



Digital Image Processing

Lecture #6
Ming-Sui (Amy) Lee

Announcement

Class Information

- The following schedule

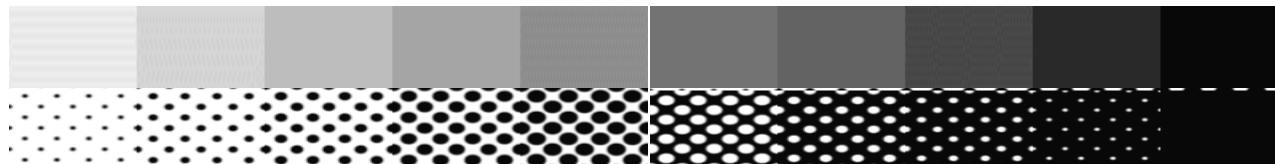
| | | | |
|-------|-----------|-------|-------------------|
| 03/23 | Lecture 5 | 05/11 | proposal |
| 03/30 | Lecture 6 | 05/18 | Lecture 9 |
| 04/06 | Lecture 7 | 05/25 | Lecture 10 |
| 04/13 | RealSense | 06/01 | Lecture 11 |
| 04/20 | midterm | 06/08 | Demo |
| 04/27 | RealSense | 06/15 | Demo |
| 05/04 | Lecture 8 | 06/22 | Final Package Due |

Digital Halftoning

Digital Halftoning

■ Goal

- Render the illusion of a continuous-tone image based on two-tone (half-tone) display



- Applications

- Computer hardcopies

- Laser printers/dot-matrix printers/color printers
 - Fax machine

- Implementation

- Thresholding at $1/2$?

Digital Halftoning



Gray-level image

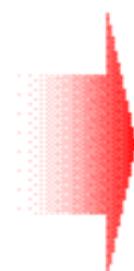


Half-toned images

Digital Halftoning

■ Color Printer

Continuous Image



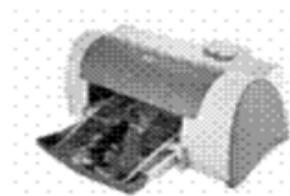
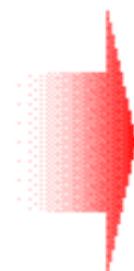
Binary Image



CMY channel



Black channel



Digital Halftoning

■ Basic idea

- Spatial modulation

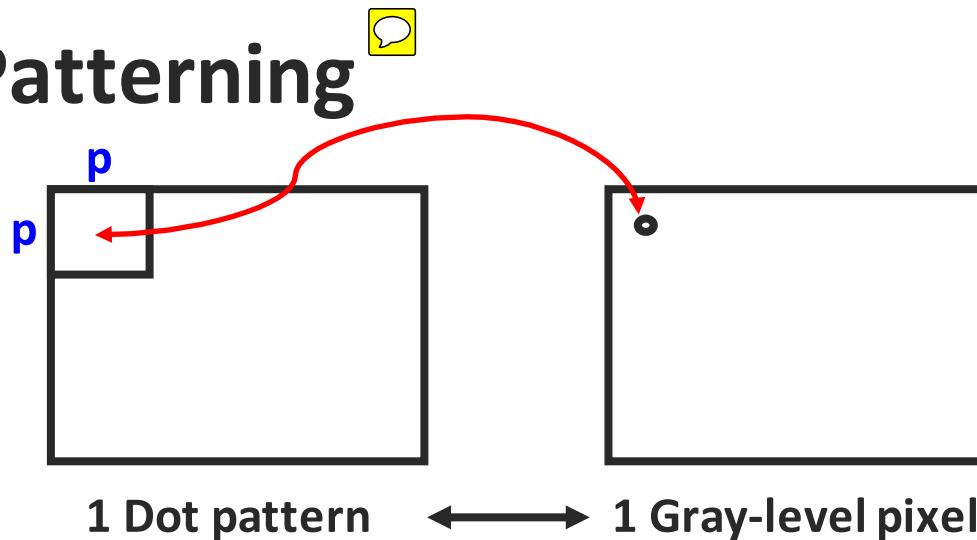
- Gray-level ↔ black/white
- Darker area ↔ denser black points per area
- Whiter area ↔ sparser black points per area

- Three approaches

- Patterning
- Dithering
- Error Diffusion

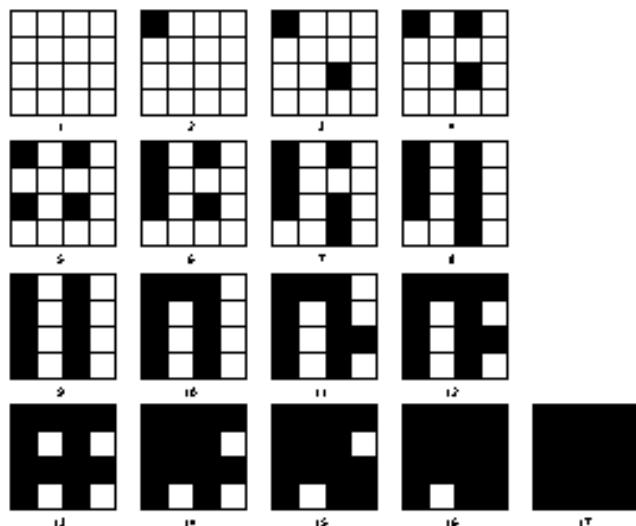
Digital Halftoning

Patterning



If $p=4$

- 16 binary pixels
- 17 levels (0~16)
- 256 gray levels
- Quantization



Rylander's recursive
patterning matrices

Digital Halftoning

■ Patterning

○ Four steps

- Read in the given grey-level image
- Quantization
- Design the patterning table
- Map each pixel to its corresponding pattern

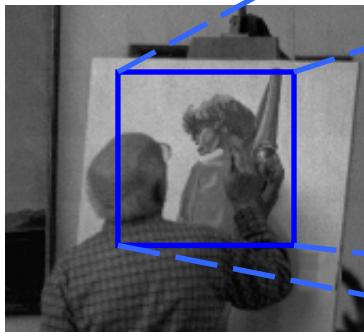
○ Simplest way

- Generates image with higher spatial resolution than the source image

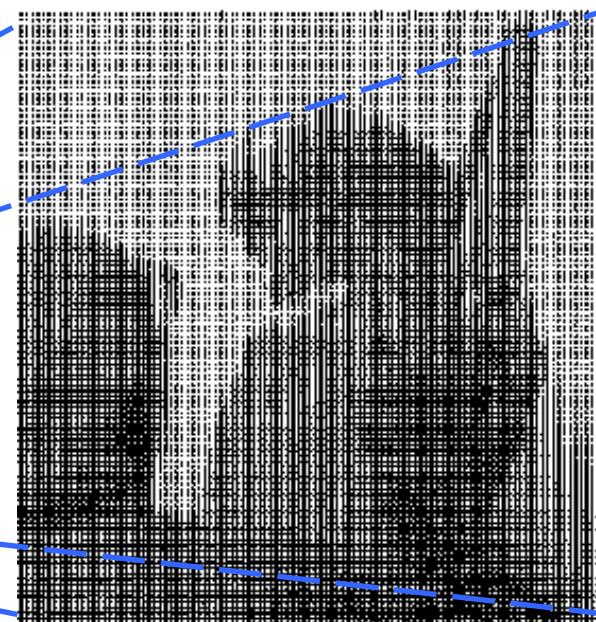
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■ Patterning

○ Example



Original gray-level image

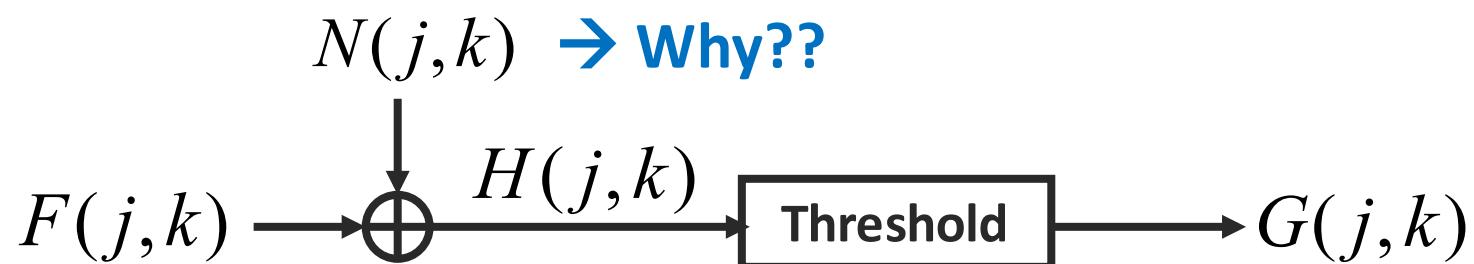


Half-toned image: patterning

Digital Halftoning

■ Dithering

- Create an image with the same number of dots as the number of pixels in the source image
- Idea



Digital Halftoning

■ Dithering

- Why adding noise?

- Under fixed thresholding → taking MSB

- E.g. before and after adding noise



- To break the monotonicity of accumulated error in the area of constant (nearly constant) gray level
 - White noise, pink noise, blue noise and green noise

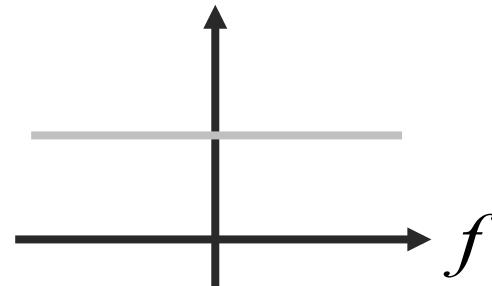
Digital Halftoning

■ Dithering

- Noise Type

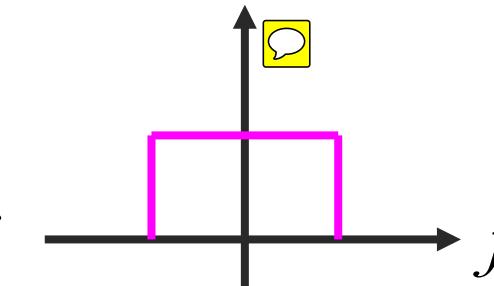
- Power spectral density

- White noise



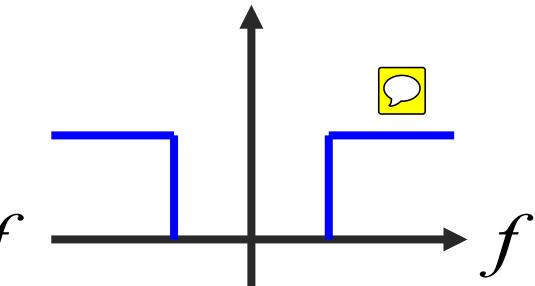
Grainy appearance

- Pink noise



Low-frequency noise

- Blue noise



High-frequency noise

- Robert Ulichney, “Digital Halftoning”

- <http://www.hpl.hp.com/people/u/>

Digital Halftoning

■ Dithering

- **Adaptive thresholding**

- Generate a threshold matrix according to a dither matrix
- Whenever the pixel value of the image is greater than the value in the threshold matrix, the pixel is turned on

- **Notes**

- No randomness 
- Region-to-region mapping
- Recursive definition allowed

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■ Dithering



Dither matrix



$$I_2(i, j) = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}; \quad I_2(i, j) = \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}$$

- 0 → lowest threshold
- 3 → highest threshold

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■ Dithering

- The general form of the NxN dither matrix

- $2 \times 2 \rightarrow 4 \times 4 \rightarrow 8 \times 8 \rightarrow 16 \times 16 \dots$

$$I_{2n}(i,j) = \begin{bmatrix} 4I_n(i,j)+1 & 4I_n(i,j)+2 \\ 4I_n(i,j)+3 & 4I_n(i,j)+0 \end{bmatrix}$$

- Eg. What is $I_4(i,j)$ if $I_2(i,j) = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$?

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■ Dithering

- Determine the threshold matrix

$$T(i, j) = 255 \cdot \frac{I(i, j) + 0.5}{N^2}$$

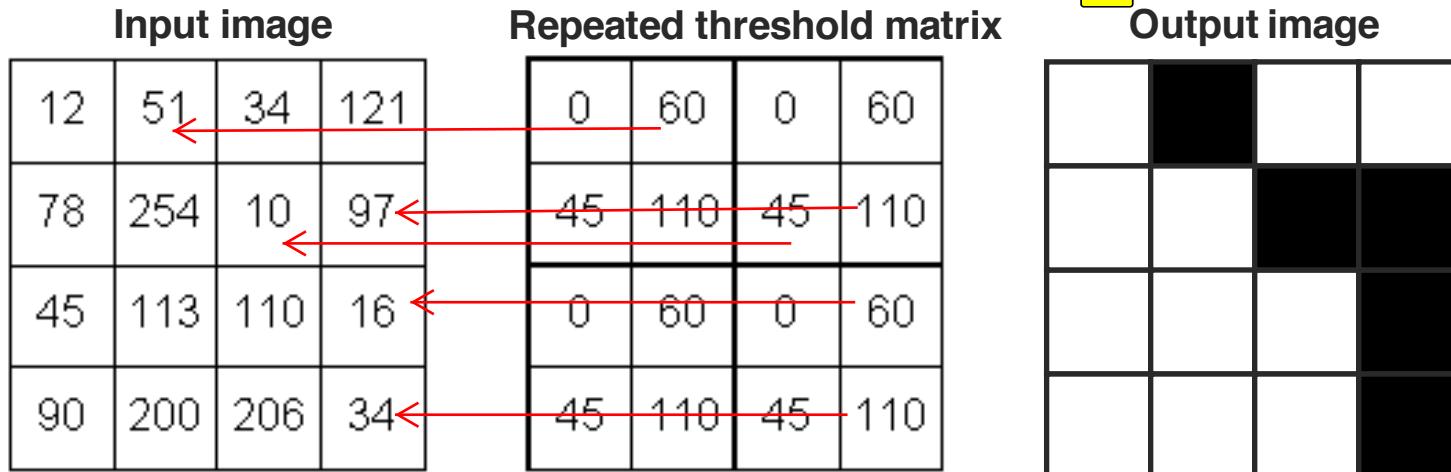
■ Eg. N=4

$$I_4(i, j) = \begin{bmatrix} 5 & 9 & 13 & 1 \\ 15 & 3 & 7 & 11 \\ 12 & 0 & 4 & 8 \\ 14 & 2 & 6 & 10 \end{bmatrix}, \quad T_4(i, j) = ?$$

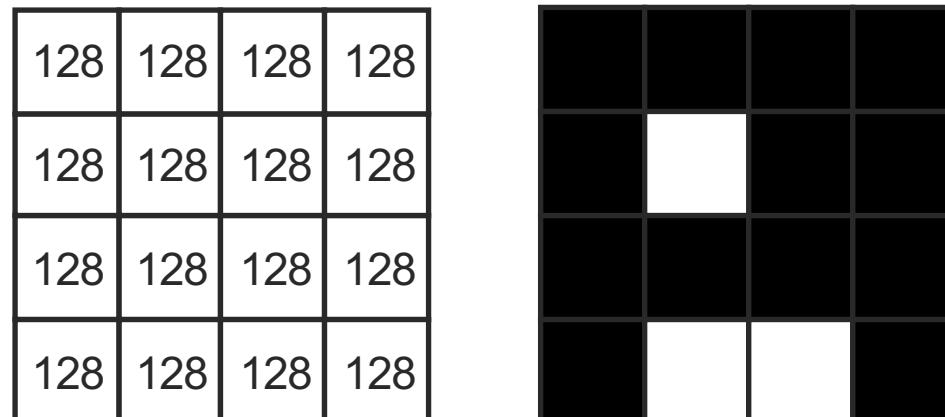
The matrix $I_4(i, j)$ is a 4x4 grid of values. Above the matrix, two red boxes are labeled '+1' and '+2'. Below the matrix, two red boxes are labeled '+3' and '+0'. A red arrow points from the '+1' box to the top-left cell of the matrix. Another red arrow points from the '+2' box to the top-right cell of the matrix. A third red arrow points from the '+3' box to the bottom-left cell of the matrix. A fourth red arrow points from the '+0' box to the bottom-right cell of the matrix.

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Dithering



Another repeated threshold matrix

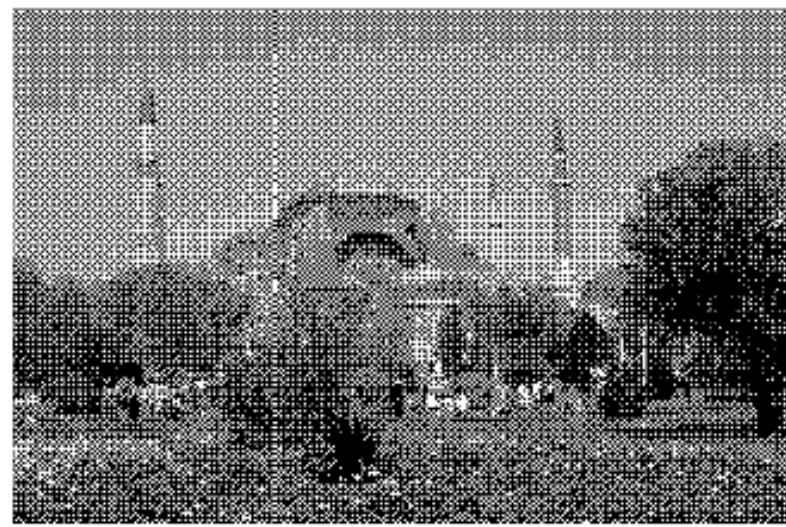


Digital Halftoning

■ Experimental results



Original Image



Dithering

Digital Halftoning

■ Experimental results



Original Image

Dithering

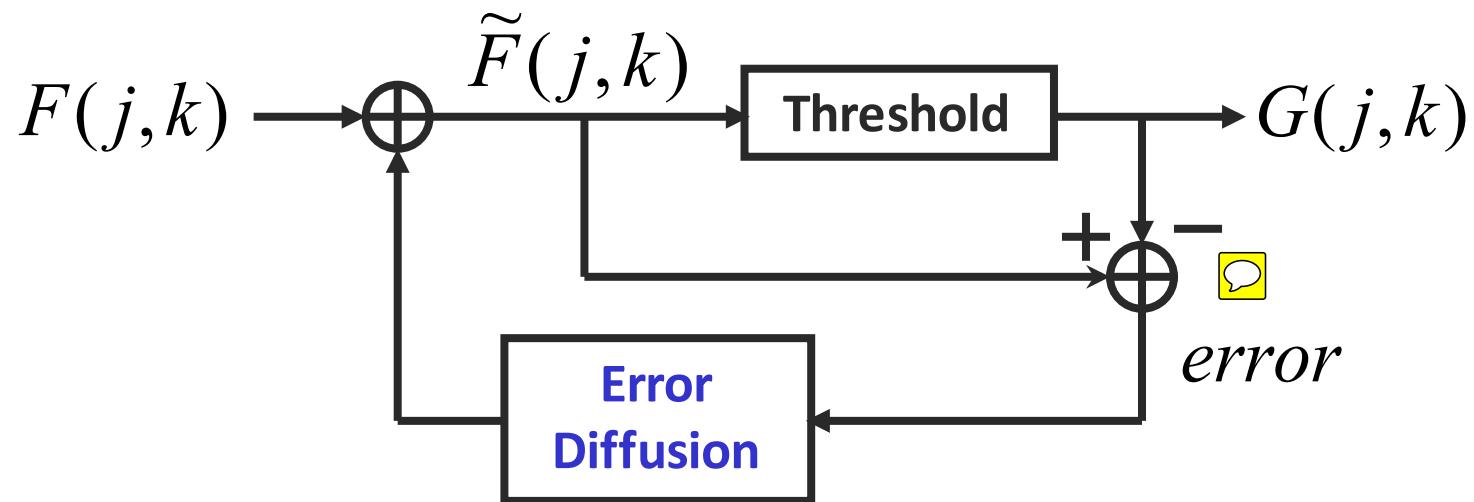
Digital Halftoning

■ Error diffusion

- 1975 Floyd & Steinberg

- A practical algorithm to implement blue noise dithering

- Framework



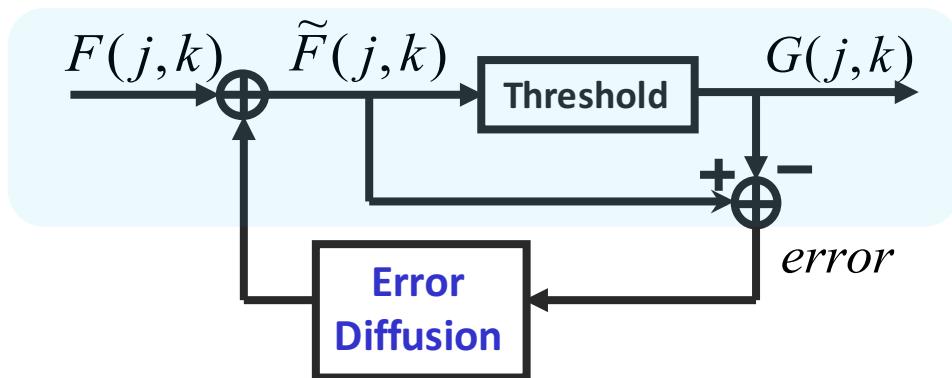
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Digital Halftoning

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■ Error diffusion

- Normalize $F(j,k)$ to lie between [0,1]
- Set threshold=0.5
- Output image: 0 or 1



if $\tilde{F}(j,k) \geq 0.5$ → $G(j,k) = 1$

if $\tilde{F}(j,k) < 0.5$ → $G(j,k) = 0$

Define $E(j,k) = \tilde{F}(j,k) - G(j,k)$

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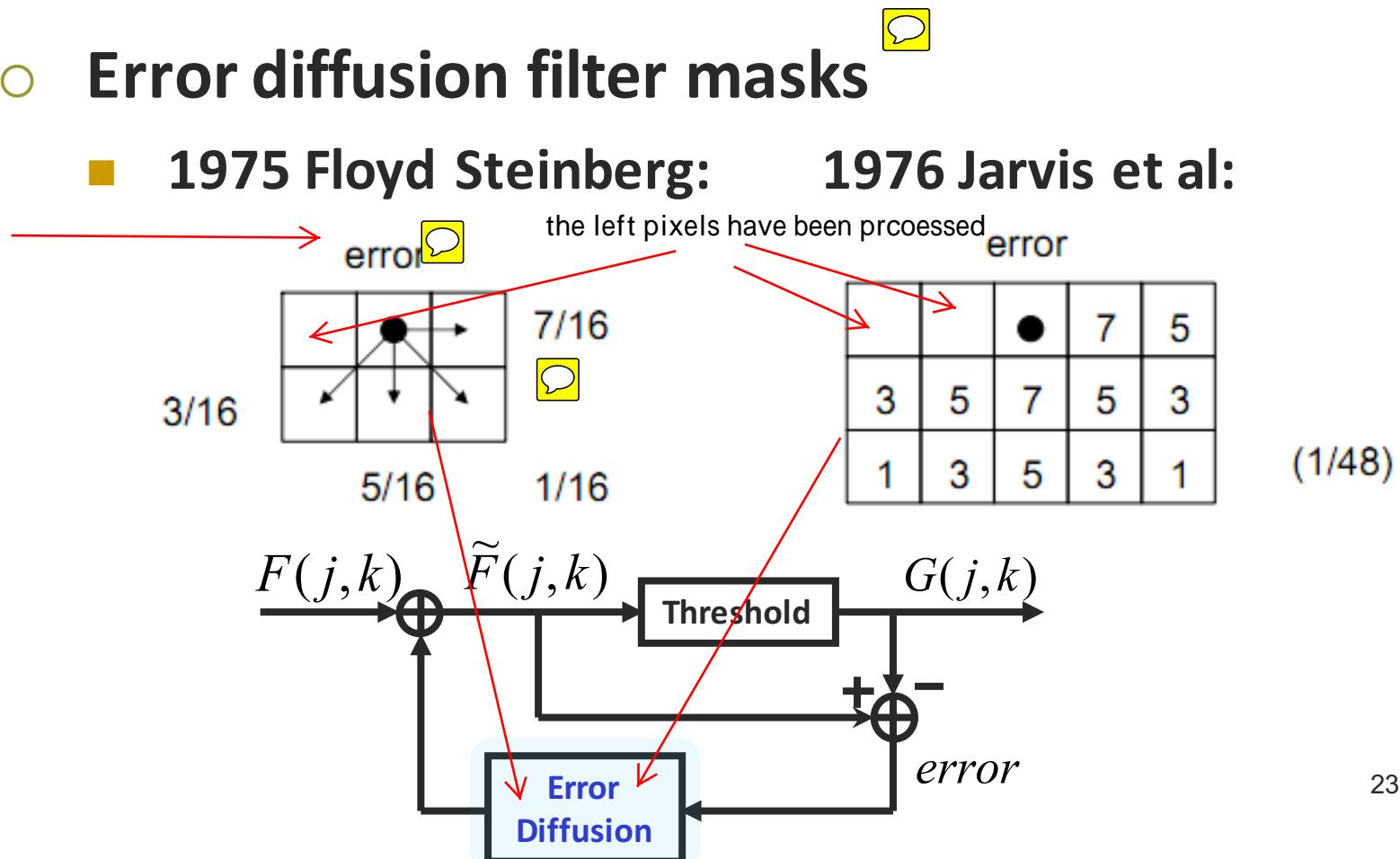
Digital Halftoning

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Error diffusion

Error diffusion filter masks

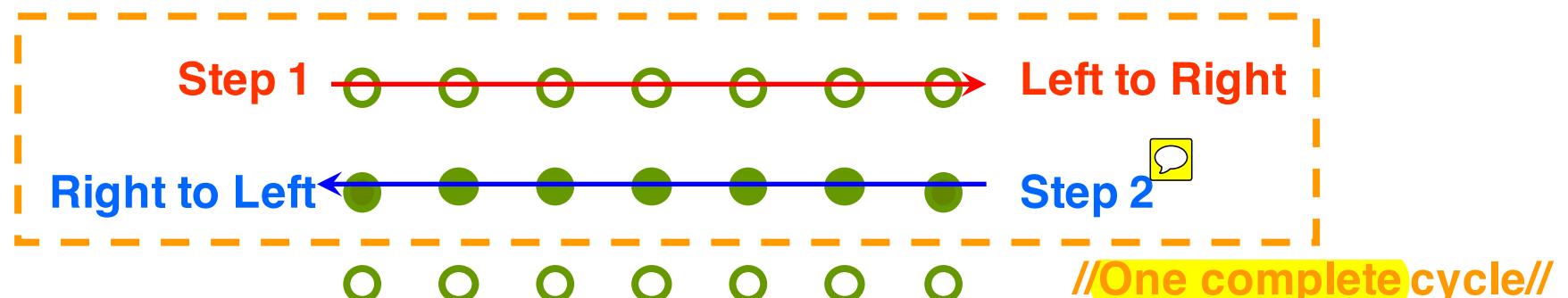
1975 Floyd Steinberg: 1976 Jarvis et al:



Digital Halftoning

Error diffusion

Error diffusion + serpentine scanning



$$\frac{1}{16} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 7 \\ 3 & 5 & 1 \end{pmatrix}$$

Left to Right

$$\frac{1}{16} \begin{pmatrix} 0 & 0 & 0 \\ 7 & 0 & 0 \\ 1 & 5 & 3 \end{pmatrix}$$

Right to Left

Digital Halftoning

■ Experimental results



Original Image

Error Diffusion

Digital Halftoning

■ Experimental results



Original Image

Floyd-Steinberg

Jarvis

Digital Halftoning

■ Multi-scale Error diffusion

- Several issues

- Region-to-region mapping
 - Multi-resolution
- Time series/causal error diffusion process
 - Easy to implement
 - Causality appears to be artificial in images
 - Is non-causal error diffusion possible?
- Quality metrics of half-toned images

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Digital Halftoning

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■ Multi-scale Error diffusion

“A multiscale error diffusion technique for digital halftoning”

Ioannis Katsavounidis and C. –C. Jay Kuo

○ Problem set-up

- Input image → $X(i, j) \in [0,1]$
- Output image → $B(i, j) \in \{0,1\}$
- Error image → $E(i, j) = X(i, j) - B(i, j)$
- Intermediate stage →

$$X_k(i_k, j_k), \quad 0 \leq k \leq r, \quad r = \log_2 N$$

$$X_k(i_k, j_k) = \sum_{i=0}^1 \sum_{j=0}^1 X_{k+1}(2i_k + i, 2j_k + j)$$

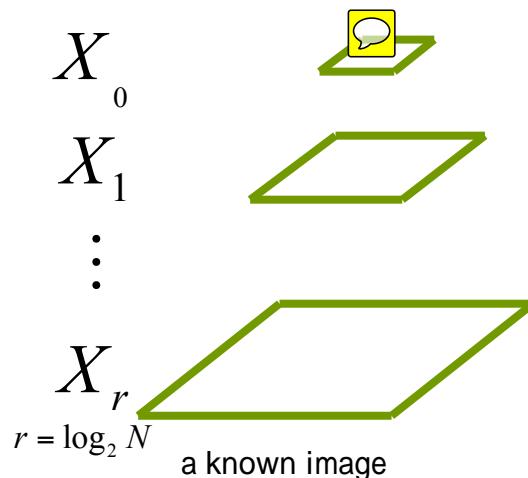
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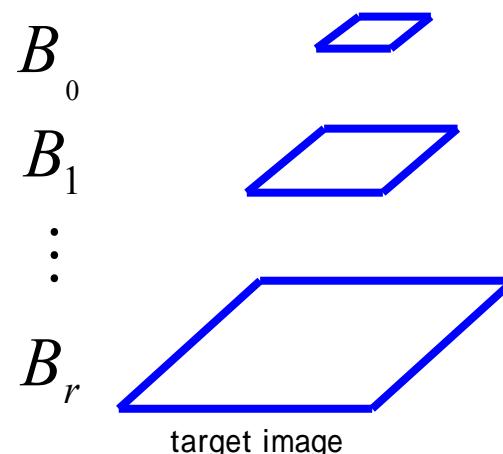
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■ Multi-scale Error diffusion

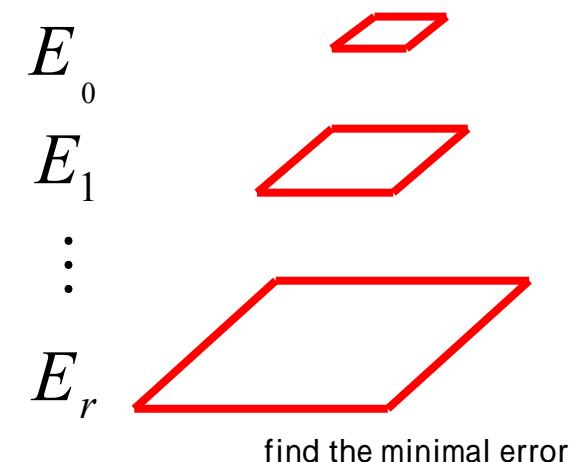
input $X(i, j) \in [0,1]$



output $B(i, j) \in \{0,1\}$



error $E(i, j) = X(i, j) - B(i, j)$



$$X_k(i_k, j_k) = \sum_{i=0}^1 \sum_{j=0}^1 X_{k+1}(2i_k + i, 2j_k + j), \quad 0 \leq k \leq r$$

$$E_k(i_k, j_k) = X_k(i_k, j_k) - B_k(i_k, j_k), \quad 0 \leq k \leq r$$

Goal: minimize the error pyramid in a certain way!

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■ Multi-scale Error diffusion

- //Step 1// Initialization
 - Set the entire output image pyramid to "0"
- multi-scale
○ //Step 2// Dot assignment
 - Find the largest error from top to bottom level
 - 1 parent node distributes its dots (integer numbers) to 4 children
- //Step 3// Error diffusion process

error diffuse

$$\frac{1}{12} \begin{pmatrix} 1 & 2 & 1 \\ 2 & -12 & 2 \\ 1 & 2 & 1 \end{pmatrix}$$

center

$$\frac{1}{8} \begin{pmatrix} 0 & 0 & 0 \\ 2 & -8 & 2 \\ 1 & 2 & 1 \end{pmatrix}$$

side

$$\frac{1}{5} \begin{pmatrix} 0 & 0 & 0 \\ 0 & -5 & 2 \\ 0 & 2 & 1 \end{pmatrix}$$

corner

Digital Halftoning

■ Multi-scale Error diffusion

○ Quality management

■ MSE vector

$$MSEV = \begin{pmatrix} MSE_0 \\ MSE_1 \\ \vdots \\ MSE_r \end{pmatrix} \quad MSE_k = \frac{1}{N^2} \sum_{i=0}^{2^k-1} \sum_{j=0}^{2^k-1} E_k^2(i, j)$$

○ Notes

- Preserve contrast of the original image
- Does not over-smooth the image 

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Digital Halftoning

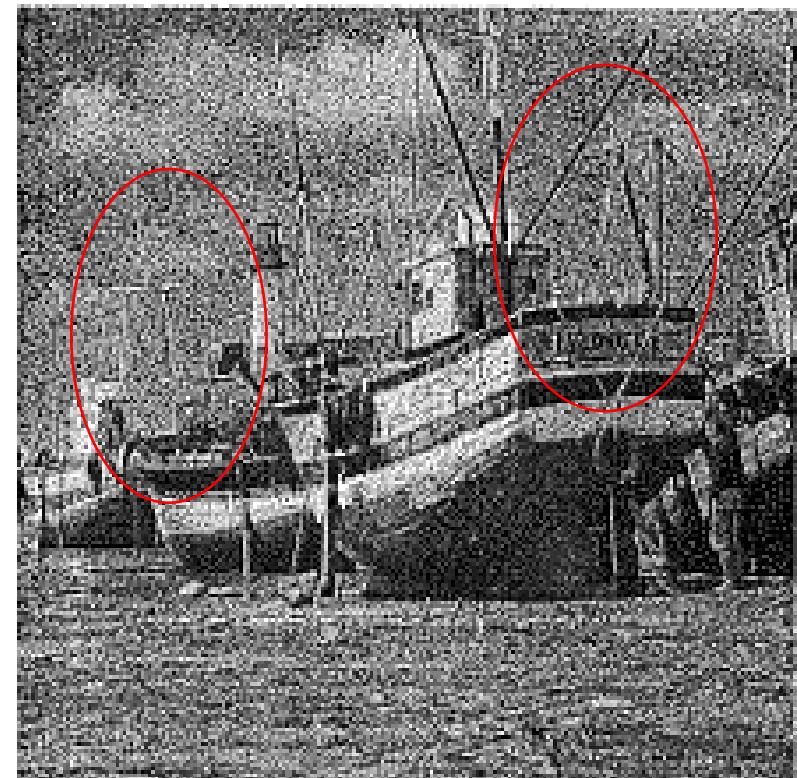
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■ Experimental results

better detail and contrast



Error Diffusion



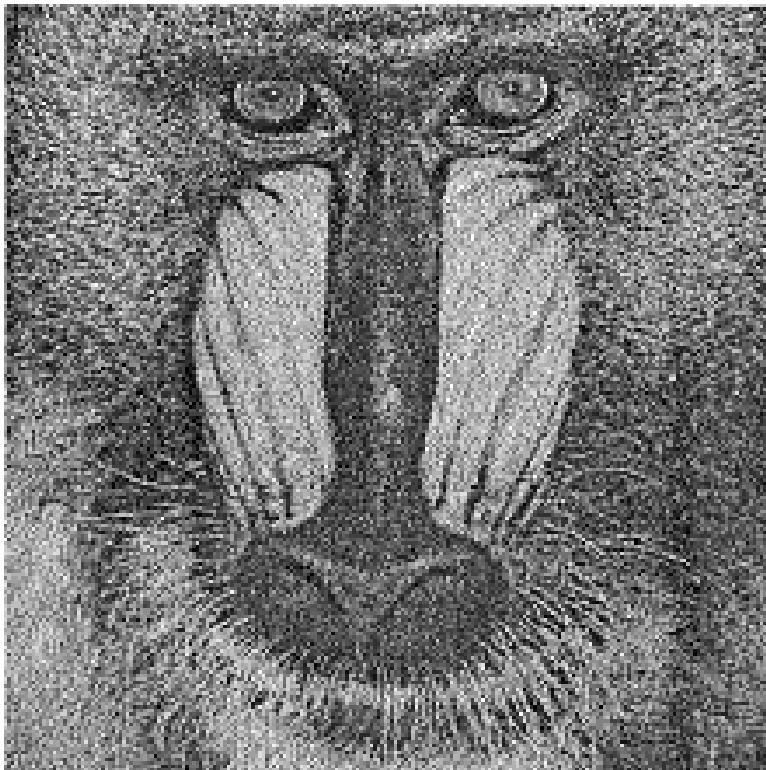
Multi-Scale Error Diffusion

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Digital Halftoning

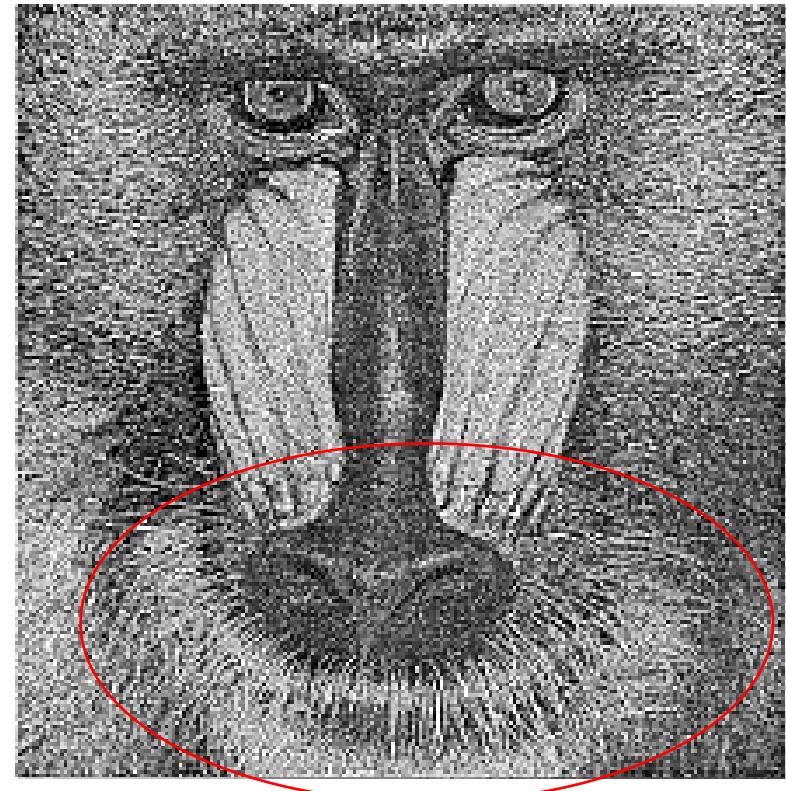
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■ Experimental results



Error Diffusion

worse contrast and detail



Multi-Scale Error Diffusion

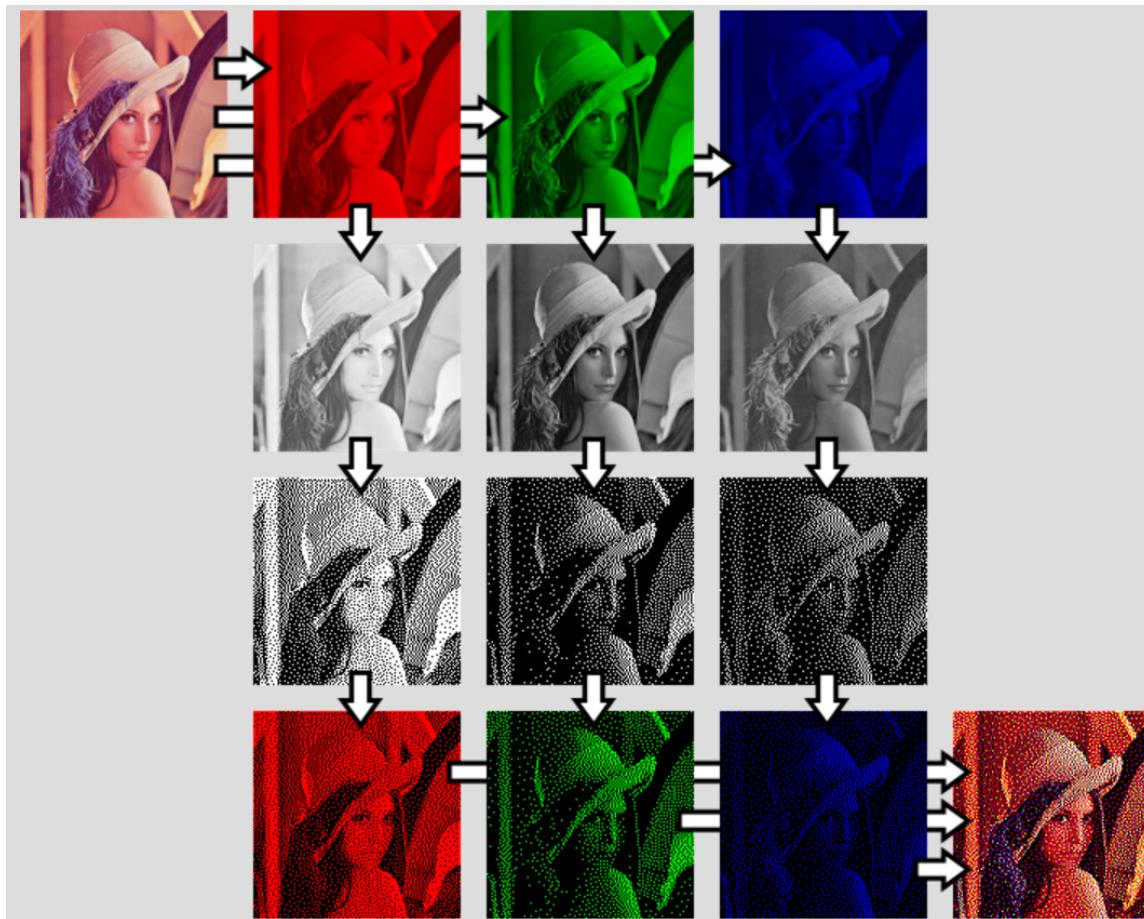
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Digital Halftoning

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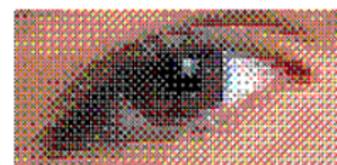
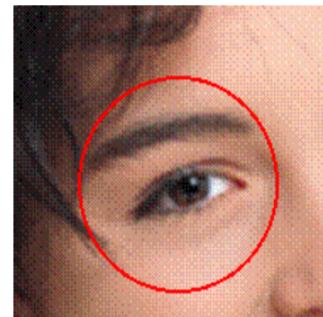
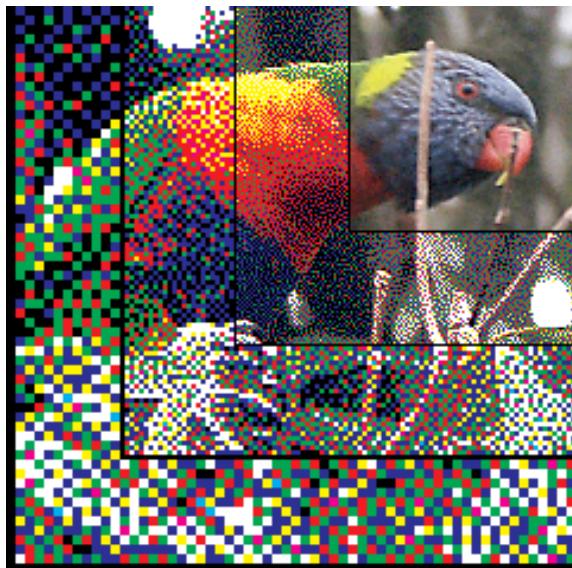
Color image

R,G,B is half-tone individually and then combined again.

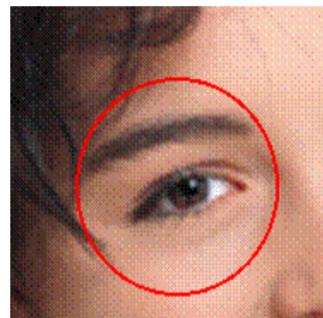


Digital Halftoning

■ Examples



Dithering



Error Diffusion

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Digital Halftoning

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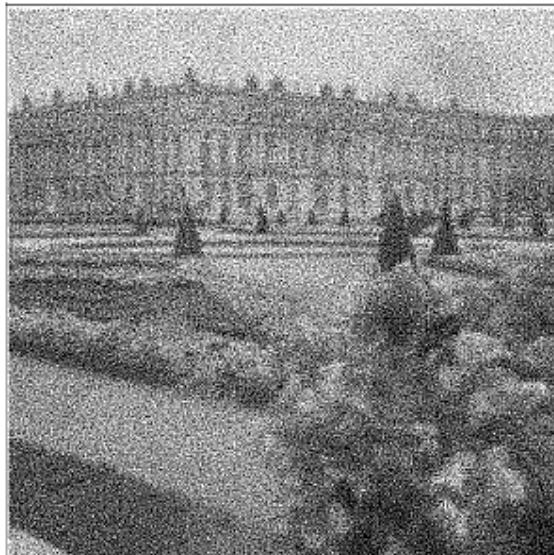
■ Application

- Visual cryptography

“visual cryptography based on void-and-cluster halftoning technique” E. Myodo, S. Sakazawa and Y. Takishima



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