

Chapter #21

Fourier Transforms

윤영우

Fourier Transform

- Fourier Transform

$$\mathcal{F}(u) = \int_{-\infty}^{\infty} f(x) e^{-i2\pi ux} dx$$

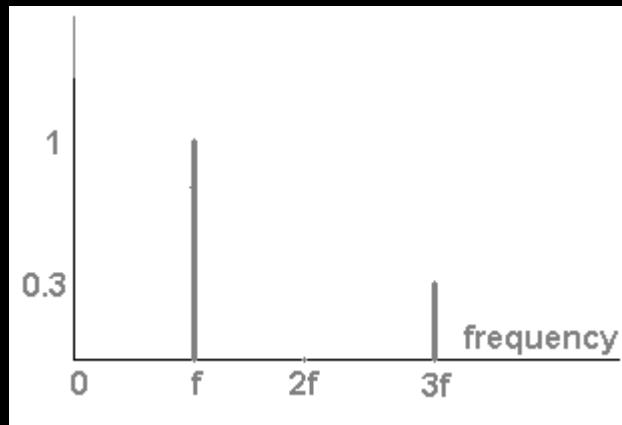
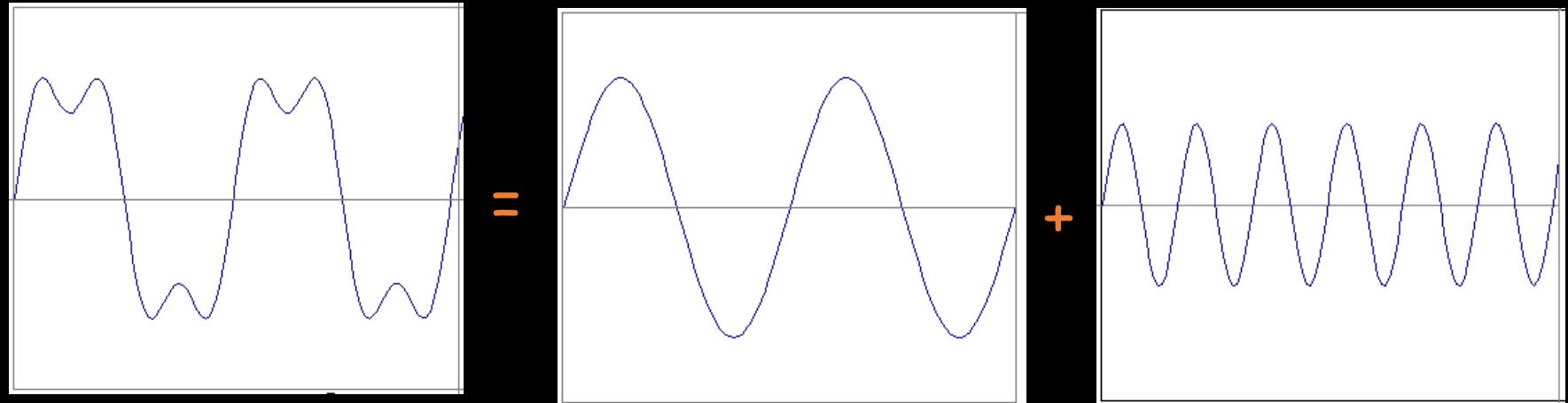
$$e^{ik} = \cos k + i \sin k \quad i = \sqrt{-1}$$

- Inverse Fourier Transform (IFT)

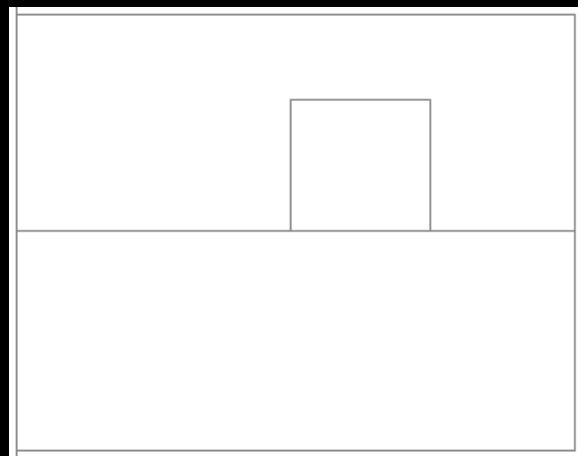
$$f(x) = \int_{-\infty}^{\infty} F(u) e^{i2\pi ux} dx$$

Frequency Spectra

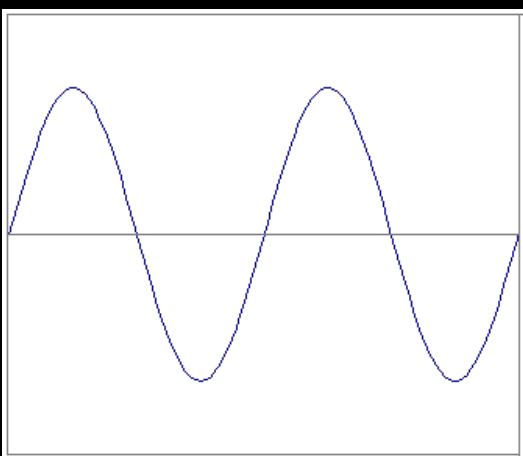
- example : $g(t) = \sin(2\pi f t) + (1/3)\sin(2\pi (3f) t)$



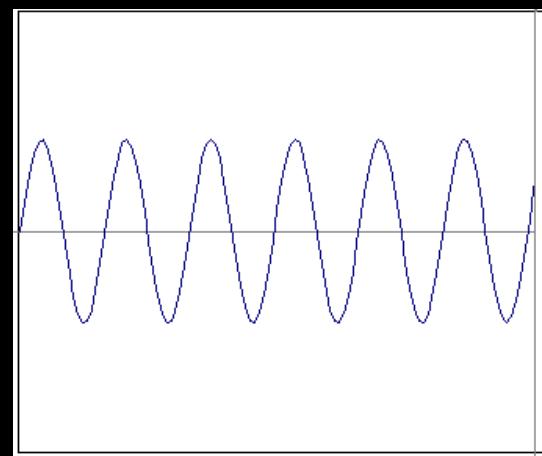
Frequency Spectra



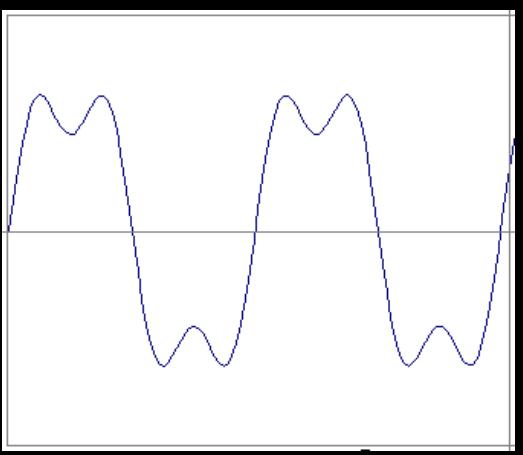
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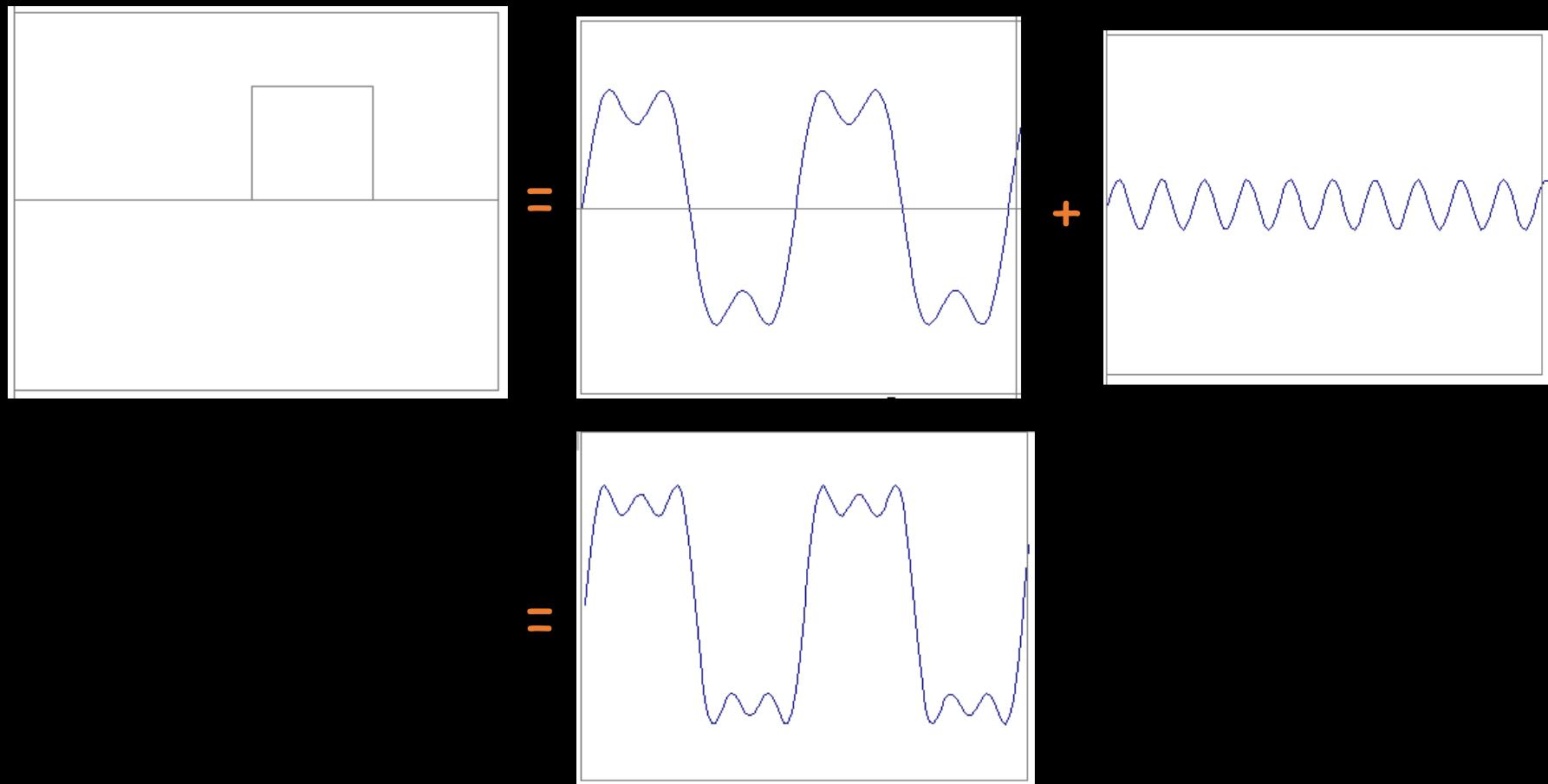
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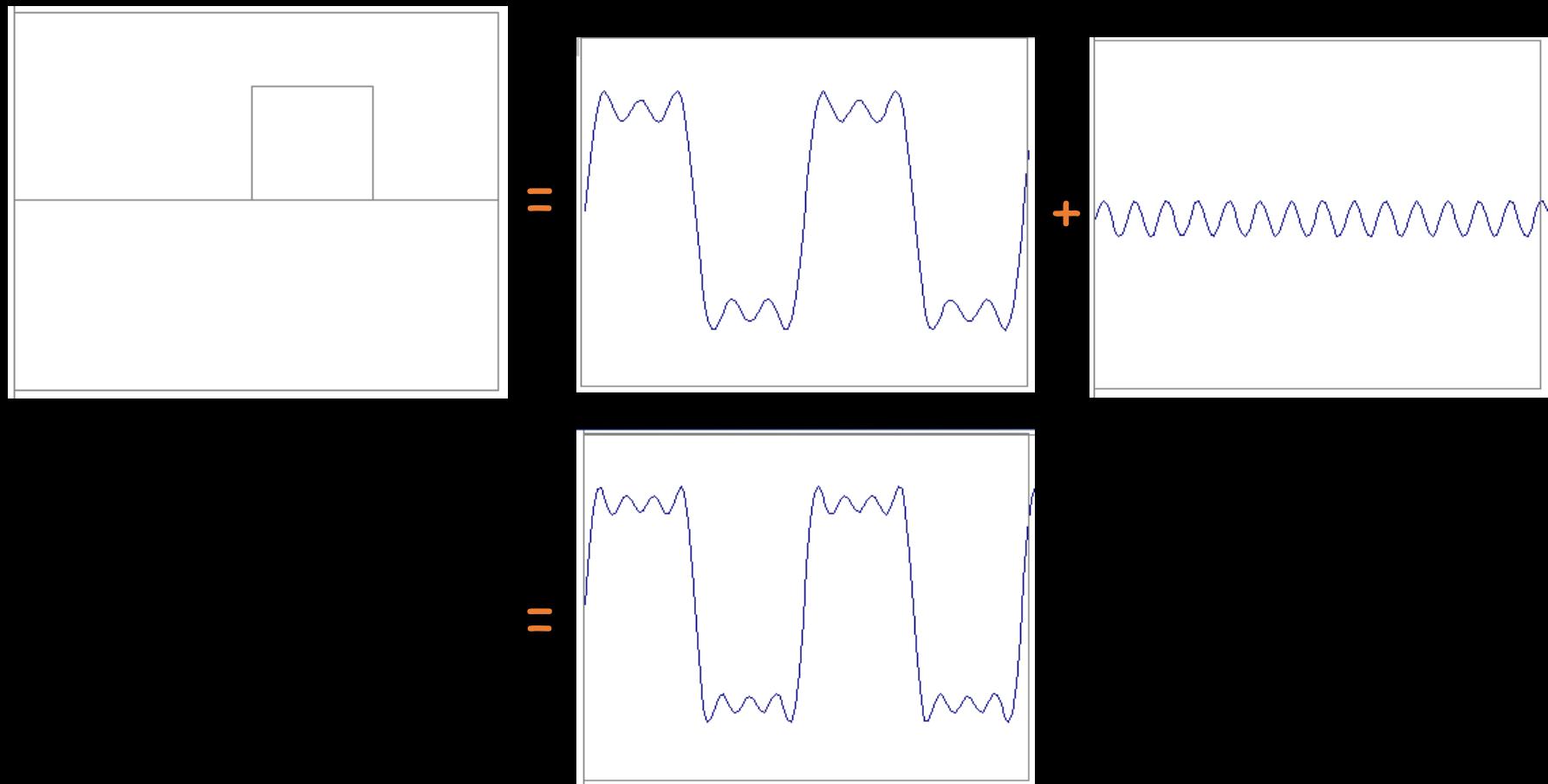
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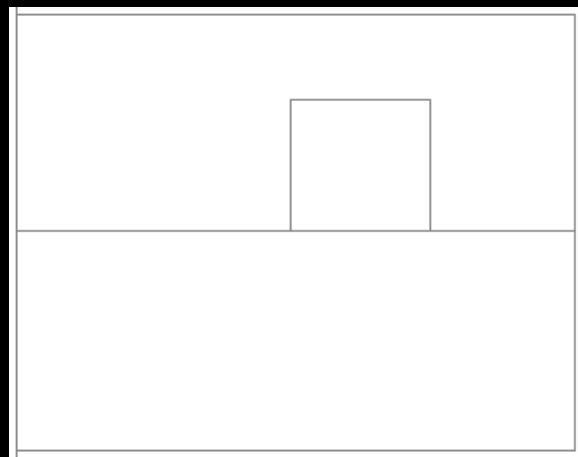
Frequency Spectra



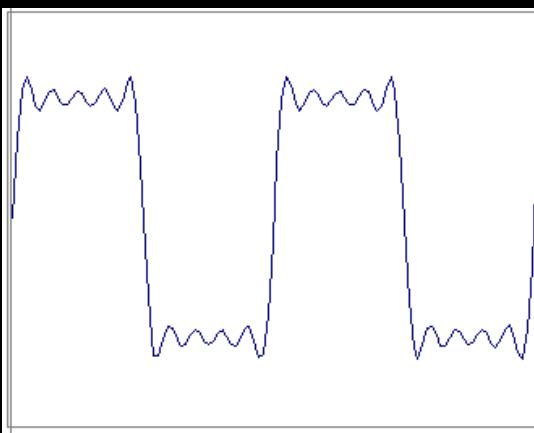
Frequency Spectra



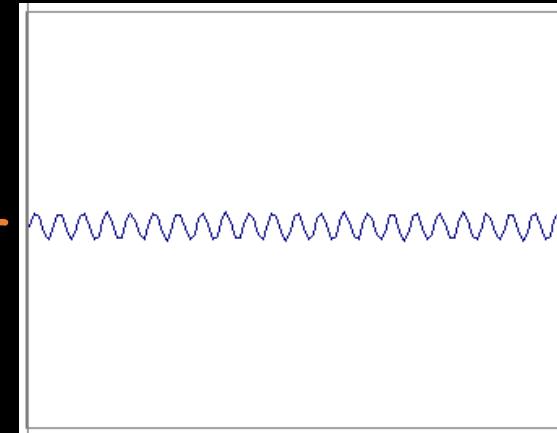
Frequency Spectra



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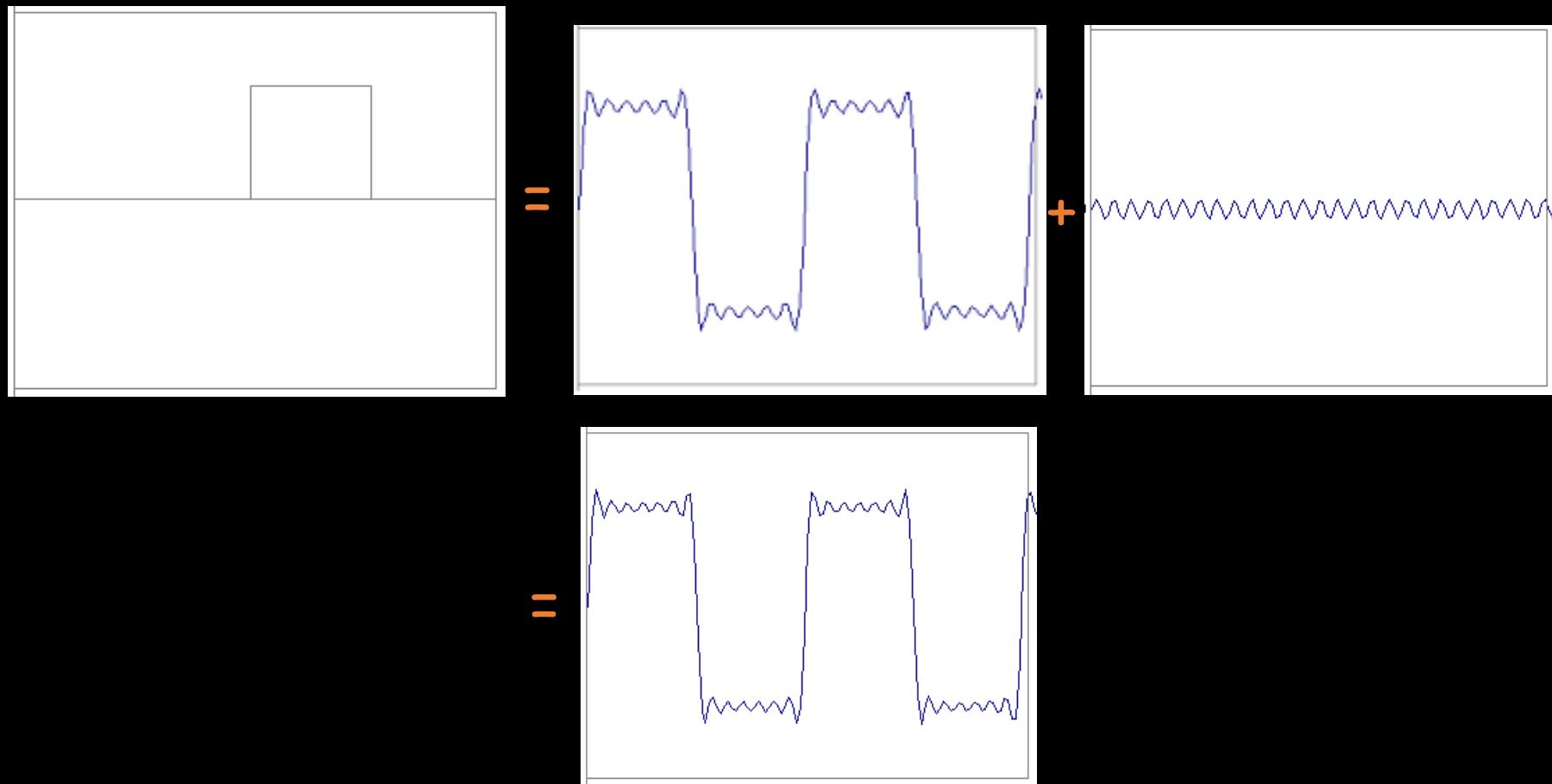
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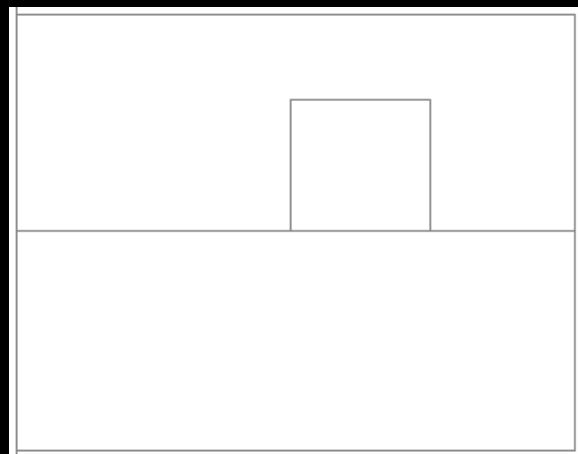
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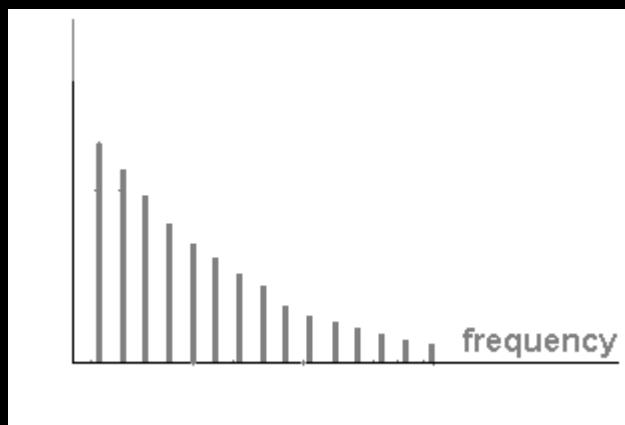
Frequency Spectra



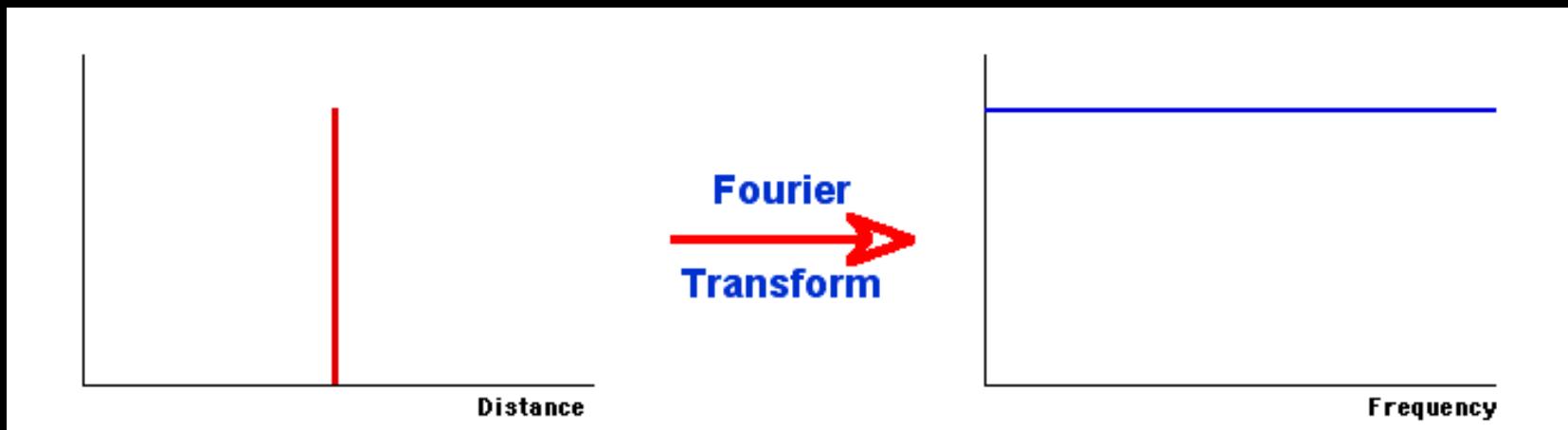
Frequency Spectra



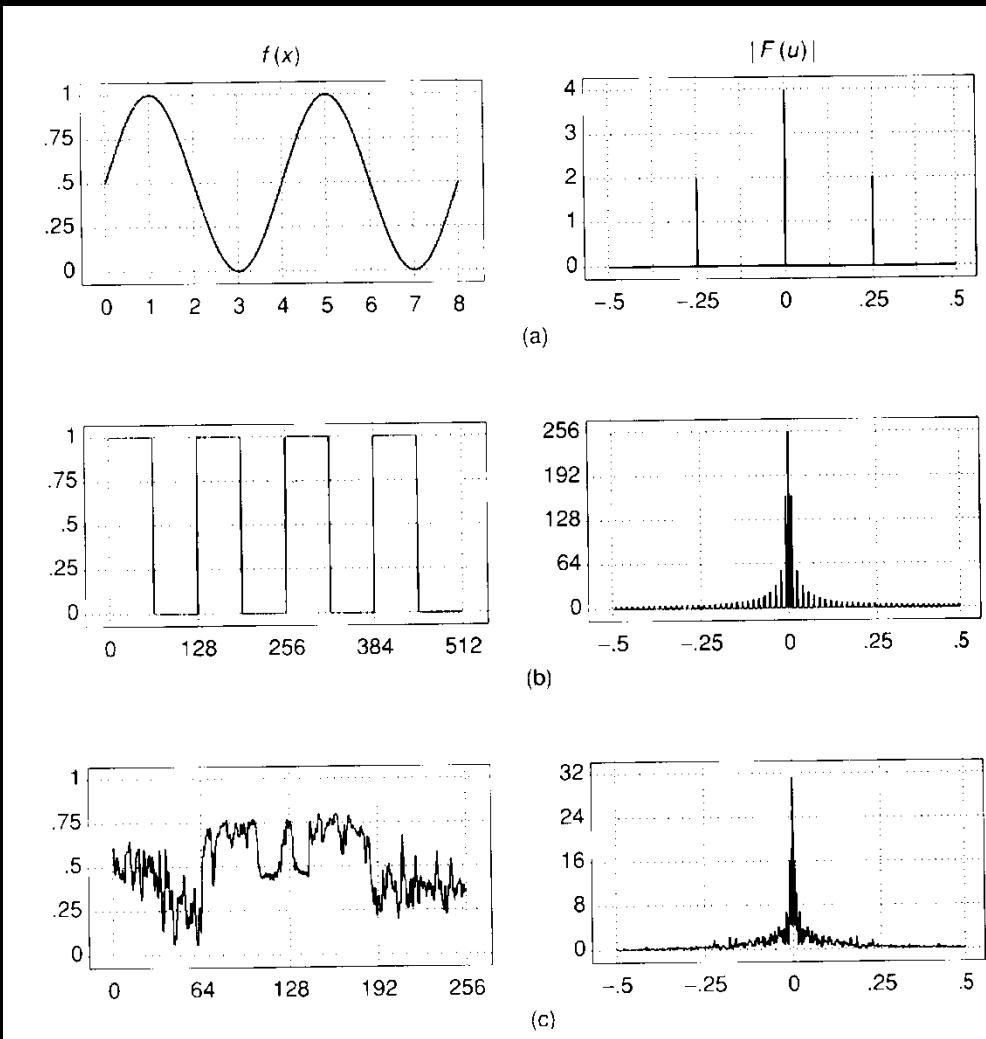
$$= A \sum_{k=1}^{\infty} \frac{1}{k} \sin(2\pi kt)$$



Frequency Spectra



Frequency Spectra



Digital Fourier Transform

- 거의 Fourier transform 과 유사
- 연속적인 신호를 시간에 따라 sampling을 한 형태를 Fourier Transform 시킴.
- 즉, 적분을 시그마로!!

$$F(w) = \int_{-\infty}^{\infty} f(x)e^{-iwx} dx$$

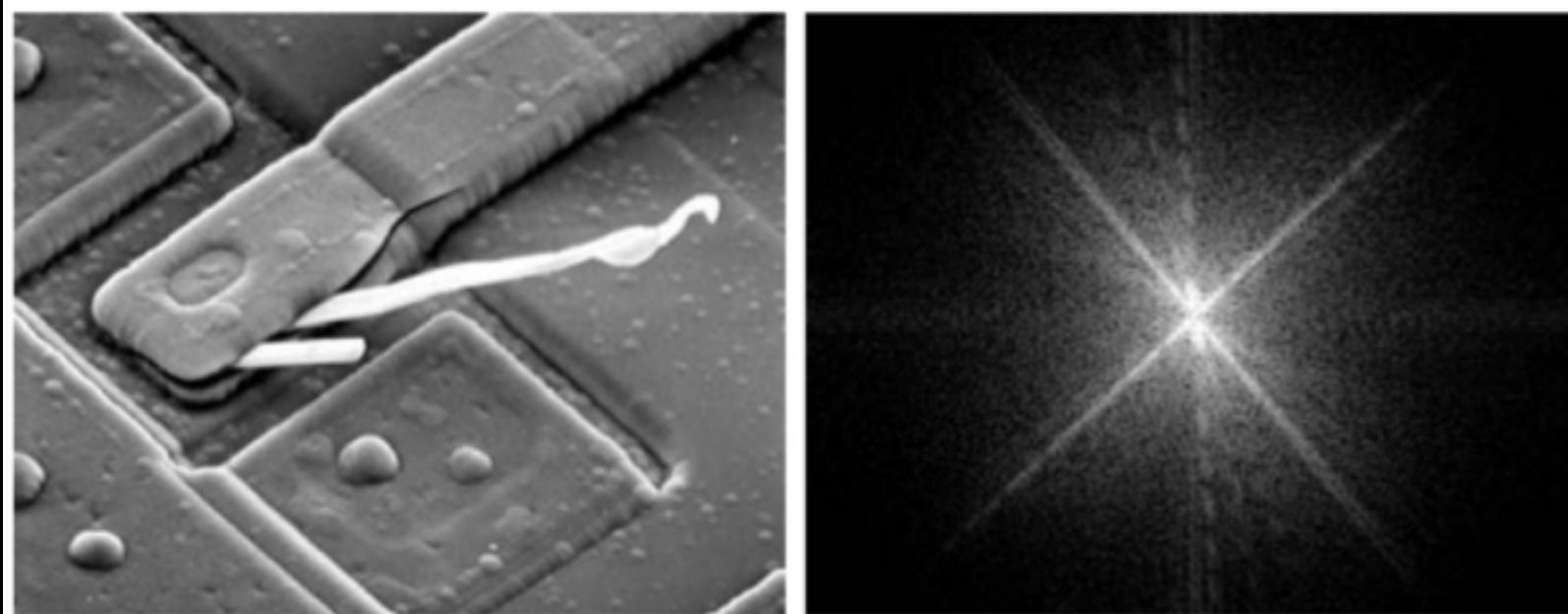
$$F[w] = \sum_{x=0}^{N-1} f[x]e^{-i2\pi x w/N}$$

Fast Fourier Transform

- DFT는 계산시간이 너무 오래 걸리는 단점이 있다
- 신호 전부를 reordering 한 후
- sampling된 신호 전부를 변환시키는 것이 아니라 필요 한 신호만을 골라서! 빠르게 Fourier Transform 시킴

Fourier Transform & Image analysis

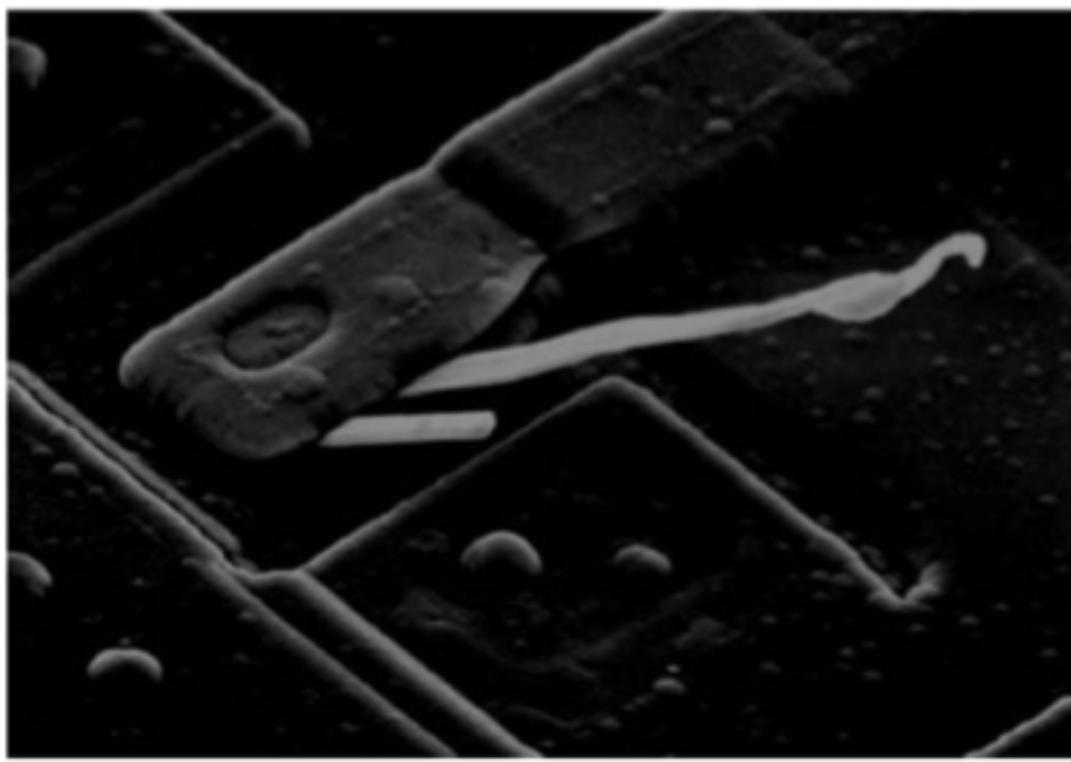
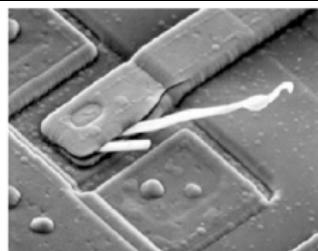
Original image



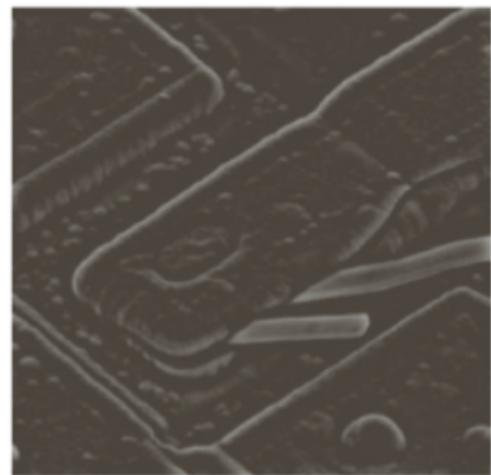
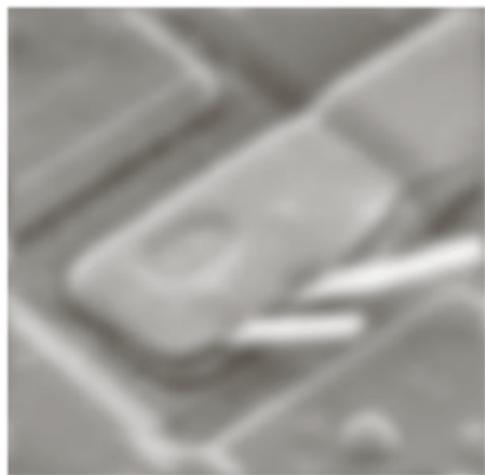
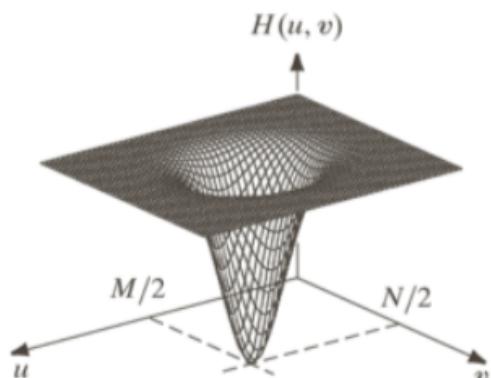
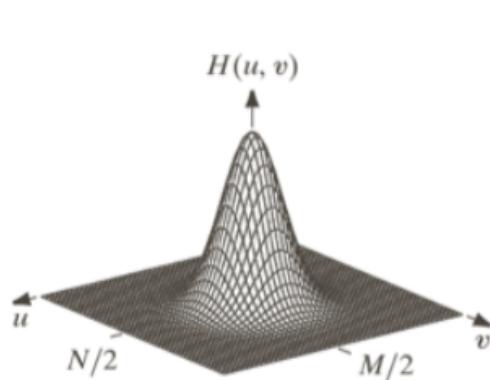
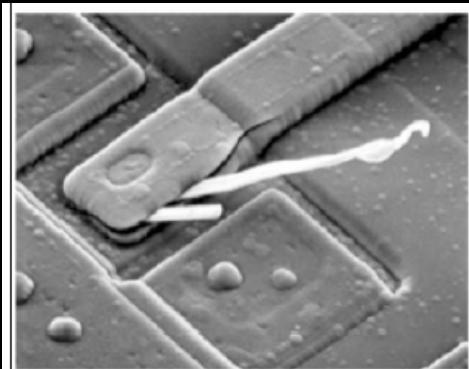
a b

FIGURE 4.29 (a) SEM image of a damaged integrated circuit. (b) Fourier spectrum of (a). (Original image courtesy of Dr. J. M. Hudak, Brockhouse Institute for Materials Research, McMaster University, Hamilton, Ontario, Canada.)

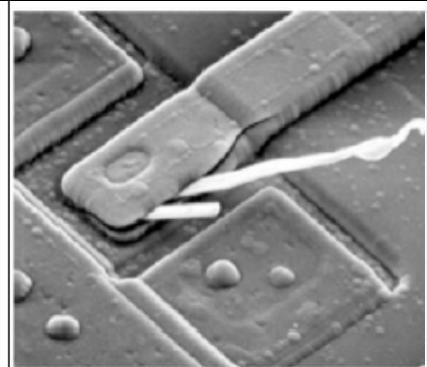
Removal of DC component



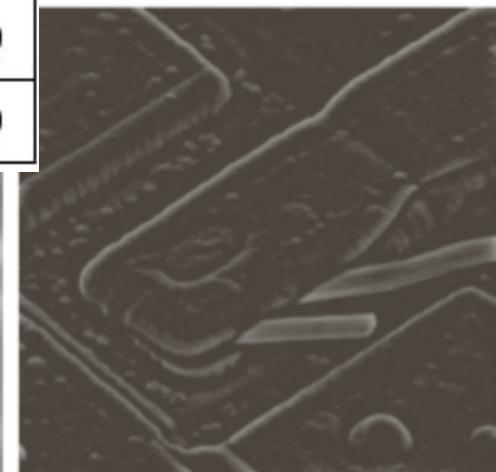
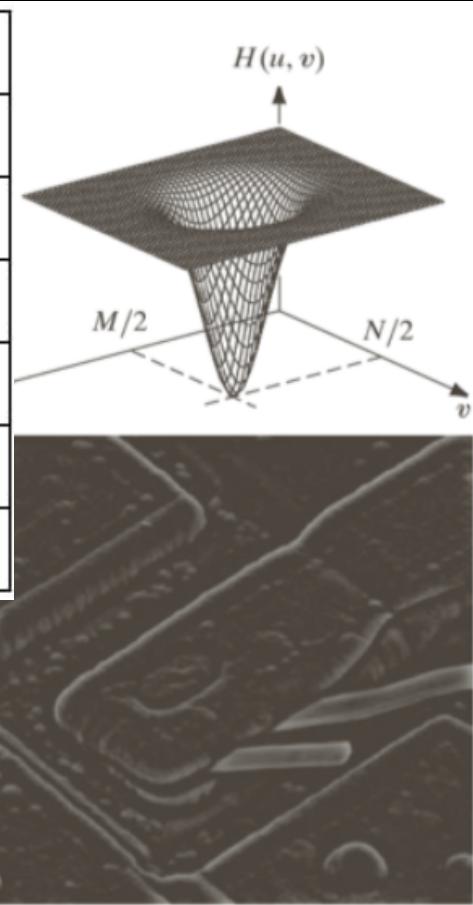
Gaussian kernel



Gaussian kernel



0	0	0	5	0	0	0
0	5	18	32	18	5	0
0	18	64	100	64	18	0
5	32	100	100	100	32	5
0	18	64	100	64	18	0
0	5	18	32	18	5	0
0	0	0	5	0	0	0

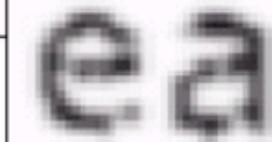


LPF

Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.



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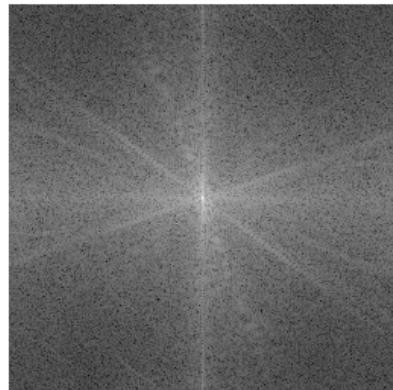


Low-pass Filtering

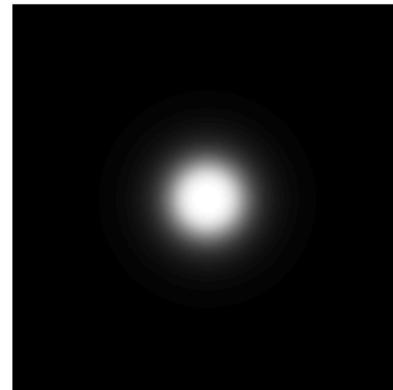
Original image



FFT of original image



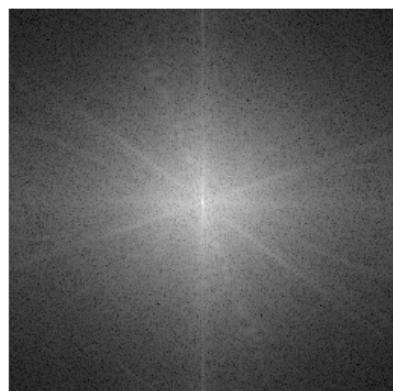
Low-pass filter



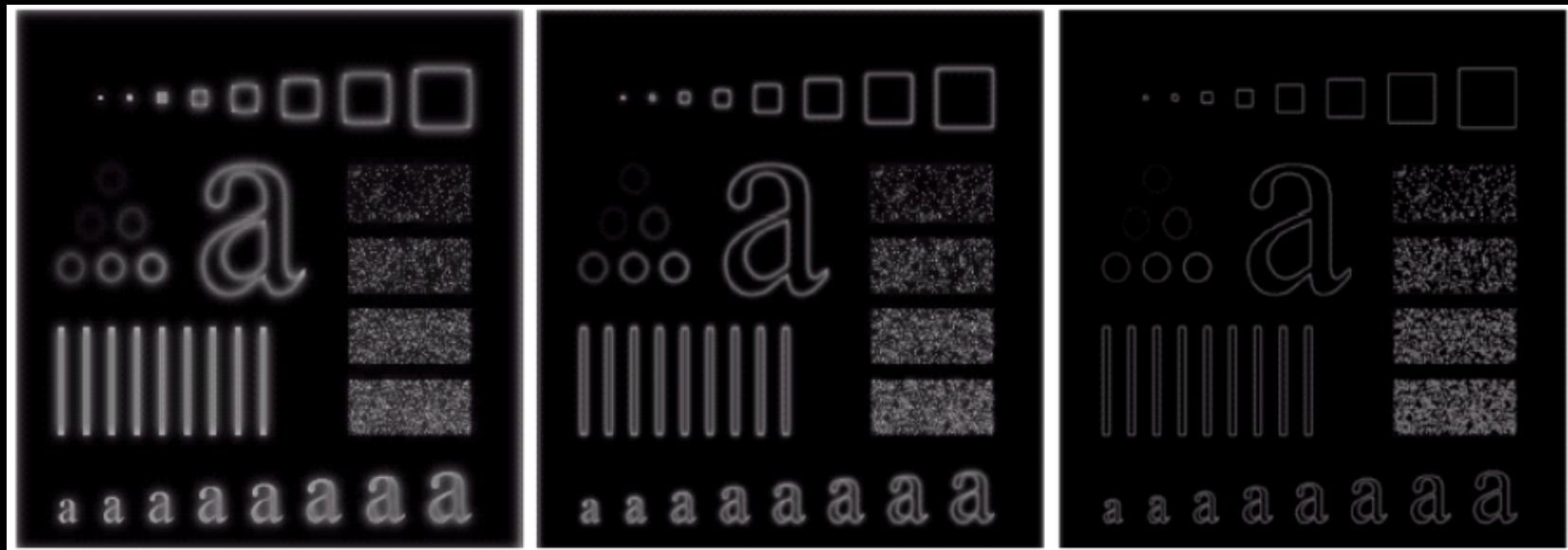
Low-pass image



FFT of low-pass image



HPF

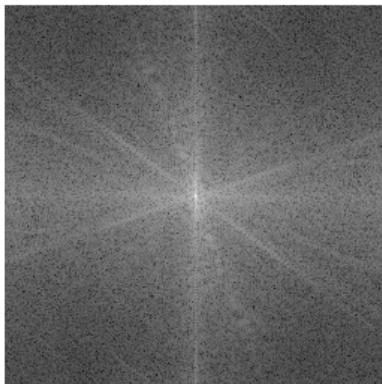


High-pass Filtering

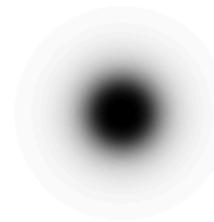
Original image



FFT of original image



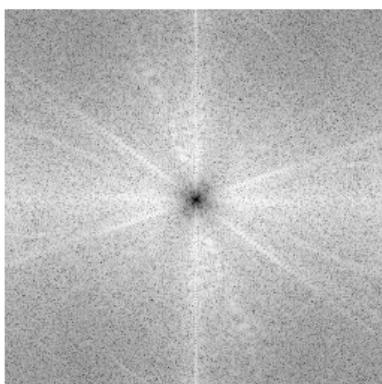
High-pass filter



High-pass image



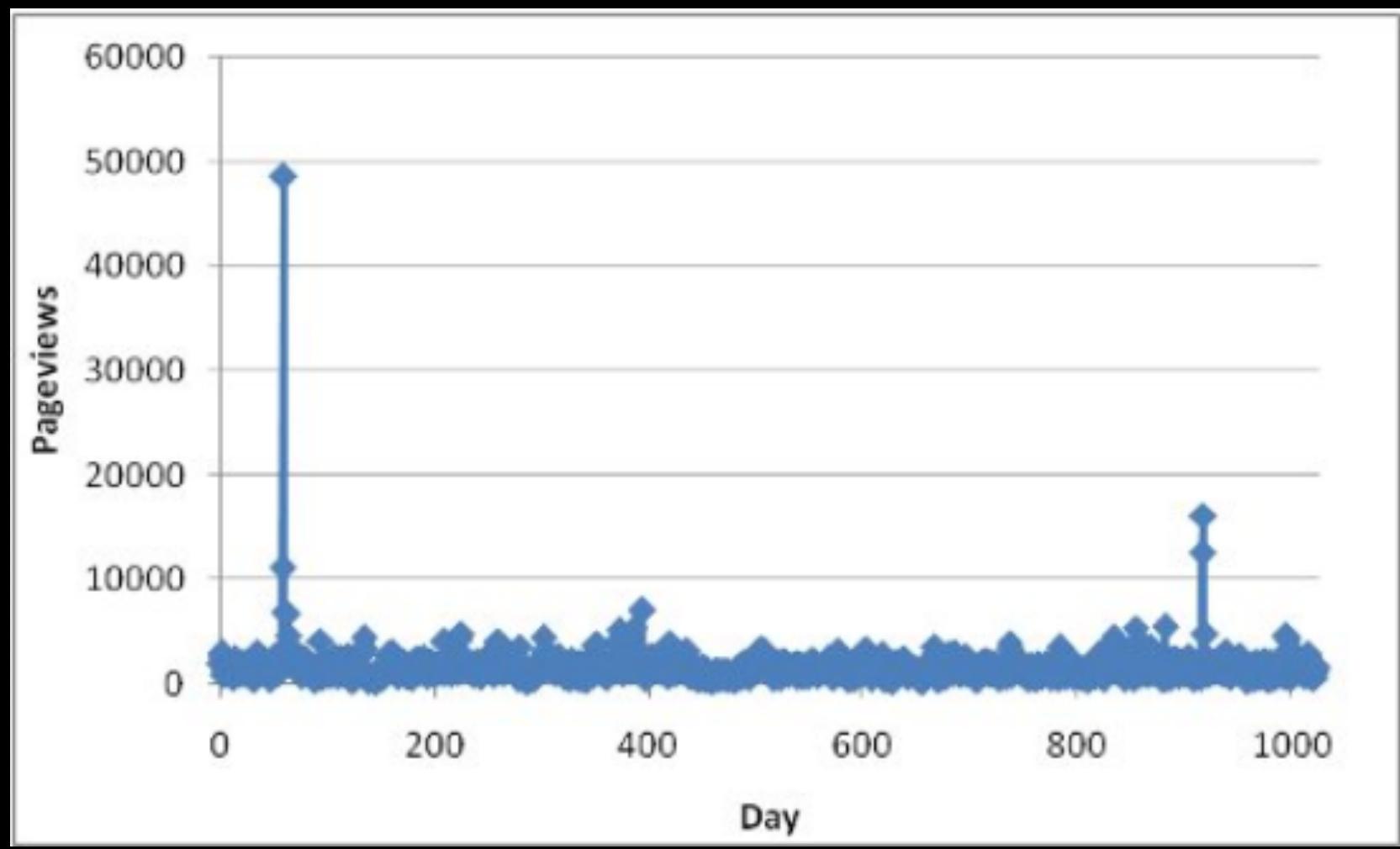
FFT of high-pass image

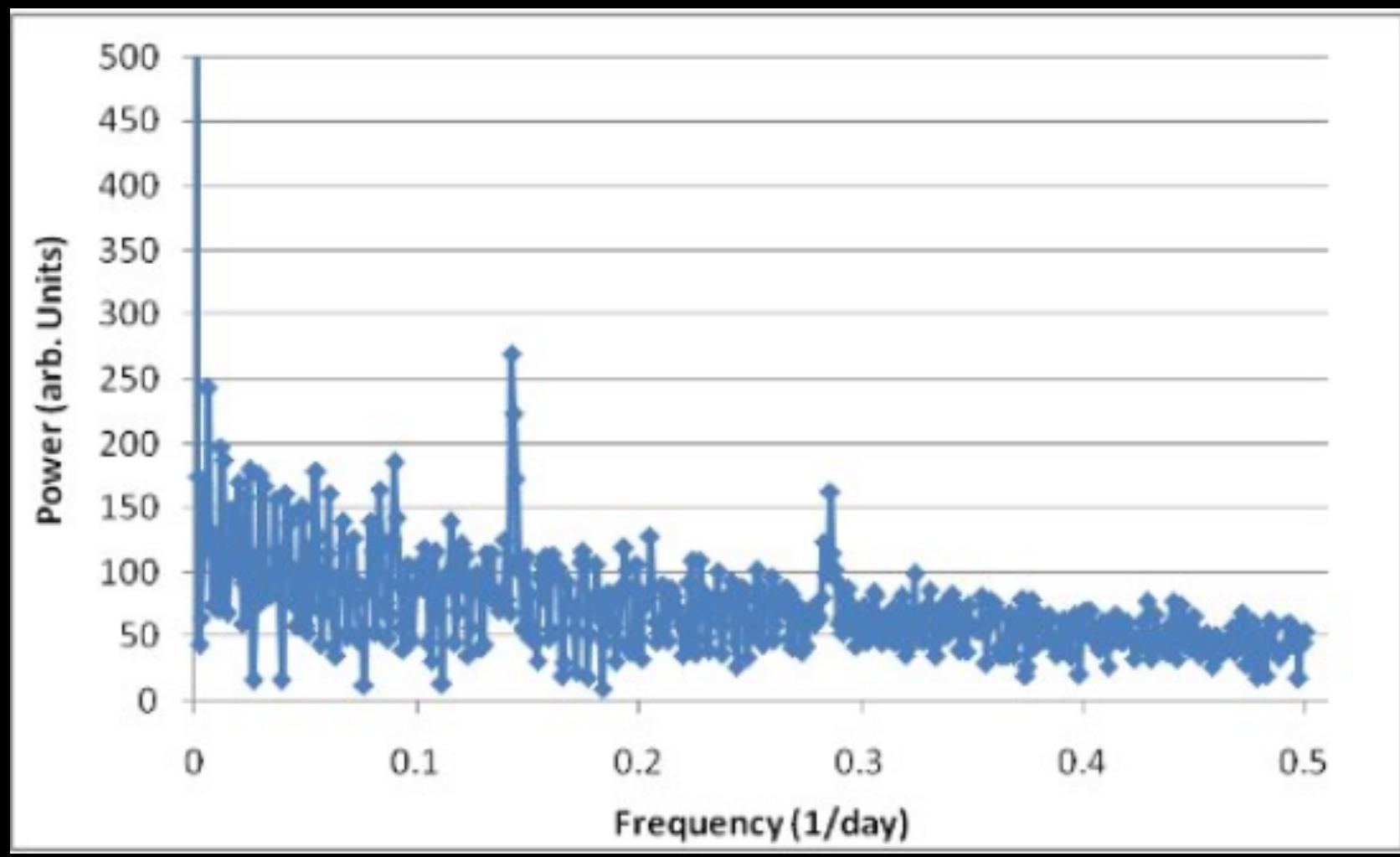


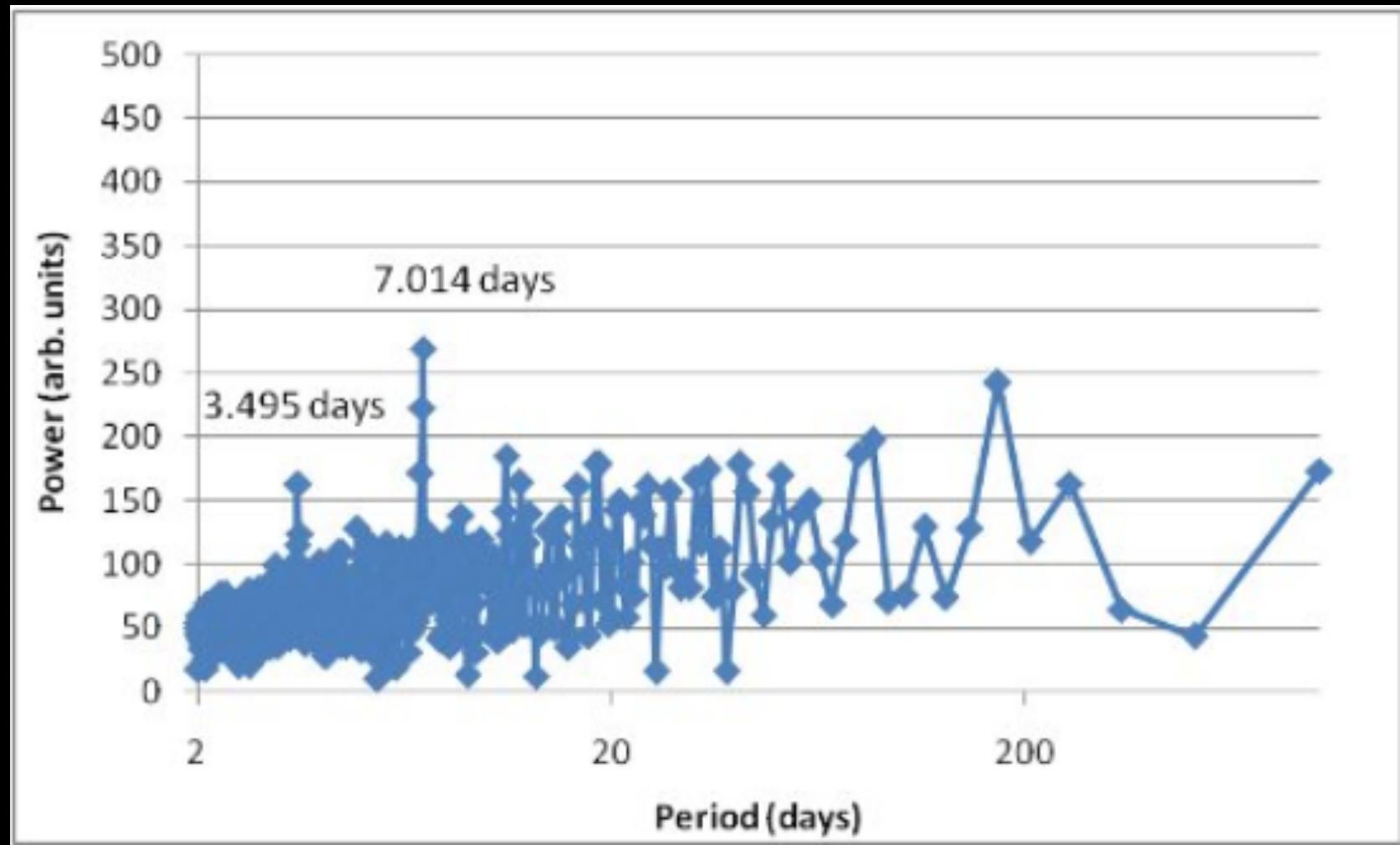
FT for data analysis

FT for data analysis

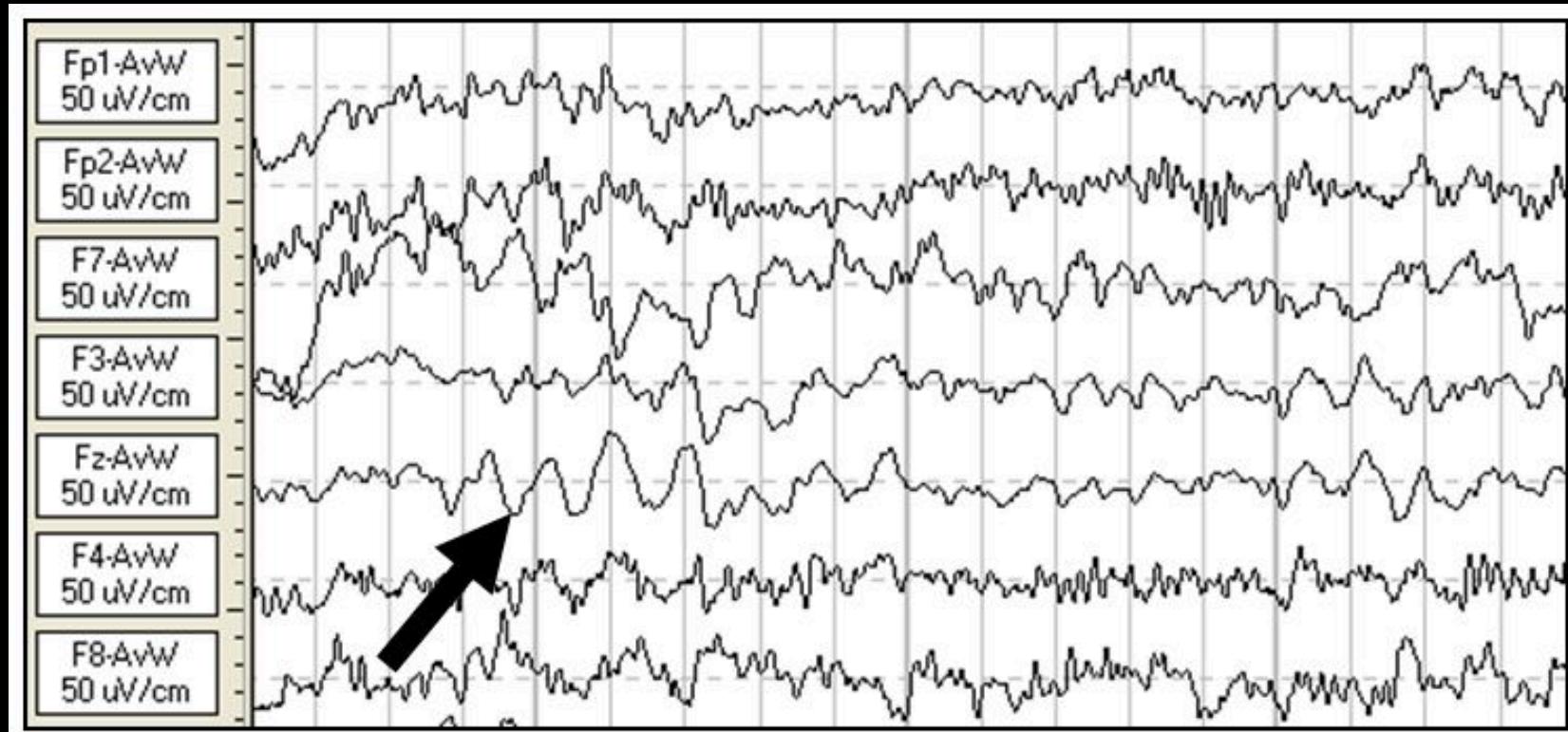
- 어떤 것의 regular한 pattern을 찾는데 있어서 fourier transform을 통해서 할 수 있다.







Raw EEG



Relative amplitude



0

10

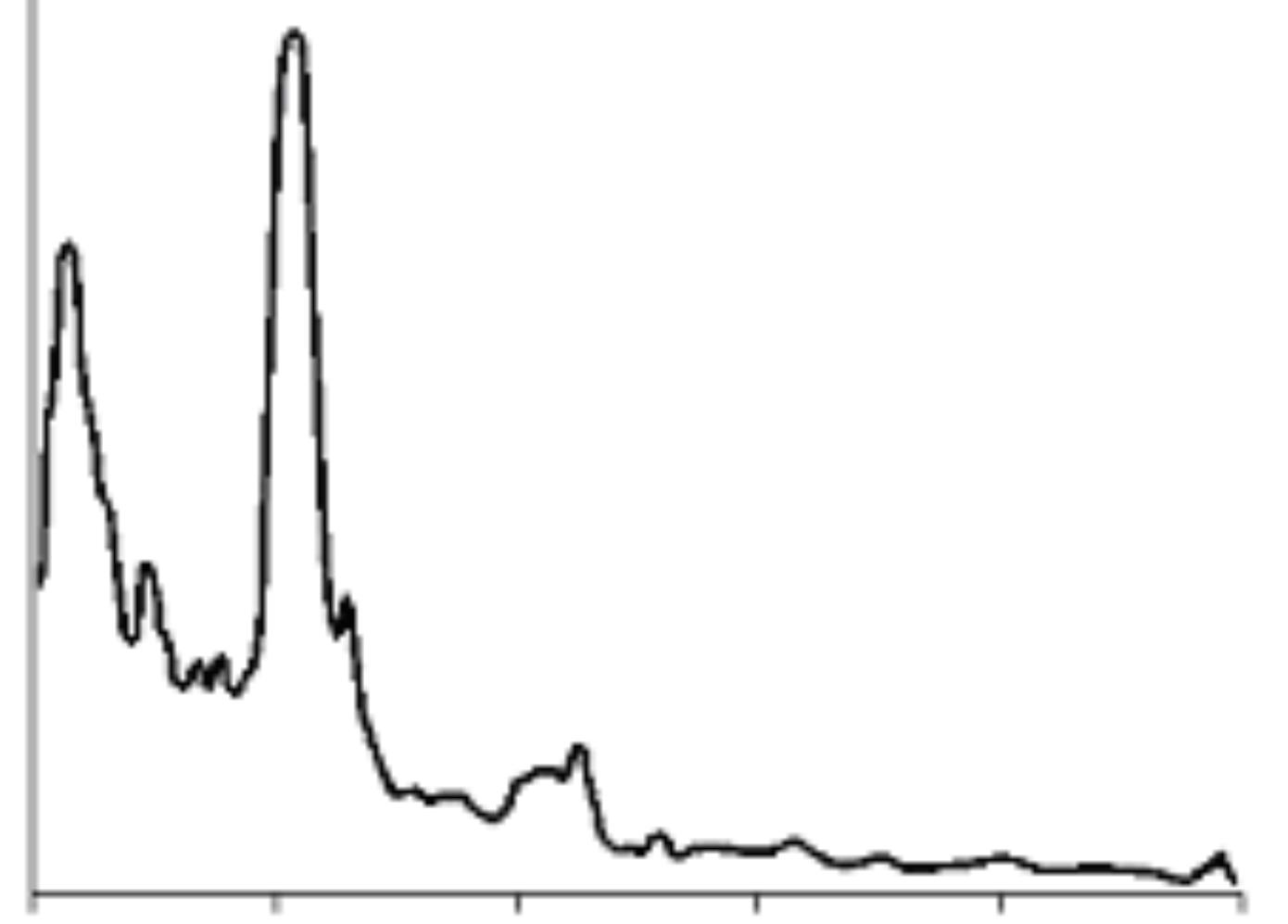
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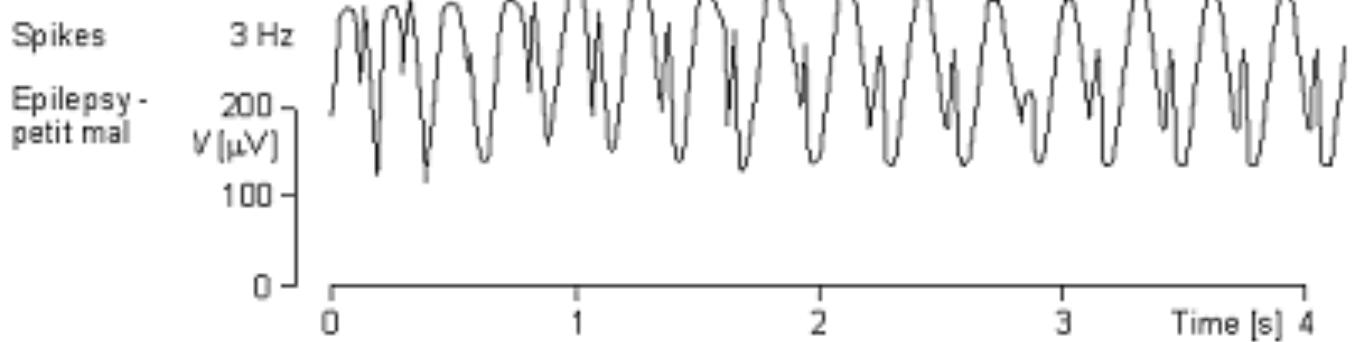
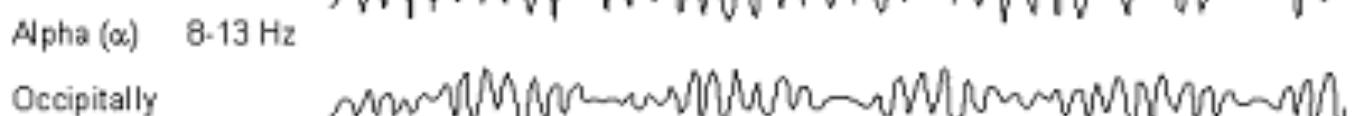
30

40

50

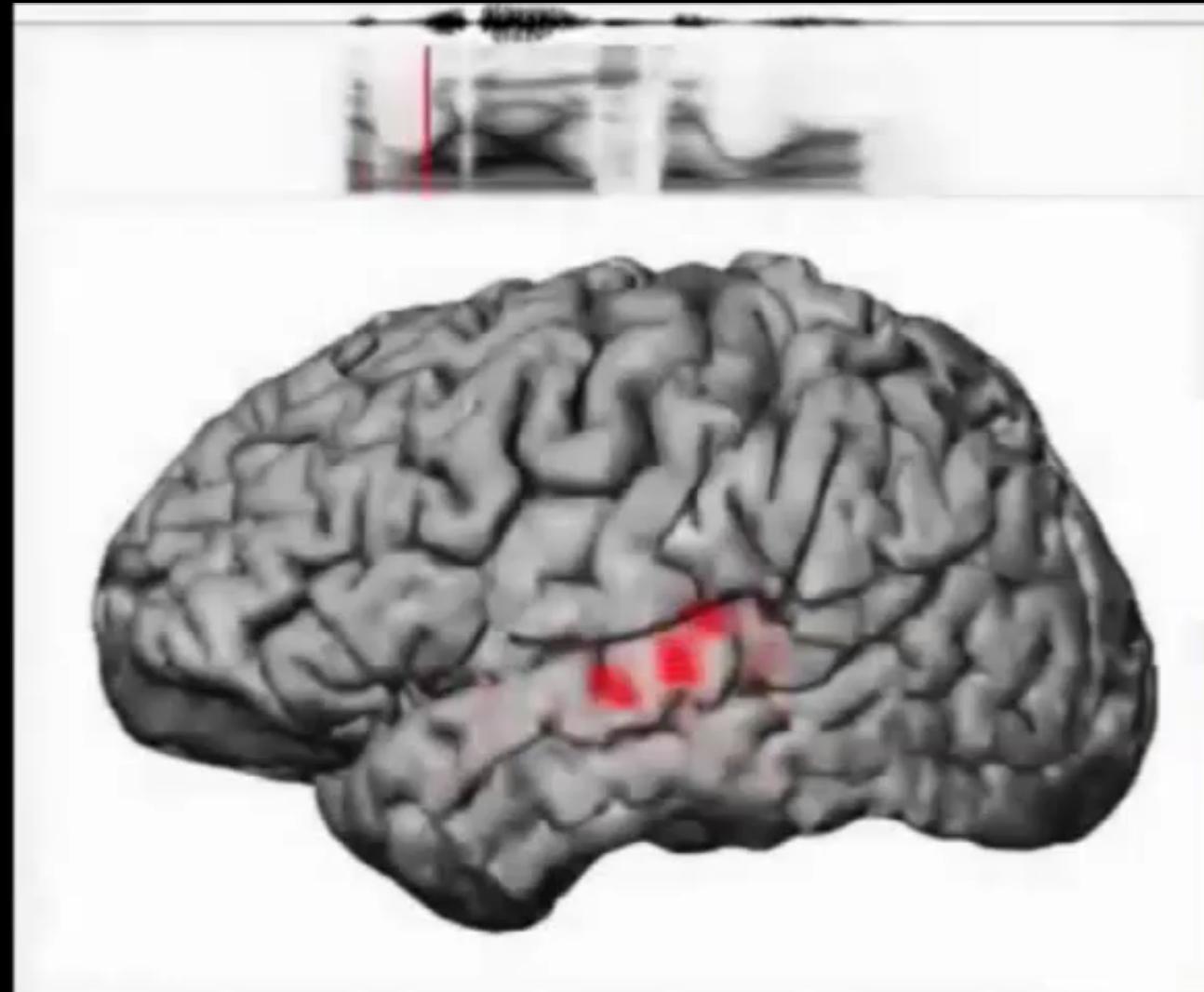
Frequency[Hz]





fMRI

우리가 어떤 것을 들을때



Resting State Functional Connectivity

Resting State

‘멍때리기 대회’ 우승자, 9세 초등학생…우승 상품 보니 ‘폭소’

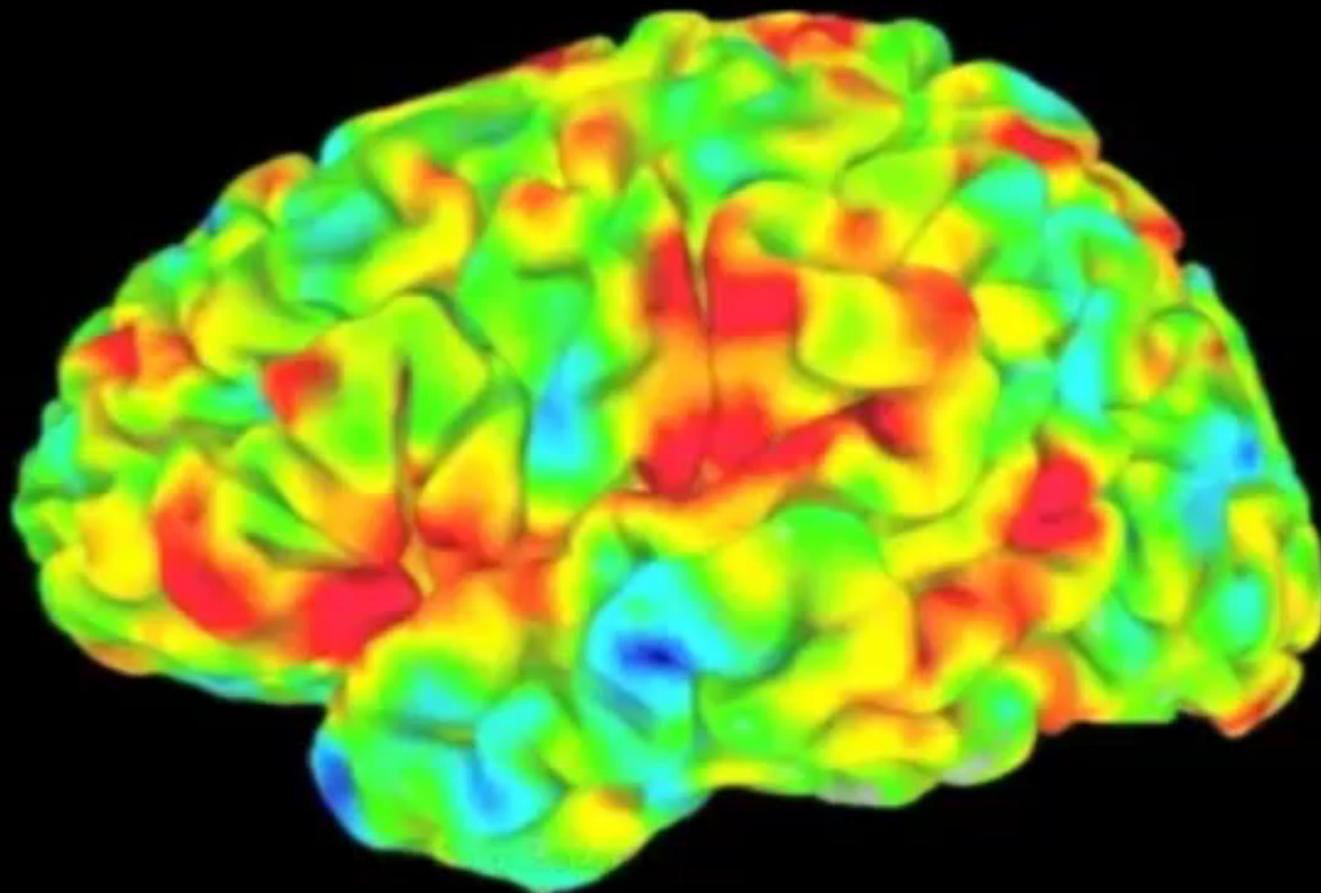
기사입력 2014-10-28 10:00

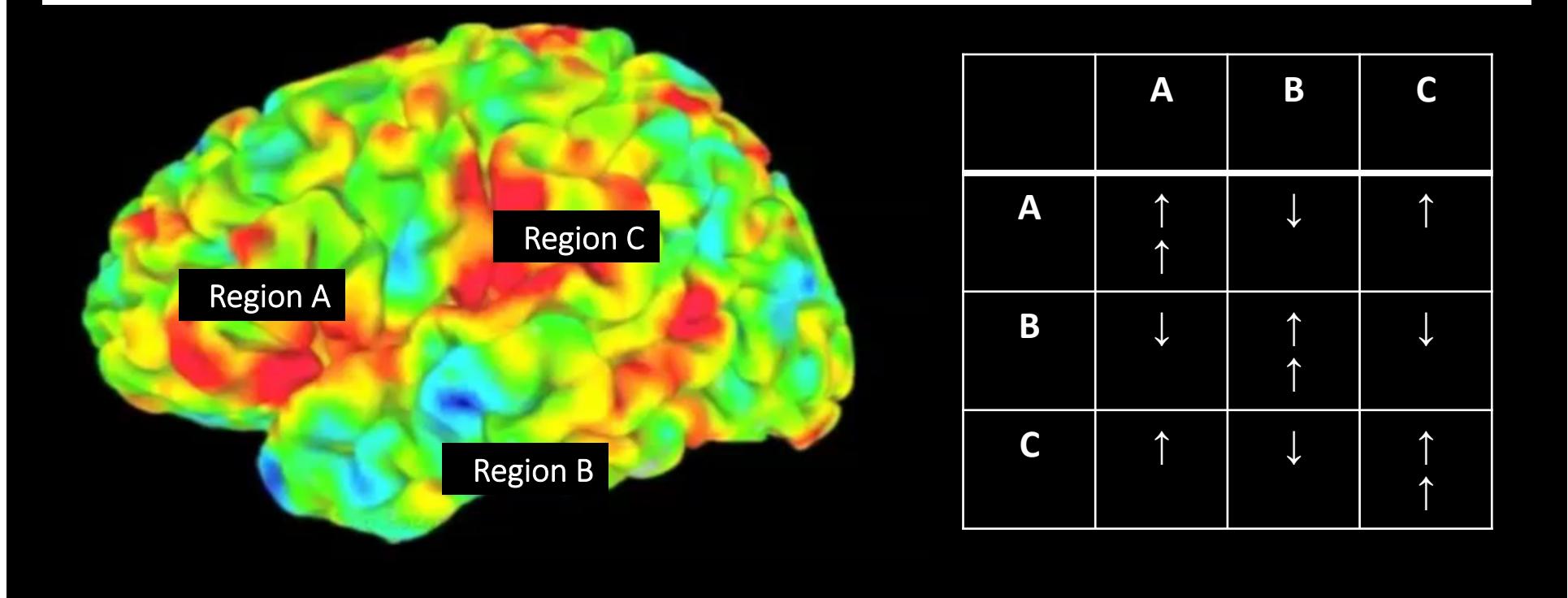
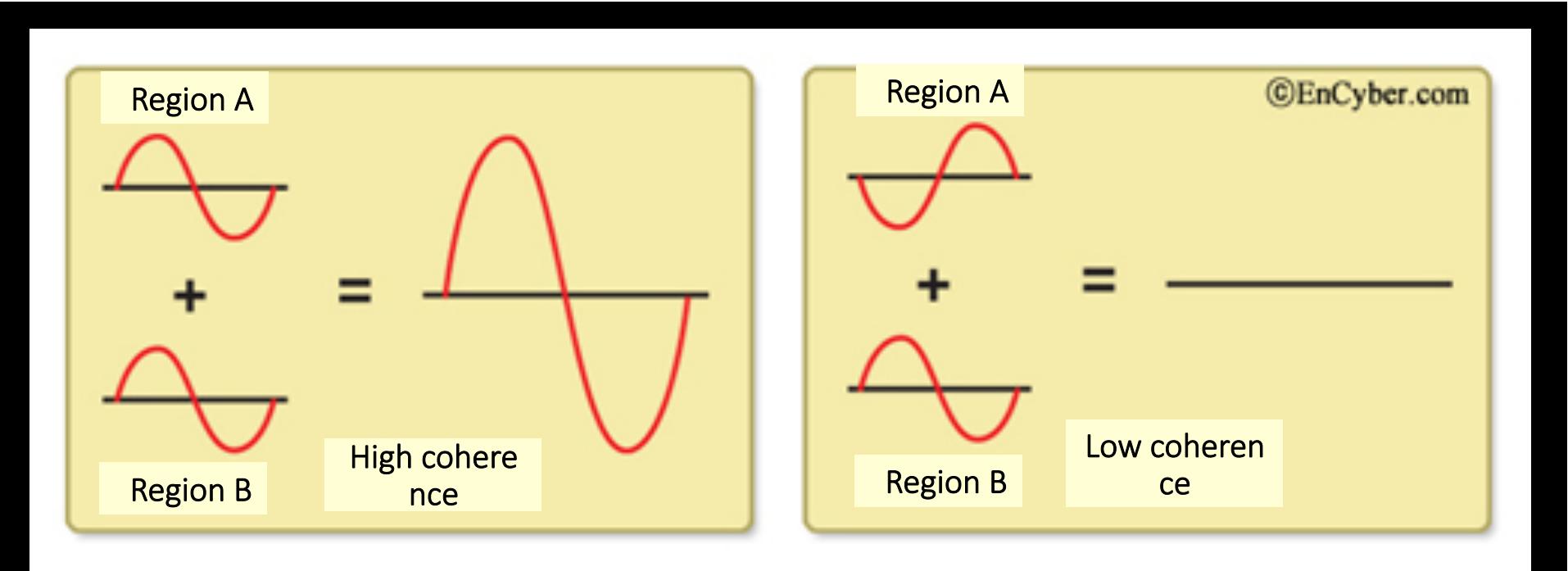
기사원문

1>



멍 때리고 있을때





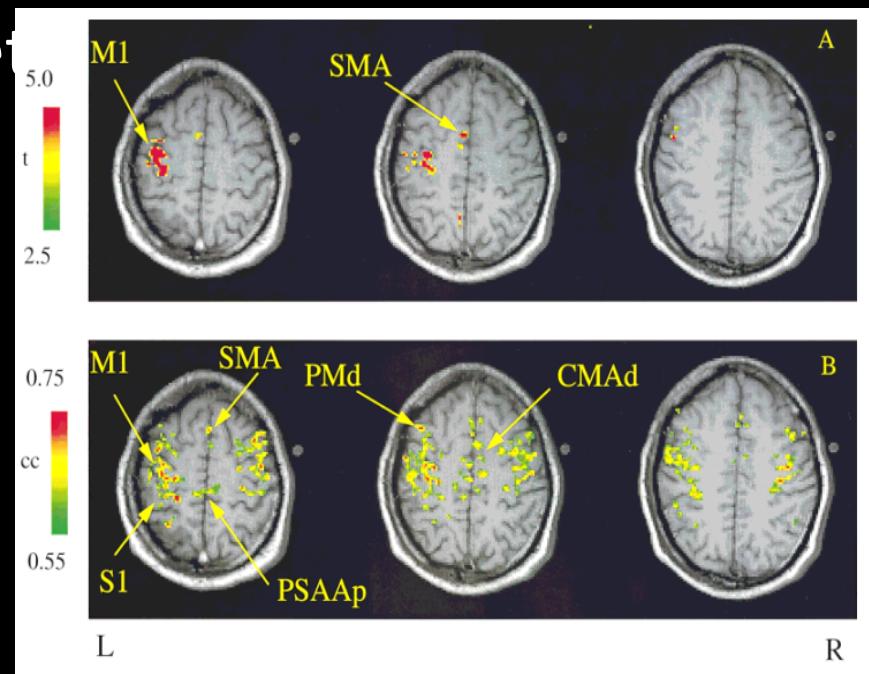
Resting State FC

- Definition
 - temporal correlation of a neurophysiological index measured in different brain areas' while subject is at resting state
 - Patients are too impaired to perform a task correctly in the scanner environment
- ⇒ free from individual's motivation (Fornito and Bullmore, 2010)



Resting State FC

- Advantages
 - task-independent function al network analysis can ide ntify submerged neural net works
 - comprehensive properties of brain physiology
- (Xiong et al., 1999)



Fourier Transform & fMRI analysis

Research Article

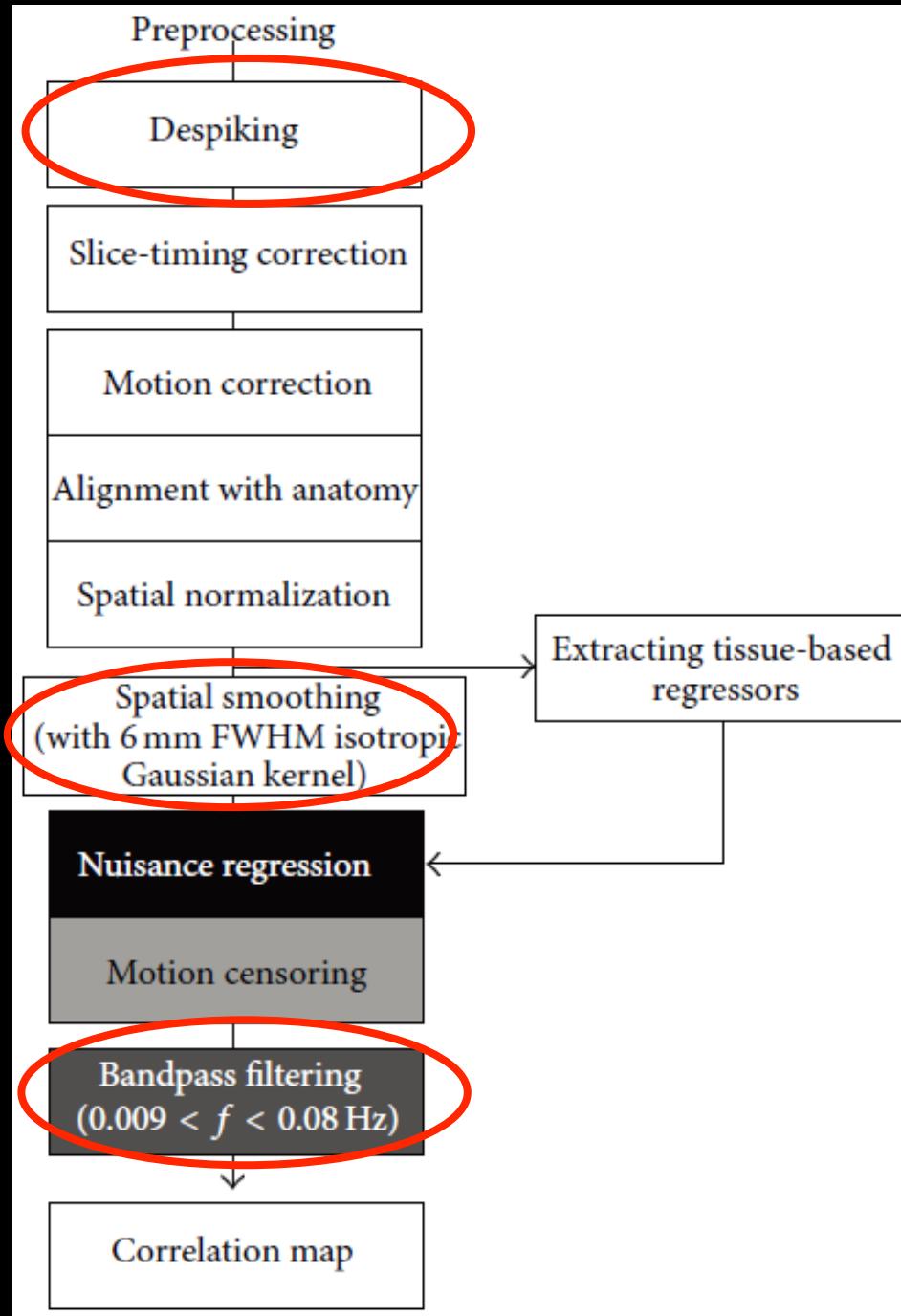
Effective Preprocessing Procedures Virtually Eliminate Distance-Dependent Motion Artifacts in Resting State fMRI

Hang Joon Jo,¹ Stephen J. Gotts,² Richard C. Reynolds,³ Peter A. Bandettini,¹ Alex Martin,² Robert W. Cox,³ and Ziad S. Saad³

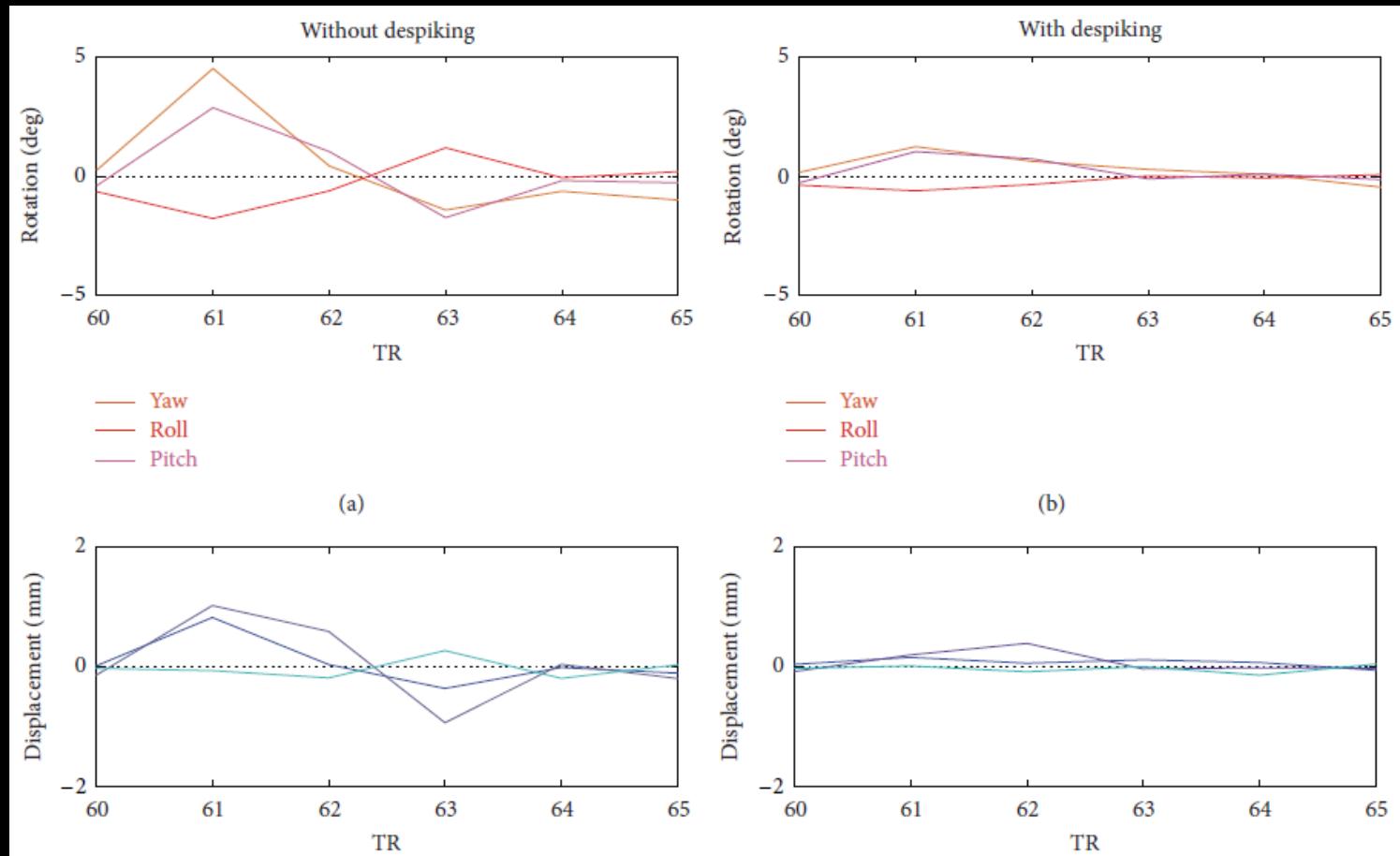
¹ *Section on Functional Imaging Methods, Laboratory of Brain and Cognition, National Institute of Mental Health, National Institutes of Health, Bethesda, MD 20892-1148, USA*

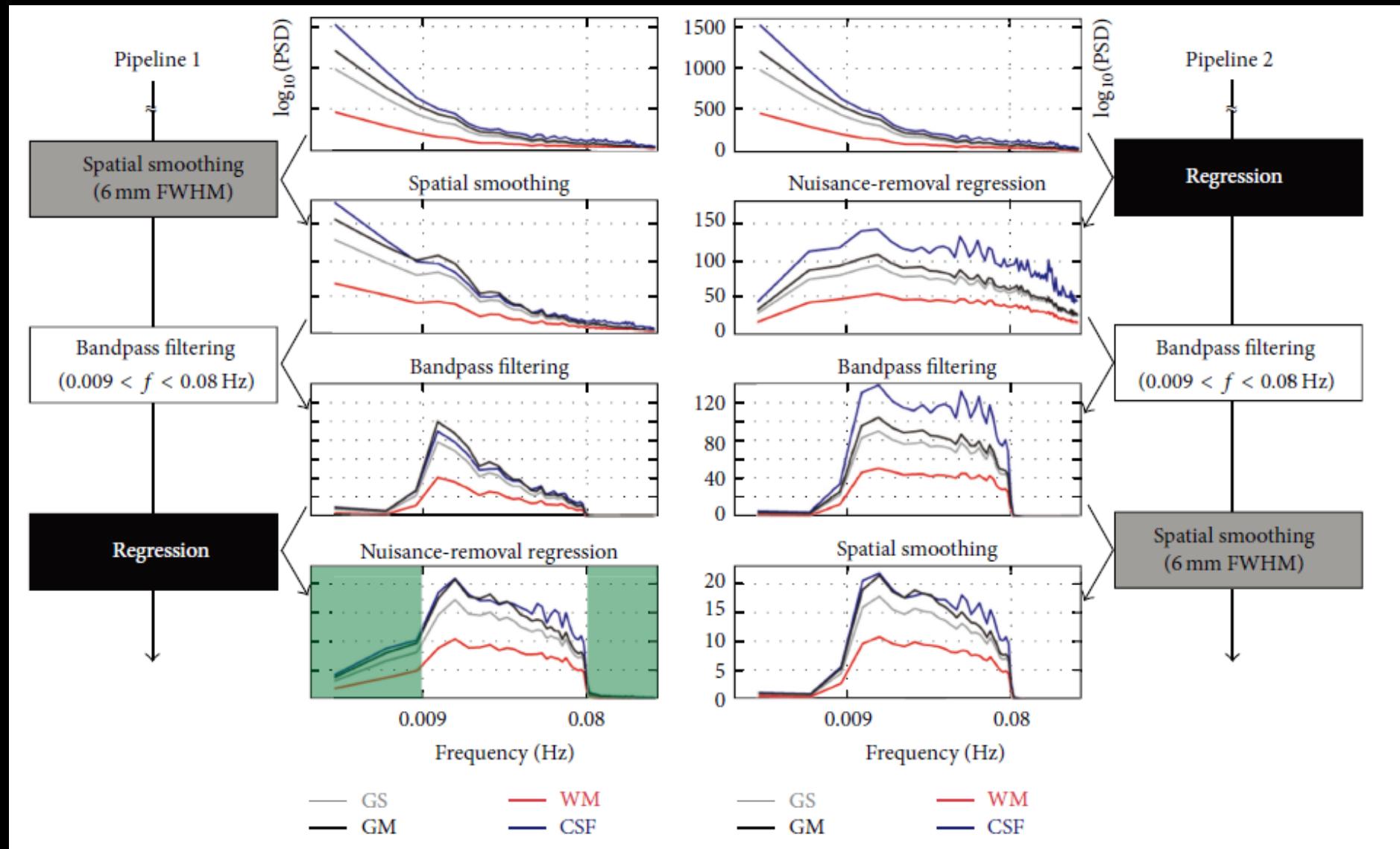
² *Section on Cognitive Neuropsychology, Laboratory of Brain and Cognition, National Institute of Mental Health, National Institutes of Health, Bethesda, MD 20892-1366, USA*

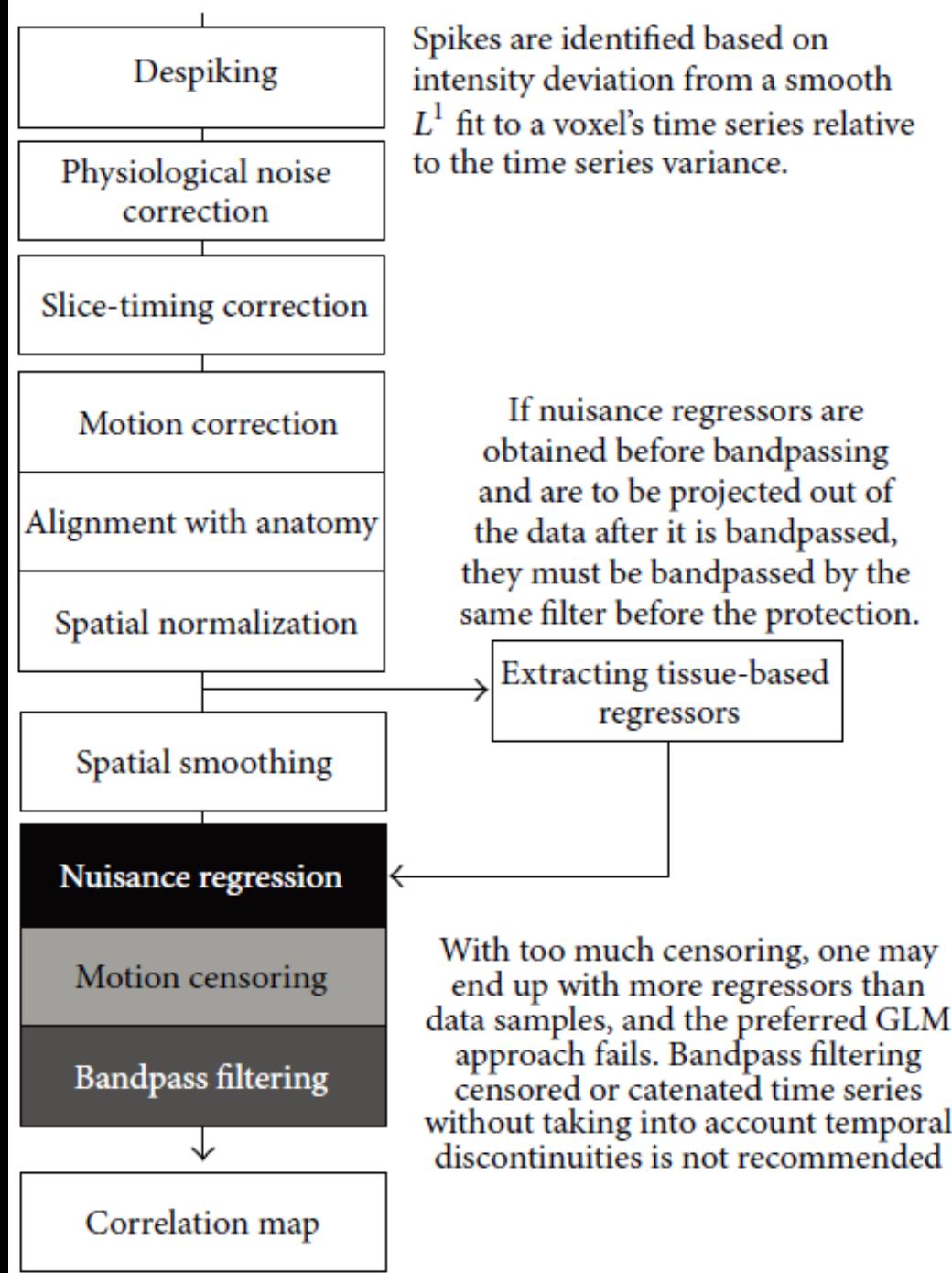
³ *Scientific and Statistical Computing Core, National Institute of Mental Health, National Institutes of Health, Bethesda, MD 20892-1148, USA*



Despiking







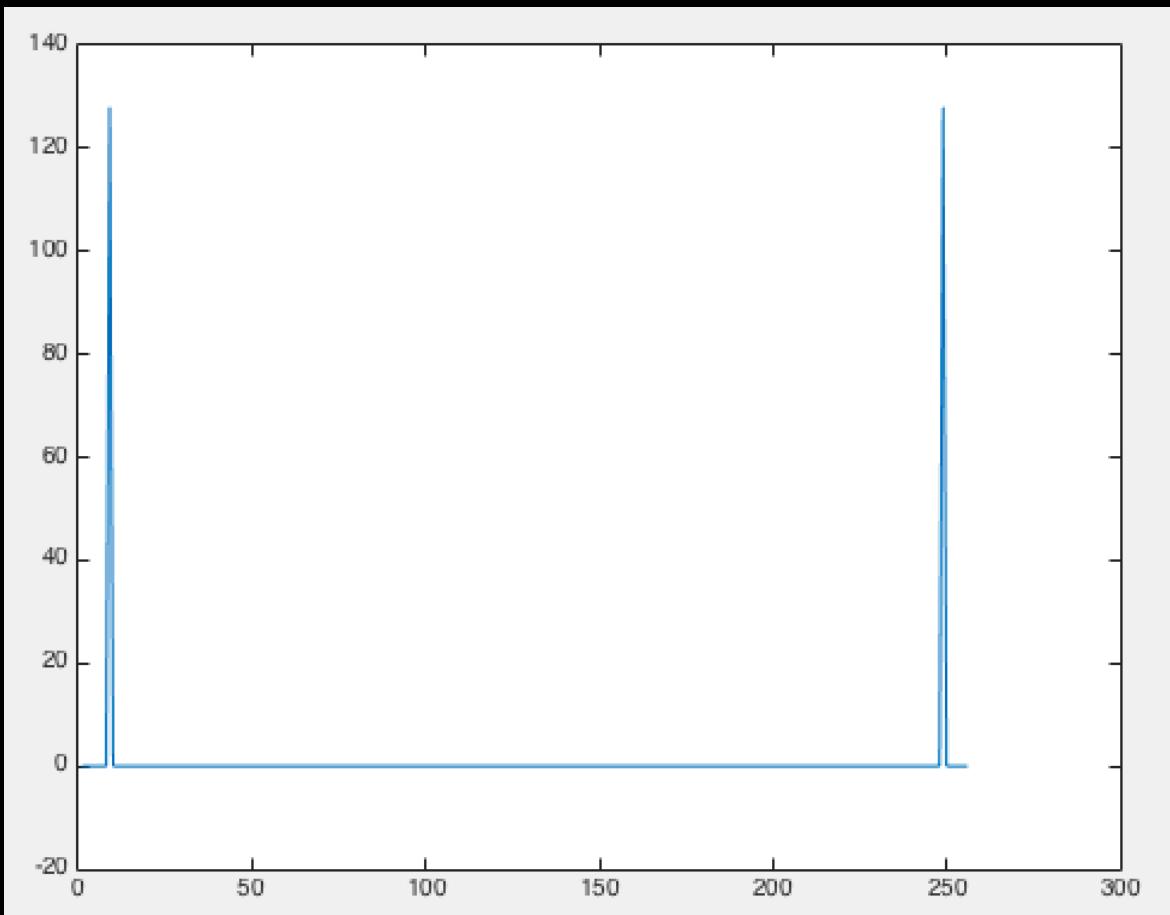
실습 #1 – fftpack 사용

$$F[0] = \sum_x f[x].$$

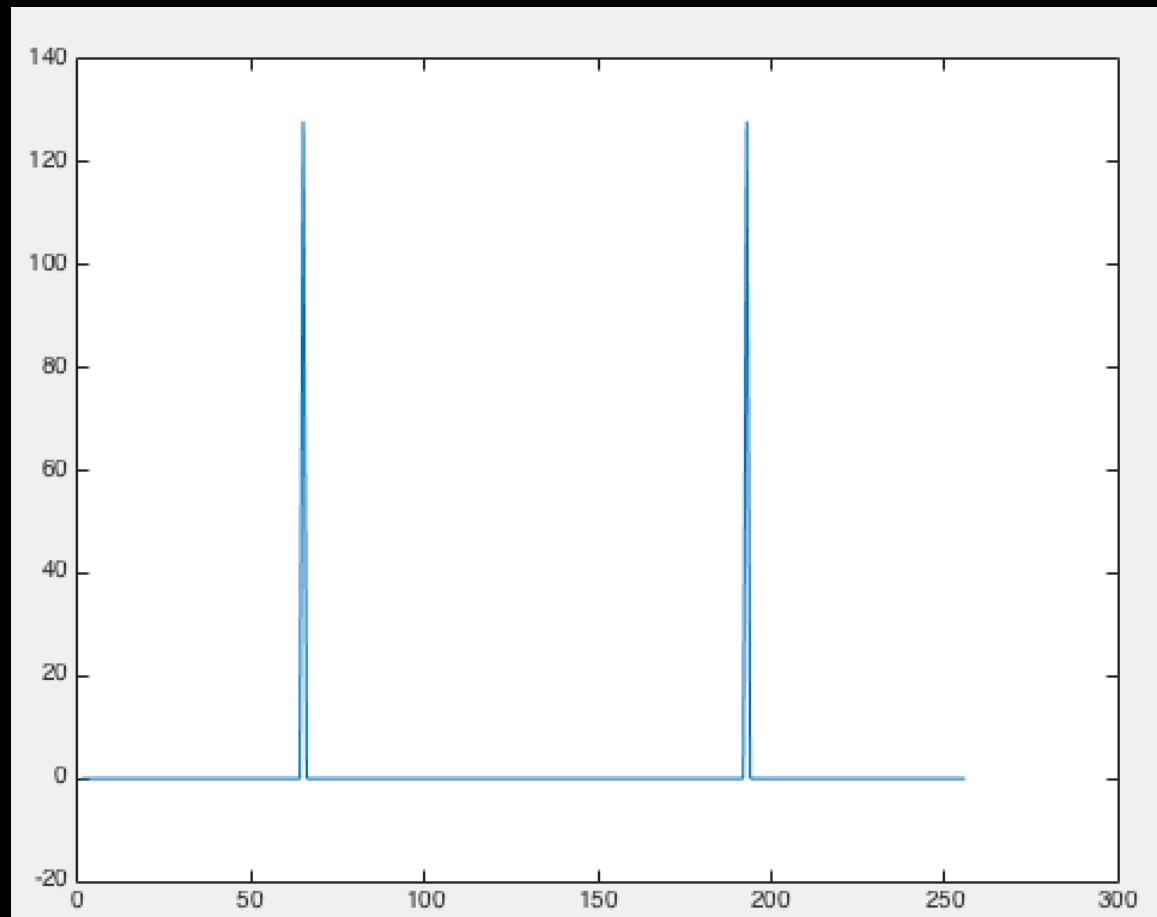
$$f[x] = F^{-1}\{F\{f[x]\}\}$$

$$\sum |f[x]|^2 = \frac{1}{N} \sum |F[w]|^2$$

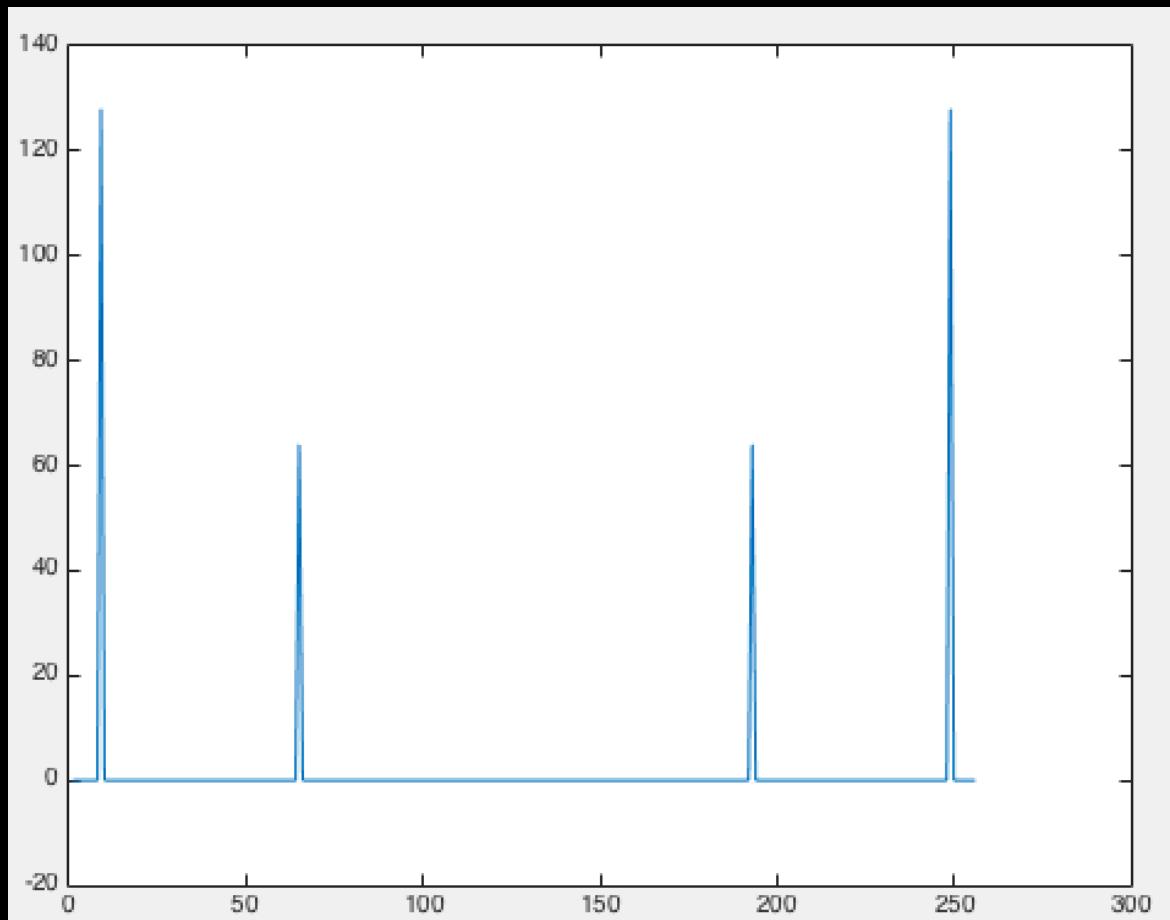
실습 #2 – fig16_7a



실습 #2 – fig16_7b



실습 #2 – fig16_7c



감사합니다