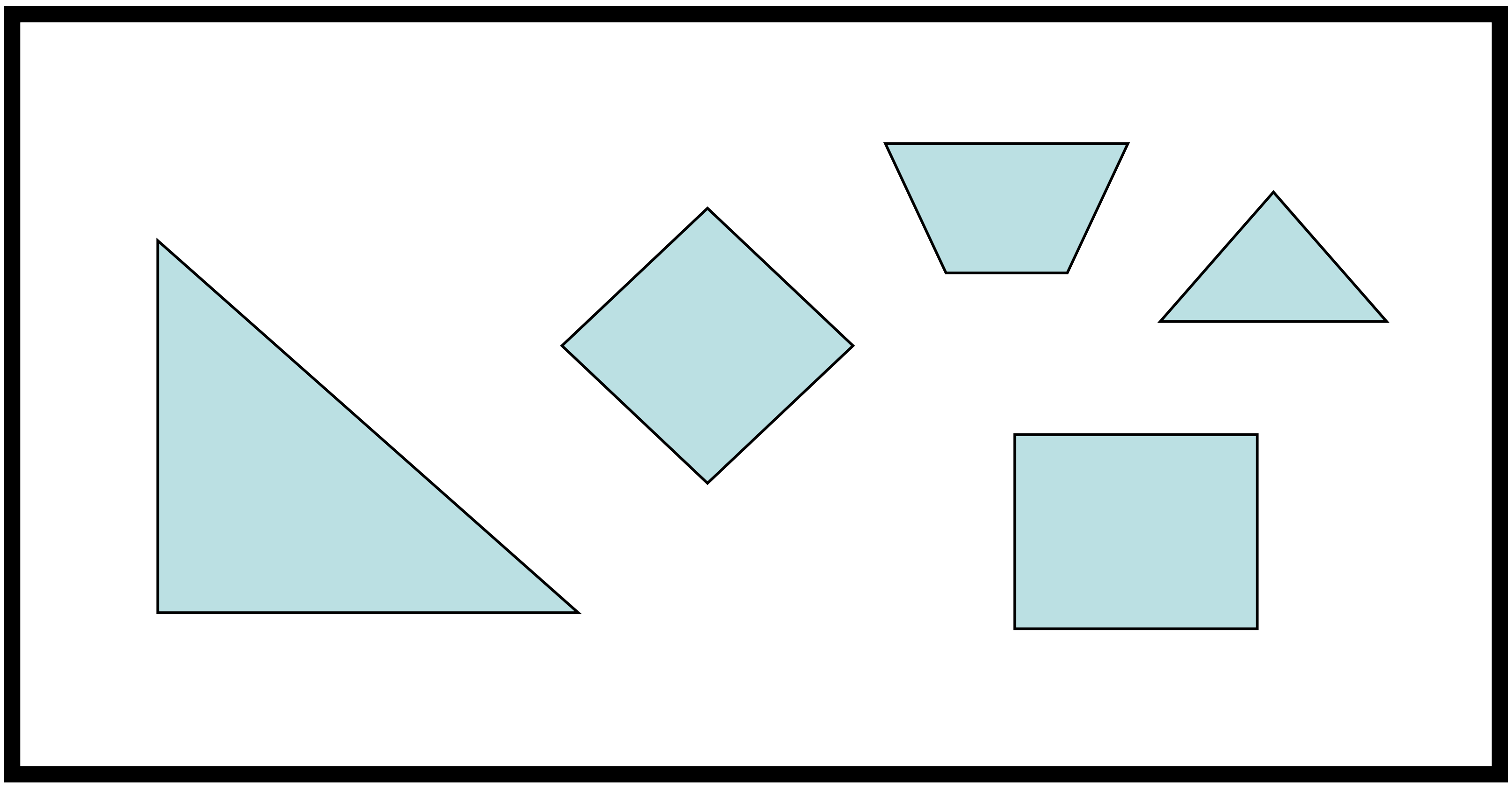
Cell Decomposition

Alfredo Weitzenfeld

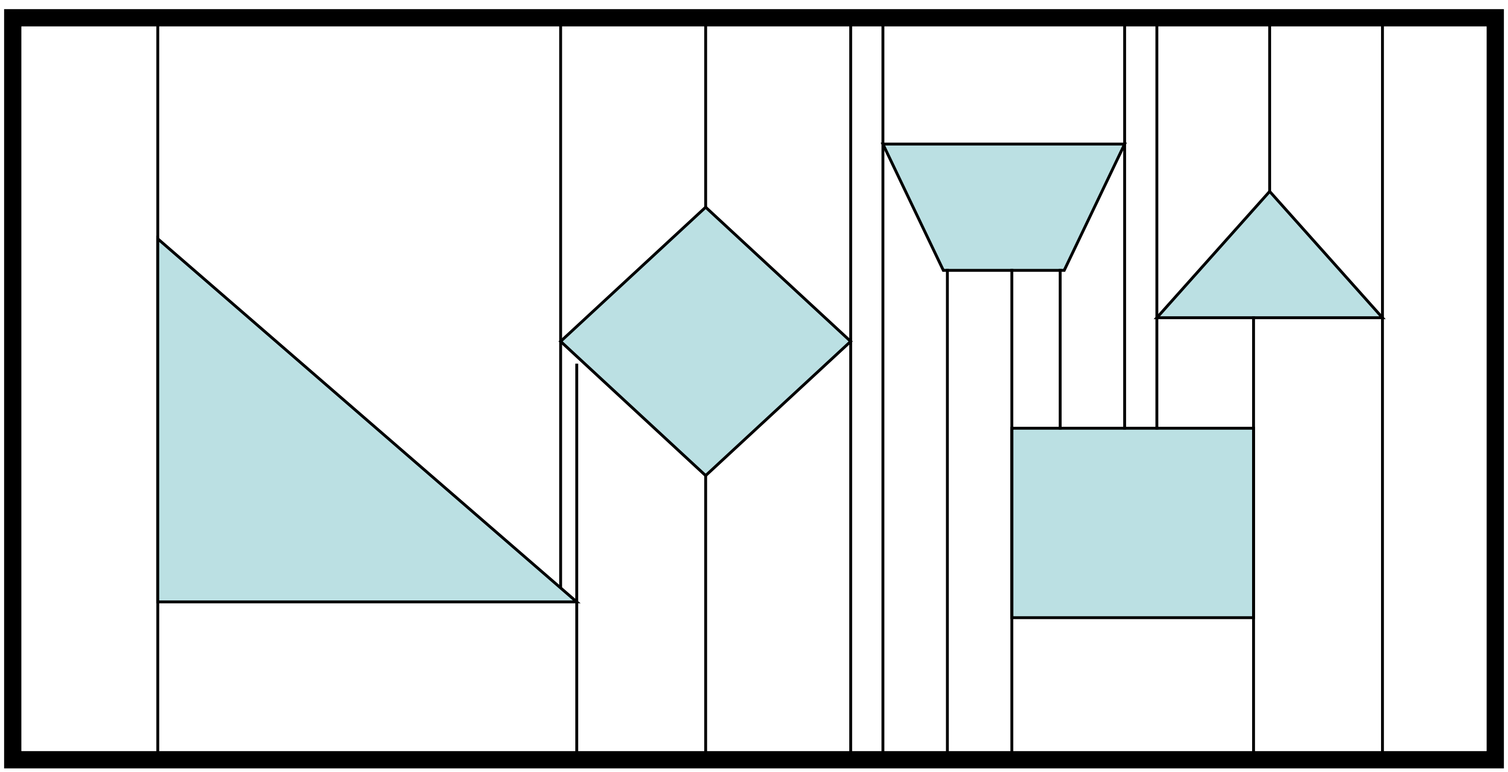
Trapezoidal Decomposition

- A way to divide the world into smaller regions
- Assume a polygonal world



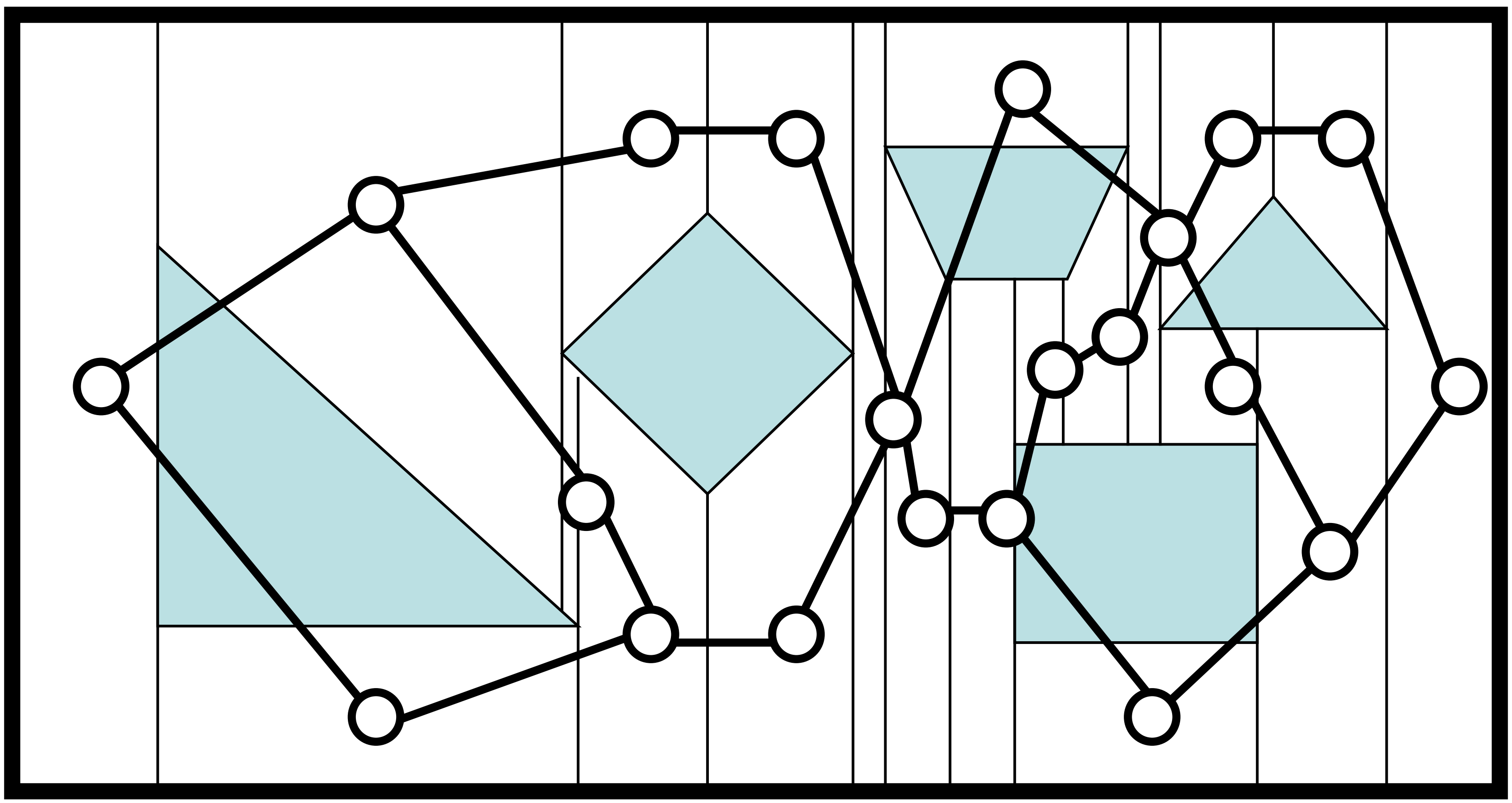
Exact Cell Decomposition

- Free space is represented by the “exact” union of simple trapezoidal regions or cells, while obstacles are represented by polygons.
- Draw a vertical line from each vertex up or down until you hit an obstacle. This reduces the world to a union of trapezoid-shaped cells
- This decomposition can be applied using vertical, horizontal or both type of lines.



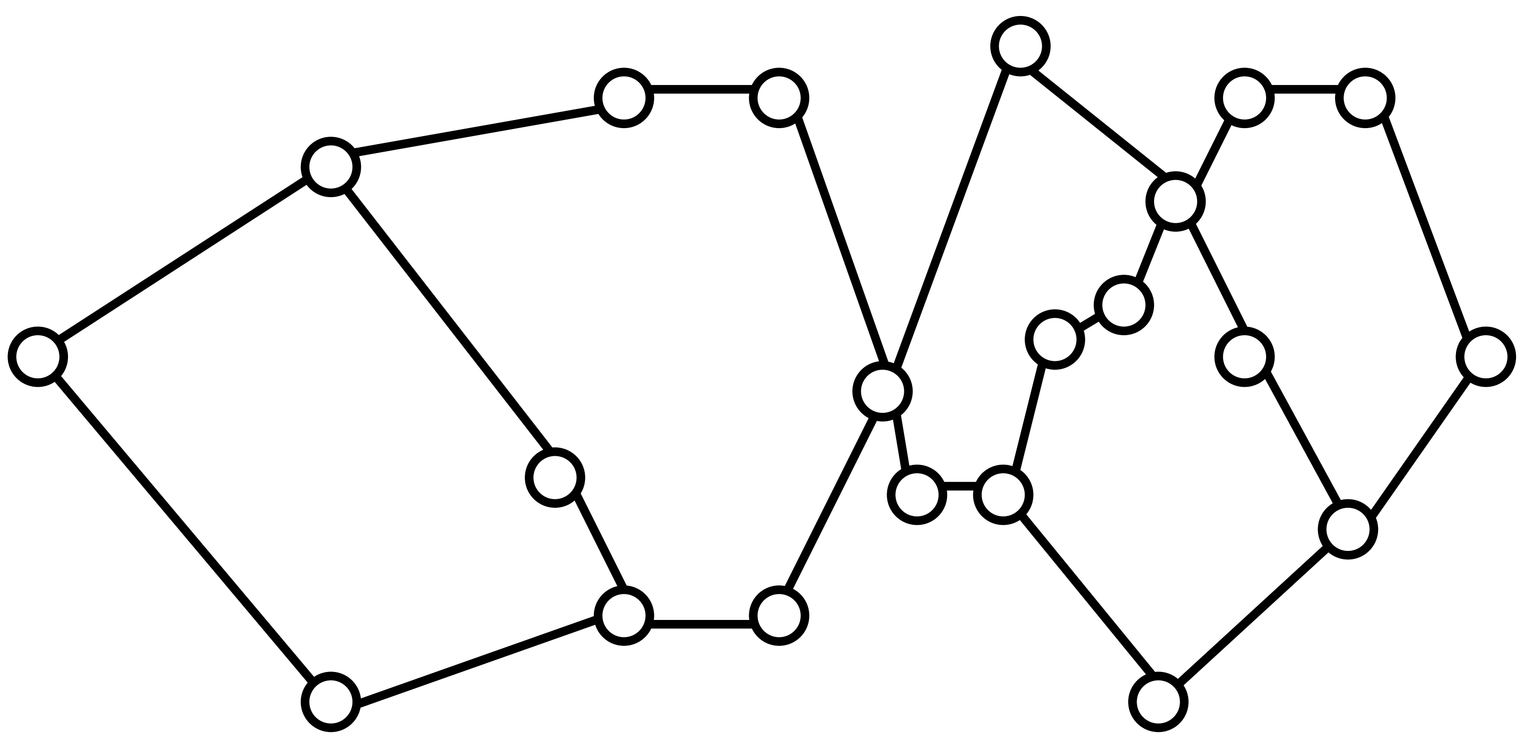
Exact Cell Decomposition

- Adjacent cells represent the connectivity of the free space by the adjacency graph of cells sharing a common boundary.
- The Topological Map encodes the adjacency relationship of cells, where nodes correspond to cells and edges connect nodes of adjacent cells.



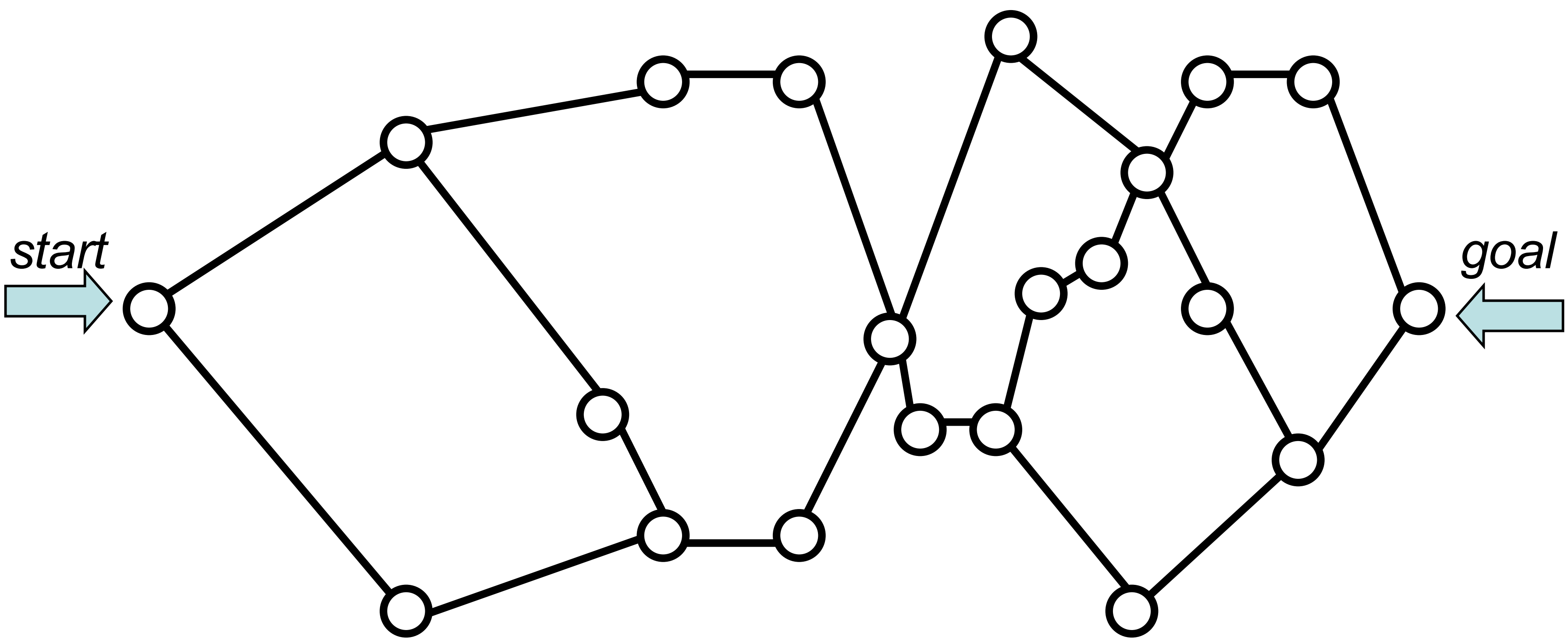
Find a path

- Abstract the world to a topological map.



Find a path

- A path from start to goal can be found by simple traversal



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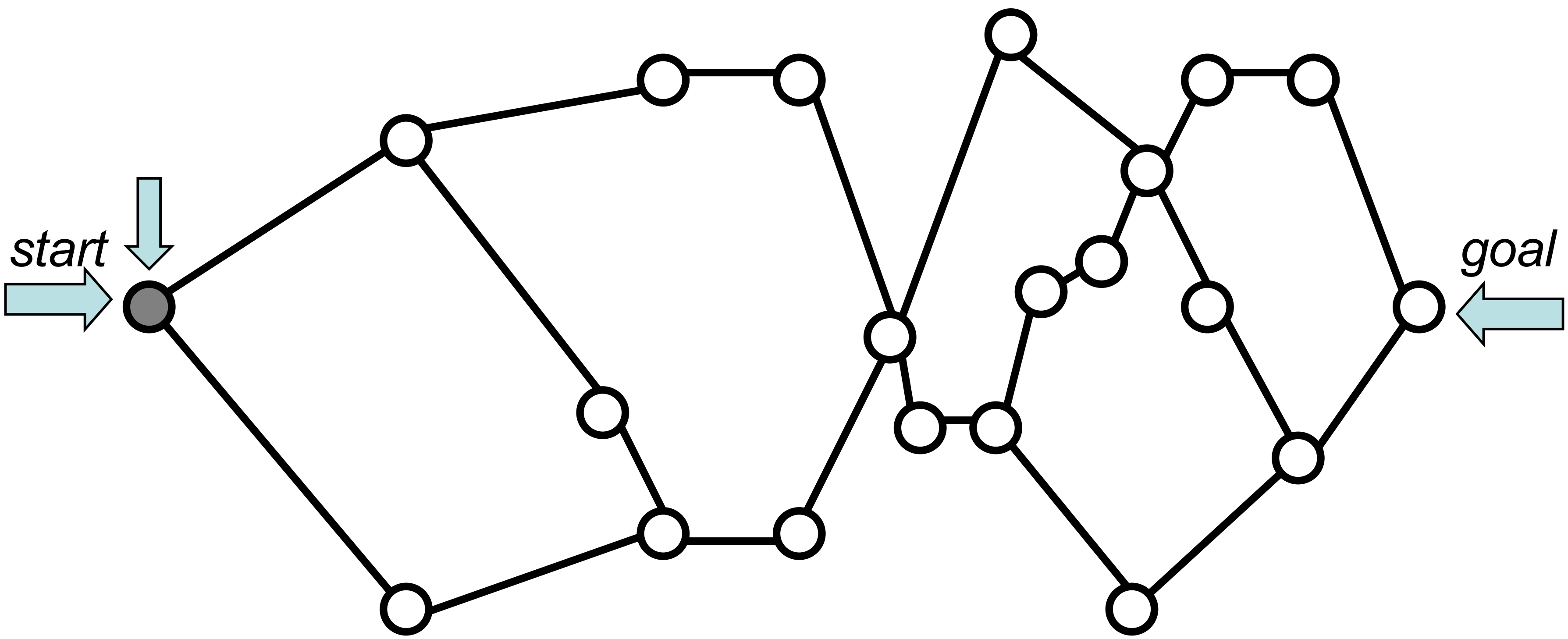

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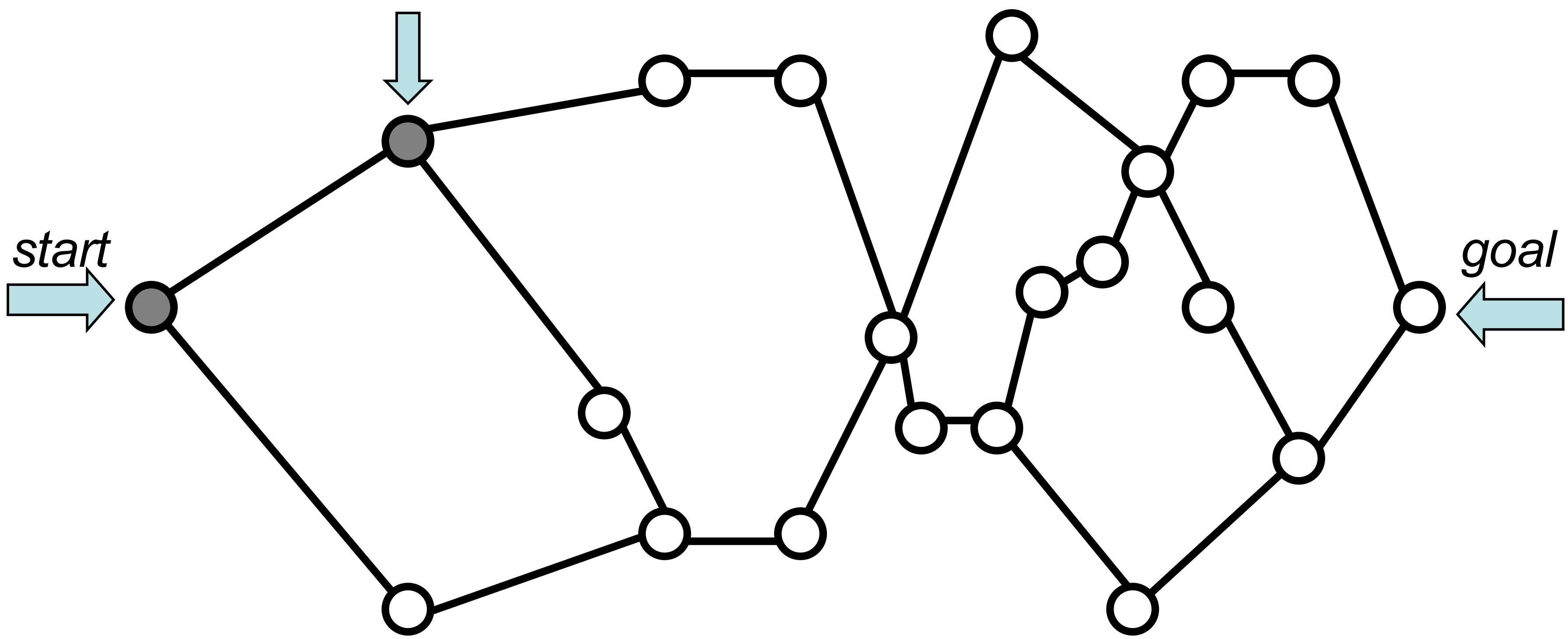

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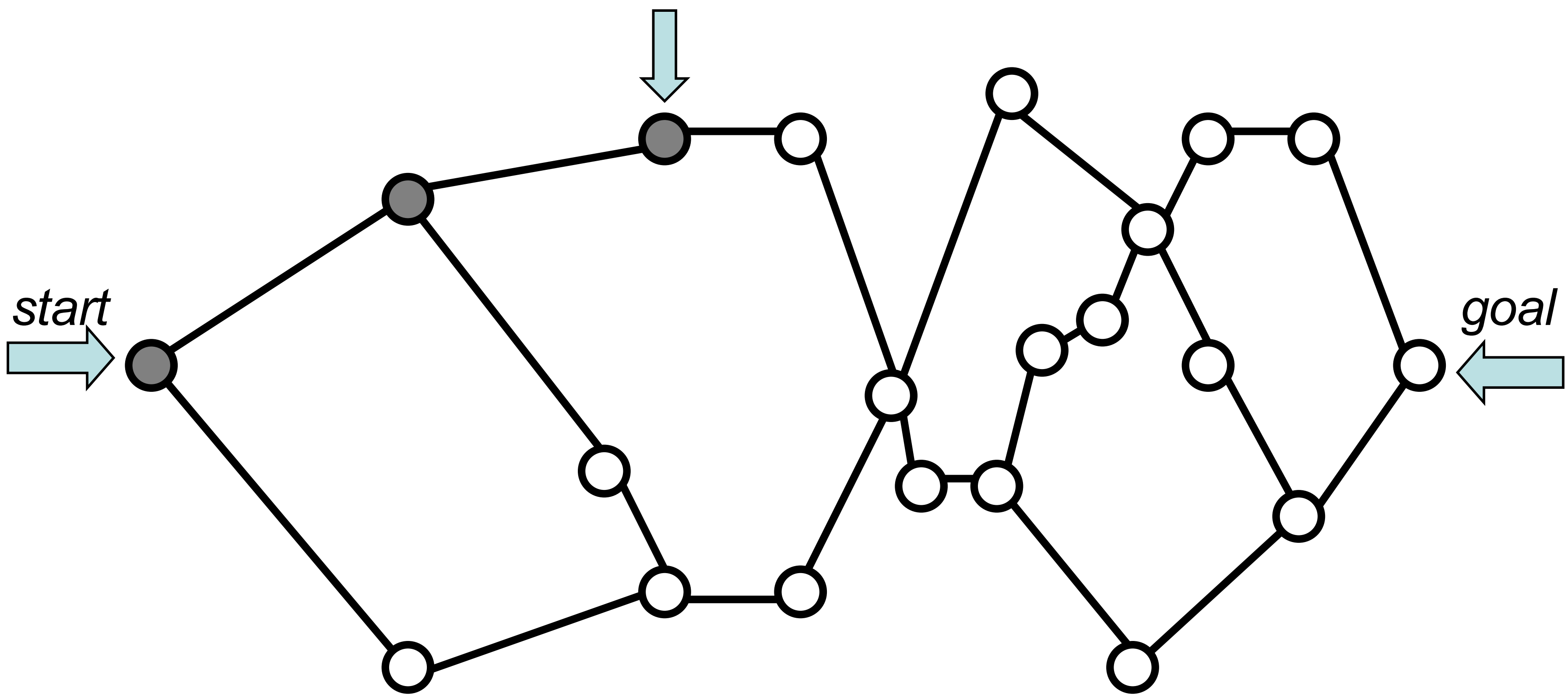

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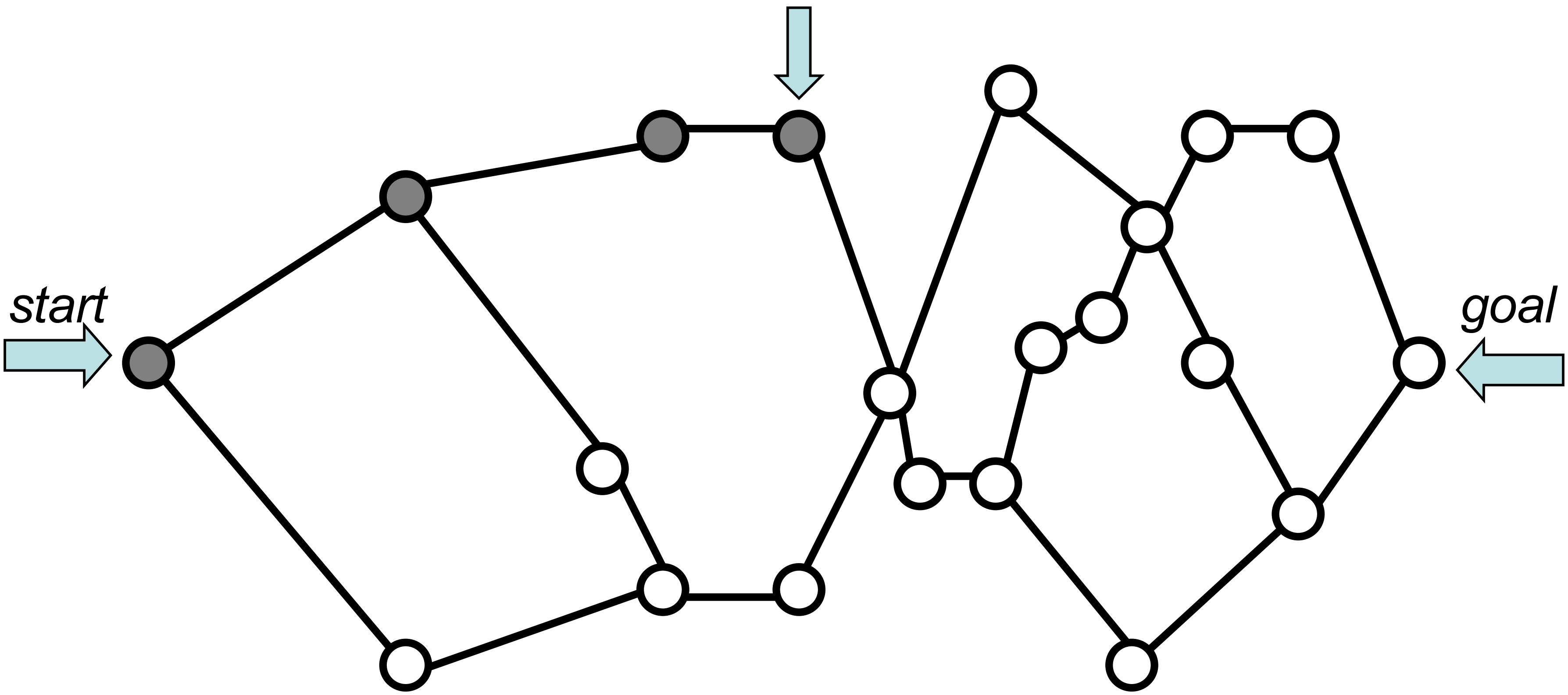

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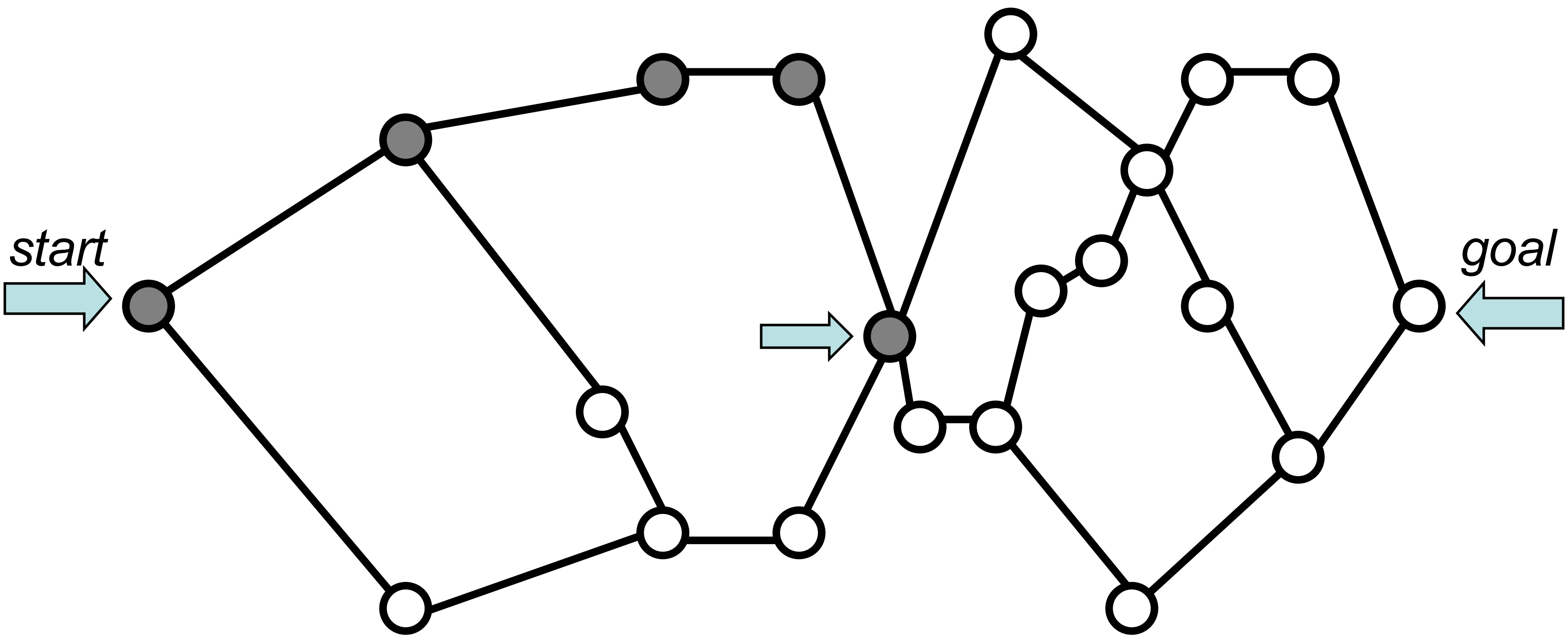

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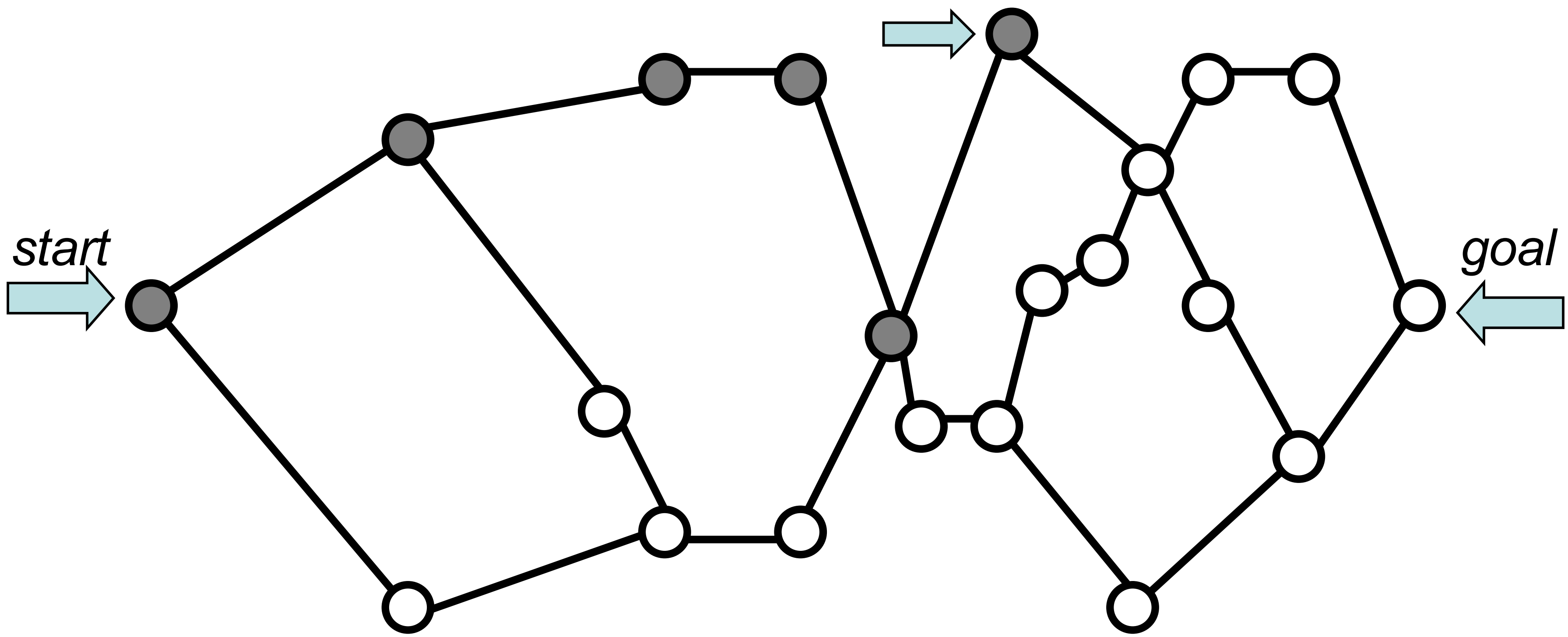

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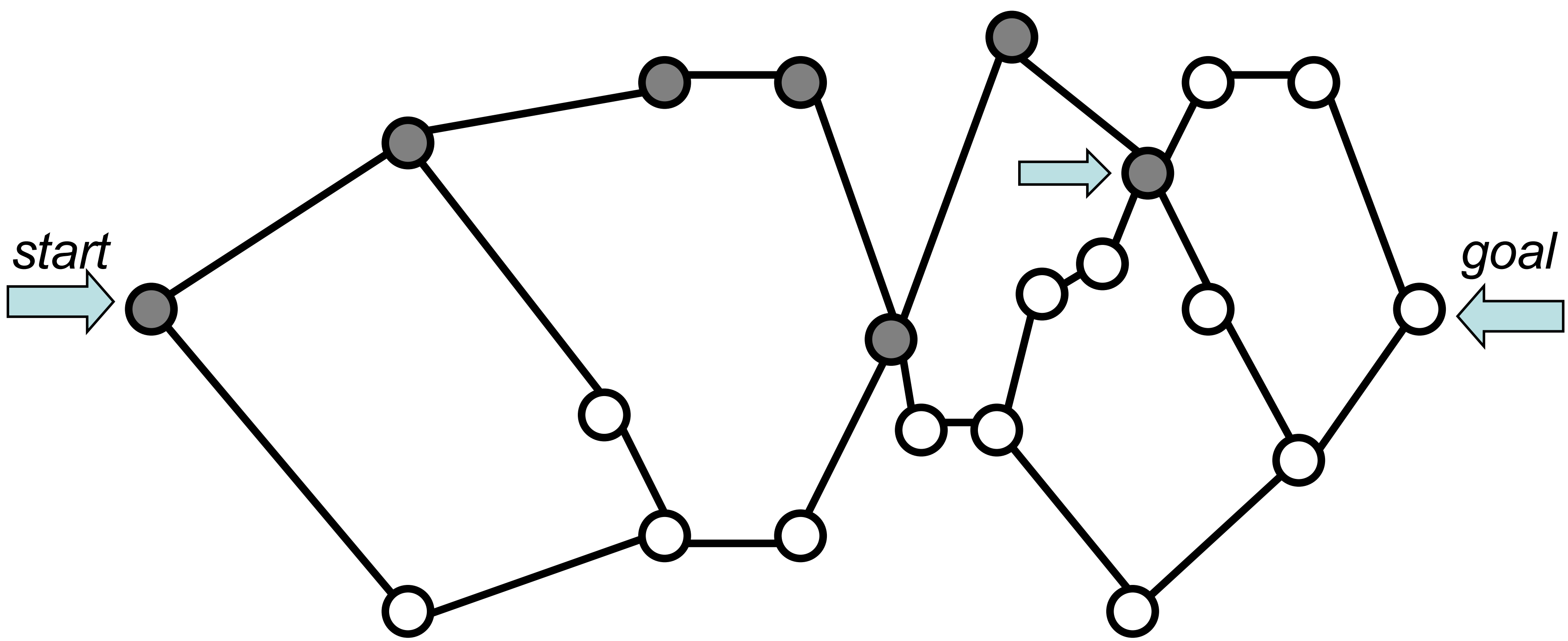

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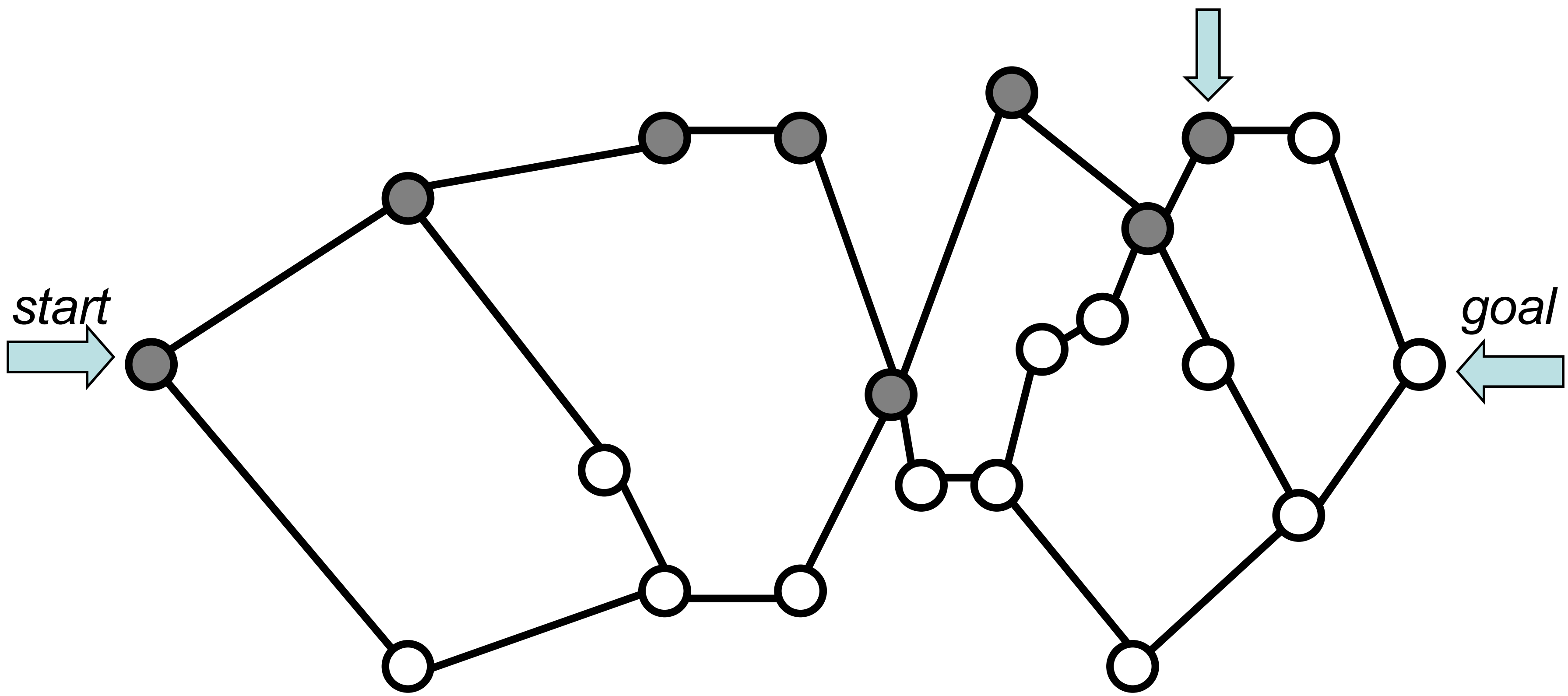

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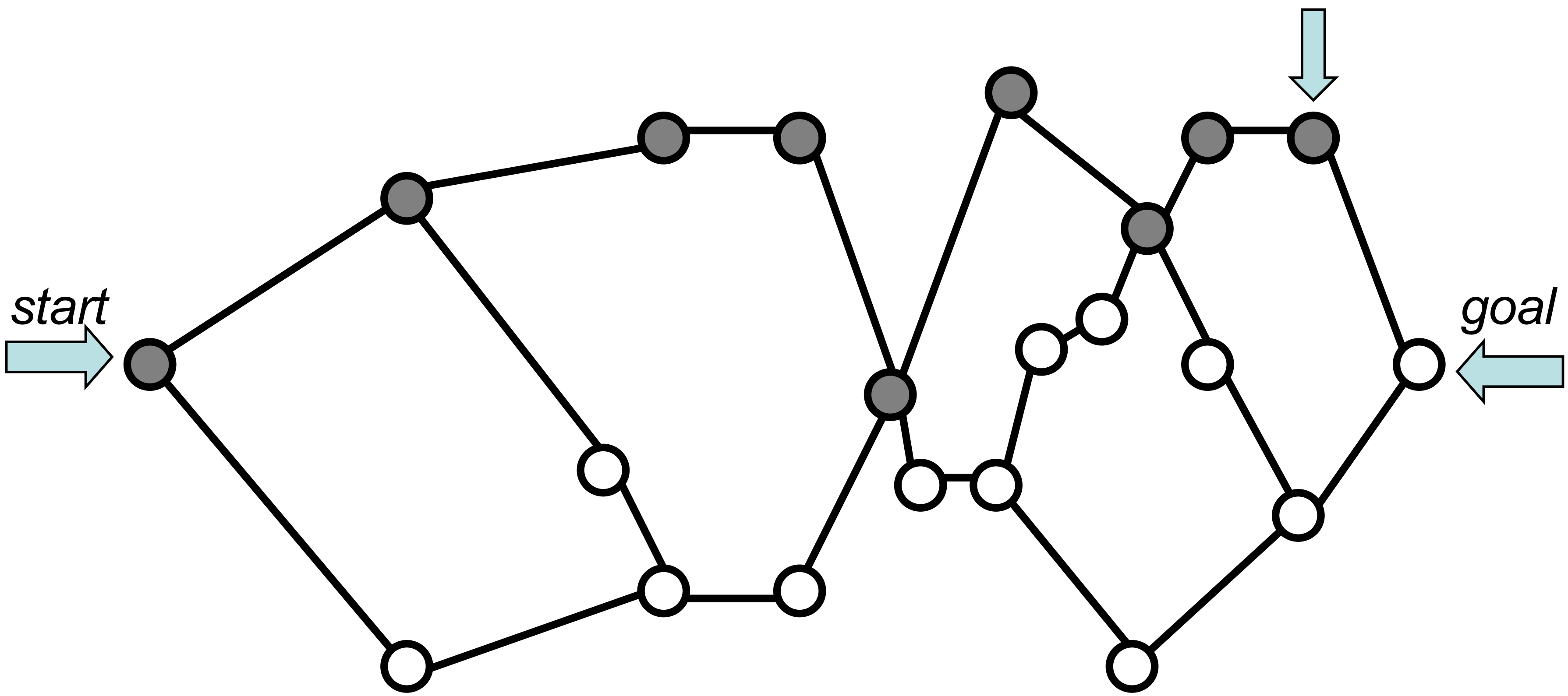

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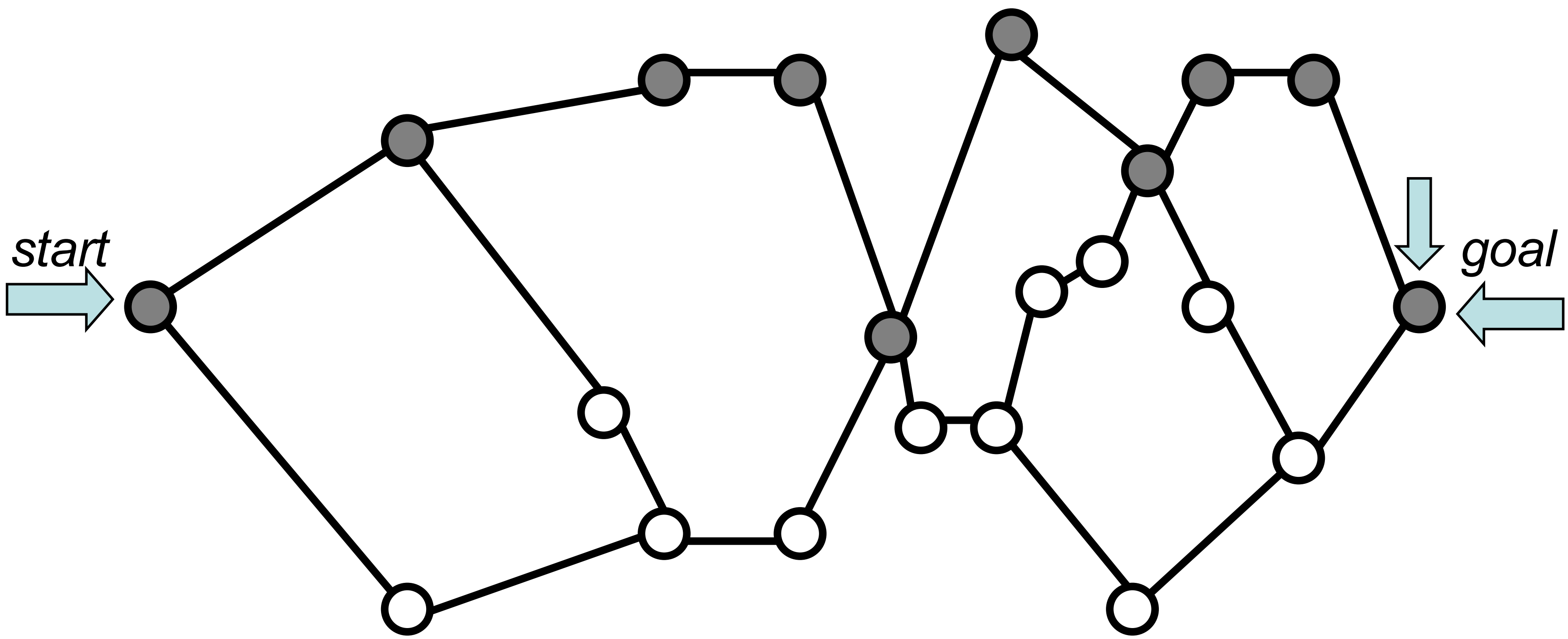
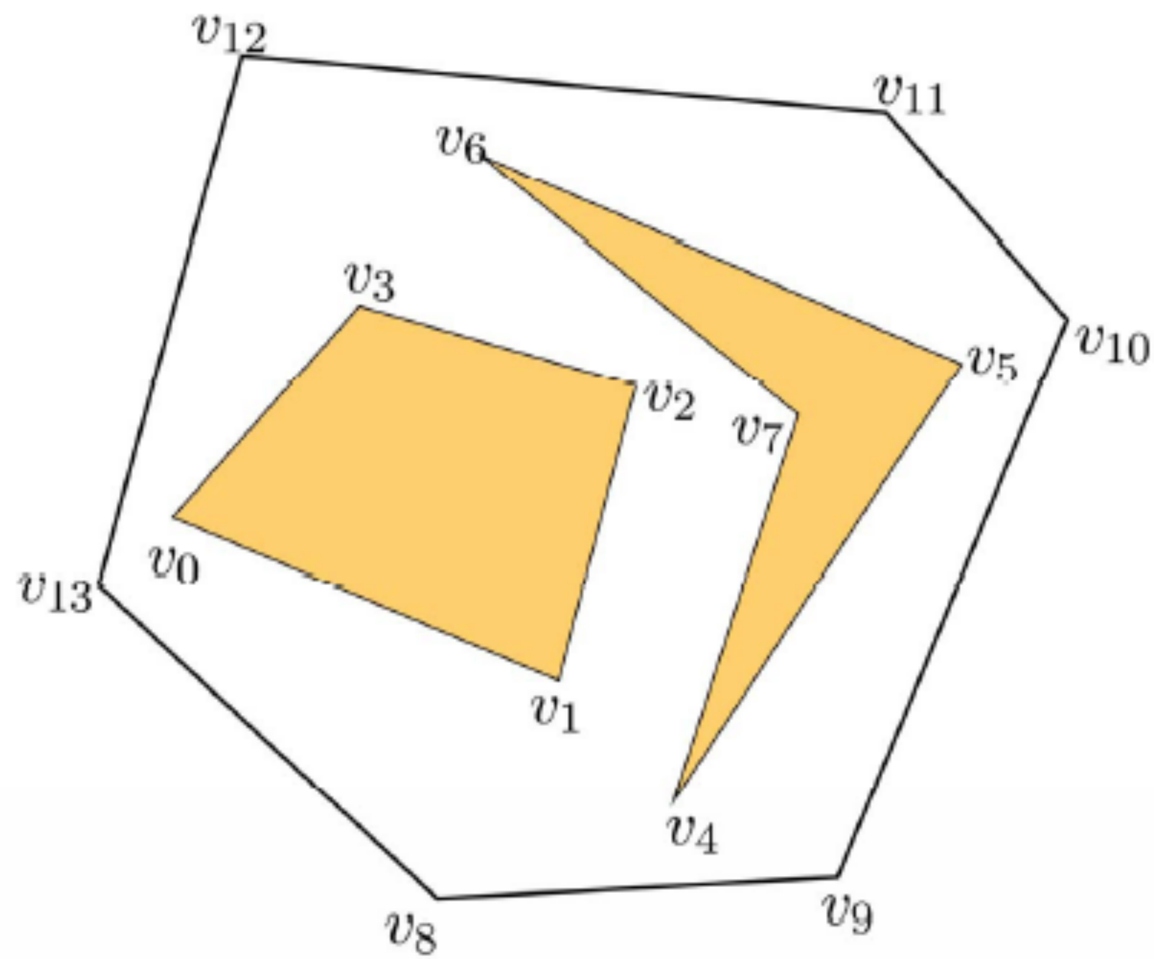


Trapezoidal Decomposition

- To form the decomposition, at each vertex *v* draw two segments:
  - an upper vertical extension (increasing *y*)
  - a lower vertical extension (decreasing *y*)
- Upper and lower vertical extensions start and terminate when they first intersect an edge of the polygon that lies immediately above or below *v*, respectively.
- Many vertices will have either just an upper or a lower vertical extension.
- Edges are defined as connections between adjacent vertices.

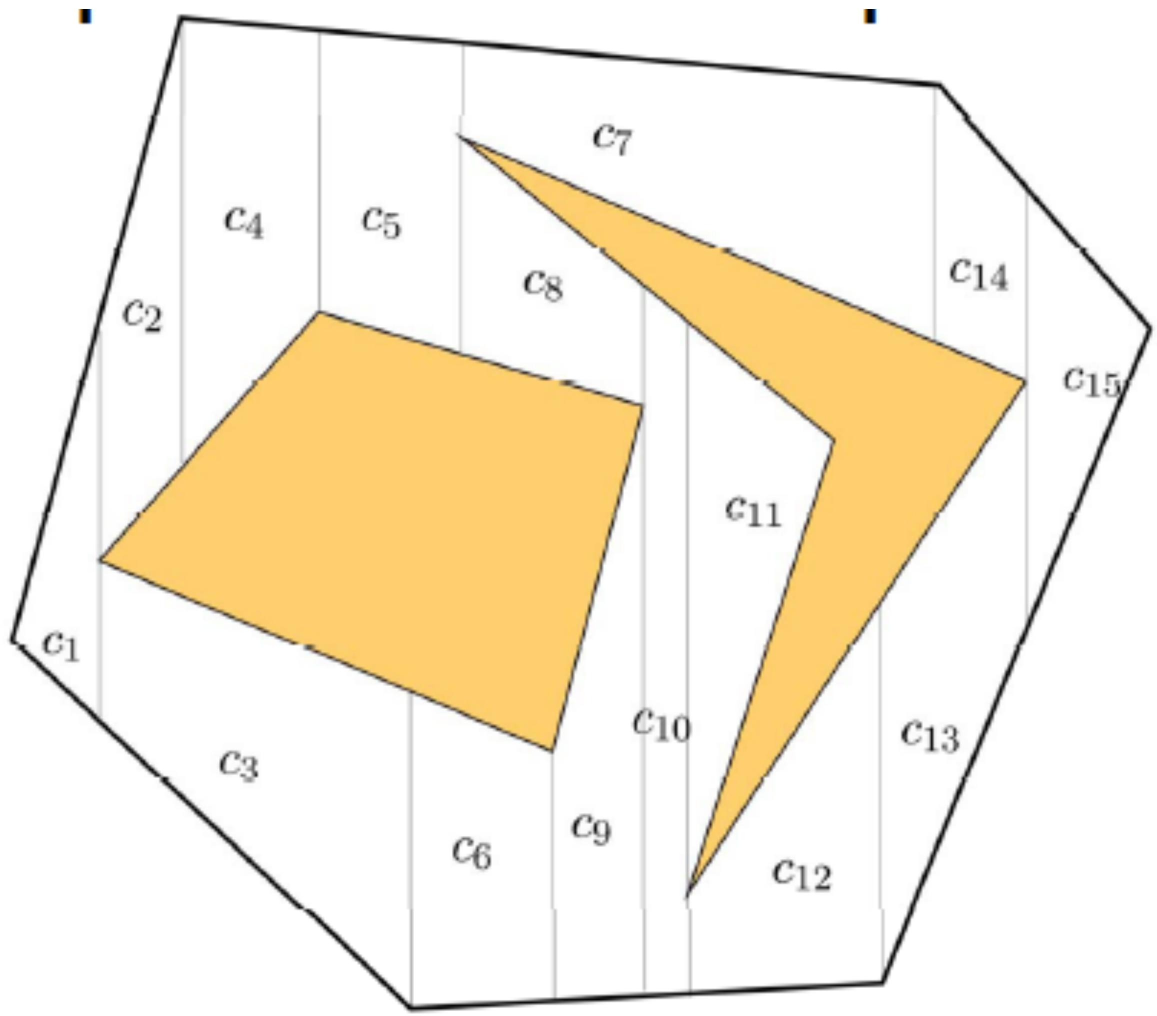
Trapezoidal Decomposition

- Assume a simple (*x,y*) coordinate system for the planar configuration space
- The free space is bounded by a polygon and all obstacles are polygonal
- Each vertex v_i on all the polygons has a unique coordinate.



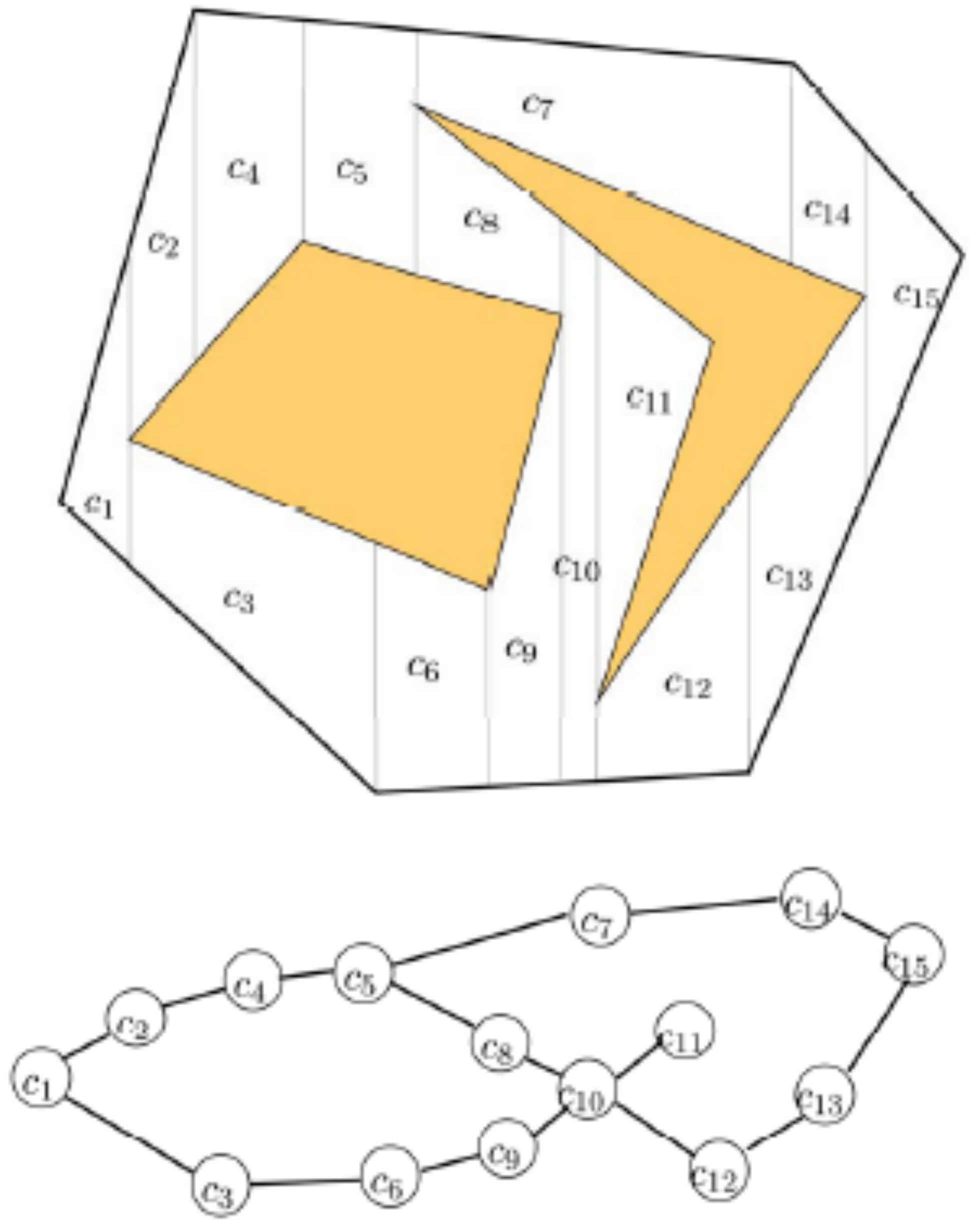
Trapezoidal Decomposition

- Trapezoidal decomposition configuration space



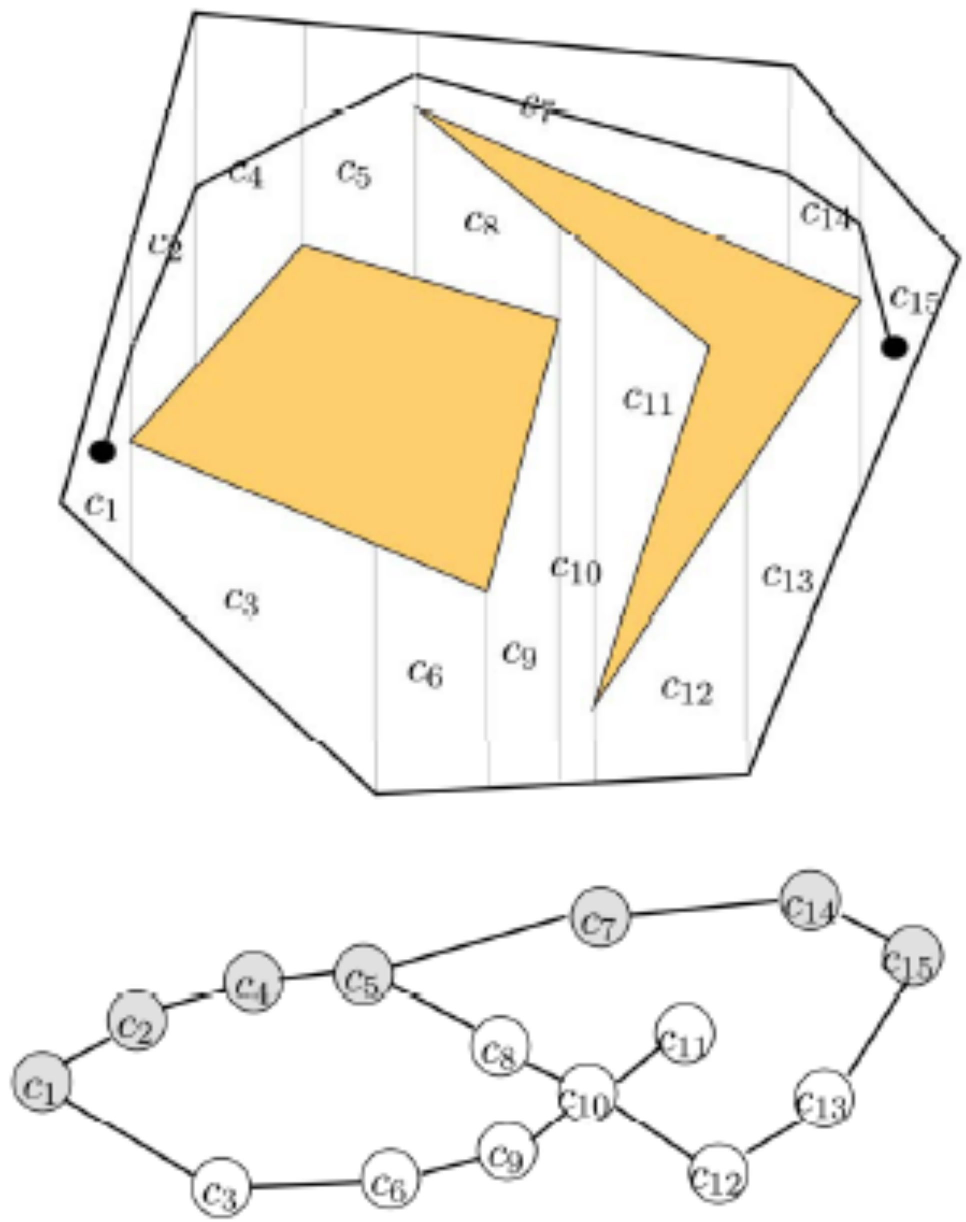
Trapezoidal Decomposition

- Adjacency graph for trapezoidal decomposition configuration space.
- Node corresponds to a cell.
- Edge connects nodes of adjacent cells.
- Two cells are adjacent if they share a common boundary.



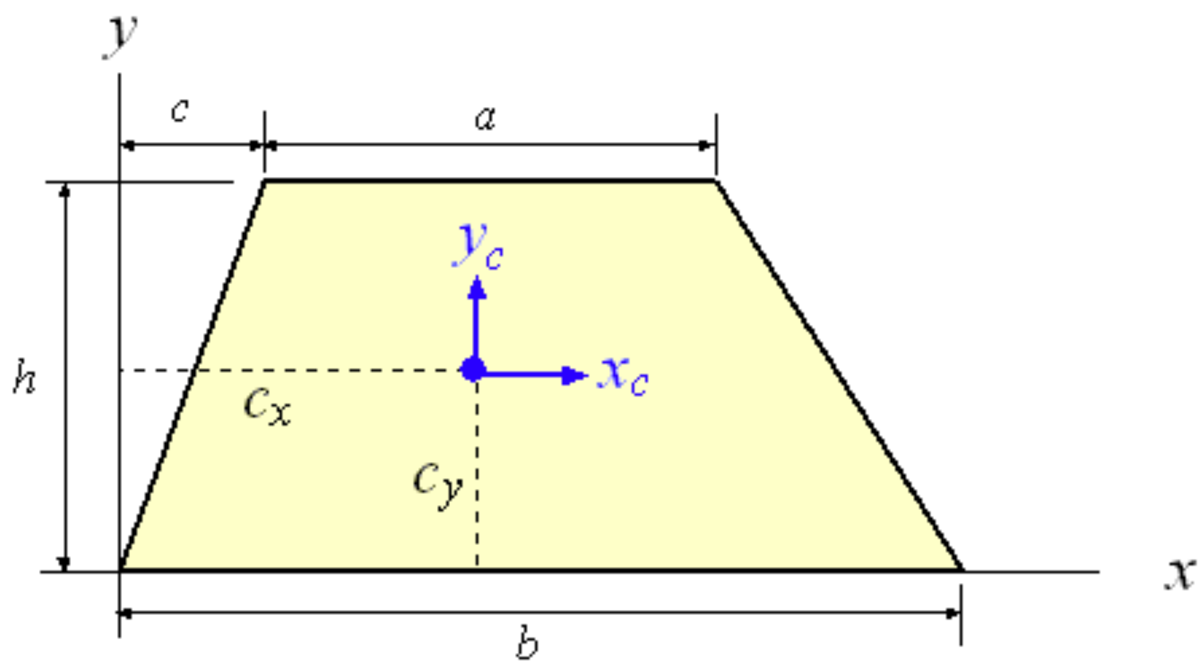
Trapezoidal Decomposition Path

- Collision-free path through the free space derived from the adjacency graph between start and goal points.
- Path is derived by connecting *midpoints* of the vertical extensions to the *centroid* of each trapezoid.



Trapezoid Centroid

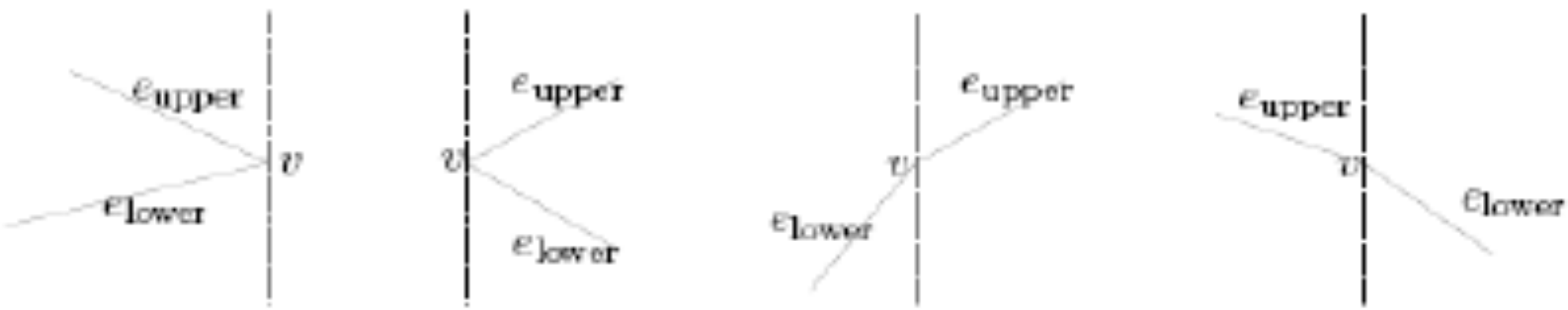
- A trapezoid is a quadrilateral with two sides parallel. The centroid of a trapezoid lies between the two bases. For any trapezoid with parallel sides a and b, where h is the height of the trapezoid.



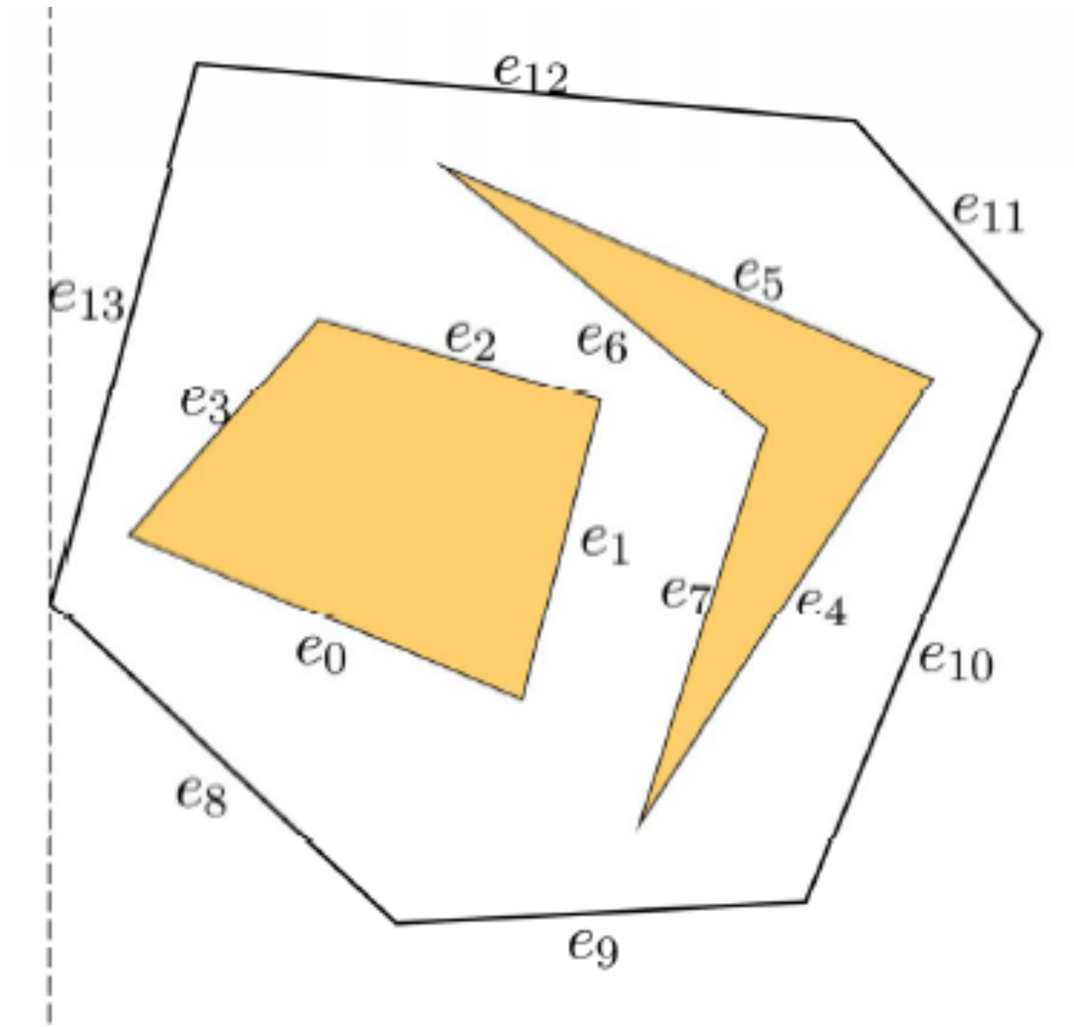
C_x	C_y	Area
$\dfrac{2ac + a^2 + cb + ab + b^2}{3(a+b)}$	$\dfrac{h(2a+b)}{3(a+b)}$	$\dfrac{h(a+b)}{2}$

Sweep Lines

- Let e_{lower} and e_{upper} be the two polygon edges that contain *v.*
- The *sweep line* is the vertical lines that goes up or down through v.
- There are four types of events that can occur and the type of event determines the appropriate action to take on the list:
  1. e_{lower} and e_{upper} are both to the left of the sweep line: delete e_{lower} and e_{upper} from the list
  2. e_{lower} and e_{upper} are both to the right of the sweep line: insert e_{lower} and e_{upper} into the list
  3. e_{lower} is to the left and e_{upper} is to the right of the sweep line: delete e_{lower} from the list and insert e_{upper} into the list
  4. e_{lower} is to the right and e_{upper} is to the left of the sweep line: insert e_{lower} into the list and delete e_{upper} from the list

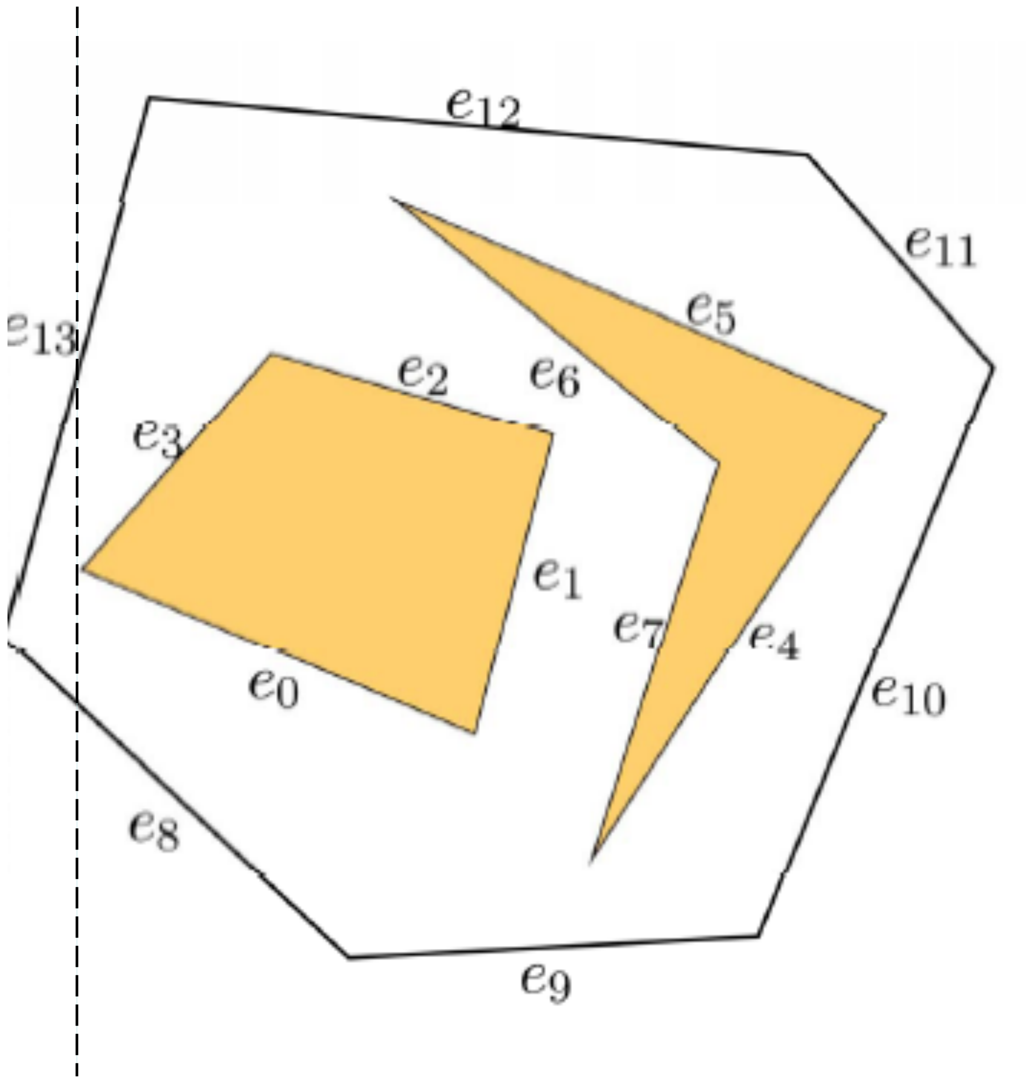


Trapezoidal Decomposition



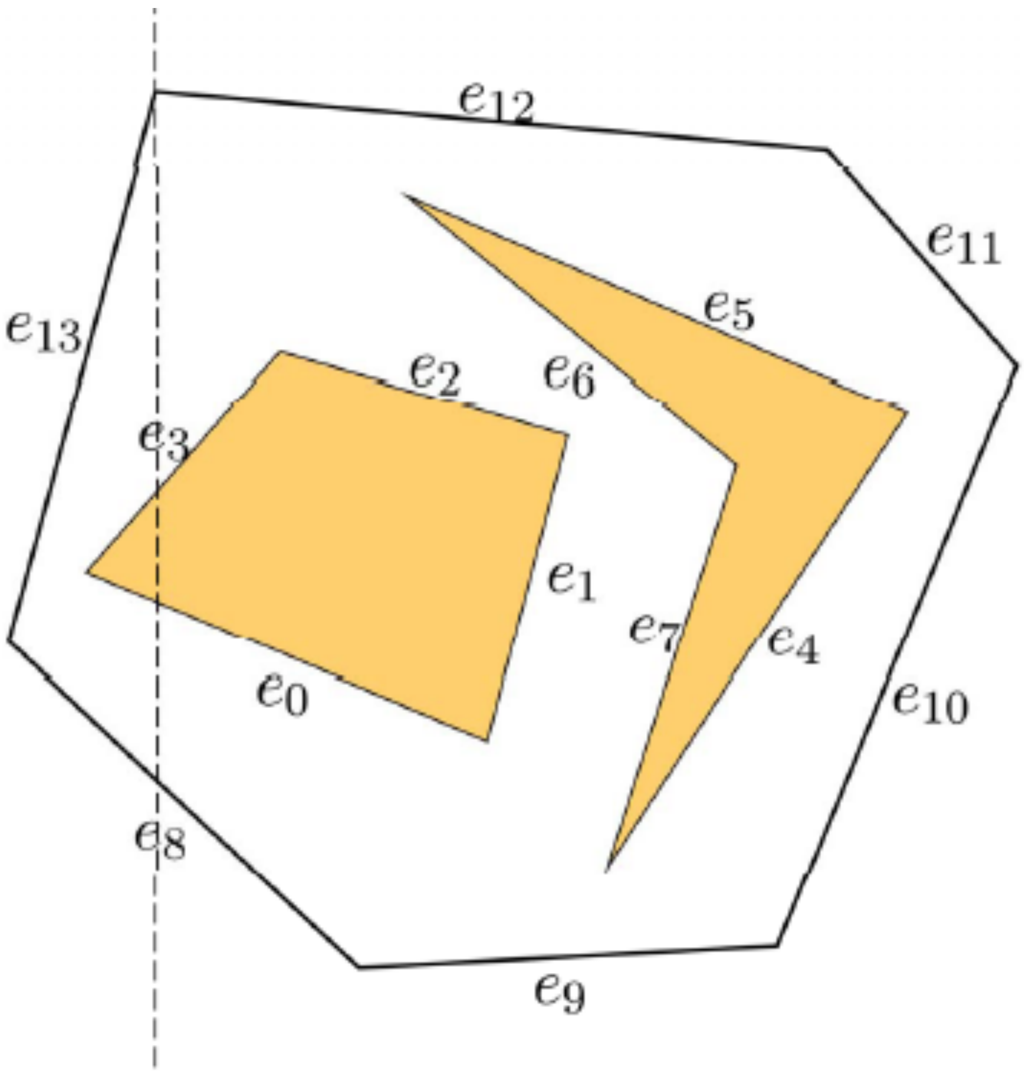
$$L : \varnothing \rightarrow \{e_8, e_{13}\}$$

Trapezoidal Decomposition



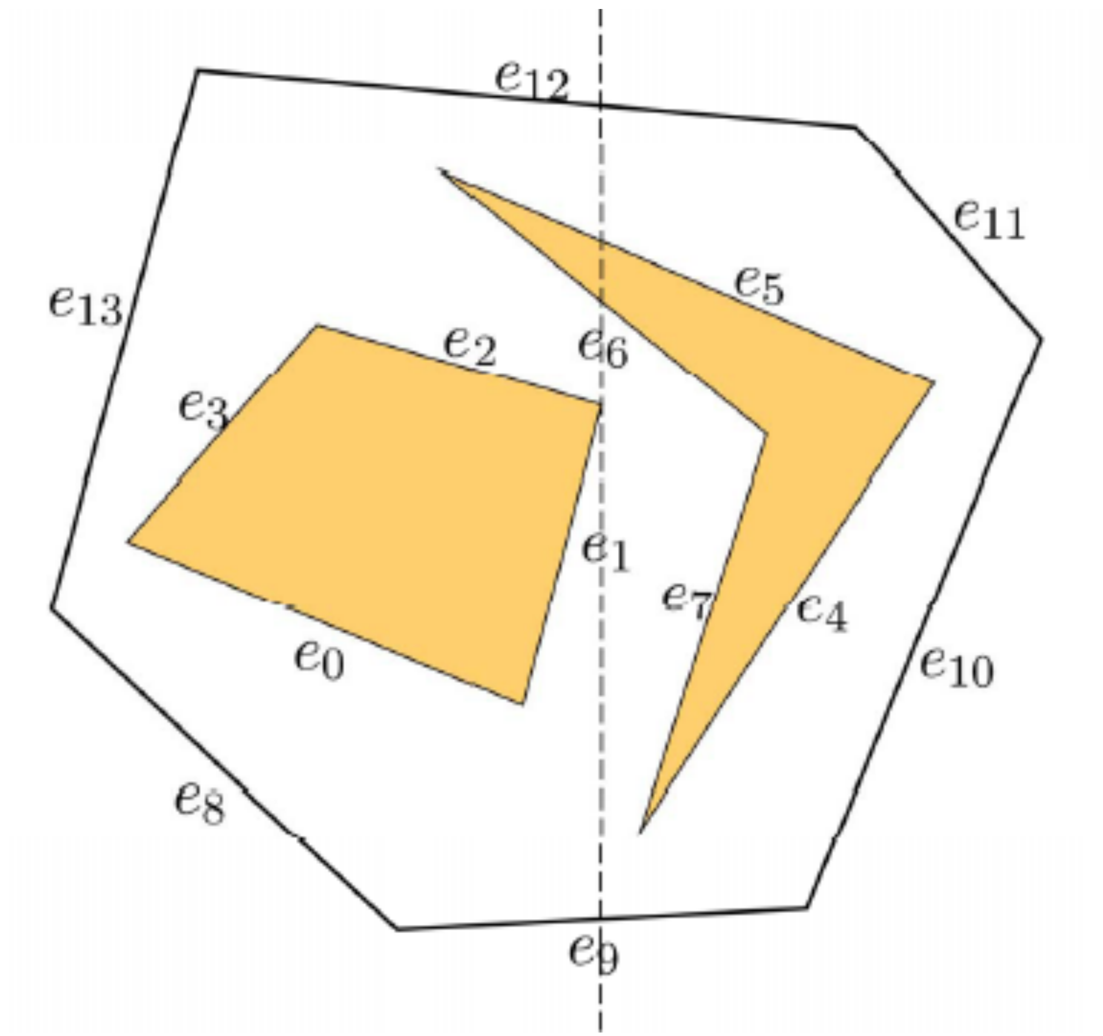
$$L:\{e_8, e_{13}\} \rightarrow \{e_8, e_0, e_3, e_{13}\}$$

Trapezoidal Decomposition



$$L:\left\{e_8, e_0, e_3, e_{13}\right\} \rightarrow \left\{e_8, e_0, e_3, e_{12}\right\}$$

Trapezoidal Decomposition

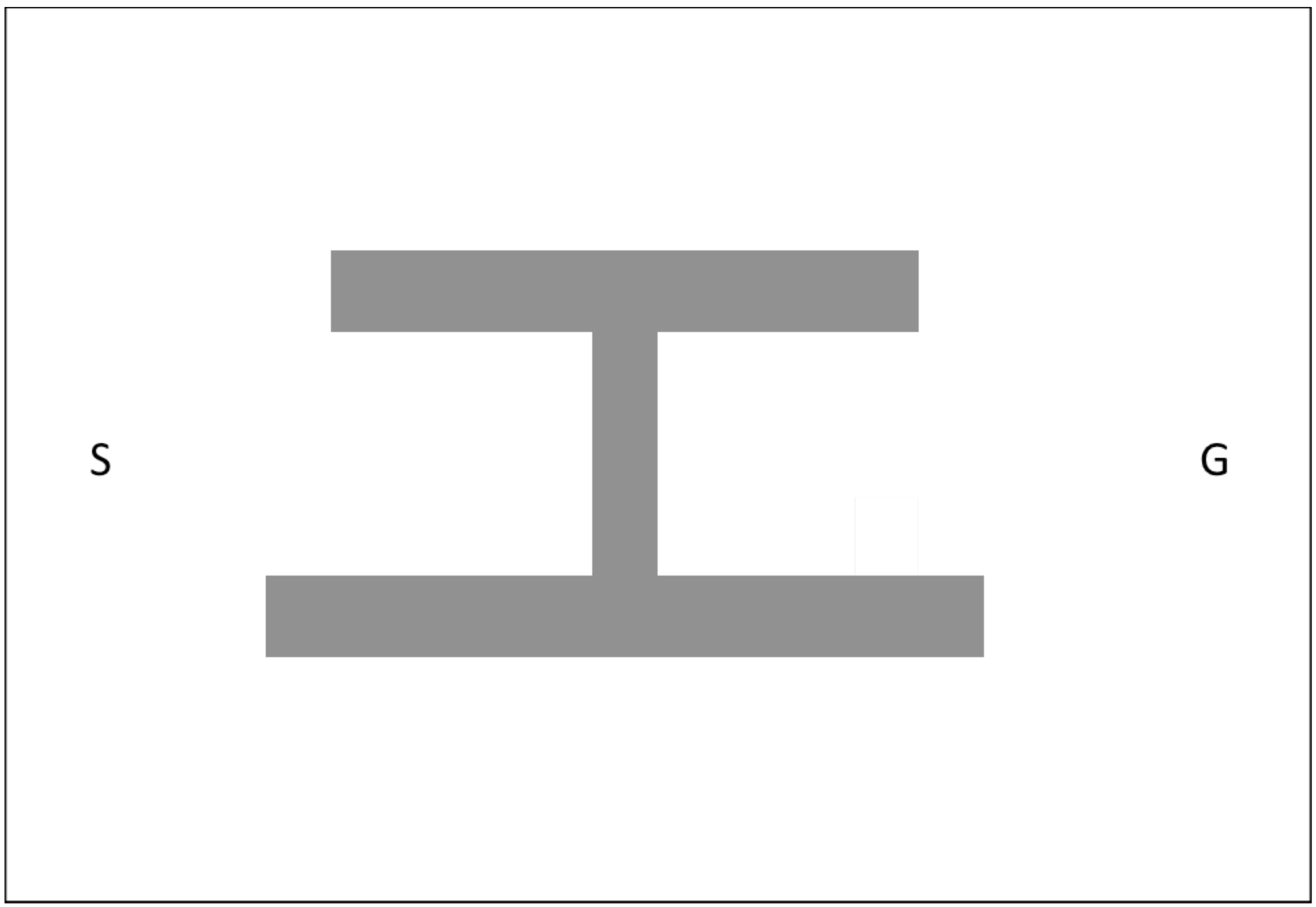


$$L:\left\{e_9,e_1,e_2,e_6,e_5,e_{12}\right\}\rightarrow\left\{e_9,e_6,e_5,e_{12}\right\}$$

Exact Cell Decomposition

Spring 2017

Apply the Exact Cell Decomposition algorithm and show the Coverage (Topological Map) to the scene below where darker elements correspond to obstacles, surrounding rectangle represents a wall, “S” represents the start location, and “G” represents the goal location. Show the complete trapezoidal cell decomposition. Show the topological map including final shortest path from start to goal in terms of number of adjacent cells using the trapezoidal cell decomposition.



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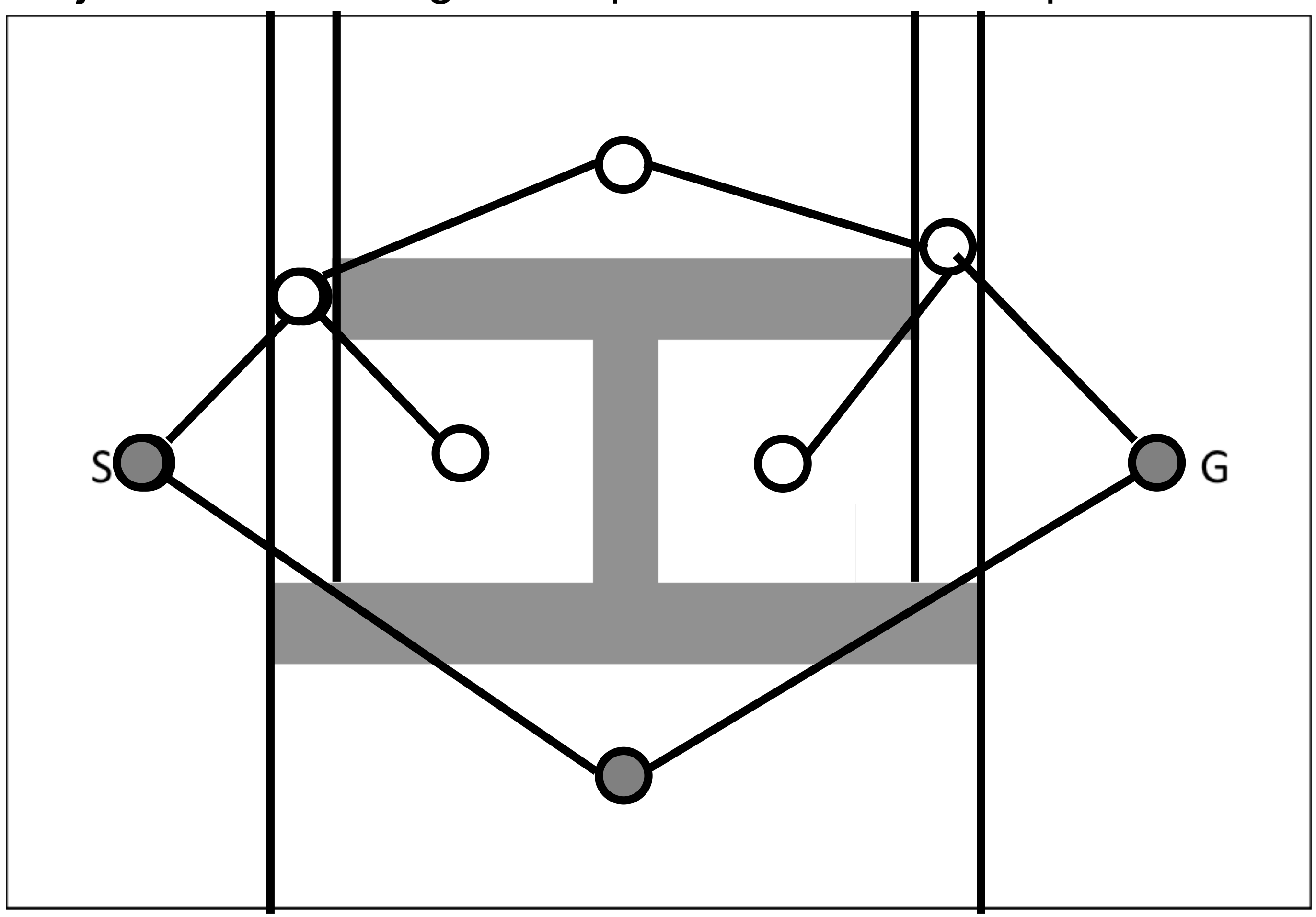
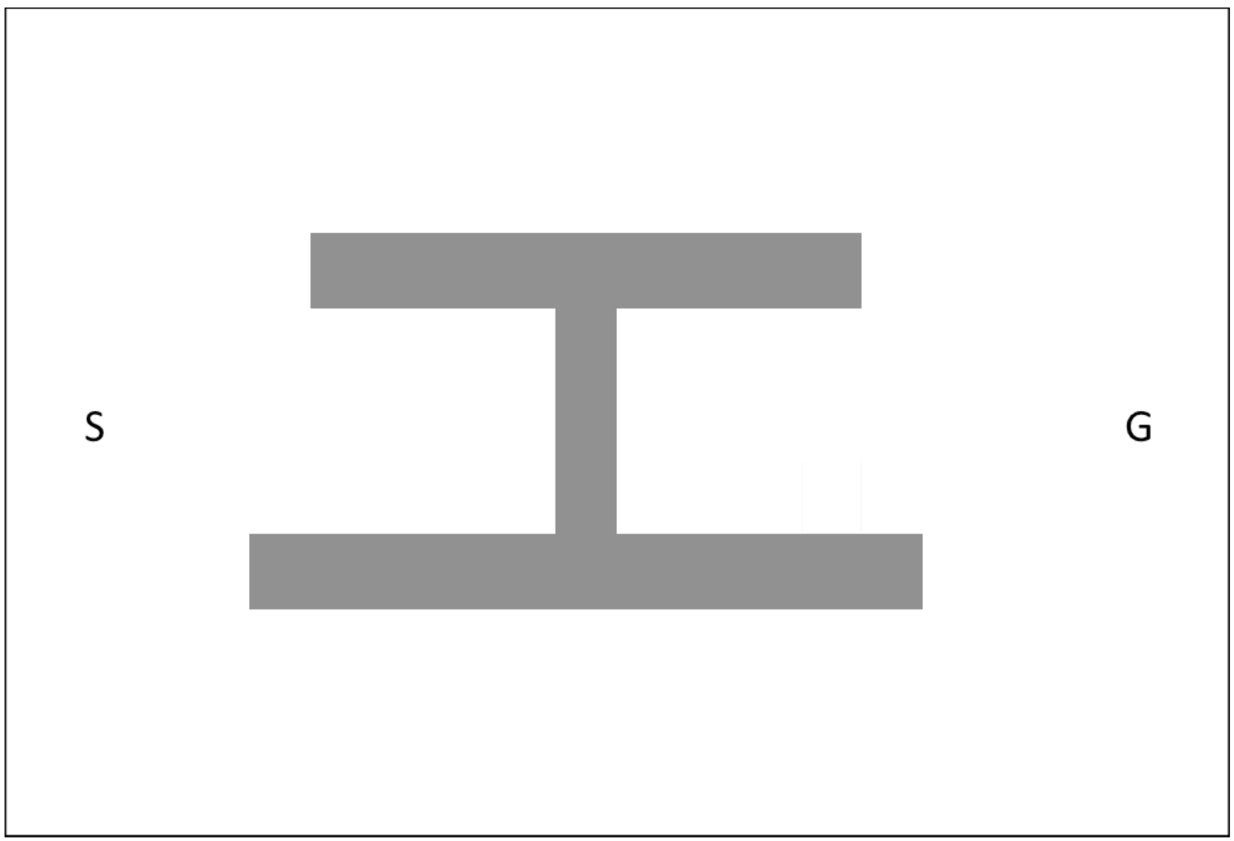


Extended Exact Cell Decomposition

Spring 2017

Apply the Exact Cell Decomposition algorithm, using both vertical and horizontal region borders, and show the Coverage (Topological Map) to the scene below where darker elements correspond to obstacles, surrounding rectangle represents a wall, “S” represents the start location, and “G” represents the goal location. Show the complete trapezoidal cell decomposition. Show the topological map including final shortest path from start to goal in terms of number of adjacent cells using the trapezoidal cell decomposition.



Extended Exact Cell Decomposition

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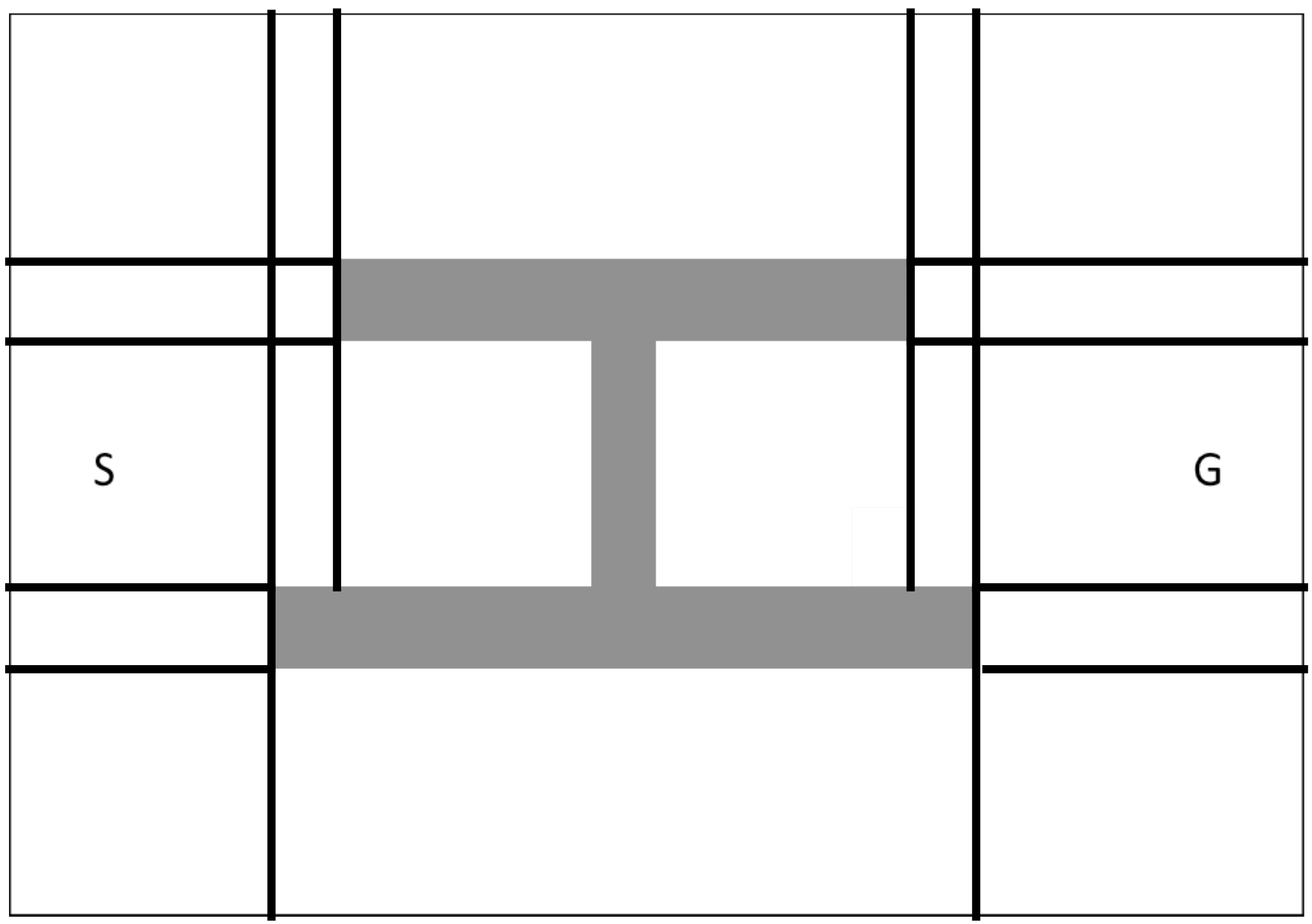
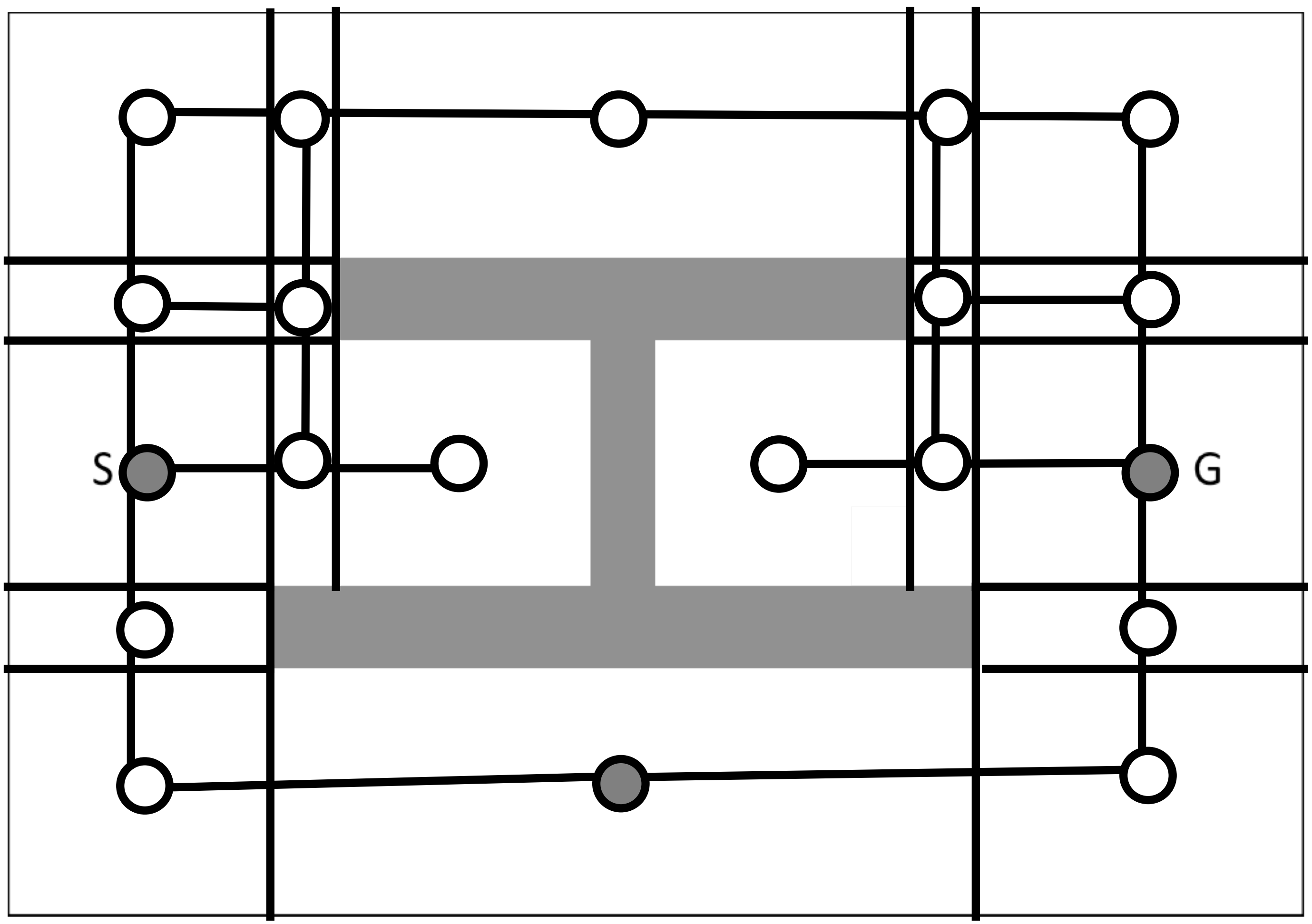
Apply the Exact Cell Decomposition algorithm, using both vertical and horizontal region borders to the scene below where darker elements correspond to obstacles, surrounding rectangle represents a wall. Show the complete trapezoidal cell decomposition.



Extended Exact Cell Decomposition

Spring 2017

Apply the Exact Cell Decomposition algorithm, using both vertical and horizontal region borders and show the Coverage (Topological Map) to the scene below. Show the topological map including final shortest path from start to goal in terms of number of adjacent cells using the trapezoidal cell decomposition.



Cell Decomposition

Spring 2017

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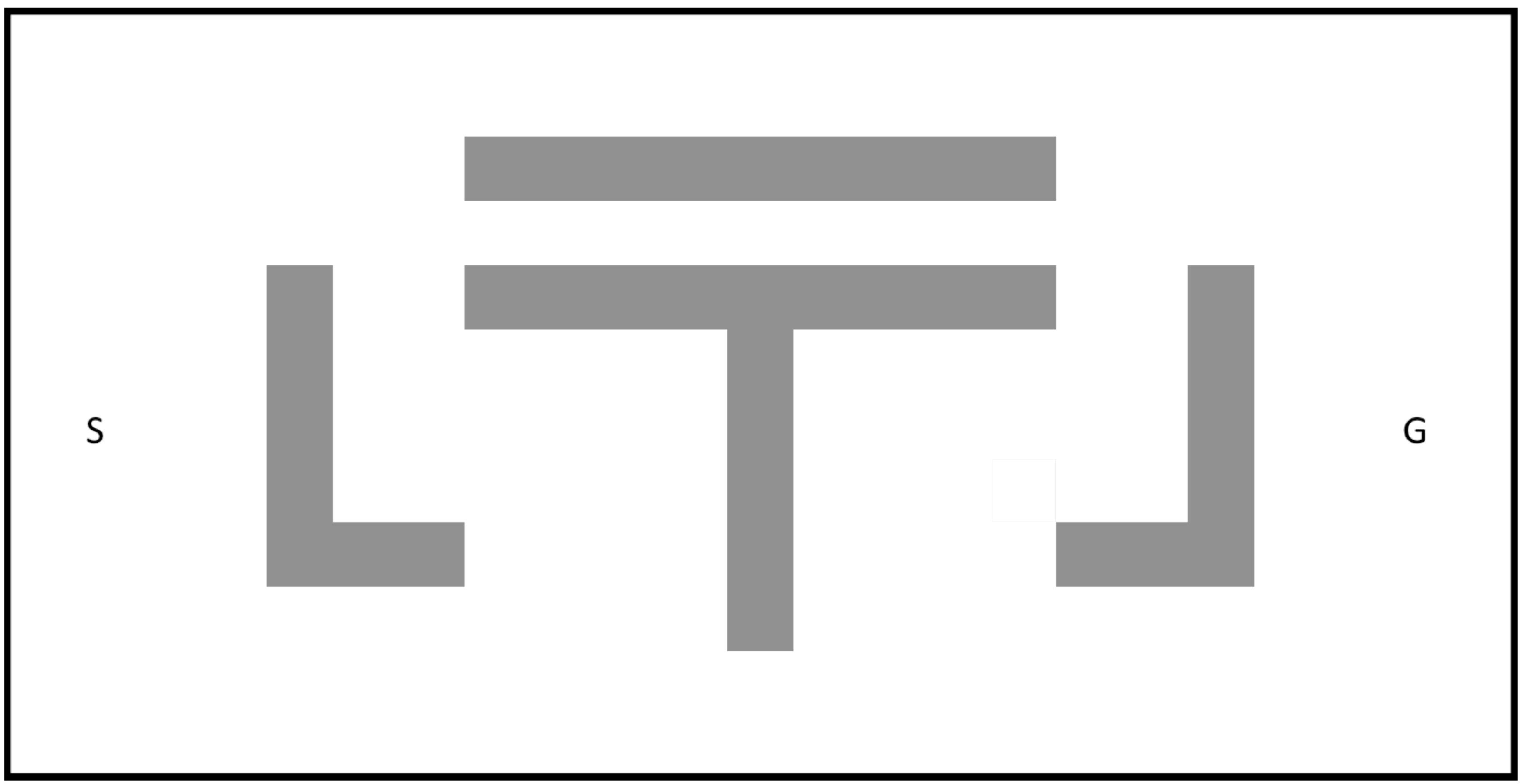

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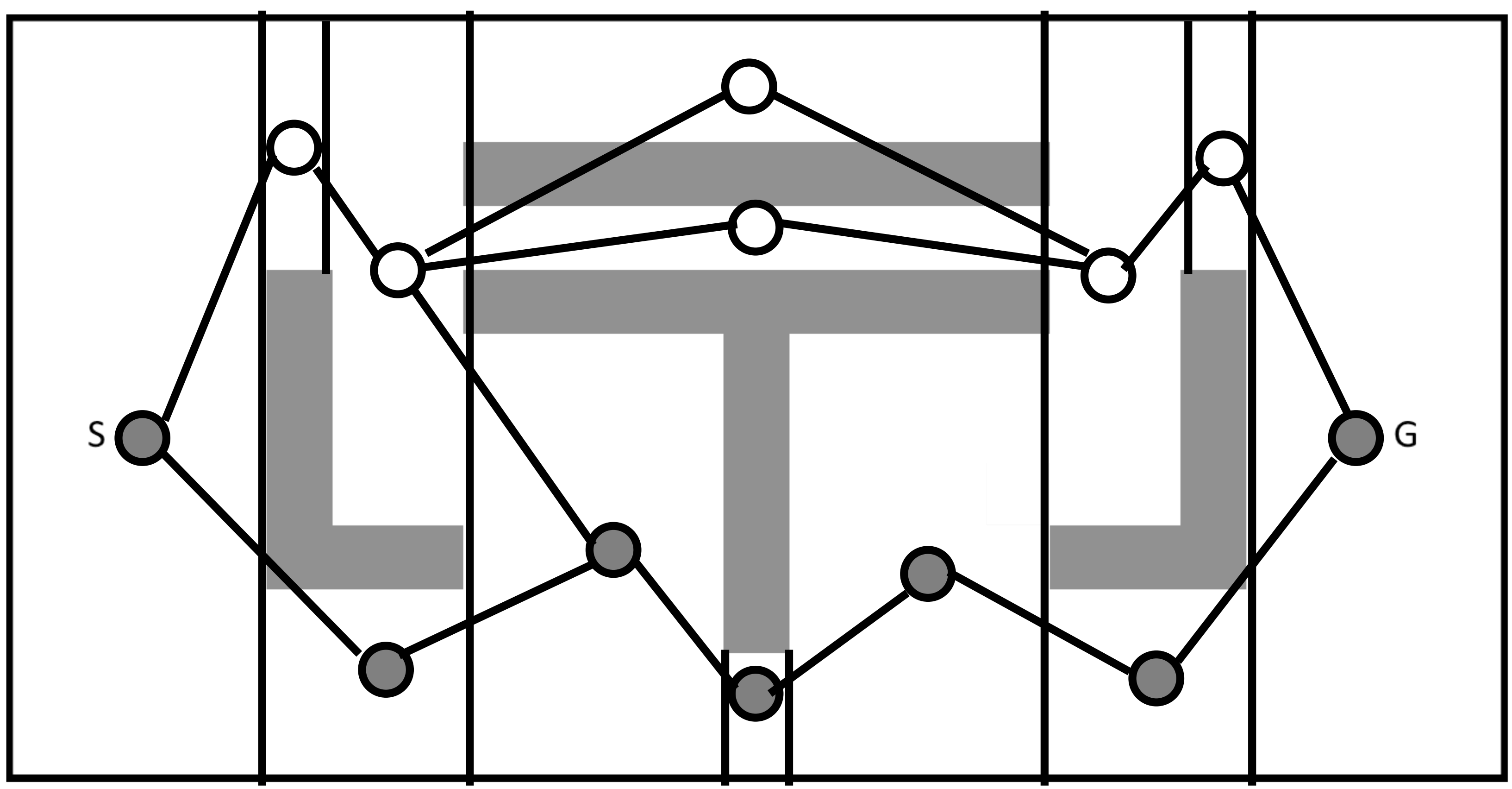


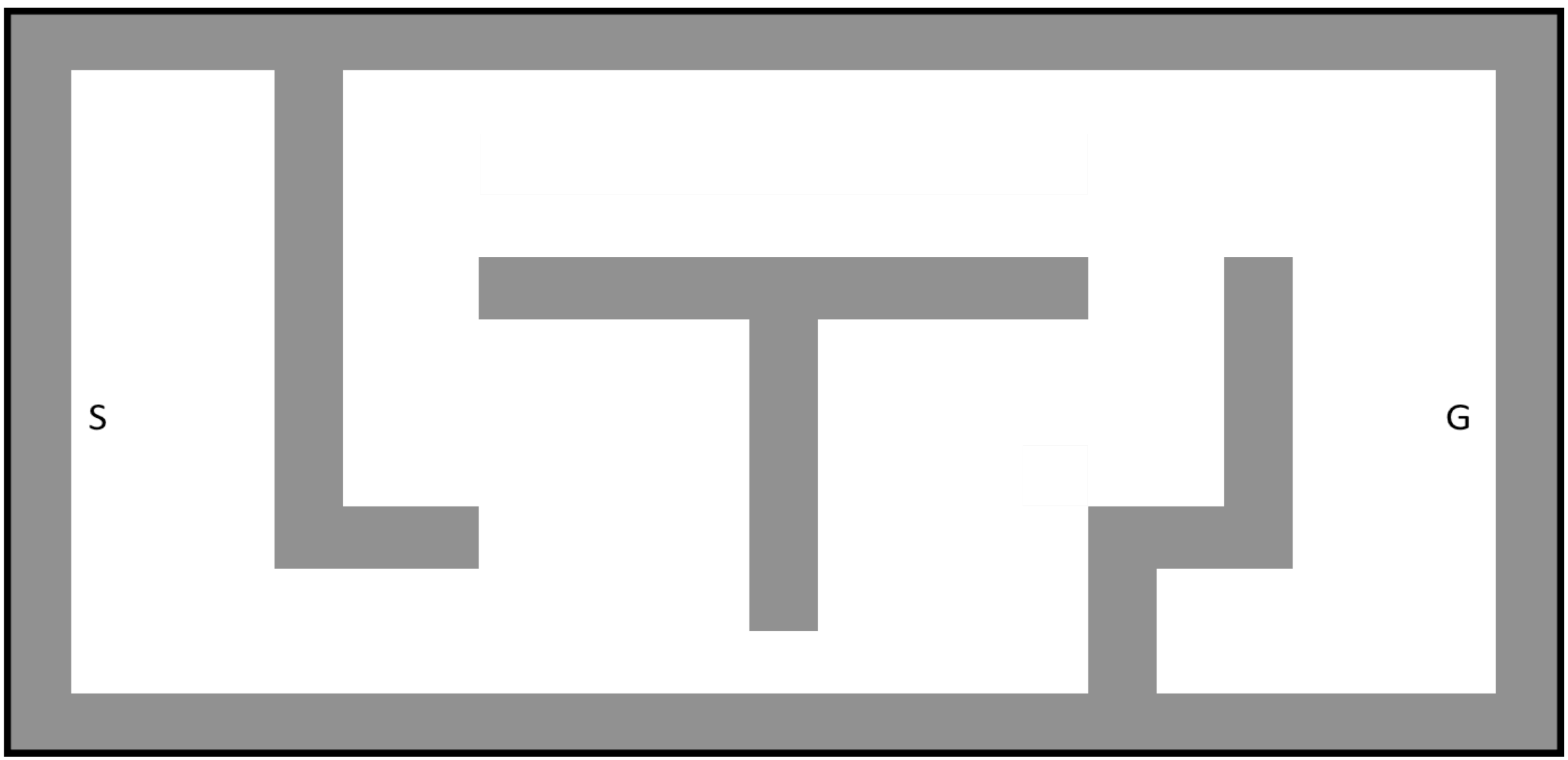
Cell Decomposition

Fall 2017

Apply the Exact Cell Decomposition algorithm and show the “Topological Map” to the scene below where darker elements correspond to obstacles, “S” represents the start location, and “G” represents the goal location.

Show the complete trapezoidal cell decomposition.

Show the topological map including a final shortest path from start “S” to goal “G” based on the least number of individual nodes.



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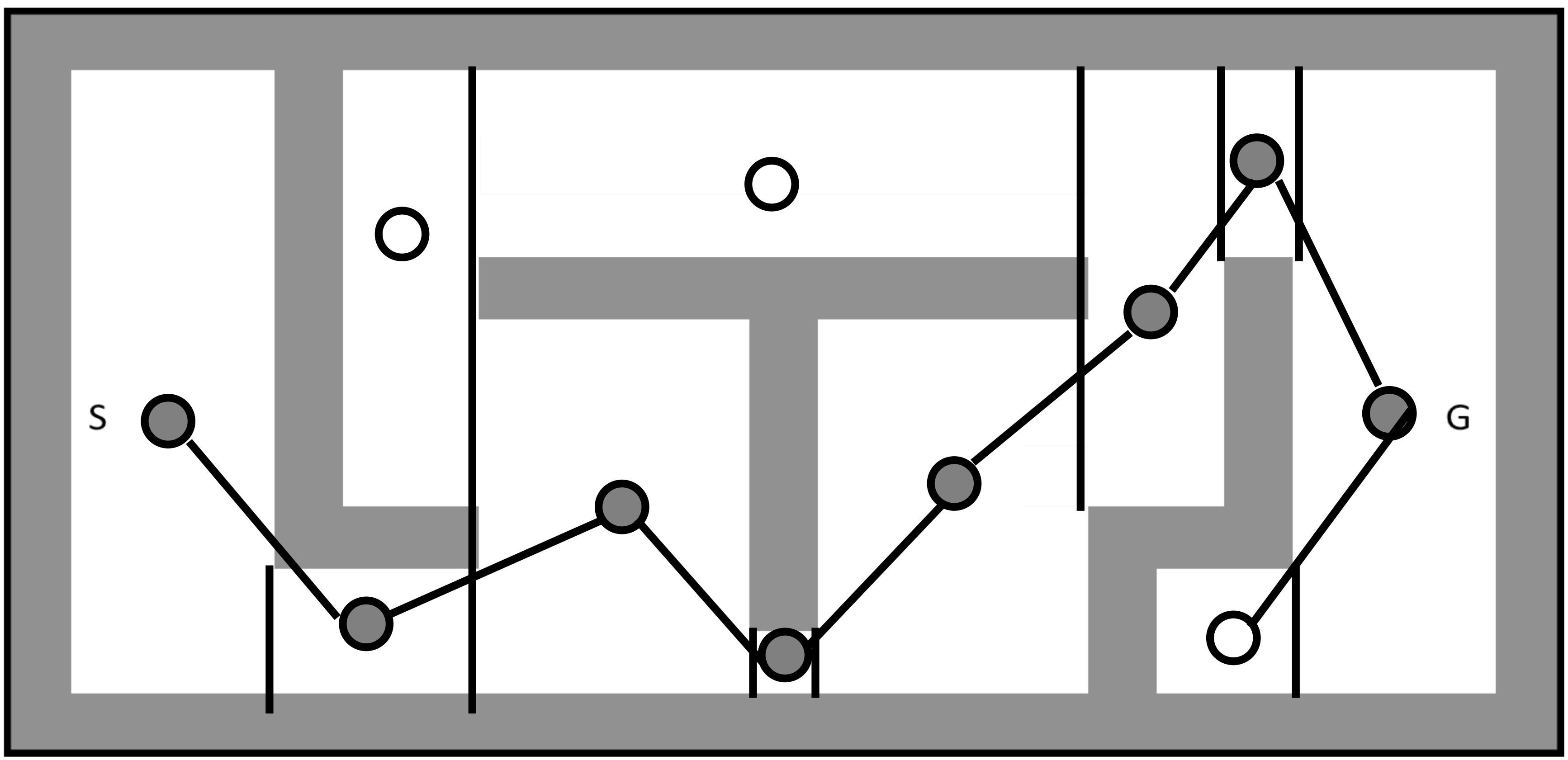

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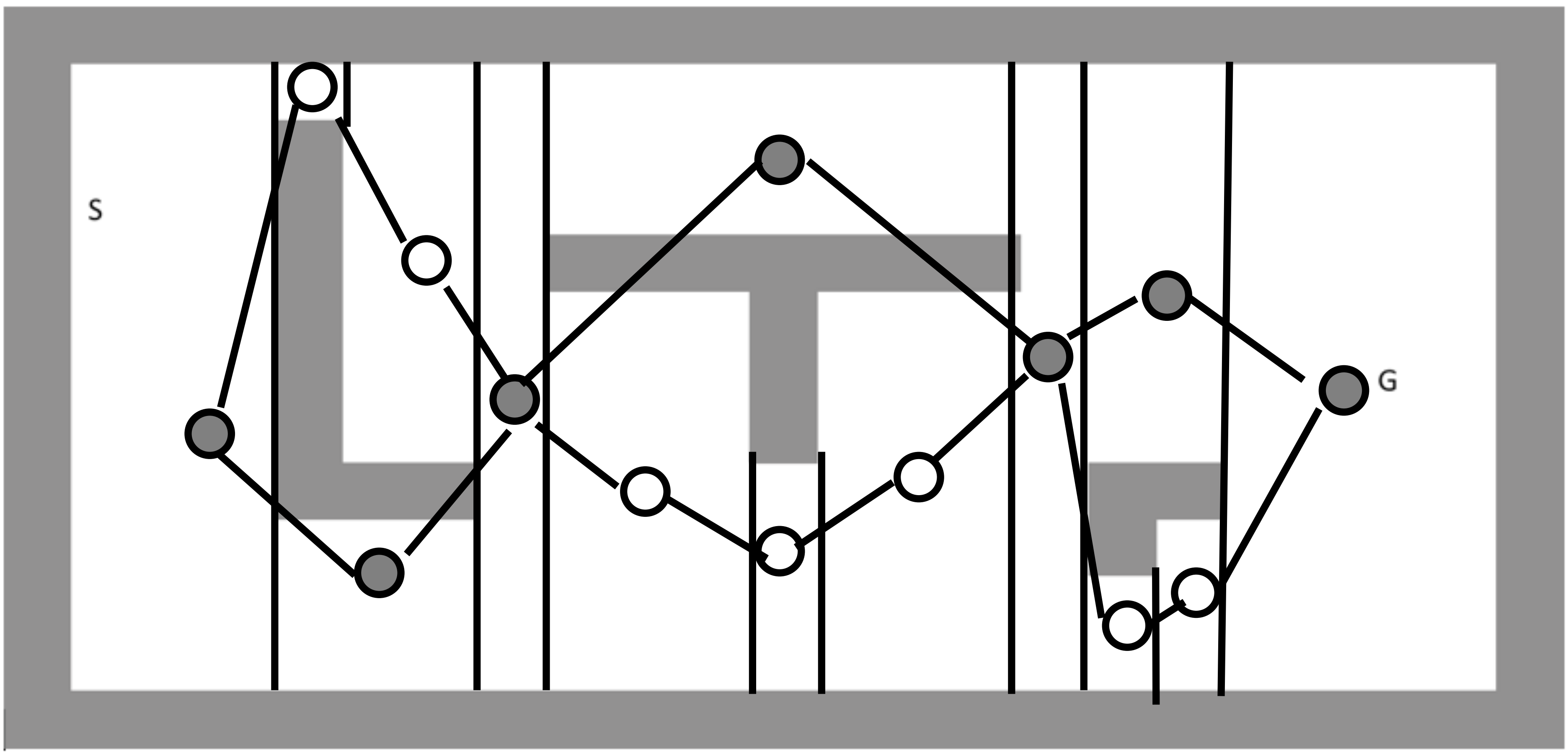
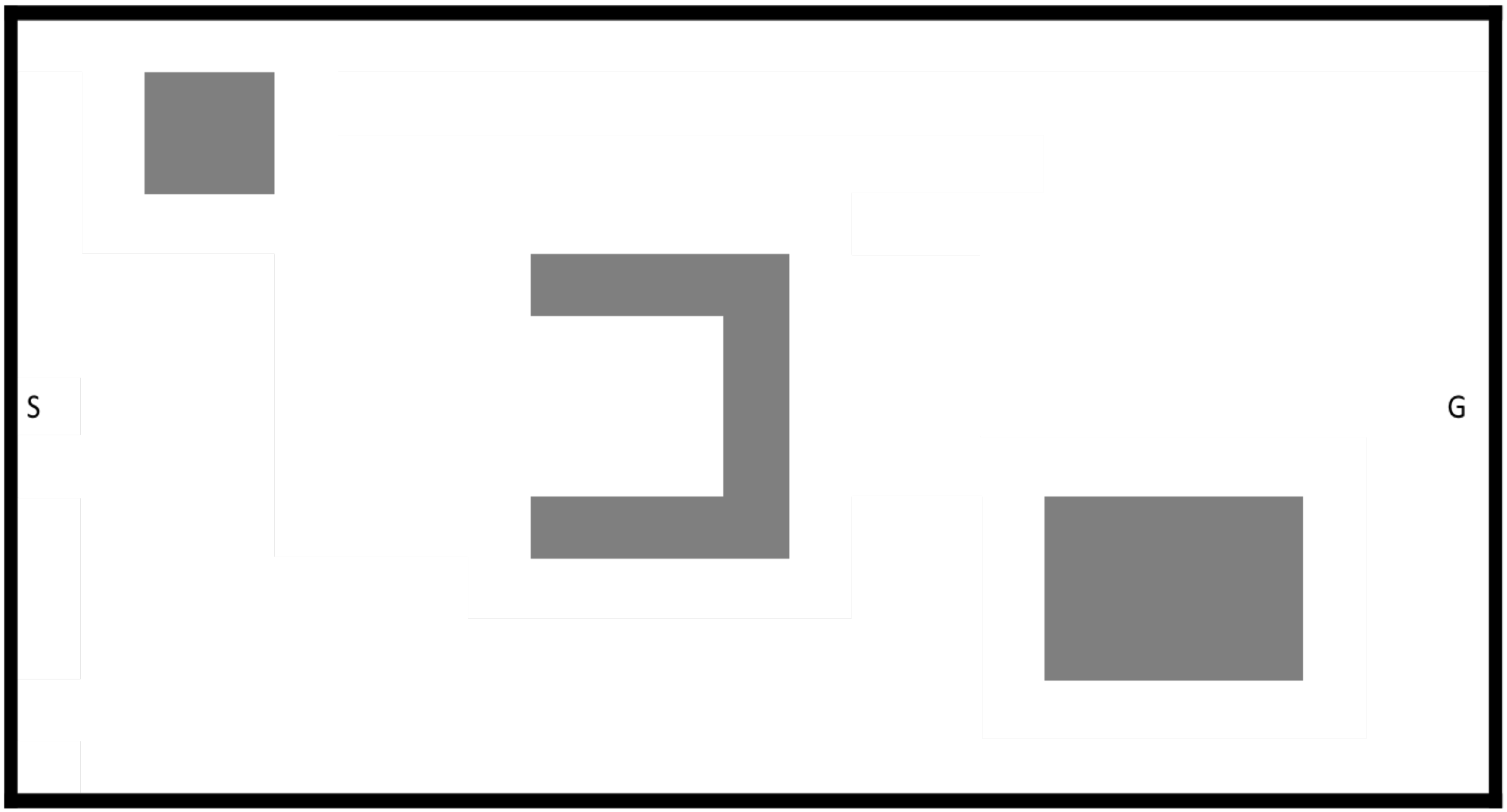


Exact Cell Decomposition

Fall 2018

Apply the Exact Cell Decomposition algorithm to the scene below. Add a topological map and then highlight the shortest path from “S” to “G” based on the least number of individual nodes.



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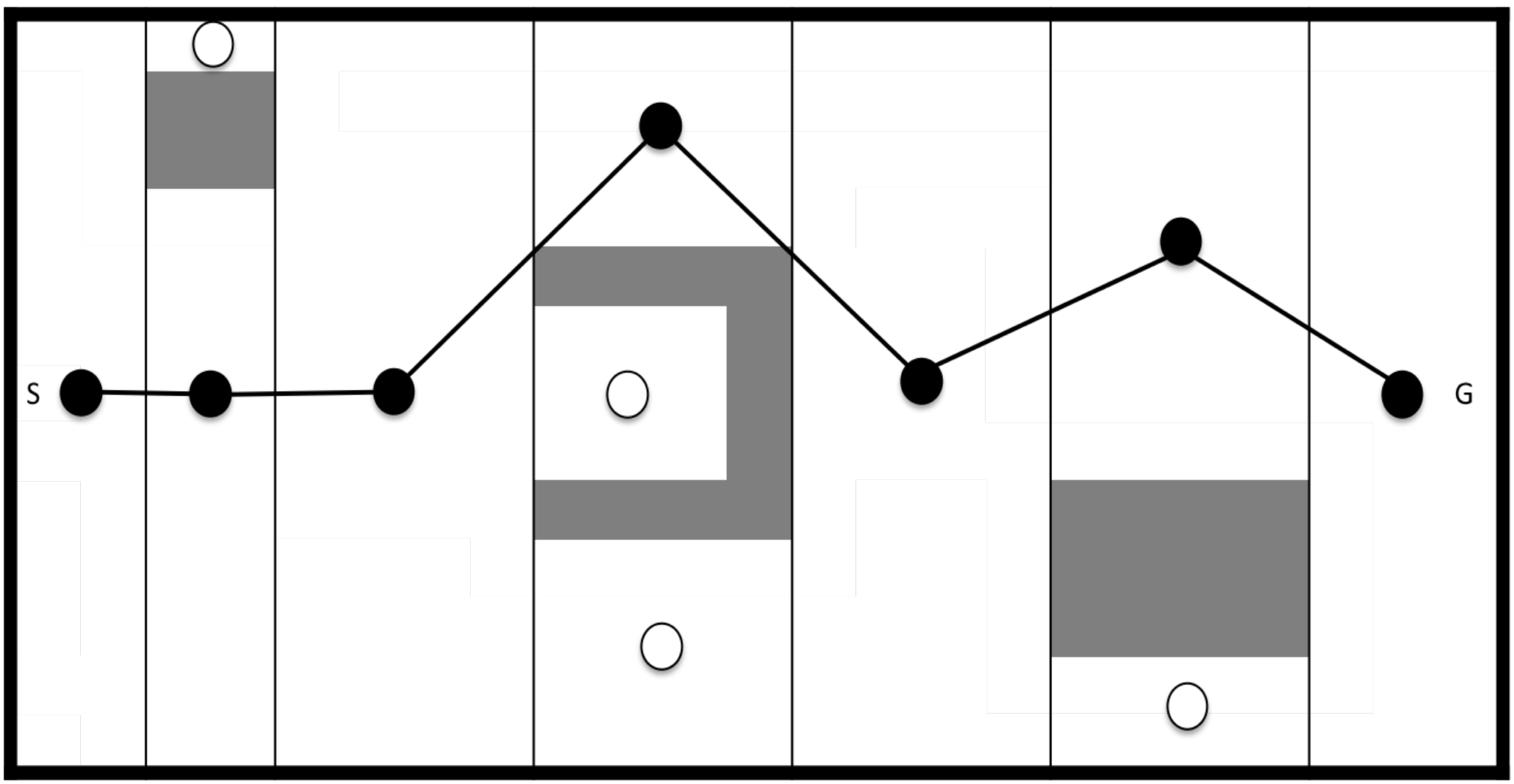
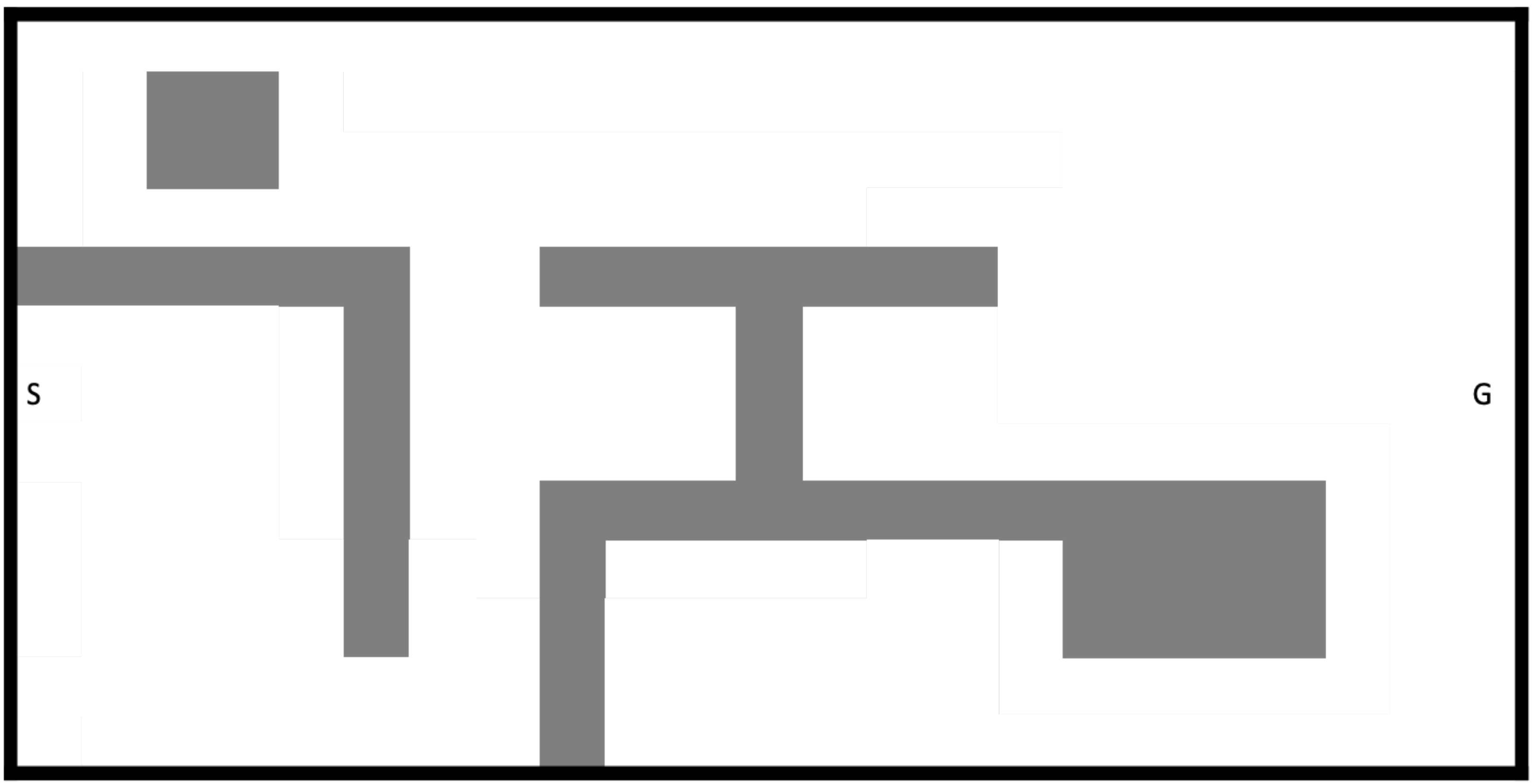


Exact Cell Decomposition

Fall 2018

Apply the Exact Cell Decomposition algorithm to the scene below. Add a topological map and then highlight the shortest path from start “S” to goal “G” based on the least number of individual nodes. <u>Show midpoint connections in the diagram</u>.



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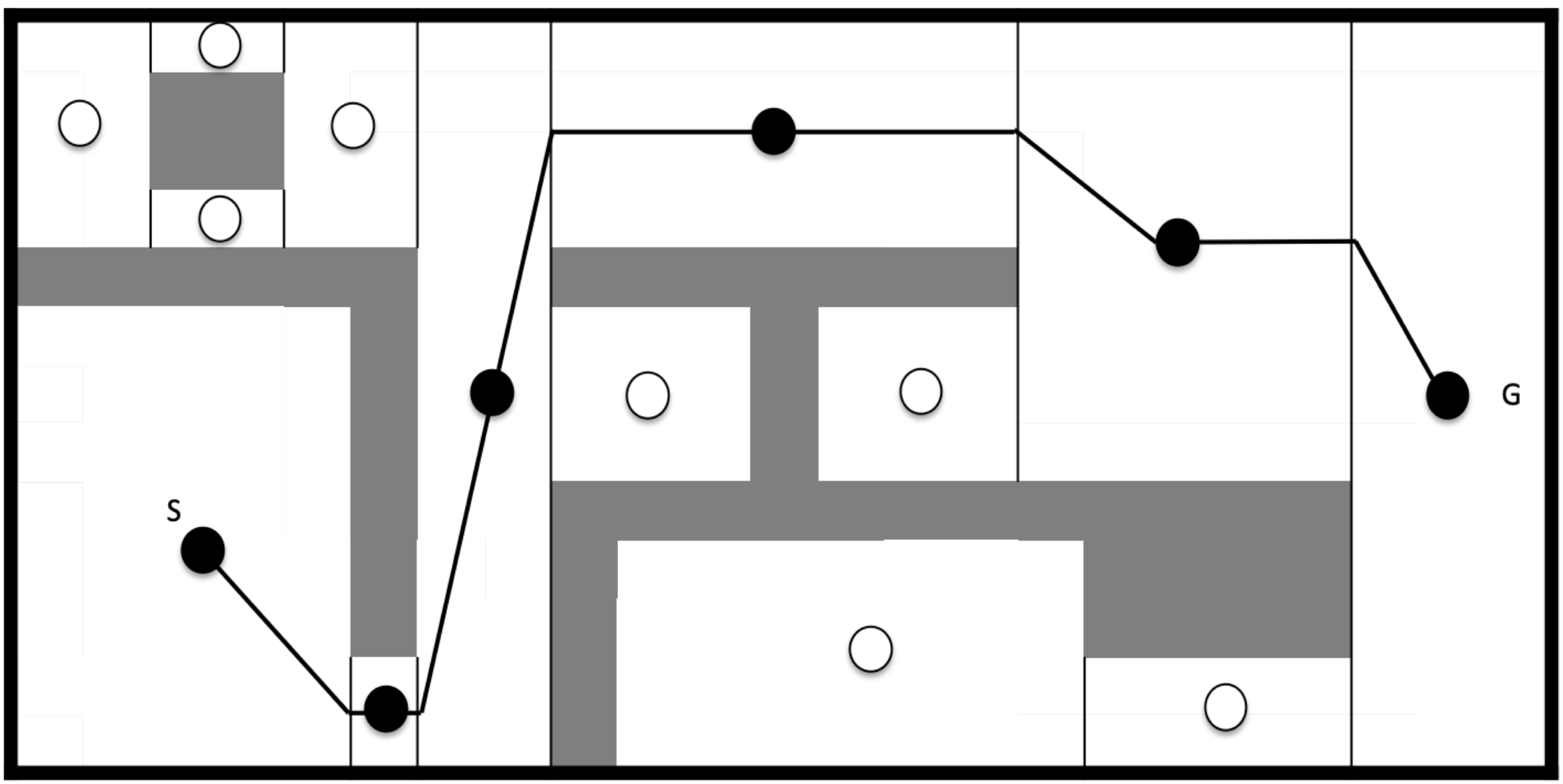
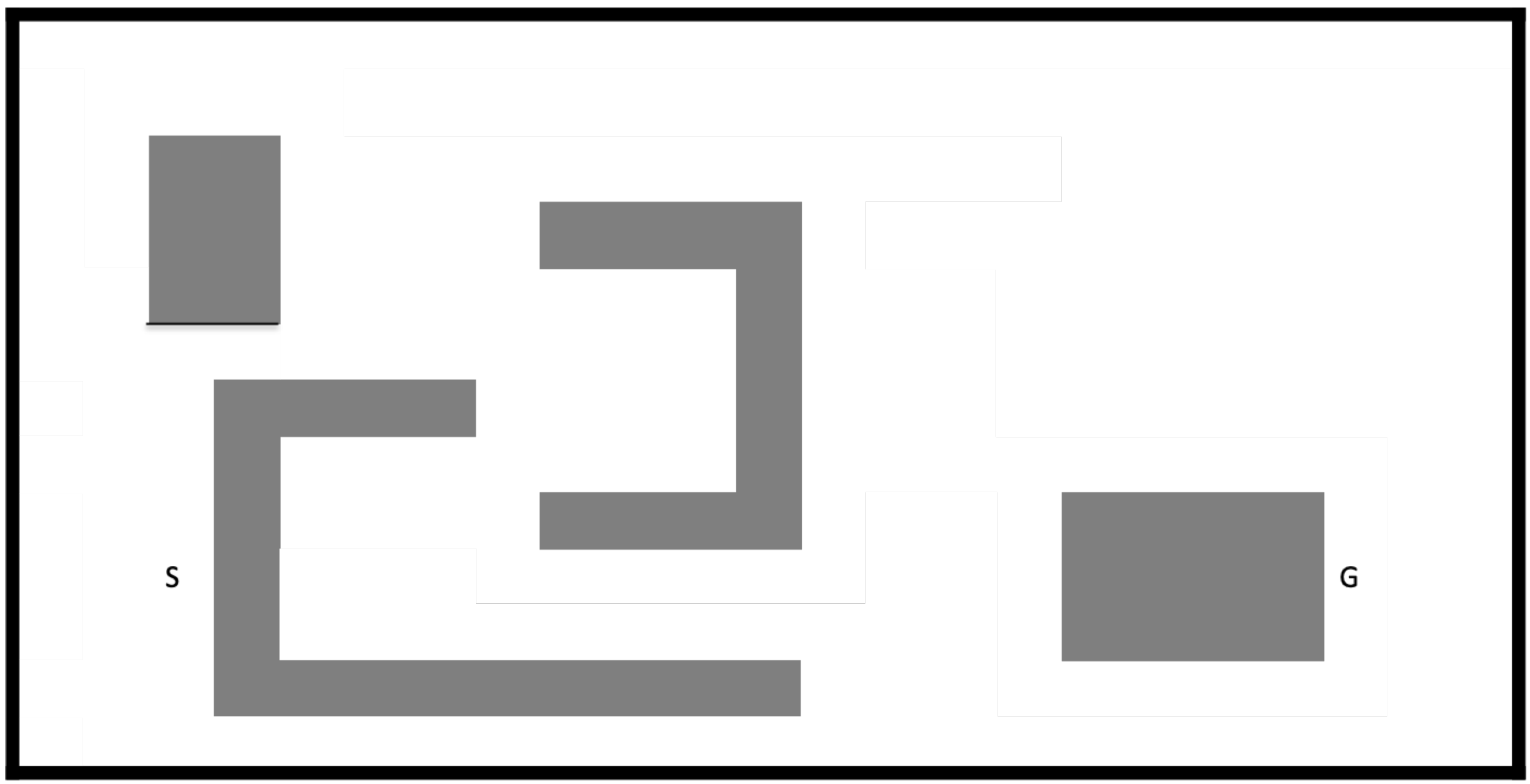


Exact Cell Decomposition

Spring 2019

1. Apply the Exact Cell Decomposition algorithm to the diagram below. Note the outer wall in the diagram.
2. Add a topological map and then highlight the shortest path from “S” to “G” based on the least number of nodes.



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Spring 2019

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