

Digital Image Processing

Ming-Sui (Amy) Lee
Mar. 02, 2016

[

Announcement

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■ Class Information

○ Teaching Assistant

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Office Hours: 14:00 ~ 16:00, Tuesday

■ 簡均容 @532

Office Hours: 10:00 ~ 12:00, Thursday

■ Email: dip.mslee@gmail.com

[

Announcement

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■ Class Information

- Class website
 - <https://ceiba.ntu.edu.tw/1042DIP>
 - Syllabus
 - Lecture #1
 - Lecture #2
 - Submission guideline
 - Sample codes
 - Homework #1

Announcement

- Class Information
 - Homework
 - Please be sure to read the guideline carefully
 - Submission guideline
 - Homework #1
 - Sample codes
 - Deadline: 11:59 am on Mar. 15, 2016

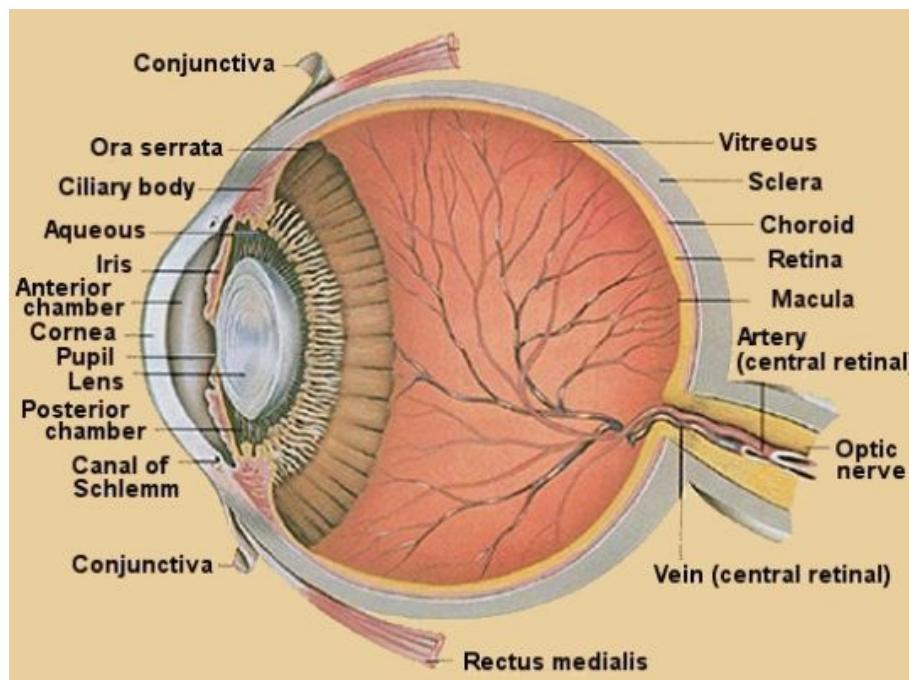
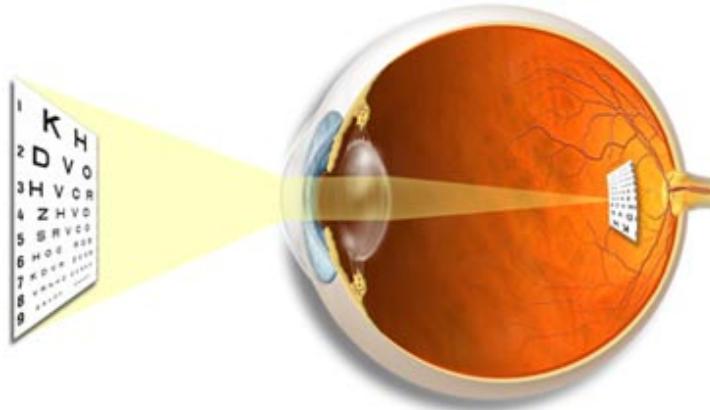
Digital Image Fundamentals

Image Quality

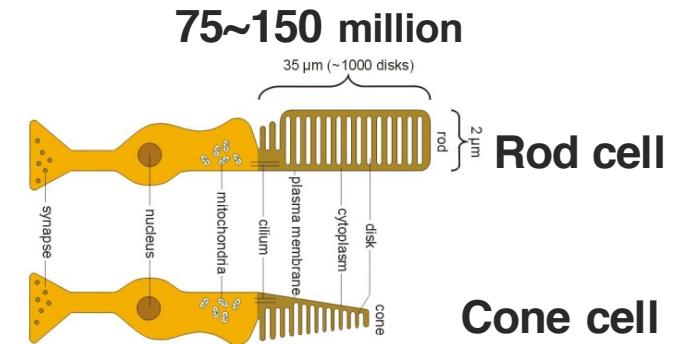
- Objective / subjective
 - Machine/human beings
 - Mathematical and Probabilistic/
human intuition and perception



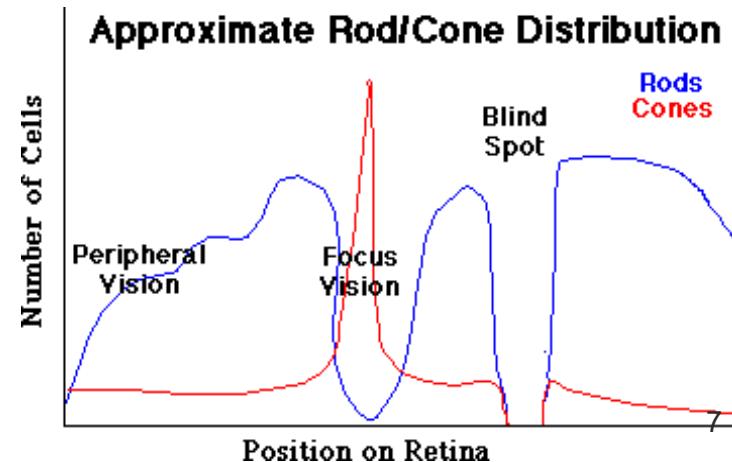
Structure of the Human Eye



photoreceptor cells

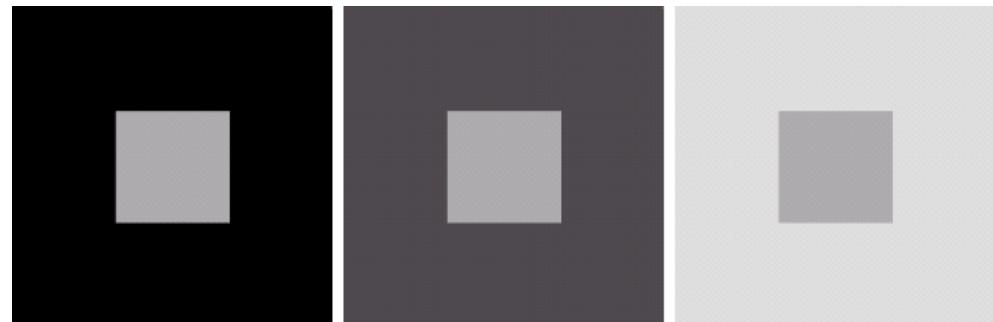
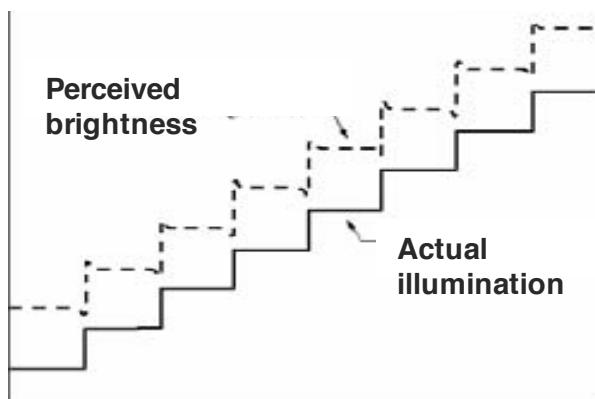
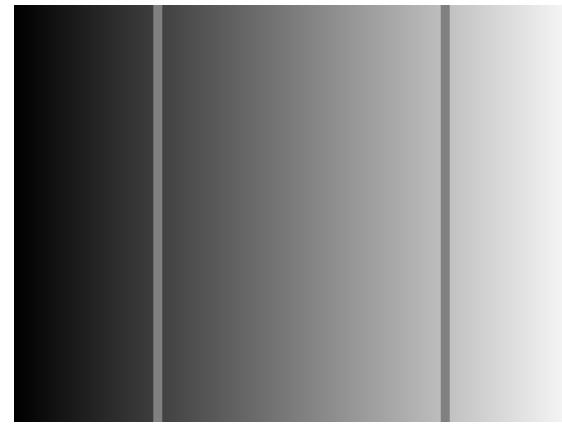
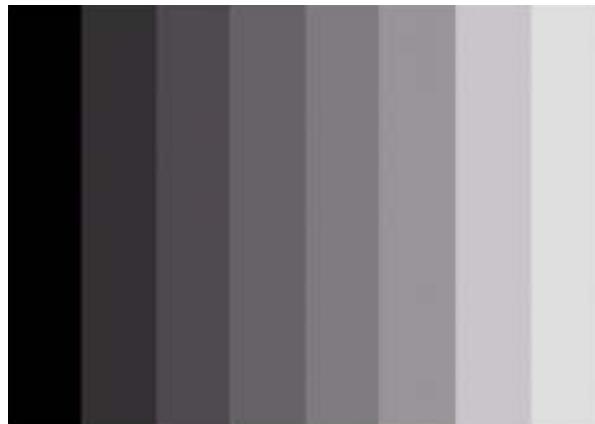


6~7 million



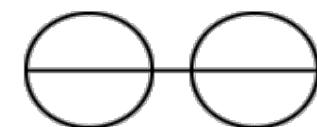
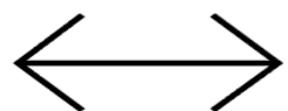
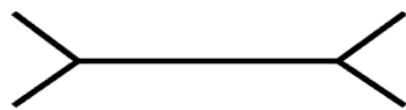
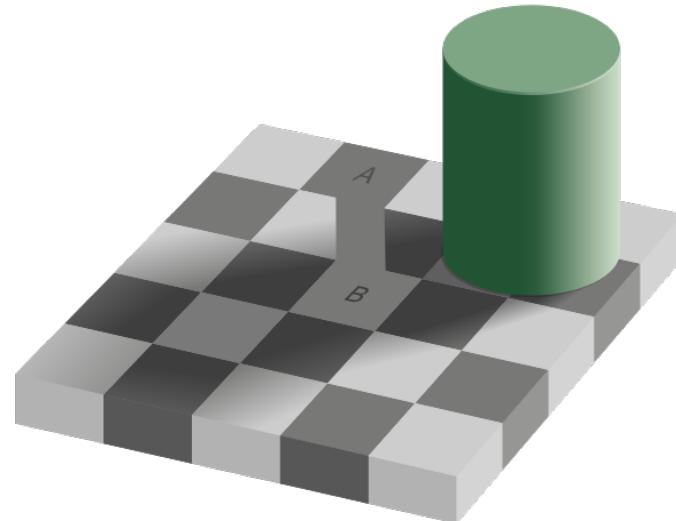
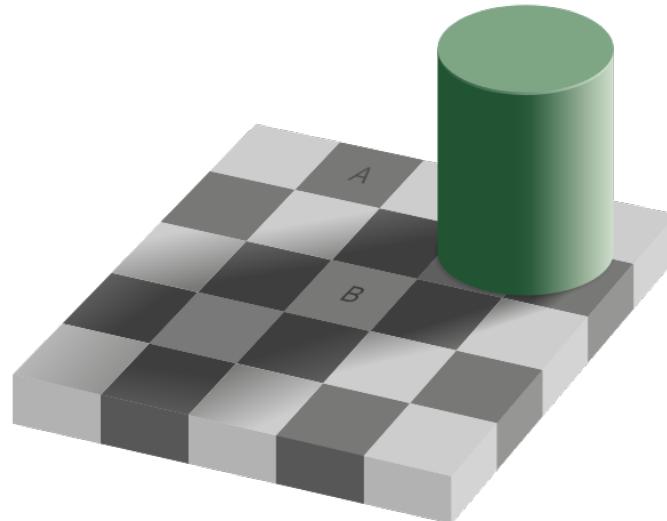
Human Visual Perception

- Perceived brightness is NOT a simple function of intensity



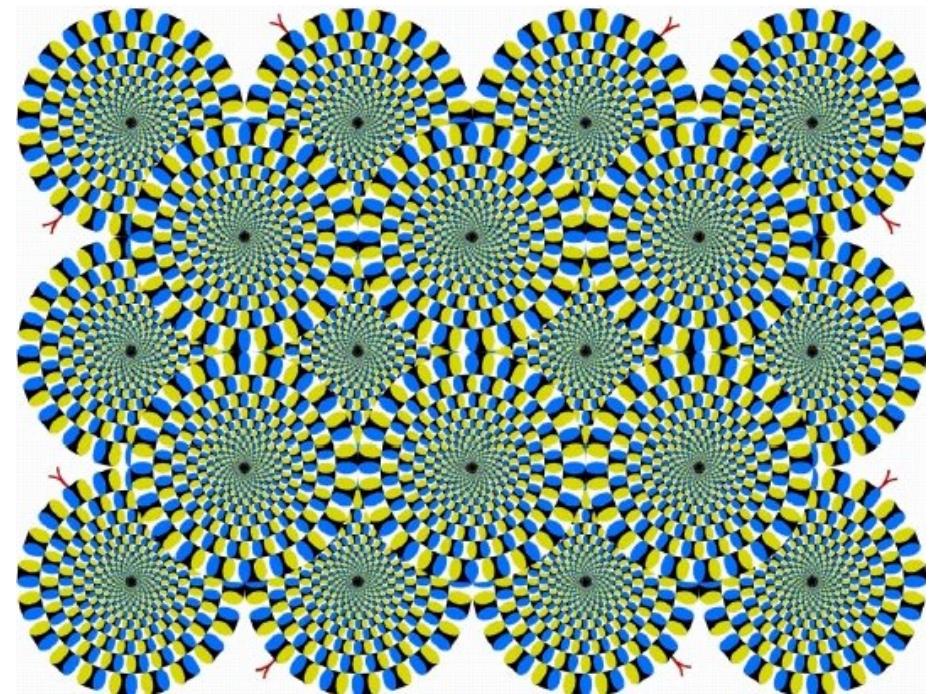
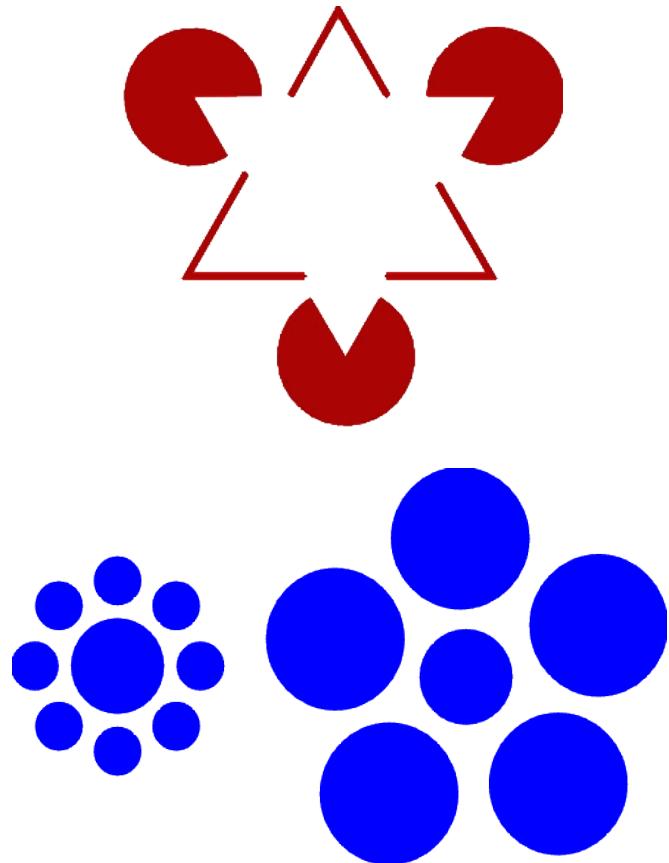
Human Visual Perception

■ Optical Illusion



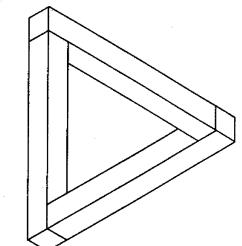
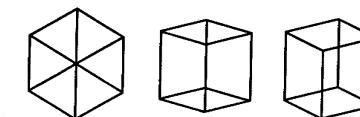
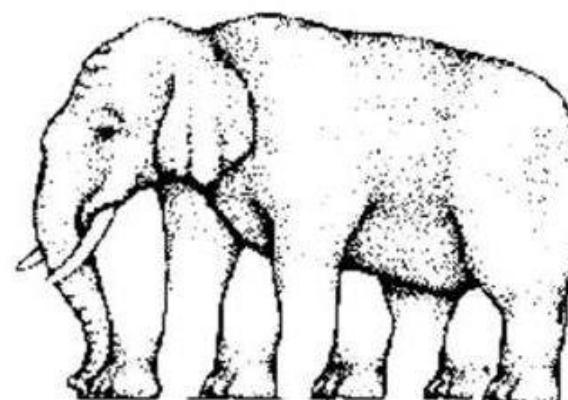
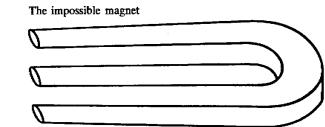
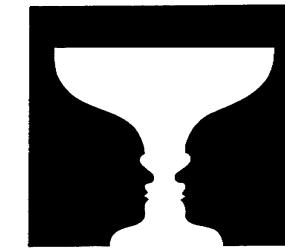
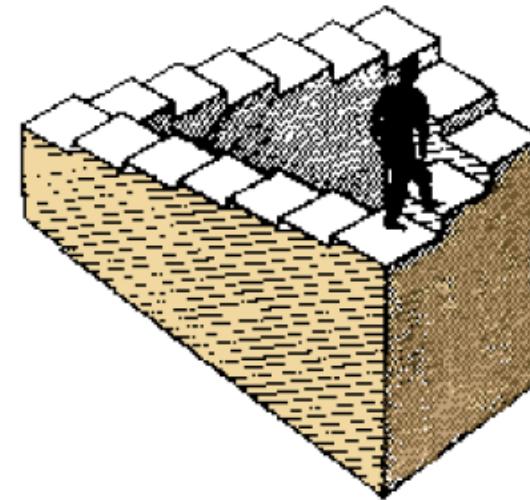
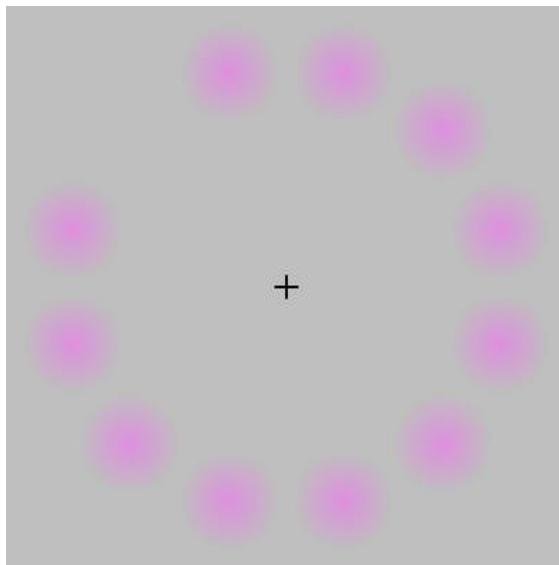
Human Visual Perception

■ Optical Illusion



Human Visual Perception

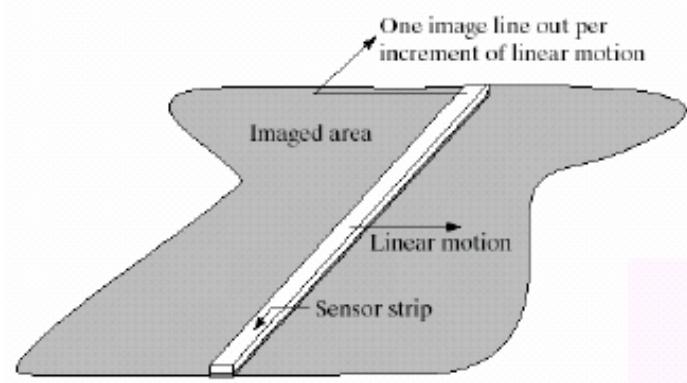
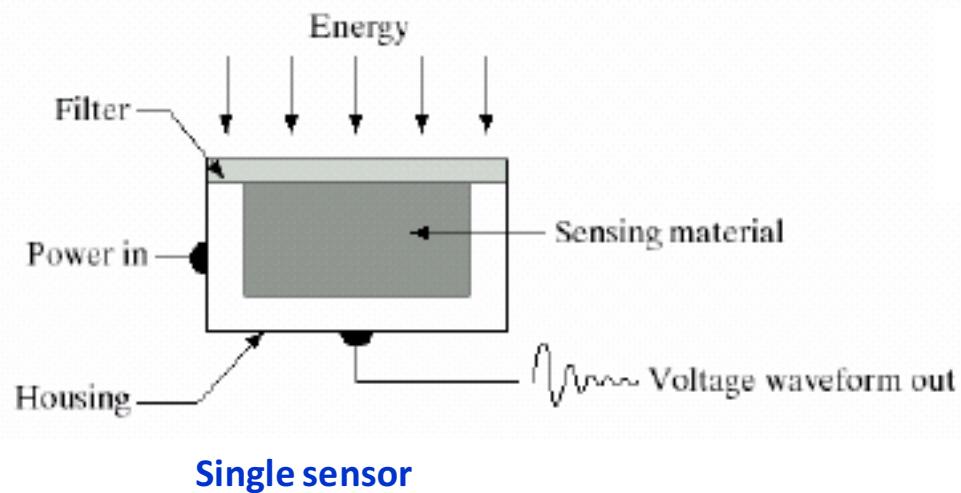
■ Optical Illusion



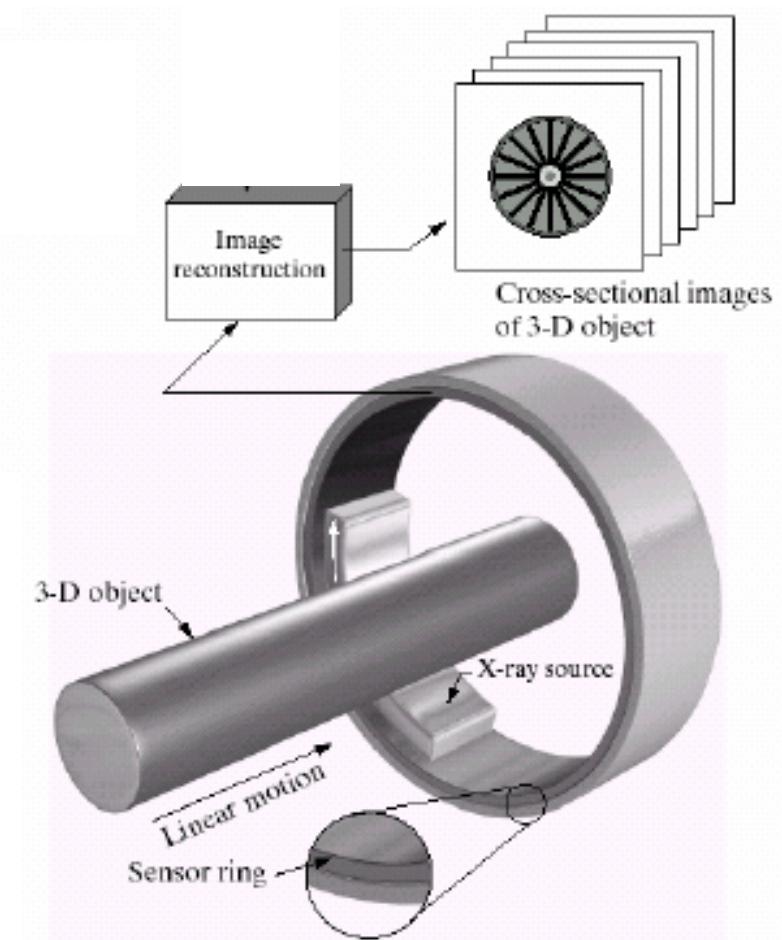
[Image Sensing and Acquisition]

- **Illumination Source**
 - EM energy, ultrasound, synthesized, ...
- **Scene Element**
 - Objects, human organs, buried mineral,...
- **Sensing Material**
 - Single sensor: photodiode
 - Sensor strips: require extensive processing
 - Sensor arrays: CCD & CMOS

[Image Sensing and Acquisition]



Sensor Strip



Circular Sensor Strip

[Image Sensing and Acquisition]

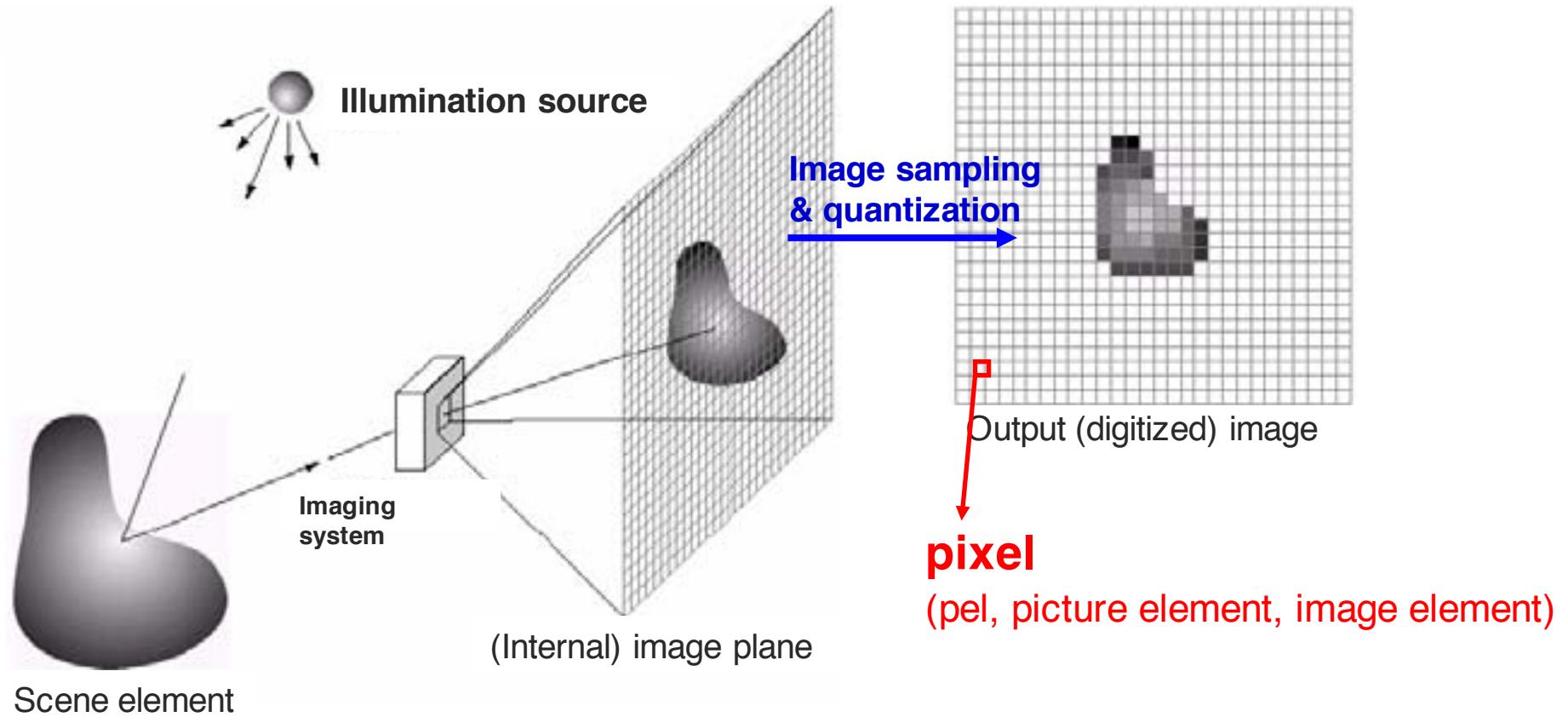
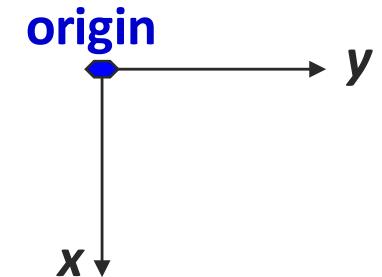


Image Formation Model

- An image → 2D function

$$0 < f(x, y) < \infty$$

where x and y are spatial coordinates



- Categorized by two components

$$f(x, y) = i(x, y)r(x, y)$$

- **Illumination:** $0 < i(x, y) < \infty$
- **Reflectance:** $0 < r(x, y) < 1$
 - black velvet/ flat-white wall paint/ snow/ silver-plated metal

0.1

0.8

0.93

0.9

Image Sampling & Quantization

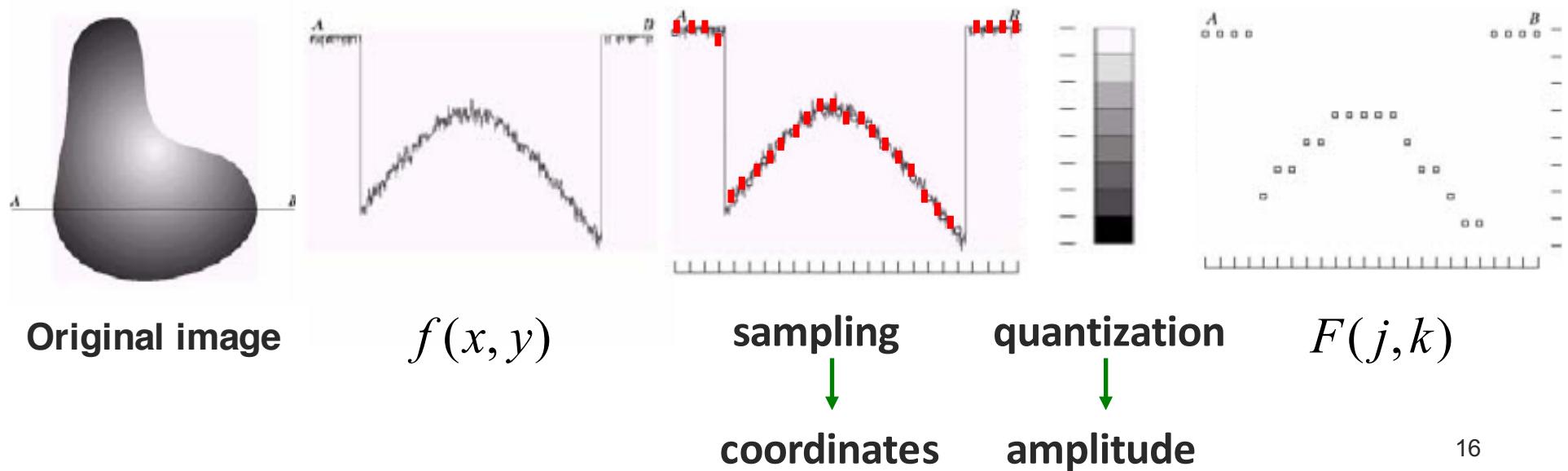
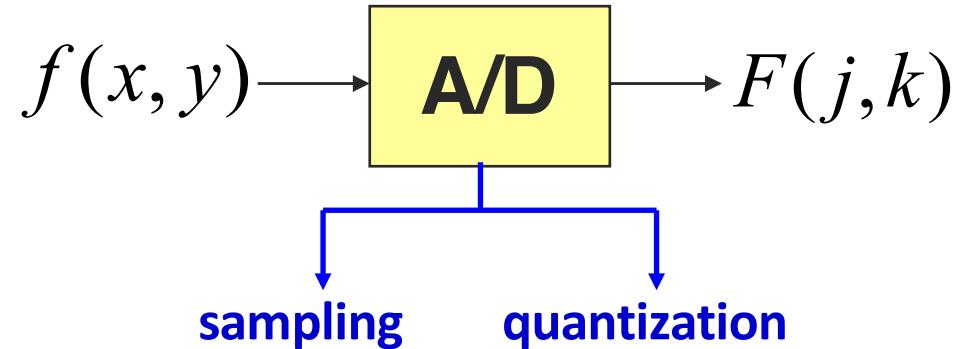
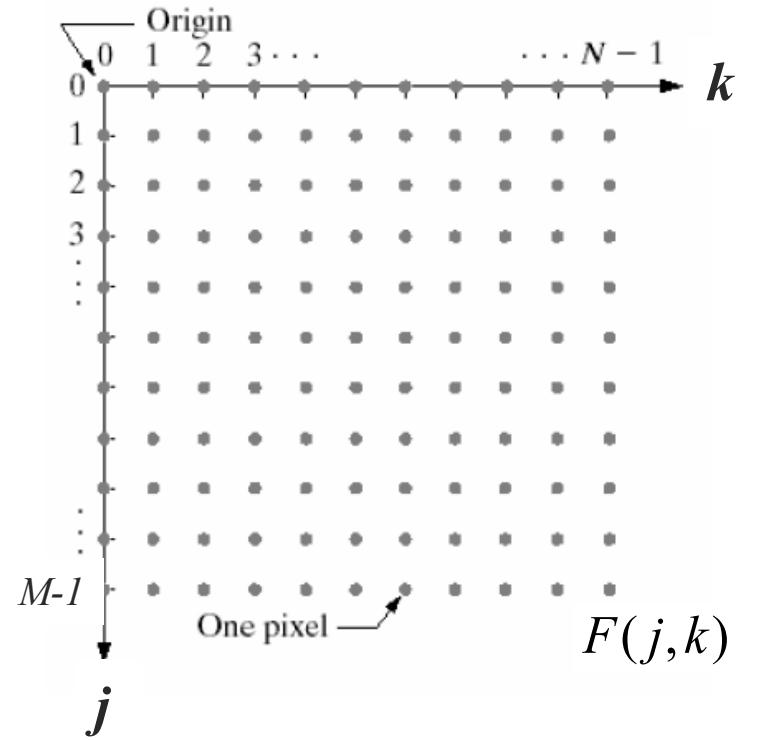
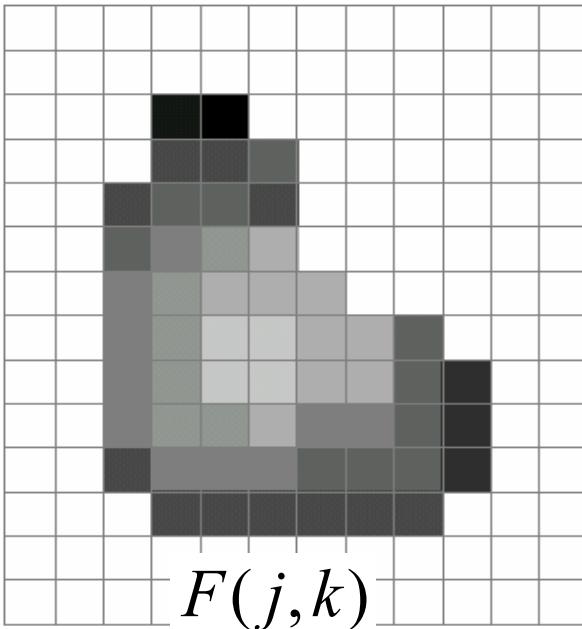
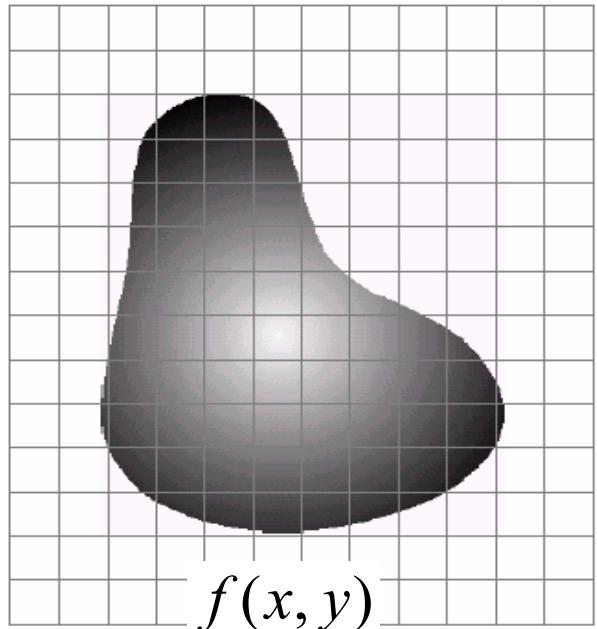


Image Sampling & Quantization



$$A = \begin{bmatrix} a_{0,0} & a_{0,1} & \cdots & a_{0,N-1} \\ a_{1,0} & a_{1,1} & \cdots & a_{1,N-1} \\ \vdots & \vdots & \ddots & \vdots \\ a_{M-1,0} & a_{M-1,1} & \cdots & a_{M-1,N-1} \end{bmatrix}$$

$$F(j, k) = \begin{bmatrix} F(0,0) & F(0,1) & \cdots & F(0,N-1) \\ F(1,0) & F(1,1) & \cdots & F(1,N-1) \\ \vdots & \vdots & \ddots & \vdots \\ F(M-1,0) & F(M-1,1) & \cdots & F(M-1,N-1) \end{bmatrix}$$

Digital Image Representation

■ Dynamic Range

- The range of values spanned by the gray scale

$$\{0, 1, \dots, L - 1\} \quad L = 2^k$$

■ Image Size

- for a square image, $M = N$

total number of bits required to store the image: $b = N^2 \cdot k$

N/k	1 ($L = 2$)	2 ($L = 4$)	3 ($L = 8$)	4 ($L = 16$)	5 ($L = 32$)	6 ($L = 64$)	7 ($L = 128$)	8 ($L = 256$)
32	1,024	2,048	3,072	4,096	5,120	6,144	7,168	8,192
64	4,096	8,192	12,288	16,384	20,480	24,576	28,672	32,768
128	16,384	32,768	49,152	65,536	81,920	98,304	114,688	131,072
256	65,536	131,072	196,608	262,144	327,680	393,216	458,752	524,288
512	262,144	524,288	786,432	1,048,576	1,310,720	1,572,864	1,835,008	2,097,152
1024	1,048,576	2,097,152	3,145,728	4,194,304	5,242,880	6,291,456	7,340,032	8,388,608
2048	4,194,304	8,388,608	12,582,912	16,777,216	20,971,520	25,165,824	29,369,128	33,554,432
4096	16,777,216	33,554,432	50,331,648	67,108,864	83,886,080	100,663,296	117,440,512	134,217,728
8192	67,108,864	134,217,728	201,326,592	268,435,456	335,544,320	402,653,184	469,762,048	536,870,912

Downsampling

- $1024 \times 1024 \rightarrow 32 \times 32$
 - Downsampled by a factor of 2



64^2

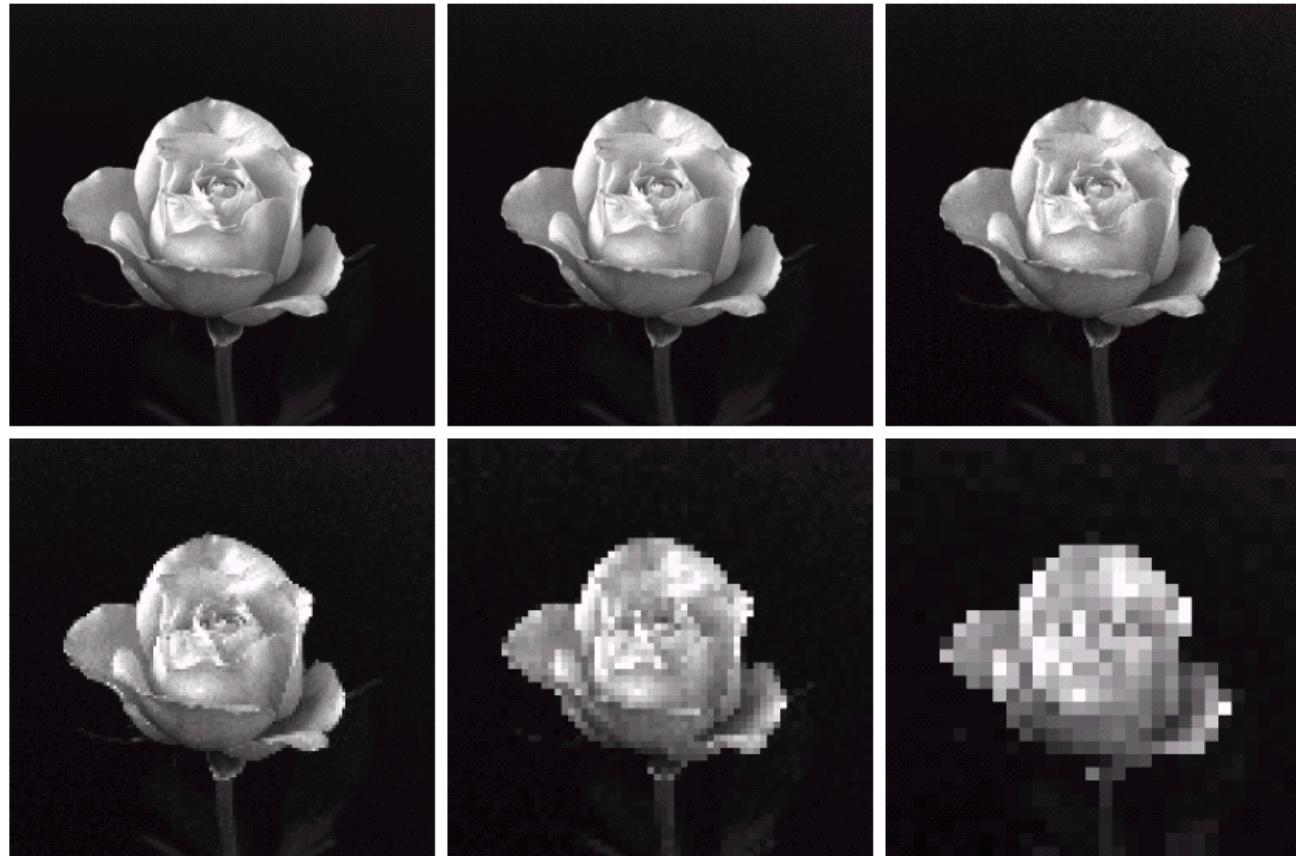
128

256

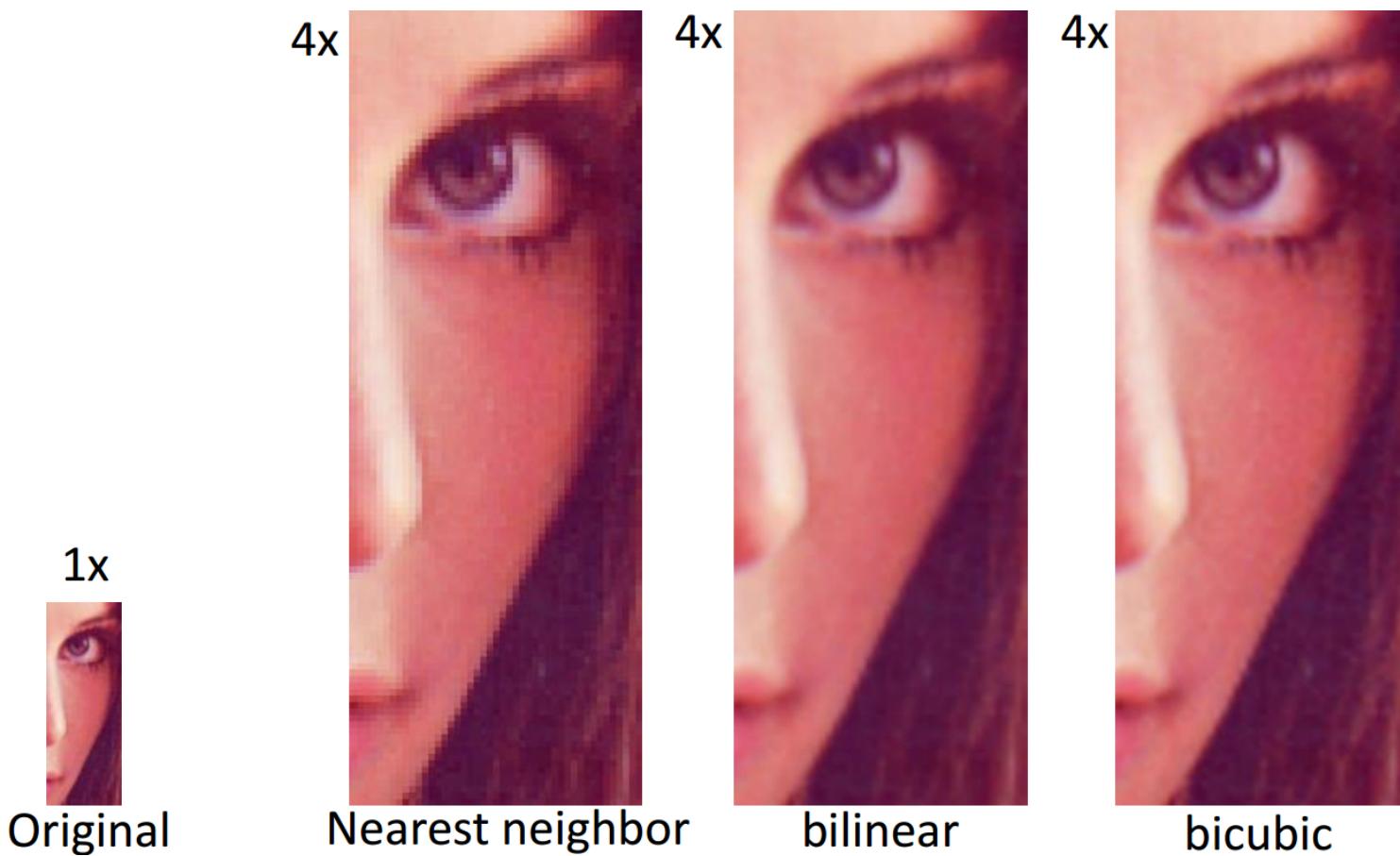
512

Re-Sampling

- Zero-Order-Hold Method (ZOH)
 - Row and column duplication



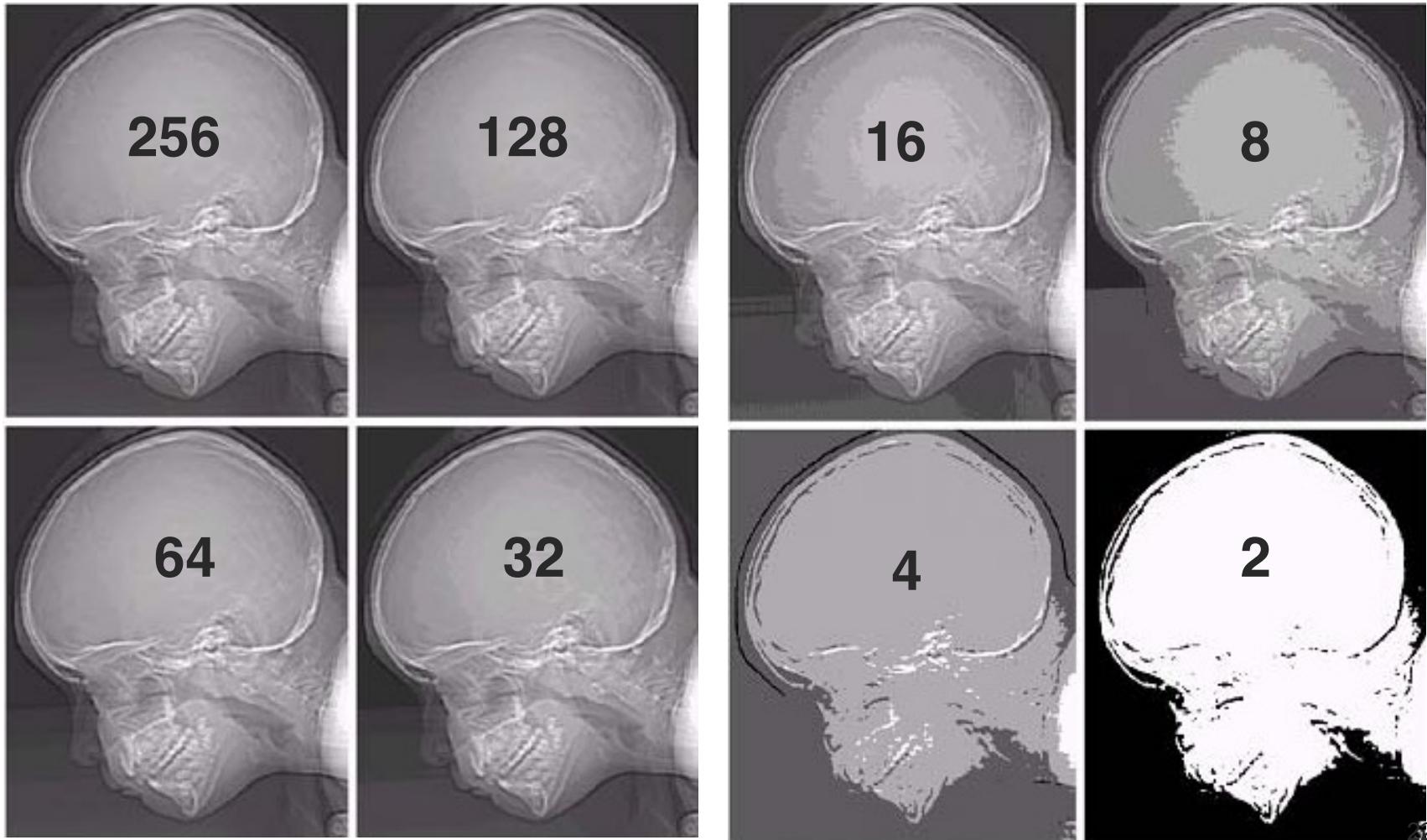
Re-Sampling



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L=256,128,64,32,16,8,4,2

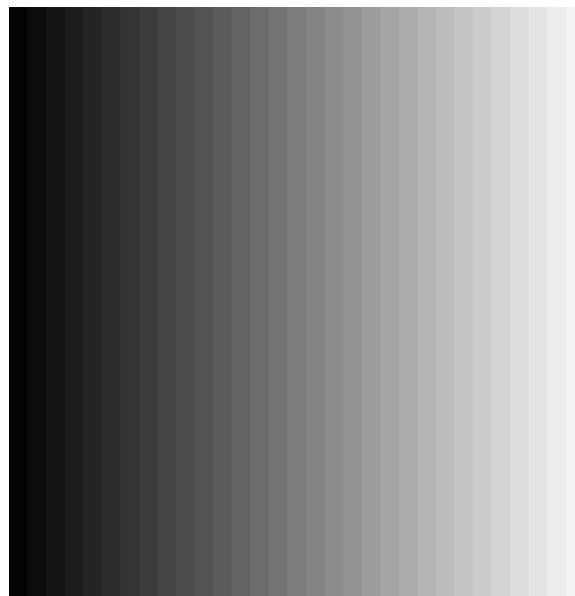
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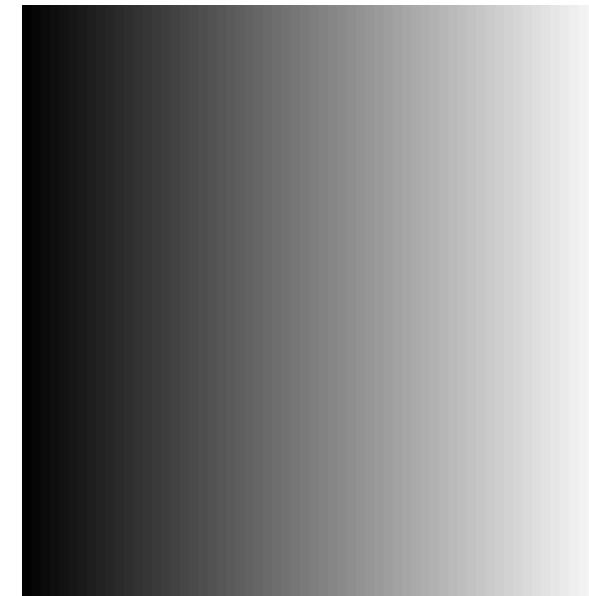
Digital Image Representation

- 8-bit image is commonly used

- Storage
 - Human perception



32 steps (5 bits) in gray level



64 steps (6 bits) in gray level

Image Enhancement

Image Enhancement

■ Goal of Image Enhancement

- make images more appealing
- no theory, ad-hoc rules, derived with insights

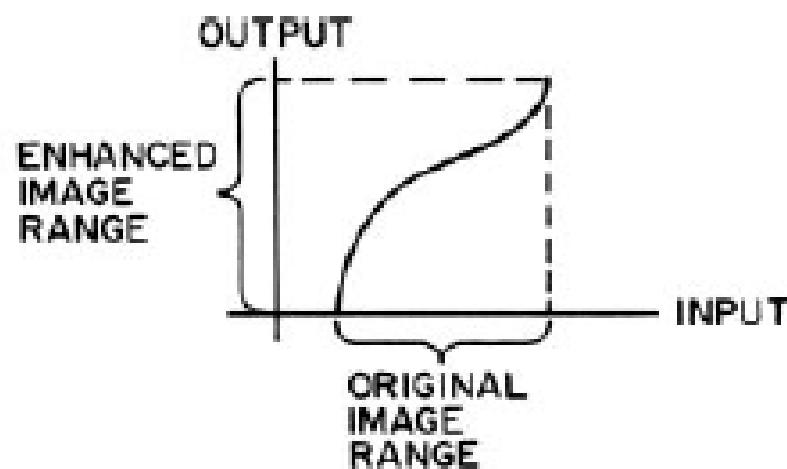
■ Two Approaches

- Contrast Manipulation
- Histogram Modification

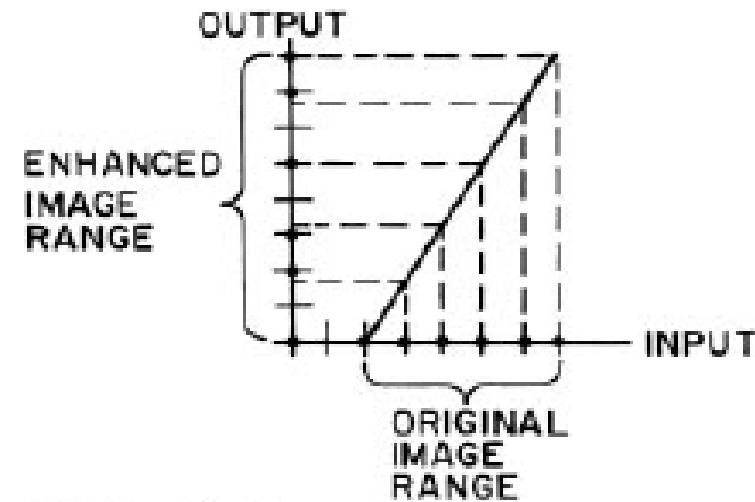
Contrast Manipulation

Transfer Function

- Linear
- Nonlinear
- Piecewise



Continuous Image



Quantized Image

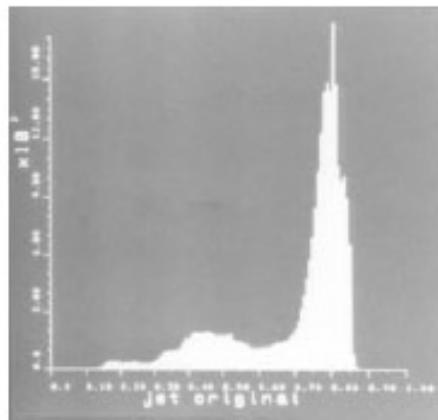
Contrast Manipulation

Linear scaling and clipping

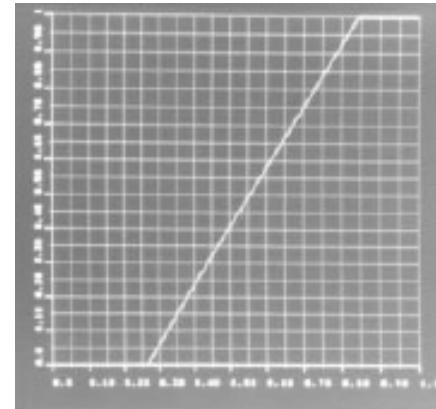
$$G(j,k) = T[F(j,k)] \quad 0 \leq F(j,k) \leq 1$$



(a) Original



(b) Original histogram



(c) Transfer function



(d) Contrast stretched

[

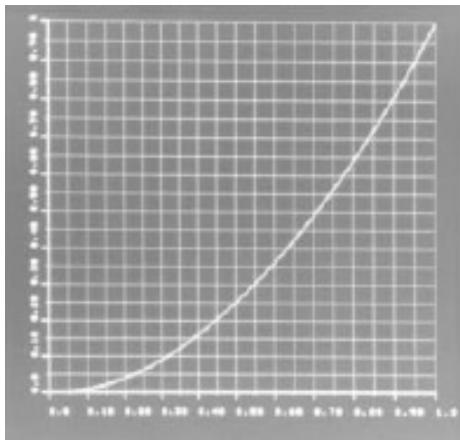
Contrast Manipulation

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■ Power-Law



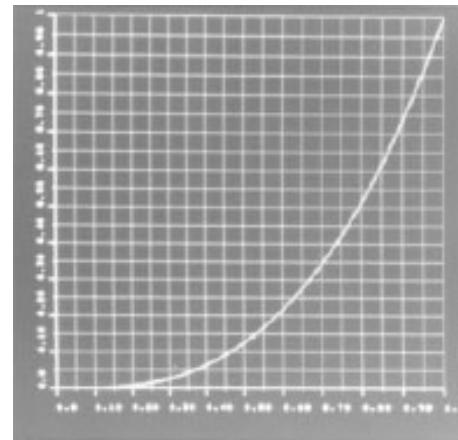
$$G(j,k) = [F(j,k)]^p \quad 0 \leq F(j,k) \leq 1$$



(a) Square function



(b) Square output



(c) Cube function



(d) Cube output 28

[

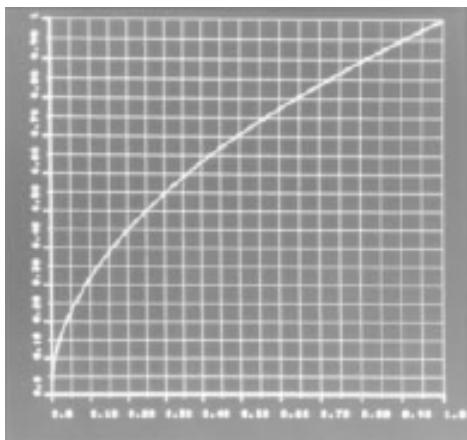
Contrast Manipulation

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■ Power-Law



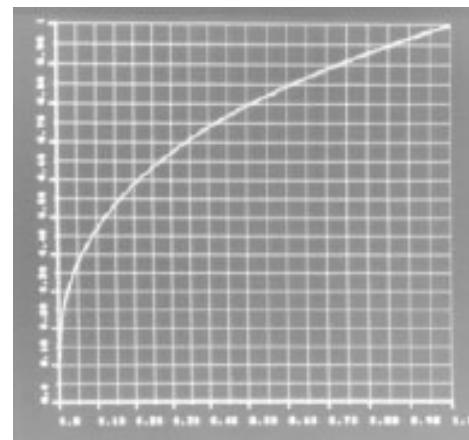
$$G(j,k) = [F(j,k)]^p \quad 0 \leq F(j,k) \leq 1$$



(a) Square root function



(b) Square root output



(c) Cube root function

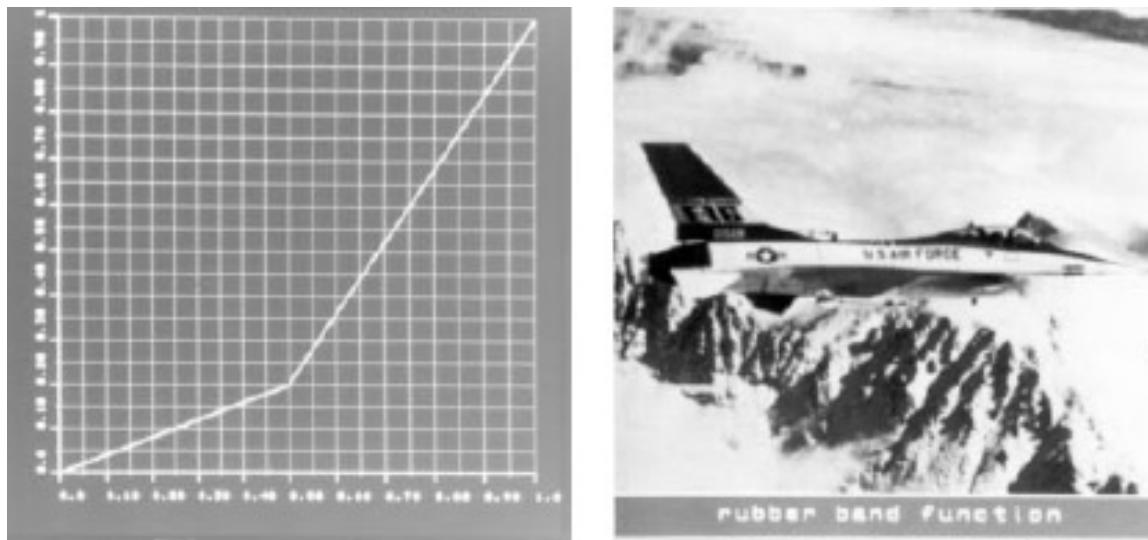


(d) Cube root output

Contrast Manipulation

■ Rubber Band Transfer Function

- Piecewise linear transformation
- Inflection point (control point)



Can choose the area where we want to stretch or reduce the contrast³⁰

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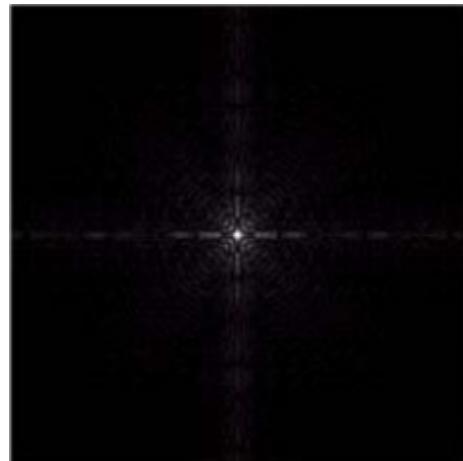
Contrast Manipulation

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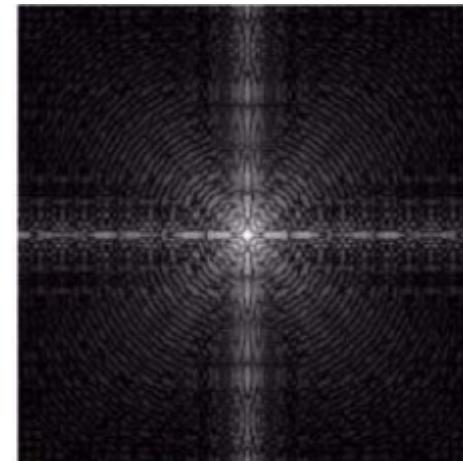
■ Logarithmic Point Transformation

$$G(j, k) = \frac{\log_e \{1 + aF(j, k)\}}{\log_e \{2.0\}} \quad 0 \leq F(j, k) \leq 1$$

Fourier Spectrum



$0 \sim 1.5 \times 10^6$ →



$0 \sim 6.2$

Useful for scaling image arrays with a very wide dynamic range

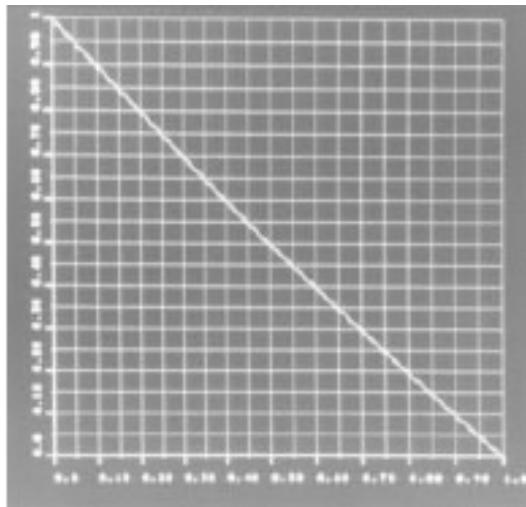
[

Contrast Manipulation

]

■ Reverse Function

$$G(j,k) = 1 - F(j,k) \quad 0 \leq F(j,k) \leq 1$$



(a) Reverse function



(b) Reverse function output

Able to see more details in dark areas of an image

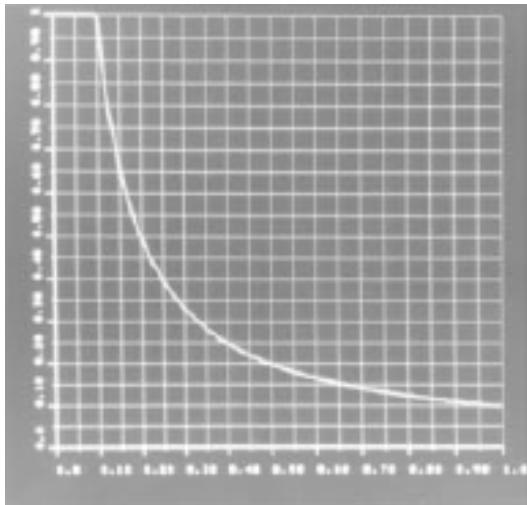
[

Contrast Manipulation

]

■ Inverse Function

$$G(j,k) = \begin{cases} 1 & 0 \leq F(j,k) \leq 0.1 \\ \frac{0.1}{F(j,k)} & 0.1 \leq F(j,k) \leq 1 \end{cases}$$



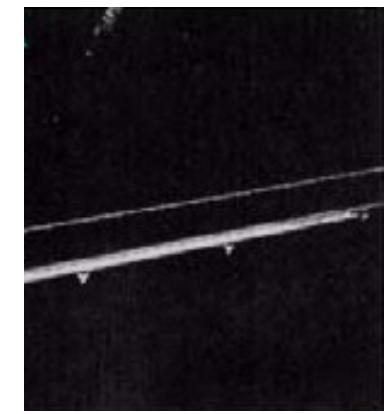
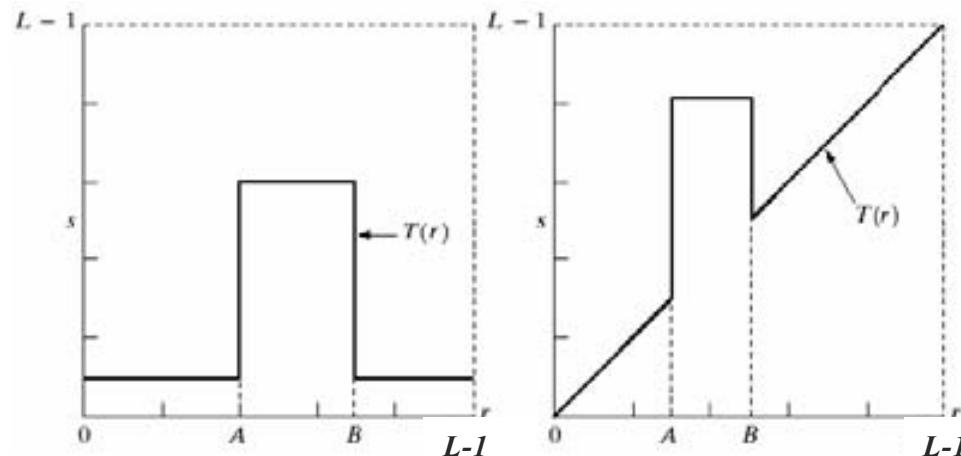
(c) Inverse function



(d) Inverse function output

Contrast Manipulation

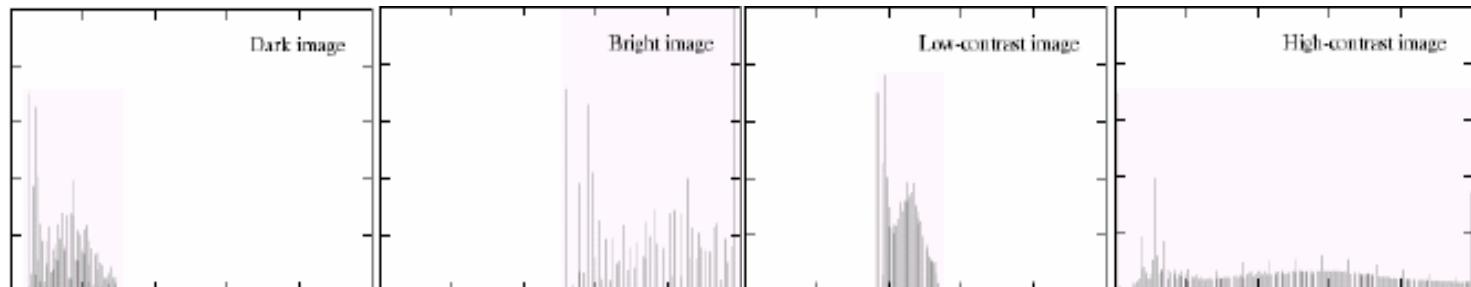
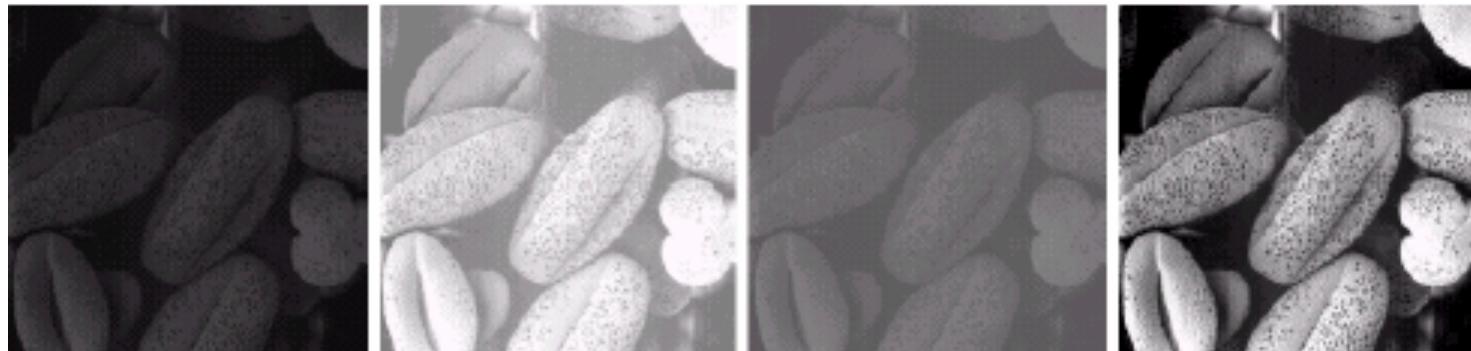
■ Amplitude-Level Slicing (Gray-Level Slicing)



Histogram Modification

■ Goal

- Rescale the original image so that the histogram of the enhanced image follows some desired form



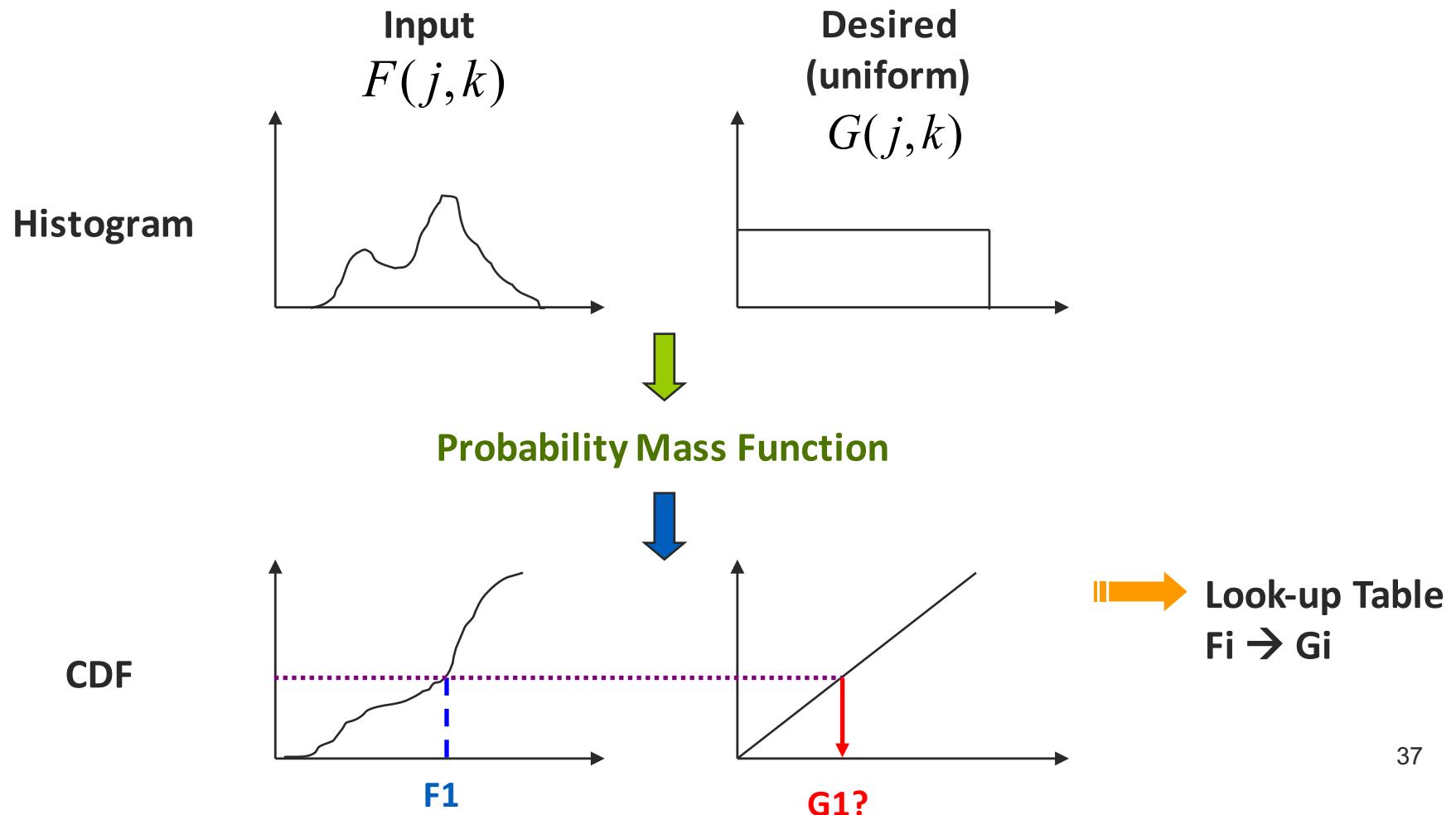
Histogram Modification

■ Histogram Equalization

- make the output histogram to be uniformly distributed
 - Transfer function
 - Bucket filling

Histogram Equalization

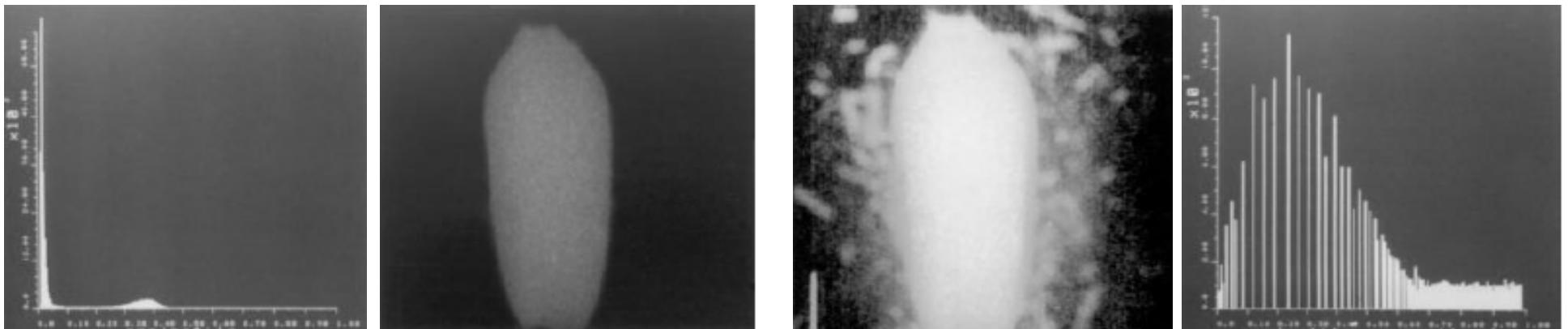
Transfer Function



Histogram Equalization

■ Transfer Function

- Output histogram not really uniformly distributed
- Still keep the shape
- More flat than the original histogram



Histogram Equalization

Bucket Filling

arbitrary

$F(j,k)$	# of pixels
0	1
1	2
2	5
:	:
255	3

uniform

$G(j,k)$	# of pixels
0	$N/256$
1	$N/256$
2	$N/256$
:	:
255	$N/256$

N : # of total pixels

- Not 1-1 mapping
- Accumulated probability may not end exactly at the boundary of a bin → split it out

Noise Cleaning

Noise Cleaning

■ Noise

- electrical sensor noise
- photographic grain noise
- channel error
- etc.

■ Characteristics of the noise

- discrete
- not spatially correlated
- higher spatial frequency



Noise Cleaning

- Two types of noise
 - Uniform Noise
 - Additive uniform noise, Gaussian noise
 - Impulse Noise
 - Salt and pepper noise
- Solutions
 - Uniform Noise → low-pass filtering
 - Impulse Noise → non-linear filtering

Basics of Spatial Filtering

■ Mask

- filter, kernel, template
- $m \times n$
 - $m=2a+1, n=2b+1,$
where a and b are nonnegative integers
 - e.g. 3x3 mask

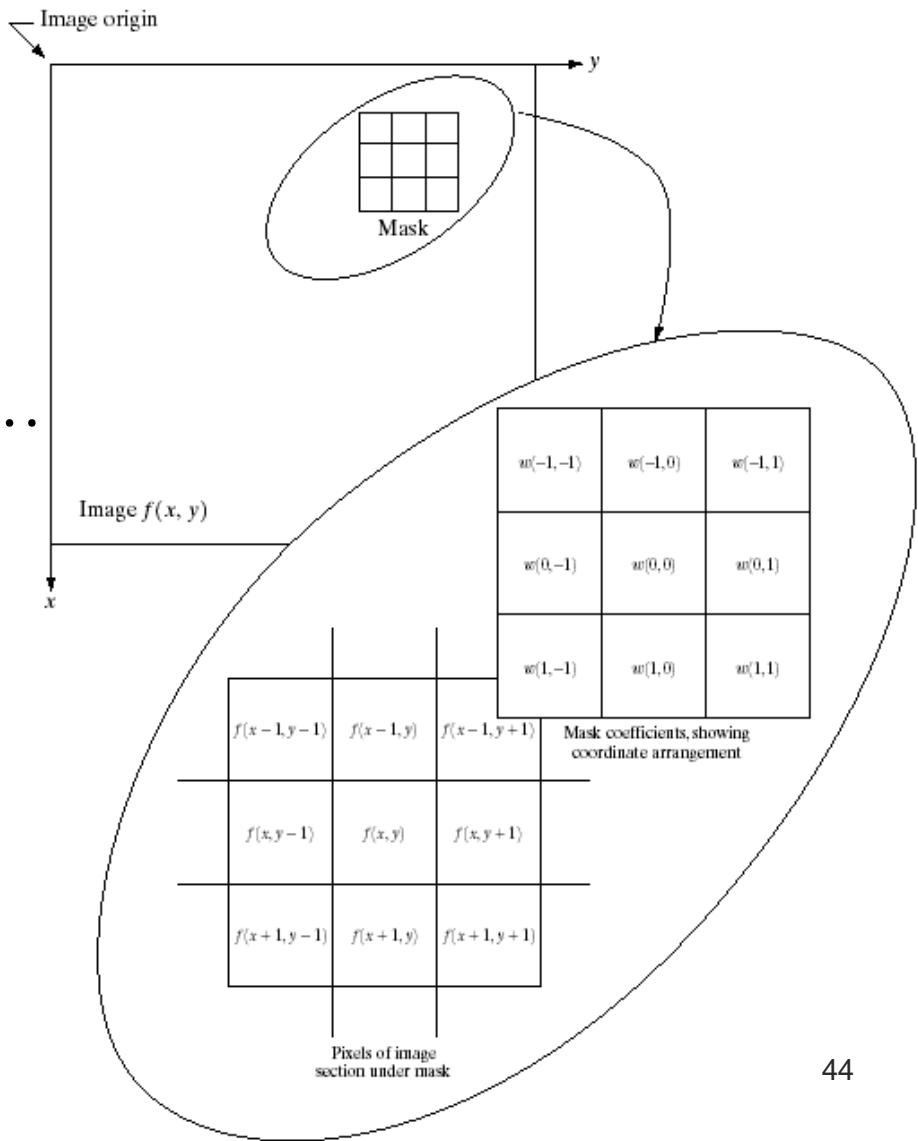
$w(-1,-1)$	$w(-1,0)$	$w(-1,1)$
$w(0,-1)$	$W(0,0)$	$w(0,1)$
$w(1,-1)$	$w(1,0)$	$w(1,1)$

■ Spatial Filtering/Convolution

$$\begin{aligned} G(j,k) &= w(-1,-1)F(j-1, k-1) + w(-1,0)F(j-1, k) + \dots \\ &\quad + w(0,0)F(j, k) + \dots \\ &\quad + w(1,0)F(j+1, k) + w(1,1)F(j+1, k+1) \end{aligned}$$

Basics of Spatial Filtering

$$\begin{aligned}
 G(j, k) = & w(-1, -1)F(j-1, k-1) \\
 & + w(-1, 0)F(j-1, k) + \dots \\
 & + w(0, 0)F(j, k) + \dots \\
 & + w(1, 0)F(j+1, k) \\
 & + w(1, 1)F(j+1, k+1)
 \end{aligned}$$

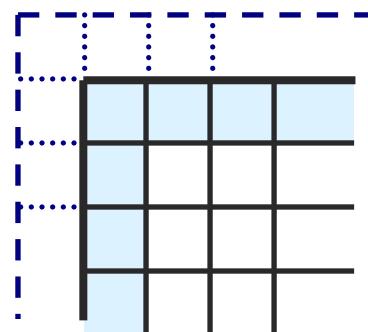
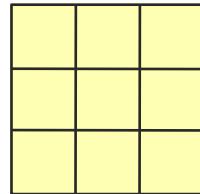


Q: Boundary pixels?

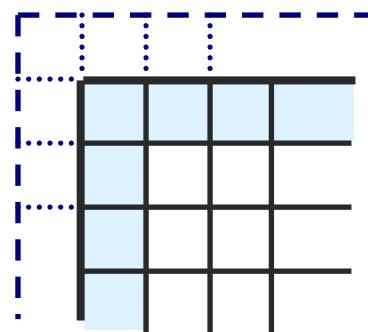
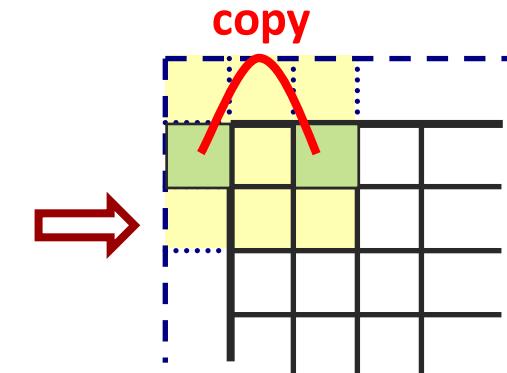
Basics of Spatial Filtering

■ Boundary Extension (3x3 mask)

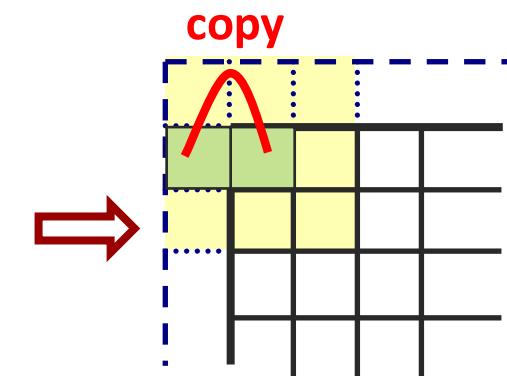
e.g.
3x3 mask, w



odd



even



Q: 5x5 mask?

[

Noise Cleaning

]

■ Uniform noise

- Perform low-pass filtering

$$H = \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \quad H = \frac{1}{10} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix} \quad H = \frac{1}{16} \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

- General form

$$H = \frac{1}{(b+2)^2} \begin{bmatrix} 1 & b & 1 \\ b & b^2 & b \\ 1 & b & 1 \end{bmatrix}$$

e.g.

$$F = \begin{bmatrix} 0 & 0 & 180 & 180 \\ 0 & 0 & 180 & 180 \\ 0 & 0 & 180 & 180 \\ 0 & 0 & 180 & 180 \end{bmatrix}$$

High Frequency Noise Removal

■ Low-pass filtering

- Normalized to unit weighting
- Averaging
- Smaller/Larger filter size ?



3x3



7x7

Noise Cleaning

- Impulse noise
 - black: pixel value =0 → dead sensor
 - white: pixel value=255 → saturated sensor
- Solutions
 - Outlier detection
 - Median filtering
 - Pseudo-median filtering (PMED)

Impulse Noise Removal

■ Outlier detection

O ₁	O ₂	O ₃	
O ₈	X	O ₄	
O ₇	O ₆	O ₅	

$$\text{if } \left| x - \frac{1}{8} \sum_{i=1}^8 O_i \right| > \varepsilon \quad \text{then } x = \frac{1}{8} \sum_{i=1}^8 O_i$$

How to choose ε ?

Larger window?

Impulse Noise Removal

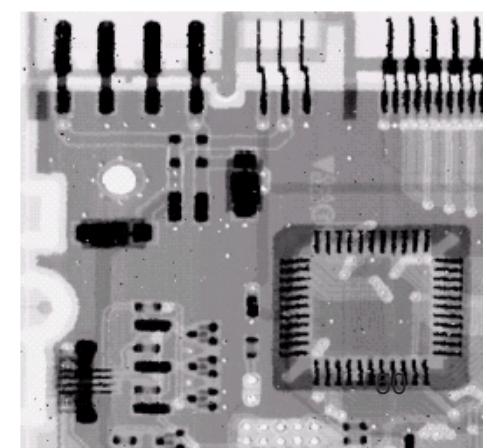
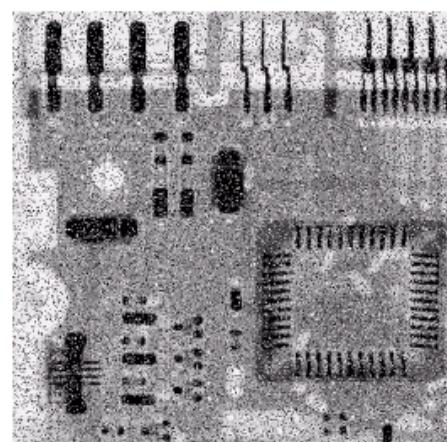
Median filtering

a_1, \dots, a_N where N is odd

- sort those values in order
- pick the middle one in the sorted list
- e.g. 3x3 mask:

$$I = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 8 & 7 \\ 1 & 5 & 6 \end{bmatrix}$$

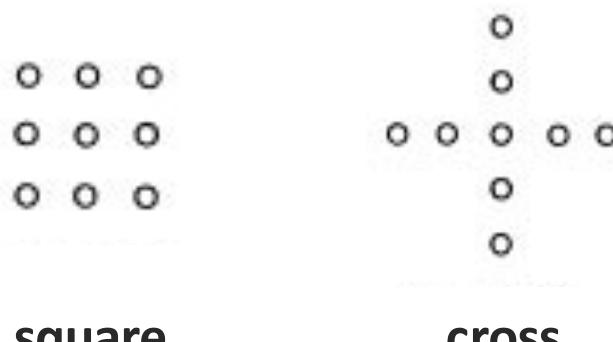
→ Median is 3



[Impulse Noise Removal]

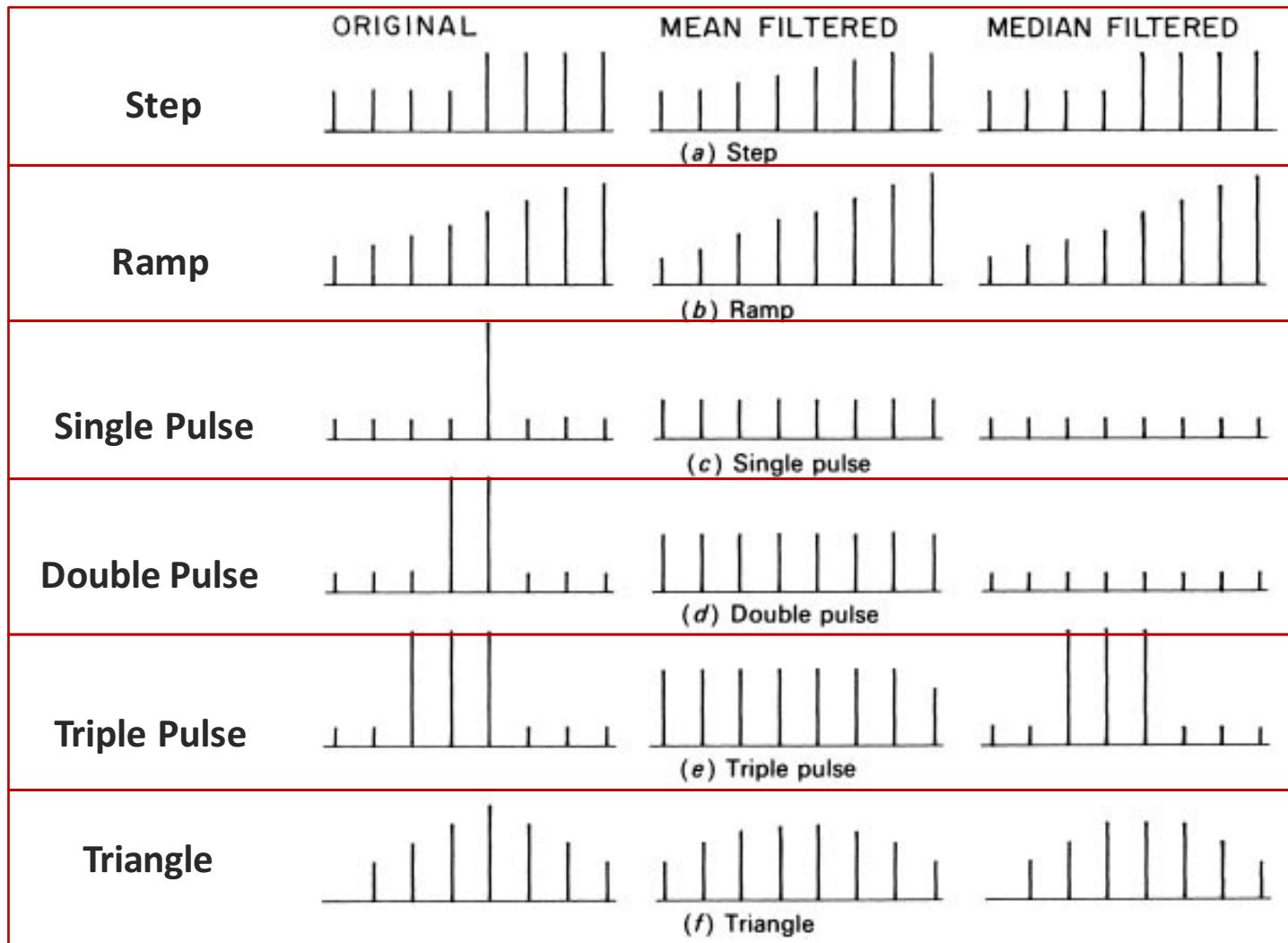
■ Median filtering

- Preserve sharp edges
- Effective in removing impulse noise
- 1D/2D (directional)
 - e.g. 2D



Impulse Noise Removal

- e.g. 1D (window size = 5)



Impulse Noise Removal

■ Median filtering

- Fast computation

- Approximation of median

- e.g. 5-element filter

a, b, c, d, e

→ MED(a, b, c, d, e)

= $\max(\min(a,b,c), \min(a,b,d), \dots)$

= $\min(\max(a,b,c), \max(a,b,d), \dots)$

→ there are 10 possible choices

→ could be narrowed down

[Impulse Noise Removal]

■ Pseudomedian filtering (PMED)

- e.g. 5-element filter

a, b, c, d, e → spatially ordered

MAXMIN = A (under estimated)

$$= \max(\min(a,b,c) , \min(b,c,d) , \min(c,d,e))$$

MINMAX = B (over estimated)

$$= \min(\max(a,b,c) , \max(b,c,d) , \max(c,d,e))$$

→ PMED(a, b, c, d, e)

$$= 0.5 * (A + B) = \underline{0.5 * (MAXMIN + MINMAX)}$$

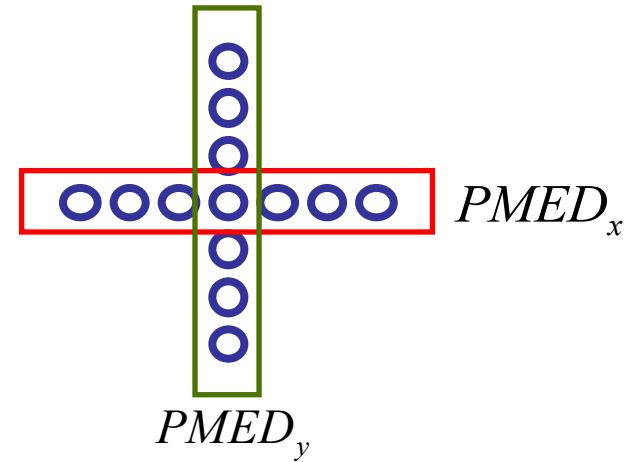
$$\sim \text{MED}(a, b, c, d, e)$$

Impulse Noise Removal

■ Pseudomedian filtering (PMED)

○ 2D case

$$PMED = \frac{1}{2} (PMED_x + PMED_y)$$



$$\begin{aligned} PMED &= \frac{1}{2} \max(MINMAX(x_c), MAXMIN(y_R)) \\ &+ \frac{1}{2} \min(MAXMIN(x_c), MINMAX(y_R)) \end{aligned}$$

[Impulse Noise Removal]

- Pseudomedian filtering (PMED)
 - MAXMIN
 - Remove salt noise
 - MINMAX
 - Remove pepper noise
 - May cascade two operations
 - Remove salt and pepper noise

Impulse Noise Removal



Original noisy image



MAXMIN



MINMAX of MAXMIN



MINMAX



MAXMIN of MINMAX

Q: same results?

Quality Measurement

■ Peak signal-to-noise ratio (PSNR)

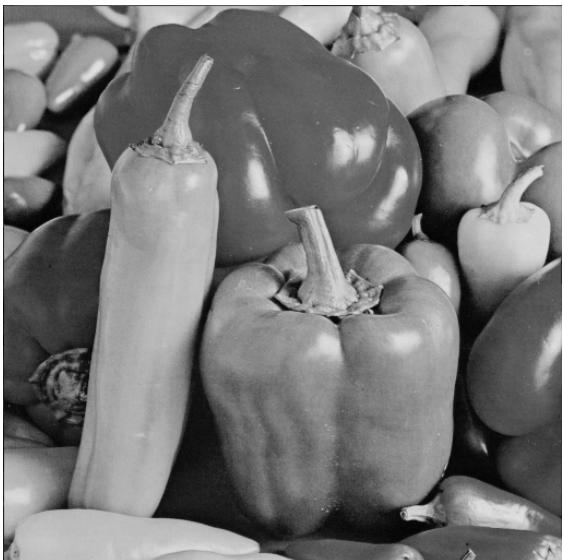
- Mean squared error (MSE)

$$MSE = \frac{1}{w^* h} \sum_j \sum_k [F(j, k) - F'(j, k)]^2$$

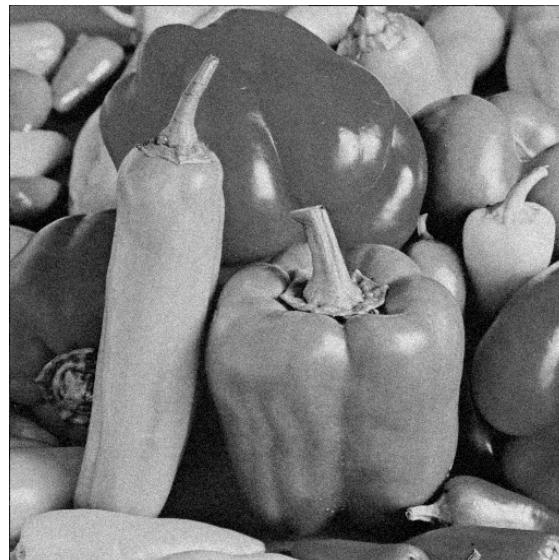
- The PSNR is defined as

$$PSNR = 10 \times \log_{10} \left(\frac{255^2}{MSE} \right)$$

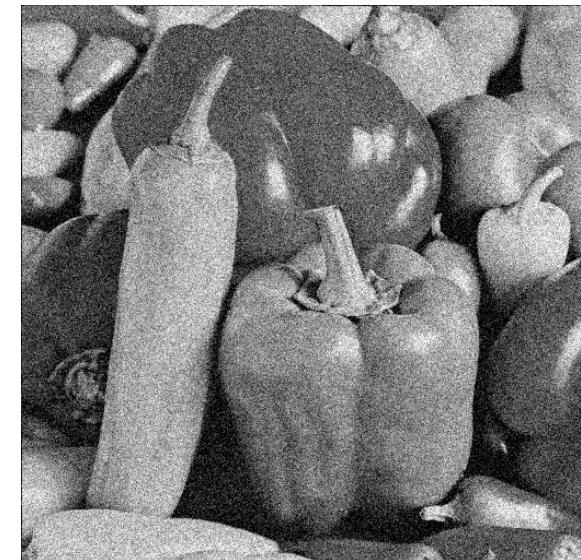
Example



Original image



Gaussian noise ($\sigma=10$)
PSNR : 28.18dB



Gaussian noise ($\sigma=30$)
PSNR : 18.81dB

Q: Represent perceived visual quality?