Digital Image Processing

Lecture #10

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Announcement

- **Class Information**
  - **The following schedule**

03/23	Lecture 5	05/11	proposal
03/30	Lecture 6 & 7	05/18	Lecture 11
04/06	Lecture 8	05/25	Lecture 12
04/13	RealSense	06/01	Lecture 13
04/20	midterm	06/08	Demo
04/27	RealSense & Lecture 9	06/15	Demo
05/04	Lecture 9 & Lecture 10	06/22	Final Package Due

Announcement

- Proposal
  - Oral presentation
    - 6 minutes for each team
  - May 11, 2016
    - Motivation
    - Proposed idea
    - Algorithm
    - Reference
      - Authors, Paper Title, Conference/Journal, Page Number, Year
      - Complete link

Image Restoration

Image Restoration

- Attempt to reconstruct or recover an image that has been degraded
- Model the degradation and apply the inverse process

- A priori modeling
  - measurements on the physical imaging system, digitizer and display by hardware
- A posteriori modeling
  - measurements of a particular image to be restored

Image Restoration

Original input image X → Degradation H → Observed image Y

- Forward Problem
  - Given X & H → find Y

    This is easier.
- System Identification Problem
  - Given X & Y → find H

    If H can be found, the restoration is possible.
    But it's very hard to find the real H.
- Inverse Problem
  - Given H & Y → find X

⇨ Image enhancement v.s. image restoration

Image Restoration

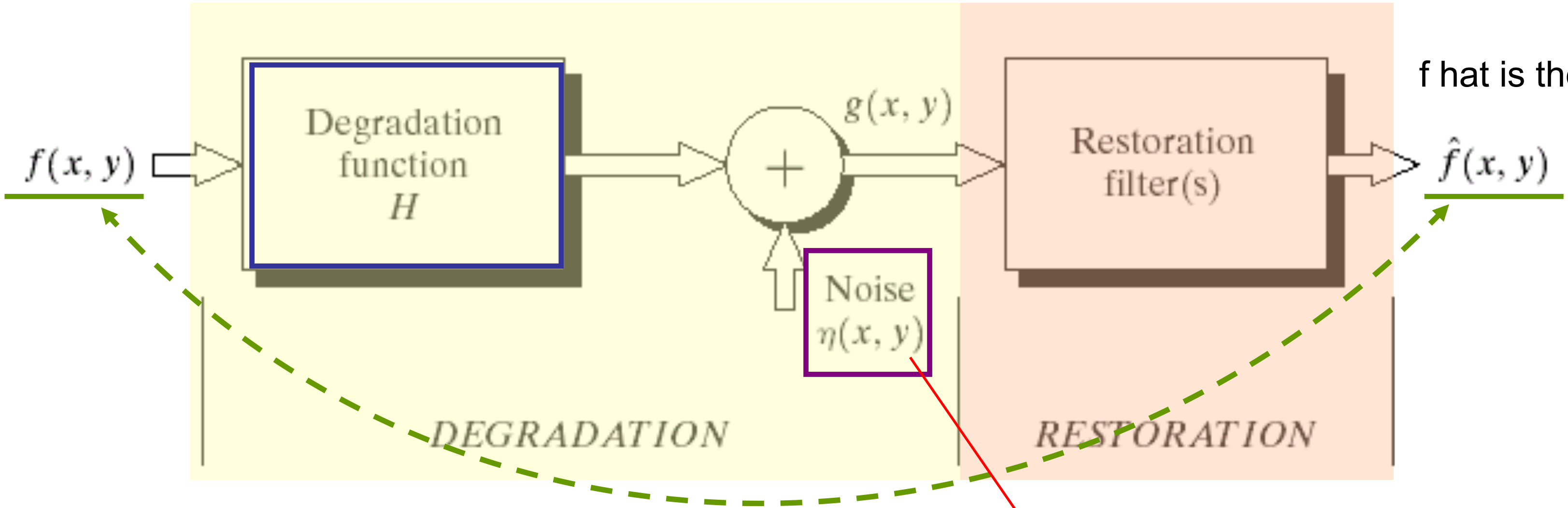
- Image Enhancement v.s. Image Restoration
  - Image enhancement
    - Largely subjective 主觀
    - No benchmark
    - Basically a heuristic procedure
  - Image restoration
    - More objective 客觀
    - With a ground truth
    - Usually formulate a criterion of goodness

Image Restoration

- Image observation models
  - An acquired image could be degraded by the sensing environment
    - Sensor noise
    - Optical system aberrations
    - Image motion blur
    - Atmospheric turbulence effects
  - Model the degradation and apply the inverse process
    - A priori modeling
    - A posteriori modeling

Image Restoration

- **Image observation models**



f hat is the restored f.

$$g(x,y) = h(x,y) * f(x,y) + \eta(x,y)$$

$$G(u,v) = H(u,v)\,F(u,v) + N(u,v)$$

Find H and N, because they are unknown.

Image Restoration

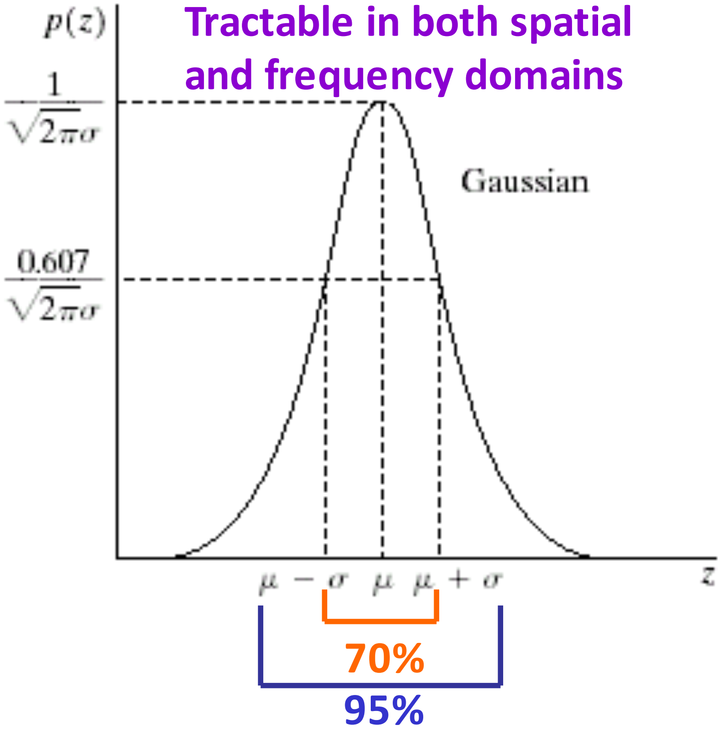
- **Noise models**
  - Gaussian Noise
  - Rayleigh Noise
  - Gamma (Erlang) Noise
  - Exponential Noise
  - Uniform Noise
  - Impulse (Salt-and-Pepper) Noise

//Note// assume that noise is independent of spatial coordinates and uncorrelated w.r.t. the image itself (except periodic noise)

Image Restoration

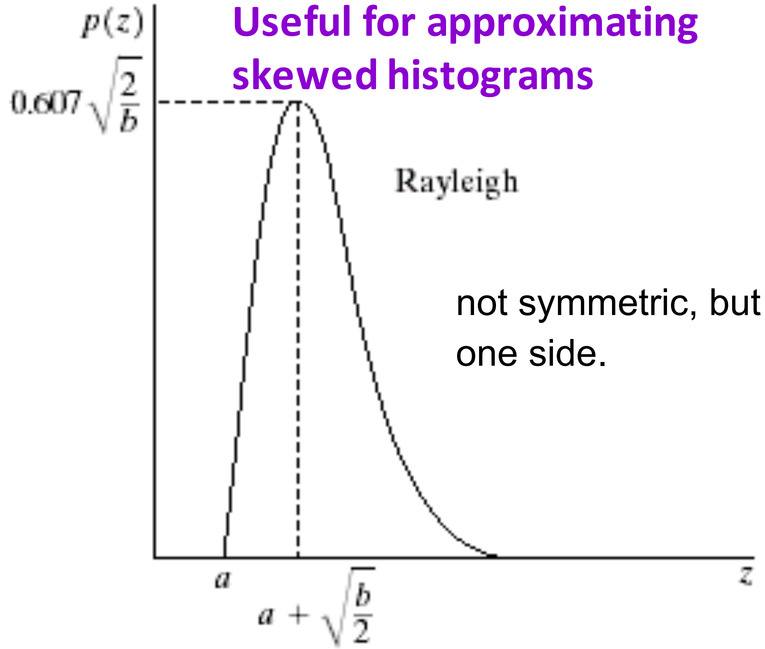
- **Gaussian noise v.s. Rayleigh noise**

Tractable in both spatial and frequency domains



$$p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(z-\mu)^2/2\sigma^2}$$

Useful for approximating skewed histograms

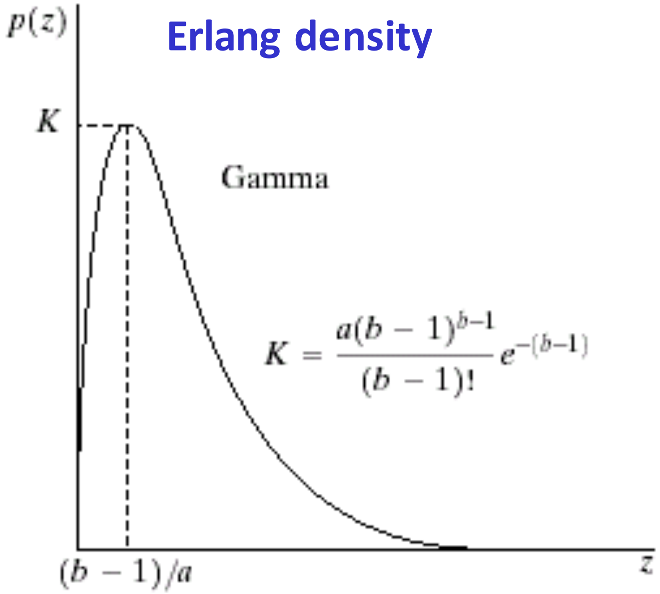


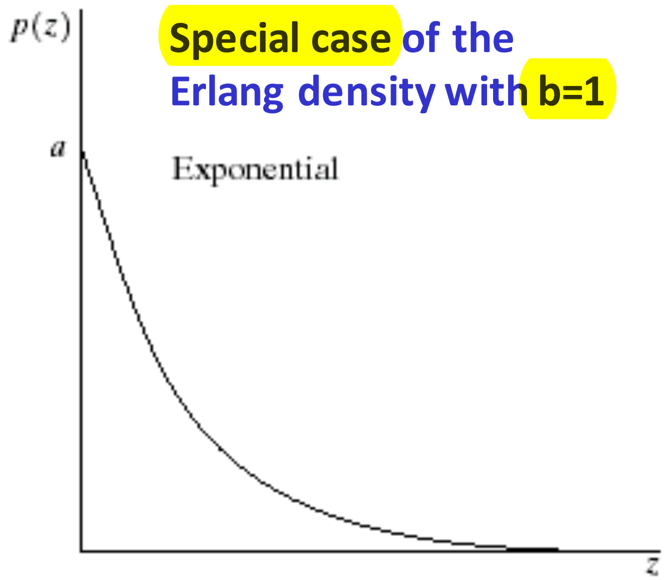
not symmetric, but skewed in one side.

$$p(z) = \begin{cases} \frac{2}{b}(z-a)e^{-(z-a)^2/b} & for\ z \geq a \\ 0 & for\ z < a \end{cases}$$

Image Restoration

- **Gamma noise v.s. Exponential noise**



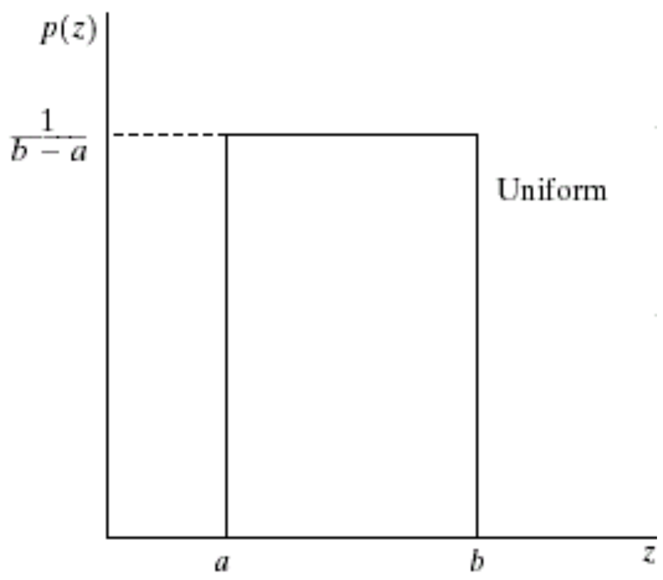


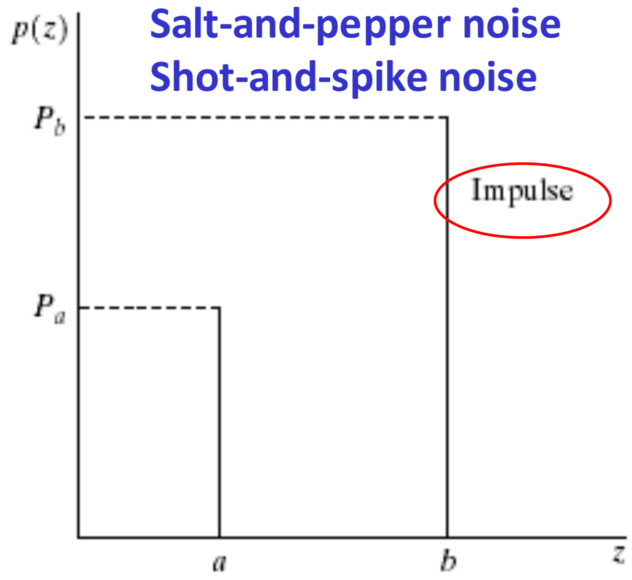
$$p(z) = \begin{cases} \frac{a^b z^{b-1}}{(b-1)!}e^{-az} & for\ z \geq 0 \\ 0 & for\ z < 0 \end{cases}$$

$$p(z) = \begin{cases} ae^{-az} & for\ z \geq 0 \\ 0 & for\ z < 0 \end{cases}$$

Image Restoration

- **Uniform noise v.s. Impulse noise**





$$p(z) = \begin{cases} \frac{1}{b-a} & if\ a \leq z \leq b \\ 0 & otherwise \end{cases}$$

$$p(z) = \begin{cases} P_a & for\ z = a \\ P_b & for\ z = b \\ 0 & otherwise \end{cases}$$

Image Restoration

- **Noise models**
  - Gaussian Noise
    - Electronic noise and sensor noise due to poor illumination and/or high temperature
  - Rayleigh Noise
    - Helpful in characterizing noise phenomena in range imaging
  - Gamma (Erlang) Noise & Exponential Noise
    - Applications in laser imaging
  - Uniform Noise
    - As basis for random number generators
  - Impulse (Salt-and-Pepper) Noise
    - Quick transient (faulty switching) during imaging

Image Restoration

- Test pattern
  - Three-leveled gray-scale image

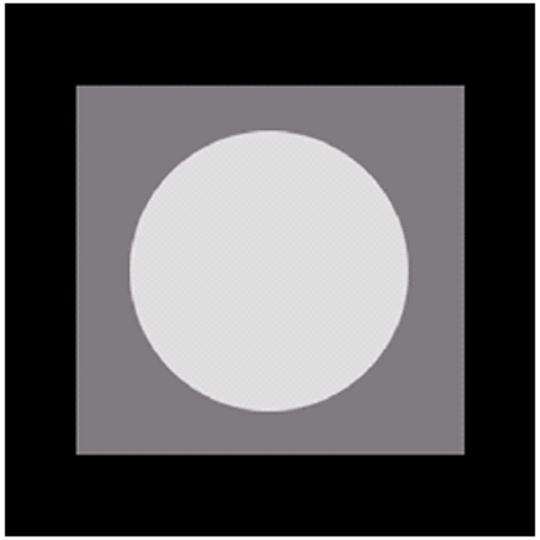
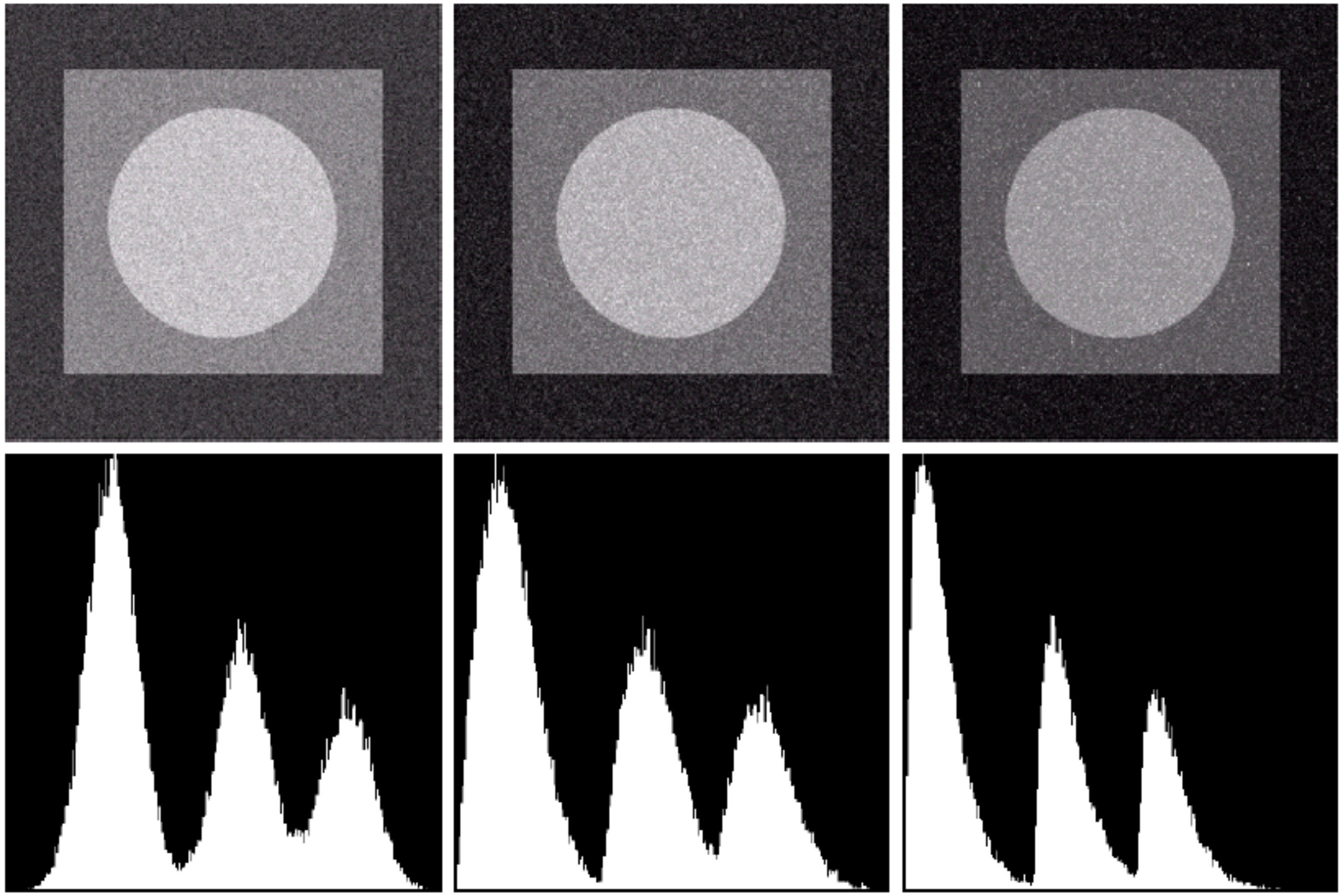


Image Restoration

- **Experimental results** It's hard to tell which one is.

image

histogram



Gaussian Rayleigh Gamma

Image Restoration

- **Experimental results**

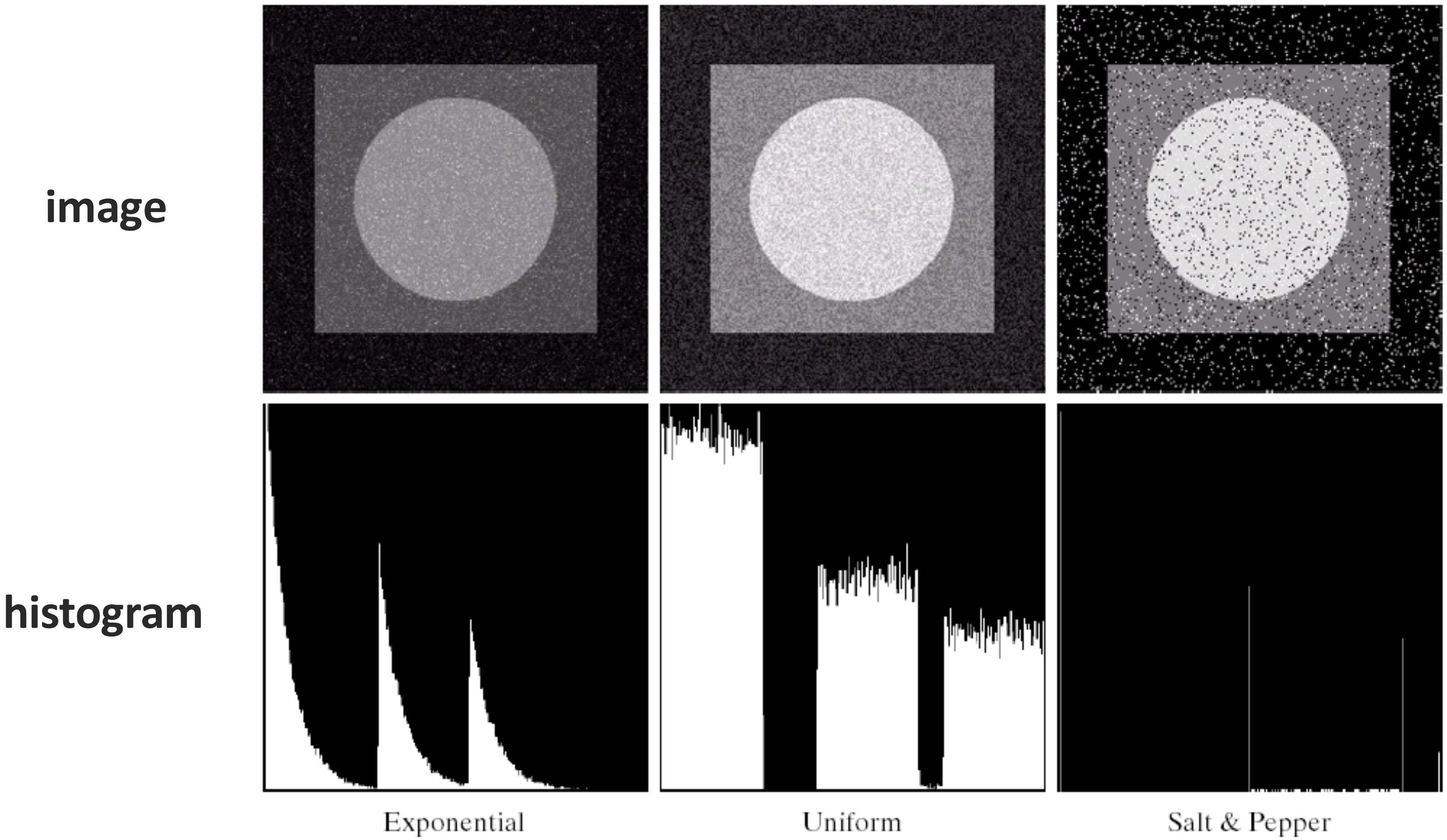
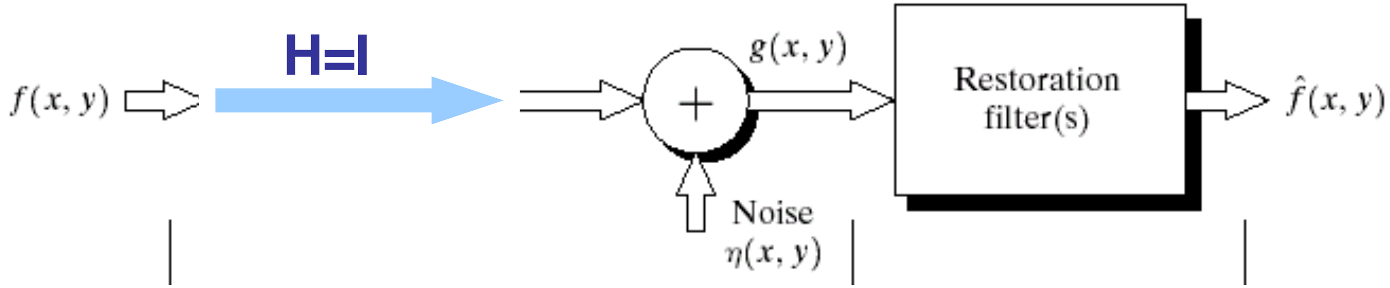


Image Restoration

- Restoration with only additive noise

only noise degrades the image. No other filter is used.



$$g(x,y) = f(x,y) + \eta(x,y)$$

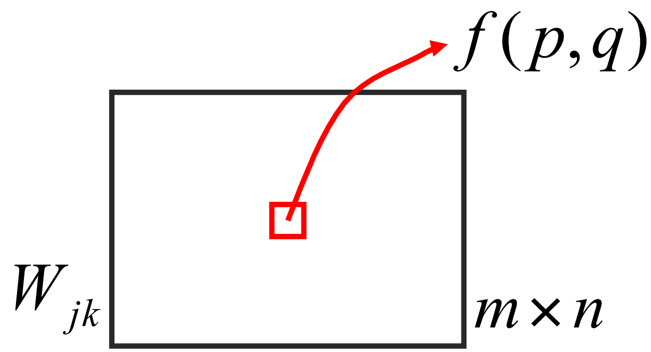
$$G(u,v) = F(u,v) + N(u,v)$$

- Spatial filtering: three solutions
  - Mean filters
  - Order-statistics filters
  - Adaptive filters

Image Restoration

- **Mean filters**
  - **Arithmetic mean filter**

$$g(j,k) = \frac{1}{mn} \sum_{(p,q)\in W_{jk}} f(p,q)$$



Smooth local variations as a result of blurring

  - **Geometric mean filter**

$$g(j,k) = \left[\prod_{(p,q)\in W_{jk}} f(p,q) \right]^{\frac{1}{mn}}$$

Tend to lose less image detail in the process

Image Restoration

- **Example**
  - **Arithmetic mean filter v.s. Geometric mean filter**

lose less image detail

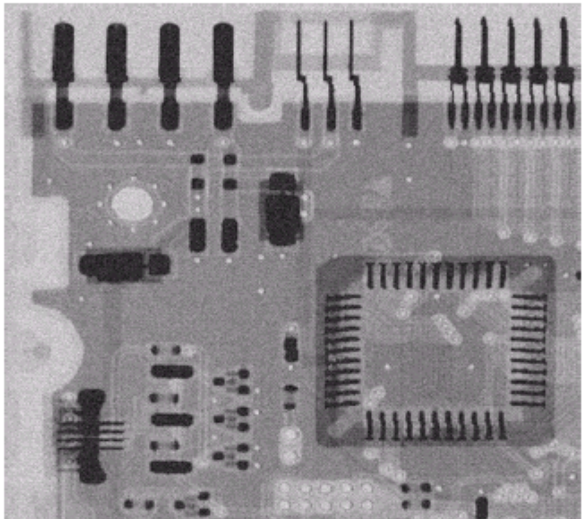
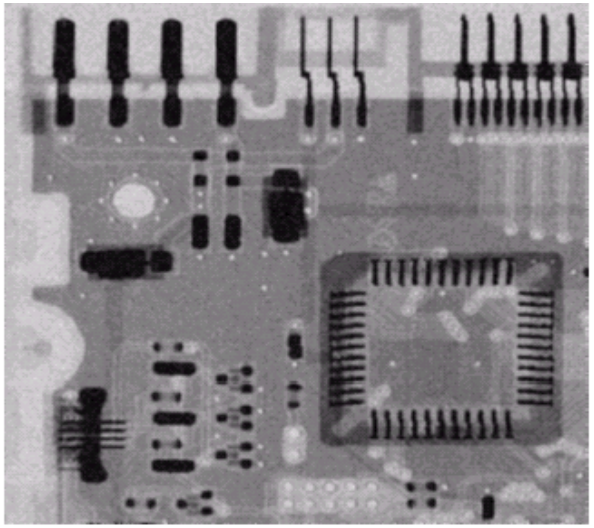
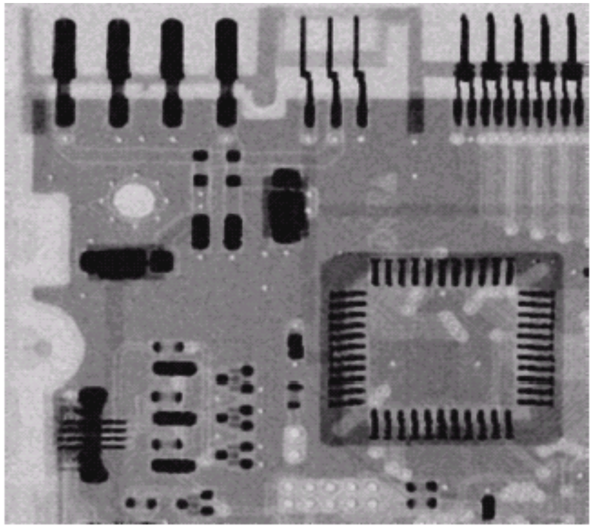


Image corrupted by additive Gaussian noise



Arithmetic mean filter
Of size 3x3



Geometric mean filter
Of size 3x3

Image Restoration

- Mean filters
  - Harmonic mean filter

$$g(j,k) = \frac{mn}{\sum_{(p,q)\in W_{jk}} \frac{1}{f(p,q)}}$$

→Good for salt noise
→Gaussian noise
→Fails for pepper noise

  - Contraharmonic mean filter

$$g(j,k) = \frac{\sum_{(p,q)\in W_{jk}} f(p,q)^{Q+1}}{\sum_{(p,q)\in W_{jk}} f(p,q)^{Q}}$$

order

→Q>0: pepper noise removed
Q<0: salt noise removed
→Q=0? →Arithmetic ()/m*n
→Q=-1? →Harmonic

Image Restoration

- Example
  - Contraharmonic mean filter with order=1.5

    (eliminate pepper noise)

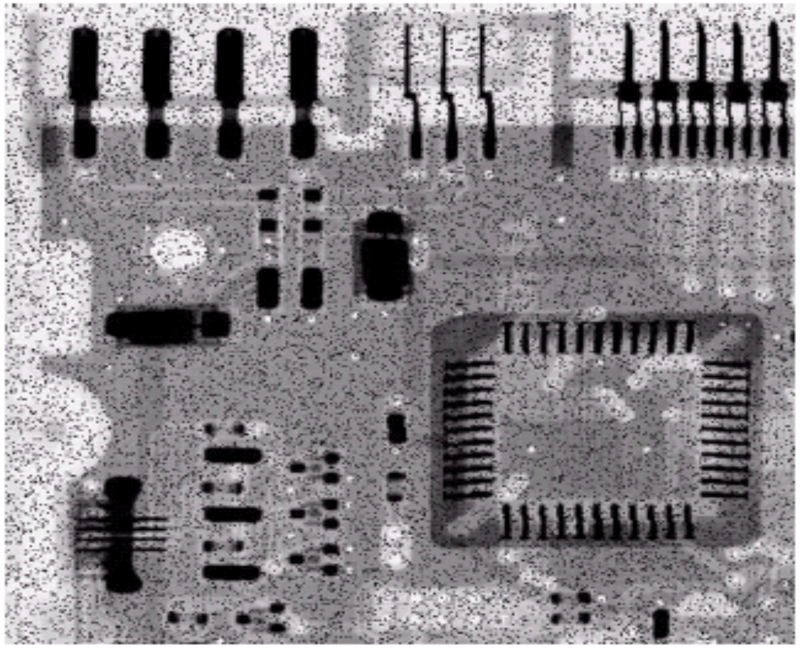
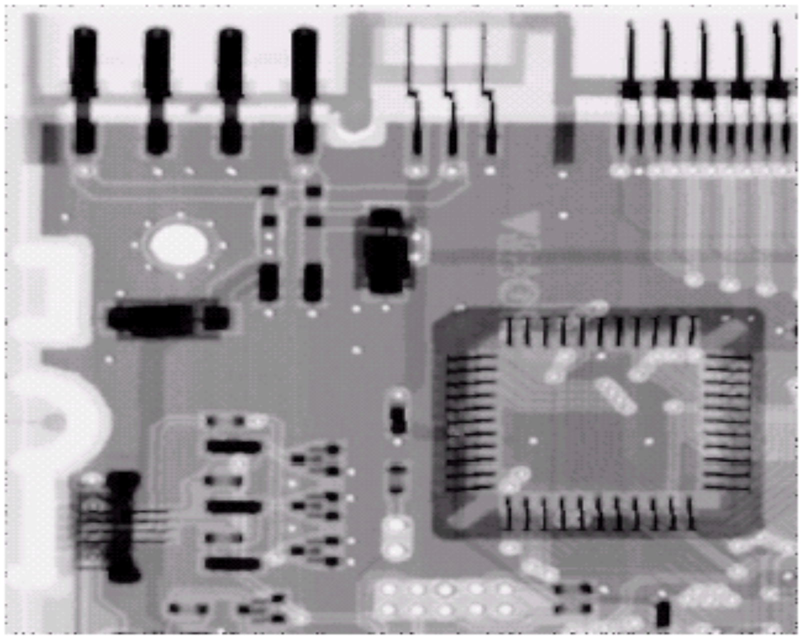


Image corrupted by pepper noise



3x3 contraharmonic filter of order 1.5

Image Restoration

- Example
  - Contraharmonic mean filter with order=-1.5 (eliminate salt noise)

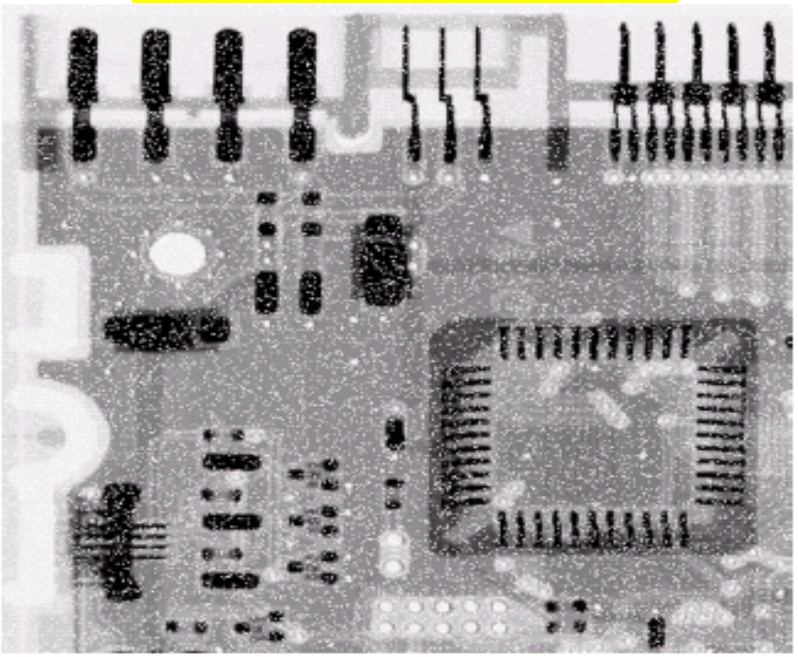
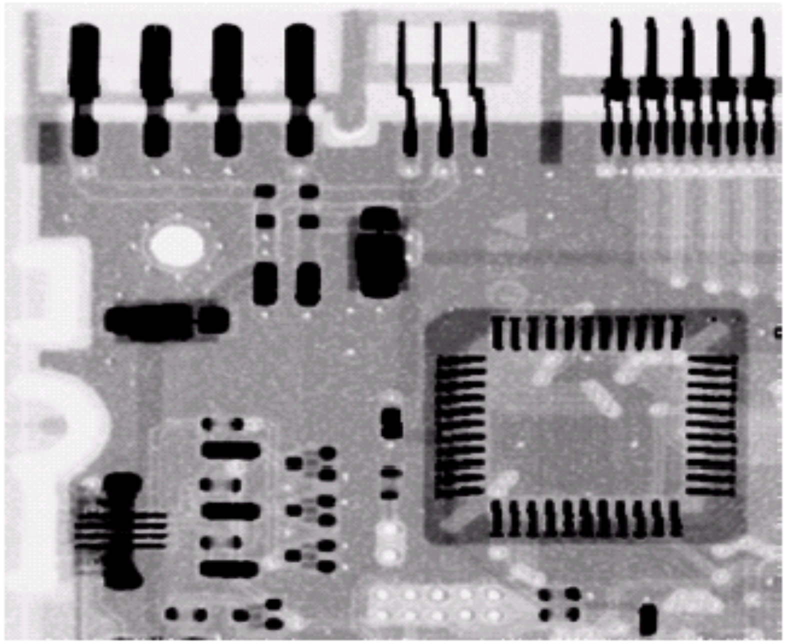


Image corrupted by salt noise

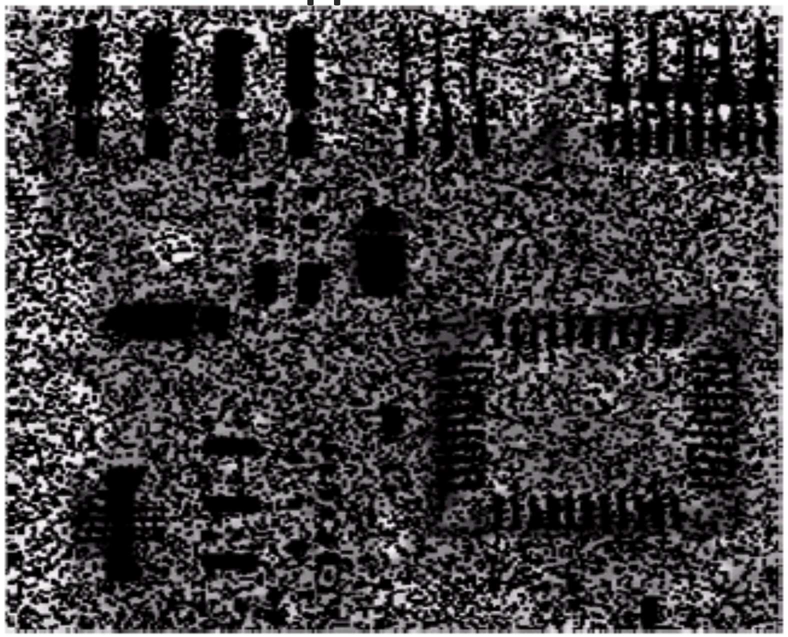


3x3 contraharmonic filter of order -1.5

Image Restoration

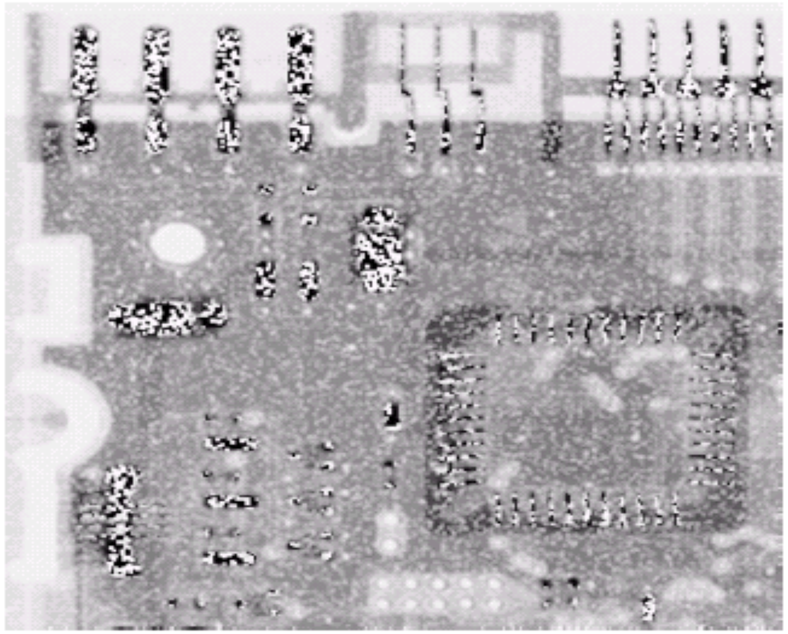
- **Contraharmonic mean filters**
  - Choose wrong sign of the filter

Pepper noise



3x3 contraharmonic filter
of order -1.5

Salt noise



3x3 contraharmonic filter
of order 1.5

Image Restoration

- **Order-statistics filters**
  - with response that is based on ordering the pixels contained in the image area encompassed by the filter

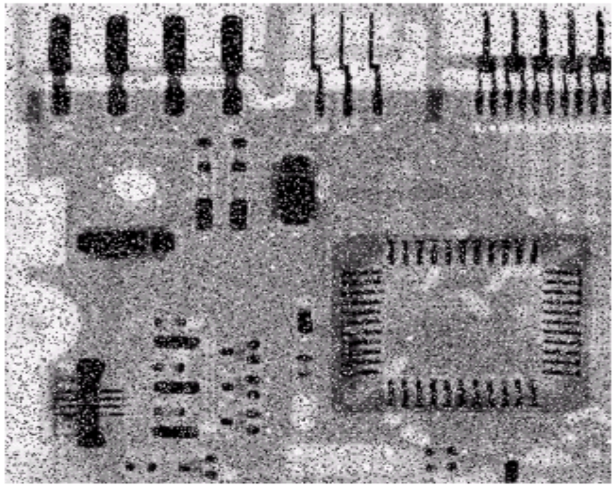
  - Median filter

$$g(j,k) = \underset{(p,q)\in W_{jk}}{median}\{f(p,q)\}$$

    - Effective in the presence of impulse noise
    - Less blurring than smoothing filters

Image Restoration

- **Example**
  - **Median filter**



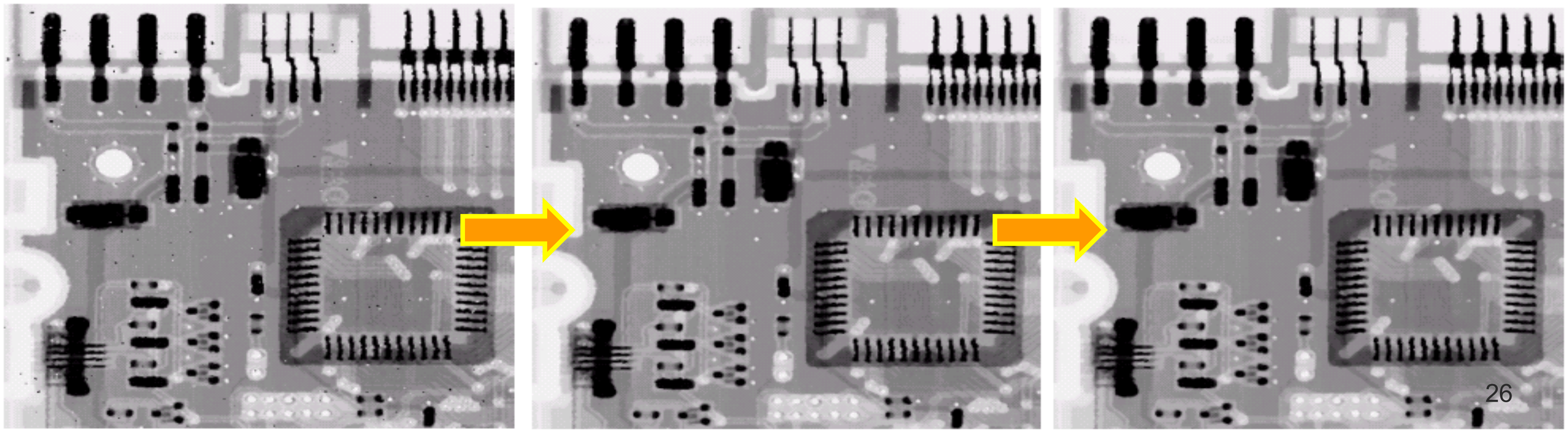


Image Restoration

- **Order-statistics filters**
  - **Max and min filters**

$$g(j,k) = \max_{(p,q)\in W_{jk}} \{f(p,q)\}$$ → **Remove pepper noise** pepper is black, low pixel value

$$g(j,k) = \min_{(p,q)\in W_{jk}} \{f(p,q)\}$$ → **Remove salt noise** salt is white, high pixel value

  - **Midpoint filter**

$$g(j,k) = \frac{1}{2}\left[\max_{(p,q)\in W_{jk}} \{f(p,q)\} + \min_{(p,q)\in W_{jk}} \{f(p,q)\}\right]$$

- **Combine order statistics and averaging**
- **Best for randomly distributed noise, eg. Gaussian or uniform noise**

Image Restoration

- **Order-statistics filters**
  - **Alpha-trimmed mean filter**
    - Suppose we delete d/2 lowest and d/2 highest gray-level values of $f(p,q)$ in the neighborhood W_{jk}

d/2 + d/2 = d, total "d" pixels are removed.

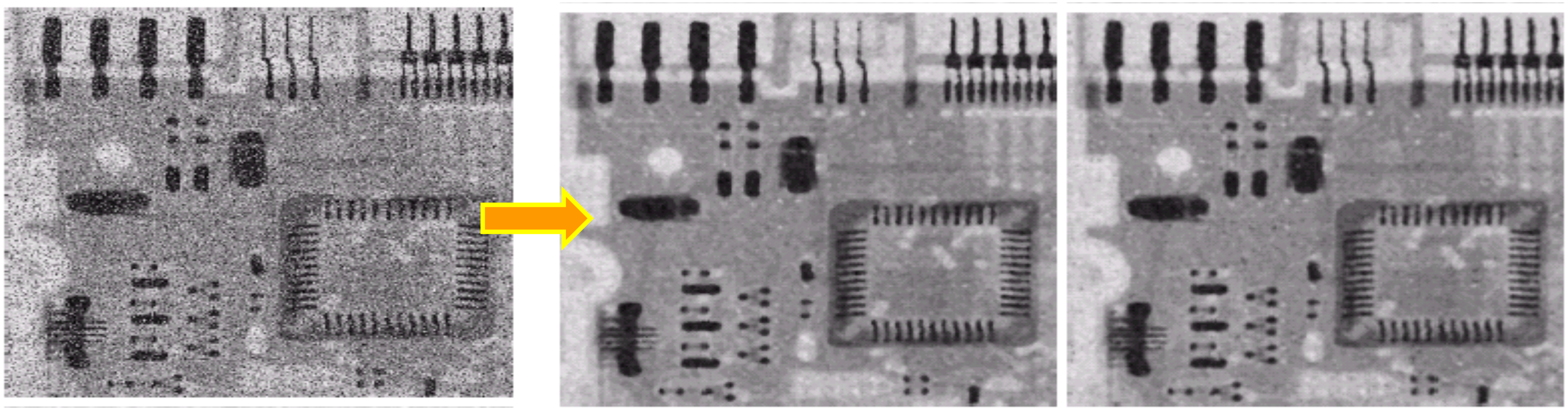
$$g(j,k) = \frac{1}{mn-d} \sum_{(p,q)\in W_{jk}} f_r(p,q)$$

    - where $f_r(p,q)$ represents the remaining $mn-d$ pixels
    - *d* ranges from 0 to $mn-1$
    - Useful for removing multiple types of noise

Image Restoration

- Example
  - Median and alpha-trimmed filters
    - Image corrupted by both uniform and salt-and-pepper noise

alpha-trimmed is "better" than median"



median

Alpha-trimmed

Image Restoration

- Adaptive filters
  - Adaptive, local noise reduction filter (Adaptive mean filter)
    - σ_η^2 : noise variance
    - σ_L^2 : local variance of the pixels in S_{xy}
    - m_L : local mean of the pixels in S_{xy}
  - If $\sigma_L^2 >> \sigma_\eta^2$, return a value close to $f(j,k)$

$$g(j,k) = f(j,k) - \frac{\sigma_\eta^2}{\sigma_L^2}\left(f(j,k) - m_L\right)$$

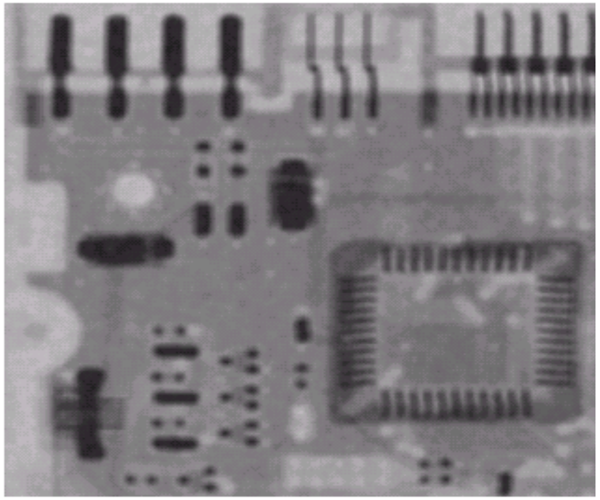
  - If $\sigma_L^2 \approx \sigma_\eta^2$, return arithmetic mean of the pixels in S_{xy}

Image Restoration

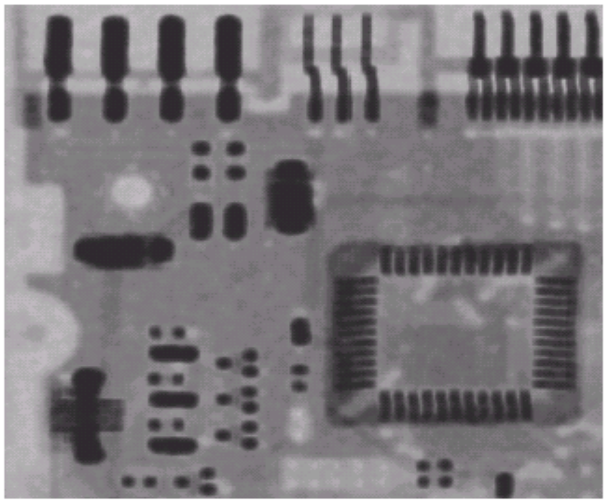
- Example
  - Adaptive noise reduction filter
    - Compared with arithmetic and geometric mean filters

better

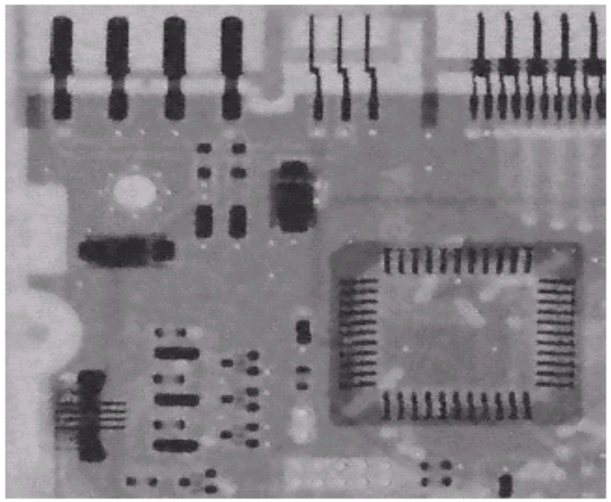
Best



arithmetic



geometric



adaptive noise reduction

Image Restoration

- Adaptive filters
  - Adaptive median filter

Wj,k is a window of the filter to process

$z_{\min}$ = minimum gray level value in $W_{j,k}$

$z_{\max}$ = maximum gray level value in $W_{j,k}$

z_{med} = median of gray levels in $W_{j,k}$

z_{jk} = gray level at coordinates (j,k) pixel value at (j,k)

$W_{\max}$ = maximum allowed size of W_{jk} window size is resizable.

$Level\quad A:$

$A_1 = z_{med} - z_{\min}$

$A_2 = z_{med} - z_{\max}$

$if\quad A_1 > 0\quad \text{and}\quad A_2 < 0,\quad \text{go to } Level\quad B$

$else\quad \text{increase the window } size$

$if\quad \text{window size} \le W_{\max},\quad \text{repeat Level } A$

$else\quad \text{output}\quad z_{jk}$

$Level\quad B:$

$B_1 = z_{jk} - z_{\min}$

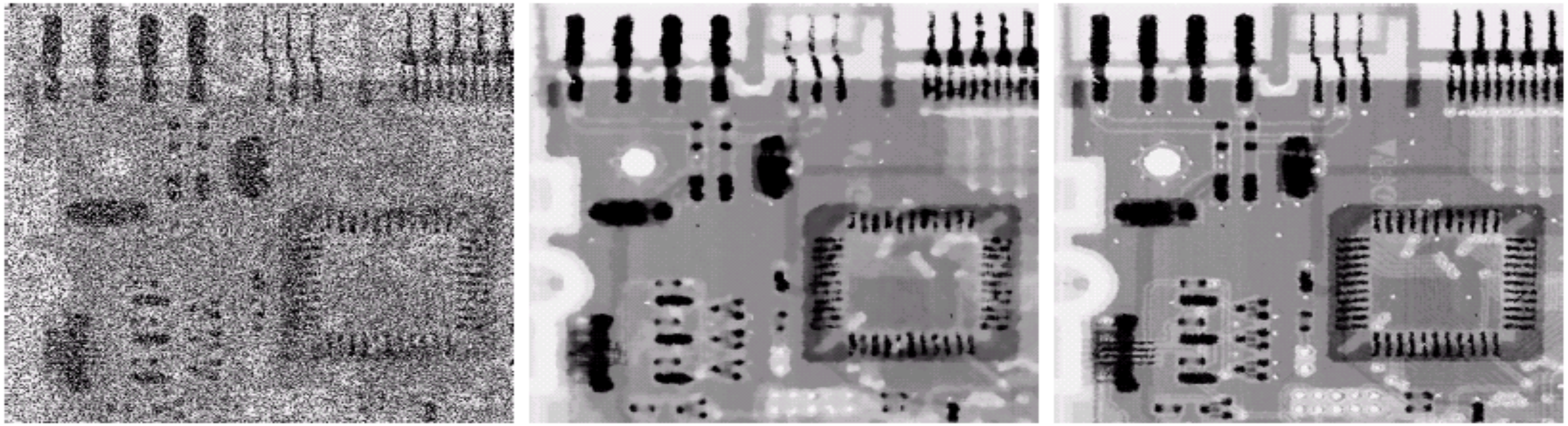
$B_2 = z_{jk} - z_{\max}$

$if\quad B_1 > 0\quad \text{and}\quad B_2 < 0,\quad \text{output}\quad z_{jk}$

$else\quad \text{output}\quad z_{med}$

Image Restoration

- Example
  - Adaptive median filter



salt-and-pepper median adaptive median

→Handle impulse noise with larger probabilities
→Preserve detail while smoothing nonimpulse noise
→Change the size of filter

Image Restoration

- Periodic Noise Reduction by Frequency Domain Filtering
  - Example

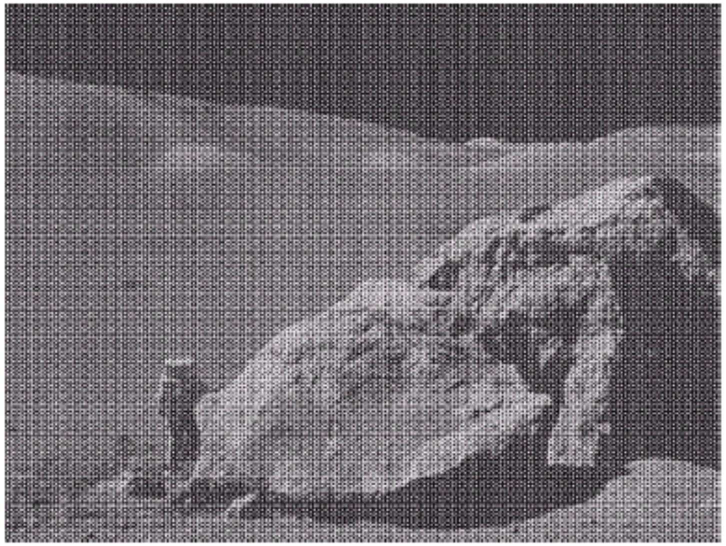
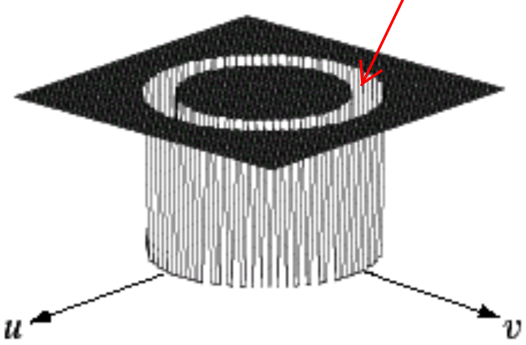


Image corrupted by sinusoidal noise

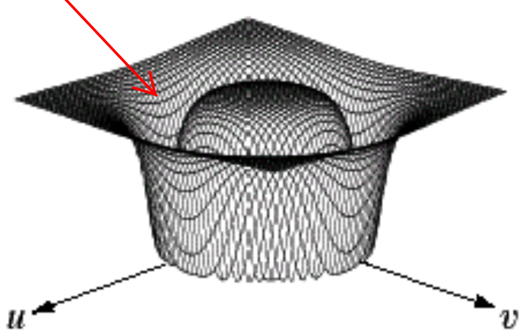
Spectrum of the corrupted image

Image Restoration

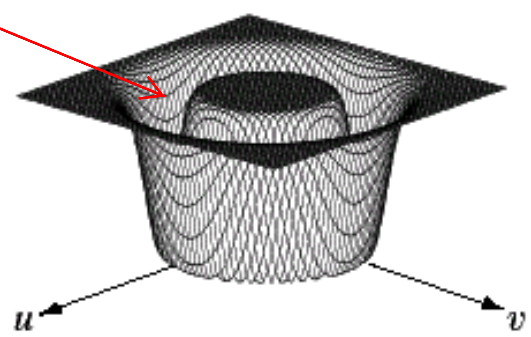
- **Periodic Noise Reduction by Frequency Domain Filtering**
  - **Band-reject Filters**



ideal



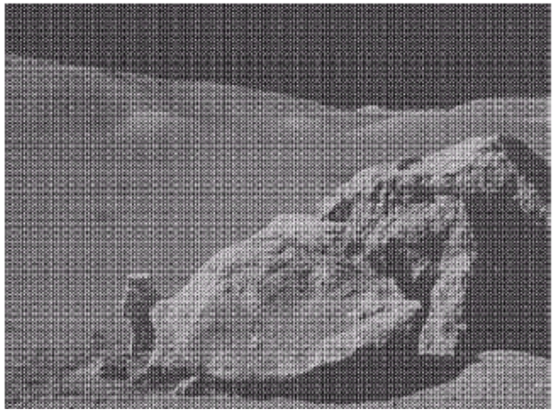
Butterworth (of order 1)



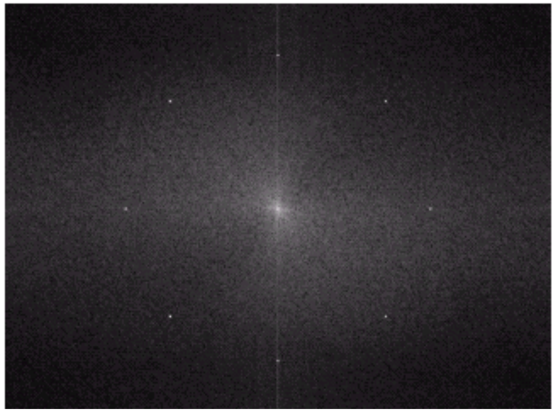
Gaussian

Image Restoration

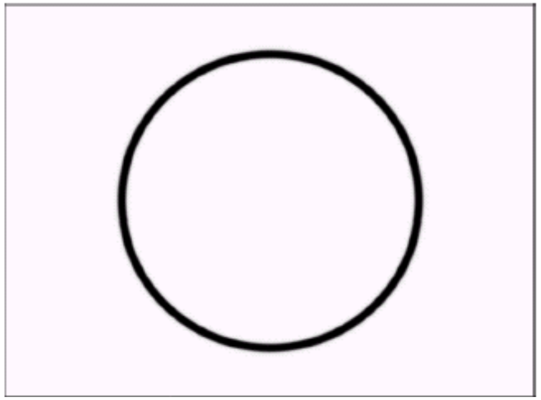
- **Example**



Degraded image



Spectrum



Butterworth (of order 1)

Restored image

Image Restoration

- **Periodic Noise Reduction by Frequency Domain Filtering**
  - **Notch Filters** Low frequency is still preserved

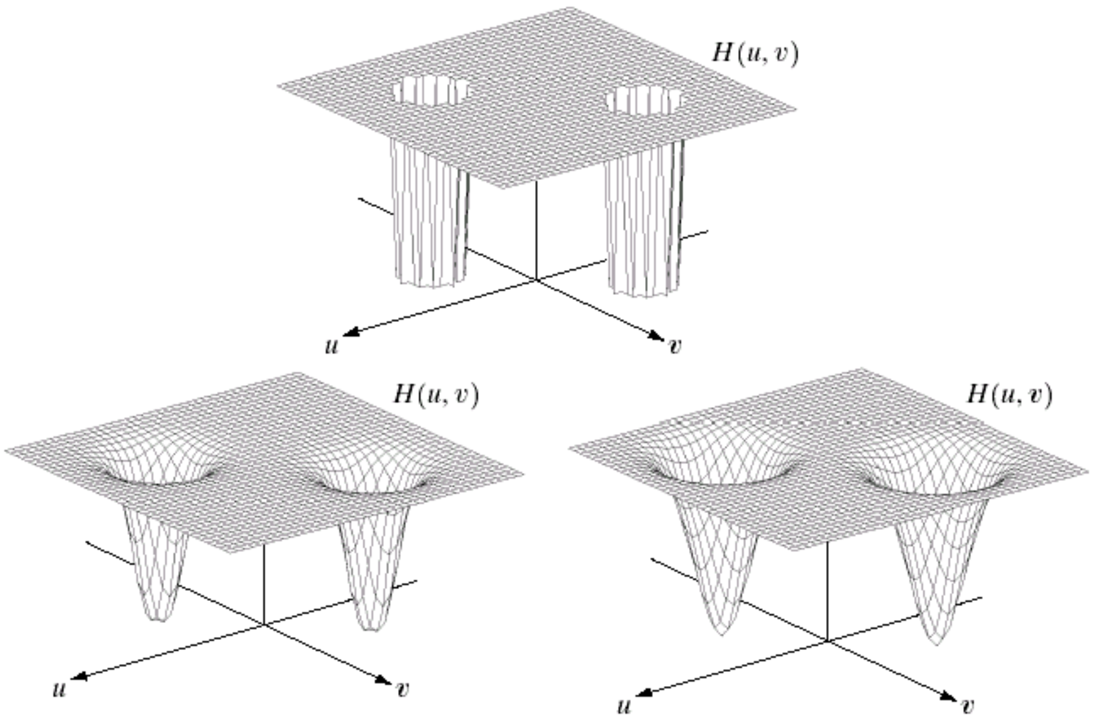
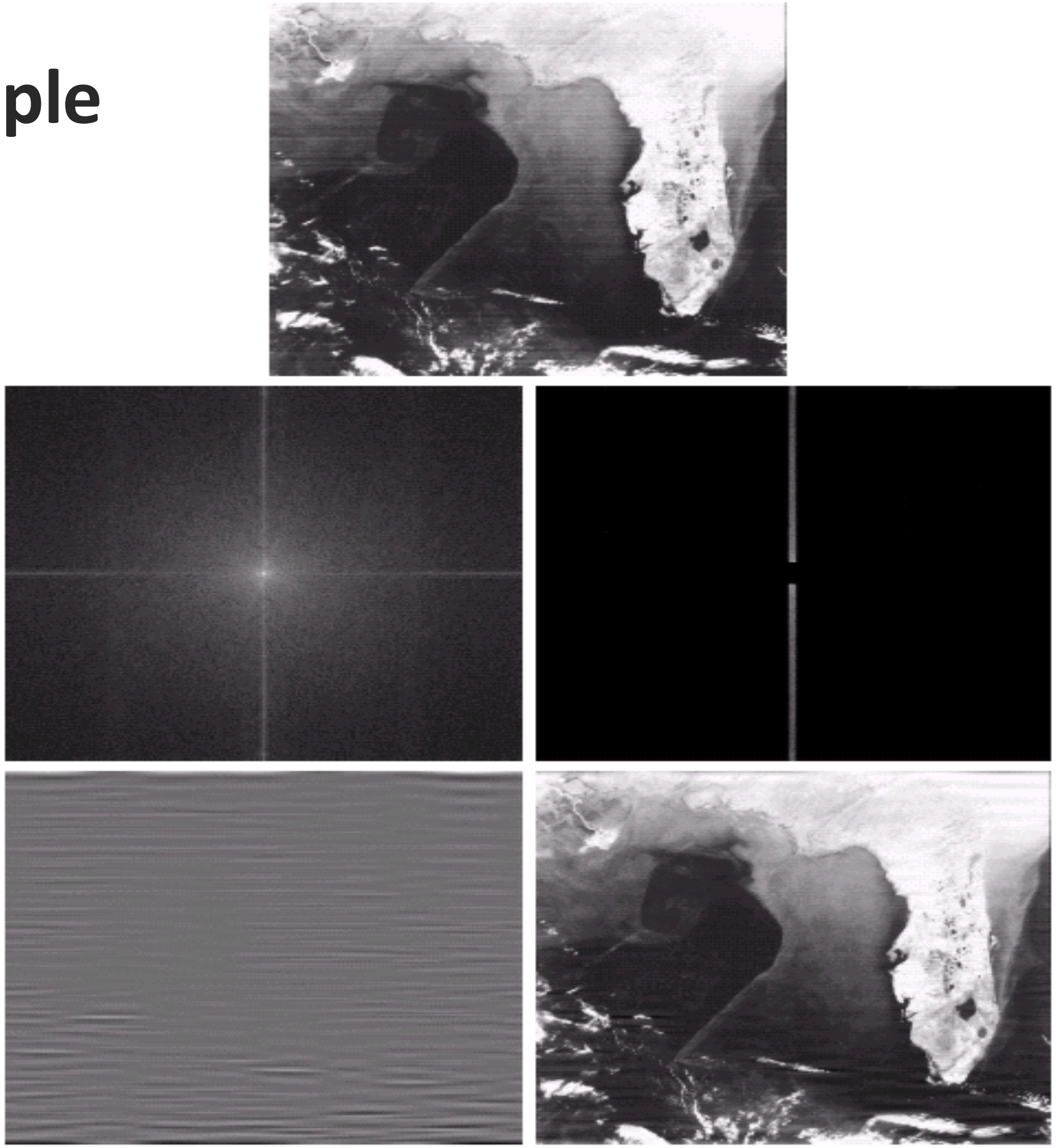


Image Restoration

- **Example**



Fourier spectrum

Notch filter

spatial domain

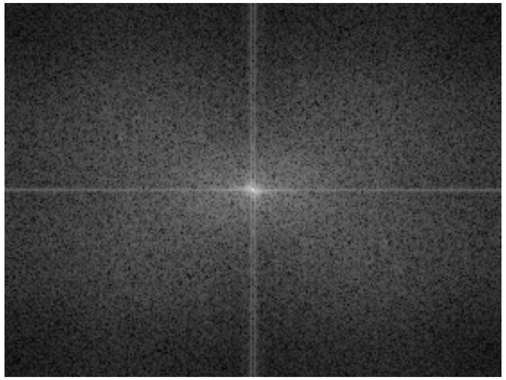
Noise pattern

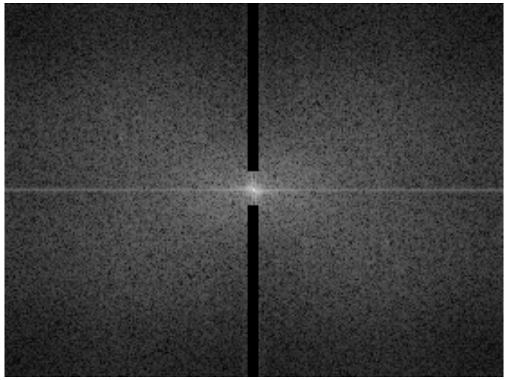
Result

Image Restoration

- **Example**

Fourier spectrum





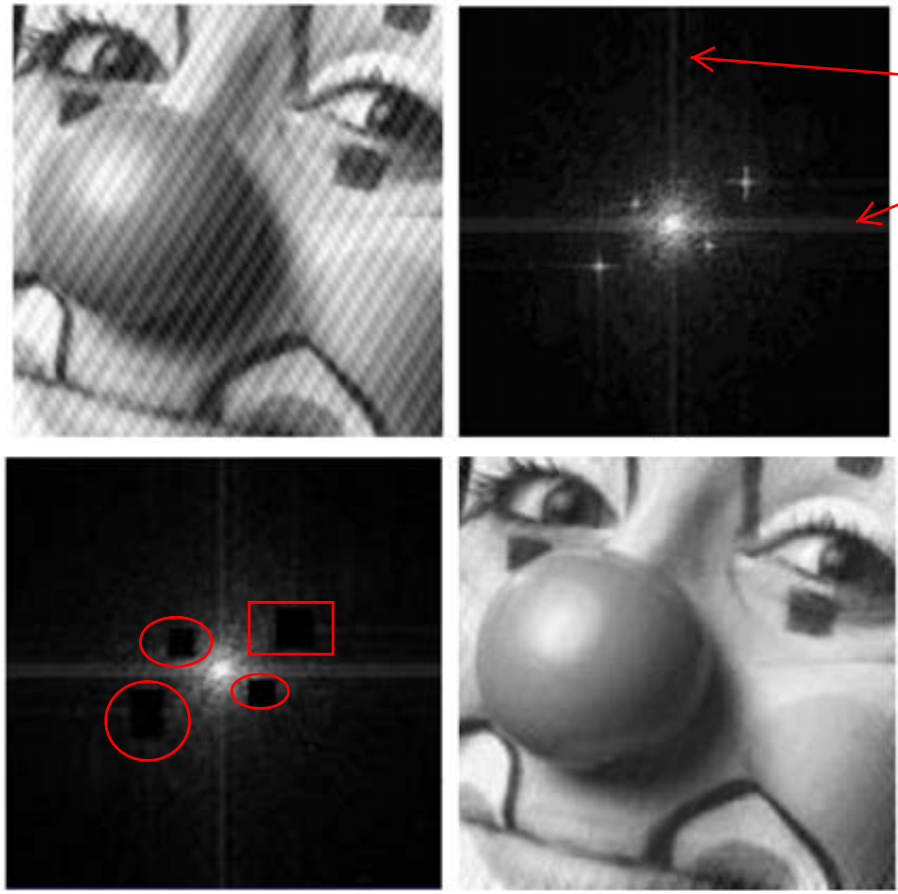
Notch filter

spatial domain
Noise pattern

Result

Image Restoration

- **Example**



try to filter them by energy threshold in frequency domain

Fourier spectrum

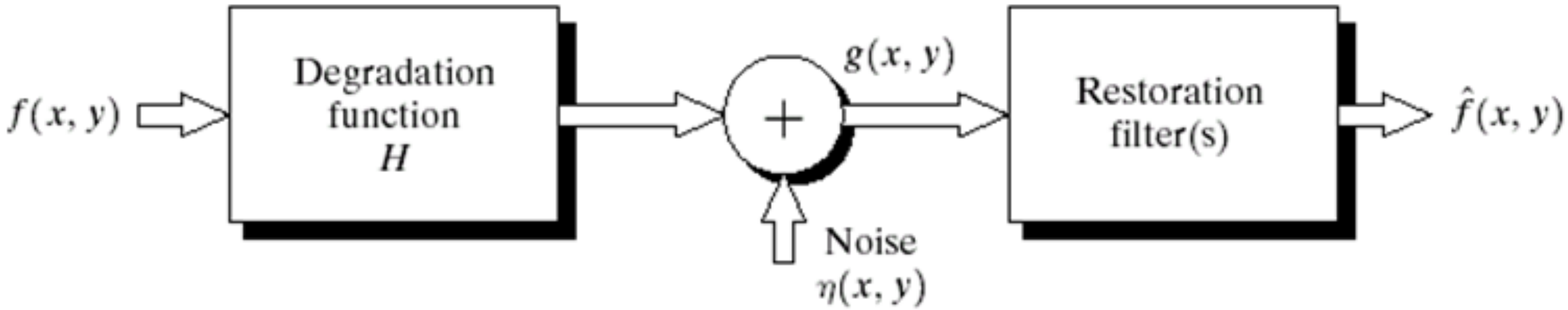
Notch filter

Result

Moallemi, Payman, and Majid Behnampourii. "Adaptive Optimum Notch Filter for Periodic Noise Reduction in Digital Images." *Amirkabir International Journal of Electrical & Electronics Engineering* 42.1 (2010): 1-7.

Image Restoration

- **Inverse Filter**



$$g(x,y) = h(x,y) * f(x,y) + \eta(x,y)$$

$$G(u,v) = H(u,v)\,F(u,v) + N(u,v)$$

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)} = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

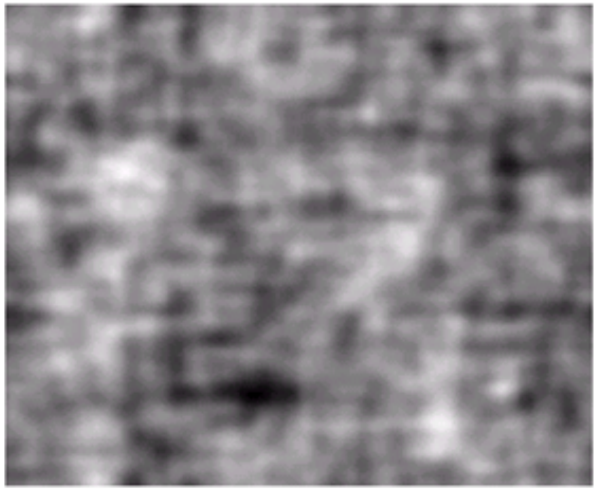
Image Restoration

- **Examples**

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)} = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

H should not be zero or near zero.

degraded image

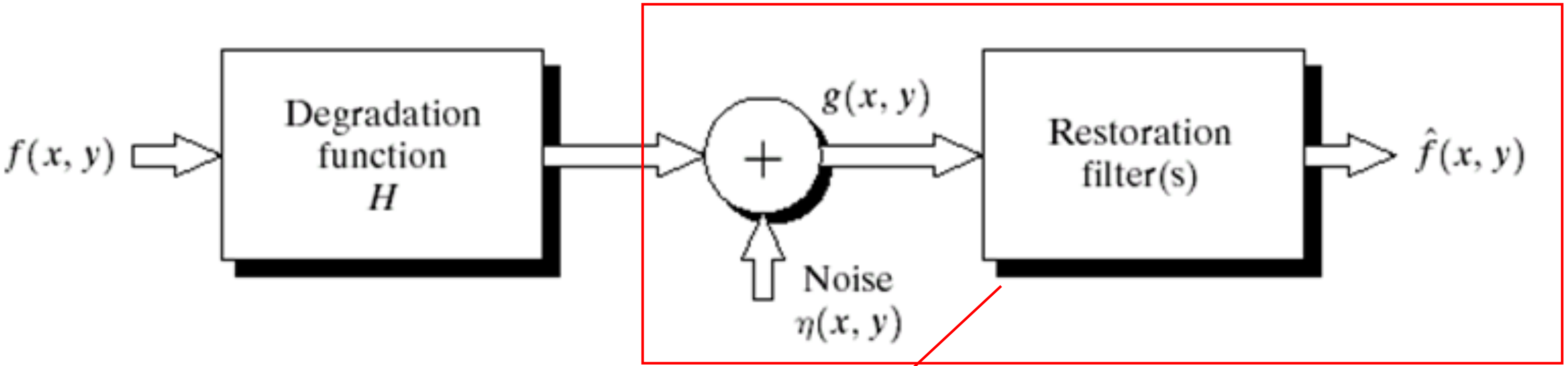


full inverse filter

limited inverse filter

Image Restoration

- **Wiener Filter**



$$g(x,y) = h(x,y) * f(x,y) + \eta(x,y)$$
$$G(u,v) = H(u,v)\,F(u,v) + N(u,v)$$
$$\hat{F}(u,v) = H_{Wiener}(u,v)\,G(u,v)$$

- **The objective is to find $H_{Wiener}(u,v)$ with minima MSE between original image and received one.**

$$\varepsilon = E[(F(u,v) - \hat{F}(u,v))^2]$$

Image Restoration

- **Wiener Filter**

$$\varepsilon = E[(F - \hat{F})^2]$$
$$= E[(F - H_{Wiener}G)^2]$$
$$= E[(F - H_{Wiener}(HF + N))^2]$$
$$= E[((1 - HH_{Wiener})F - H_{Wiener}N))^2]$$
$$= (1 - HH_{Wiener})\overline{(1 - HH_{Wiener})}E[F^2] - (1 - HH_{Wiener})\overline{H_{Wiener}}E[F\overline{N}]$$
$$- \overline{(1 - HH_{Wiener})}H_{Wiener}E[\overline{F}N] + H_{Wiener}\overline{H_{Wiener}}E[N^2]$$

Image Restoration

- **Wiener Filter**
  - Noise is independent to signal, and its mean is zero

    $$E[\overline{F}N] = 0 = E[F\overline{N}]$$

  - Define power spectral densities (PSD) as follows

    $$D_F(u,v) = E[F(u,v)^2] \quad \text{signal PSD}$$

    $$D_N(u,v) = E[N(u,v)^2] \quad \text{noise PSD}$$

  - Hence

    $$\varepsilon = (1 - HH_{Wiener})\overline{(1 - HH_{Wiener})}D_F + H_{Wiener}\overline{H_{Wiener}}D_N$$

Image Restoration

- **Wiener Filter**
  - **To minimize the error, let the derivative to be zero**

$$\frac{d\varepsilon}{dH_{Wiener}} = 0 = -H\overline{(1 - HH_{Wiener})}D_F + \overline{H_{Wiener}}D_N$$

  - **Then**

$$\boxed{H_{Wiener}(u,v) = \frac{\overline{H}(u,v)D_F(u,v)}{|H(u,v)|^2 D_F(u,v) + D_N(u,v)}}$$

Image Restoration

- **Examples**

Original image

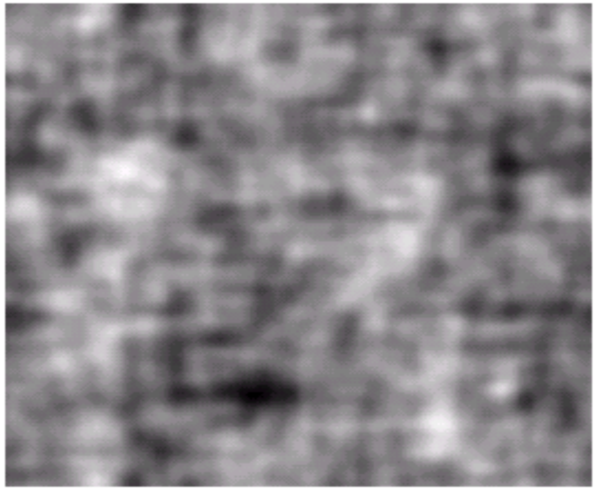
Add motion blur
and Gaussian noise

Restored by
Wiener filter

Image Restoration

- **Other examples**

degraded image



full inverse filter

limited inverse filter

Wiener filter

Best

Image Restoration

- **Other examples**

with motion blur & additive noise | **inverse filtering** | **Wiener filter**