Morphological Image Processing: Basic Algorithms

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Preliminaries

When dealing with binary images, one of the principal applications of morphology is extracting image components that are useful in the representation and description of shape.

We consider morphological algorithms for extracting boundaries, connected components, the convex hull, and the skeleton of a region. We also develop methods (region filling, thinning, thickening, and pruning) that are frequently used in conjunction with these algorithms as pre- or post-processing steps.

Boundary Extraction

The ***boundary*** of a set A, denoted as $\beta(A)$, can be obtained by first eroding A by B and then performing the set difference between A and its erosion as follows:

$$\beta(A) = A - (A \ominus B)$$

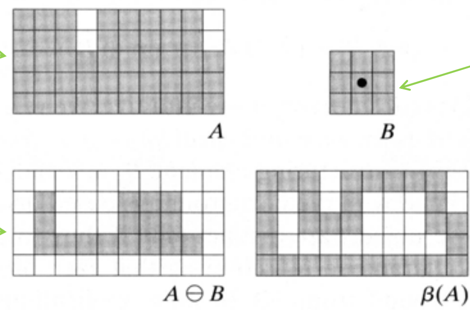
where B is a suitable structuring element.

Set A

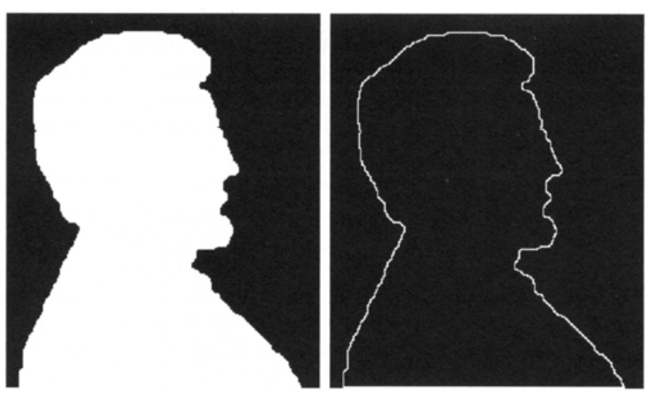
Structuring element B

Erosion of A by B

Boundary: set difference between A and its erosion



Boundary Extraction



A binary image

Boundary extracted using a 3x3 structuring element of ones

Size of structuring element defines the boundary being 1 pixel thick.

Hole Filling

A ***hole*** may be defined as a background region surrounded by a connected border of foreground pixels.

Let denote by A a set whose elements are 8-connected boundaries, each boundary enclosing a background region (a hole). Given a point in each hole, the objective is to fill all the holes with ones (for binary images).

We start from forming an array X_0 of zeros (the same size as the array containing A), except at the locations in X_0 corresponding to the given point in each hole, which is set to one. Then, the following procedure fills all the holes with ones:

$$X_k = \left(X_{k-1} \oplus B \right) \cap A^c \qquad k = 1, 2, 3, \ldots$$

where B is the symmetric structuring element.

Hole Filling

The algorithm terminates at the iteration step k if $X_k = X_{k-1}$. The set X_k then contains all the filled holes; the union of X_k and A contains all the filled holes and their boundaries.

The dilation would fill the entire area if left unchecked. However, the intersection at each step with the complement of A limits the result to inside the region of interest. This is an example of how a morphological process can be ***conditioned*** to meet a desired property. In the current application, it can be called a ***conditional dilation***.

Hole Filling

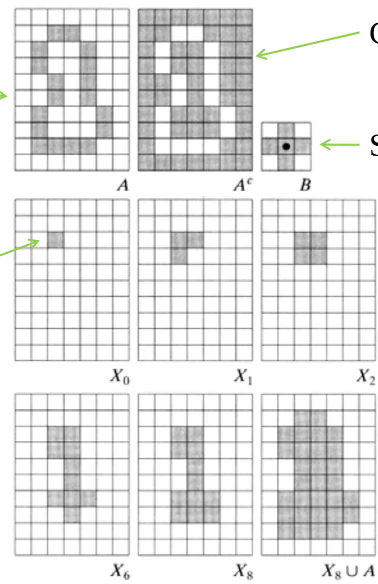
Set A

Complement of A

Structuring element

Set X_0; a grey dot indicates location of a hole

A set with a filled hole



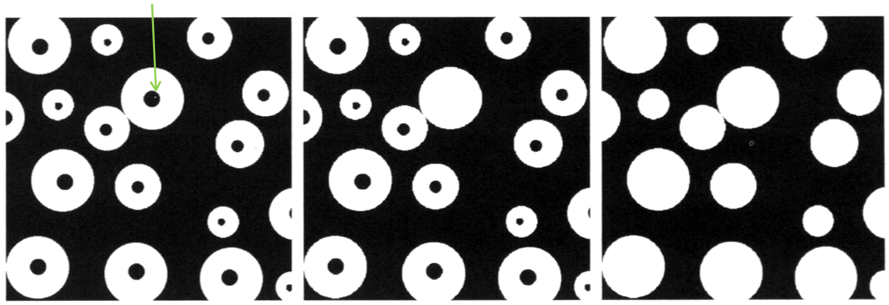
Hole Filling

An image that could result from thresholding to 2 levels a scene containing polished spheres (ball bearings). Dark spots could be results of reflections. The objective is to eliminate reflections by hole filling…

A (white) point selected inside one sphere

Result of filling <u>that</u> component

Result of filling all the spheres



Extraction of connected components

We discussed concepts of connectivity and connected components earlier. Extraction of connected components from a binary image is important for many automated image analysis applications.

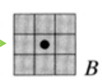
Let *A* be a set containing one or more connected components. We form an array X_0 (of the same size as the array containing *A*), whose elements are zeros (background values), except at each location known to correspond to a point in each connected component in *A*, which we set to one (foreground value). The objective is to start with X_0 and find all the connected components by the following iterative procedure:

$$X_k = \left(X_{k-1} \oplus B\right) \cap A \qquad k = 1, 2, 3, \ldots$$

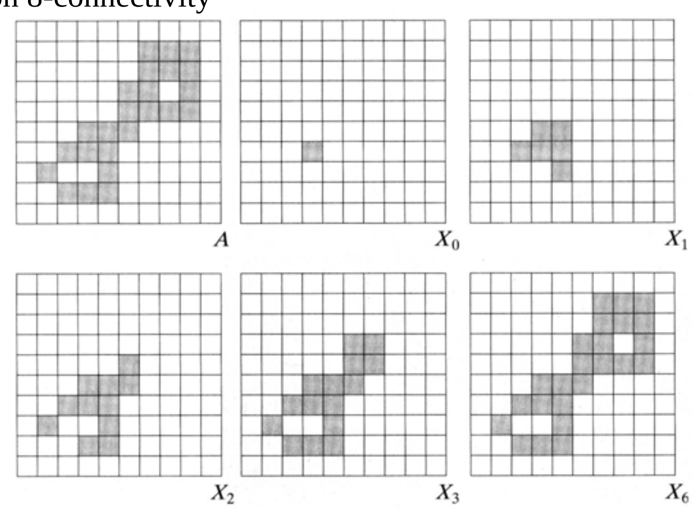
where *B* is a suitable structuring element. The procedure terminates when $X_k = X_{k-1}$ with X_k containing all the connected components of *A*.

Extraction of connected components

Structuring element based on 8-connectivity



Set *A*



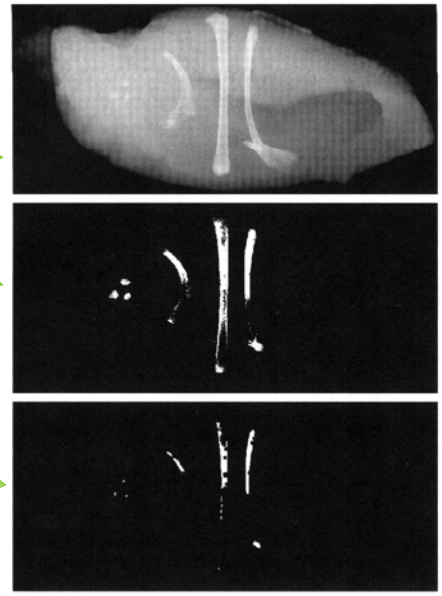
Extraction of connected components

Connected components are often used for automated inspection.

X-ray image of chicken filet with bone fragments

Thresholded image

Image eroded with a 5x5 structuring element



Extraction of connected components

It is of interest to be able to detect foreign fragments in processed food before packaging or shipping.

In this case, the density of the bones is such that their intensity values are different from background. After thresholding, we observe that the points that remain are clustered into objects (bones). Therefore, we can make sure that only objects of "significant" size remain by eroding the thresholded image. for erosion, a 5x5 structuring element was selected.

Next, we analyze the size of objects that remain. We identify them by extracting the connected components in the image. As a result, 15 connected components were found with 3 of them being dominant in size (133, 674, and 743 pixels). This is good enough to determine that significant undesirable objects are contained in the image.

Convex Hull

A set A is said to be ***convex*** if the straight line segment joining any two points in A lies entirely within A. The ***convex hull H*** of an arbitrary set S is the smallest convex set containing S.

The difference $H - S$ is called the ***convex deficiency*** of S. The convex hull and convex deficiency are useful for object description.

Let B^i, $i=1,2,3,4$ represent the 4 structuring elements. The procedure consists of implementing the equation:

$$X_k^i = \left(X_{k-1} \otimes B^i \right) \cup A \qquad i = 1,2,3,4 \quad k = 1,2,3,\ldots$$

with $X_0^i = A$

When the procedure converges ($X^i_k = X^i_{k-1}$), we let $D^i = X^i_k$. The convex hull of A is then

$$C(A) = \bigcup_{i=1}^{4} D_i$$

Convex Hull

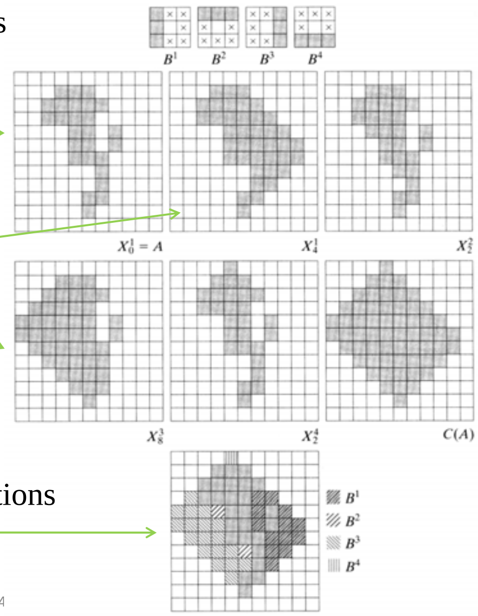
Therefore, the method consists of iteratively applying the hit-or-miss transform to A with B^1; when no further changes occur, we perform the union with A and call the result D^1. The procedure is repeated with B^2(applied to A) until no further changes occur, and so on… the union of the four resulting Ds is the convex hull of A.

Convex Hull

Structuring elements: “x” indicates “do not care” conditions

A set A

Results of convergence with structural elements



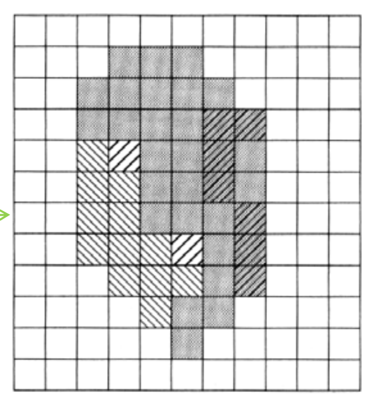
Convex hull showing the contributions of each structuring element

Convex Hull

One shortcoming of the procedure is that the convex hull can grow beyond the minimum dimensions required to guarantee convexity.

One simple approach to reduce this effect is to limit growth such that it does not extend past the vertical and horizontal dimensions of the original set.

The result of this limitation on the previous image: a convex hull limited to the dimensions of the original set.



Thinning

The ***thinning*** of a set A by a structuring element B is defined in terms of the hit-or-miss transform:

$$A \textcircled{N} B = A - \left(A \otimes B\right) = A \cap \left(A \otimes B\right)^c$$

So far, we were interested only in pattern matching with the structuring elements, so no background operation is required in the hit-or-miss transform. A more useful expression for thinning A symmetrically is based on a sequence of structuring elements:

$$\left\{B\right\} = \left\{B^1, B^2, \ldots B^n\right\}$$

where B^i are rotated versions of B^{i-1}. The thinning by a sequence of SEs:

$$A \textcircled{N} \left\{B\right\} = \left(\ldots\left(\left(A \textcircled{N} B^1\right) \textcircled{N} B^2\right)\ldots\right) \textcircled{N} B^n$$

Thinning

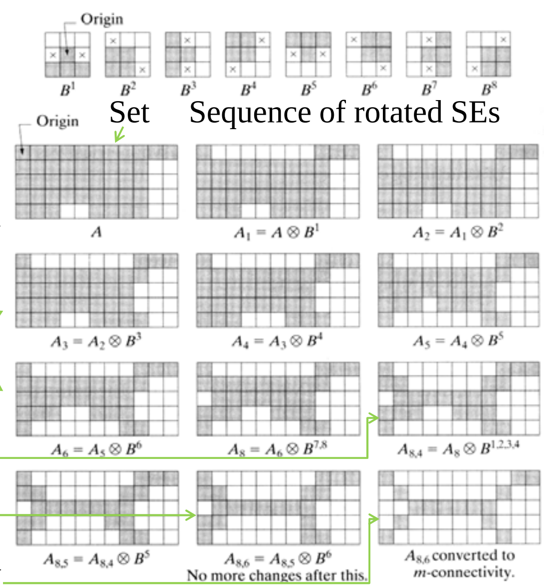
The process is to thin A by one pass with B^1, then thin the result with one pass of B^2, and so on, until A is thinned with one pass of B^n. The entire process is repeated until no further changes occur.

Results of thinning with each SE one after another

Using 4 first SEs again

Result after convergence

Conversion to m-connectivity



Thickening

Thickening is a morphological dual of a thinning and is defined as

$$A \odot B = A \cup \left(A \otimes B\right)^c$$

where B is a structuring element suitable for thickening. As in thinning, thickening can be defined as a sequential operation:

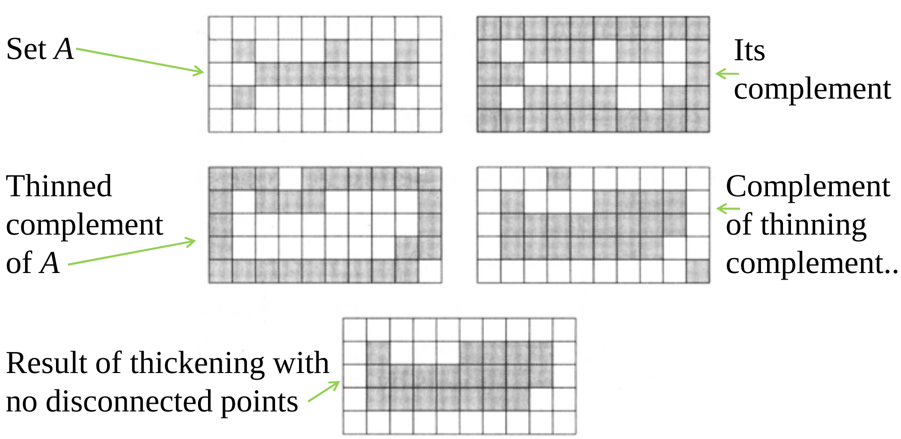
$$A \odot \{B\} = \left(\ldots\left(\left(A \odot B^1\right) \odot B^2\right)\ldots\right) \odot B^n$$

The structuring element used for thickening has the same form as one used for thinning but with all ones and zeros interchanged.

However, the usual procedure is to thin the background of the set to be processed and then complement the result. Therefore, to thicken a set A, we form its complement, thin it, and then complement the result.

Thickening

Depending on the nature of A, the thickening procedure may result in disconnected points. Therefore, this method is usually followed by post-processing to remove disconnected points.



Skeletons

A skeleton $S(A)$ of a set A can be viewed as:

a) If z is a point of $S(A)$ and $(D)_z$ is the largest disk centered at z and contained in A, one cannot find a larger disk (not necessarily centered at z) containing $(D)_z$ and included in A. The disk $(D)_z$ is called a ***maximum disk***.

b) The disk $(D)_z$ touches the boundary of A at two or more different places.

The skeleton of A can be expressed in terms of erosions and openings:

$$S(A) = \bigcup_{k=0}^{K} S_k(A)$$

Structuring element

$$S_k(A) = (A \ominus kB) - (A \ominus kB) \circ B$$

k successive erosions of A

Skeletons

$$(A \ominus kB) = (\ldots((A \ominus B) \ominus B) \ominus \ldots) \ominus B$$

K is the last iterative step before A erodes to an empty set:

$$K = \max\{k \mid (A \ominus kB) \neq \varnothing\}$$

$S(A)$ can be obtained as the union of the ***skeleton subsets*** $S_k(A)$. Also, we can show that A can be reconstructed from these subsets by:

$$A = \bigcup_{k=0}^{K} (S_k(A) \oplus kB)$$

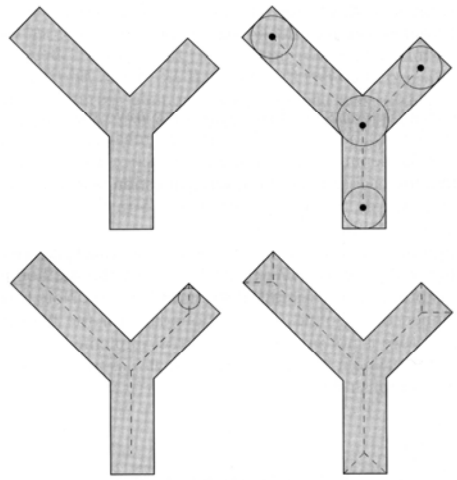
k successive dilations of A

$$(A \oplus kB) = (\ldots((A \oplus B) \oplus B) \oplus \ldots) \oplus B$$

Skeletons

Set A

Various positions of maximum disks with centers on the skeleton of A



Another maximum disk on a different segment of the skeleton of A

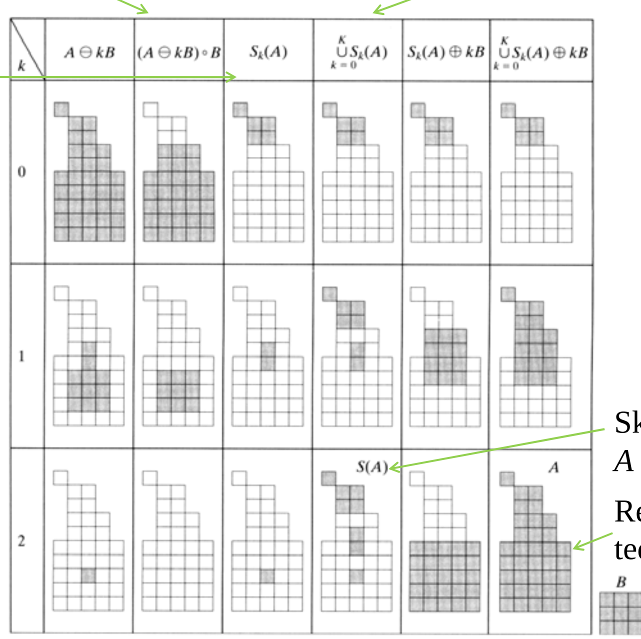
Complete skeleton of A

Skeletons

Openings by B

2 partial skeletons and final

Set differences between 1st and 2nd columns



Set A

Erosion 1 of A

Erosion 2 of A: next erosion will be $\varnothing$

Skeleton of A (not connected)

Reconstructed A

SE

Pruning

Pruning methods are an essential component to thinning and skeletonizing algorithms since these procedures tend to leave parasitic components that need to be "cleaned up" by post-processing. We start with a pruning problem and then develop a morphological solution.

A common approach in the automated recognition of printed characters is to analyze the shape of the skeleton of each character. These skeletons often are characterized by "spurs" (parasitic components). Spurs are created during the erosion by non uniformities in the strokes composing the characters. We develop a morphological technique for handling this problem, starting with the assumption that the length of a parasitic component does not exceed a specific number of pixels.

Pruning

A parasitic component

Original image A: a skeleton of a printed "a"

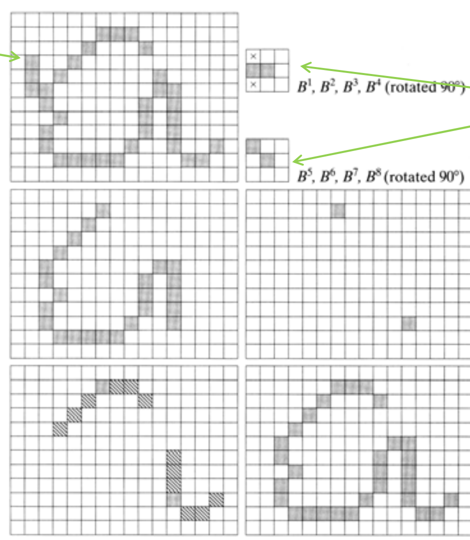
Structuring elements: "x" – don't care

3 runs of thinning: X_1

End points: X_2

Dilations of end points conditioned on A: X_3

Pruned image: X_4



Pruning

The solution is based on suppressing a parasitic branch by successively eliminating its end point. This also shortens (or eliminates) other branches in the character. The assumption is that, in the absence of other structural information, any branch with 3 or less pixels should be eliminated. Thinning of an input set *A* with a sequence of structuring elements designed to detect only end points achieves the desired result.

Let $$X_1 = A \otimes \{B\}$$

where $\{B\}$ is the sequence of structuring elements. This sequence consists of two different structures, each of which is rotated 90^0 for a total of 8 elements.

Applying the equation 3 times yields the set X_1 and the next step is to "restore" the character to its original form but with the parasitic branches removed.

Pruning

To do this, we first form a set X_2 containing all end points in X_1:

$$X_2 = \bigcup_{k=1}^{8} \left(X_1 \otimes B^k \right)$$

where B^k are the same structuring elements. Next step is dilation of the end points 3 times using set *A* as a delimiter:

$$X_3 = \left(X_2 \oplus H \right) \cap A$$

where *H* is a 3x3 structuring element of ones and the intersection with *A* as applied after each step. This type of conditional dilation prevents appearance of non-zero elements outside the region of interest.

Finally, the union of X_3 and X_1 yields the desired result:

$$X_4 = X_1 \cup X_3$$

Pruning

In more complex situations, X_3 sometimes includes the "tips" of some parasitic branches. This can occur when the end points of these branches are near the skeleton.

Although, they may be eliminated in X_1, these elements may show up during the dilation since they are valid points in A. The entire parasitic elements are rarely picked up again since they are usually short compared to the valid strokes. Therefore, their detection and elimination is easy since they are disconnected regions.

Morphological reconstruction (MR)

Morphological reconstruction is a morphological transform involving 2 images and a structuring element. One image, the ***marker***, contains the starting points for the transformation. The other image, the ***mask***, constrains the transformation. The structuring element is used to define connectivity.

MR: geodesic dilation and erosion

Central to morphological reconstruction are the concepts of geodesic dilation and geodesic erosion. Let F denote the marker image and G the mask image (assuming that both are binary images and that $F \subseteq G$). The ***geodesic dilation of size 1*** of the marker image with respect to the mask is

$$D_G^{(1)}(F) = (F \oplus B) \cap G$$

Here $\cap$ denotes the set intersection and may be interpreted as logical AND since they are the same for binary sets.

The ***geodesic dilation of size n*** of F with respect to G is

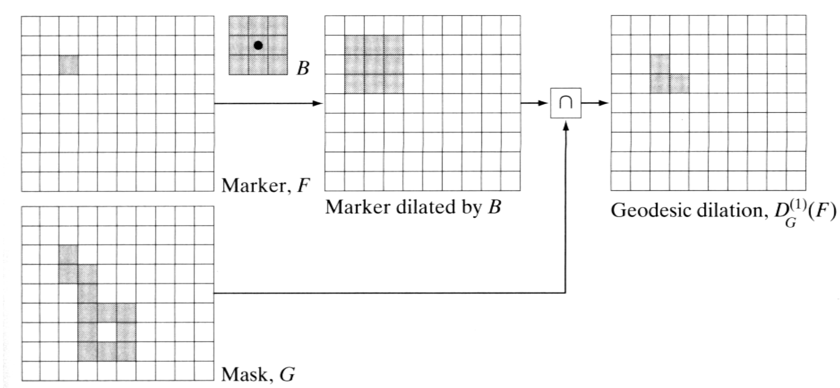
$$D_G^{(n)}(F) = D_G^{(1)}\left(D_G^{(n-1)}(F)\right)$$

with

$$D_G^{(0)}(F) = F$$

MR: geodesic dilation and erosion

In the last expression, the set intersection is performed at each step. The intersection operator guarantees that mask G will limit the growth (dilation) of marker F.



Geodesic erosion

MR: geodesic dilation and erosion

Similarly, the ***geodesic erosion of size 1*** of the marker image F with respect to the mask G is

$$E_G^{(1)}(F) = (F \ominus B) \cup G$$

where $\cup$ denotes the union (OR operation). The geodesic erosion of size n of F with respect to G is defined as

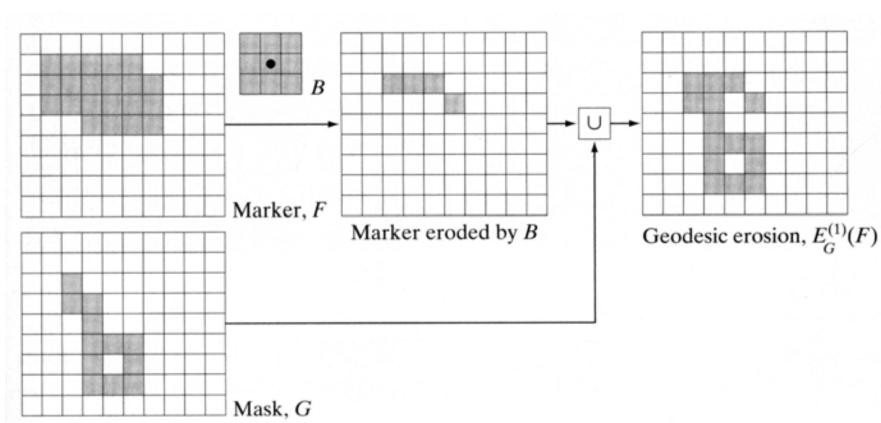
$$E_G^{(n)}(F) = E_G^{(1)}\left(E_G^{(n-1)}(F)\right)$$

with

$$E_G^{(0)}(F) = F$$

The union operation is performed at each iterative step and guarantees that geodesic erosion of an image remains greater then or equal to its mask image. Geodesic dilation and erosion are duals with respect to set complementation.

MR: geodesic dilation and erosion



Geodesic dilation

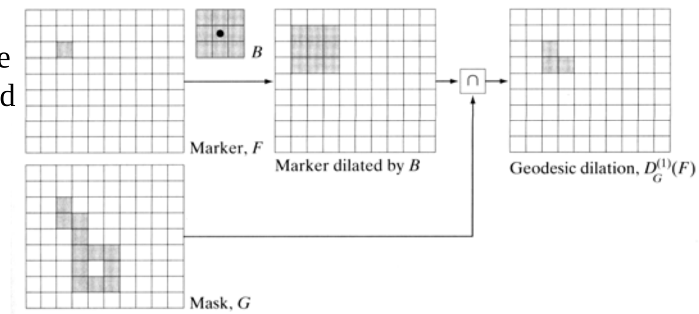
MR by dilation and erosion

Morphological reconstruction by dilation of a mask image G from a marker image F is defined as the geodesic dilation of F with respect to G, iterated until stability is achieved:

$$R_G^D(F) = D_G^{(k)}(F)$$

with k such that $D_G^{(k)}(F) = D_G^{(k+1)}(F)$.

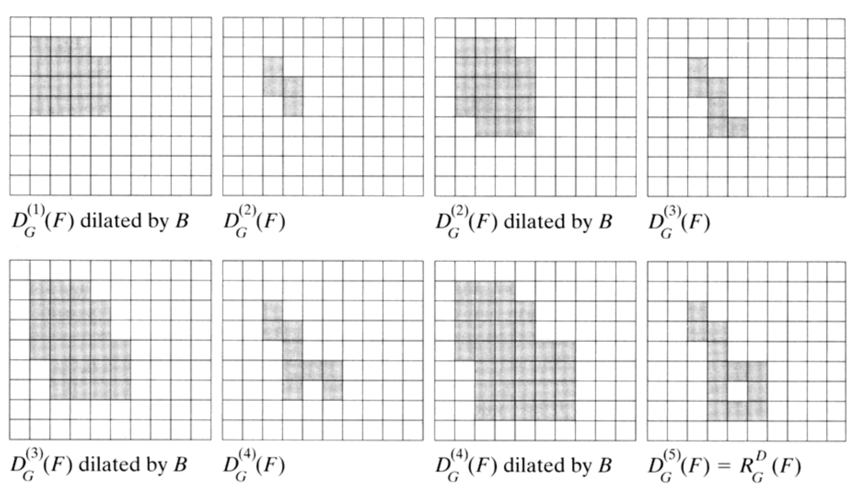
Considering these marker, mask, and structuring element…



MR by dilation and erosion

Dilation dilated with SE

Result AND G



MR by dilation and erosion

The morphological reconstructed image is $D_G^{(5)}(F)$, which is identical to the mask F since F contained a single 1-valued pixel (this is analogous to convolution of an image with an impulse, which simply copies an image at the location of the impulse).

The ***morphological reconstruction by erosion*** of a mask G from a marker image F is defined as the geodesic erosion of F with respect to G, iterated until stability:

$$R_G^E(F) = E_G^{(k)}(F)$$

with k such that $E_G^{(k)}(F) = E_G^{(k+1)}(F)$.

MR: sample applications

Morphological reconstruction has a broad spectrum of practical applications, each determined by the selection of the marker and mask images, by the structuring elements used, and by combinations of the primitive operations as defined above.

Opening by reconstruction:

In a morphological opening, erosion removes small objects and the subsequent dilation attempts to restore the shape of objects that remain. However, the accuracy of this restoration is highly dependent on the similarity of the shapes of the objects and the structuring element used.

Opening by reconstruction restores exactly the shapes of the objects that remain after erosion.

MR: sample applications

The ***opening by reconstruction*** of size *n* of an image *F* is defined as the reconstruction by dilation of *F* from the erosion of size *n* of *F*:

$$O_R^{(n)}\left(F\right) = R_F^D\left[\left(F \ominus nB\right)\right]$$

n erosions of *F* by *B*

We observe that *F* is used as the mask in this application.

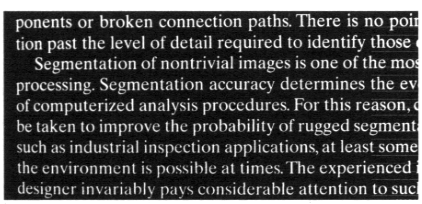
A similar expression can be written for ***closing by reconstruction***:

$$C_R^{(n)}\left(F\right) = R_F^E\left[\left(F \oplus nB\right)\right]$$

n dilations of *F* by *B*

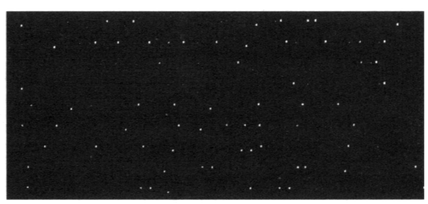
MR: sample applications

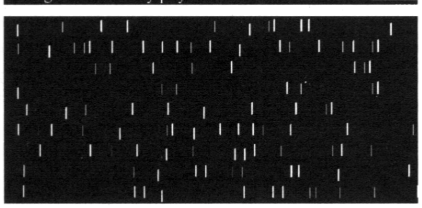
Text image of size 918x2018; average height of tall characters is 50



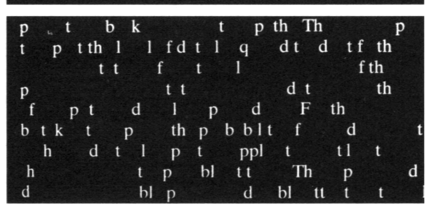
ponents or broken connection paths. There is no poi
tion past the level of detail required to identify those
Segmentation of nontrivial images is one of the mos
processing. Segmentation accuracy determines the ev
of computerized analysis procedures. For this reason,
be taken to improve the probability of rugged segment
such as industrial inspection applications, at least some
the environment is possible at times. The experienced
designer invariably pays considerable attention to suc

Erosion with a SE of size 51x1 pixels





Opening of the image with the same SE (for reference)



Opening by reconstruction: interest to extract characters with long vertical strokes

MR: sample applications

Filling holes:

We develop next a fully automatic procedure for hole filling based on morphological reconstruction. Let $I(x,y)$ be a binary image, assume that we form a marker image F that is 0 everywhere , except at the image border, where it is set to 1 – I:

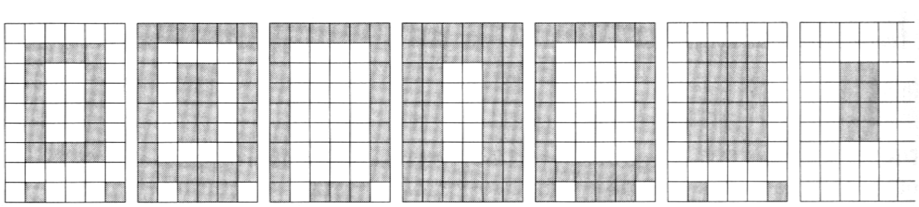
$$F(x,y) = \begin{cases} 1 - I(x,y) & if\ (x,y)\ is on the border of\ I \\ 0 & otherwise \end{cases}$$

Then

$$H = \left[R_{I^c}^{D}(F) \right]^c$$

Is a binary image equal to I with all holes filled.

MR: sample applications

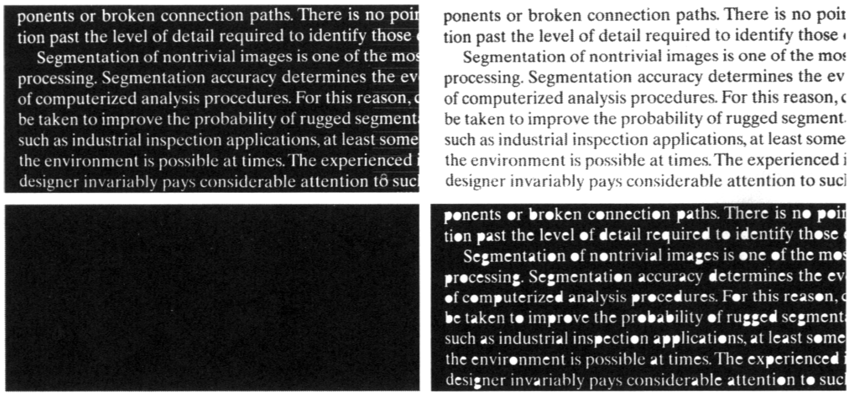


I	I^c	F	$F \oplus B$	$F \oplus B \cap I^c$	H	$H \cap I^c$
Original image with a hole	Its complement		F dilated by a 3x3 SE of ones	Intersec-tion of dilation with comple-ment	Its comple-ment	Intersec-tion with comple-ment

MR: sample applications

Text image of size 918x2018;
average height of tall characters is 50

Its complement



Marker image *F*

Result of hole filling

MR: sample applications

Border cleaning:

The extraction of objects from an image for subsequent shape analysis is a fundamental task in automated image processing. An algorithm for removing objects that touch (connected to) the border is useful since:

1) it can be used to screen the image such that only complete objects remain;
2) It can be used to detect partial objects that are present in the field of view.

We develop a border-cleaning procedure based on morphological reconstruction.

MR: sample applications

We use the original image as the mask and the following marker image:

$$F(x,y)=\begin{cases} I(x,y) & \text{if } (x,y) \text{ is on the border of } I \\ 0 & \text{otherwise} \end{cases}$$

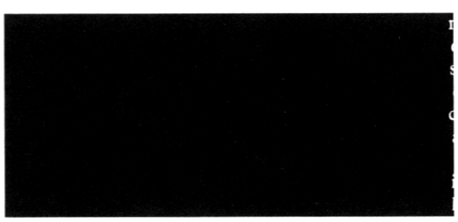
The border-cleaning algorithm first computes the morphological reconstruction $R_I^D(F)$, which extracts the objects touching the border, and then computes the difference

$$X = I - R_I^D(F)$$

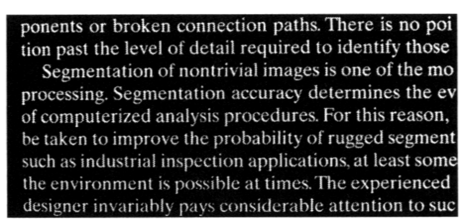
To obtain an image X with no objects touching the border.

MR: sample applications

For the same text image, we need to eliminate the incomplete characters (ones touching the border). This may be used before automatic character recognition.



Marker image F obtained using a 3x3 SE of ones



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The image with no objects touching the border

Structuring elements used

Basic types of structuring elements used in binary morphology: