Morphological Operators

CS/BIOEN 4640: Image Processing Basics

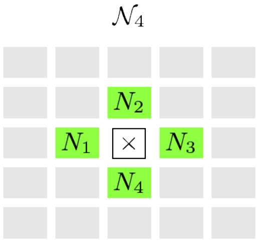
February 23, 2012

Common Morphological Operations

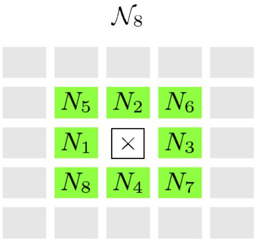
- Shrinking the foreground ("erosion")
- Expanding the foreground ("dilation")
- Removing holes in the foreground ("closing")
- Removing stray foreground pixels in background ("opening")
- Finding the outline of the foreground
- Finding the skeleton of the foreground

Pixel Neighborhoods

Remember the two definitions of “neighbors” that we’ve discussed:

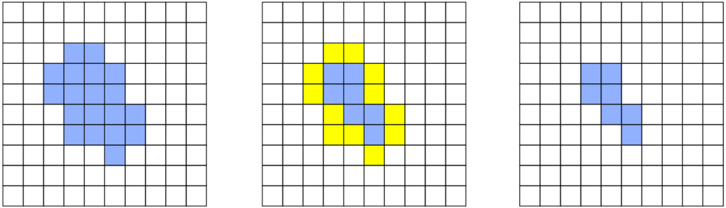


4 Neighborhood



8 Neighborhood

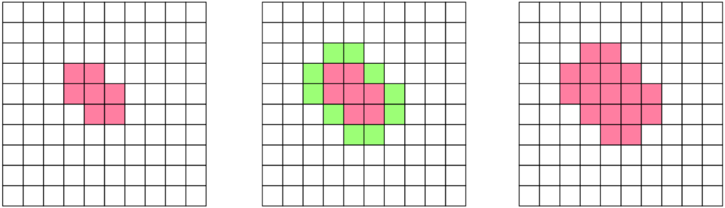
Erosion Example



(a) (b) (c)

Change a foreground pixel to background if it has a background pixel as a 4-neighbor.

Dilation Example



(a) (b) (c)

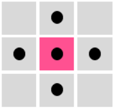
Change a background pixel to foreground if it has a foreground pixel as a 4-neighbor.

Structuring Element

Definition

A **structuring element** is simply a binary image (or mask) that allows us to define arbitrary neighborhood structures.

Example:

$H =$ 

origin (hot spot)

This is the structuring element for the 4-neighborhood.

Binary Images as Sets

We can think of a binary image $I(u, v)$ as the set of all *pixel locations* in the foreground:

$$\mathcal{Q}_I = \{(u, v) \mid I(u, v) = 1\}$$

To simplify notation, we'll use a single variable for a coordinate pair, $\mathbf{p} = (u, v)$. So,

$$\mathcal{Q}_I = \{\mathbf{p} \mid I(\mathbf{p}) = 1\}$$

Set Operations = Point Operations

- **Complement = Inversion**
  Let $\bar{I}$ denote image inversion (pointwise NOT)
  $$\mathcal{Q}_{\bar{I}} = \overline{\mathcal{Q}}_I = \{\mathbf{p} \in \mathbb{Z}^2 \mid \mathbf{p} \notin \mathcal{Q}_I\}$$
- **Union = OR**
  Let $I_1 \vee I_2$ be pointwise OR operation
  $$\mathcal{Q}_{I_1 \vee I_2} = \mathcal{Q}_{I_1} \cup \mathcal{Q}_{I_2}$$
- **Intersection = AND**
  Let $I_1 \wedge I_2$ be pointwise AND operation
  $$\mathcal{Q}_{I_1 \wedge I_2} = \mathcal{Q}_{I_1} \cap \mathcal{Q}_{I_2}$$

More Image Operations

(Instead of $\mathcal{Q}_I$, we'll just use I to denote the set)

- **Translation:** Let $\mathbf{d} \in \mathbb{Z}^2$

$$I_{\mathbf{d}} = \{(\mathbf{p} + \mathbf{d}) \mid \mathbf{p} \in I\}$$

- **Reflection:**

$$I^* = \{-\mathbf{p} \mid \mathbf{p} \in I\}$$

Dilation

Definition

A **dilation** of an image I by the structure element H is given by the set operation

$$I \oplus H = \{(\mathbf{p} + \mathbf{q}) \mid \mathbf{p} \in I, \mathbf{q} \in H\}$$

Alternative definition: Take the union of copies of the structuring element, $H_{\mathbf{p}}$, centered at every pixel location $\mathbf{p}$ in the foreground:

$$I \oplus H = \bigcup_{\mathbf{p} \in I} H_{\mathbf{p}}$$

Dilation Algorithm

Uses equivalent formula $I \oplus H = \bigcup_{\mathbf{q} \in H} I_{\mathbf{q}}$:

Input: Image I, structuring element H
Output: Image $I' = I \oplus H$

1. Start with all-zero image I'
2. Loop over all $\mathbf{q} \in H$
3. Compute shifted image $I_{\mathbf{q}}$
4. Update $I' = I' \vee I_{\mathbf{q}}$

Erosion

Definition

A **erosion** of an image I by the structure element H is given by the set operation

$$I \ominus H = \{\mathbf{p} \in \mathbb{Z}^2 \mid (\mathbf{p} + \mathbf{q}) \in I, \text{for every } \mathbf{q} \in H\}$$

Alternative definition: Keep only pixels $\mathbf{p} \in I$ such that $H_{\mathbf{p}}$ fits inside I:

$$I \ominus H = \{\mathbf{p} \mid H_{\mathbf{p}} \subseteq I\}$$

Duality of Erosion and Dilation

Erosion can be computed as a dilation of the background:

$$I \ominus H = \overline{(\bar{I} \oplus H^*)}$$

Same duality for dilation:

$$I \oplus H = \overline{(\bar{I} \ominus H^*)}$$

Erosion Algorithm

Uses dual, $I \ominus H = \overline{(\bar{I} \oplus H^*)}$

Input: Image I, structuring element H
Output: Image $I' = I \ominus H$

1. Start with inversion, $I' = \bar{I}$
2. Dilate I' with reflected structure element, H^*
3. Invert I'

Properties of Dilation

Similar to convolution properties, we need to assume the image domains are large enough that operations don't "fall off" the edges.

Commutativity:

$$I \oplus H = H \oplus I$$

Means we can switch the roles of the structuring element and the image

Properties of Dilation

Associativity:

$$I_1 \oplus (I_2 \oplus I_3) = (I_1 \oplus I_2) \oplus I_3$$

Means that we can sometimes break up a big structuring element into smaller ones:

That is, if $H = H_1 \oplus H_2 \oplus \ldots \oplus H_n$, then

$$I \oplus H = (((I \oplus H_1) \oplus H_2) \oplus \ldots \oplus H_n)$$

Properties of Erosion

- It is NOT commutative:

$$I \ominus H \neq H \ominus I$$

- It is NOT associative, but:

$$(I \ominus H_1) \ominus H_2 = I \ominus (H_1 \oplus H_2)$$

Some Particular Dilation Operators

- **Identity:** $\mathrm{id} = \{(0,0)\}$

$$I \oplus \mathrm{id} = \mathrm{id} \oplus I = I$$

- **Shift by k pixels in x:** $S_x = \{(k,0)\}$
- **Shift by k pixels in y:** $S_y = \{(0,k)\}$

Opening

Opening operation is an erosion followed by a dilation:

$$I \circ H = (I \ominus H) \oplus H$$

Stray foreground structures that are smaller than the H structure element will disappear. Larger structures will remain.

Closing

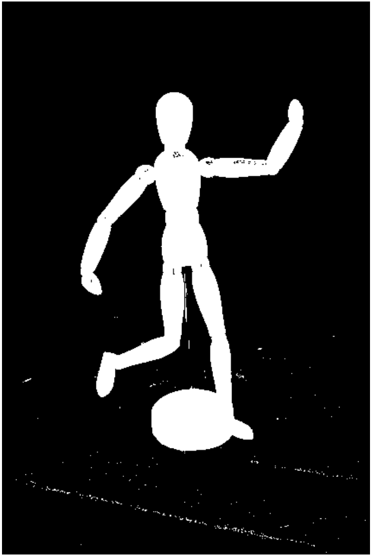
Closing operation is a dilation followed by an erosion:

$$I \bullet H = (I \oplus H) \ominus H$$

Holes in the foreground that are smaller than H will be filled.

Improving a Segmentation

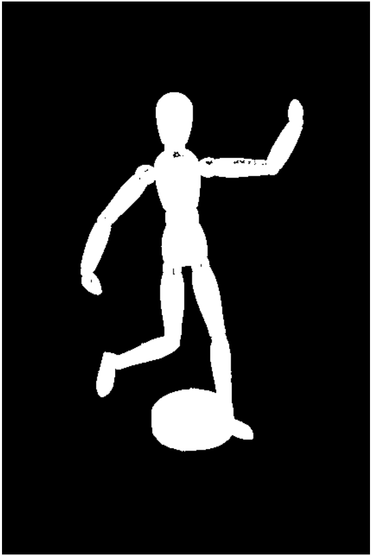
Original image



Initial threshold

Improving a Segmentation

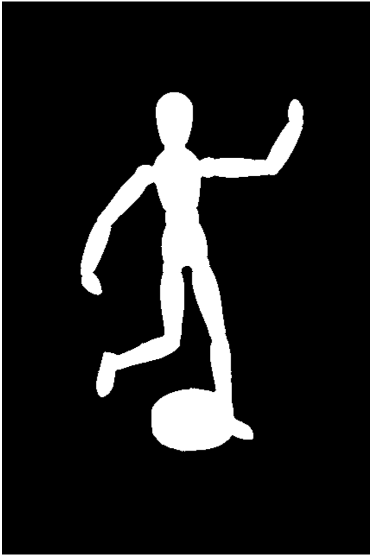
Original image



After opening

Improving a Segmentation

Original image



After closing

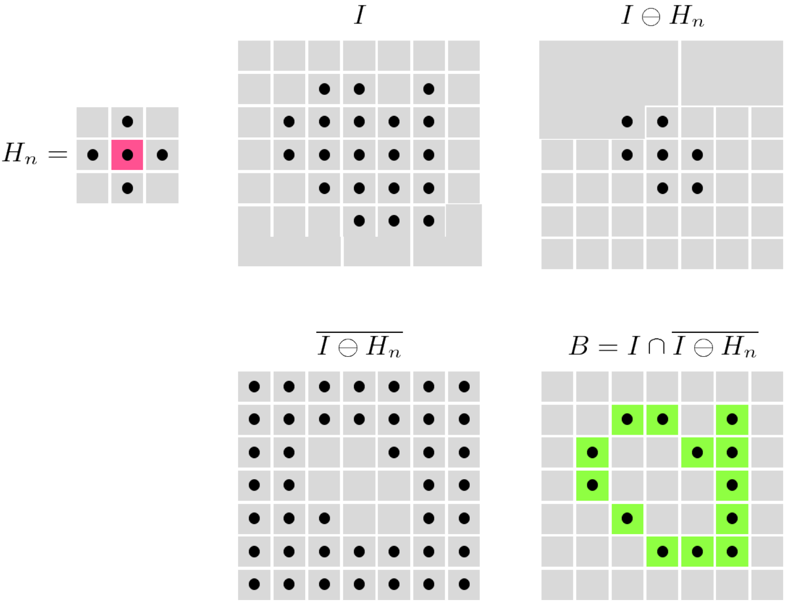
Outline

The outline image $B(u, v)$ of a binary object can be computed using a dilation followed by a subtraction (or XOR operation):

$$I' = I \ominus H$$

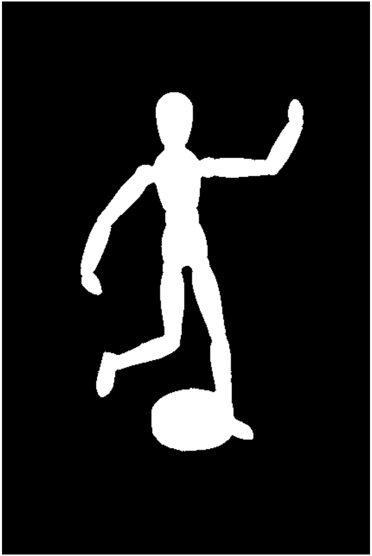
$$B(u, v) = \mathrm{XOR}(I'(u, v), I(u, v))$$

Outline

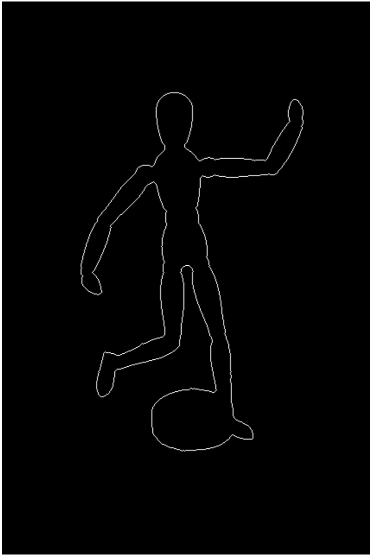


$H_n =$
I
$I \ominus H_n$
$\overline{I \ominus H_n}$
$B = I \cap \overline{I \ominus H_n}$

Outline Example

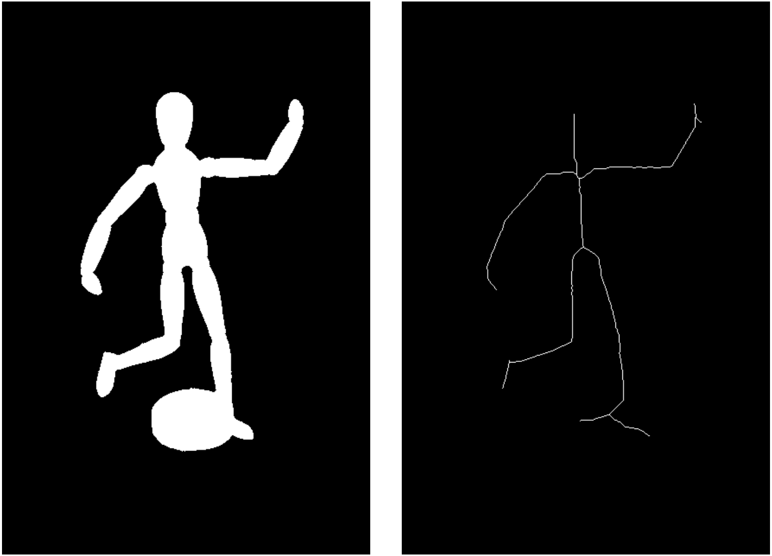


Binary segmentation



After outline operation

Skeletonize



Repeatedly run erosion, stop when 1-pixel thick

Grayscale Morphology

- We can also apply morphological operators to grayscale images.
- Now our structuring elements are real-valued $H(i,j) \in \mathbb{R}$. That is, they are grayscale images.
- Need to make a distinction between 0 and "don't care" entries.

Grayscale Morphology

Dilation:

$$(I \oplus H)(u,v) = \max_{(i,j)\in H} \{I(u+i, v+j) + H(i,j)\}$$

Erosion:

$$(I \ominus H)(u,v) = \min_{(i,j)\in H} \{I(u+i, v+j) + H(i,j)\}$$

Link to ImageJ Morphology Package

http://rsbweb.nih.gov/ij/plugins/gray-morphology.html