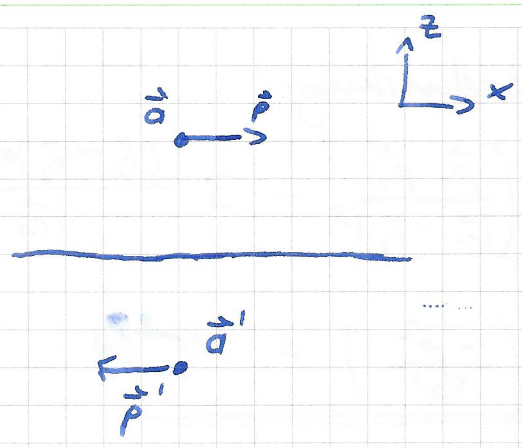
A1

a) Skizze:



Aus Überlegung:

$\vec{p}\,' = -\vec{p}$

$\vec{a}\,' = -\vec{a}$

$$\Rightarrow \Phi(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\left(\frac{\vec{p}\,(\vec{r}-\vec{a})}{|\vec{r}-\vec{a}|^3} - \frac{\vec{p}\,(\vec{r}+\vec{a})}{|\vec{r}+\vec{a}|^3}\right) = \frac{1}{4\pi\varepsilon_0}\left(\frac{px}{\sqrt{x^2+y^2+(z-a)^2}^3} - \frac{px}{\sqrt{x^2+y^2+(z+a)^2}^3}\right)$$

überprüfen der Randbedingung:

$$\phi(\vec{r})\Big|_{z=0} = \frac{1}{4\pi\varepsilon_0}\left(\frac{px}{\sqrt{x^2+y^2+a^2}^3} - \frac{px}{\sqrt{x^2+y^2+a^2}^3}\right) = 0 \checkmark$$

b)

Influenzierte Flächenladungsdichte:

$$\sigma(x,y) = \varepsilon_0 \vec{n}\cdot\vec{E}\Big|_{z=0} = \varepsilon_0 \vec{e}_z \vec{\nabla}\phi\Big|_{z=0} = \varepsilon_0 \frac{\partial\phi}{\partial z}\Big|_{z=0}$$

$$\frac{\partial\phi}{\partial z} = \frac{1}{4\pi\varepsilon_0}\left(\frac{-px\,3\,(x^2+y^2+(z-a)^2)\frac{1}{2\sqrt{x^2+y^2+(z-a)^2}}\,2(z-a)}{\sqrt{x^2+y^2+(z-a)^2}^6} + \frac{px\,3\,(x^2+y^2+(z+a)^2)\frac{1}{2\sqrt{x^2+y^2+(z+a)^2}}\,2(z+a)}{\sqrt{x^2+y^2+(z+a)^2}^6}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\left(-\frac{3px(z-a)}{\sqrt{x^2+y^2+(z-a)^2}^5} + \frac{3px(z+a)}{\sqrt{x^2+y^2+(z+a)^2}^5}\right)$$

$$\Rightarrow \sigma(x,y) = \frac{1}{4\pi}\left(\frac{6pxa}{\sqrt{x^2+y^2+a^2}^5}\right) = \frac{3pxa}{2\pi\sqrt{x^2+y^2+a^2}^5}$$

c) Dipol-Dipol-Wechselwirkung:

$$W_{12} = \frac{1}{4\pi\varepsilon_0}\left(\frac{\vec{p}_1\cdot\vec{p}_2}{|\vec{r}_1-\vec{r}_2|^3} - 3\,\frac{(\vec{r}_1-\vec{r}_2)\circ\vec{p}_1\cdot(\vec{r}_1-\vec{r}_2)\circ\vec{p}_2}{|\vec{r}_1-\vec{r}_2|^5}\right) =$$

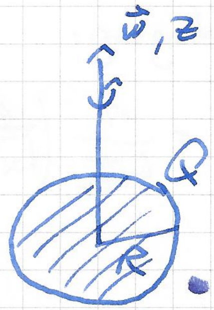
$$= \underline{\underline{\frac{1}{4\pi\varepsilon_0}\left(\frac{-p^2}{8a^3}\right) = W_{12}(a)}}$$

mit $\vec{p}_1 = \begin{pmatrix} p \\ 0 \\ 0 \end{pmatrix}$, $\vec{p}_2 = \begin{pmatrix} -p \\ 0 \\ 0 \end{pmatrix}$, $\vec{r}_1 = \begin{pmatrix} 0 \\ 0 \\ a \end{pmatrix}$, $\vec{r}_2 = \begin{pmatrix} 0 \\ 0 \\ -a \end{pmatrix}$

$\vec{F} \sim \vec{e}_z \qquad \vec{F} = -\vec{\nabla} W_{12}$

$$\Rightarrow \vec{F} = -\frac{d}{da} W_{12}\,\vec{e}_z = \frac{1}{4\pi\varepsilon_0}\,p^2\,\frac{-3}{8a^4}\,\vec{e}_z =$$

$$= \underline{\underline{-\frac{3p^2}{32\pi\varepsilon_0 a^4}\,\vec{e}_z}}$$



A2

a) $\vec{j}(\vec{r}) = \rho(\vec{r})\cdot\vec{v}(\vec{r})$

$\rho(\vec{r}) = \frac{Q}{\pi R^2}\,\delta(z)\,\theta(R-\rho)$

$\vec{v}(\vec{r}) = \vec{\omega}\times\vec{r}$

$$\underline{\underline{\vec{j}(\vec{r}) = \frac{Q}{\pi R^2}\,\delta(z)\,\theta(R-\rho)\,\vec{\omega}\times\vec{r}}}$$

Divergenz-Freiheit

$$\vec{\nabla}\cdot\vec{j}(\vec{r}) = \frac{Q}{\pi R^2}\,\delta(z)\,\theta(R-\rho)\,\vec{\nabla}(\vec{\omega}\times\vec{r}) =$$

$$= \frac{Q}{\pi R^2}\,\delta(z)\,\theta(R-\rho)\left[\vec{r}\,\underbrace{(\vec{\nabla}\times\vec{\omega})}_{=0} - \vec{\omega}\,\underbrace{(\vec{\nabla}\times\vec{r})}_{=0}\right] = 0 \checkmark$$

$$\Rightarrow \underline{\vec{\nabla}\cdot\vec{j}(\vec{r}) = 0}$$

b) $\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int_V \vec{j}(\vec{r}') \times \frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3} d^3r' =$ $\quad \vec{r}' = \begin{pmatrix} \rho' \cos\varphi' \\ \rho' \sin\varphi' \\ z' \end{pmatrix}$

$\vec{r} = \begin{pmatrix} 0 \\ 0 \\ z \end{pmatrix}$

$$= \frac{\mu_0}{4\pi} \int_0^\infty \int_0^\infty \int_0^{2\pi} \frac{Q}{\pi R^2} \delta(z) \Theta(R-\rho) (\vec{\omega} \times \vec{r}') \times \frac{\begin{pmatrix} -\rho' \cos\varphi' \\ -\rho' \sin\varphi' \\ z-z' \end{pmatrix}}{\sqrt{\rho'^2 + z^2}^3} \rho' d\rho' d\varphi' dz' =$$

$$= \frac{\mu_0 Q}{4\pi^2 R^2} \int_0^{2\pi} \int_0^R \frac{(\vec{\omega} \times \vec{r}') \times \begin{pmatrix} -\rho' \cos\varphi' \\ -\rho' \sin\varphi' \\ z \end{pmatrix} \rho'}{\sqrt{\rho'^2 + z^2}^3} d\rho' d\varphi' =$$

NR:

$$\begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} \rho' \cos\varphi' \\ \rho' \sin\varphi' \\ 0 \end{pmatrix} = \begin{pmatrix} -\omega\rho' \sin\varphi' \\ \omega\rho' \cos\varphi' \\ 0 \end{pmatrix}$$

$$= \frac{\mu_0 Q}{4\pi^2 R^2} \int_0^{2\pi} \int_0^R \frac{\begin{pmatrix} -\omega\rho' \sin\varphi' \\ \omega\rho' \cos\varphi' \\ 0 \end{pmatrix} \times \begin{pmatrix} -\rho' \cos\varphi' \\ -\rho' \sin\varphi' \\ z \end{pmatrix} \rho'}{\sqrt{\rho'^2 + z^2}^3} d\rho' d\varphi'$$

$$= \frac{\mu_0 Q}{4\pi^2 R^2} \int_0^{2\pi} \int_0^R \frac{\begin{pmatrix} \omega\rho' \cos\varphi' \\ \omega\rho' \sin\varphi' \\ \omega\rho'^2 \end{pmatrix} \rho'}{\sqrt{\rho'^2 + z^2}^3} d\rho' d\varphi' =$$

Hinweis:

$$= \frac{\mu_0 Q}{4\pi^2 R^2} \int_0^R 2\pi \vec{e}_z \omega \frac{\rho'^3}{\sqrt{\rho'^2 + z^2}^3} d\rho' = \underline{\underline{\vec{e}_z \frac{\mu_0 Q \omega}{2\pi R^2} \left(\frac{R^2 + 2z^2}{\sqrt{R^2 + z^2}} - 2|z| \right)}} = \vec{B}(\vec{r} = \begin{pmatrix} 0 \\ 0 \\ z \end{pmatrix})$$

c) $\vec{m} = \frac{1}{2} \int d^3r' \, \vec{r}' \times \vec{j}(\vec{r}') = \frac{1}{2} \int_0^\infty \int_0^\infty \int_0^{2\pi} \vec{r}' \times (\vec{\omega} \times \vec{r}') \frac{Q}{\pi R^2} \delta(z) \Theta(R-\rho) \rho' d\rho' dz' d\varphi' =$

$$= \frac{Q}{\pi R^2} \frac{1}{2} \int_0^{2\pi} \int_0^R \begin{pmatrix} \rho' \cos\varphi' \\ \rho' \sin\varphi' \\ 0 \end{pmatrix} \times \begin{pmatrix} -\omega\rho' \sin\varphi' \\ \omega\rho' \cos\varphi' \\ 0 \end{pmatrix} \rho' d\rho' d\varphi' =$$

$$= \frac{1}{2} \int_0^R \int_0^{2\pi} \frac{Q}{\pi R^2} \vec{e}_z \rho'^3 \omega \, d\rho' d\varphi' = \frac{Q}{2\pi R^2} \omega 2\pi \vec{e}_z \int_0^R \rho'^3 d\rho' = \frac{Q\omega}{R^2} R^4 \frac{1}{4} \vec{e}_z =$$

$$= \underline{\underline{\frac{Q\omega R^2}{4} \vec{e}_z}} = \vec{m}$$

Berechne $\vec{B}_{dip}$: $\hat{r} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ für $\vec{r} = \begin{pmatrix} 0 \\ 0 \\ z \end{pmatrix}$

$$\vec{B}_{dip}(\vec{r}) = \frac{\mu_0}{4\pi|z|^3}\left(3\hat{r}(\vec{m}\cdot\hat{r}) - \vec{m}\right) = \frac{\mu_0}{4\pi|z|^3}\left(3\vec{e}_z\frac{Q\omega R^4}{4} - \frac{Q\omega R^4}{\frac{1}{4}}\vec{e}_z\right)$$

$$= \underline{\underline{\frac{\mu_0 Q\omega R^4}{8\pi|z|^3}\vec{e}_z = \vec{B}_{dip}(\vec{r})}}$$

Taylorentwicklung des $\vec{B}$-Feldes aus b)

$$\vec{B}(z) = \vec{e}_z\frac{\mu_0 Q\omega}{2\pi R^2}\left(\frac{R^2+2z^2}{\sqrt{R^2+z^2}} - 2|z|\right) = \vec{e}_z\frac{\mu_0 Q\omega}{2\pi R^2}\left(\frac{R^2+2z^2}{|z|\sqrt{1+\frac{R^2}{z^2}}} - 2|z|\right) =$$

Taylorentw. bei $\frac{R^2}{z^2} \approx 0$ für $z \gg R$

$$\approx \vec{e}_z\frac{\mu_0 Q\omega}{2\pi R^2}\left[\frac{R^2+2z^2}{|z|}\left(1 - \frac{R^2}{2z^2} + \frac{3R^4}{8z^4}\right) - 2|z|\right] =$$

$$= \vec{e}_z\frac{\mu_0 Q\omega}{2\pi R^2}\left[\frac{R^2}{|z|} - \frac{R^4}{2|z|^3} + \frac{3R^6}{8|z|^5} + 2|z| - \frac{R^2}{|z|} + \frac{6R^4}{8|z|^3} - 2|z|\right]$$

$$= \vec{e}_z\frac{\mu_0 Q\omega}{2\pi R^2}\left(\frac{1}{4}\frac{R^4}{|z|^3} + \underbrace{\frac{3R^6}{8|z|^5}}_{\text{vernachl.}}\right) \approx \underline{\underline{\vec{e}_z\frac{\mu_0 Q\omega R^2}{8\pi|z|^3} = \vec{B}_{dip}}}$$

$\boxed{A3}$

a) Verschiebungsstrom : $\mu_0\varepsilon_0\frac{d\vec{E}}{dt}$

$$\Rightarrow \vec{\nabla}\times\vec{B} = \mu_0\varepsilon_0\frac{d\vec{E}}{dt} \qquad \text{mit } \vec{B} = B(\varrho)\,\vec{e}_\varphi$$

$$\Rightarrow \vec{\nabla}\times\vec{B} = \frac{1}{\varrho}\frac{\partial}{\partial\varrho}(\varrho B(\varrho))\vec{e}_z = \mu_0\varepsilon_0 K\vec{e}_z$$

$$\Rightarrow \frac{\partial}{\partial\varrho}(\varrho B(\varrho)) = \mu_0\varepsilon_0 K\varrho$$

$$\varrho B(\varrho) = \frac{\mu_0\varepsilon_0 K\varrho^2}{2}$$

$$\Rightarrow B(\varrho) = \frac{\mu_0\varepsilon_0 K\varrho}{2} \Rightarrow \underline{\underline{\vec{B}(\vec{r}) = \frac{\mu_0\varepsilon_0 K\varrho}{2}\vec{e}_\varphi}}$$

b) $\vec{S}(\vec{r},t) = \frac{1}{\mu_0}\vec{E}\times\vec{B} =$ mit $E(0)=0$: $\frac{d\vec{E}}{dt} = k\vec{e}_z \Rightarrow \vec{E}(\vec{r},t) = kt\,\vec{e}_z$

$$= \frac{1}{\mu_0} kt\,\vec{e}_z \times \frac{\mu_0\varepsilon_0 k\rho}{2}\vec{e}_\varphi =$$

$$= -\frac{\varepsilon_0 k^2 \rho t}{2}\vec{e}_\rho$$

Energiefluss:

$$J = \int_A \vec{S}(\vec{r},t)\, d\vec{F} \leftarrow$$

Fläche mit nach innen gerichtetem Normalenvektor (−) da reinfließende Energie positiv ist.

Auf Rand

$$= \int_0^l \int_0^{2\pi} \frac{\varepsilon_0 k^2 \rho t}{2}\vec{e}_\rho \vec{e}_\rho\, R\, d\varphi\, dz =$$

$$= \varepsilon_0 k^2 R^2 t l \pi = J$$

Energie im Kondensator:

Energiedichte:

$$w_{em} = \frac{\varepsilon_0}{2}\vec{E}^2 + \frac{1}{2\mu_0}\vec{B}^2 = \frac{\varepsilon_0}{2}k^2t^2 + \frac{\mu_0\varepsilon_0 k^2\rho^2}{8}$$

$$\mathcal{E}_{em} = \int_{V_{Kond}} w_{em}\, d^3r = \int_0^l\int_0^{2\pi}\int_0^R \left(\frac{\varepsilon_0}{2}k^2t^2 + \frac{\mu_0\varepsilon_0}{8}k^2\rho^2\right)\rho\, d\rho\, d\varphi\, dz =$$

$$= l\pi R^2 \frac{\varepsilon_0}{2}k^2t^2 + 2\pi l \frac{\mu_0\varepsilon_0}{32}k^2R^4 = \mathcal{E}_{em}(t)$$

$$\frac{d\mathcal{E}_{em}}{dt} = l\pi R^2 \varepsilon_0 k^2 t = J \quad \square$$

$\boxed{A4}$

a) $\vec{j}(\vec{r}) = \left[\frac{I}{\pi R_1^2}\,\theta(R_1 - \varrho) - \frac{I}{2\pi R_2}\,\delta(\varrho - R_2)\right]\vec{e}_z$

b) $\vec{\nabla}\times\vec{B} = \mu_0 \vec{j}$

$\Rightarrow \vec{\nabla}\times(\vec{\nabla}\times\vec{A}) = \mu_0\vec{j}$

$\vec{\nabla}\underbrace{(\vec{\nabla}\cdot\vec{A})}_{=0 \text{ nach Coulomb-Eichung}} \underline{\underline{- \Delta\vec{A} = \mu_0\vec{j}}}$

$\Rightarrow$ Hinweis $\frac{1}{\varrho}\frac{\partial}{\partial\varrho}\left[\varrho A'(\varrho)\right]\vec{e}_z = -\mu_0\,\vec{j}(\vec{r})$

$\varrho < R_1$: $\Rightarrow \frac{1}{\varrho}\frac{\partial}{\partial\varrho}\left[\varrho A'(\varrho)\right] = -\frac{\mu_0 I}{\pi R_1^2}$

$$-\frac{\mu_0 I}{\pi R_1^2}\varrho^2 + c = \varrho A'(\varrho)$$

$$-\frac{\mu_0 I}{2\pi R_1^2}\varrho + \frac{C_{1i}}{\varrho} = A'(\varrho)$$

$$\Rightarrow \underline{-\frac{\mu_0 I}{4\pi R_1^2}\varrho^2 + C_{1i}\ln(\varrho) + C_{2i} = A(\varrho)}$$

$R_1 < \varrho < R_2$:

$$\frac{1}{\varrho}\frac{d}{d\varrho}\left[\varrho A'(\varrho)\right] = 0$$

$$\varrho A'(\varrho) = C_{1m}$$

$$\underline{A(\varrho) = C_{1m}\ln(\varrho) + C_{2m}}$$

$R_2 < \varrho$, kein $\vec{B}$-Feld $\Rightarrow A(\varrho) = \text{const} = C_{aA}$

$$\Rightarrow A(\varrho) = \begin{cases} -\frac{\mu_0 I}{4\pi R_1^2}\varrho^2 + C_{1i}\ln(\varrho) + C_{2i} & \varrho < R_1 \\ C_{1m}\ln(\varrho) + C_{2m} & R_1 < \varrho < R_2 \\ C_{1A} & R_2 < \varrho \end{cases}$$

- Keine singulären Terme,

$$\Rightarrow C_{1i} = 0$$

- Setze $C_{2i} = 0$ da 1 Konst. frei wählbar

• Stetigkeit bei $\rho = R_1$:

$$-\frac{\mu_0 I}{4\pi} = C_{1m} \ln(R_1) + C_{2m}$$

• Differenzierbarkeit bei $\rho = R_1$:

$$-\frac{\mu_0 I}{2\pi R_1} = \frac{C_{1m}}{R_1} \Rightarrow C_{1m} = \frac{-\mu_0 I}{2\pi}$$

In Stetigkeitsbedingung:

$$\frac{-\mu_0 I}{4\pi} = -\frac{\mu_0 I}{2\pi} \ln(R_1) + C_{2m}$$

$$\Rightarrow C_{2m} = \frac{-\mu_0 I}{4\pi} + \frac{\mu_0 I}{2\pi} \ln(R_1)$$

$$\Rightarrow A(\rho) = -\frac{\mu_0 I}{4\pi} - \frac{\mu_0 I}{2\pi} \ln\left(\frac{\rho}{R_1}\right) = \underline{\underline{\frac{-\mu_0 I}{4\pi}\left(1 + \ln\left(\frac{\rho^2}{R_1^2}\right)\right)}}_{R_1 < \rho < R_2}$$

Stetigkeit bei R_2:

$$\Rightarrow A(\rho) = \underline{\underline{-\frac{\mu_0 I}{4\pi}\left(1 + \ln\left(\frac{R_2^2}{R_1^2}\right)\right)}} \quad \rho > R_2$$

$$\Rightarrow A(\rho) = \begin{cases} \frac{-\mu_0 I \rho^2}{4\pi R_1^2} & \rho < R_1 \\ \frac{-\mu_0 I}{4\pi}\left(1 + 2\ln\left(\frac{\rho}{R_1}\right)\right) & R_1 < \rho < R_2 \\ \frac{-\mu_0 I}{4\pi}\left(1 + 2\ln\left(\frac{R_2}{R_1}\right)\right) & R_2 < \rho \end{cases}$$

c) Selbstinduktivität pro Längeneinheit $\frac{L}{l}$:

$$\frac{L}{l} = \frac{1}{I^2} \int_S dF \, \vec{j}(\vec{r}) \vec{A}(\vec{r}) =$$

$$= \frac{1}{I^2} \left[\int_0^{R_1} \int_0^{2\pi} \frac{I}{\pi R_1^2} \left(-\frac{\mu_0 I \rho^2}{4\pi R_1^2} \rho \, d\rho \, d\varphi - \int_{R_1}^{R_2} \int_0^{2\pi} \frac{I}{2\pi R_2} \delta(\rho - R_2) \left[\frac{-\mu_0 I}{4\pi} \left(1 + 2\ln\left(\frac{\rho}{R_1}\right)\right) \right] \rho \, d\rho \, d\varphi \right]$$

$$= \frac{1}{I^2} \left[- \int_0^{R_1} \int_0^{2\pi} \frac{I^2 \mu_0 \rho^3}{4\pi^2 R_1^4} + \int_0^{2\pi} \frac{I^2 \mu_0}{8\pi^2 R_2} \left(1 + 2\left(\ln\left(\frac{R_2}{R_1}\right)\right)\right) R_2 \, d\varphi \right] =$$

$$= -\frac{\mu_0}{8\pi} + \int_0^{2\pi} \frac{\mu_0}{8\pi^2} \left(1 + 2\ln\left(\frac{R_2}{R_1}\right)\right) d\varphi =$$

$$= -\frac{\mu_0}{8\pi} + \frac{\mu_0}{4\pi} \left(1 + 2\ln\left(\frac{R_2}{R_1}\right)\right) =$$

$$= \underline{\underline{\frac{\mu_0}{2\pi} \left[\ln\left(\frac{R_2}{R_1}\right) + \frac{1}{4} \right] = \frac{L}{l}}}$$

5.

$$\vec{E}_j(z,t) = \left\{E_j^+ e^{i(k_j z-\omega t)} + E_j^- e^{i(-k_j z-\omega t)}\right\}\vec{e}_x$$

Stetigkeitsbed:

$z>0$:

$$E_1^+ e^{i(k_1 z-\omega t)} + E_1^- e^{i(-k_1 z-\omega t)}$$

$$n_1(\hat{k}\times\vec{E}_1^+)e^{i(k_1 z-\omega t)} - n_1(\hat{k}\times E_1^-)e^{i(-k_1 z-\omega t)}$$

$z>0$:

$$E_2^+ e^{i(k_2 z-\omega t)} + E_2^- e^{i(-k_2 z-\omega t)}$$

$$-n_2(\hat{k}\times E_2^-)e^{i(-k_2 z-\omega t)} + n_2(\hat{k}\times E_2^+)e^{i(k_2 z-\omega t)}$$

$z=0$

$$E_1^+ + E_1^- = E_2^+ + E_2^-$$

$$n_1(E_1^+ - E_1^-) = n_2(E_2^+ - E_2^-)$$

$$E_1^+ = E_2^+ + E_2^- - E_1^-$$

$$n_1(E_2^+ + E_2^- - 2E_1^-) = n_2(E_2^+ - E_2^-)$$

$$-2n_1E_1^- = E_2^+(n_2-n_1) - E_2^-(n_1+n_2)$$

$$E_1^- = E_2^+\left(\frac{1}{2}-\frac{n_2}{2n_1}\right) + E_2^-\left(\frac{1}{2}+\frac{n_2}{2n_1}\right)$$

$$E_1^+ = E_2^+ + E_2^- - E_2^+\left(\frac{1}{2}-\frac{n_2}{2n_1}\right) - E_2^-\left(\frac{1}{2}+\frac{n_2}{2n_1}\right)$$

$$E_1^+ = E_2^+\cdot\left[1-\frac{1}{2}+\frac{n_2}{2n_1}\right] + E_2^-\cdot\left[1-\frac{1}{2}-\frac{n_2}{2n_1}\right]$$

$$E_1^+ = E_2^+ \cdot \left[\frac{1}{2} + \frac{n_2}{2n_1}\right] + E_2^- \cdot \left[\frac{1}{2} - \frac{n_2}{2n_1}\right]$$

$$\begin{pmatrix} E_1^+ \\ E_1^- \end{pmatrix} = \begin{pmatrix} \frac{1}{2} + \frac{n_2}{2n_1} & \frac{1}{2} + \frac{n_2}{2n_1} \\ \frac{1}{2} + \frac{n_2}{2n_1} & \frac{1}{2} + \frac{n_2}{2n_1} \end{pmatrix} \begin{pmatrix} E_2^+ \\ E_2^- \end{pmatrix} \quad]$$

b) $E_2^- = 0$

$$\Rightarrow E_1^+ = E_2^+ \left[\frac{n_1 + n_2}{2n_1}\right]$$

$$\Rightarrow E_1^- = E_2^+ \left[\frac{n_1 - n_2}{2n_1}\right]$$

$$\langle S_1^+ \rangle = \frac{1}{2} \varepsilon_0 n_1^2 E_2^{+2} \left[\frac{n_1 + n_2}{2n_1}\right]^2 \cdot \frac{c}{n_1} \vec{e_z}$$

$$= \frac{1}{2} \varepsilon_0 n_1 E_2^{+2} \left[\frac{n_1 + n_2}{2n_1}\right]^2 \cdot c \vec{e_z}$$

$$\langle S_1^- \rangle = -\frac{1}{2} \varepsilon_0 n_1 E_2^{+2} \left[\frac{n_1 - n_2}{2n_1}\right]^2 \cdot c \vec{e_z}$$

$$\langle S_2^+ \rangle = \frac{1}{2} \varepsilon_0 n_2 E_2^{+2} c \vec{e_z}$$

$$R = \frac{\langle S_1^- \rangle}{\langle S_1^+ \rangle} = \left(\frac{n_1 - n_2}{2n_1} \cdot \frac{2n_1}{n_1 + n_2}\right)^2 = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2$$

$$T = \frac{\langle S_2^+ \rangle}{\langle S_1^+ \rangle} = \frac{n_2}{n_1} \left[\frac{2n_1}{n_1 + n_2}\right]^2$$

$$R + T = \frac{(n_1 - n_2)^2 n_1 + n_2 [2n_1]^2}{(n_1 + n_2)^2 \cdot n_1} = 1$$

$$(n_1 - n_2)^2 n_1 + n_2 [2n_1]^2 = (n_1 + n_2)^2 \cdot n_1$$

(13)

$$\frac{u_1^2 - 2u_1u_2 + u_2^2 + u_2 4u_1}{u_1^2 + 2u_1u_2 + u_2^2} = 1$$