Digital Image Processing

Lecture #3

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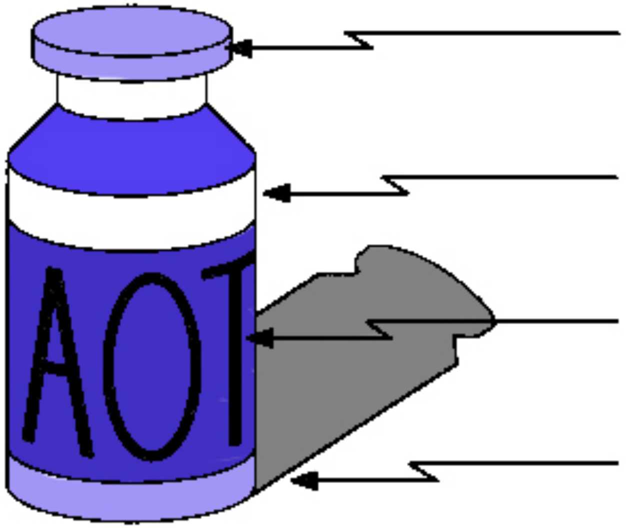
Announcement

- Class Information
  - Homework #1
    - Package posted on the course website
      - Homework assignment #1
      - Sample codes
      - Submission guideline
      - Bonus #1
    - Deadline: 11:59 a.m. on Mar. 15, 2016
    - Upload to CEIBA
      - Electronic version
      - Written report
      - Bonus

Edge Crispening

Edges

- Edges are caused by a variety of factors



Surface normal discontinuity

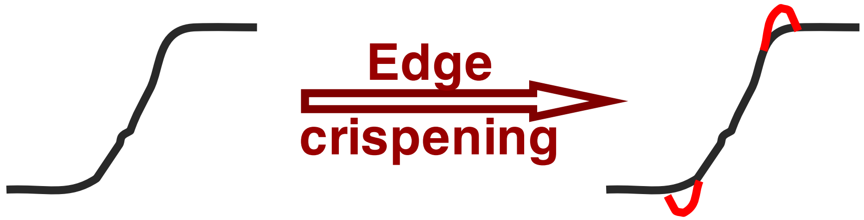
Depth discontinuity

Surface color discontinuity

Illumination discontinuity

Edge Crispening

- **Motivation**
  - **A photograph with accentuated edges look more appealing**



$$H = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

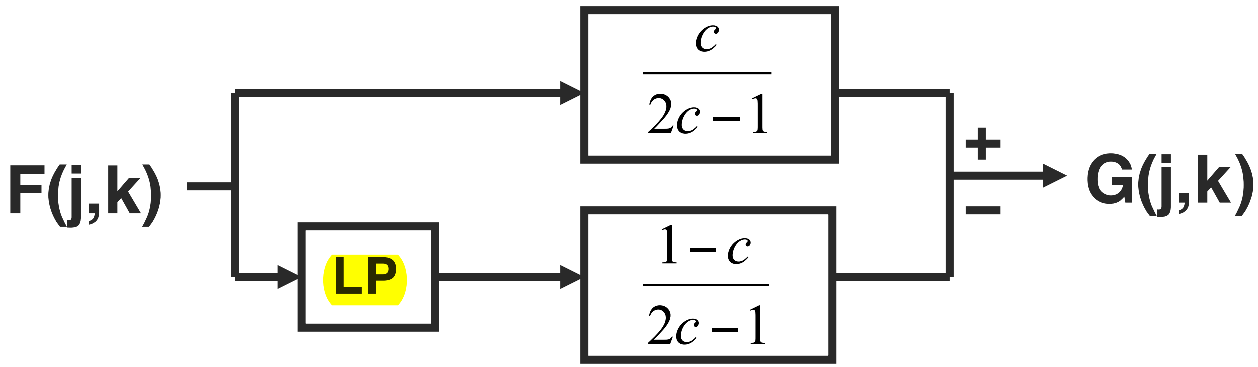
  - **Edge → high frequency**
  - **High pass filtering**

$$H = \begin{bmatrix} -1 & -1 & -1 \\ -1 & 9 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

→ amplify the noise at the same time

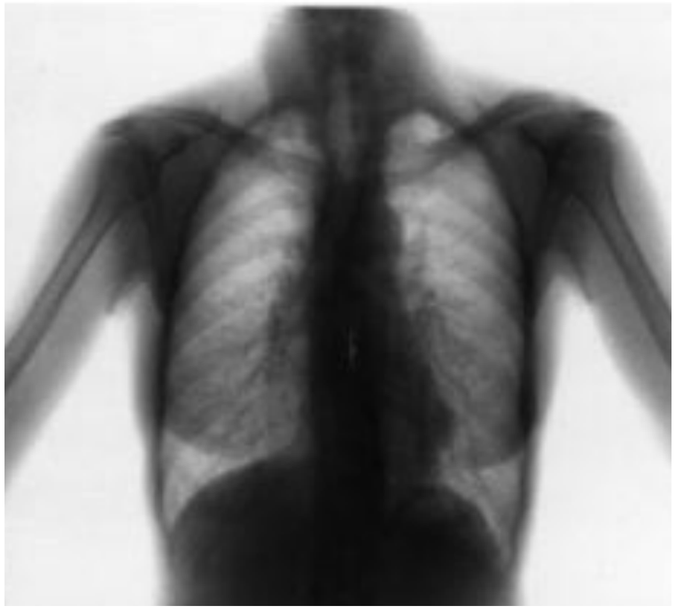
Edge Crispening

- **Unsharp Masking**
  - **Appropriate combination of all-pass and low-pass filters**

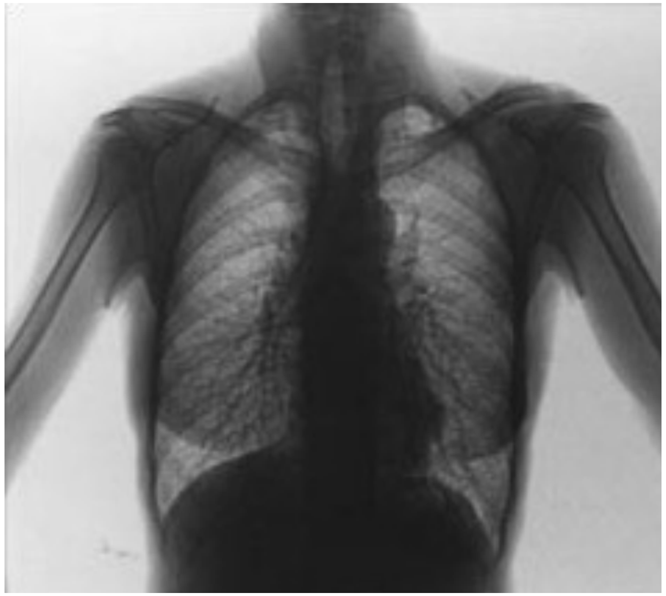


$$G(j,k) = \frac{c}{2c-1} F(j,k) - \frac{1-c}{2c-1} F_L(j,k), \quad where \quad \frac{3}{5} \le c \le \frac{5}{6}$$

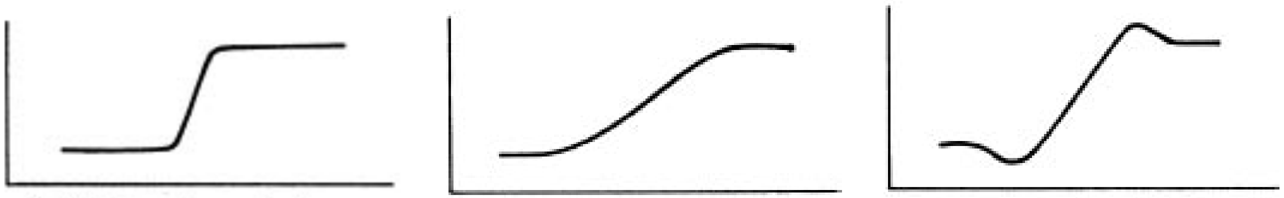
Edge Crispening



Original image



After sharpening
L=7, c=0.6



(a) Normal resolution (b) Low resolution (c) Unsharp masking

Edge Detection

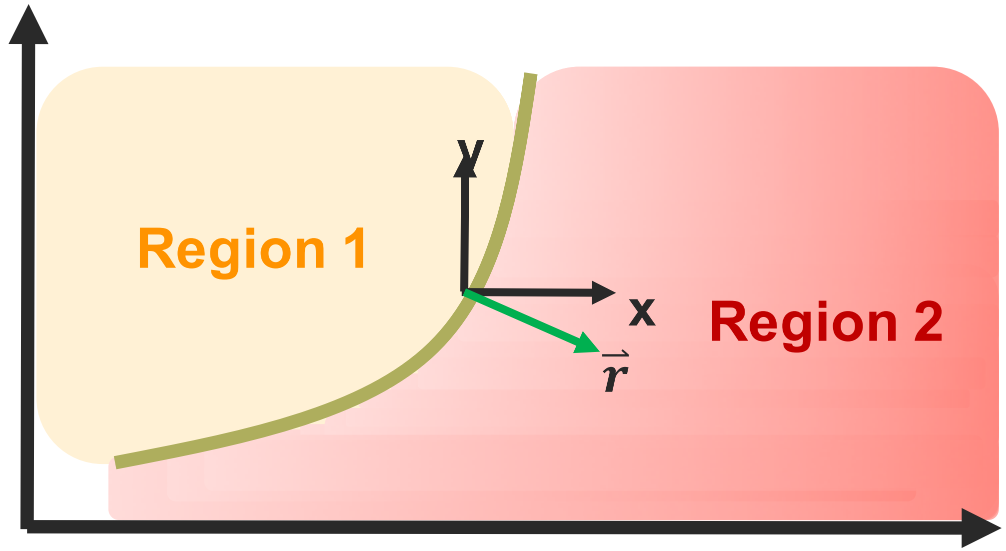
Edge Detection

- Motivation
  - Human eyes are more sensitive to edges
  - Characterize object boundaries
  - Fundamental step in image analysis
    - Segmentation, registration, identification, etc.

- Edge description
  - Model-based methods
    - Rarely used
  - Non-parametric approaches
    - 1st and 2nd order derivatives

Edge Detection (1st order)

- **Orthogonal gradient generation**



$$\frac{dF}{dr} = \frac{\partial F}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial F}{\partial y}\frac{\partial y}{\partial r} = \begin{pmatrix} \frac{\partial F}{\partial x} & \frac{\partial F}{\partial y} \end{pmatrix} \cdot \begin{pmatrix} \frac{\partial x}{\partial r} \\ \frac{\partial y}{\partial r} \end{pmatrix} = \begin{pmatrix} \frac{\partial F}{\partial x} & \frac{\partial F}{\partial y} \end{pmatrix} \cdot \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$$

Edge Detection (1st order)

- **Orthogonal gradient generation**
  - When $\begin{pmatrix} \frac{\partial F}{\partial x} \\ \frac{\partial F}{\partial y} \end{pmatrix}$ (gradient direction) is **parallel** to $\begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$

    $\left\| \frac{dF}{dr} \right\|$ has **maximum** value

$$\Rightarrow \left\| \frac{dF}{dr} \right\| = \left\| \begin{pmatrix} \frac{\partial F}{\partial x} \\ \frac{\partial F}{\partial y} \end{pmatrix} \right\| = \sqrt{\left(\frac{\partial F}{\partial x}\right)^2 + \left(\frac{\partial F}{\partial y}\right)^2} \qquad \theta = \tan^{-1}\left(\frac{\frac{\partial F}{\partial y}}{\frac{\partial F}{\partial x}} \right)$$

Magnitude

Orientation

Edge Detection (1st order)

- Discrete case
  - Approximation I – 3 points
    - Row gradient

$$\frac{\partial F}{\partial x}(j,k) \cong F(j,k) - F(j,k-1) = G_R(j,k)$$

    - Column gradient

$$\frac{\partial F}{\partial y}(j,k) \cong F(j,k) - F(j+1,k) = G_C(j,k)$$

A_0	A_1	A_2
A_7	$F(j,k)$	A_3
A_6	A_5	A_4

$$\Rightarrow \quad G(j,k) = \sqrt{G_R^2(j,k) + G_C^2(j,k)} \qquad \theta(j,k) = \tan^{-1}\left(\frac{G_C(j,k)}{G_R(j,k)}\right)$$

Edge Detection (1st order)

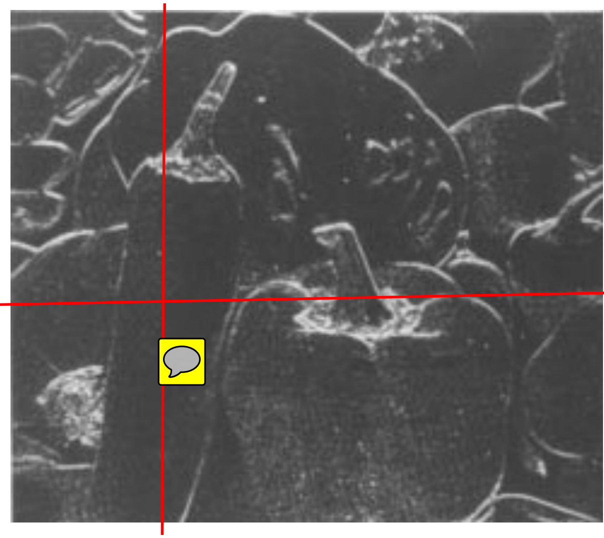
- **Example**

$$G_R(j,k) = F(j,k) - F(j,k-1)$$

$$G_C(j,k) = F(j,k) - F(j+1,k)$$

Original image

Horizontal magnitude



Vertical magnitude

Edge Detection (1st order)

- **Discrete case**
  - **Approximation II – 4 points**
  - **Roberts cross differentiation** (0~90→45~135)

A_0	A_1	A_2
A_7	$F(j,k)$	A_3
A_6	A_5	A_4

- Row gradient

$$G_1(j,k) = F(j,k) - F(j+1,k+1)$$

- Column gradient

$$G_2(j,k) = F(j,k+1) - F(j+1,k)$$

$$\Rightarrow G(j,k) = \sqrt{G_1^2(j,k) + G_2^2(j,k)} \qquad \theta(j,k) = \tan^{-1}\left(\frac{G_2(j,k)}{G_1(j,k)}\right) + \frac{\pi}{4}$$

Edge Detection (1st order)

- Discrete case
  - Approximation III –9points
    - Row gradient

$$G_R(j,k) = \frac{1}{K+2}\left[\left(A_2 + KA_3 + A_4\right) - \left(A_0 + KA_7 + A_6\right)\right]$$

    - Column gradient

$$G_C(j,k) = \frac{1}{K+2}\left[\left(A_0 + KA_1 + A_2\right) - \left(A_6 + KA_5 + A_4\right)\right]$$

A_0	A_1	A_2
A_7	$F(j,k)$	A_3
A_6	A_5	A_4

$$\Rightarrow G(j,k) = \sqrt{G_R^2(j,k) + G_C^2(j,k)} \qquad \theta(j,k) = \tan^{-1}\left(\frac{G_C(j,k)}{G_R(j,k)}\right)$$

- **k=1: Prewitt Mask ; k=2: Sobel Mask**

Edge Detection (1st order)

- Compute row and column gradients
- Analyze the statistics of the magnitude (histogram)

- Pick a threshold T
- If G(j,k)>=T
  → set it as an edge point
  otherwise (If G(j,k)<T)
  → non-edge point
- **Q: How to select T?**
- Examine the cumulative distribution function

Canny Edge Detector

- **Why Canny?**
  - **Good Detection**
    - The optimal detector must minimize the probability of false positives as well as false negatives
  - **Good Localization**
    - The edges detected must be as close as possible to the true edges
  - **Single Response Constraint**
    - The detector must return one point only for each edge point

Canny Edge Detector

- **Five Steps:**
  - Noise reduction
  - Compute gradient magnitude and orientation
  - Non-maximal suppression
  - Hysteretic thresholding
  - Connected component labeling method

Smooth the image with a Gaussian filter

Example:
5x5 Gaussian filter with $\sigma = 1.4$

$$F_{NR} = \frac{1}{159}\begin{bmatrix} 2 & 4 & 5 & 4 & 2 \\ 4 & 9 & 12 & 9 & 4 \\ 5 & 12 & 15 & 12 & 5 \\ 4 & 9 & 12 & 9 & 4 \\ 2 & 4 & 5 & 4 & 2 \end{bmatrix} * F$$

Canny Edge Detector

- **Five Steps:**
  - Noise reduction
  - Compute gradient magnitude and orientation
  - Non-maximal suppression
  - Hysteretic thresholding
  - Connected component labeling method

$$G(j,k) = \sqrt{G_R^2(j,k) + G_C^2(j,k)}$$

$$\theta(j,k) = \tan^{-1}\left(\frac{G_C(j,k)}{G_R(j,k)}\right)$$

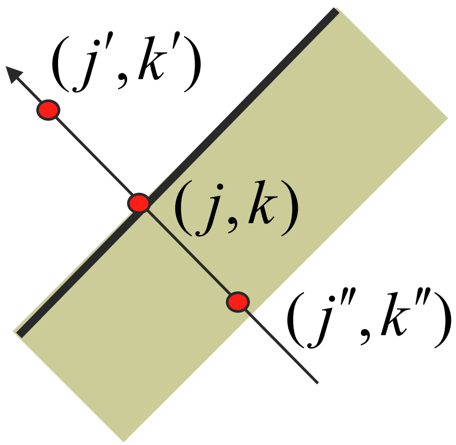
Magnitude

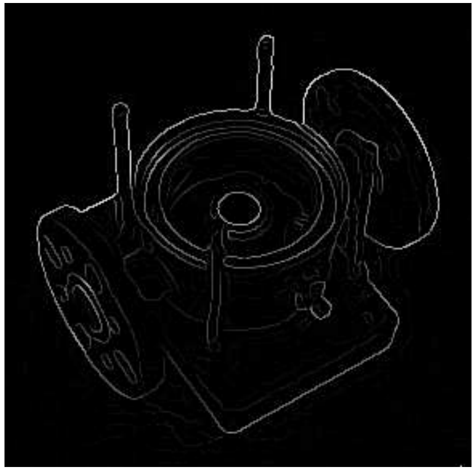
Canny Edge Detector

- **Five Steps:**
  - Noise reduction
  - Compute gradient magnitude and orientation
  - Non-maximal suppression
  - Hysteretic thresholding
  - Connected component labeling method

Search the nearest neighbors (j',k') and (j'',k'') along the edge normal

$$G_N(j,k) = \begin{cases} G(j,k) & if\ G(j,k) > G(j',k') \\ & and\ G(j,k) > G(j'',k'') \\ 0 & \end{cases}$$





Canny Edge Detector

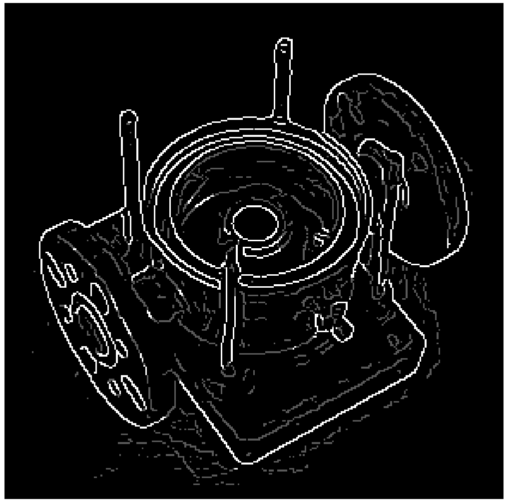
- **Five Steps:**
  - Noise reduction
  - Compute gradient magnitude and orientation
  - Non-maximal suppression
  - Hysteretic thresholding
  - Connected component labeling method

Label each pixels according to two threshold: T_H, T_L

$G_N(x,y) \geq T_H$ Edge Pixel

$T_H > G_N(x,y) \geq T_L$ Candidate Pixel

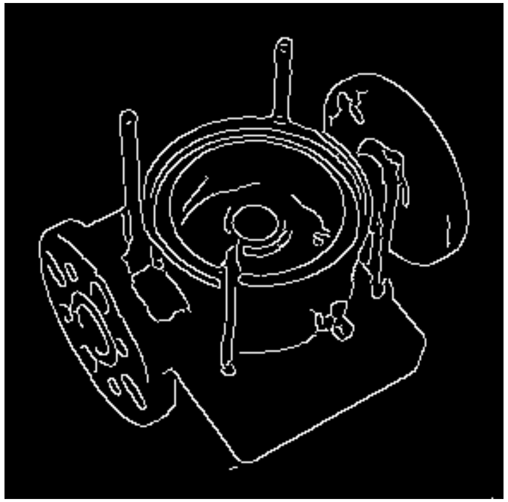
$G_N(x,y) < T_L$ Non-edge Pixel



Canny Edge Detector

- **Five Steps:**
  - Noise reduction
  - Compute gradient magnitude and orientation
  - Non-maximal suppression
  - Hysteretic thresholding
  - Connected component labeling method

If a candidate pixel is connected to an edge pixel directly or via another candidate pixel then it is declared as an edge pixel



Edge Map

Edge Detection – Part II

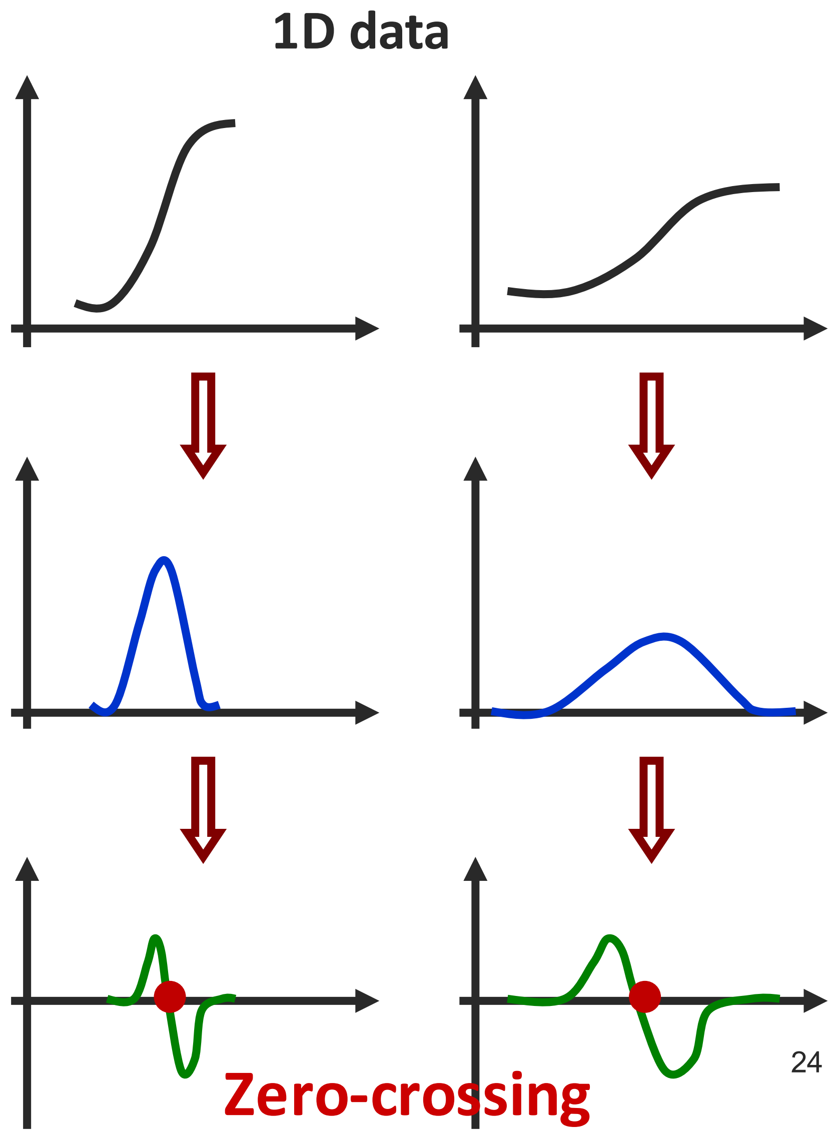
Edge Detection (2^{nd} order)

- **Why 2nd order?**
  - Significant spatial change occurs

F

1st order derivative $\frac{\partial F}{\partial x}$

2nd order derivative $-\frac{\partial^2 F}{\partial x^2}$



Edge Detection (2nd order)

- **Laplacian Generation**

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \Rightarrow \nabla^2 F(x,y) = \frac{\partial^2 F(x,y)}{\partial x^2} + \frac{\partial^2 F(x,y)}{\partial y^2}$$

- **Discrete Approximation**

$$-\frac{\partial^2}{\partial x^2} \cong \begin{bmatrix} -1 & 2 & -1 \end{bmatrix} \Rightarrow 2f(x) - (f(x+h) + f(x-h))$$

  - **By Taylor series expansion**

$$2f(x) - (f(x+h) + f(x-h))$$

$$= \quad 2f(x) - \left[f(x) + hf'(x) + \frac{h^2}{2} f''(x) + f(x) + (-h)f'(x) + \frac{h^2}{2} f''(x) + \cdots \right]$$

$$\cong \quad \boxed{-h^2 f''(x)}$$

Edge Detection (2^{nd} order)

- **Discrete Approximation**

$$-\frac{\partial^2}{\partial x^2} \cong \begin{bmatrix} -1 & 2 & -1 \end{bmatrix} \qquad -\frac{\partial^2}{\partial y^2} \cong \begin{bmatrix} -1 & 2 & -1 \end{bmatrix}^T$$

  - **combine together**

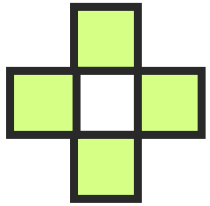
$$-\nabla^2 \cong \frac{1}{4}\begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}; \qquad \nabla^2 \cong \frac{1}{4}\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

gain-normalized

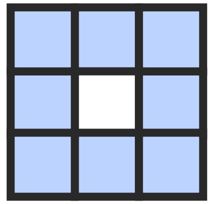
Edge Detection (2^{nd} order)

- Laplacian impulse response
  - four-neighbor

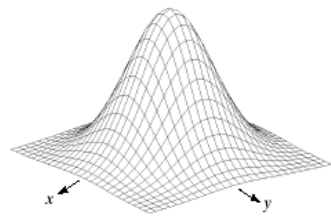
$$H = \frac{1}{4}\begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$



4-neighbor



8-neighbor



  - eight-neighbor

$$H_1 = \frac{1}{8}\begin{bmatrix} -1 & 2 & -1 \\ -1 & 2 & -1 \\ -1 & 2 & -1 \end{bmatrix}$$

$$H = \frac{1}{8}\begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

non-separable

$$H = \frac{1}{8}\begin{bmatrix} -2 & 1 & -2 \\ 1 & 4 & 1 \\ -2 & 1 & -2 \end{bmatrix}$$

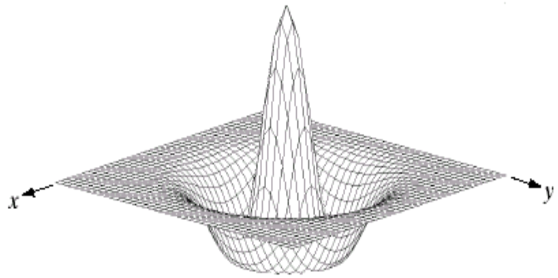
separable

$$H_2 = \frac{1}{8}\begin{bmatrix} -1 & -1 & -1 \\ 2 & 2 & 2 \\ -1 & -1 & -1 \end{bmatrix}$$

Edge Detection (2nd order)

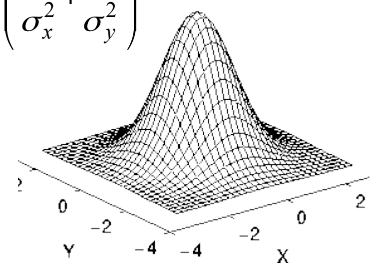
- **Laplacian of Gaussian (LOG) – p.474**

Gaussian Low-pass Filter → 2nd-order Derivative (Laplacian) →



Mexican Hat Low-pass Filter

$$g(x,y) = Ae^{-\frac{1}{2}\left(\frac{x^2}{\sigma_x^2}+\frac{y^2}{\sigma_y^2}\right)}$$



$\frac{1}{273}$

1	4	7	4	1
4	16	26	16	4
7	26	41	26	7
4	16	26	16	4
1	4	7	4	1

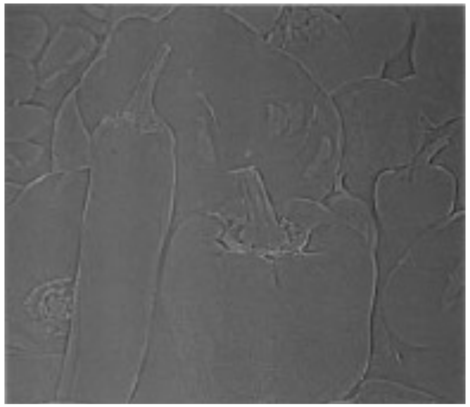
- **Difference of Gaussians (DOG)**

Gaussian Low-pass Filter → 1st-order Derivative →

Edge Detection (2nd order)

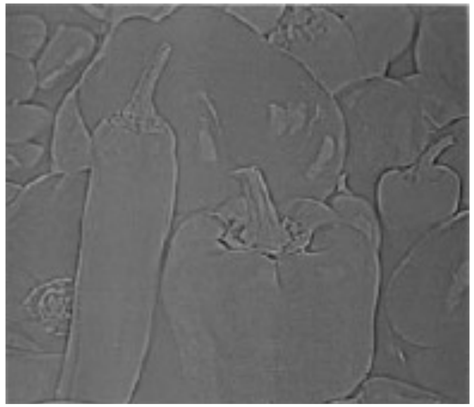
- Examples

Four-neighbor



Eight-neighbor

Separable eight-neighbor



Laplacian of Gaussian (LOG)

Are we done yet?

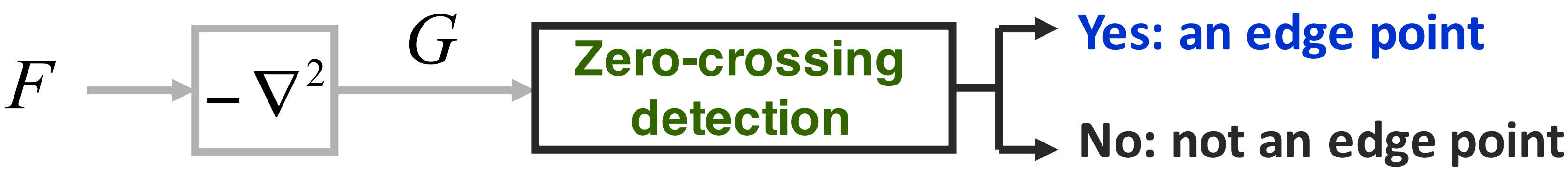
Edge Detection (2nd order)

- 2nd Order Edge Detection

  - How to detect zero-crossing?
    - many ways

Edge Detection (2nd order)

- **Zero-crossing**

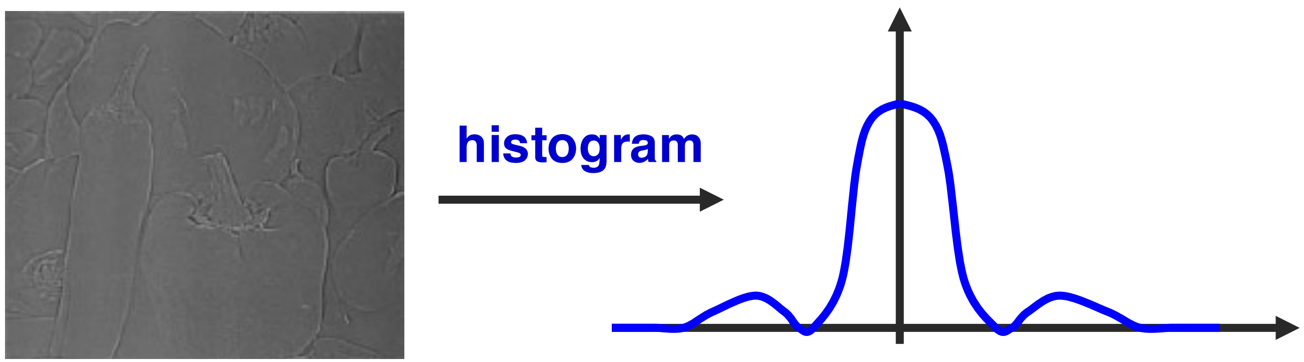


3 steps:

- Generate the histogram of G
- Set up a threshold to separate zero and non-zero, G’
- For G’(j,k)=0, decide whether (j,k) is a zero-crossing point

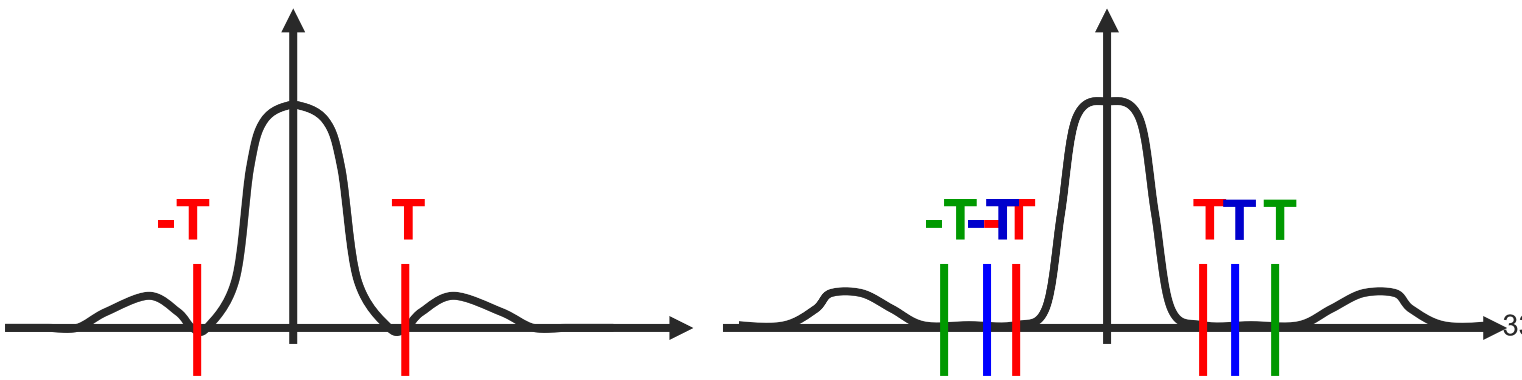
Edge Detection (2^{nd} order)

- **Zero-crossing**
  - **3 steps:**
    - **Generate the histogram of G**
    - **Set up a threshold to separate zero and non-zero to get G'**
    - **For G'(j,k)=0, decide whether (j,k) is a zero-crossing point**



Edge Detection (2nd order)

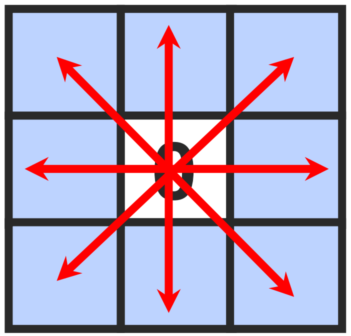
- Zero-crossing
  - 3 steps:
    - Generate the histogram of G
    - **Set up a threshold to separate zero and non-zero to get G'** $|G(j,k)| \le T \Rightarrow G'(j,k) = 0$
    - For G'(j,k)=0, decide whether (j,k) is a zero-crossing point



Edge Detection (2^{nd} order)

- **Zero-crossing**
  - **3 steps:**
    - Generate the histogram of G
    - Set up a threshold to separate zero and non-zero to get G'
    - **For G'(j,k)=0, decide whether (j,k) is a zero-crossing point → edge map**

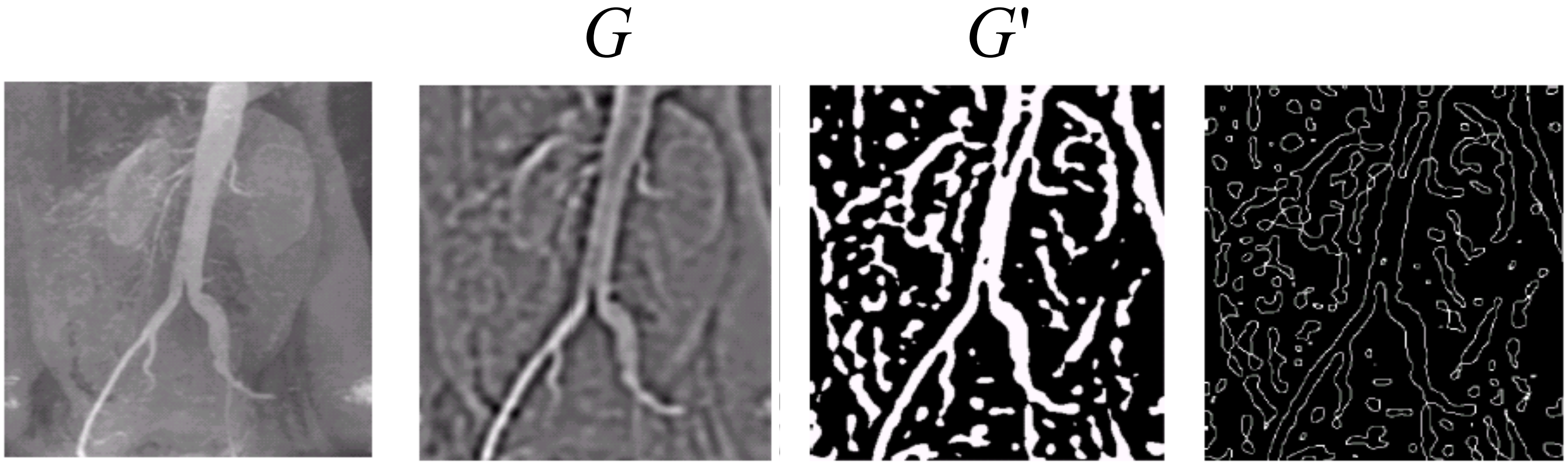
$$G'(j,k) = 0$$



$$\{-1,0,1\}$$

Edge Detection (2nd order)

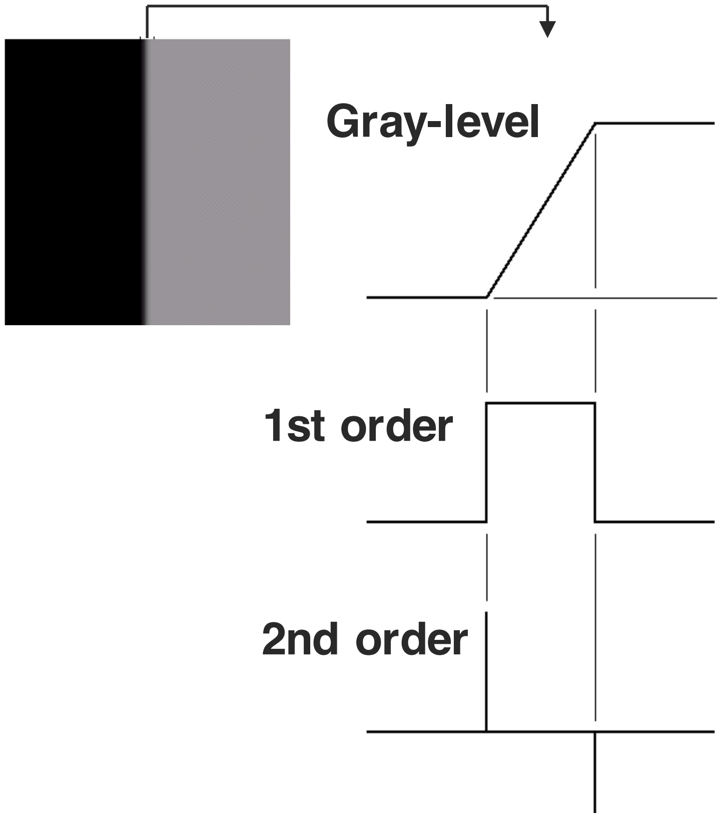
- **Example**

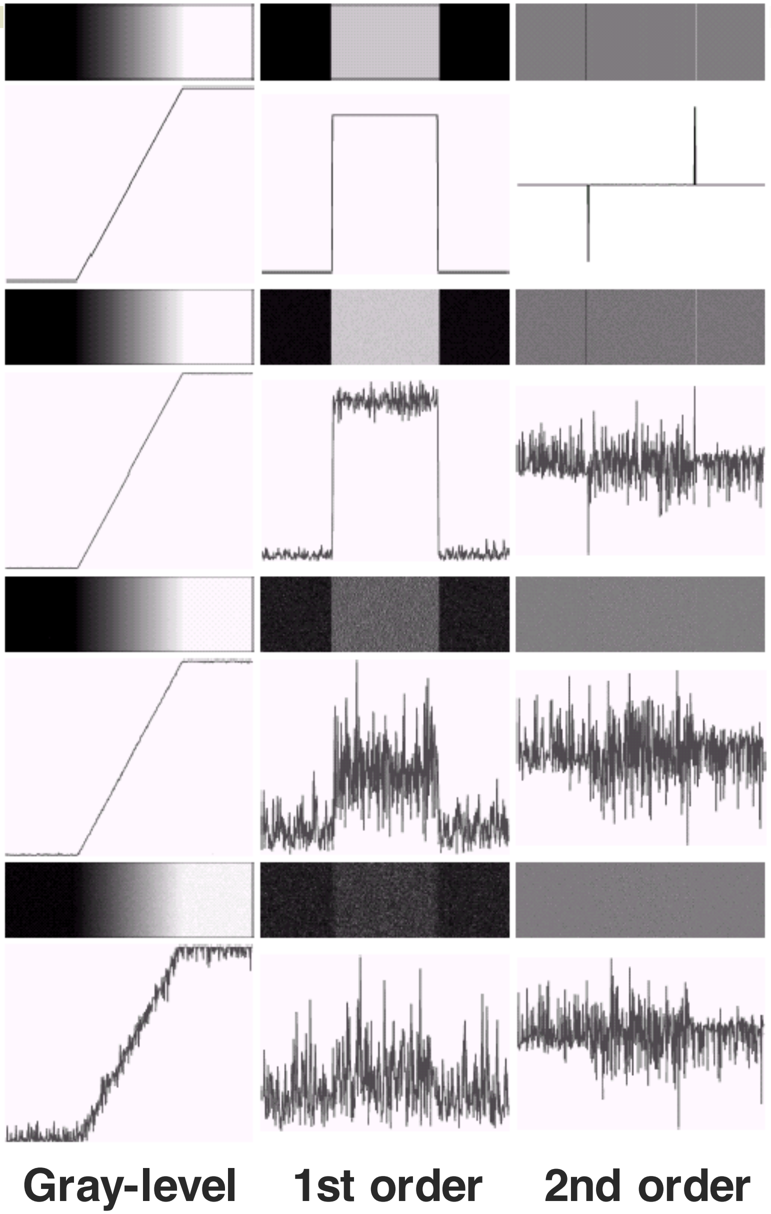


Original | Laplacian | Thresholded | Edge map

Edge Detection

- Noisy image





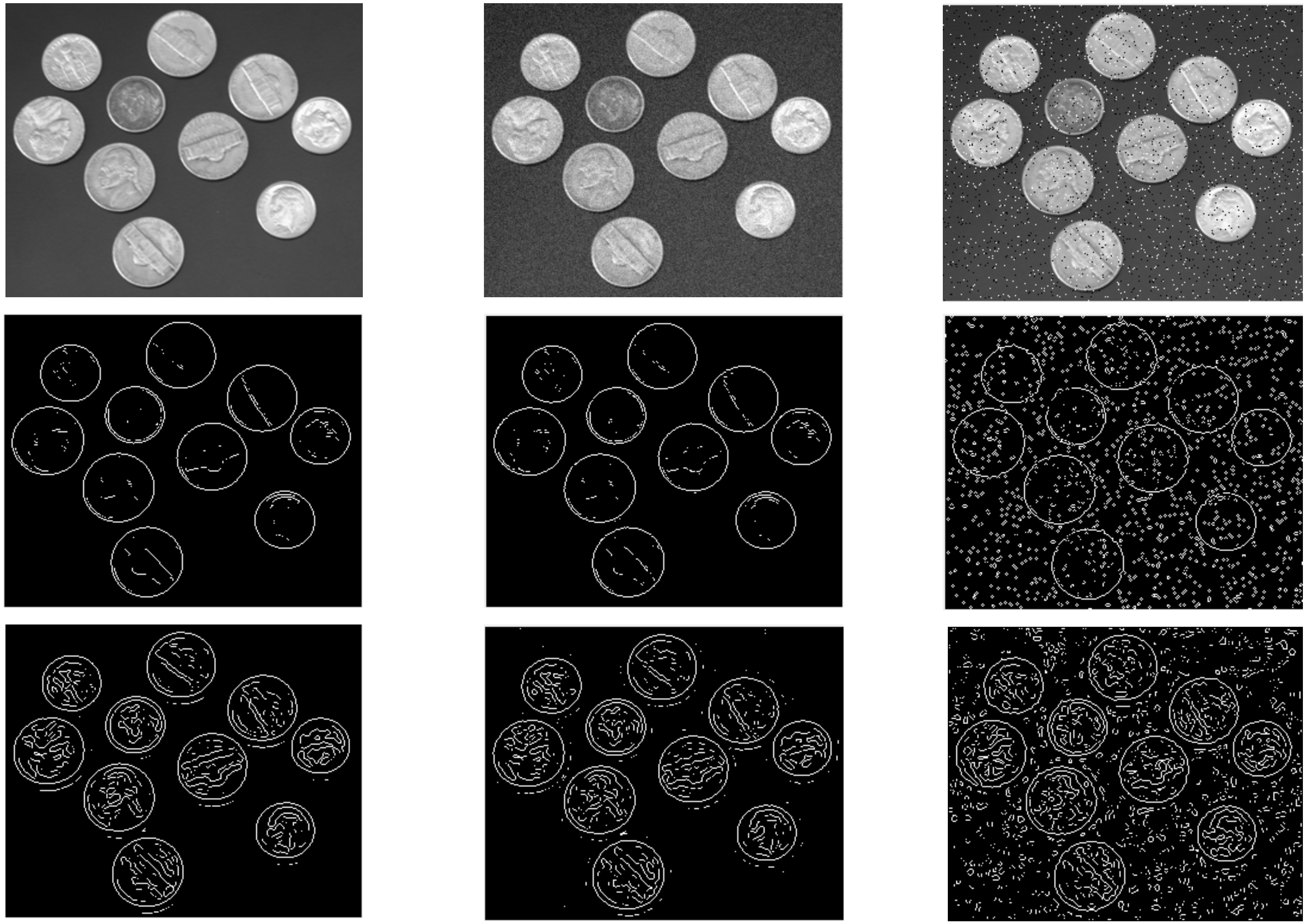
"Pre-processing" ⇐

Edge Detection

I | I + Gaussian noise | I + impulse noise

1st order

2nd order



Edge Detection

- **Post-Processing**

Edge Map	Binary image Edge points → 1; non-edge points → 0
↓	
Edge Linking	Local processing: Based on edge orientation Global processing: Hough transform
↓	
Boundary Curves	A complete edge contour

Review

Review

- Noise Cleaning
  - Uniform noise → low-pass filtering
  - Impulse noise → non-linear filtering
  - Mixed noise → ?
- Edge Crispening
  - Unsharp masking
- Edge Detection
  - 1^{st}-order edge detection -- threshold
  - 2^{nd}-order edge detection -- zero-crossing

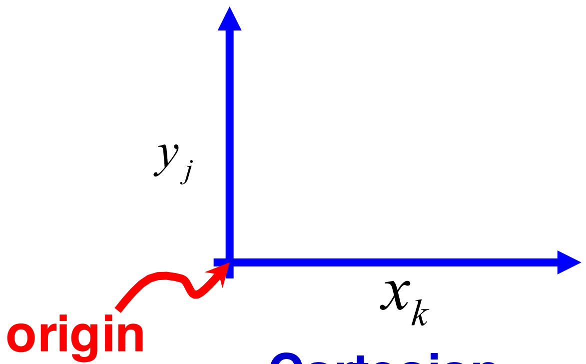
Geometrical Modification

Geometrical Modification

- Goal
  - Translate, scale, rotate, reflect or nonlinear warp an image
- Applications
  - Zoom-in/zoom-out
  - Image registration
  - Image mosaicking
  - Special effects
    - Use 2D image to simulate the 3D environment
    - http://www.erich3d.com/
  - Etc.

Geometrical Modification

- Coordinates
  - Geometrical transformations
    - Cartesian coordinates
  - Discrete image
    - Cartesian coordinates v.s. Image coordinates



Cartesian Coordinates

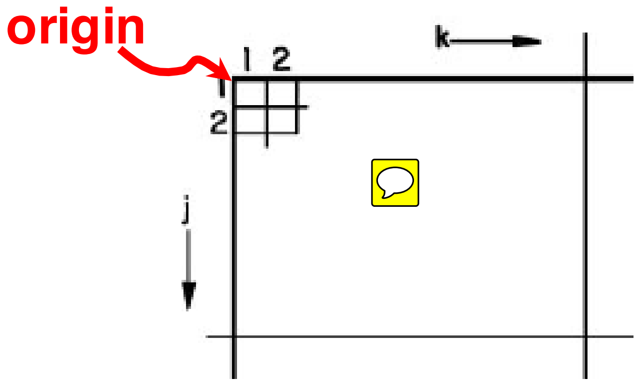


Image Coordinates

Geometrical Modification

- **Linear/Affine coordinates transformation**

$F(p,q)$

$P \times Q$

$1 \le p \le P$

$1 \le q \le Q$

$G(j,k)$

$J \times K$

$1 \le j \le J$

$1 \le k \le K$

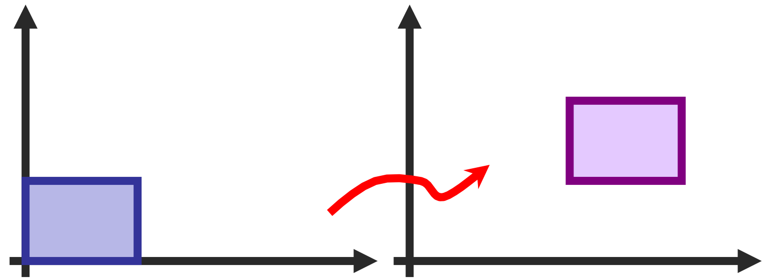
$$\begin{cases} u_q = q - \boxed{\dfrac{1}{2}} \\ v_p = P + \boxed{\dfrac{1}{2}} - p \end{cases} \qquad \begin{cases} x_k = k - \boxed{\dfrac{1}{2}} \\ y_j = J + \boxed{\dfrac{1}{2}} - j \end{cases}$$

<< Cartesian Coordinates >>

Geometrical Modification

- **Translation (Shift)**

$$\begin{cases} x_k = u_q + t_x \\ y_j = v_p + t_y \end{cases}$$



substitute $\begin{cases} u_q = q - \frac{1}{2} \\ v_p = P + \frac{1}{2} - p \end{cases}$ **and** $\begin{cases} x_k = k - \frac{1}{2} \\ y_j = J + \frac{1}{2} - j \end{cases}$

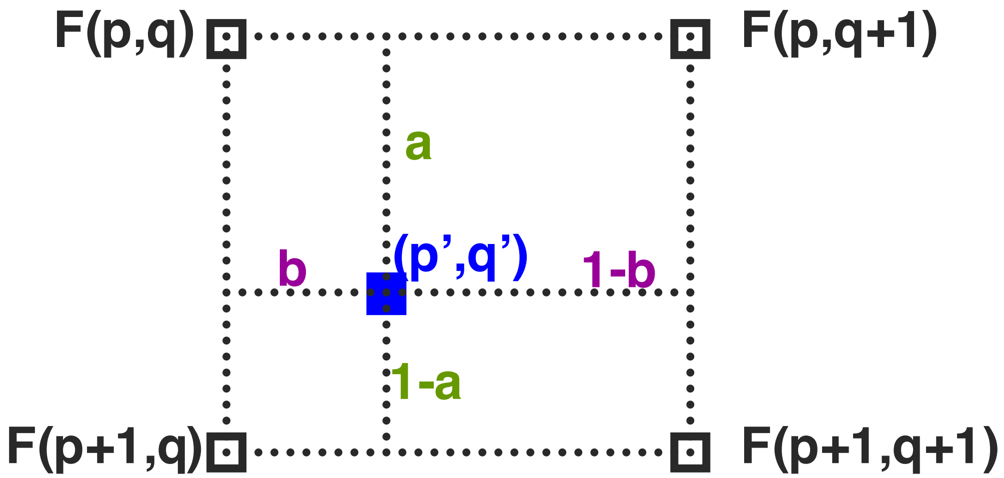
$\Rightarrow \begin{cases} k' = q + t_x \\ j' = p - (P - J) - t_y \end{cases}$ $\begin{cases} k = q' + t_x \\ j = p' - (P - J) - t_y \end{cases}$

Forward treatment **Backward treatment**

Better

Geometrical Modification

- **Translation (Shift)**
  - **Non-integer pixel positions**
  - **i.e. How to compute p’ and q’?**
    - **Bilinear interpolation**

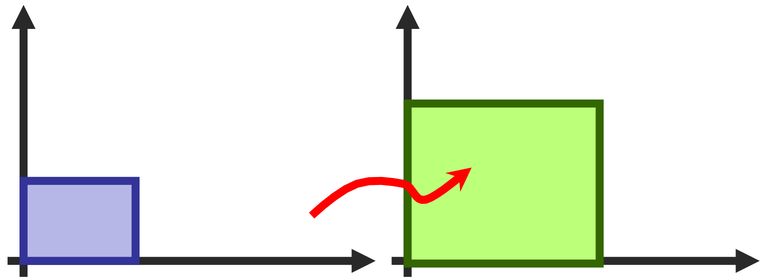


$$
\begin{aligned}
F(p',q') &= (1-a)(1-b)F(p,q) \\
&+ (1-a)bF(p,q+1) \\
&+ a(1-b)F(p+1,q) \\
&+ abF(p+1,q+1)
\end{aligned}
$$

Geometrical Modification

- **Scaling**

$$\begin{cases} x_k = s_x u_q \\ y_j = s_y v_p \end{cases}$$



where $s_x \,\&\, s_y$ are scaling parameters, and $s_x \,\&\, s_y > 0$

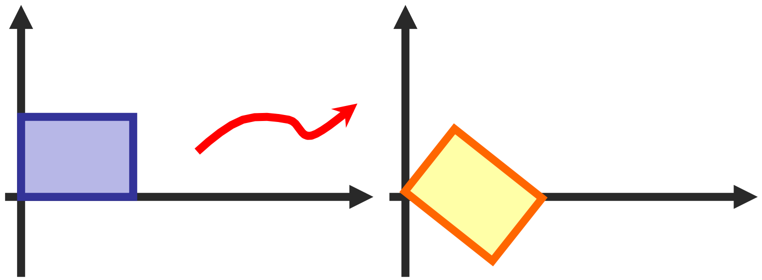
$$\begin{cases} s_x \,\&\, s_x > 1: \quad magnification \\ s_x \,\&\, s_x < 1: \quad \min ification \end{cases}$$

$$\Rightarrow \begin{cases} p' = \frac{1}{s_y}(j - J - \frac{1}{2}) + P + \frac{1}{2} \\ q' = \frac{1}{s_x}(k - \frac{1}{2}) + \frac{1}{2} \end{cases}$$

Geometrical Modification

- **Rotation**

$$\begin{cases} x_k &= u_q \cos\theta - v_p \sin\theta \\ y_j &= u_q \sin\theta + v_p \cos\theta \end{cases}$$



Rotate by an angle with respect to the origin of the Cartesian coordinates

What if the reference point is not the origin of the Cartesian coordinate?

Geometrical Modification

- **Generalized Linear Geometrical Transformations**
  - **translation** $$\begin{bmatrix} x_k \\ y_j \end{bmatrix} = \begin{bmatrix} u_q \\ v_p \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$
  - **scaling** $$\begin{bmatrix} x_k \\ y_j \end{bmatrix} = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} u_q \\ v_p \end{bmatrix}$$
  - **rotation** $$\begin{bmatrix} x_k \\ y_j \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} u_q \\ v_p \end{bmatrix}$$

Geometrical Modification

- **Generalized Linear Geometrical Transformations**
  - **Compound operator**

$$\begin{bmatrix} u_q \\ v_p \end{bmatrix} \rightarrow \text{translation} \rightarrow \text{scaling} \rightarrow \text{rotation} \rightarrow \begin{bmatrix} x_k \\ y_j \end{bmatrix}$$

$$\begin{bmatrix} x_k \\ y_j \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \left[\begin{pmatrix} u_q \\ v_p \end{pmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix} \right]$$

How to convert the above affine system to a linear one?

Geometrical Modification

- **Generalized Linear Geometrical Transformations**
  - **Expand the system from 2D to 3D**

$$T(t_x,t_y)=\begin{bmatrix}1 & 0 & t_x\\ 0 & 1 & t_y\\ 0 & 0 & 1\end{bmatrix}\quad S(s_x,s_y)=\begin{bmatrix}s_x & 0 & 0\\ 0 & s_y & 0\\ 0 & 0 & 1\end{bmatrix}\quad R(\theta)=\begin{bmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$\begin{bmatrix}x_k\\ y_j\\ 1\end{bmatrix}=R(\theta)S(s_x,s_y)T(t_x,t_y)\begin{bmatrix}u_q\\ v_p\\ 1\end{bmatrix}\Rightarrow\begin{bmatrix}u_q\\ v_p\\ 1\end{bmatrix}=T^{-1}(t_x,t_y)S^{-1}(s_x,s_y)R^{-1}(\theta)\begin{bmatrix}x_k\\ y_j\\ 1\end{bmatrix}$$

Geometrical Modification

- Exercise
  - Write down a linear system which represents the following operation:
    - Rotate an image by an angle of θ w.r.t. a pivot point (x_c, y_c)

$$H = \begin{bmatrix} 1 & 0 & x_c \\ 0 & 1 & y_c \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -x_c \\ 0 & 1 & -y_c \\ 0 & 0 & 1 \end{bmatrix}$$

Geometrical Modification Part II

Geometrical Modification

- **Non-linear Coordinates Transformation and Spatial Warping**
  - Non-linear address mapping
    - Forward $\begin{cases} x = X\{u,v\} \\ y = Y\{u,v\} \end{cases}$
    - Backward (reverse) $\begin{cases} u = U\{x,y\} \\ v = V\{x,y\} \end{cases}$

$$\begin{cases} u_q = q - \frac{1}{2} \\ v_p = P + \frac{1}{2} - p \end{cases} \boxed{\text{input}} \Longrightarrow \boxed{\text{output}} \begin{cases} x_k = k - \frac{1}{2} \\ y_j = J + \frac{1}{2} - j \end{cases}$$

Geometrical Modification

- **Polynomial Warping (2nd-order)**

$$\begin{cases} u = a_0 + a_1 x + a_2 y + a_3 x^2 + a_4 xy + a_5 y^2 \\ v = b_0 + b_1 x + b_2 y + b_3 x^2 + b_4 xy + b_5 y^2 \end{cases}$$

$$\Rightarrow \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} a_0 & a_1 & a_2 & a_3 & a_4 & a_5 \\ b_0 & b_1 & b_2 & b_3 & b_4 & b_5 \end{bmatrix} \begin{bmatrix} 1 \\ x \\ y \\ x^2 \\ xy \\ y^2 \end{bmatrix}$$

Geometrical Modification

- **Polynomial Warping**

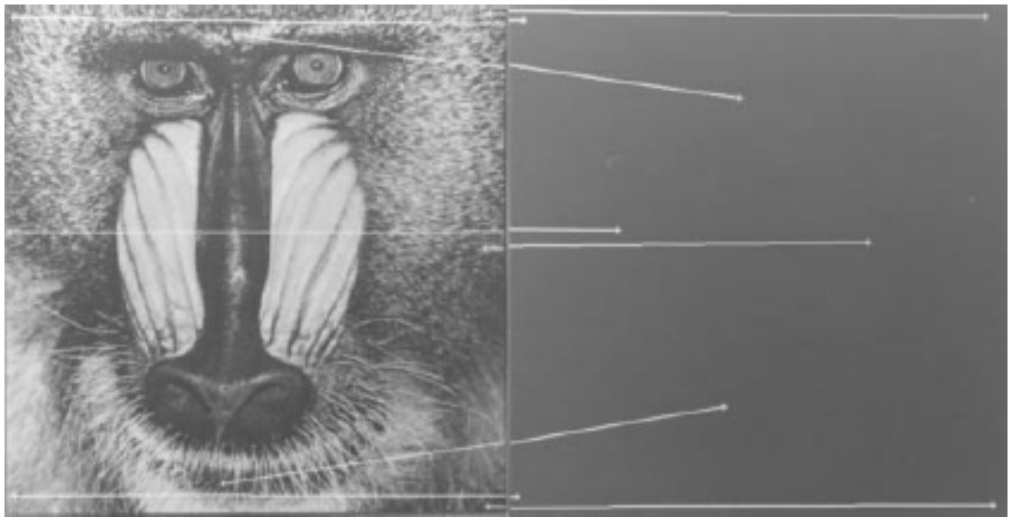
  - Rubber-sheet stretching

  - **Identify spatial distortion**
    - **Calibration → test patterns**
    - **Two steps:**
      - **Based on 'known' input and output pairs (control points), compute the coefficients 'a' and 'b' (either exact or least squares solution)**
      - **Use the spatial warping matrix to compute all the output points from their corresponding input points**

→ proper interpolation is necessary

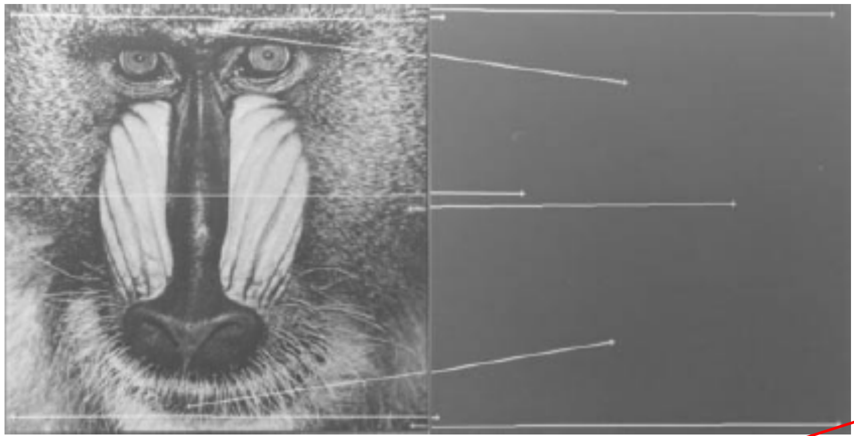
Geometrical Modification

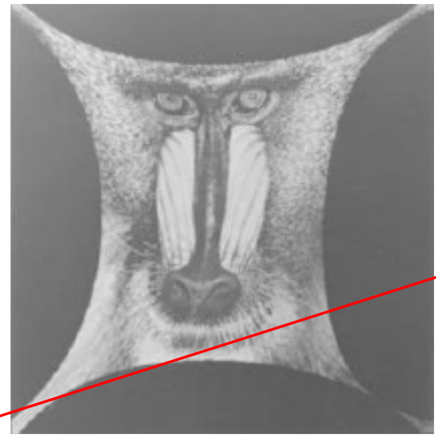
- **Polynomial Warping**
  - **Example**



Geometrical Modification

- **Example**



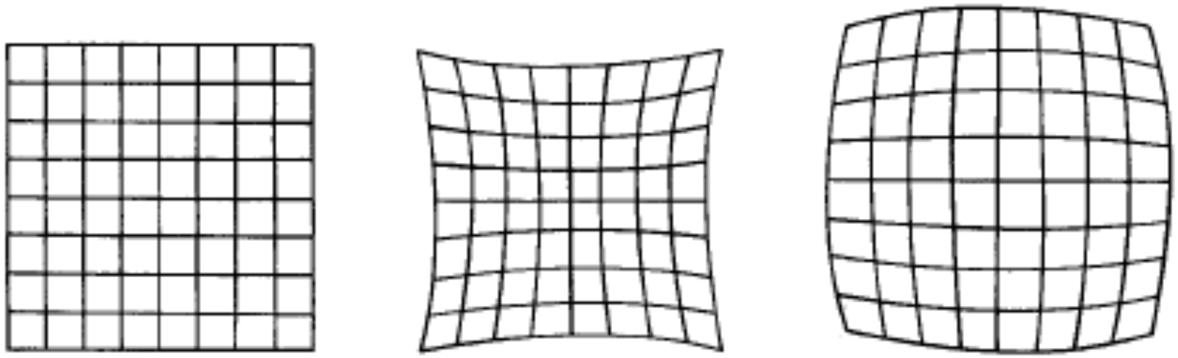


$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} a_0 & a_1 & a_2 & a_3 & a_4 & a_5 \\ b_0 & b_1 & b_2 & b_3 & b_4 & b_5 \end{bmatrix} \begin{bmatrix} 1 \\ x \\ y \\ x^2 \\ xy \\ y^2 \end{bmatrix}$$

$$\begin{bmatrix} u_1 & u_2 & \cdots & u_k \\ v_1 & v_2 & \cdots & v_k \end{bmatrix} = \begin{bmatrix} a_0 & a_1 & a_2 & a_3 & a_4 & a_5 \\ b_0 & b_1 & b_2 & b_3 & b_4 & b_5 \end{bmatrix} \begin{bmatrix} 1 & 1 & \cdots & 1 \\ x_1 & x_2 & \cdots & x_k \\ y_1 & y_2 & \cdots & y_k \\ x_1^2 & x_2^2 & \cdots & x_k^2 \\ x_1 y_1 & x_2 y_2 & \cdots & x_k y_k \\ y_1^2 & y_2^2 & \cdots & y_k^2 \end{bmatrix}$$

Geometrical Modification

- **Polynomial Warping**
  - **Useful to compensate the spatial distortion caused by the limitation of a physical imaging system**



Original　　Pincusion distortion　　Barrel distortion

Geometrical Modification

- Examples

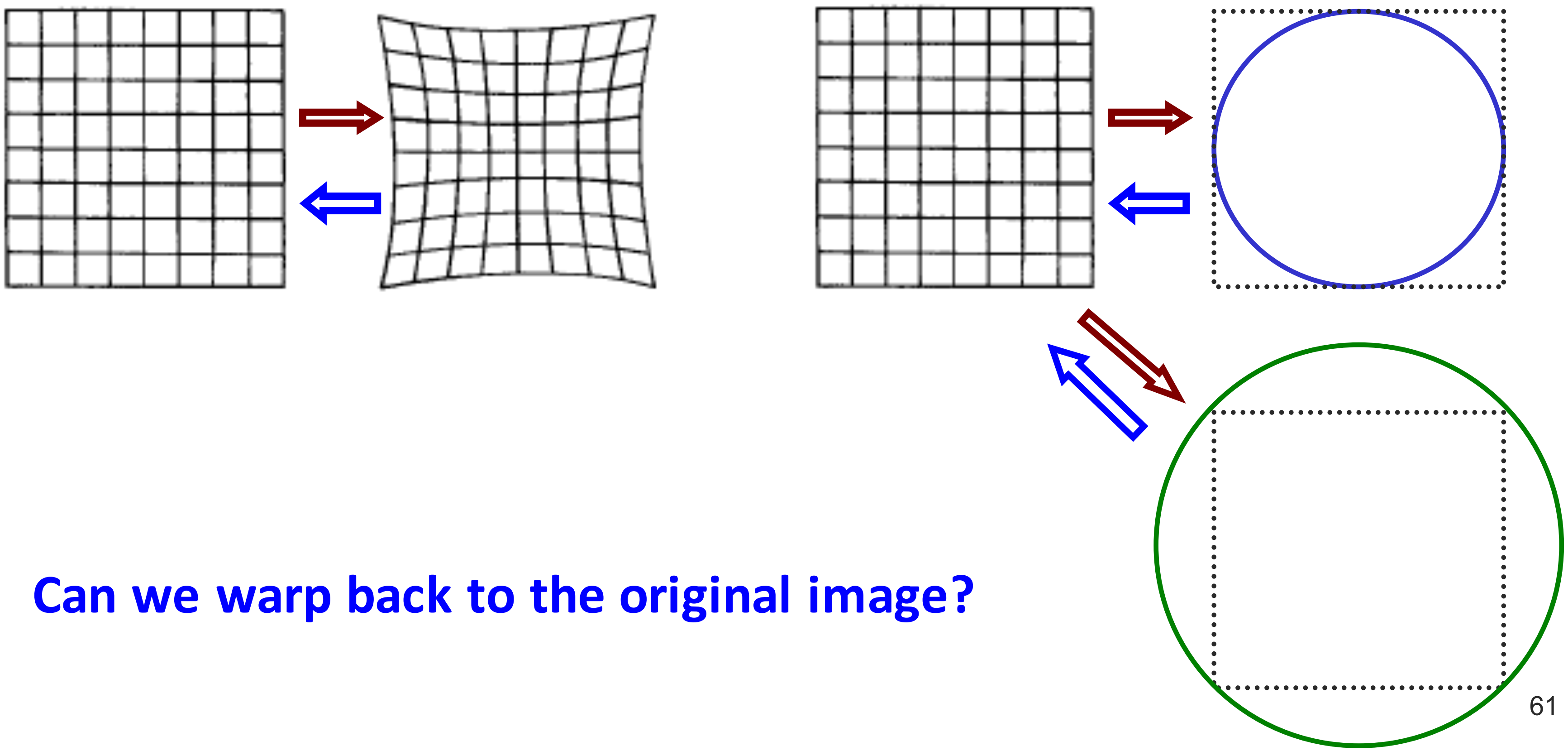
Original

Barrel distortion

Pincusion distortion

Geometrical Modification

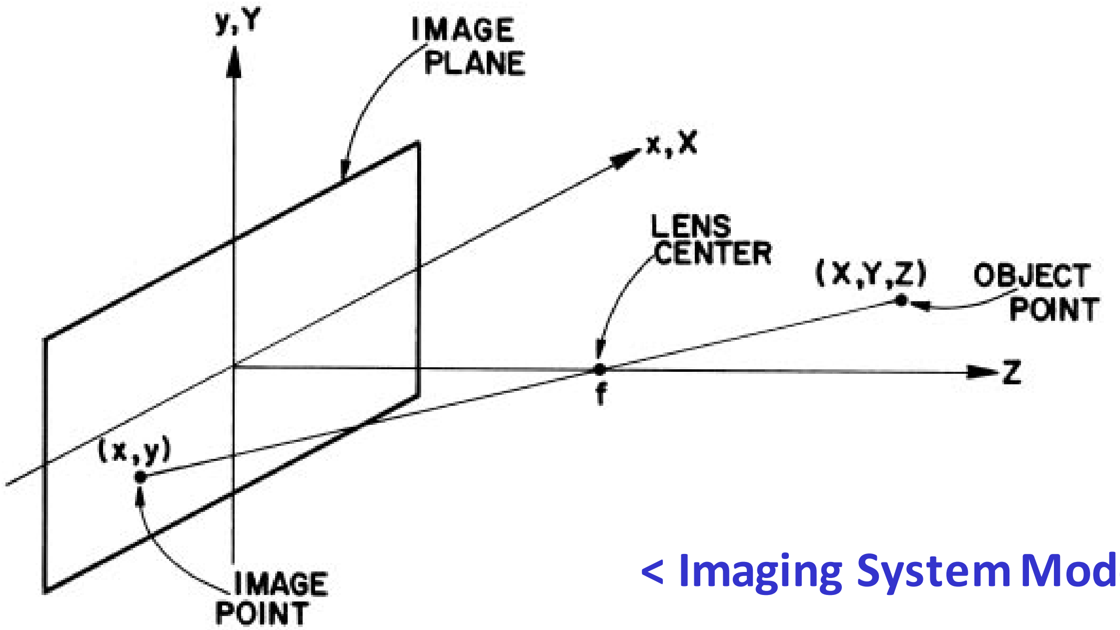
- **Example**



Can we warp back to the original image?

Geometrical Modification

- Perspective Transformation
  - Imaging in the 3D space
    - Fundamentals of computer graphics



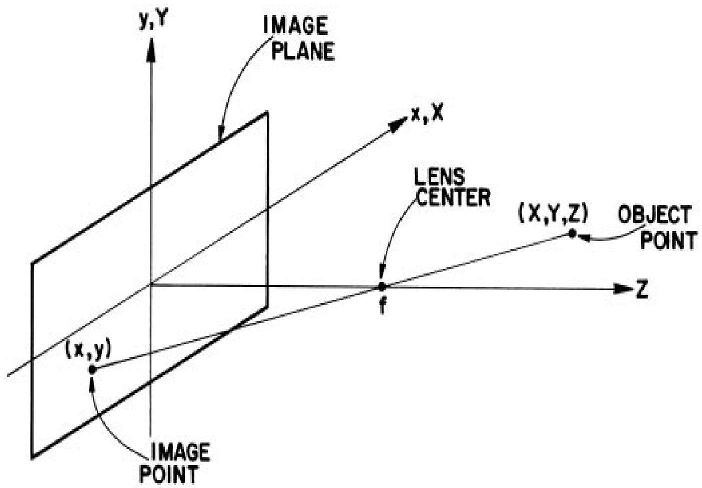
< Imaging System Model >

Geometrical Modification

- **Perspective Transformation**
  - **Cartesian to image coordinates**
    - **Similar triangle property**

$$\frac{X}{-x} = \frac{Z-f}{f} \Rightarrow x = \frac{fX}{f-Z};$$

$$y = \frac{fY}{f-Z}$$



→ Many-to-one mapping

Geometrical Modification

- **Perspective Transformation**
  - **Image to Cartesian coordinates**
    - **Need another degree of freedom**

$$X = \frac{f x_i}{f + z_i}; \quad Y = \frac{f y_i}{f + z_i}; \quad Z = \frac{f z_i}{f + z_i}$$

z_i is a free variable

    - **Given Z, we may compute z_i and then X & Y via**

$$X = \frac{x_i}{f}(f - Z) \qquad Y = \frac{y_i}{f}(f - Z)$$

Geometrical Modification

- **Perspective Transformation**

P is a perspective transformation matrix,

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1/f & 1 \end{bmatrix}$$

$$\tilde{v} = s\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

homogeneous vector
3D object
s: scaling factor

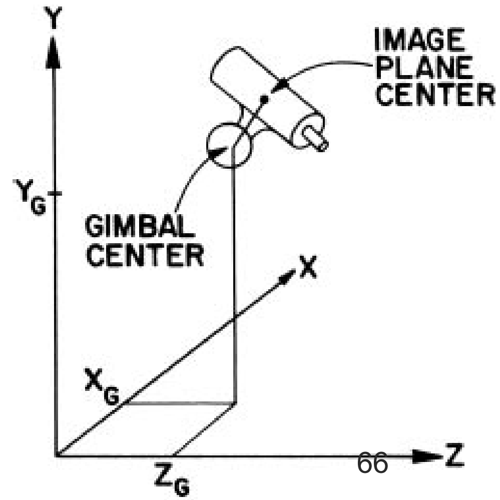
$$\tilde{w} = \begin{bmatrix} x_i \\ y_i \\ z_i \\ 1 \end{bmatrix}$$

homogeneous image position vector

$$\tilde{w} = P\tilde{v} = \begin{bmatrix} sX \\ sY \\ sZ \\ s - sz/f \end{bmatrix} \qquad \Rightarrow s = \frac{f}{f - z}$$

Geometrical Modification

- **Camera Imaging Model**
  - Camera is supported by a gimbal (X_G, Y_G, Z_G)
  - Gimbal can do 3D movements
    - panning (θ) /tilting (ϕ)
  - Offset between the gimbal support and the image plane center is (X_0, Y_0, Z_0)
  - The complete camera imaging model can be derived by sequentially operating on the homogeneous vector



$$\tilde{w} = P T_c R T_G \tilde{v}$$