Digital Image Processing

Lecture #11

Ming-Sui (Amy) Lee

Announcement

- **Class Information**
  - **The following schedule**

03/23	Lecture 5	05/11	proposal
03/30	Lecture 6 & 7	05/18	Lecture 11
04/06	Lecture 8	05/25	Lecture 12
04/13	RealSense	06/01	Lecture 13
04/20	midterm	06/08	Demo
04/27	RealSense & Lecture 9	06/15	Demo
05/04	Lecture 9 & Lecture 10	06/22	Final Package Due

Announcement

- Class Information
  - Homework #4
    - Due Date: 11:59 a.m. on May 31, 2016

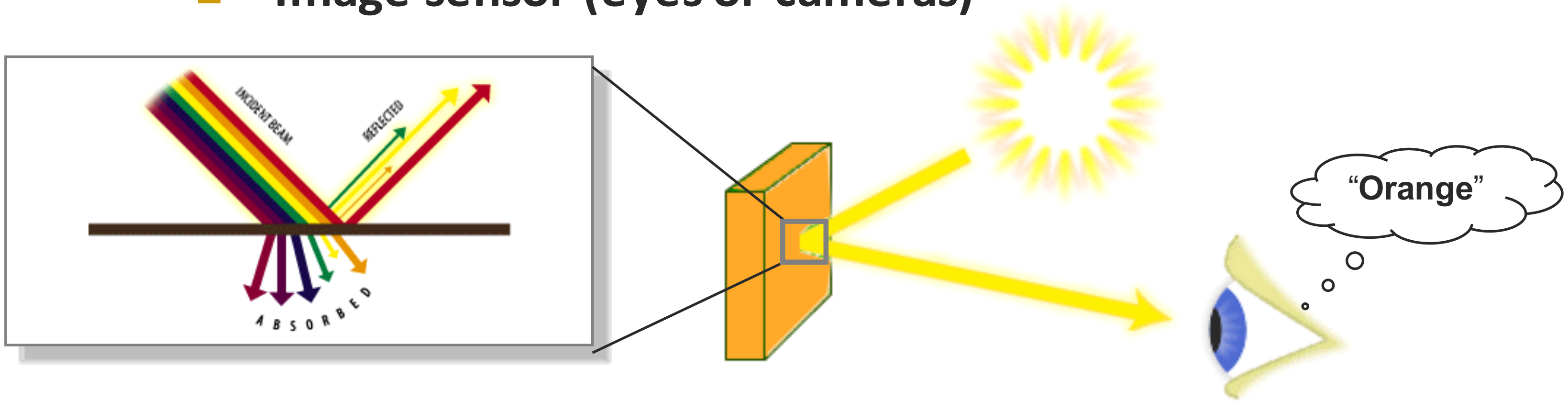
Color Image Processing

Color Image Processing

- Introduction
  - Color Vision
    - What is color
    - Visible spectrum
    - Spectral colors
    - Color perception
    - Color systems
    - Tri-stimulus colors
    - CIE primaries
    - Color mixing
  - Color models
    - Overview
    - RGB
    - CMY/CMYK
    - YIQ/YUV
    - HSI/HSV
    - L*a*b
    - Transform
    - Comparison
  - Color image processing
    - Individual vs. vector
    - Tonal correction
    - Color correction
    - Hist. equalization
    - Sharpening
    - Color quantization
    - Pseudo-coloring
    - Re-coloring
    - Multi-spectral images

Color Image Processing

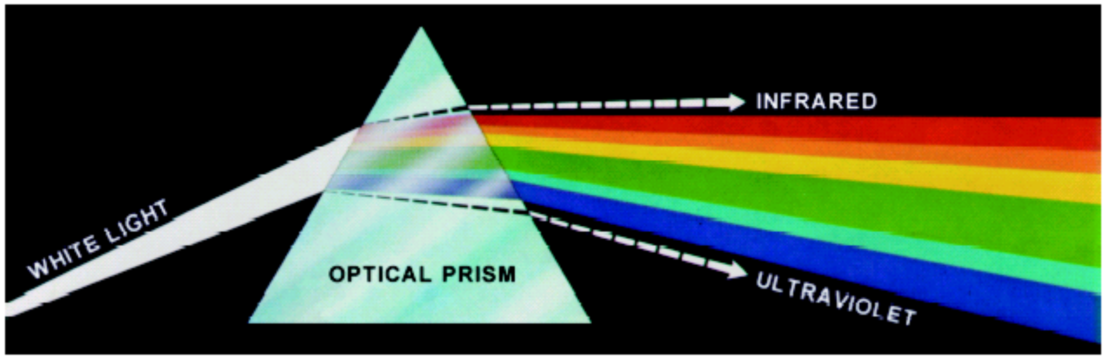
- Human perception, not directly measurable
  - Related to the light spectrum of a stimulus
  - Depends on
    - Light source
    - Reflectance (Reflecting objects)
    - Image sensor (eyes or cameras)



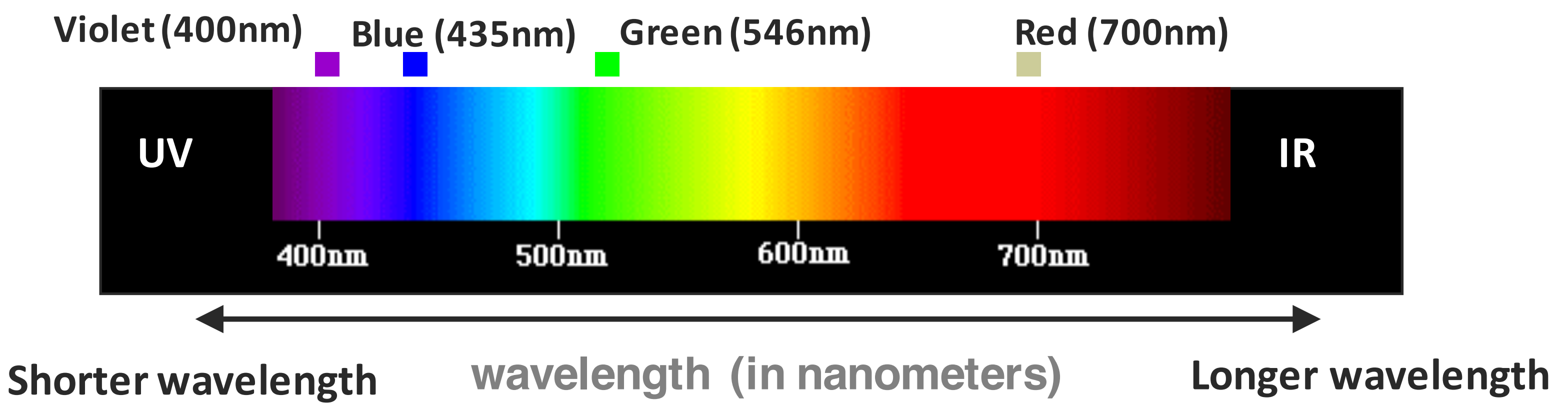
- Some or all of the light may be absorbed
- Dominate wavelength reflected by objects determines the color tone

Color Image Processing

- Newton's experiment

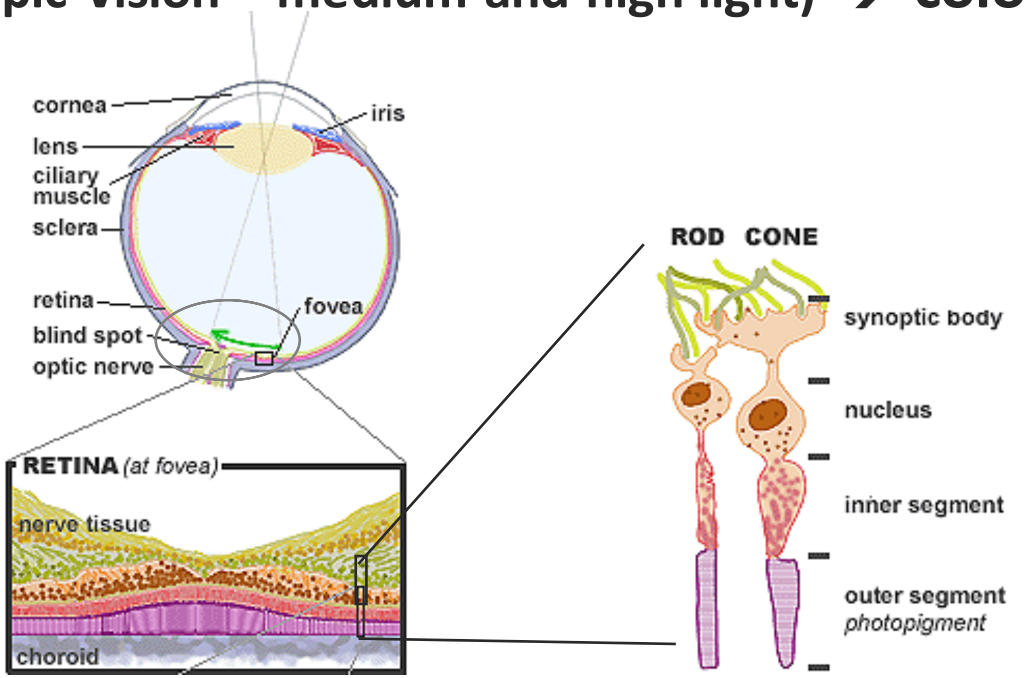


- Visible spectrum



Color Image Processing

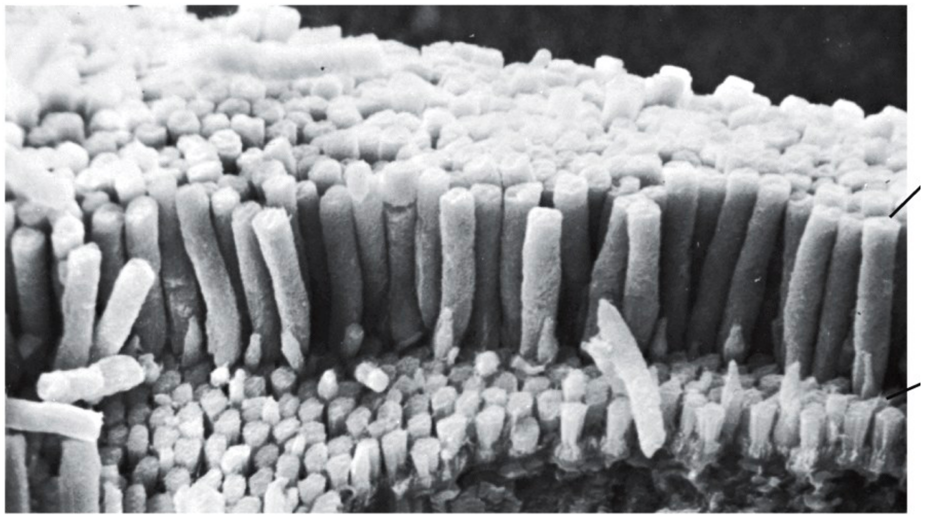
- **Retina**
  - a light-sensitive layer at the back of the eye
- **Photosensitive cells**
  - **Rods** (scotopic vision – low light) → **luminance**
  - **Cones** (photopic vision – medium and high light) → **color**



Rods: 75~150 millions
Cones: 6-7 millions

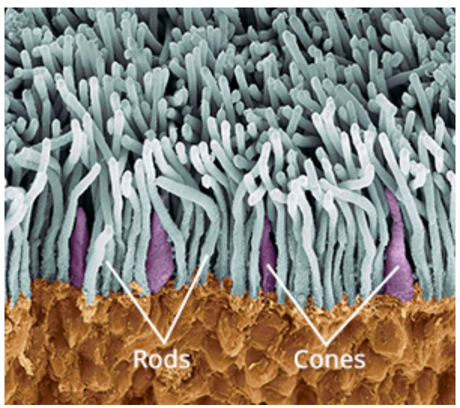
Color Image Processing

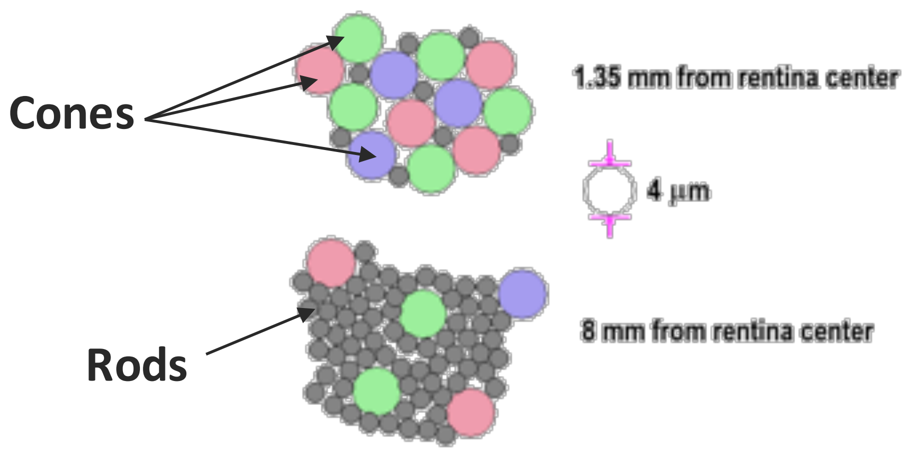
- Rods 
  - Low illumination levels (scotopic vision)
  - Provide our night vision ability
  - A thousand times more sensitive to light than cones
  - Much slower to respond to light than cones
  - Distribute primarily in the periphery of the visual field



Color Image Processing

- Cones 
  - High illumination levels (Photopic vision)
  - Less sensitive than rods
  - Not evenly distributed - density decreases with distance from fovea

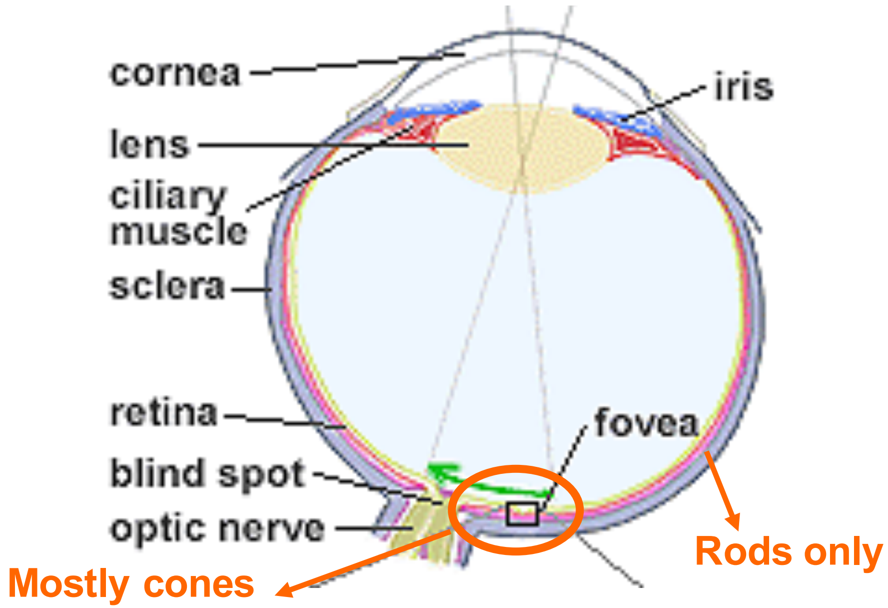


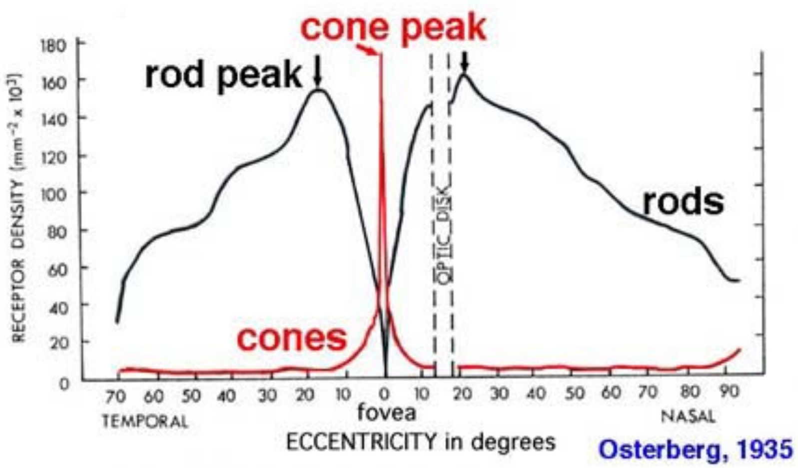


- "red" cones (64%)
- "green" cones (32%)
- "blue" cones (2%)

//Note// Bayer filter
use twice as many green elements as red or blue to mimic the human eye's greater resolving power with green light

Color Image Processing

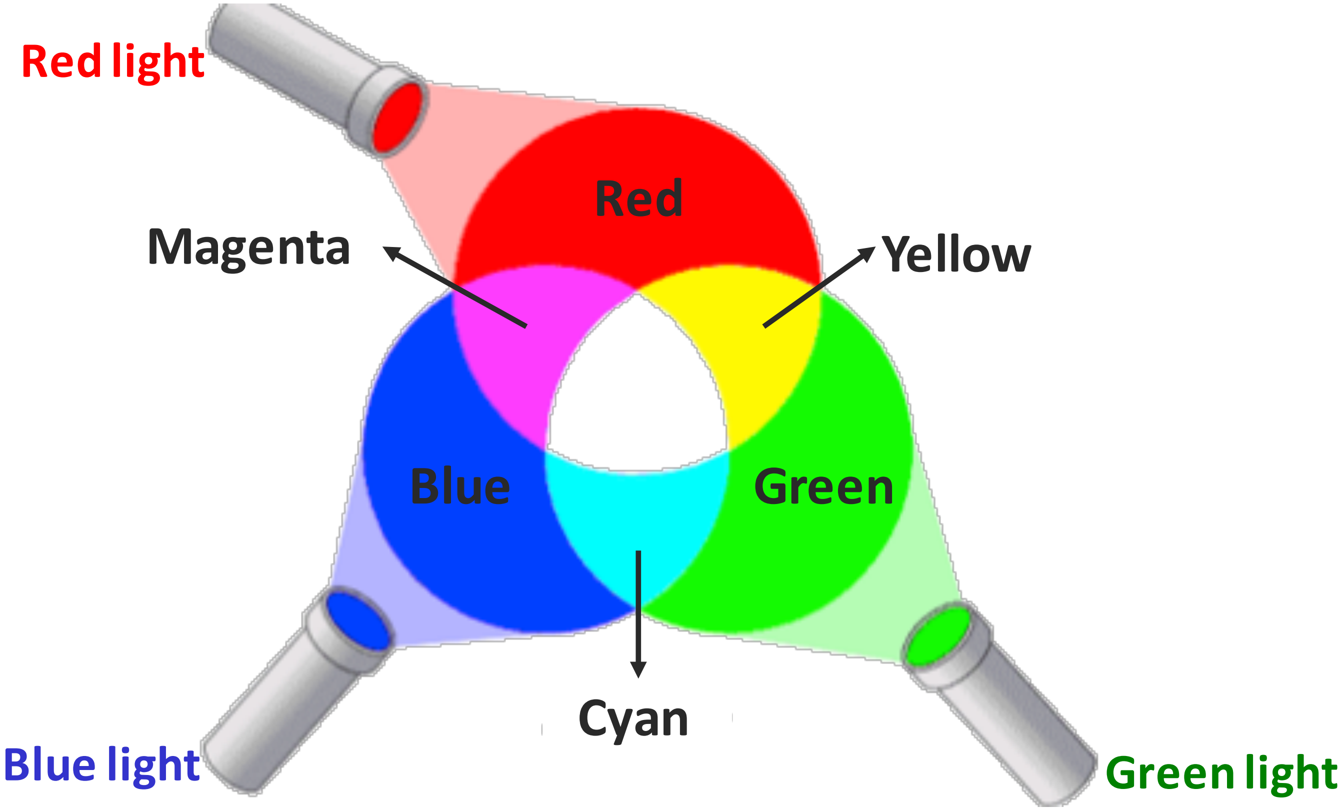




- Rod and cone densities
  - The peak number of cones occurs in the fovea
  - The rods peak about 20 degrees from the center
  - Rods: 75~150 (120) millions
  - Cones: 6~7 (7) millions

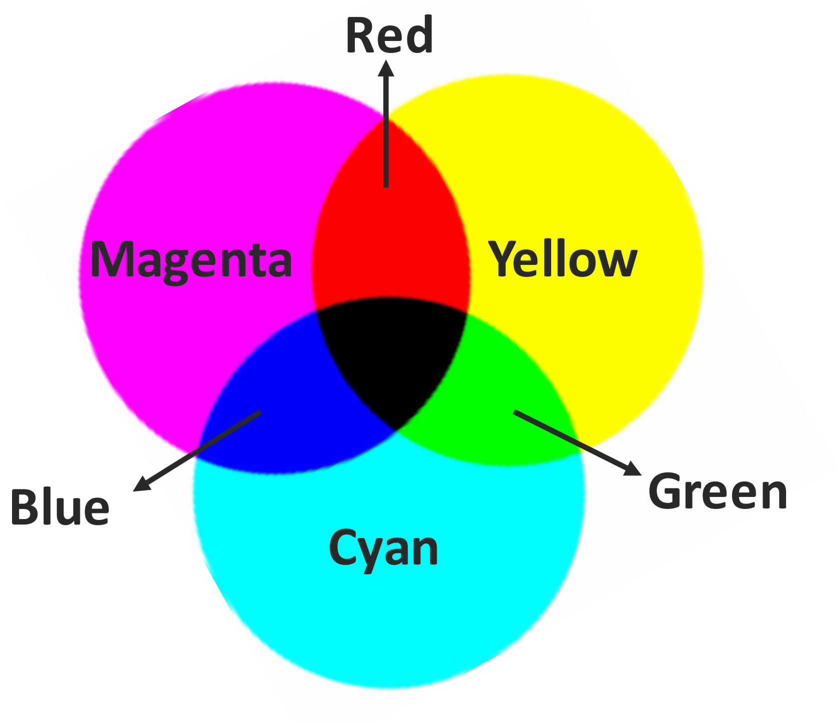
Color Image Processing

- **Additive Color Matching**
  - Primary colors can be combined to generate different composite colors



Color Image Processing

- **Subtractive Color Matching**
  - Composite color is the difference between two added colors
  - Used for painting and printing



Chromaticity

- **Chromaticity**

  = Hue (色調) + Saturation (飽和度)

  - **Hue: dominant wavelength and color**
  - **Saturation: relative purity or the amount of white light mixed with a hue**
- **Color = Brightness + Chromaticity**
- **Tristimulus values**

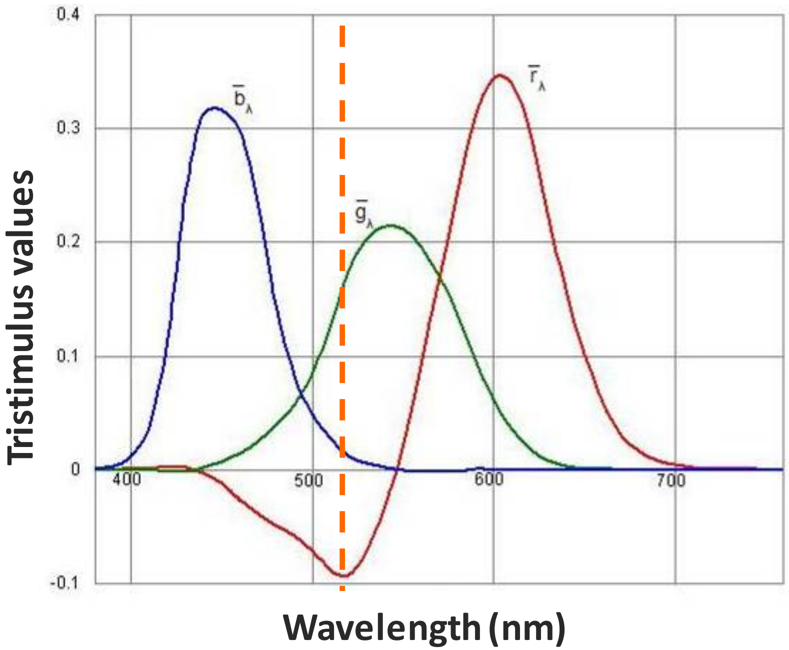
  the amount of R, G, and B needed to form any particular color : X, Y, Z

  - **Trichromatic coefficients**

$$x = X/(X+Y+Z) \qquad y = Y/(X+Y+Z) \qquad z = Z/(X+Y+Z)$$

Color Image Processing

- **Tristimulus**
  - The primary sources recommended by CIE
    - CIE: International Commission on Illuminations 1931
    - 700nm(red), 546.1nm(green) and 435.8nm(blue)
    - Be able to match all the wavelengths of the visible spectrum



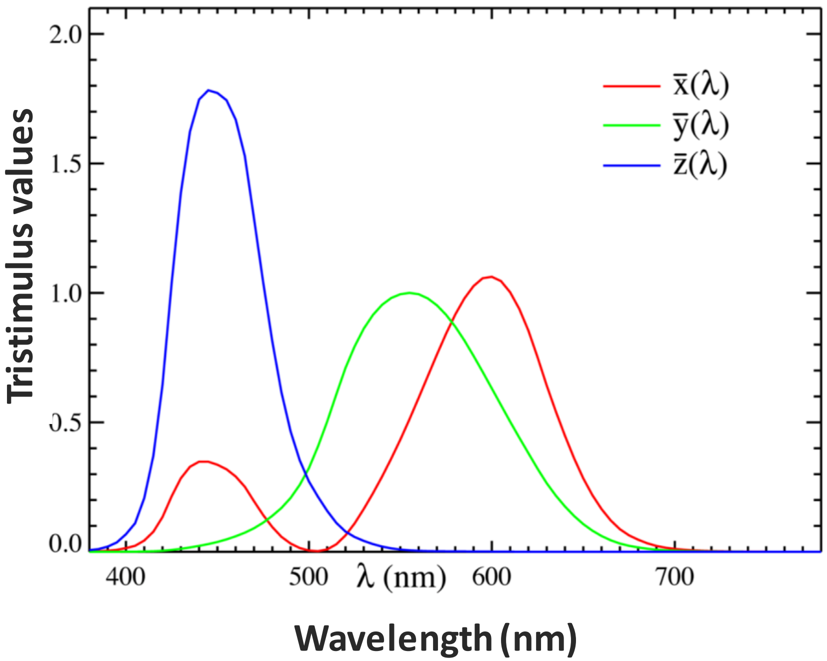
α : The amount of the k-th primary needed to produce a color C

β : reference white color

α / β : the tristimulus value of color C

Color Image Processing

- **CIE XYZ System**
  - Three “tristimulus” of a color: *X, Y* and *Z*
  - Three color matching functions: $\bar{x}\quad \bar{y}\quad \bar{z}$
  - Make all tristimulus values all positive



$$X = \int_{\lambda} I_{\lambda}\bar{x}(\lambda)d\lambda$$

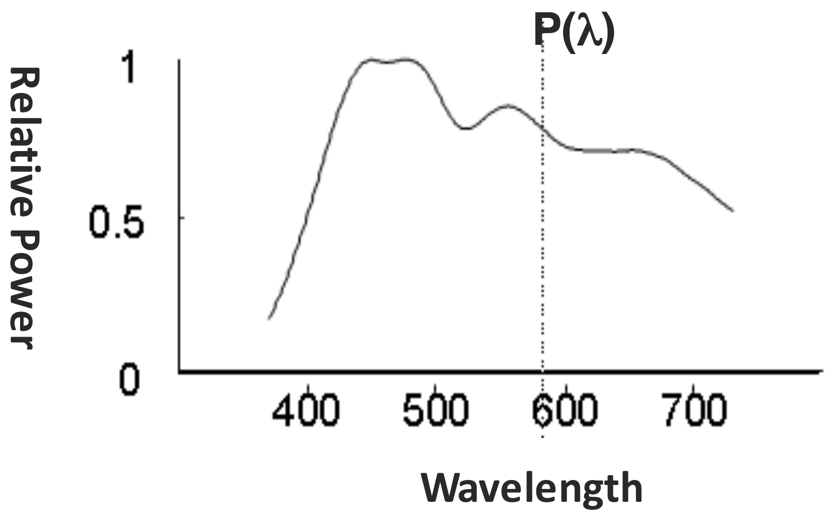
$$Y = \int_{\lambda} I_{\lambda}\bar{y}(\lambda)d\lambda$$

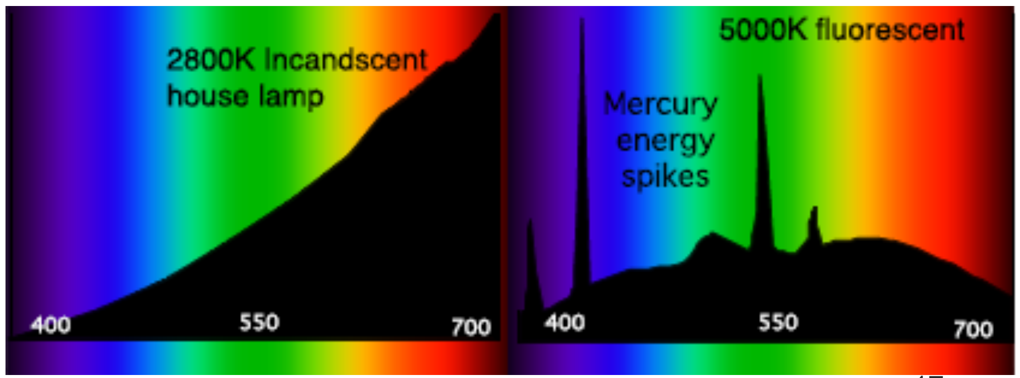
$$Z = \int_{\lambda} I_{\lambda}\bar{z}(\lambda)d\lambda$$

Spectral power distribution

Color Image Processing

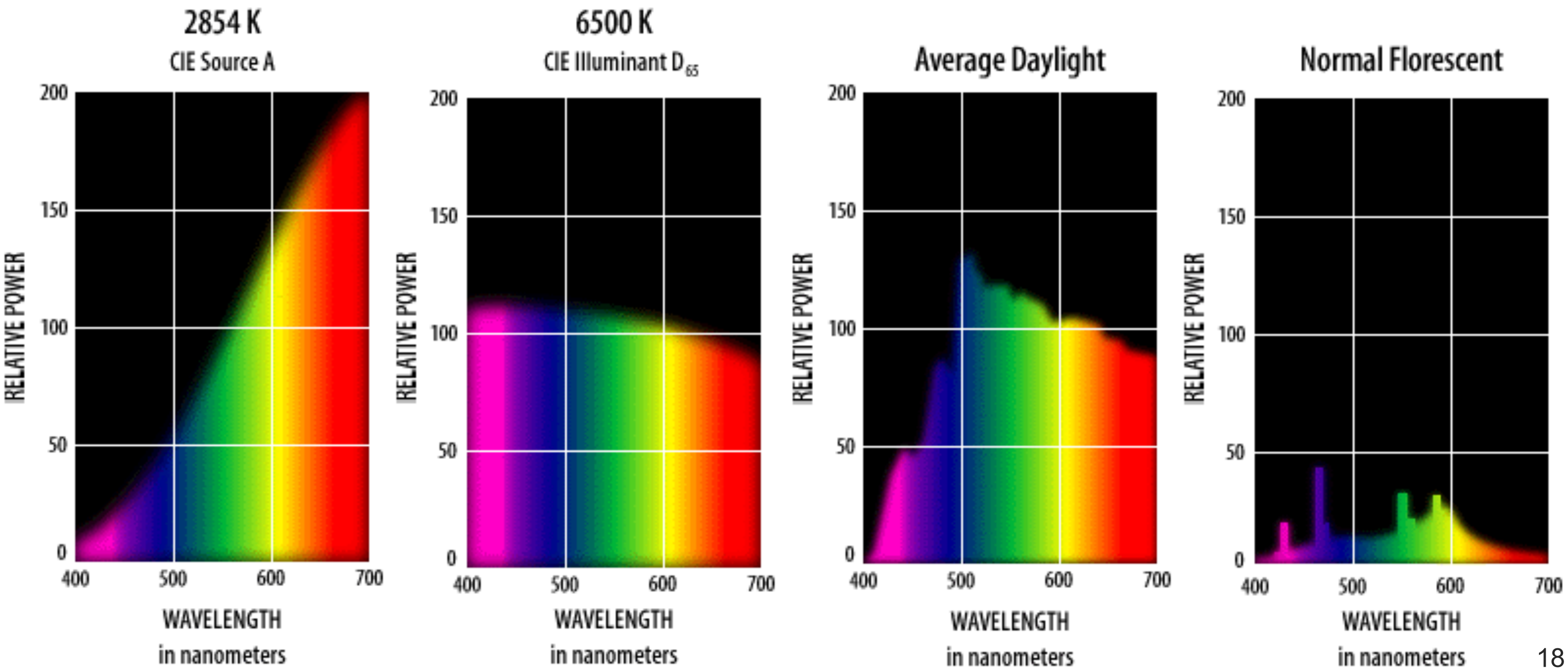
- **Spectral Power Distribution (SPD)**
  - Light may be precisely characterized by giving the power of the light at each wavelength in the visible spectrum
  - SPD is a function $P(\lambda)$ that defines the power of the light at each wavelength





Color Image Processing

- **Spectral Power Distribution (SPD)**
  - **Examples**



Color Image Processing

- **Spectral Power Distribution (SPD)**

  The color appearance change due to different illumination may be quantified in terms of the SPD of the light

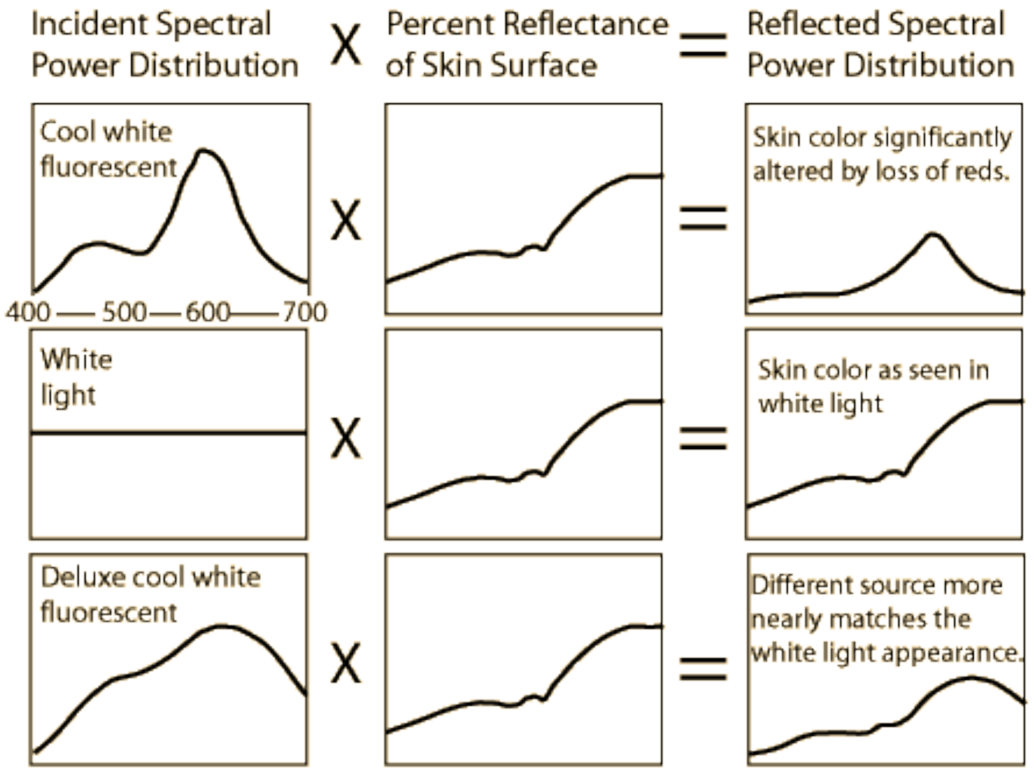
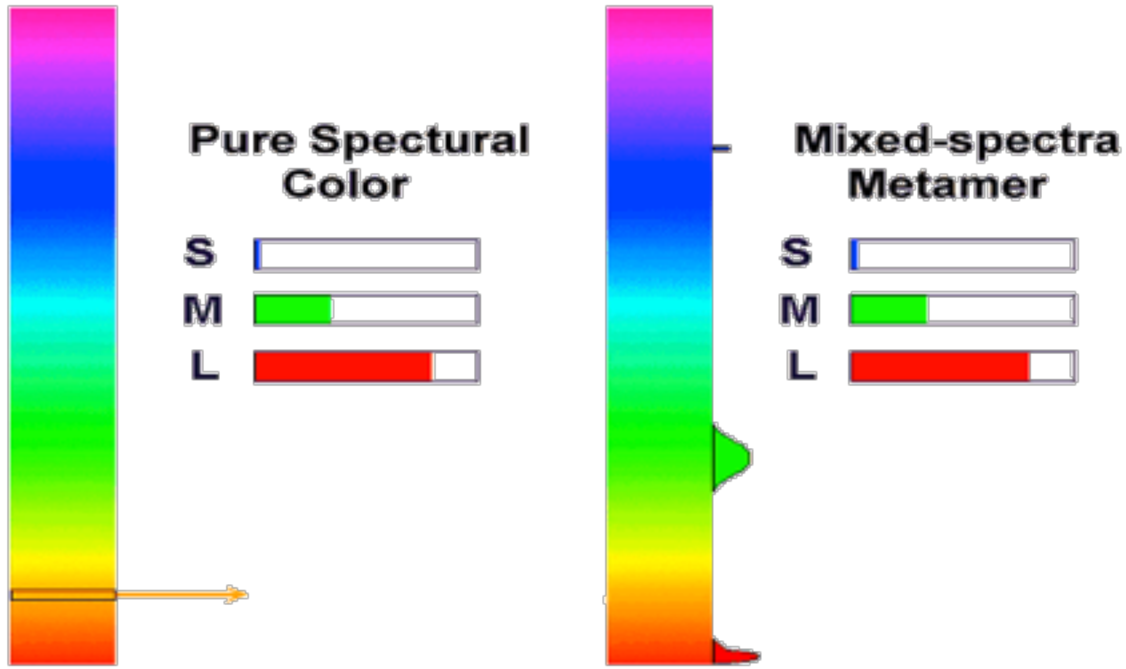


*Image source: *HyperPhysics**

Color Image Processing

- **Metamer**
  - Two colors that appear the same visually might have different SPD's (with different spectral composition)



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https://www.youtube.com/watch?v=I9HG5ewTrJ8

Color Image Processing

- **Chromaticity Diagram**
  - Any color can be defined by its tristimulus values (X, Y, Z) or chromaticity coordinates (x,y,z)
  - Mathematical conversions between x,y,z and X,Y,Z

$$x = \frac{X}{X+Y+Z}; \qquad y = \frac{Y}{X+Y+Z}$$

$$z = \frac{Z}{X+Y+Z} = 1 - x - y$$

  - x,y,z represent the proportions of the X primary, Y primary and Z primary respectively in a given color mixture

Color Image Processing

C.I.E. Chromaticity Diagram (1931)

- Commission Internationale d’Eclairage (international commission on illumination)
- Develop light measurement standard (most widely used standard today)

$$x = \frac{X}{X+Y+Z}$$

$$y = \frac{Y}{X+Y+Z}$$

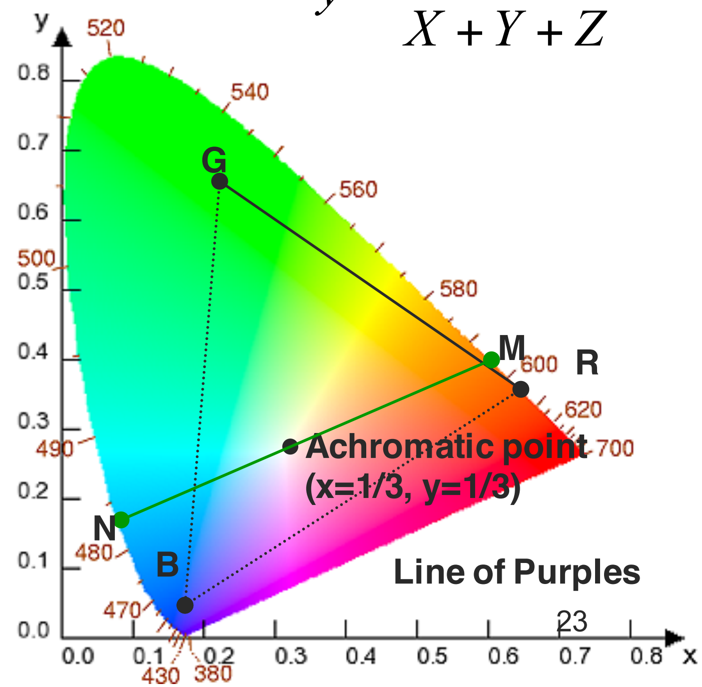
- Achromatic light

  “white” or uncolored light
- Chromatic light

  colored light
- Monochromatic light

  contained only one wavelength

  (laser light)



Color Image Processing

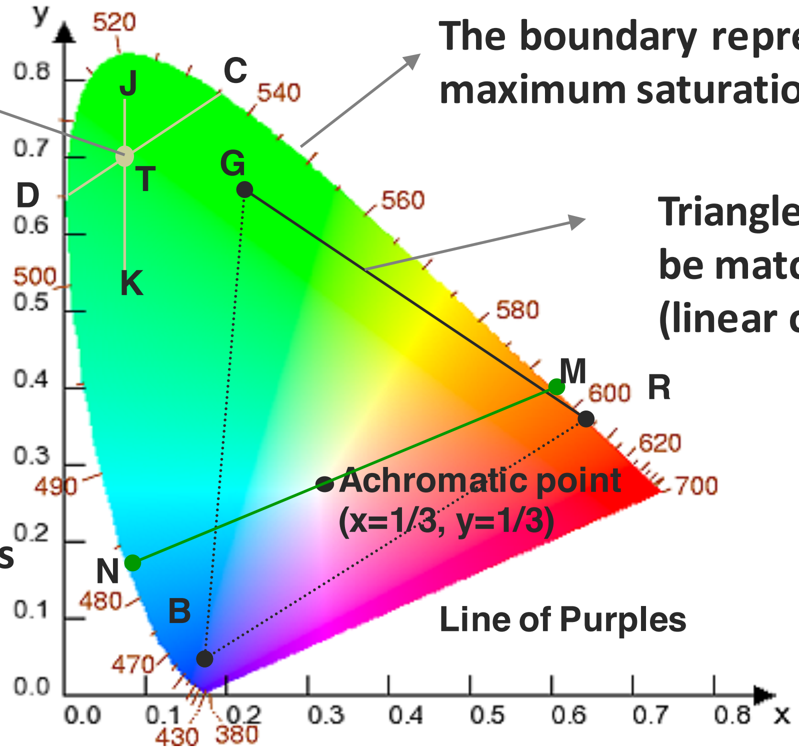
C.I.E. Chromaticity Diagram

The pairs (J,K) and (C,D) can produce the same color T if combined properly

The boundary represents maximum saturation for the spectral colors

Triangle of colors that can be matched by R, G, B (linear combination)

No three colors in the diagram can span the whole color space



The connecting line pass through achromatic point form a complementary color pair, e.g. (M,N)

All colors that can be humanly perceived can be plotted within this space

CIE 1931 *xy* Chromaticity Diagram

Color Image Processing

- **Color Reproduction**
  - CIE chromaticity diagram is used to show color reproduction of various color imaging methods
  - Gamut
    - The range of colors accessible to a given process
    - Color coverage examples:

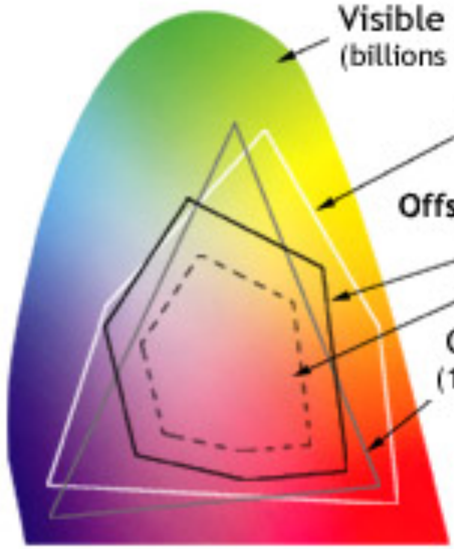


Image source: www.worgx.com

Color Image Processing

- **Color Reproduction**

http://ap.viewsonic.com/me/products/lcd/superclear/

Color Image Processing

http://genk.vn/dien-thoai/anh-tai-dai-chien-camera-phone-2013-20131122233531025.chn

Color Spaces

Color Spaces

- RGB
  - used in CRT monitors
- YIQ, YUV
  - formerly used in NTSC (National Television System Committee) television broadcasts, employed mainly in North and Central America, and Japan
- YCbCr
  - used in image and video compression: JPEG and MPEG
- CMYK
  - used in the printing process
- HSI, HSV
  - used by artists as it is more intuitive to think about a color in terms of hue and saturation
- Lab
  - Commonly used for astronomical image

Color Spaces

- **Linear Transformation of RGB color spaces**
  - **RGB: used in CRT monitors**
  - **YIQ: used in NTSC TV,** $T = \begin{pmatrix} 0.299 & 0.587 & 0.114 \\ 0.596 & -0.274 & 0.322 \\ 0.211 & -0.523 & 0.312 \end{pmatrix}$
  - **YUV: used in PAL, SECAM,** $T = \begin{pmatrix} 0.299 & 0.587 & 0.114 \\ -0.147 & -0.289 & 0.436 \\ 0.615 & -0.515 & 0.312 \end{pmatrix}$
  - **YCbCr: used for JPEG, MPEG,** $T = \begin{pmatrix} 0.299 & 0.587 & 0.114 \\ 0.500 & -0.4187 & -0.0813 \\ -0.1687 & -0.3313 & 0.5 \end{pmatrix}$

Color Spaces

- **Other transformations**
  - **CMY: cyan, magenta, yellow**
    - **Complementary to red, green and blue, respectively**
    - **Useful in color printers and copiers**
    - **Used for subtractive synthesis from white in color printers**

      (C,M,Y)=(1,1,1)-(R,G,B)
  - **CMYK: like CMY, uses black (K)**
    - **Given (C,M,Y)**

      K=min(C,M,Y); C=C-K; M=M-K; Y=Y-K

Image source: www.worgx.com

Color Spaces

- **RGB to CIEXYZ**

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.412453 & 0.357580 & 0.180423 \\ 0.212671 & 0.715160 & 0.072169 \\ 0.019334 & 0.119193 & 0.950227 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

- **CIEXYZ to CIELAB**

$\mathrm{L}^* = 116\mathrm{f}(\frac{Y}{Y_N})\text{-}16$; $\mathrm{a}^* = 500\left[\mathrm{f}\left(\frac{X}{X_N}\right) - \mathrm{f}\left(\frac{Y}{Y_N}\right)\right]$; $\mathrm{b}^* = 200\left[\mathrm{f}\left(\frac{Y}{Y_N}\right) - \mathrm{f}\left(\frac{Z}{Z_N}\right)\right]$

where $f(t) = \begin{cases} (t)^{\frac{1}{3}}, & if\ t > (\frac{6}{29})^3 \\ \frac{1}{3}(\frac{29}{6})^2 t + \frac{4}{29}, & otherwise \end{cases}$

and X_N, Y_N, Z_N are the CIEXYZ tristimulus values of the reference white point. Under Illuminant D65, the values are:
$X_N = 95.047,\ Y_N = 100.000, Z_N = 108.883$

Color Spaces

- **CIELAB to CIEXYZ**

$$X = X_N f^{-1}\left(\frac{1}{116}(\mathrm{L}^* + 16) + \frac{1}{500}\mathrm{a}^*\right)$$

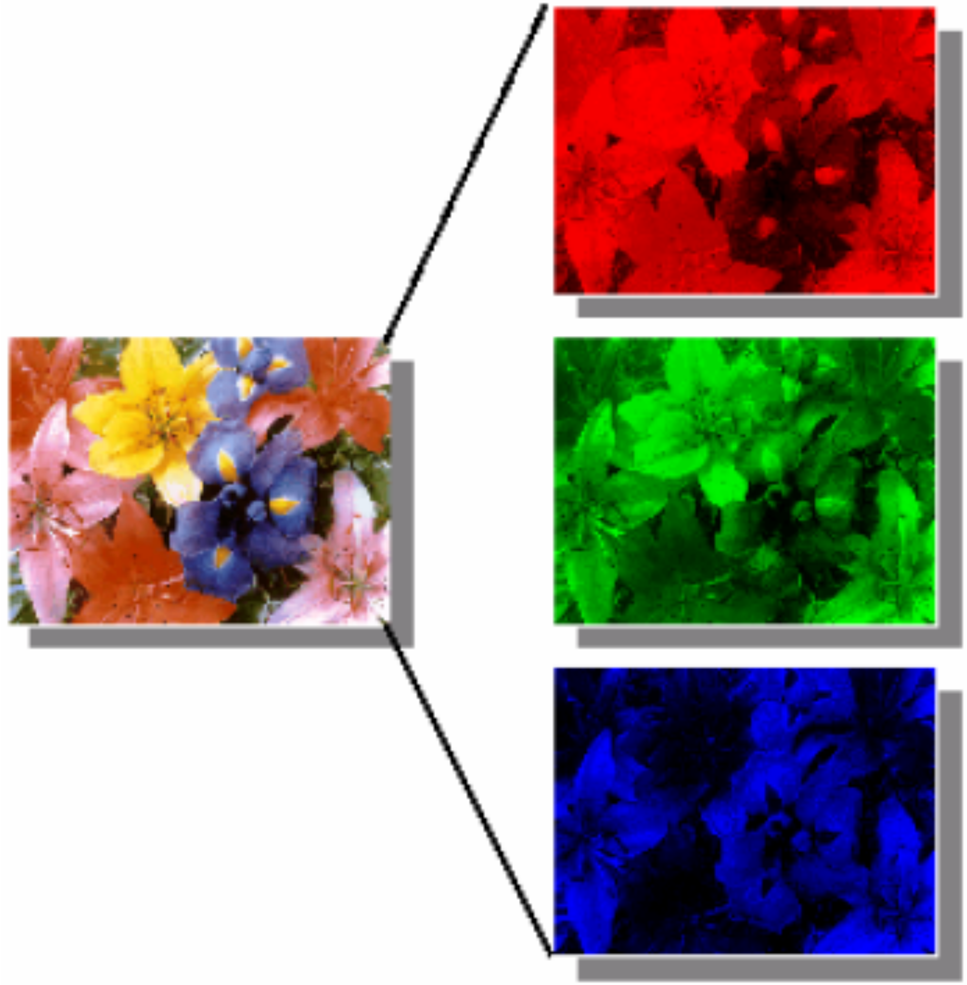
$$Y = Y_N f^{-1}\left(\frac{1}{116}(\mathrm{L}^* + 16)\right)$$

$$Z = \mathrm{Z}_N f^{-1}\left(\frac{1}{116}(\mathrm{L}^* + 16) - \frac{1}{200}b^*\right)$$

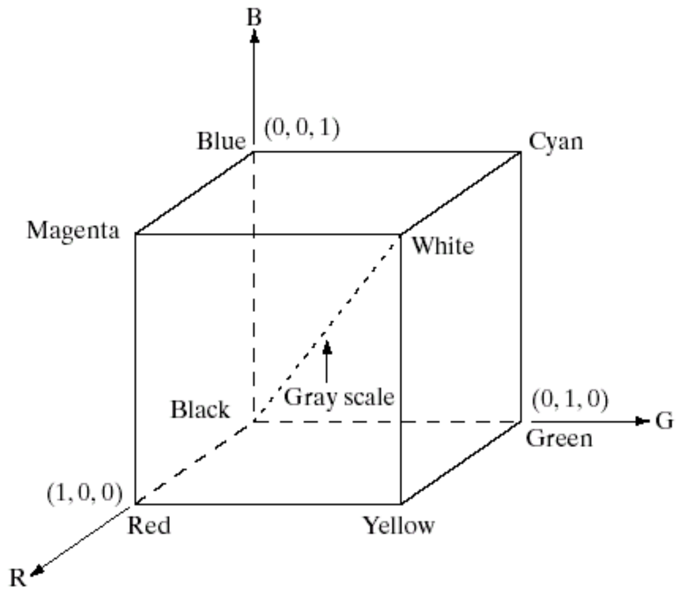
$$f^{-1}(t) = \begin{cases} t^3, & if\ t > (\frac{6}{29}) \\ 3\left(\frac{6}{29}\right)^2 (t - \frac{4}{29}), & otherwise \end{cases}$$

Color Spaces

- Color space examples - RGB

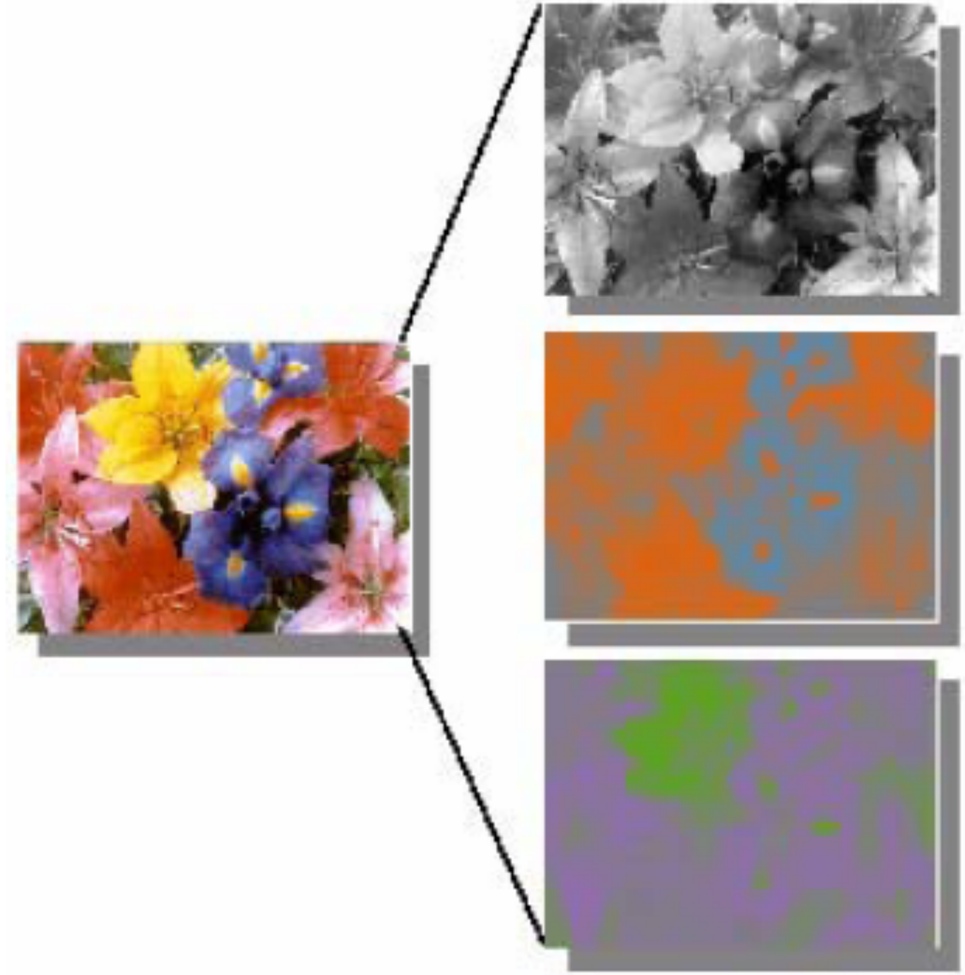


- Cartesian coordinate system (cube)
- Each color appears in its primary spectral components of R, G, and B



Color Spaces

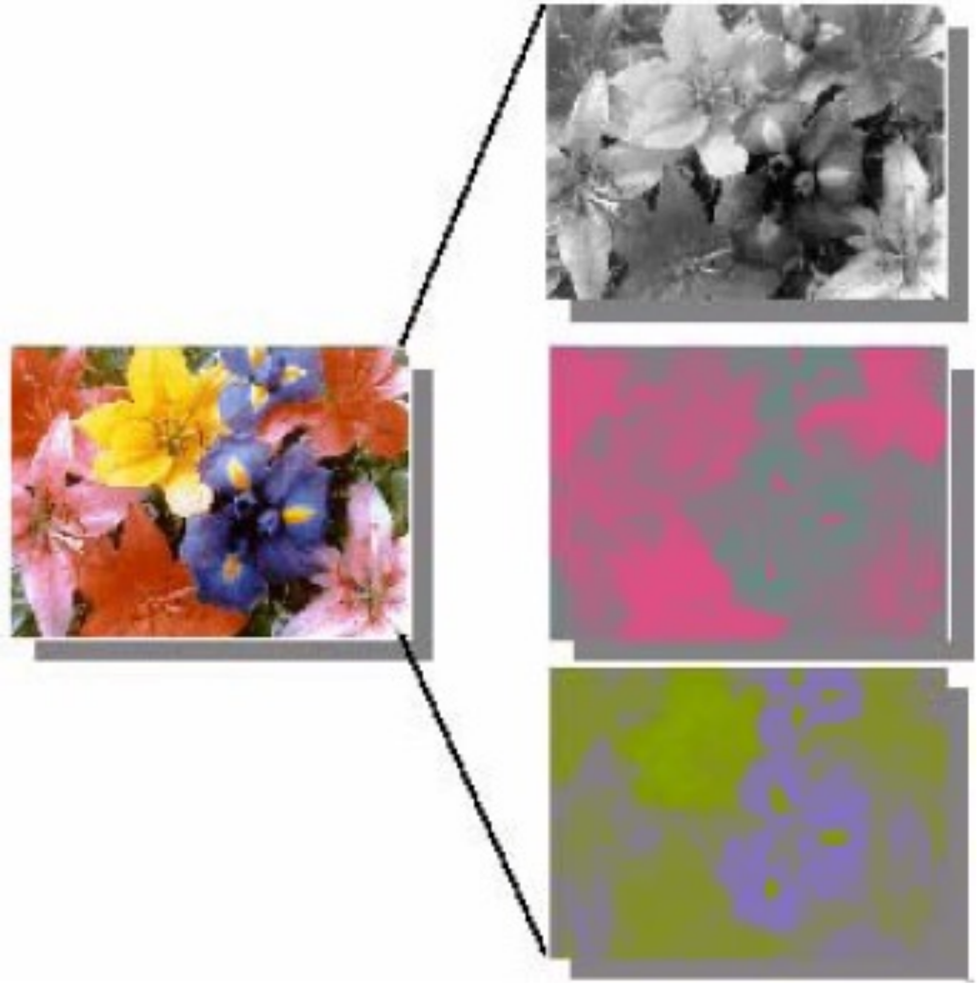
- **Color space examples - YIQ**



$$\begin{pmatrix} Y \\ I \\ Q \end{pmatrix} = \begin{pmatrix} 0.299 & 0.587 & 0.114 \\ 0.596 & -0.274 & 0.322 \\ 0.211 & -0.523 & 0.312 \end{pmatrix} \begin{pmatrix} R \\ G \\ B \end{pmatrix}$$

Color Spaces

- **Color space examples - YUV**

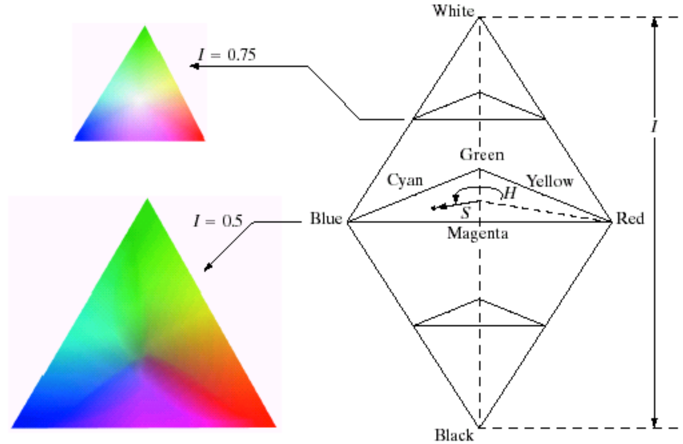


$$\begin{pmatrix} Y \\ U \\ V \end{pmatrix} = \begin{pmatrix} 0.299 & 0.587 & 0.114 \\ -0.147 & -0.289 & 0.436 \\ 0.615 & -0.515 & 0.312 \end{pmatrix} \begin{pmatrix} R \\ G \\ B \end{pmatrix}$$

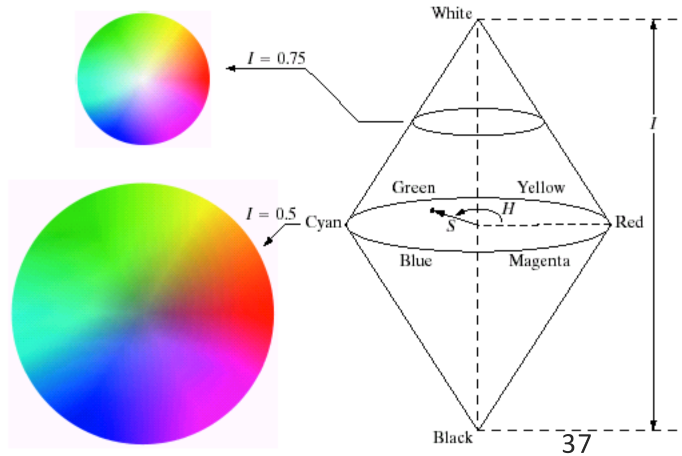
Color Spaces

- **HSI Color Model**
  - **The HSI space is represented by a vertical intensity axis, the length (saturation) of a vector from the axis to a color point, and the angle (hue) this vector makes with the red axis (*i.e.* spectral colors)**

  - **The power of HSI color model is to allow independent control over hue, saturation, and intensity**



triangular



circular

Color Spaces

- **RGB ←→ HSI Transformations**
  - **RGB→HSI**

$$H = \begin{cases} \theta & \text{if } B \le G \\ 360 - \theta & \text{if } B > G \end{cases} \qquad \theta = \cos^{-1}\left\{ \frac{\frac{1}{2}[(R-G)+(R-B)]}{[(R-G)^2+(R-B)(G-B)]^{1/2}} \right\}$$

$$S = 1 - \frac{3}{(R+G+B)}[\min(R,G,B)] \qquad I = \frac{1}{3}(R+G+B)$$

  - **HSI → RGB**

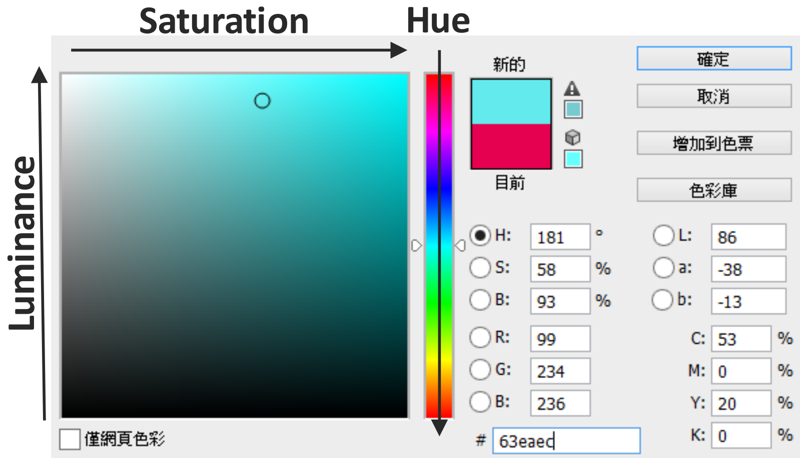
e.g. RG sector (0°<H<120°)

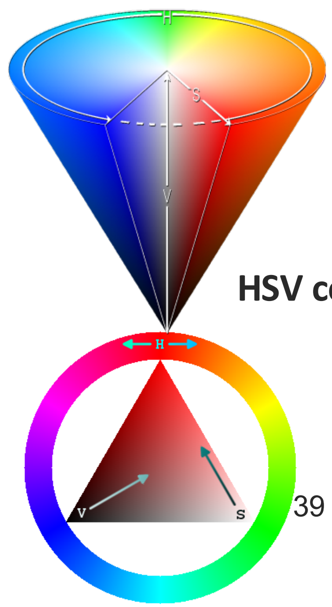
$$B = I(1-S) \qquad R = I[1 + \frac{S\cos H}{\cos(60^\circ - H)}]$$

$$G = 1 - (R+B)$$

Color Spaces

- **Attributes of color**
  - **Hue**
    - dominant wavelength of a color (*i.e.* spectral colors)
    - "tone" of a color e.g."Red" and "Green" are primarily describing hue)
  - **Saturation**
    - purity of a color (*i.e.* , how vivid a color appears)
    - fully saturated color – no mixture of white
  - **Brightness (Value)**
    - Luminance/lightness of a color





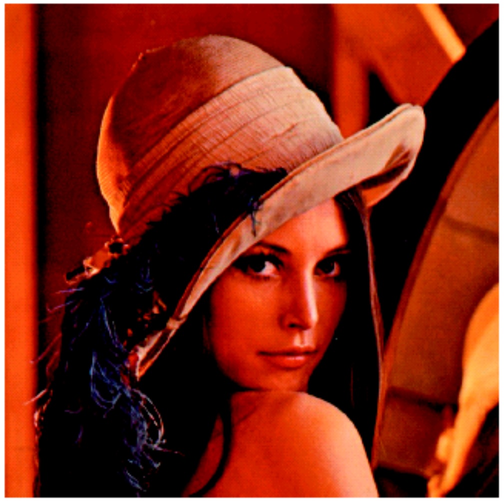
HSV cone

Color Spaces

- **Comparison of HSI & HSV**
  - **The difference between HSI and HSV (hue, saturation, value) lies in the computation of the brightness component (*I* or *V*), which determines the distribution and dynamic range of both brightness (*I* or *V*) and saturation (*S*)**

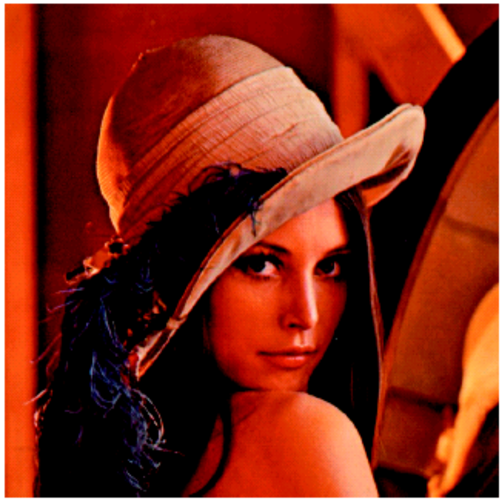
Color Spaces

- Example
  - RGB



Color Spaces

- **Example**
  - **HSI**



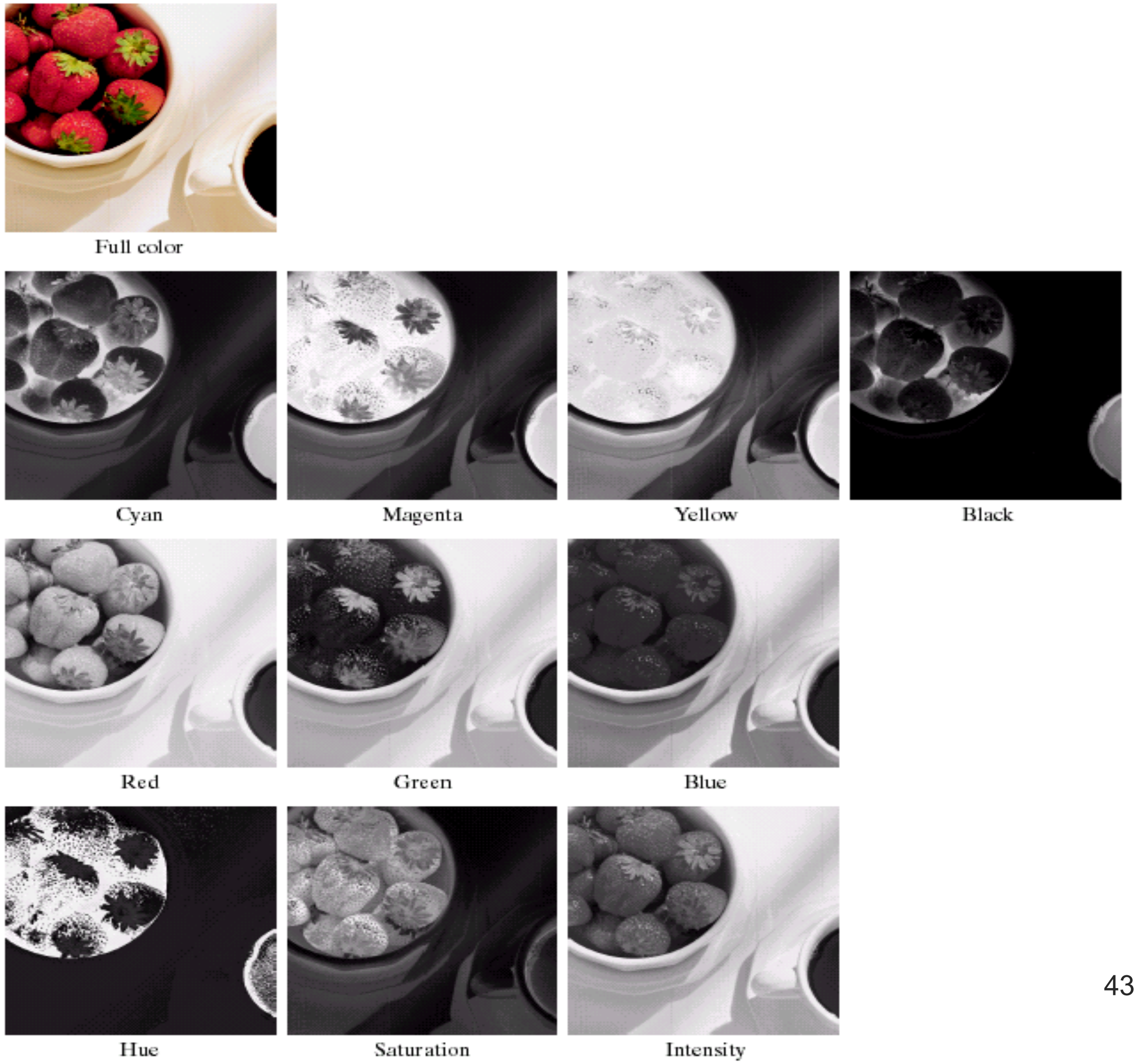
Color Spaces

- Example

CMYK

RGB

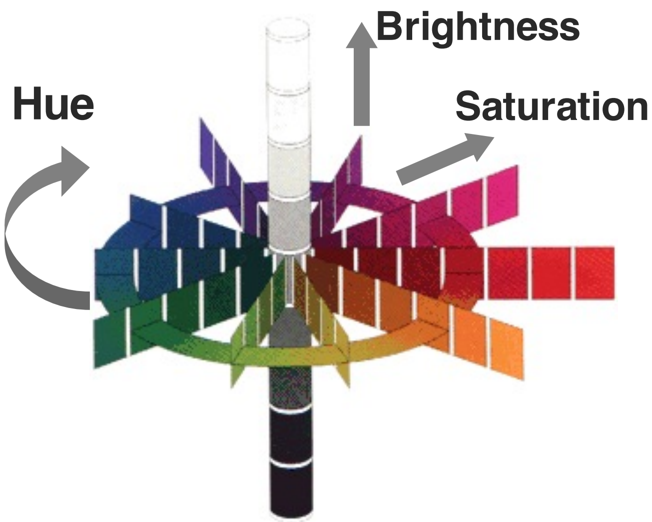
HSI

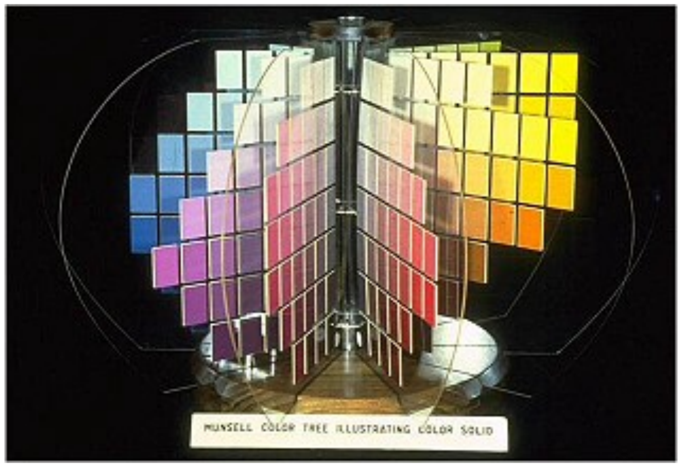


Color Spaces

- **Munsell color system**
  - Hue: 100 equally spaced hues around the circle (10 hues, each subdivided into 10 subdivisions)
  - Saturation: 0 (gray) to 10-18 (full color),

    depending on the hue
  - Brightness: values from 0 for black and 10 for white

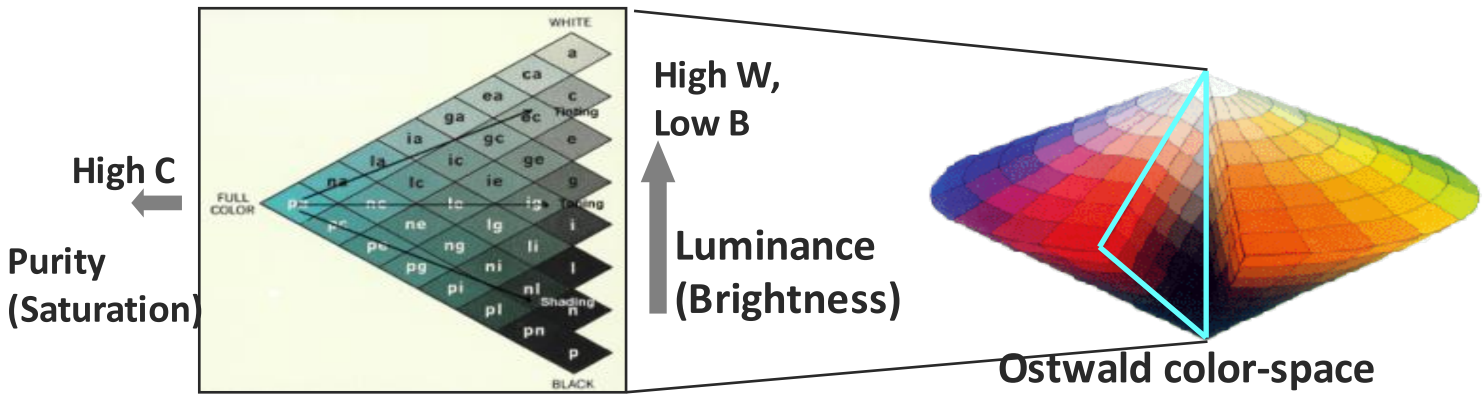




Munsell color tree

Color Spaces

- **Ostwald color system**
  - Natural color system
    - Dominant Wavelength (*Hue*)
    - Purity (*Saturation*)
    - Luminance (*Brightness*)
  - The Ostwald color space is represented by values C,W, and B to represent the percentages of the circle
    - e.g., (C,W,B)=(35,15,50) represents 35% full color, 15% white, and 50% black



Ostwald color-space

Examples of Color Image Processing

Color Image Processing

- **Full Color Image Processing**
  - Full-color and interpretations of its various color space components

- **Method 1**
  - Process each component image individually and form a composite processed color image from the individually processed components

- **Method 2**
  - Work with color pixels directly

Color Image Processing

- **Full Color Image Processing**
  - **Example**

Image Alignment Example LearnOpenCV.com

http://www.learnopencv.com/image-alignment-ecc-in-opencv-c-python/

Color Image Processing

- **Full Color Image Processing**
  - **Example**

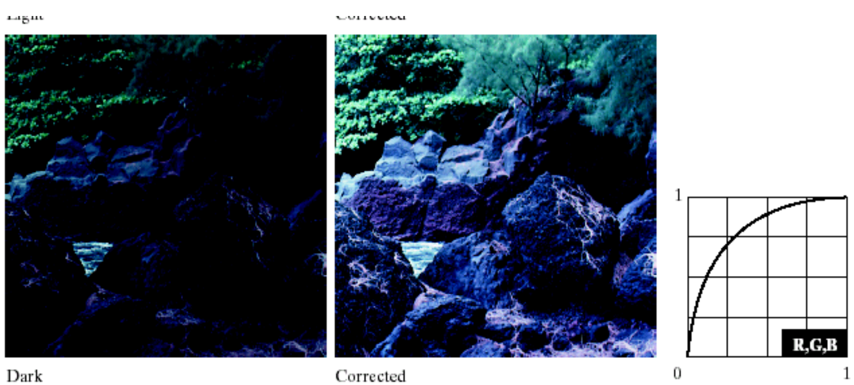
http://www.aventdudomainepublic.org/prokoudine
https://mfleisig.wordpress.com/2011/03/31/rgb-3-color-mixing-from-black-and-white/

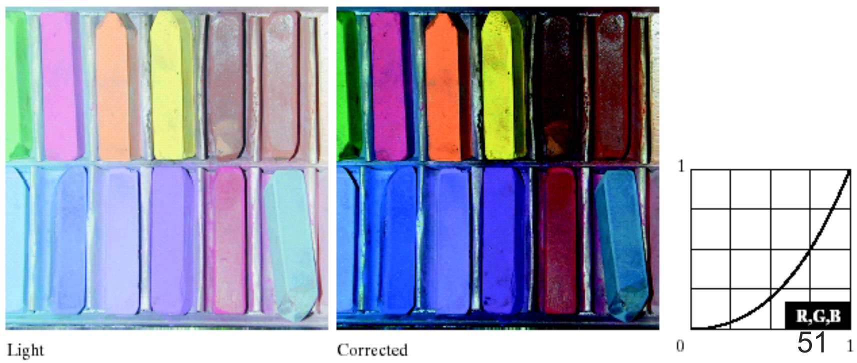
Color Image Processing

- **Color Image Tone Correction**
  - Tonal correction to provide a proper key (tone) of an image (just like to correct the brightness of a gray-level image)
  - Hue of color is not changed
  - For RGB and CMYK -- map all color components with the same transformation function
  - For HSI – only the intensity component is modified

Color Image Processing

- **Color Image Tone Correction**
  - **Example**

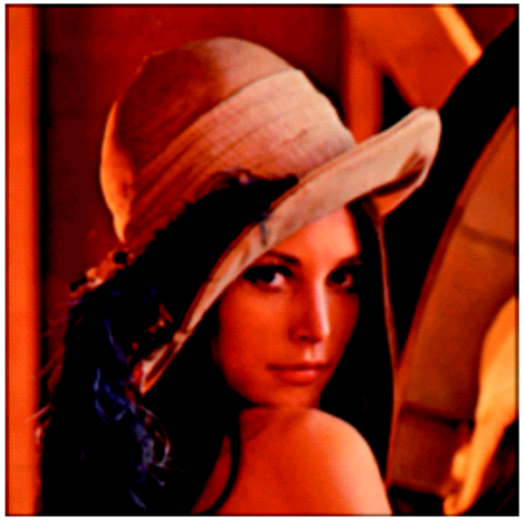
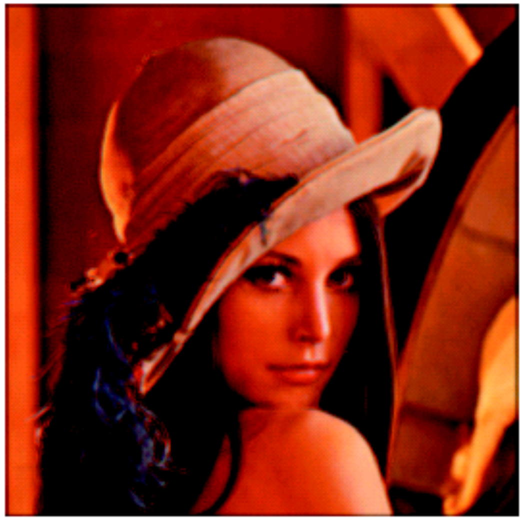
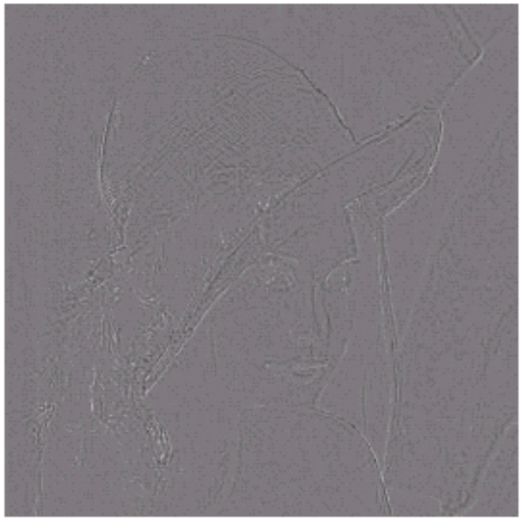




Color Image Processing

- **Color Image Smoothing**
  - **Example**

    RGB channels, Intensity component and Difference

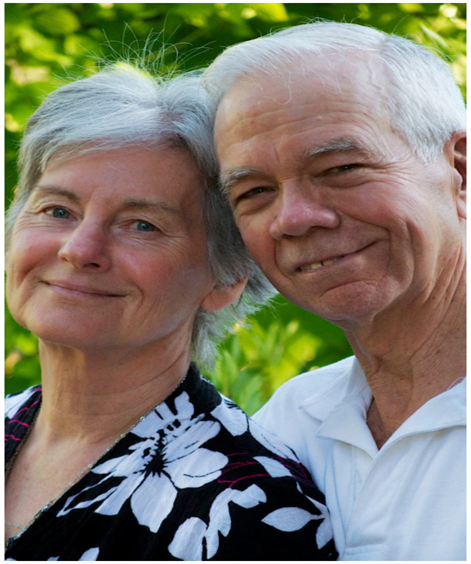
Color Image Processing

- **Color Image Sharpening**
  - **Example:**

    RGB channels, Intensity component and Difference

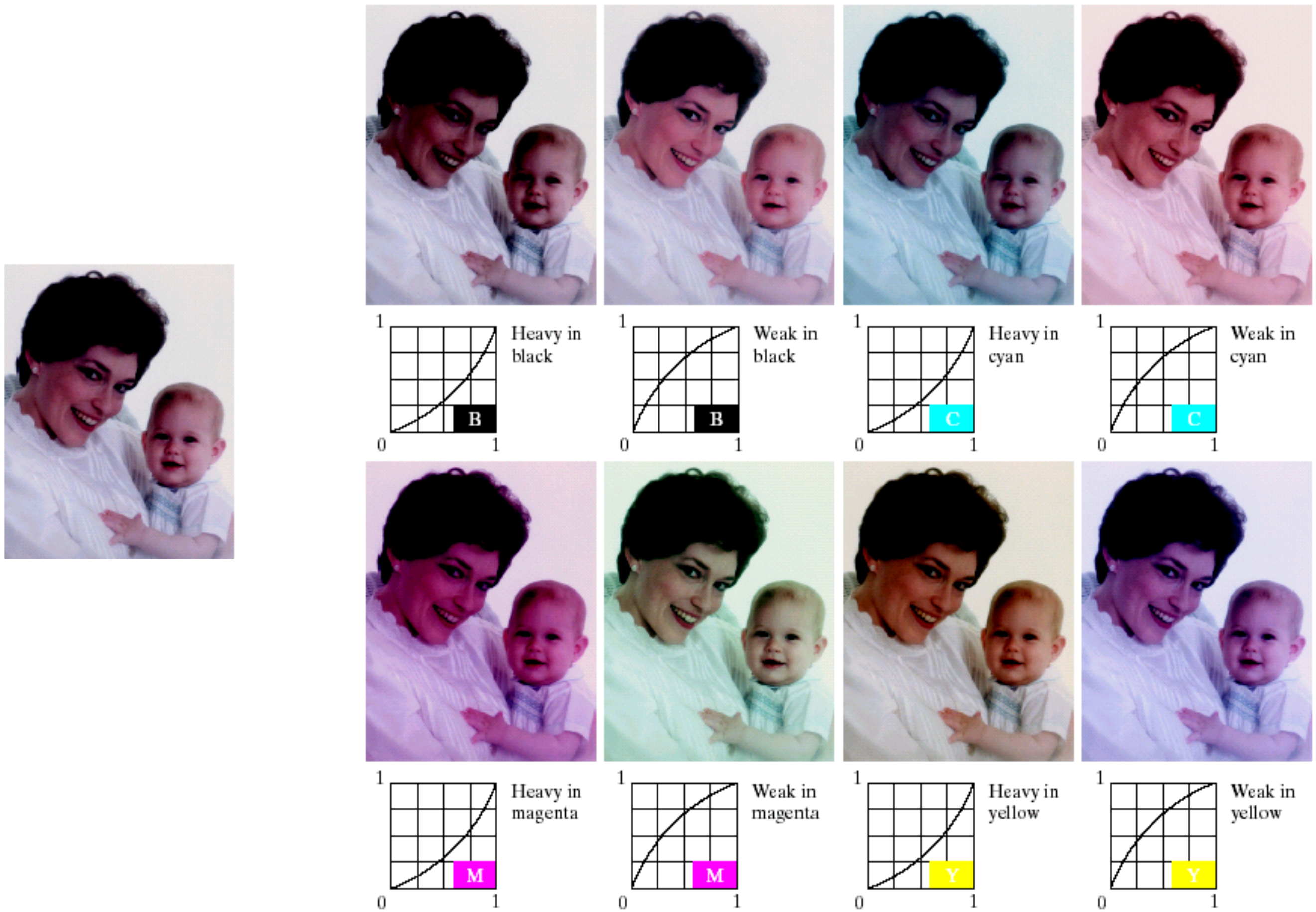
Color Image Processing

- **White Balancing**
  - Tells the camera sensor what temperature of light the camera is taking a picture of
  - Remove unrealistic color casts to get accurate colors in pictures

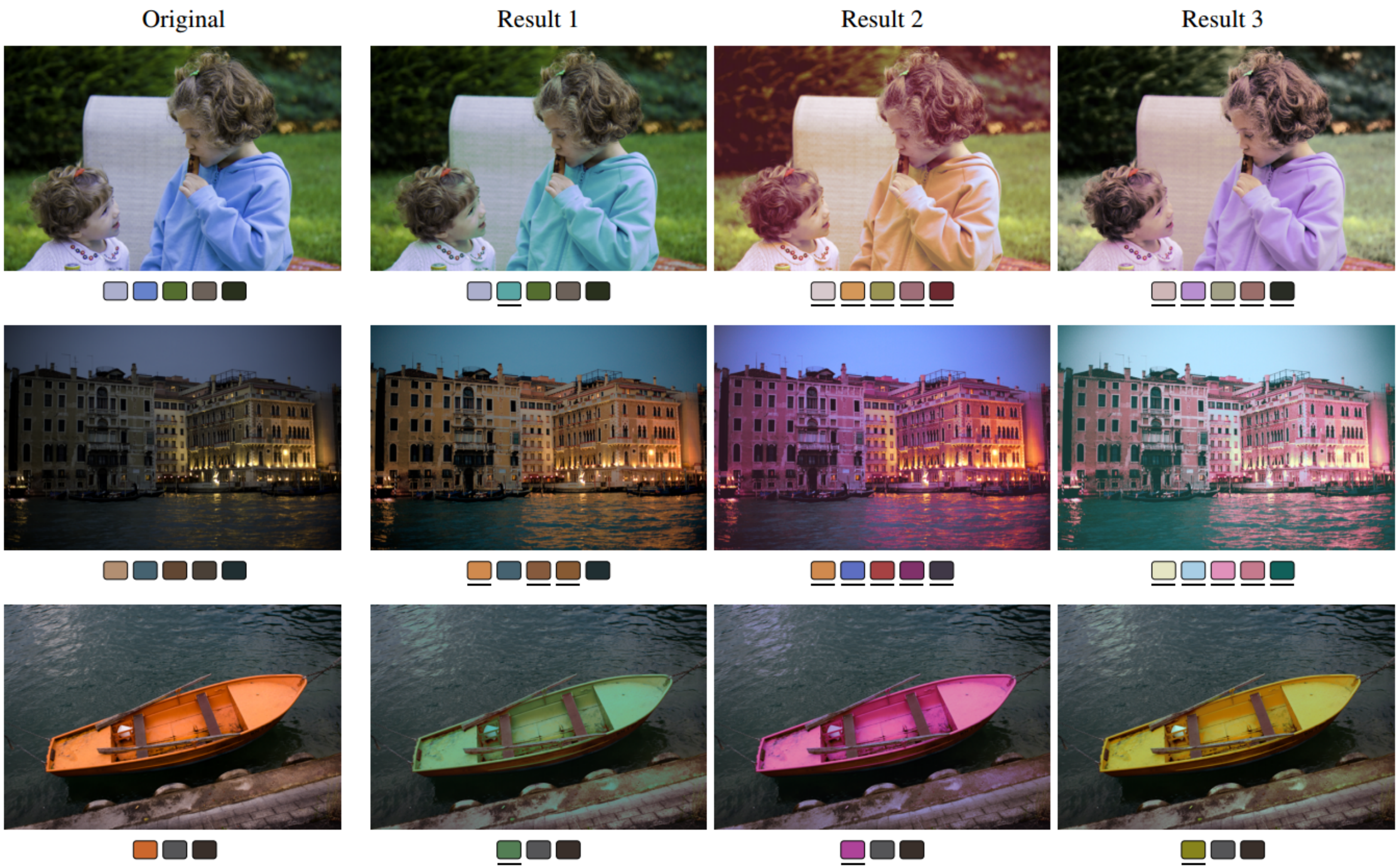


Color Image Processing

- **Color Balancing Correction**



Color Image Processing



Chang, Huiwen, et al. "Palette-based photo recoloring." ACM Transactions on Graphics (TOG) 34.4 (2015): 139.

Color Image Processing

- Image Inpainting




Color Image Processing

- **Dehazing**

http://johanneskopf.de/publications/deep_photo/

Color Image Processing

Color Image Processing

- **Mosaicking**

Color Image Processing

- **Super Resolution**

Color Image Processing

- **High-Dynamic-Range Imaging**

Image Compression

Image Compression

- ## Need for Compression

Multimedia image data	Size of image	Bits/pixel or Bits/sample	Uncompressed size (B for bytes)	Bandwidth (b for bits)	Transmission time	
					56K modem	780Kb DSL
Grayscale image	512x512	8bpp	262KB	2.1Mb per image	42 sec.	3 sec.
Color image	512x512	24bpp	786KB	6.29Mb per image	110 sec.	7.9 sec.
Medical image	2048x 1680	12bpp	5.16MB	41.3Mb per image	12 min.	51.4 sec.
SHD image	2048x 2048	24bpp	12.58MB	100Mb per image	29 min.	2 min.

Image Compression

- **Some Image Compression Formats**
  - RIFF – Resource Interchange File Format
  - GIF – Graphics Interchange File Format
  - PNG – Portable Network Graphics
  - JPEG – Joint Photographic Expert Groups
  - JPEG 2000

Image Compression

- JPEG Background
  - Make no assumptions about the type of image
    - applicable to practically any kind of continuous-tone digital source image
  - With tractable computational complexity
  - Support flexibility by allowing the following modes of operation
    - Lossless and Lossy Encoding
    - Sequential Encoding
    - Progressive Encoding
    - Hierarchical Encoding

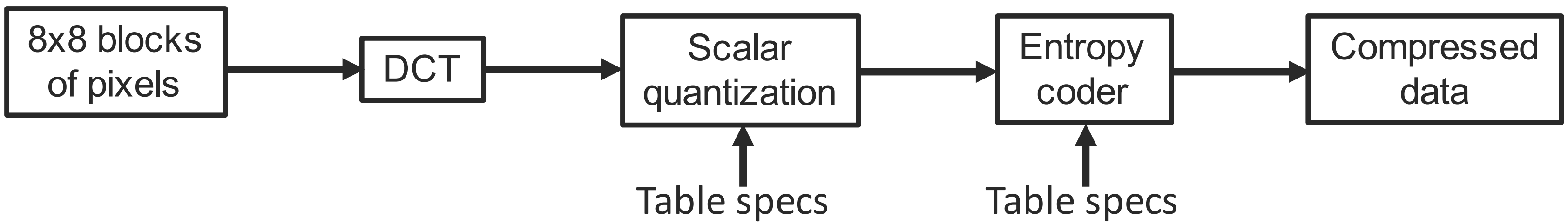
Image Compression

- **JPEG Image Coding**
  - The standard algorithm for compression of still images
    - Started in June 1987
    - Finalized and accepted in 1991
  - Reasonably low computational complexity
  - Capable of producing compressed images of high quality
  - Provide both 'lossless' and 'lossy' compression of arbitrarily sized graylevel and color images

Image Compression

- **JPEG Lossy Codec Scheme**

//Encoder//



//Decoder//

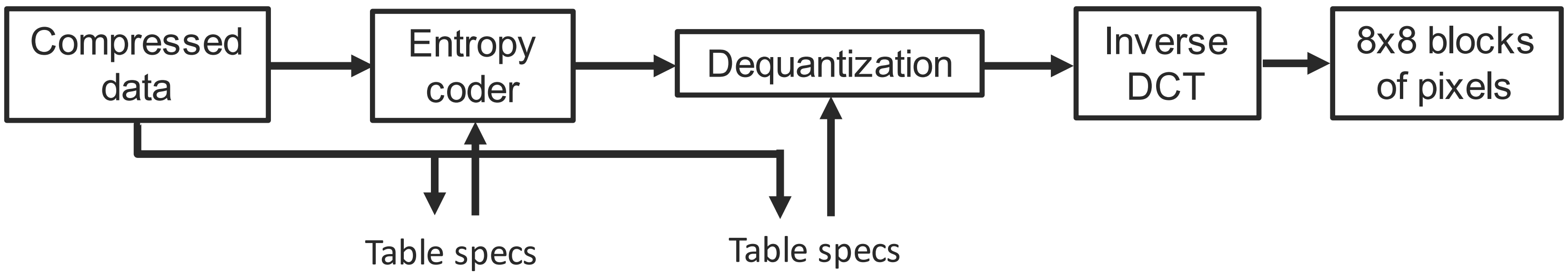


Image Compression

- **JPEG Algorithmic Overview**
  - Convert the RGB color channels of the input image into YCbCr components

    $$Y = 0.299*R + 0.587*G + 0.114*B$$
    $$Cb = - 0.299*R - 0.587*G + 0.886*B$$
    $$Cr = 0.701*R - 0.587*G - 0.114*B$$

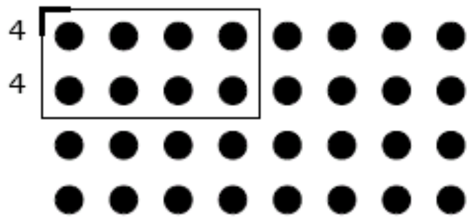
    - each of them is encoded independently
  - Each channel is divided into a series of 8x8 pixel blocks followed by being processed in a raster scan sequence from left to right, top to bottom
  - Each 8x8 block of pixels is analyzed using DCT (transform coding) and DCT coefficients (1 DC and 63 AC's) are quantized
  - After DCT and quantization, coefficients are entropy coded

Image Compression

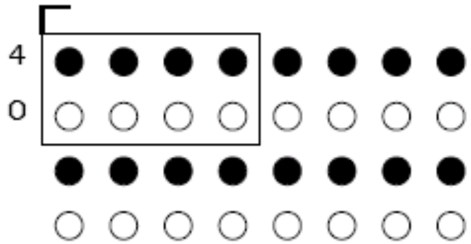
- **Why not RGB?**
  - Similarly to the processing in the human visual system, transformed color spaces such as YIQ, YUV, YCbCr represent color with luminance (Y) and chrominance (the other 2 channels)
    - We can subsample the chrominance channels (e.g. 4:2:2, 4:2:0 subsampling scheme)
    - We can quantize the chrominance channels more coarsely
    - The chrominance channels are rather uncorrelated with the luminance channel, which yields better compression

Image Compression

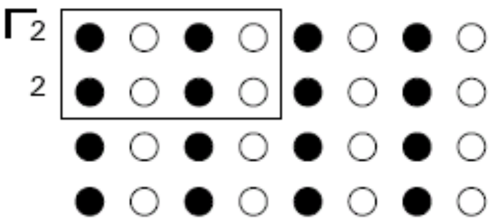
- Subsampling
  - 4:4:4



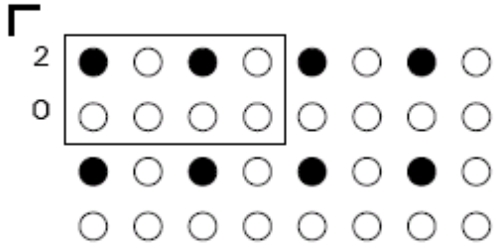
  - 4:4:0



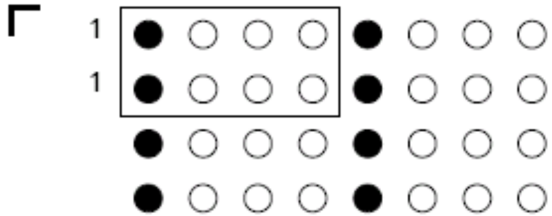
  - 4:2:2



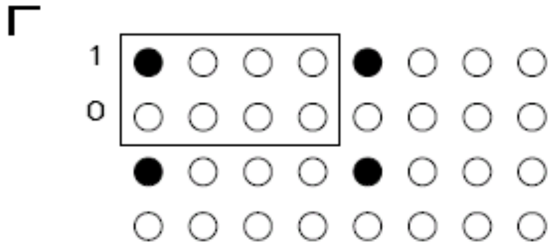
  - 4:2:0



  - 4:1:1



  - 4:1:0



DCT and IDCT

- **One-dimensional example**

Spatial Domain | **Frequency Domain**

Input Function

190	184	186	182	167	123	63	38

DCT

400.56	143.81	-67.76	-16.33	7.42	-4.73	5.49	0.05

Use all 8 DCT value for decoding

400.56	143.81	-67.76	-16.33	7.42	-4.73	5.49	0.05

Input Function computed from DCT using IDCT algorithm

190.0	184.0	186.0	182.0	167.0	123.0	63.0	38.0

Error

0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

DCT and IDCT (cont'd)

- One-dimensional example

Spatial Domain | **Frequency Domain**

Input Function

190	184	186	182	167	123	63	38

DCT

400.56	143.81	-67.76	-16.33	7.42	-4.73	5.49	0.05

Zero the last 4 DCT values

400.56	143.81	-67.76	-16.33	0.00	0.00	0.00	0.00

Input Function computed from DCT using IDCT algorithm

187.6	186.8	186.5	182.4	163.4	122.6	70.5	33.0

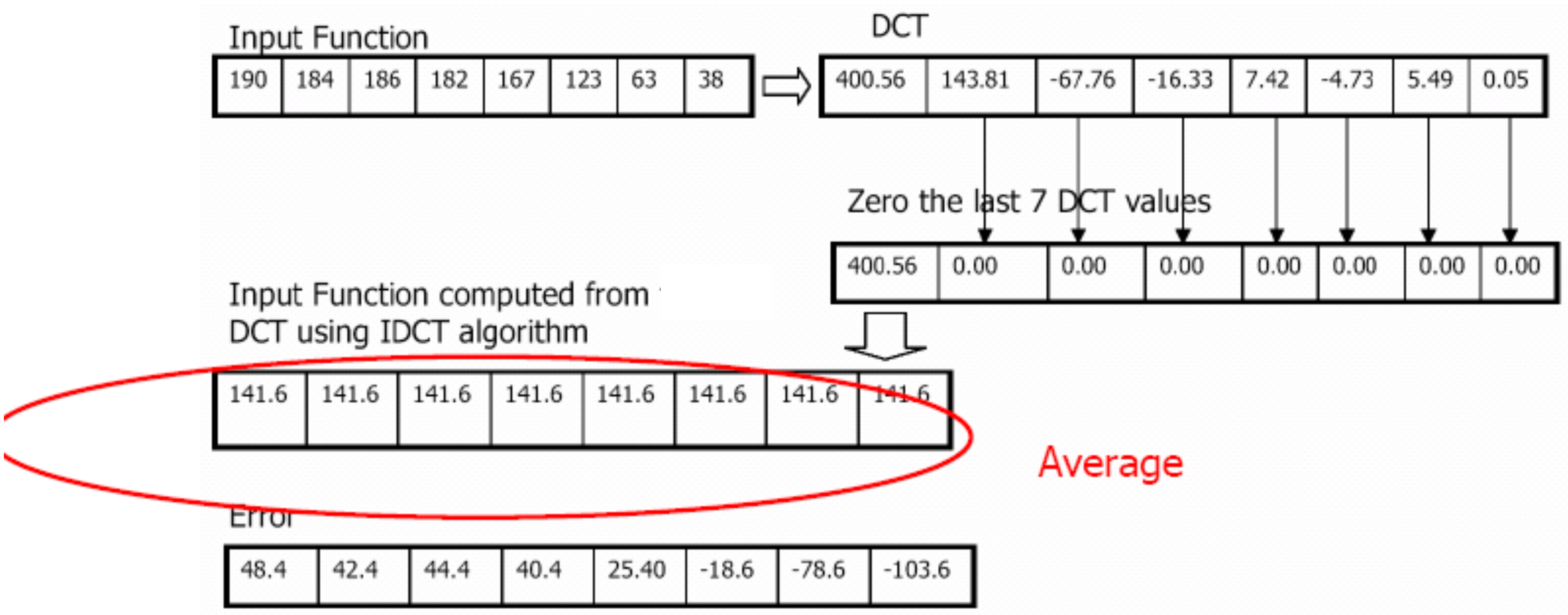
Error

2.40	-2.80	-0.50	-0.40	3.60	0.40	-7.50	5.0

DCT and IDCT (cont'd)

- One-dimensional example

Spatial Domain **Frequency Domain**



DCT and IDCT (cont'd)

- Comments
  - By zeroing out the last four values
    - the maximum error is 7 out of 255, which is 2.7 %
  - By compressing the data 50 %
    - an error of 2.7 % is introduced
    - acceptable quality

Image Compression

- **2D Discrete Cosine Transform (DCT)**
  - **Transform**

$$F(u,v) = \frac{C(u)C(v)}{4} \sum_{j=0}^{n-1} \sum_{k=0}^{n-1} f(j,k) cos\left[\frac{(2j+1)u\pi}{2n}\right] cos\left[\frac{(2k+1)v\pi}{2n}\right]$$

  - **Inverse DCT**

$$f(j,k) = \sum_{v=0}^{n-1} \sum_{u=0}^{n-1} C(u)C(v)F(u,v)\cos[\frac{(2j+1)u\pi}{2n}]\cos[\frac{(2k+1)v\pi}{2n}]$$

$$C(w) = \begin{cases} 1/\sqrt{2} & \text{if } w = 0 \\ 1 \ \text{ if } w = 1,2,\ldots,n\text{-}1 \end{cases}$$

Image Compression

- **The Meaning of DCT Coefficients**
  - **Represent the spatial frequency content within an 8x8 image block**
  - **The (0,0) coefficient is called DC coefficient**
    - **The average of the 64 image pixel values in the block**
  - **The rest of 63 coefficients are called AC coefficients**
  - **Move to the right of the block**

    → the energy in higher horizontal frequencies
  - **Move down the block**

    → the energy in higher vertical frequencies

Image Compression

- **8x8 DCT Basis Functions**

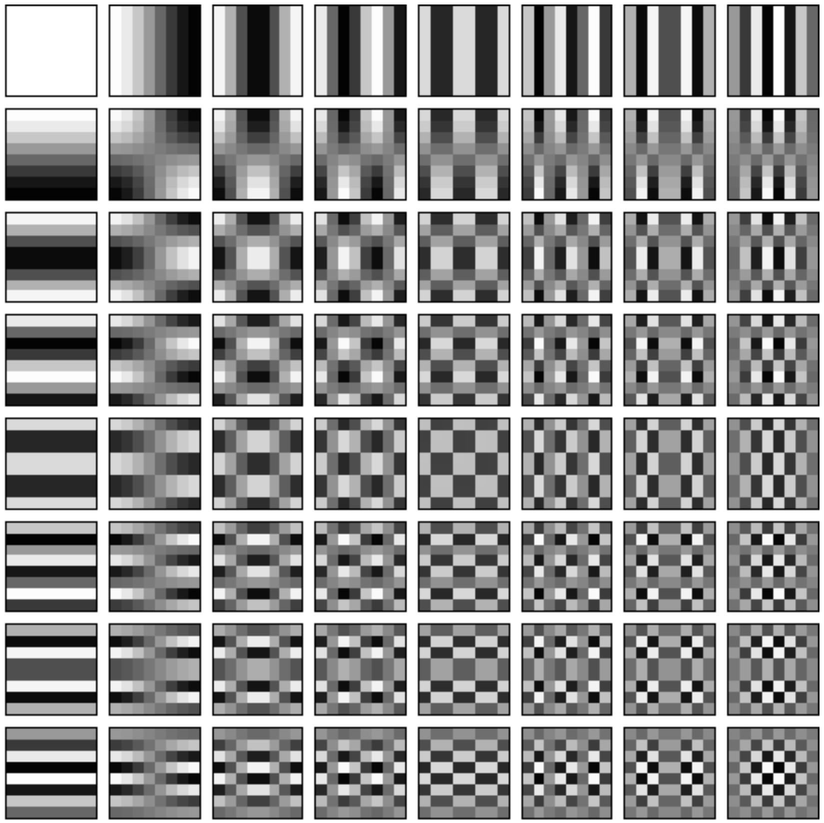


Image Compression

- **Examples of 8x8 DCT Coefficients**

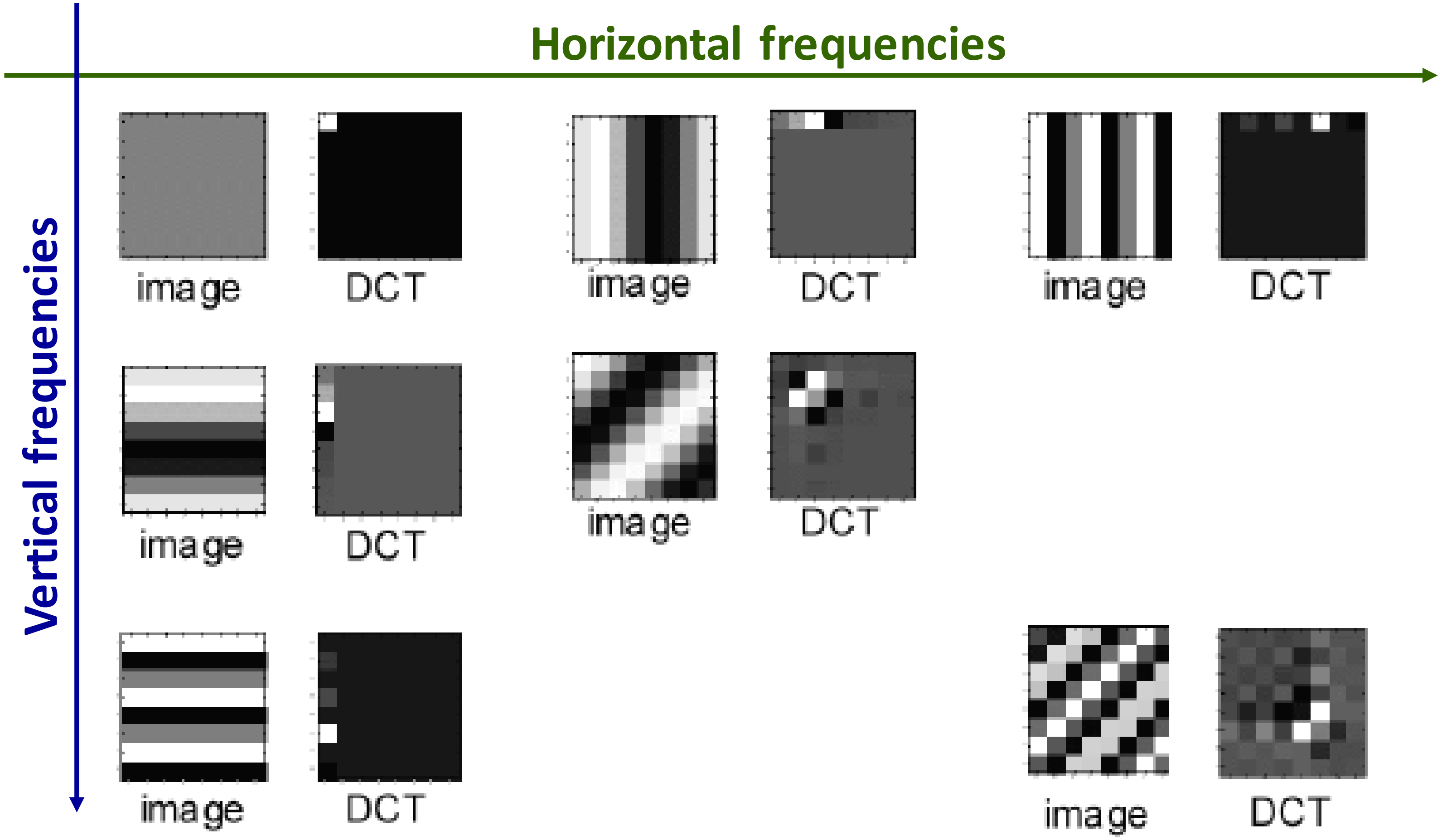


Image Compression

- **DCT Coefficient Quantization**
  - Each DCT coefficient is quantized independently
  - More quantization levels are required for
    - Low frequency coefficients
    - Luminance channel

    Because human visual system is more sensitive to the above two components
  - The quantized DC coefficient of a block is encoded as **the difference from the DC term of the previous block** in the processing order (since usually there's strong correlation between DC's of adjacent blocks)

//Note// the quantization table is not standardized

Image Compression

- **Example of Quantization**

139	144	149	153	155	155	155	155
144	151	153	156	159	156	156	156
150	155	160	163	158	156	156	156
159	161	162	160	160	159	159	159
159	160	161	162	162	155	155	155
161	161	161	161	160	157	157	157
162	162	161	163	162	157	157	157
162	162	161	161	163	158	158	158

Source image sample

235.6	-1.0	-12.1	-5.2	2.1	-1.7	-2.7	1.3
-22.6	-17.5	-6.2	-3.2	-2.9	-0.1	0.4	-1.2
-10.9	-9.3	-1.6	1.5	0.2	-0.9	-0.6	-0.1
-7.1	-1.9	0.2	1.5	0.9	-0.1	0.0	0.3
-0.6	-0.8	1.5	1.6	-0.1	-0.7	0.6	1.3
1.8	-0.2	1.6	-0.3	-0.8	1.5	1.0	-1.0
-1.3	-0.4	-0.3	-1.5	-0.5	1.7	1.1	-0.8
-2.6	1.6	-3.8	-1.8	1.9	1.2	-0.6	-0.4

DCT coefficients

16	11	10	16	24	40	51	61
12	12	14	19	26	58	60	55
14	13	16	24	40	57	69	56
14	17	22	29	51	87	80	62
18	22	37	56	68	109	103	77
24	35	55	64	81	104	113	92
49	64	78	87	103	121	120	101
72	92	95	98	112	100	103	99

Quantization table

15	0	-1	0	0	0	0	0
-2	-1	0	0	0	0	0	0
-1	-1	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

Quantized coefficients

Image Compression

- **Entropy Coding of DCT Coefficients**
  - **The quantized DCT coefficients are scanned in zigzag order**

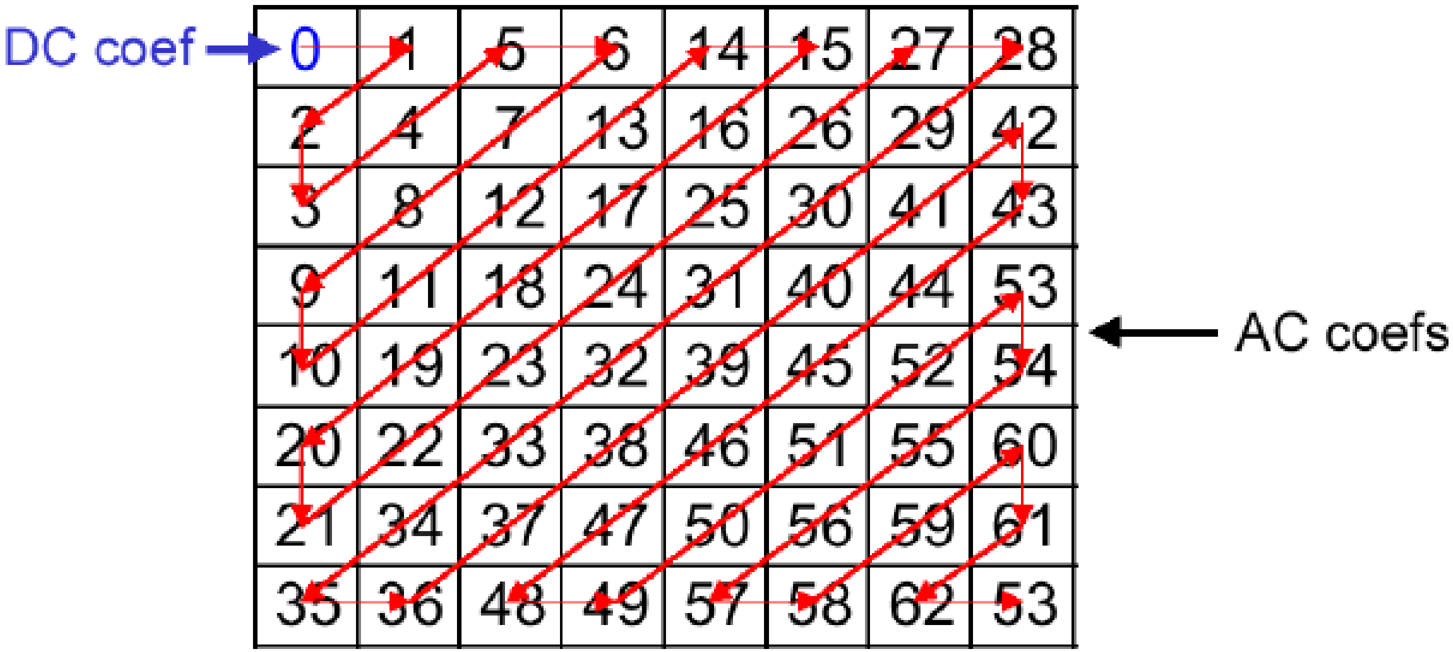


Image Compression

- **Entropy Coding of DCT Coefficients**
  - Zigzag ordering places low frequency coefficients before high frequency coefficients
    - Low frequency → more likely to be non-zero
    - High frequency → more likely to be null after quantization

→ the sequence of quantized coefficients is often a long sequence of null values

→ AC coefficients are represented in an intermediate run-length representation, which encodes the runs of zeros preceding any non-null coefficient

→ Symbol 1: (runlength, size)

→ Symbol 2: (amplitude)

Image Compression

- **Entropy Coding of DCT Coefficients**
  - **For both DC and AC coefficients,**
    - **Each symbol 1 is Huffman encoded**
      - **Variable length prefix code**
    - **Each symbol 2 is encoded with Variable Length Integer (VLI) code**
      - **Different from a Huffman code**
      - **Symbol length is known in advance**

An example

(a)8×8影像

$$\begin{bmatrix} 159 & 153 & 158 & 152 & 140 & 138 & 132 & 132 \\ 164 & 162 & 162 & 157 & 151 & 142 & 134 & 132 \\ 167 & 168 & 161 & 160 & 158 & 145 & 139 & 134 \\ 164 & 168 & 161 & 166 & 162 & 152 & 149 & 141 \\ 171 & 166 & 168 & 167 & 163 & 162 & 147 & 151 \\ 173 & 164 & 169 & 170 & 166 & 166 & 162 & 161 \\ 175 & 169 & 172 & 176 & 174 & 172 & 174 & 166 \\ 173 & 172 & 175 & 173 & 180 & 181 & 177 & 172 \end{bmatrix}$$

DCT

(b)減128後再DCT

$$\begin{bmatrix} 260 & 49 & -16 & 5 & 2 & 4 & 0 & 1 \\ -79 & 36 & -2 & -7 & 1 & -3 & -1 & -2 \\ 0 & -8 & 3 & -2 & -2 & 1 & 5 & 1 \\ -8 & -4 & 5 & -4 & 1 & 7 & 6 & -2 \\ -2 & -6 & -1 & 0 & -4 & -1 & 0 & -1 \\ -3 & -2 & -1 & -1 & 1 & 2 & -5 & -1 \\ -4 & -1 & 1 & 0 & 0 & -2 & 2 & 0 \\ -1 & 1 & 1 & 1 & -1 & 1 & 0 & 0 \end{bmatrix}$$

量化

(c)量化後系數

$$\begin{bmatrix} 16 & 4 & -2 & 0 & 0 & 0 & 0 & 0 \\ -7 & 3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Zig.Zag Scan 和 Model

(b)量化表

$$\begin{bmatrix} 16 & 11 & 10 & 16 & 24 & 40 & 51 & 61 \\ 12 & 12 & 14 & 19 & 26 & 58 & 60 & 55 \\ 14 & 13 & 16 & 24 & 40 & 57 & 69 & 56 \\ 14 & 17 & 22 & 29 & 51 & 87 & 80 & 62 \\ 18 & 22 & 37 & 56 & 68 & 109 & 103 & 77 \\ 24 & 35 & 55 & 64 & 81 & 104 & 113 & 92 \\ 49 & 64 & 78 & 87 & 103 & 121 & 120 & 101 \\ 72 & 92 & 95 & 98 & 112 & 100 & 103 & 99 \end{bmatrix}$$

長度：-, 0, 0, 1, 0, 2, 0, 54
振幅：16, 4, -7, 3, -2, -1, -1
DC

編碼：1010110100100100000110
(*)1111010111100000001010

(e)量化後係數

$$\begin{bmatrix} 16 & 4 & -2 & 0 & 0 & 0 & 0 & 0 \\ -7 & 3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

反量化

(f)DCT係數

$$\begin{bmatrix} 256 & 44 & -20 & 0 & 0 & 0 & 0 & 0 \\ -84 & 36 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -13 & 0 & 0 & 0 & 0 & 0 & 0 \\ -14 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

反DCT

(g)還原後影像

$$\begin{bmatrix} 153 & 153 & 152 & 149 & 144 & 137 & 131 & 126 \\ 158 & 158 & 157 & 154 & 149 & 142 & 135 & 131 \\ 165 & 164 & 163 & 160 & 155 & 148 & 141 & 137 \\ 167 & 167 & 167 & 164 & 159 & 153 & 147 & 143 \\ 167 & 168 & 168 & 167 & 163 & 158 & 153 & 149 \\ 166 & 168 & 169 & 170 & 168 & 165 & 161 & 159 \\ 168 & 170 & 173 & 175 & 175 & 184 & 171 & 170 \\ 169 & 172 & 176 & 179 & 181 & 180 & 179 & 177 \end{bmatrix}$$

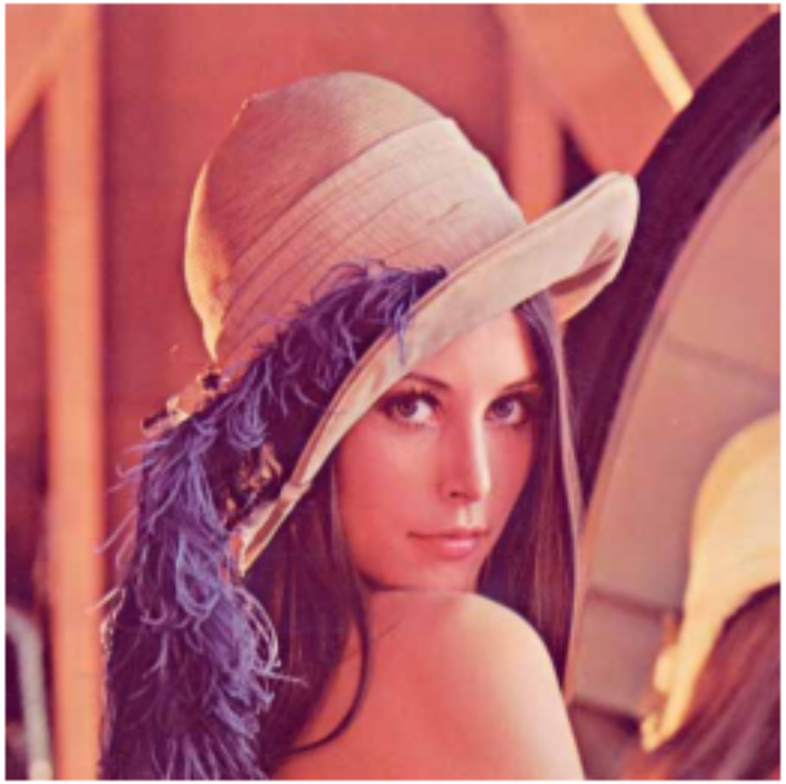
* DC值需和前一個8×8 block相減後才編碼, 此例DC編碼為1010110，size為5， Amplitude為-9。

Image Compression

- Example

$$C_r = \frac{Number\ of\ bits\ needed\ for\ the\ original\ signal}{Number\ of\ bits\ needed\ for\ the\ compressed\ signal}$$

786488 bytes



23116 bytes (Cr=34.0)

Image Compression

- Example

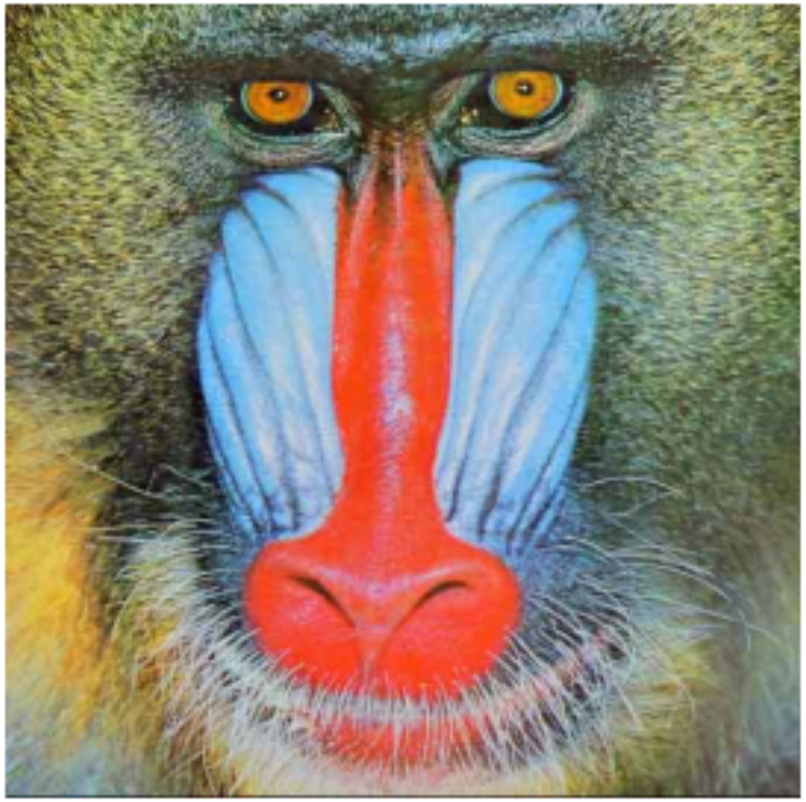
786488 bytes

11738 bytes (Cr=67.0)

Image Compression

- Example

786488 bytes



49746 bytes (Cr=15.80)

Image Compression

- Example

786488 bytes

15730 bytes (Cr=50.0)

Image Compression

- Example

786488 bytes

26614 bytes (Cr=29.55)

Image Compression

- Example

786488 bytes

7865 bytes (Cr=100.0)