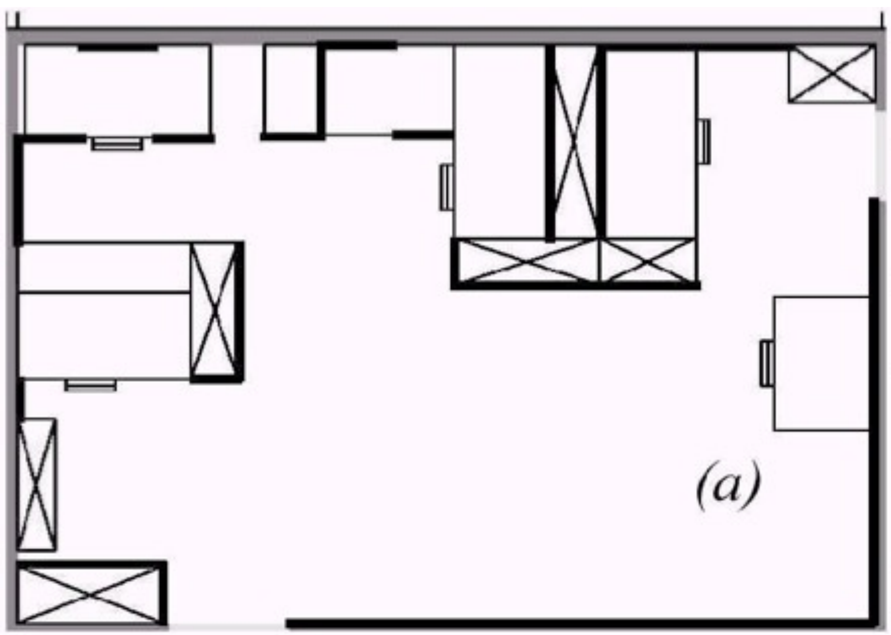
Representation

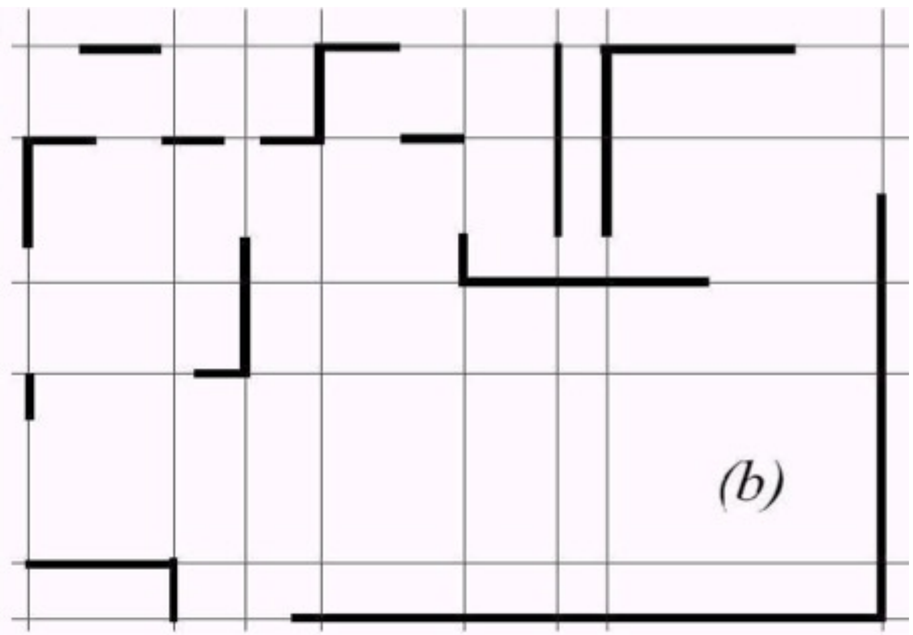
- **Representation** refers to how information is stored or encoded
- **Robot representation**
  - Represent the robot as a point (e.g. Bug Algorithms)
  - Assume robot is capable of omnidirectional motion
  - Robot in reality is of nonzero size
    - Dilation of obstacles by robot radius
    - Resulting objects are approximations
    - Leads to problems with obstacle avoidance
- **World representation**
  - Continuous
  - Discrete

Continuous Representation

a) High accuracy but can be computationally expensive

b) Map represented as series of infinite lines, e.g. using a laser ranger finder


(a)

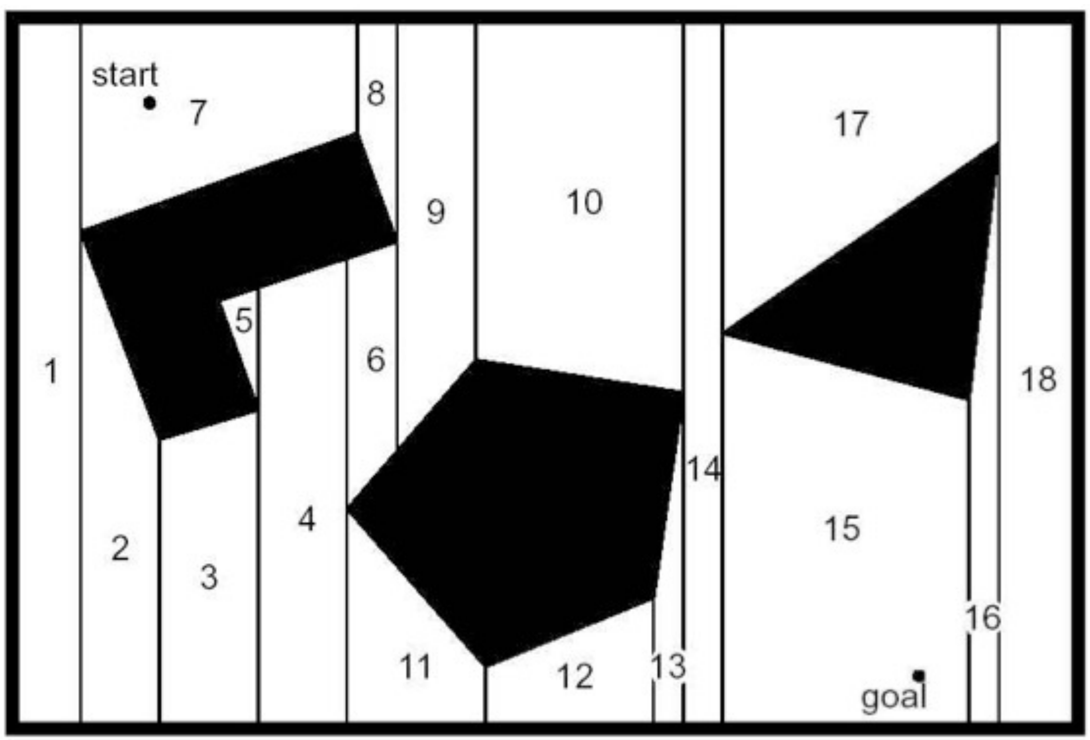

(b)

Discrete Representation

- Capture only useful features of world
- Lower accuracy but computationally less expensive
- Computationally better for reasoning, particularly if map is hierarchical
- Discrete Cell Decomposition
  - Exact Cell Decomposition
  - Fixed Cell Decomposition
  - Adaptive Cell Decomposition
- Occupancy Grid
- Topological Maps

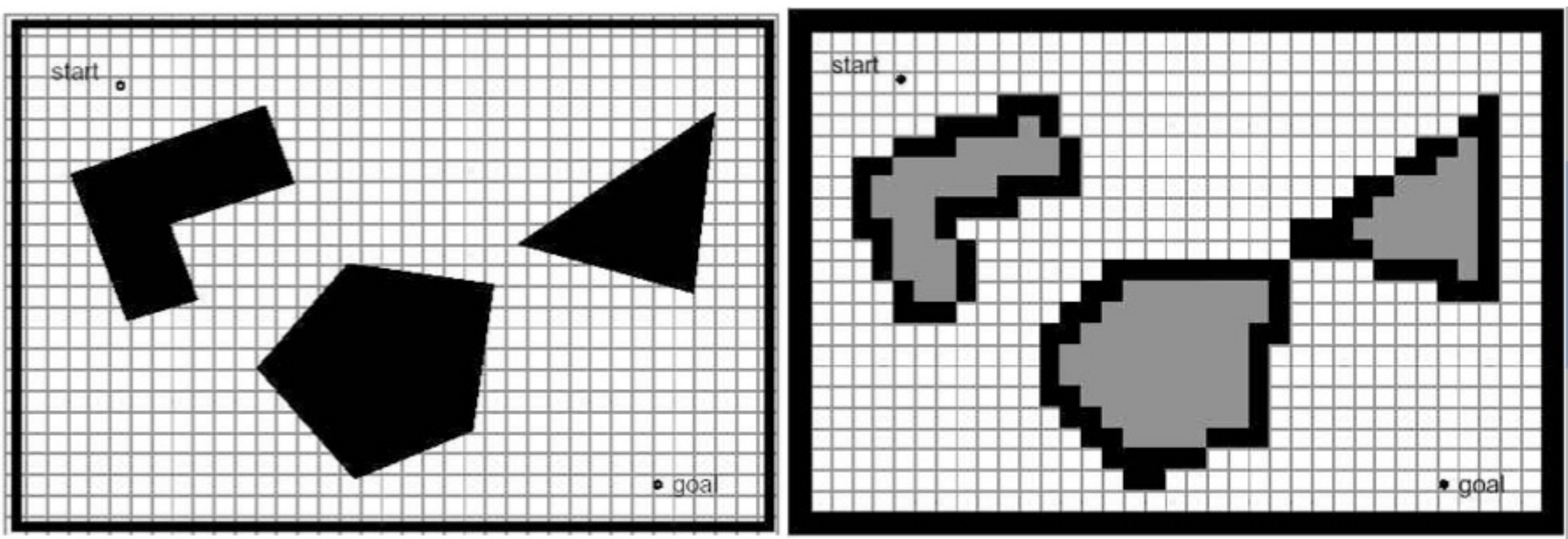
Exact Cell Decomposition

- Free space is represented by the “exact” union of simple trapezoidal regions or cells, while obstacles are represented by polygons.
- Regions or cells can be extremely compact



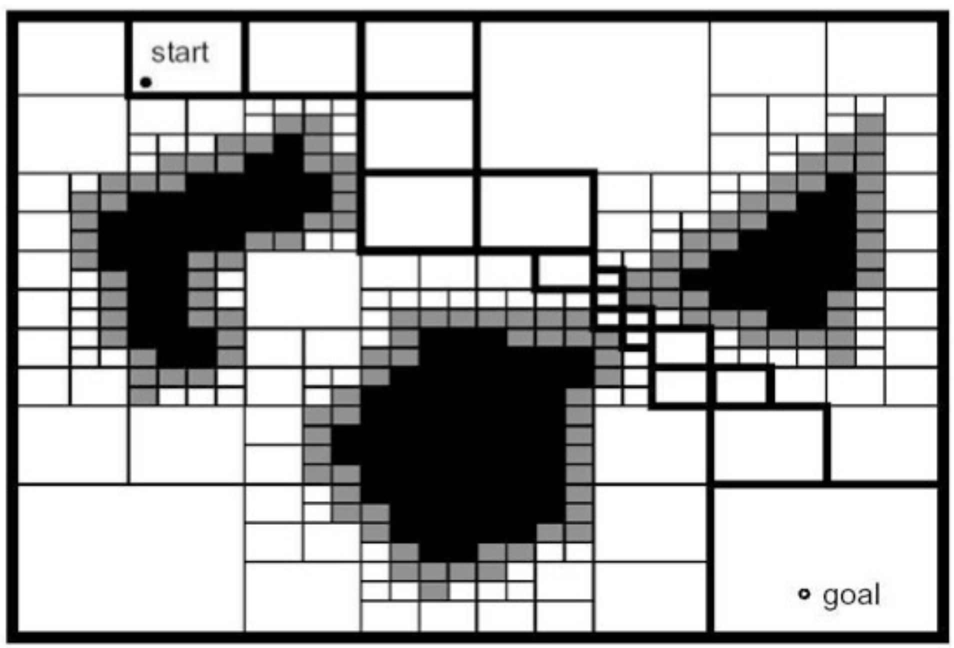
Fixed Cell Decomposition

- Free space is decomposed into cells of a fixed size
- Each cell is either empty or full (there may be loss of information such as loss of the narrow passageway)



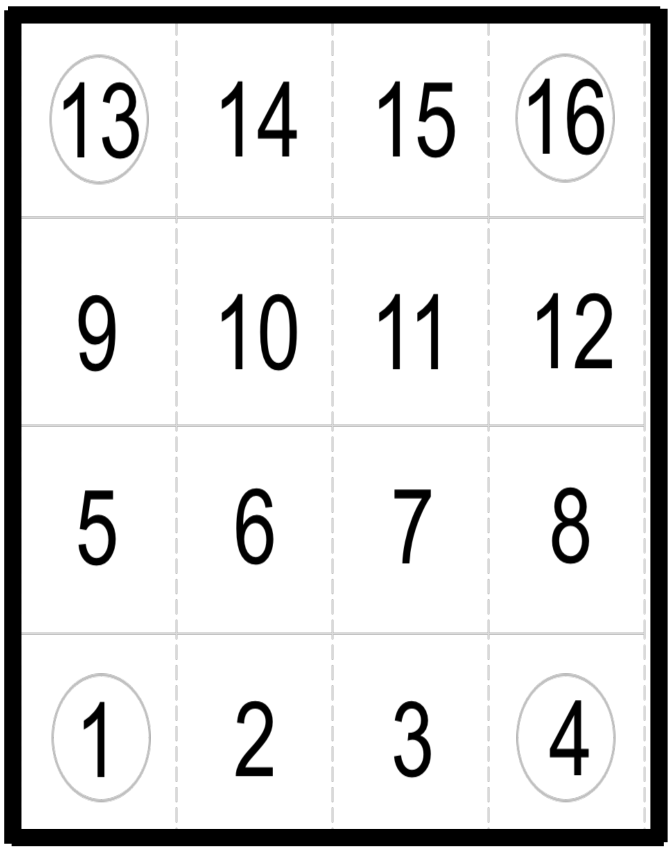
Adaptive Cell Decomposition

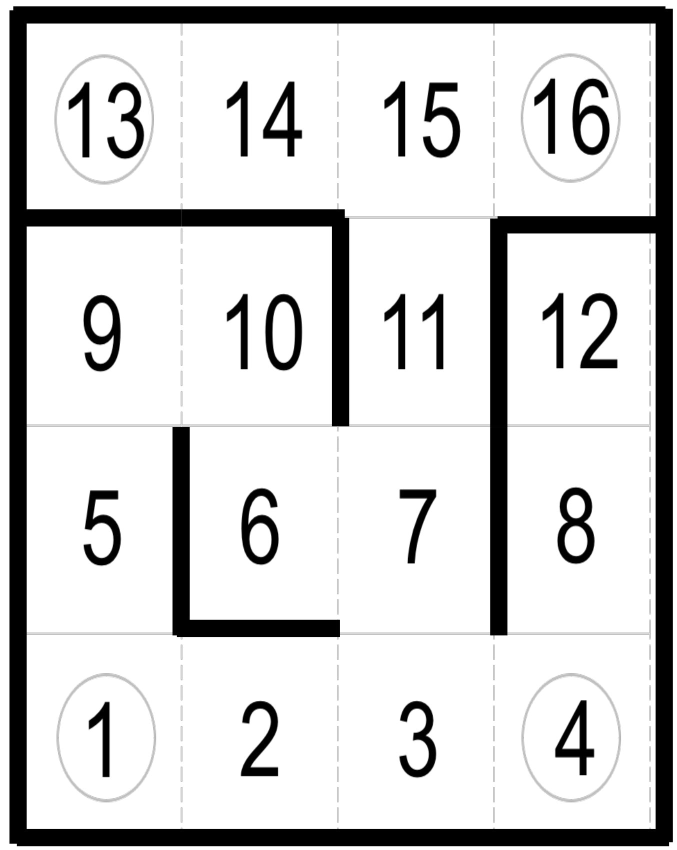
- Multiple types of adaptation: quadtree or other
- Recursively decompose free space until a cell is completely empty or full (there may be loss of information as with fixed cell decomposition)
- Space efficient if compared to fixed cell approach



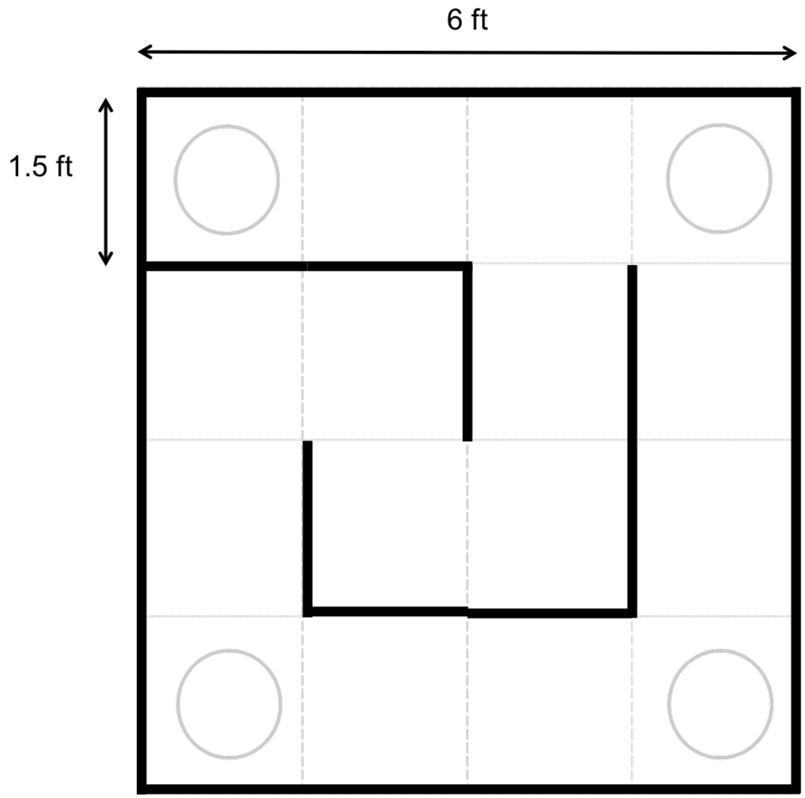
Fixed Cell Decomposition Example

- 16 fixed size cells
- No obstacles but walls separating cells





Fixed Cell Decomposition Example



W	W	W	W	W	W	W	W	W
W	1		2		3		4	W
W	W	W	W	W		W		W
W	5		6	W	7	W	8	W
W		W		W		W		W
W	9	W	10		11	W	12	W
W		W	W	W	W	W		W
W	13		14		15		16	W
W	W	W	W	W	W	W	W	W

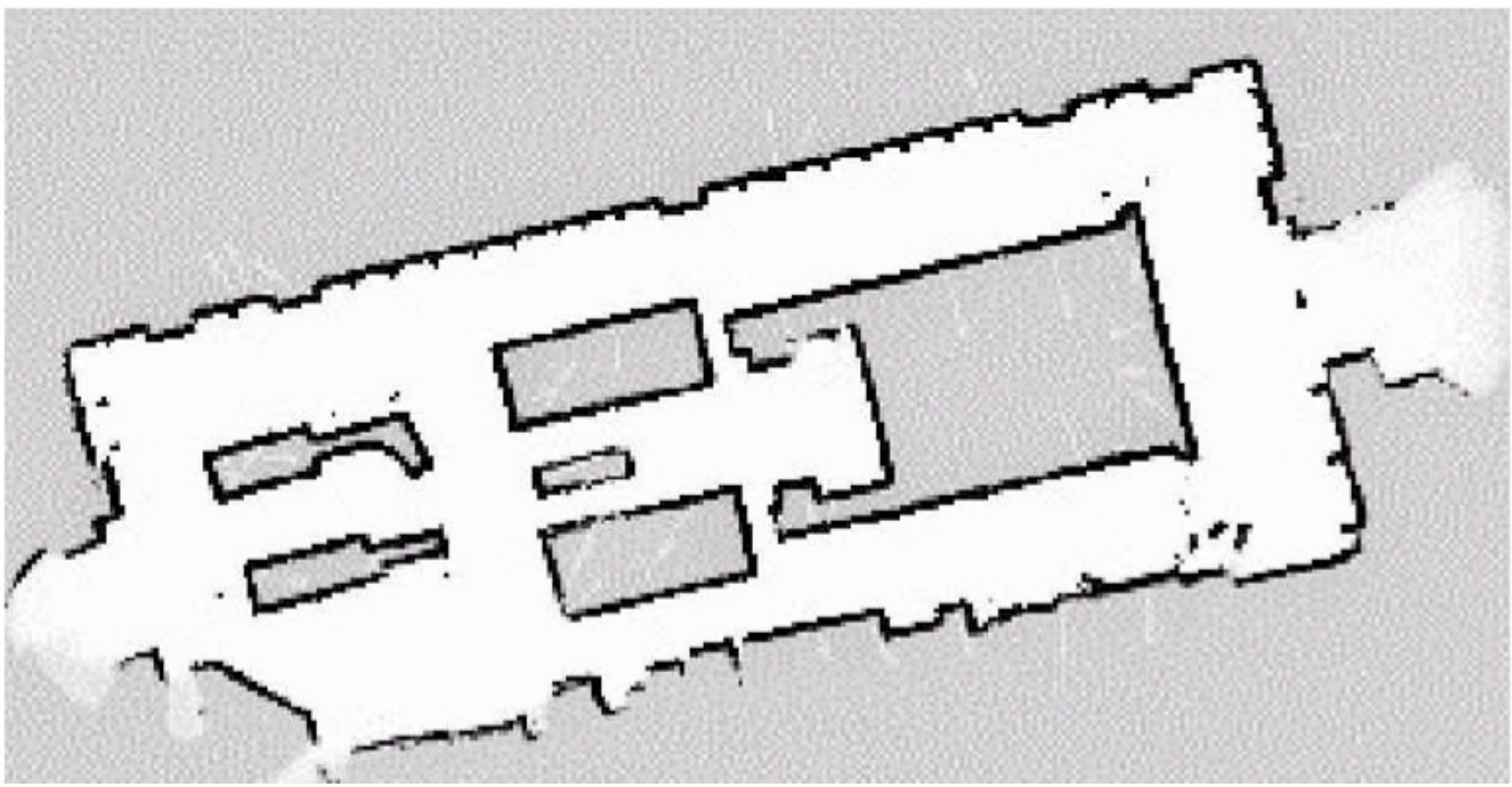
- *W*: Walls and Wall Corners
- *1-16*: Grid cell locations where robot may be found in the maze
- Empty cells are either possible locations of wall or robot

Occupancy Grid Maps

- Each cell indicates probability of being free or occupied:
  - Requires known robot pose
  - Grid cell probability distribution
  - Each variable is binary or a probability, corresponding to the degree of occupancy of the location it covers
- Particularly useful with range-based sensors
  - If sensor strikes something in a cell, higher probability
  - If sensor goes over cell and strikes something else, lower probability (presuming it is free space)
- Disadvantages
  - Map size is a function of size of environment and size of cell

Occupancy Grid Maps

- Darkness of cell proportional to cell counter value



Believes with Static States

- **Belief**: If we assume a static environment, i.e. only static objects and not affected by robot control, the belief is a function only of the measurement:

$$\mathrm{bel}_t(s) = \mathrm{p}(s \mid z_{1:t}, u_{1:t}) = \mathrm{p}(s \mid z_{1:t})$$

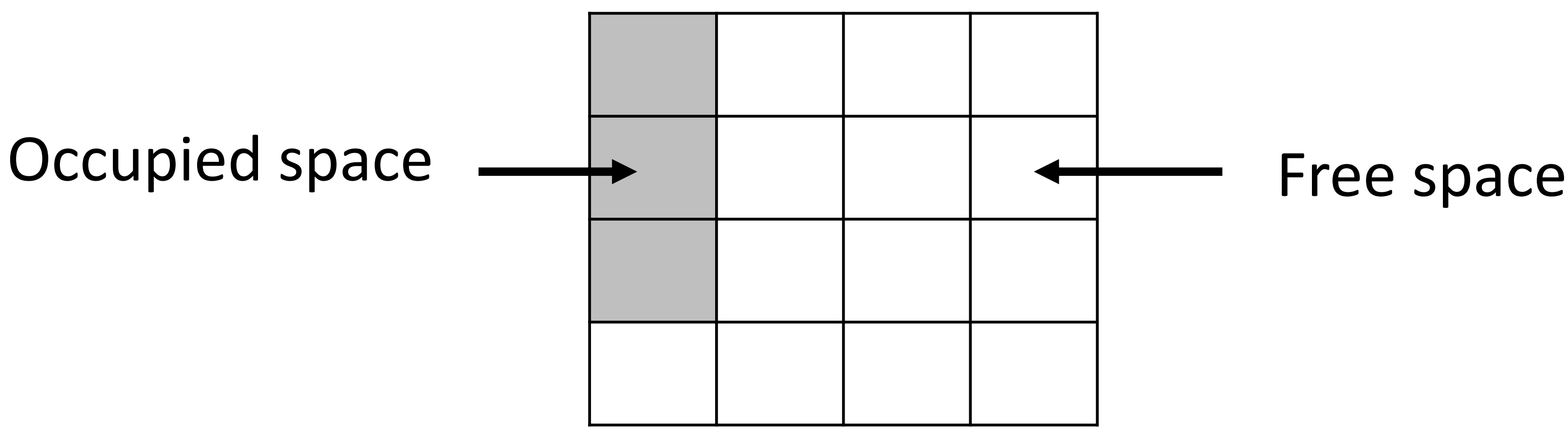
Binary Bayes Filter with Static States

- **Binary State**: The belief is defined as a binary state, i.e. occupied or not occupied:

$$\text{bel}_t(\neg s) = 1 - \text{bel}_t(s)$$

Occupancy Grid Maps

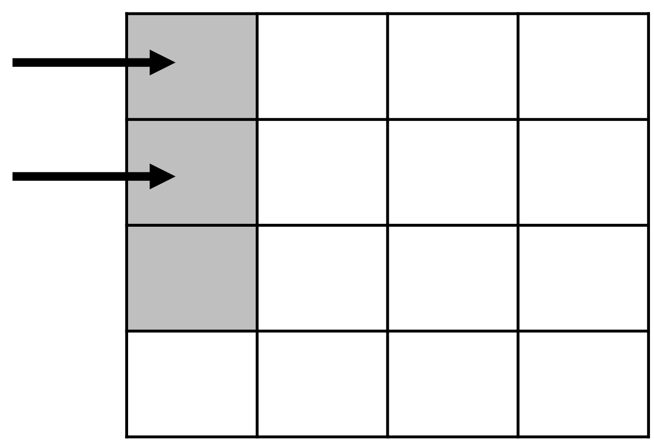
- The area that corresponds to a cell is either completely free or occupied.



Occupancy Grid Maps

- The cells (the random variables) are independent from each other.

No dependency between the cells



Occupancy Grid Maps

- Given sensor data $z_{1:t}$ and the poses $s_{1:t}$ of the sensor, the occupancy grid map m is estimated by:

$$p(m|z_{1:t}, s_{1:t})$$

- The controls $u_{1:t}$ play no role in the occupancy grid map, since the path is already known.

Occupancy Grid Maps

- Let m_i denote the grid cell with index i. The occupancy grid map partitions the space into finitely many grid cells:

$$m = \{m_i\}$$

- Each m_i has attached to it a binary occupancy value, either free ("0") or occupied ("1").
- The probability of the cell being occupied is given by

$$p(m_i = 1) = p(m_i)$$

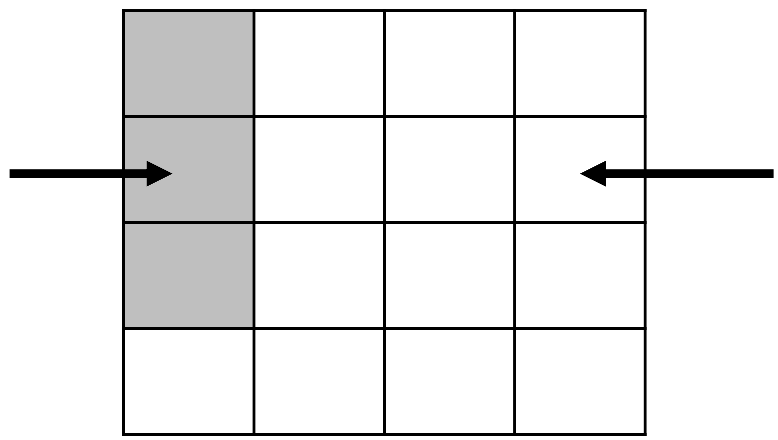
- The probability of the cell being free is given by

$$p(m_i = 0) = 1 - p(m_i)$$

Occupancy Grid Maps

- The probability of cell occupancy, $p(m_i)$, is computed from a binary variable m_i with value 0 or 1:

$p(m_i = 1) \rightarrow 1$



$p(m_i = 0) \rightarrow 0$

- p_{occupied}: Cell is occupied, $p(m_i = 1) > 0.5$
- p_{empty} or p_{free} or $p_{\text{unoccupied}}$: Cell is not occupied, $p(m_i = 0) < 0.5$
- p_{unknown}: No knowledge, $p(m_i) = 0.5$

Occupancy Grid Maps

- The occupancy grid algorithm breaks down the problem of estimating the map into a collection of separate problems of estimating each grid cell m_i:

$$p(m_i|z_{1:t}, s_{1:t})$$

- Each estimation is a binary problem with static states, and the complete map is approximated as the products of individual grid cells:

$$p(m|z_{1:t}, s_{1:t}) = \prod_i p(m_i|z_{1:t}, s_{1:t})$$

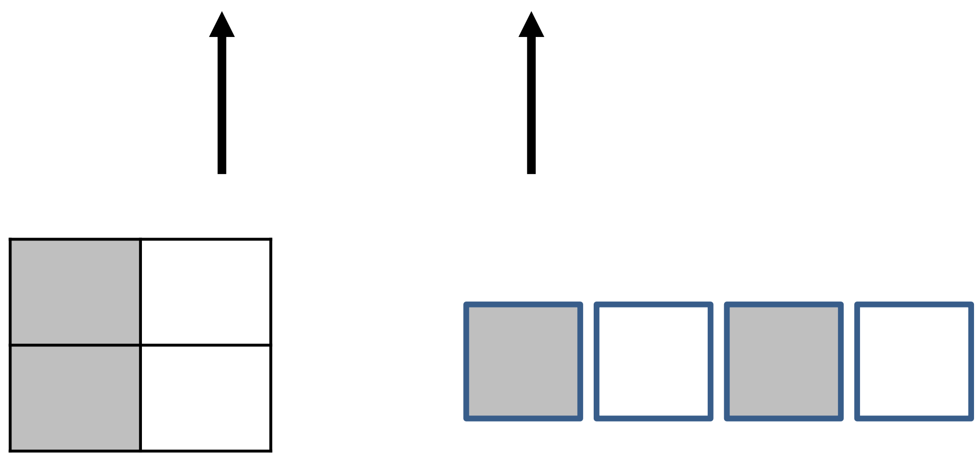
- This equation is equivalent to:

$$\underset{\substack{\uparrow \\ \text{map}}}{p(m)} = \prod_i \underset{\substack{\uparrow \\ \text{cell}}}{p(m_i)}$$

Occupancy Grid Maps

- The probability distribution of the map is given by the product over the maps.

$$p(m) = \prod_i p(m_i)$$



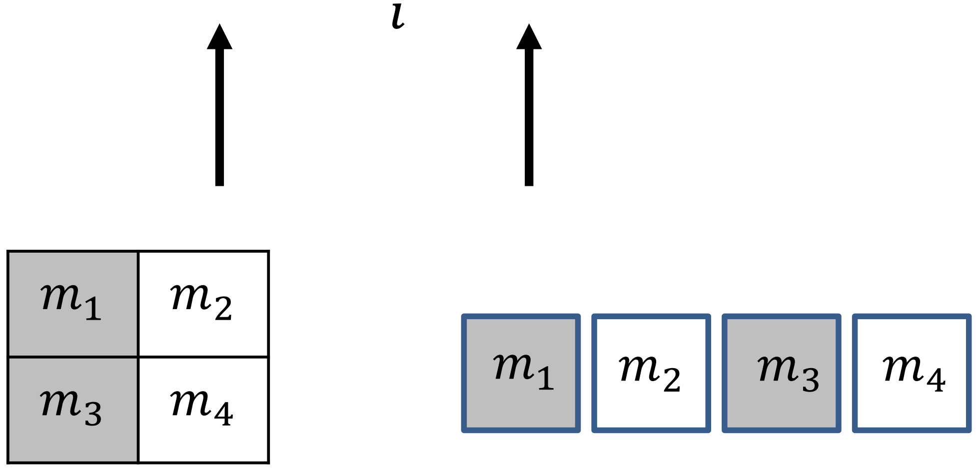
Example map
(4-dimension vector)

4 individual cells

Occupancy Grid Maps

Assume: $p(m_1)$=0.7 $p(m_2)$=0.4 $p(m_3)$=0.9 $p(m_4)$=0.2

$$p(m) = \prod_i p(m_i)$$



$$p(m) = p(m_1) * (1 - p(m_2)) * p(m_3) * (1 - p(m_4))$$
$$= 0.7 * (1 - 0.4) * 0.9 * (1 - 0.2)$$

Inverse Sensor Measurement Model

Assume:

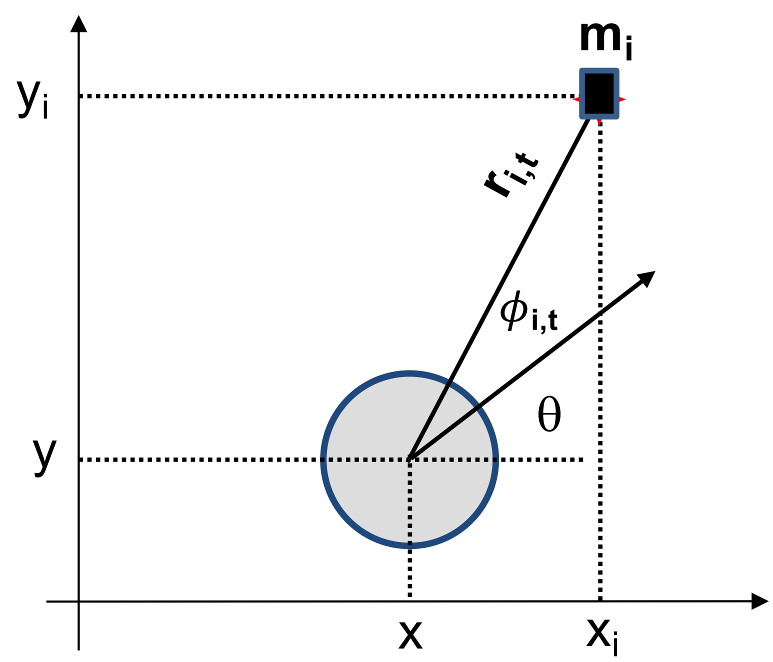
- s_t= $[x, y, \theta]^T$ is the robot pose at time t
- m_i = (x_i, y_i) is the location of the cell (can also be applied to landmarks)
- θ is the robot orientation
- $\phi_{i,t}$ is the relative orientation of grid cell m_i
- $r_{i,t}$ is the relative distance of grid cell m_i
- $z_{i,t}$ is the inverse sensor measurement from grid cell m_i to the robot:

$$z_{i,t} = (r_{i,t}, \phi_{i,t})$$

where

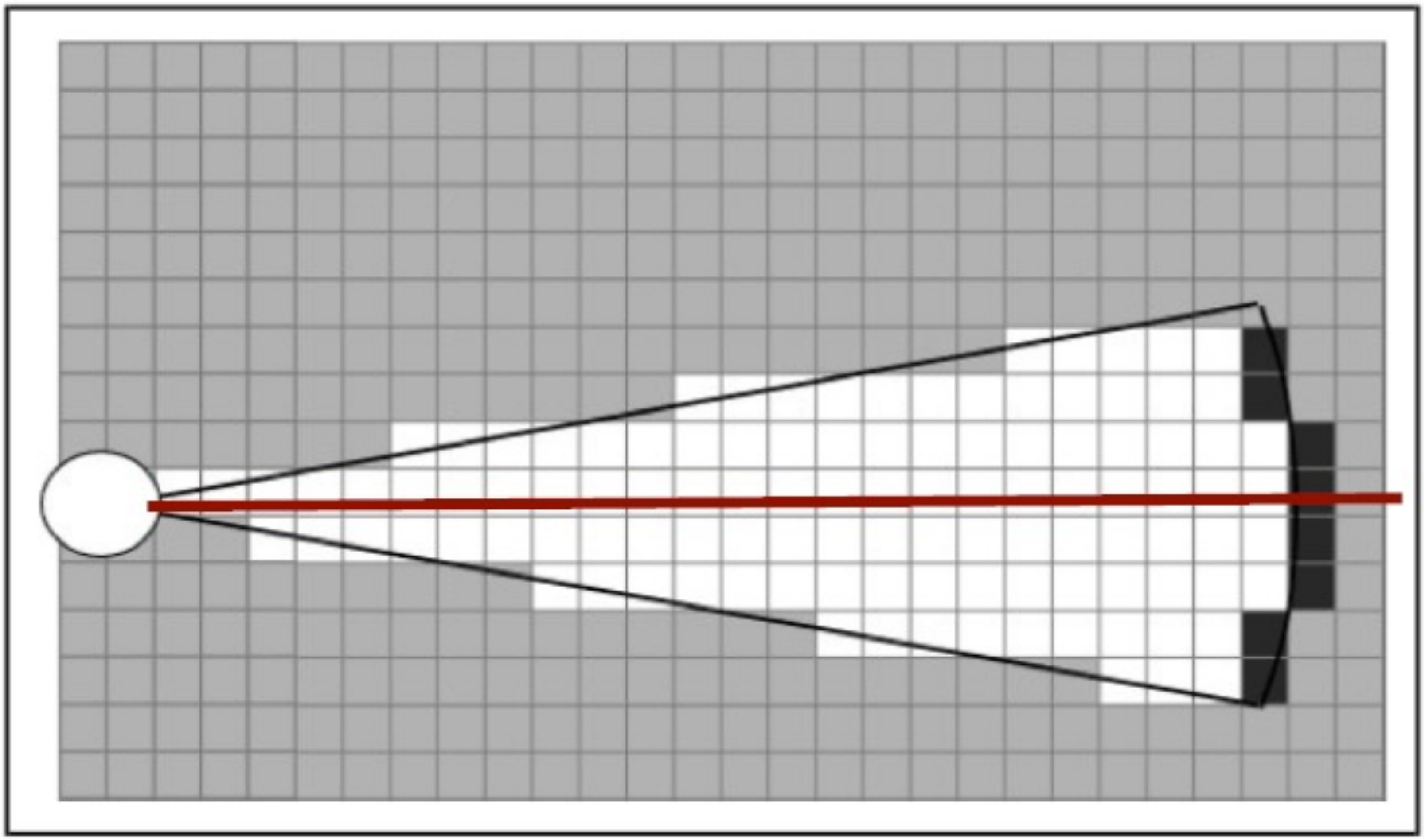
$$r_{i,t} = \sqrt{(x_i - x)^2 + (y_i - y)^2}$$

$$\phi_{i,t} = atan2\big((y_i - y), (x_i - x)\big) - \theta$$



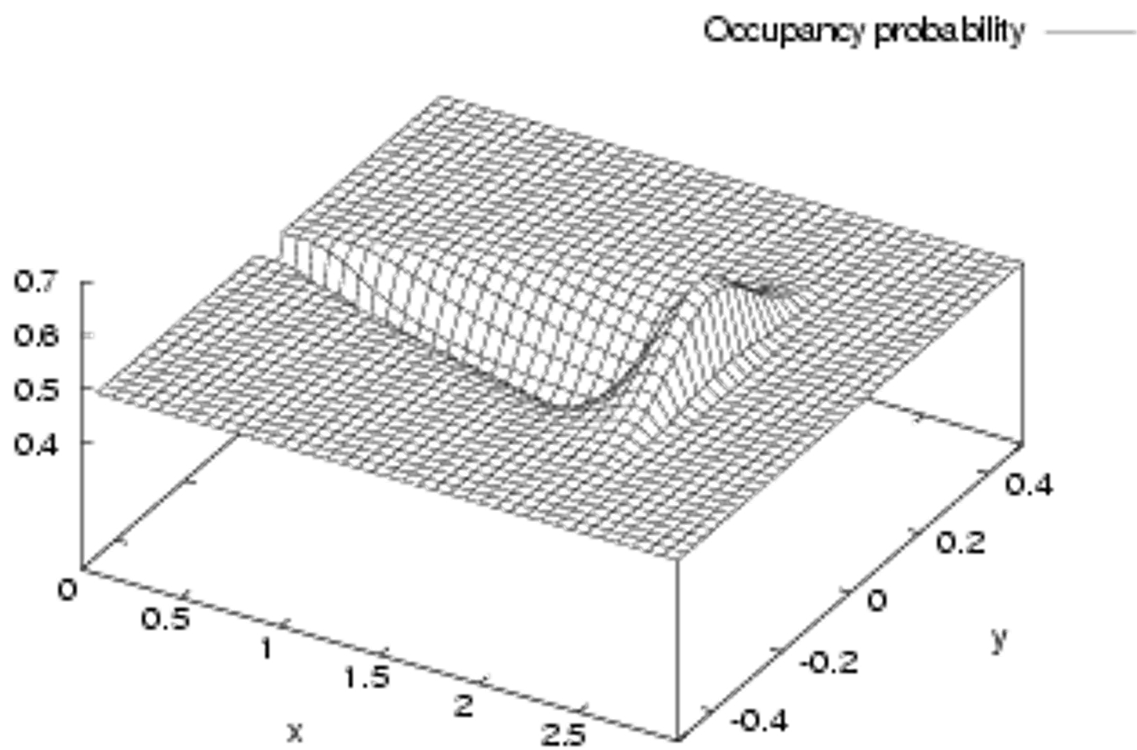
Occupancy Grid Maps

- Consider the cells along the optical axis (red lines)

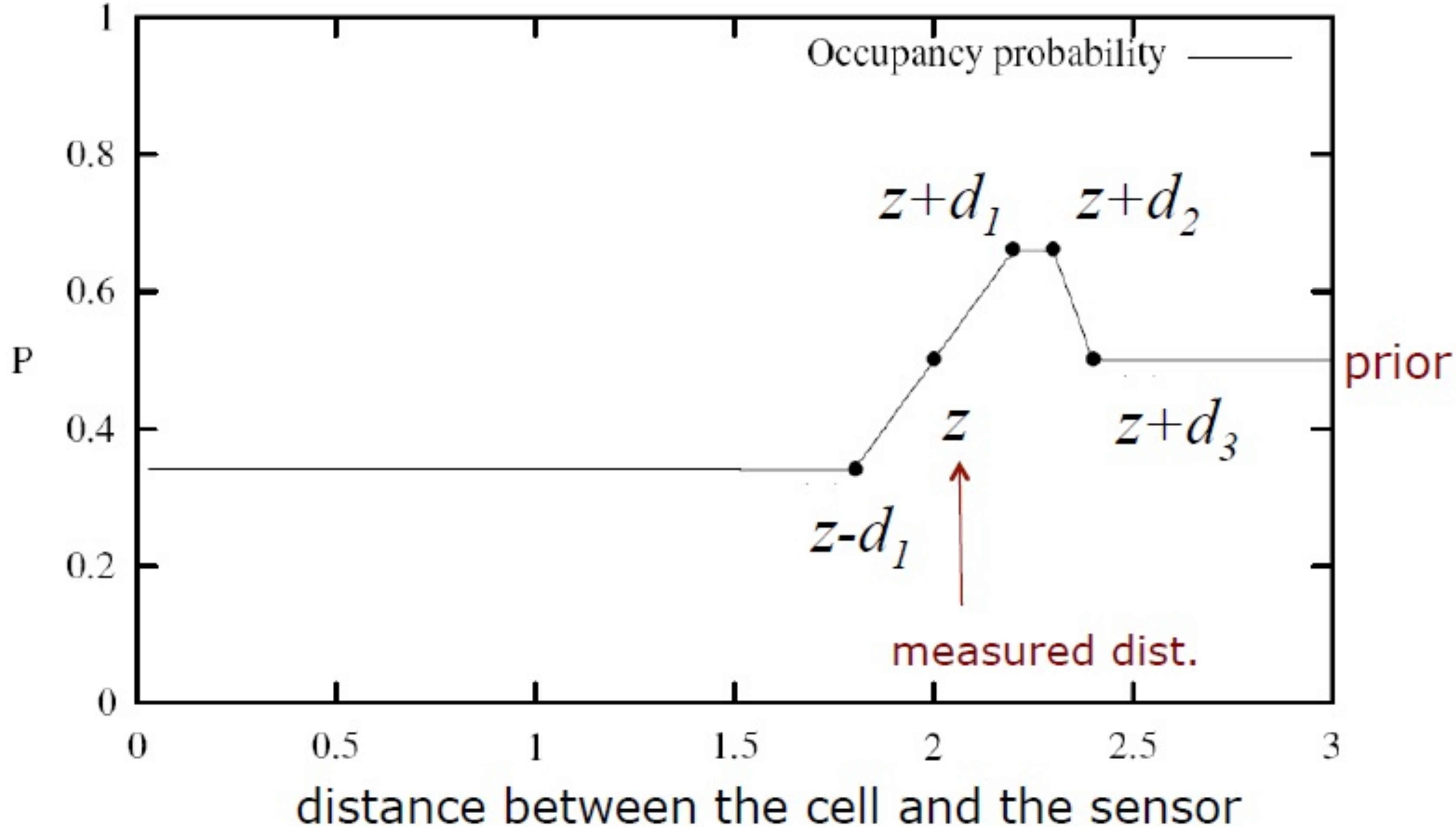


Occupancy Grid Maps

- Typical occupancy sensor model in 3D graph
- Combination of a linear function and a Gaussian



Occupancy Grid Maps
Value Depending on Measured Distance



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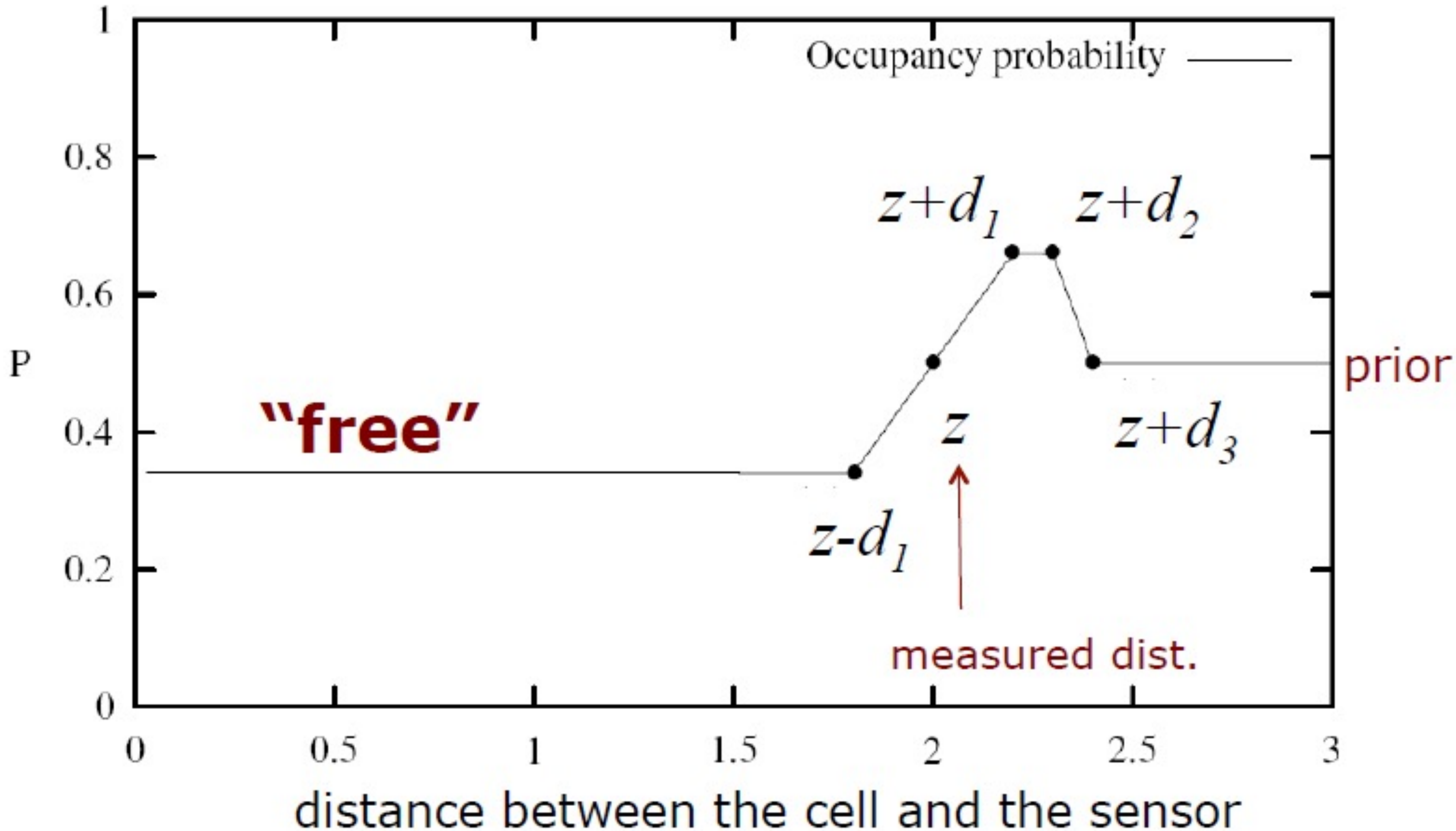

Occupancy Grid Maps
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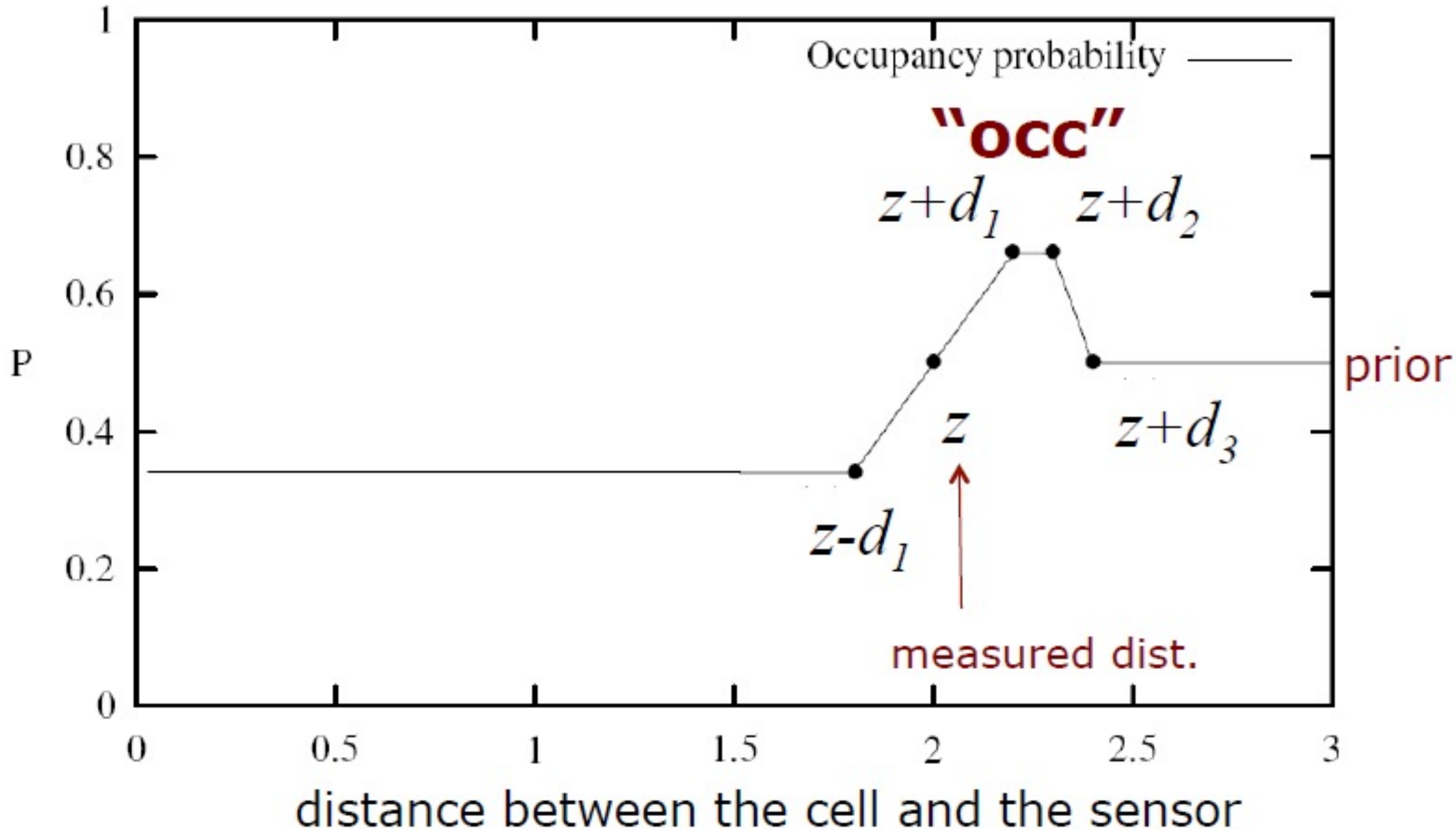
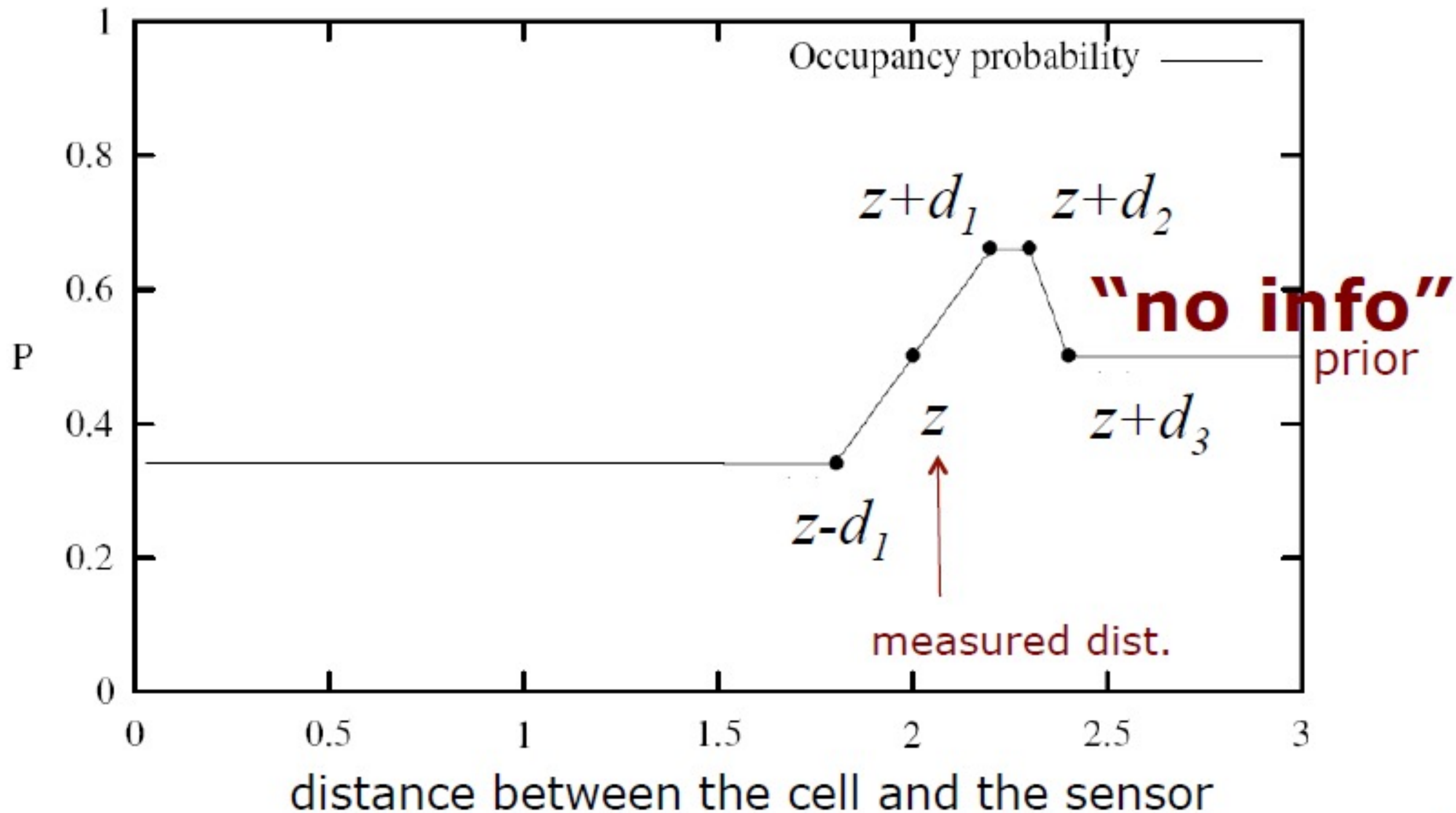
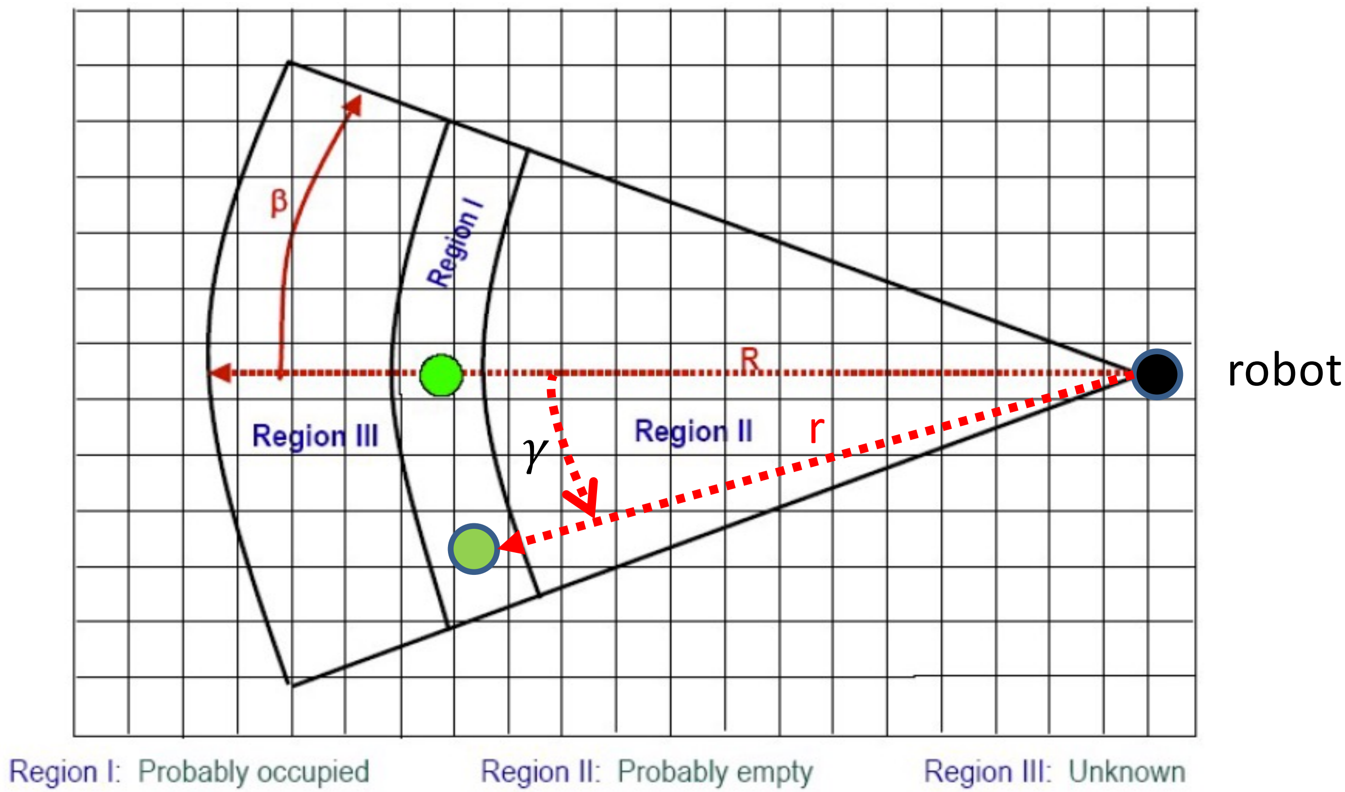


Occupancy Grid Maps
Value Depending on Measured Distance



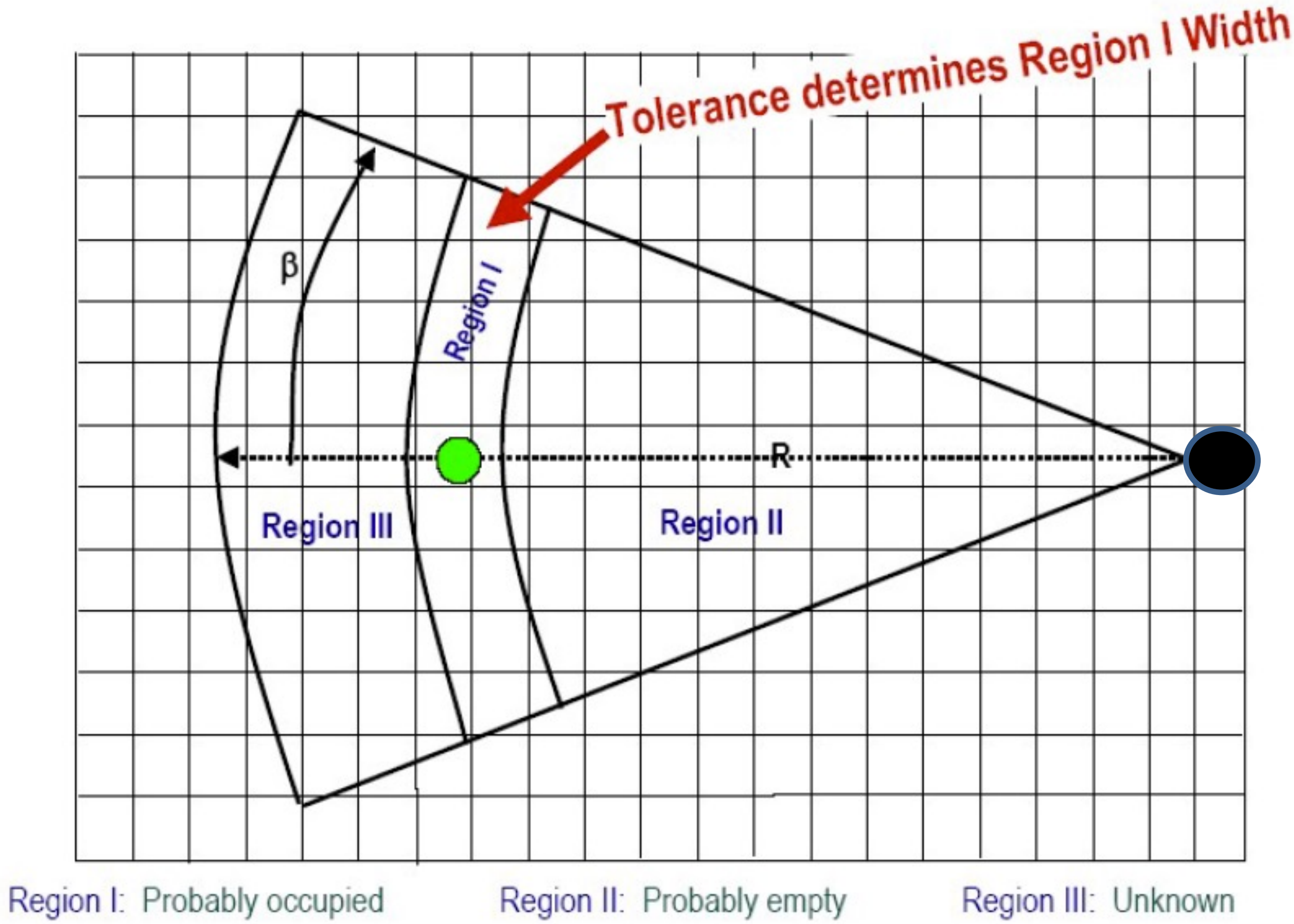
Occupancy Grid Maps

- 3 regions in relation to obstacle: (I) probably occupied by obstacle, (II) probably empty, (III) unknown



- r is the distance to the grid cell
- γ is the angle to the grid cell
- $R = z_{max}$ is the maximum range the sensor can detect
- β is the width of the sensor beam
- α is the thickness of the obstacle (Region I).

Occupancy Grid Maps



- Range readings have resolution error.
- Reading indicates range of possible values, e.g., reading of 0.87 meters with tolerance +/- 0.05 meters, is within (0.82, 0.92) meters
- Tolerance α gives width of Region I.

Occupancy Grid Maps

<u>Region I</u>:

The nearer to the sensor origin, the higher the belief

The closer to the main axis, the higher the belief

$$p_{occupied} = \frac{\frac{R-r}{R} + \frac{\beta-\gamma}{\beta}}{2}$$

$$p_{empty} = 1.0 - p_{occupied}$$

- r is the distance to the grid cell
- γ is the angle to the grid cell
- R is the maximum range the sensor can detect
- β is the width of the sensor beam

Occupancy Grid Maps

Region II:

The nearer to the sensor origin, the higher the belief

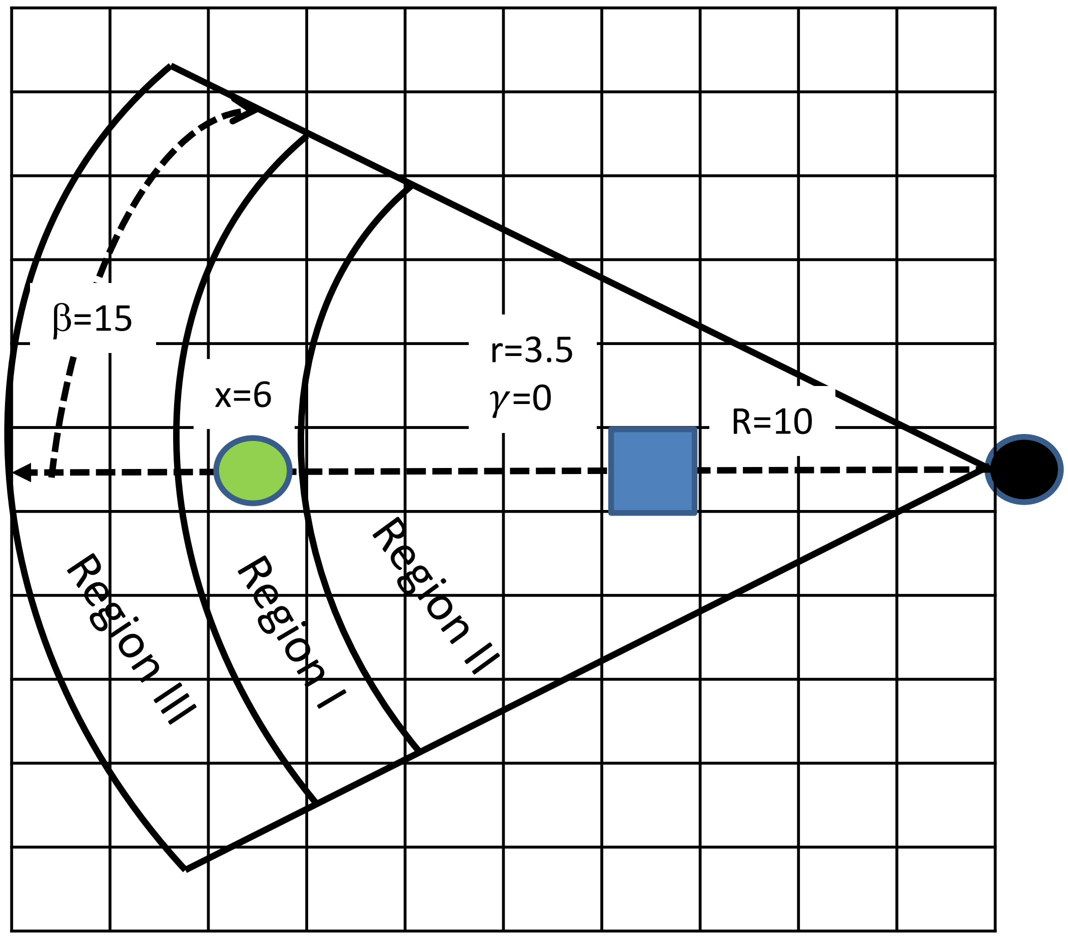
The closer to the main axis, the higher the belief

$$p_{empty} = \frac{\frac{R-r}{R} + \frac{\beta-\gamma}{\beta}}{2}$$

$$p_{occupied} = 1.0 - p_{empty}$$

- r is the distance to the grid cell
- γ is the angle to the grid cell
- R is the maximum range the sensor can detect
- β is the width of the sensor beam

Occupancy Grid Maps



Which region?
3.5 < (6.0-0.5) → Region II

$$p_{empty} = \frac{\frac{R-r}{R} + \frac{\beta-\gamma}{\beta}}{2}$$

$$= \frac{\frac{10-3.5}{10} + \frac{15-0}{15}}{2} = 0.83$$

$$p_{occupied} = 1.0 - p_{empty}$$
= (1.0-0.83) = 0.17

- x = 6 represents the distance read by the sensor to the obstacle.
- r = 3.5 represents the grid cell location where the probability is being computed
- Each square is 0.5 meters in length.

Inverse Sensor Measurement Model

inverse_sensor_model(m_i, s_t, z_t):

1. Let x_i, y_i be the center-of-mass of grid cell m_i
2. Let s_t = (x, y, θ)
3. $r = \sqrt{(x_i - x)^2 + (y_i - y)^2}$
4. $\phi = atan2\big((y_i - y), (x_i - x)\big) - \theta$
5. k = arg min$_j$ | $\phi - \theta_j$ |
6. if $r >$ min(z_{max},z_t^k+ $\alpha/2$) or | $\phi - \theta_j$ | > $\beta/2$ then
7. // no new information obtained about m_i
8. else if $z_t^k < z_{max}$ and | r - z_t^k | < $\alpha/2$
9. // measure m_i as occupied
10. else if $r \leq z_t^k$
11. // measure m_i as free

- k is the beam index.

Logs Odd Notation

- **Logs Odd Ratio** defines the ratio of the probability of an event divided by the probability of its negate:

$$\frac{p(s)}{p(\neg s)} = \frac{p(s)}{1 - p(s)}$$

- **Logs Odd** notation is defined as:

$$l(s) = \log \frac{p(s)}{1-p(s)}$$

- Retrieving $p(s)$:

$$\mathrm{bel}_t(s) = p(s) = 1 - \frac{1}{1+\exp l(s)}$$

Logs Odd Notation

- **Logs Odd Ratio** defines the ratio of the probability of an event divided by the probability of its negate:

$$\frac{p(m_i|z_{1:t}, s_{1:t})}{p(\neg m_i|z_{1:t}, s_{1:t})} = \frac{p(m_i|z_{1:t}, s_{1:t})}{1 - p(m_i|z_{1:t}, s_{1:t})}$$

- **Logs Odd** notation is defined as:

$$l(m_i|z_{1:t}, s_{1:t}) = \log \frac{p(m_i|z_{1:t}, s_{1:t})}{1 - p(m_i|z_{1:t}, s_{1:t})}$$

- Retrieving $p(m_i|z_{1:t}, s_{1:t})$:

$$p(m_i|z_{1:t}, s_{1:t}) = 1 - \frac{1}{1 + \exp l(m_i|z_{1:t}, s_{1:t})}$$

Logs Odd Notation

- Computing the ratio of both probabilities (occupied and empty), by combining Bayes Rule with Markov Assumption:

$$\frac{p(m_i|z_{1:t},s_{1:t})}{1-p(m_i|z_{1:t},s_{1:t})} = \frac{p(m_i|z_t,s_t)}{1-p(m_i|z_t,s_t)} \frac{p(m_i|z_{1:t-1},s_{1:t-1})}{1-p(m_i|z_{1:t-1},s_{1:t-1})} \frac{1-p(m_i)}{p(m_i)}$$

latest probability computation　　recursive term　　prior

- The prior defines the initial belief before processing any sensor measurements.

Logs Odd Notation

- Apply the logs odd ratio, and the product turns into a sum:

$$l(m_i|z_{1:t}, s_{1:t}) = l(m_i|z_t, s_t) + l(m_i|z_{1:t-1}, s_{1:t-1}) - l(m_i)$$

inverse sensor model | recursive term | prior

- The equation is rewritten to:

$$l_{t,i} = \text{inverse\_sensor\_model}(m_i, s_t, z_t) + l_{t-1,i} - l_0$$

$$\text{inverse\_sensor\_model}(m_i, s_t, z_t) = \log \frac{p(m_i|z_t, s_t)}{1 - p(m_i|z_t, s_t)}$$

Logs Odd Notation

- The prior defines the logs odd of the initial belief before processing any sensor measurements.

- l_0 is the prior or initial logs odd,before processing any sensor measurements:

$$l_0 = \log \frac{p(m_i=1)}{p(m_i=0)} = \log \frac{p(m_i)}{1-p(m_i)}$$

- if p_{prior}= 0.5, l_0 = log $\frac{0.5}{0.5}$ = log $1 = 0$

Occupancy Grid Maps

occupancy_grid_mapping($\{l_{t-1,i}\}$, s_t, z_t):

1. For all cells m_i do
2. if m_i in perceptual field of z_t then
3. $l_{t,i}$ = $l_{t-1,i}$ + inverse_sensor_model(m_i, s_t, z_t) - l_0
4. else
5. $l_{t,i}$ = $l_{t-1,i}$
6. endif
7. endfor
8. return $\{l_{t,i}\}$

Notes

- Line 3 uses additions only, no multiplications
- The computation is based on the inverse sensor model, p(s_t | z_t), instead of the forward model p(z_t | s_t). The inverse sensor model specifies a distribution over the (binary) state variable as a function of the measurement z_t.