E0005E - Industrial Image Analysis

The Hough Transform

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slides by Johan Carlson
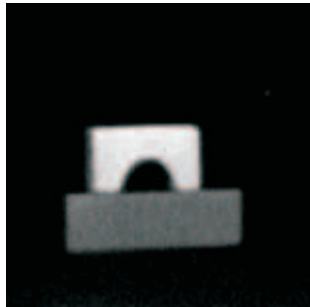
This Lecture

- The Hough transform
- Detection of lines
- Detection of other shapes (the generalized Hough transform)

Problem formulation

Let's say we have an edge detected image. How do we put this into some practical use, for example automatically <mark>detecting shapes</mark> and <mark>object</mark> sizes?
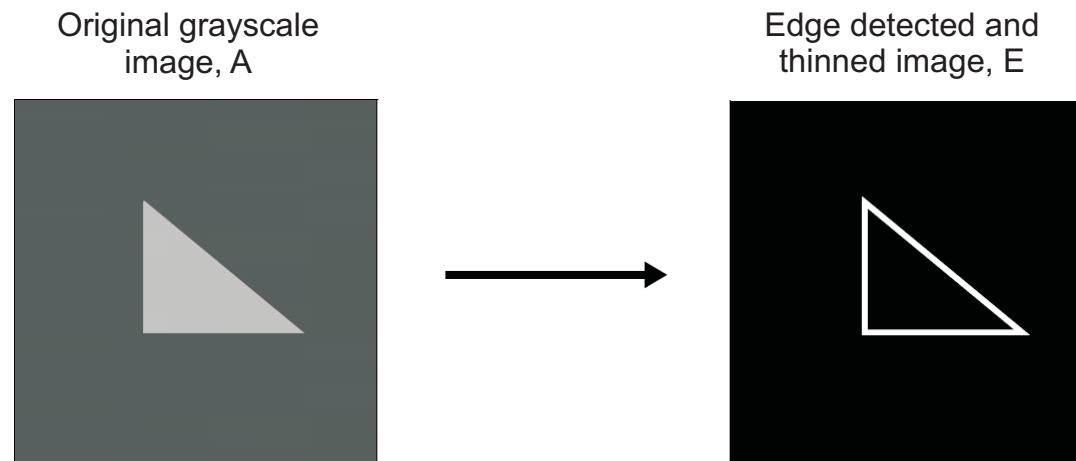
Original grayscale image		Edge detected image		Detected shape with dimensions ??

Problem formulation (cont'd...)

The first step is usually to find the edges.

Original grayscale image, A   Edge detected and thinned image, E

This can be done for example by using a gradient edge detector, followed by some thresholding and morphological processing (such as thinning the lines by erosion or opening).
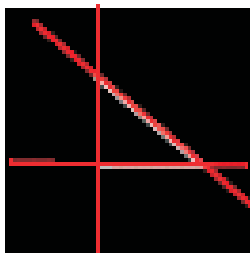
The Hough transform

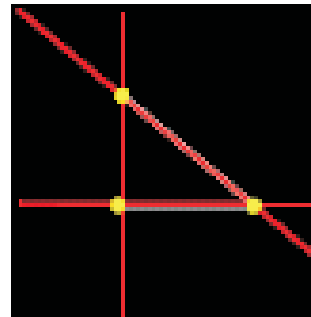The result of applying the Hough transform will be a set of parameterized lines

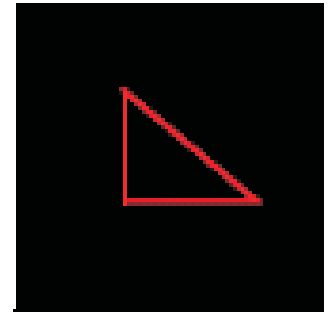Edge detected and thinned image, E
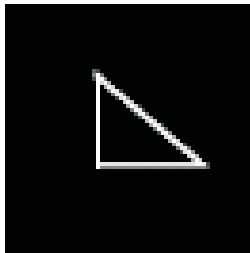
The result of Hough transform of E

Line-crossings indicate corners
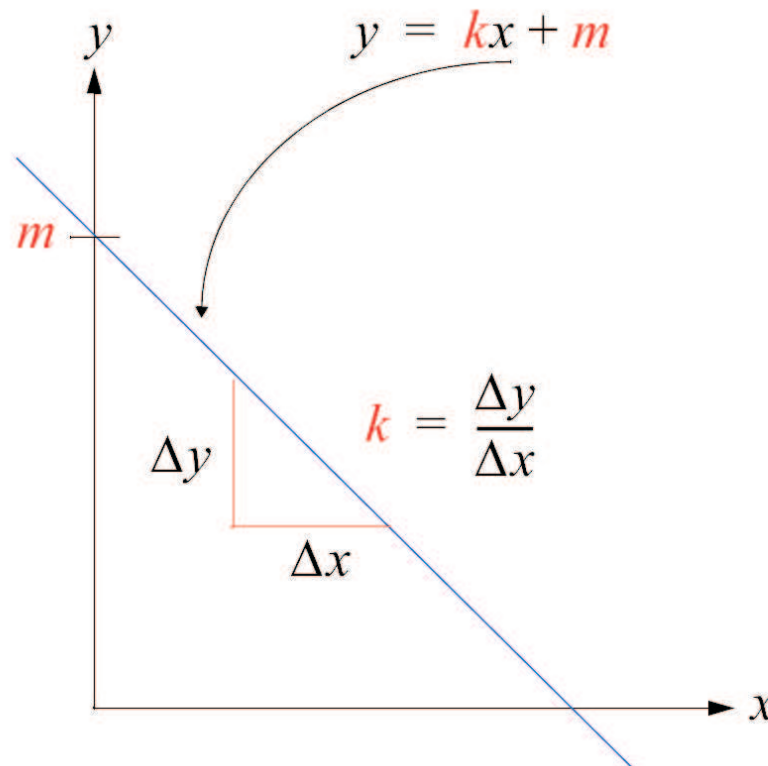
Ended lines indicate objects

The Hough transform - line equation

Recall that a straight line (in Cartesian coordinates) can be expressed as
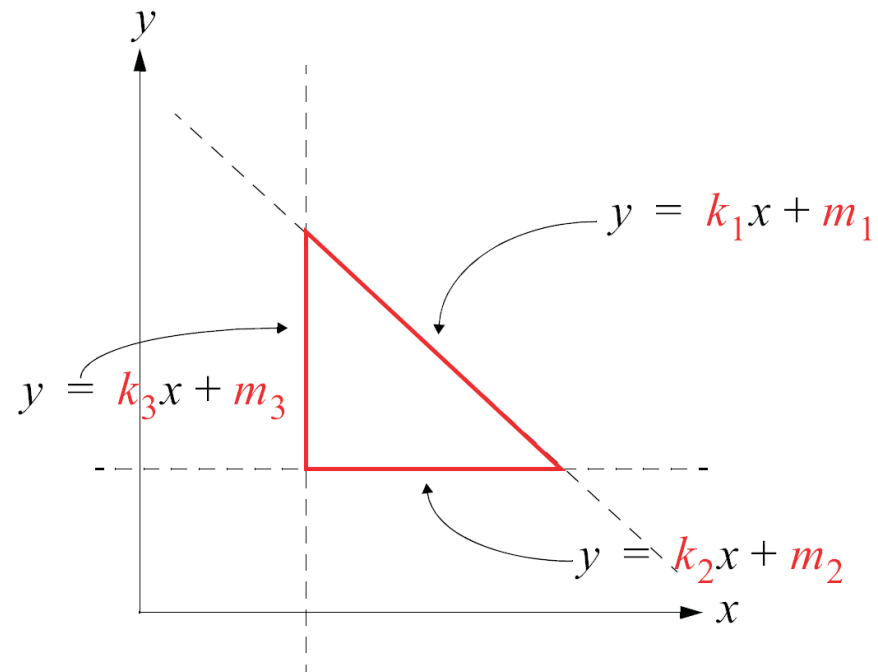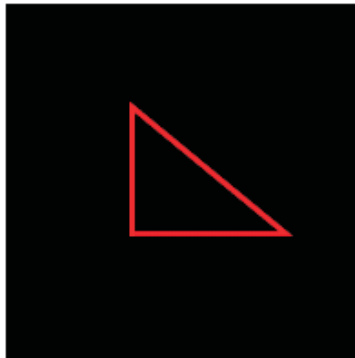
$$y = k \cdot x + m,$$

where k is the gradient and m is the intersection with the y-axis.

The Hough transform - objects and lines

Here, our original image has been transformed into a set of three parameterized lines. These can be used to characterize the detected object as a triangle.

$$y = k_1 x + m_1$$

$$y = k_3 x + m_3$$

$$y = k_2 x + m_2$$

The Hough transform - points on lines

But how do we produce these parameterized lines? Let's start by considering an example with relatively few points, where three points lie on a line and the fourth doesn't.

$$y = -x + 4$$

y	x
3	1
2	2
0	4
3	4
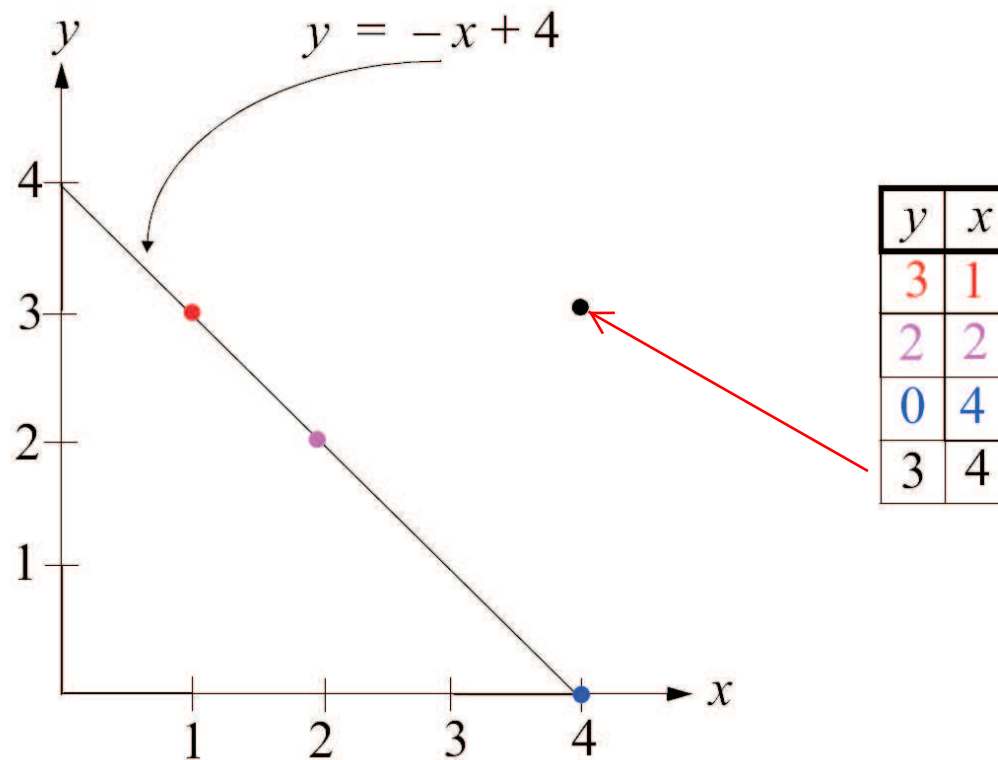
The Hough transform - (m,k) space

- We know the line parameterization

$$y = k \cdot x + m,$$

  where x and y are the variables, and k (gradient) and m (intersection) are constants.

- Let's think *backwards*, and instead express the equation as

$$m = -x \cdot k + y,$$

  and now let x and y be constants (i.e. a fixed point) and let m and k be the variables.

- We can think of this as being a transformation from (x, y)-space to (k, m)-space.

The Hough transform - (m,k) space

Our four fixed points (1,3), (2,2), (4,0), and (4,3) can be plotted in (k, m)-space

The intersection of the three lines in (k, m)-space is $m = 4, k = -1$, which is the parameters for the line in (x, y)-space.

$m = -k + 3$

$m = -2k + 2$

$m = -4k + 3$
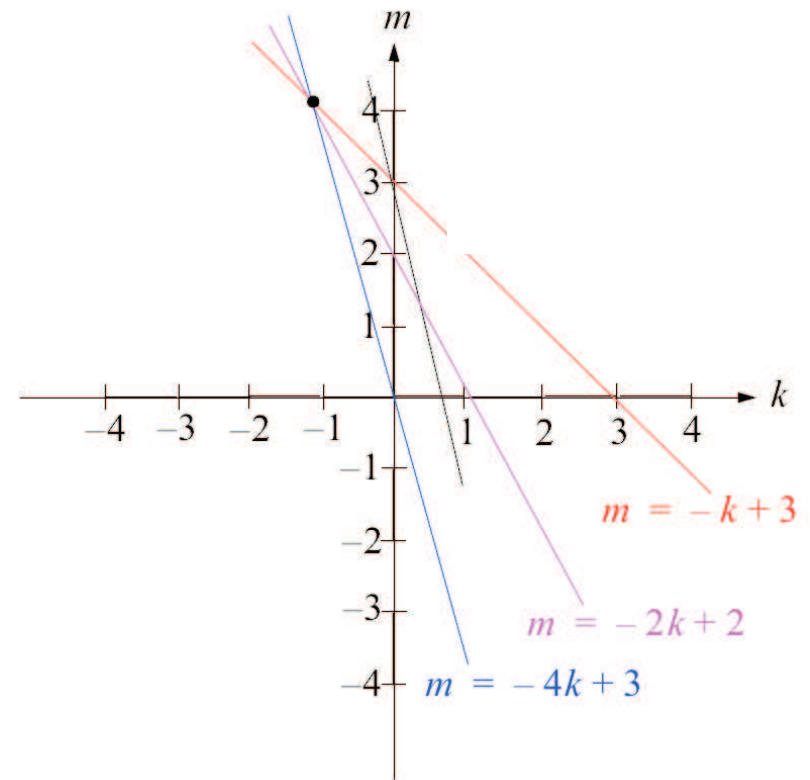
The Hough transform - (m,k) space

Problem

The (k, m)-space cannot be used. The reason is that vertical lines can not be represented (since the gradient $k \to \infty$).

Solution

Let's use polar coordinates, meaning we represent each line as an angle θ and the perpendicular distance to the origin r.
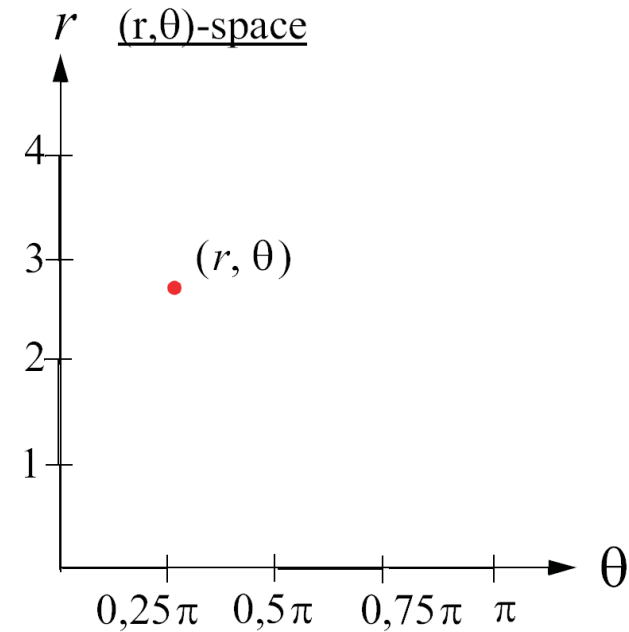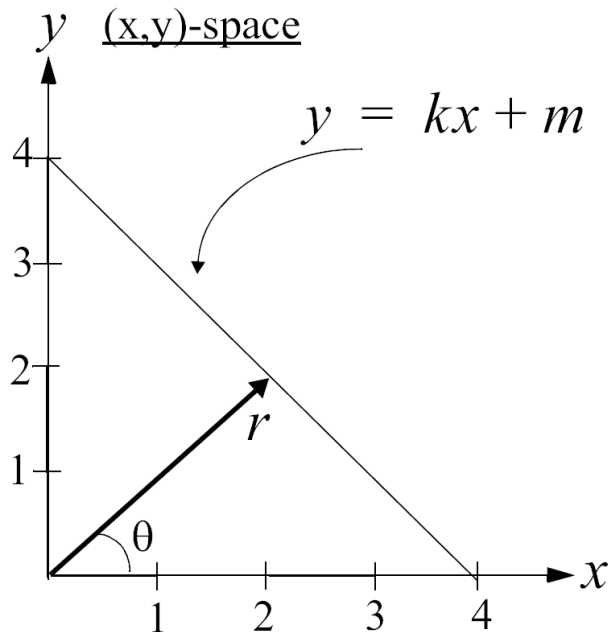
The Hough transform - polar coordinates

Our new parameter space will be (r, θ), where θ is the rotation angle and r is the perpendicular distance to the y-axis.
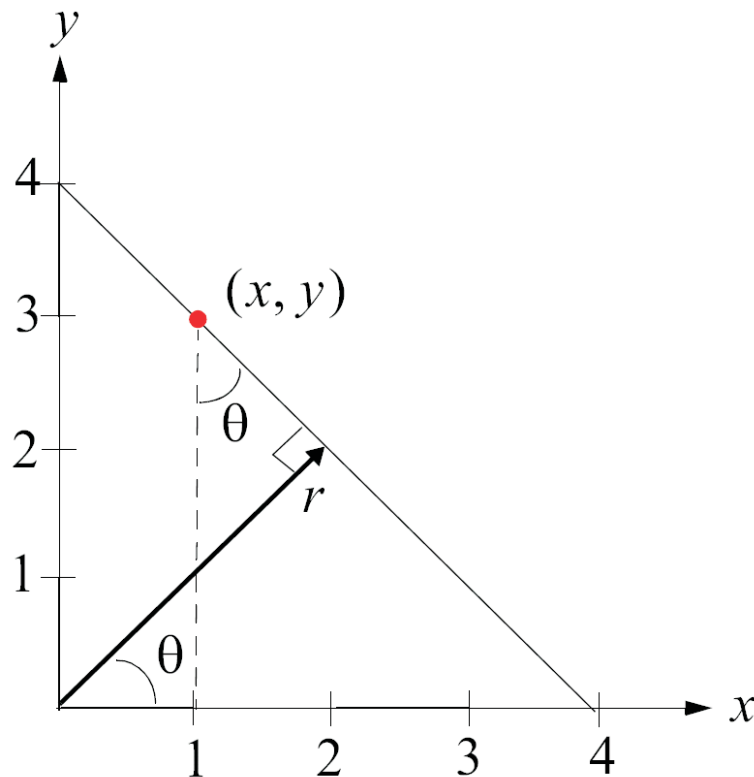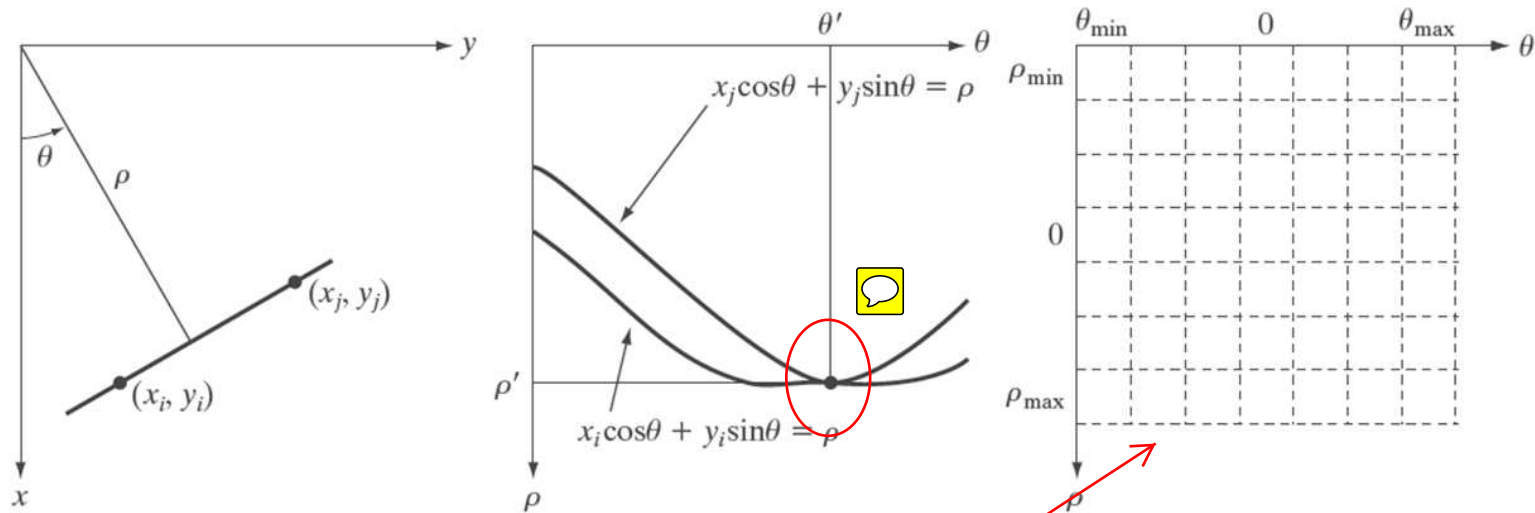
The Hough transform - polar coordinates

A line in the (x, y)-space will thus be represented by a point in (r, θ)-space.

The Hough transform - polar coordinates

a b c

FIGURE 10.32 (a) (ρ, θ) parameterization of line in the xy-plane. (b) Sinusoidal curves in the $\rho\theta$-plane; the point of intersection (ρ', θ') corresponds to the line passing through points (x_i, y_i) and (x_j, y_j) in the xy-plane. (c) Division of the $\rho\theta$-plane into accumulator cells.

r = x_j cos + y_j sin. since (x_j, y_j) is constant which is a point in x-y space, so
r = a * cos + b * sin => r = R * sin(theta+ c) where R, C are constant.
This is Angle Sum formula of trigonometric formulas.

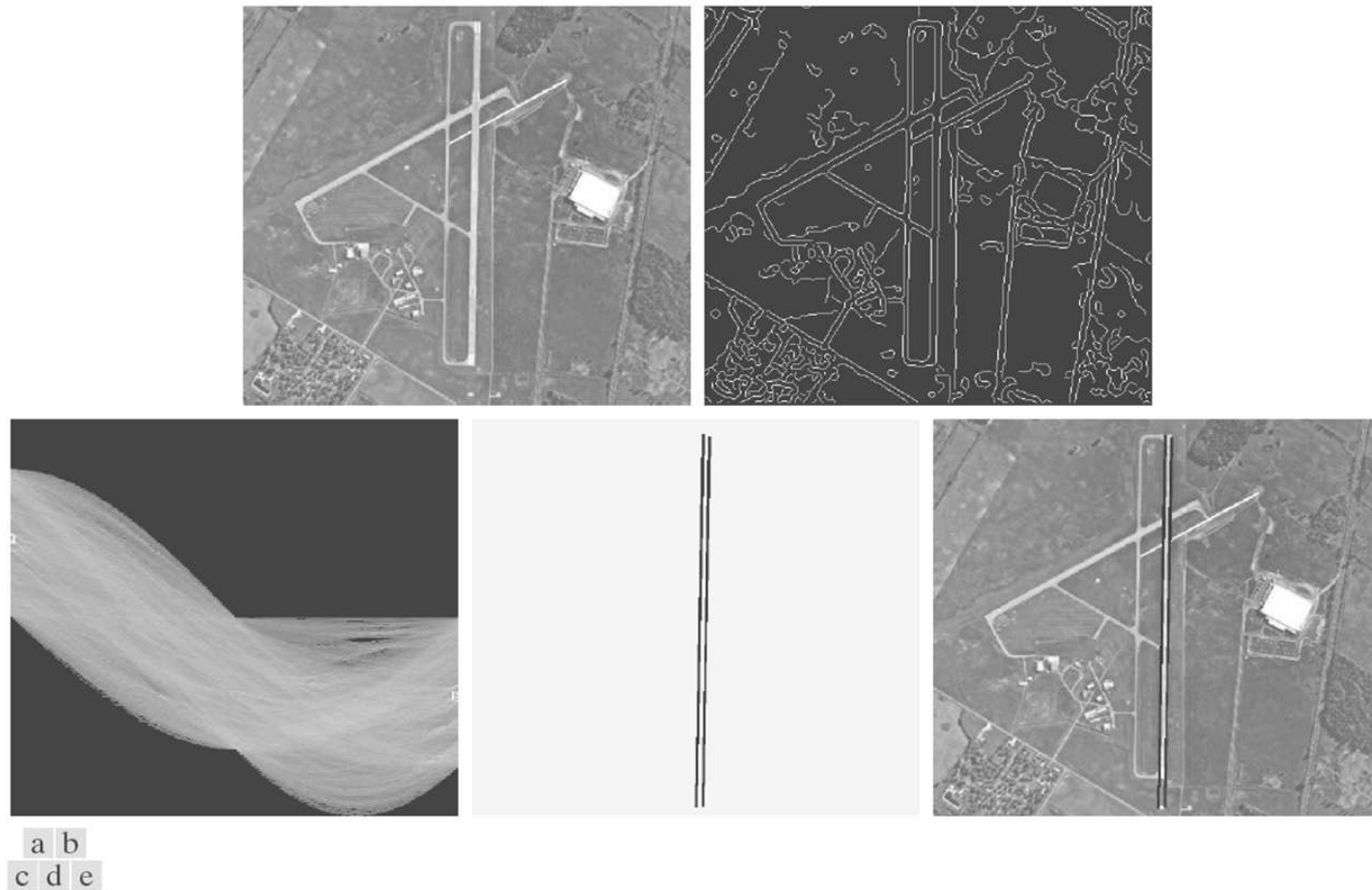# The Hough transform - An example

a b
c d e

FIGURE 10.34 (a) A 502×564 aerial image of an airport. (b) Edge image obtained using Canny's algorithm. (c) Hough parameter space (the boxes highlight the points associated with long vertical lines). (d) Lines in the image plane corresponding to the points highlighted by the boxes). (e) Lines superimposed on the original image.

The Hough transform - polar coordinates

Observation

All points on a line will have the same values of the (r, θ) parameters.

Problem

However, for any single point, infinitely many lines will pass through it, if we allow the (r, θ)-space to be continuous.

Solution

Discretize the (r, θ) space, i.e. for each points in the image, only compute the parameters for a finite set of angles $\theta = \theta_1, \theta_2, \ldots, \theta_N$. For each θ_i, we then obtain

$$r_i = x \cos \theta_i + y \sin(\theta_i)$$

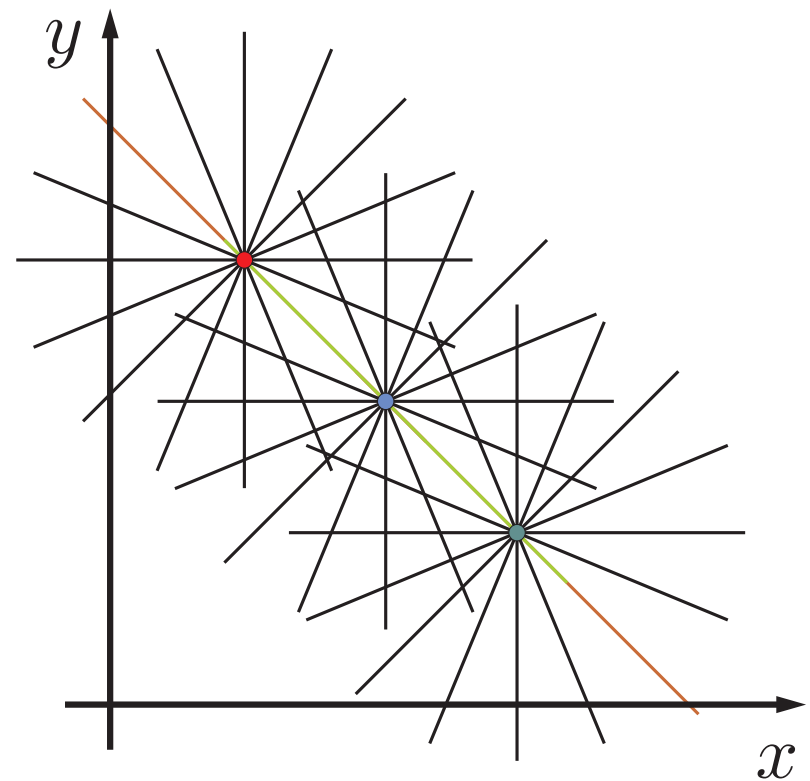# The Hough transform - accumulator matrix

The accumulator matrix (array)

- Create a matrix, called the accumulator matrix, where each column corresponds to the angles $\theta = \theta_1, \theta_2, \ldots, \theta_N$ and where the rows corresponds to *bins* or intervals of the resulting distances r.

- Then, for each point in the image, compute the r, θ values and increase the values of the corresponding elements of the accumulator matrix.

- Once we're done, the elements in the accumulator matrix corresponding to the highest values will correspond to lines in the image.

The Hough transform - accumulator matrix

The accumulator matrix (cont'd...)

To see why, lets consider the following case

- All three points have one (r, θ) pair in common.

- This (r, θ) entry of the accumulator matrix will thus be 3.

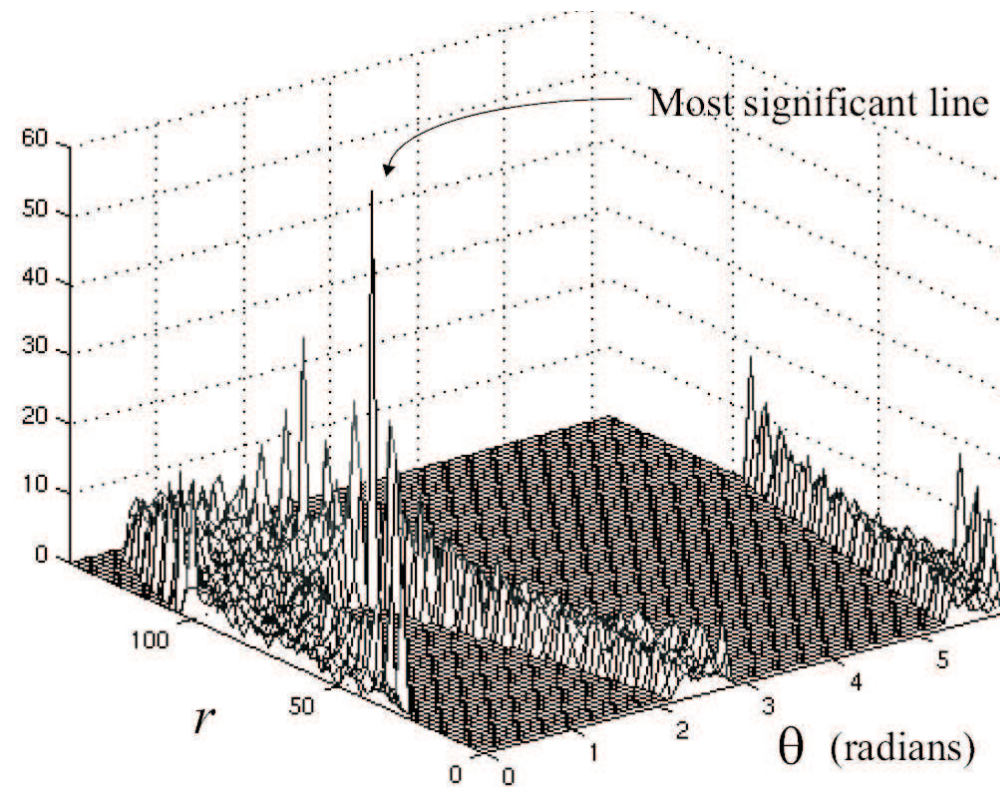- The other lines will result in the value 1 in the accumulator matrix.

The Hough transform (cont'd...)

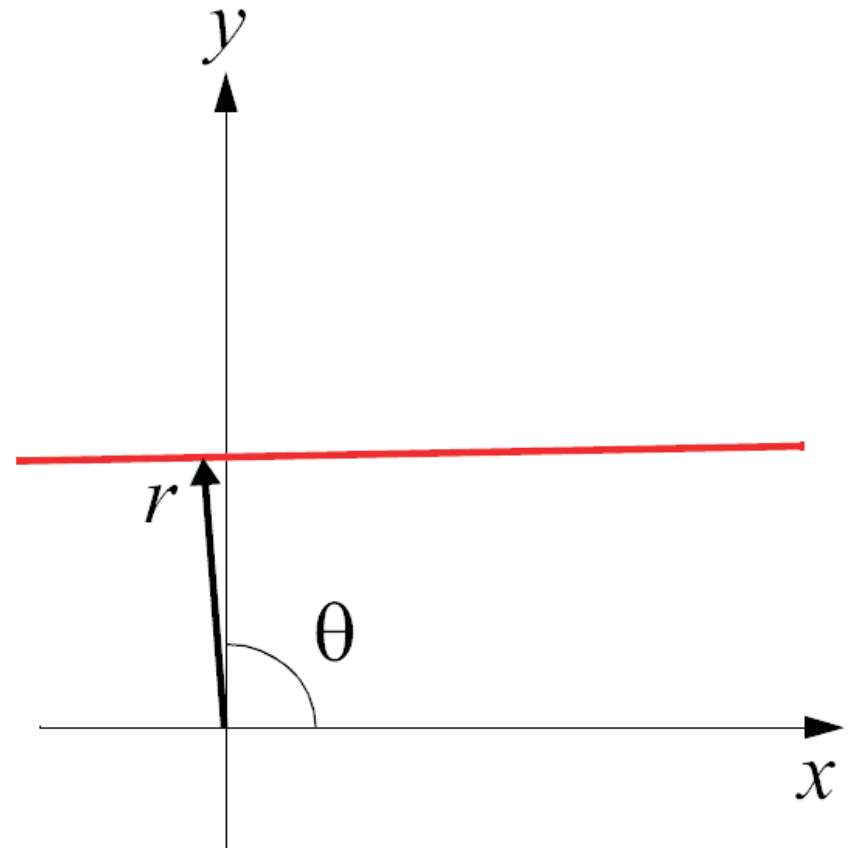## Our original example

Edge detected image

Resulting accumulator matrix

Most significant line

r

θ (radians)

The Hough transform (cont'd...)

Our original example (cont'd...)

Detection of other shapes

The generalized Hough transform

- The Hough transform can be generalized to find other shapes, e.g. circles or ellipses.

- However, the computational complexity increases drastically.

- Let's consider the example of finding a circle in an edge detected image.

Detection of other shapes (cont'd...)

The generalized Hough transform (cont'd...)

The equation of a circle can be written as

$$(x - x_0)^2 + (y - y_0)^2 - R^2 = 0,$$

where (x_0, y_0) is the center of the circle (in Cartesian coordinates) and R is its radius.
In this case, the Hough transform will be a transformation from the (x, y)-space to the (x_0, y_0, R)-space (i.e. to a three-dimensional space).

The generalized Hough transform (cont'd...)

Let's look at the simple case of five discrete points. Also, assume that we know the radius $R = 1/\sqrt{2}$, thus reducing the dimension of the parameter space to two.
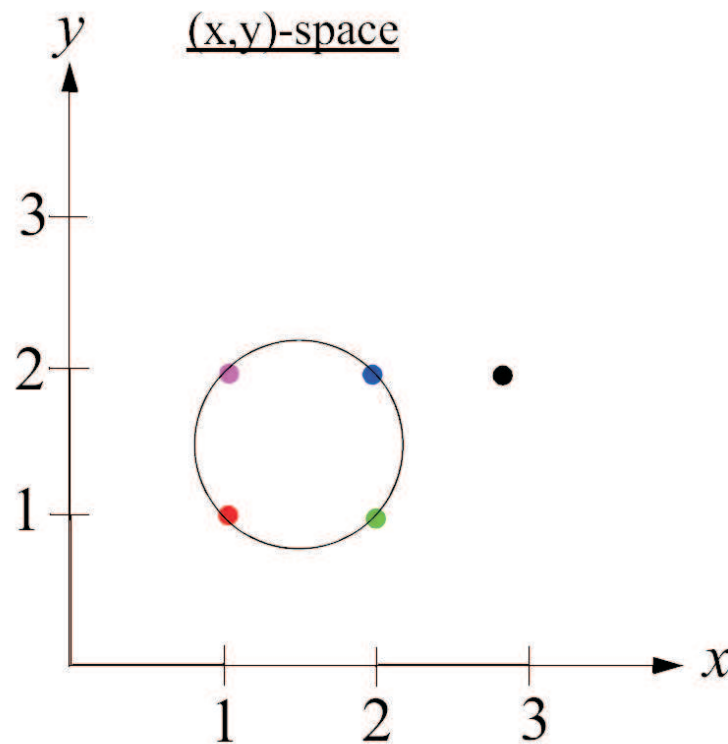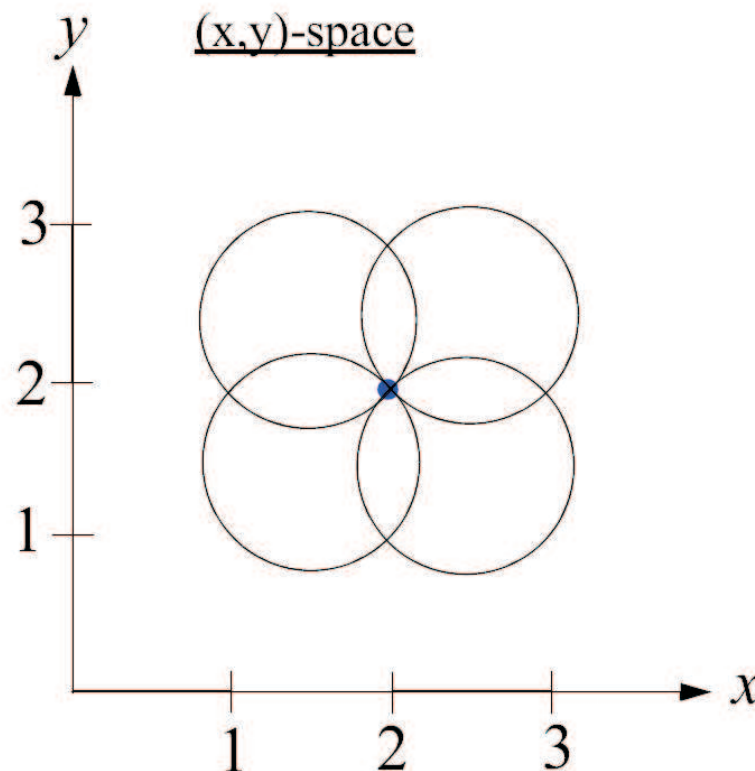

y	x
1	1
2	1
2	2
1	2
2	3

Detection of other shapes (cont'd...)

The generalized Hough transform (cont'd...)

For any fixed point (x, y) all possible circles with radius $R = 1/\sqrt{2}$ passing through the point will contribute to the accumulator matrix.

Detection of other shapes (cont'd...)

The generalized Hough transform (cont'd...)

Somewhere (at some distinct point $(x_0, y_0, 1/\sqrt{2})$) in the accumulator matrix, the largest number of corresponding circles (in the parameter space) will coincide. This is the most probable location of the circle.
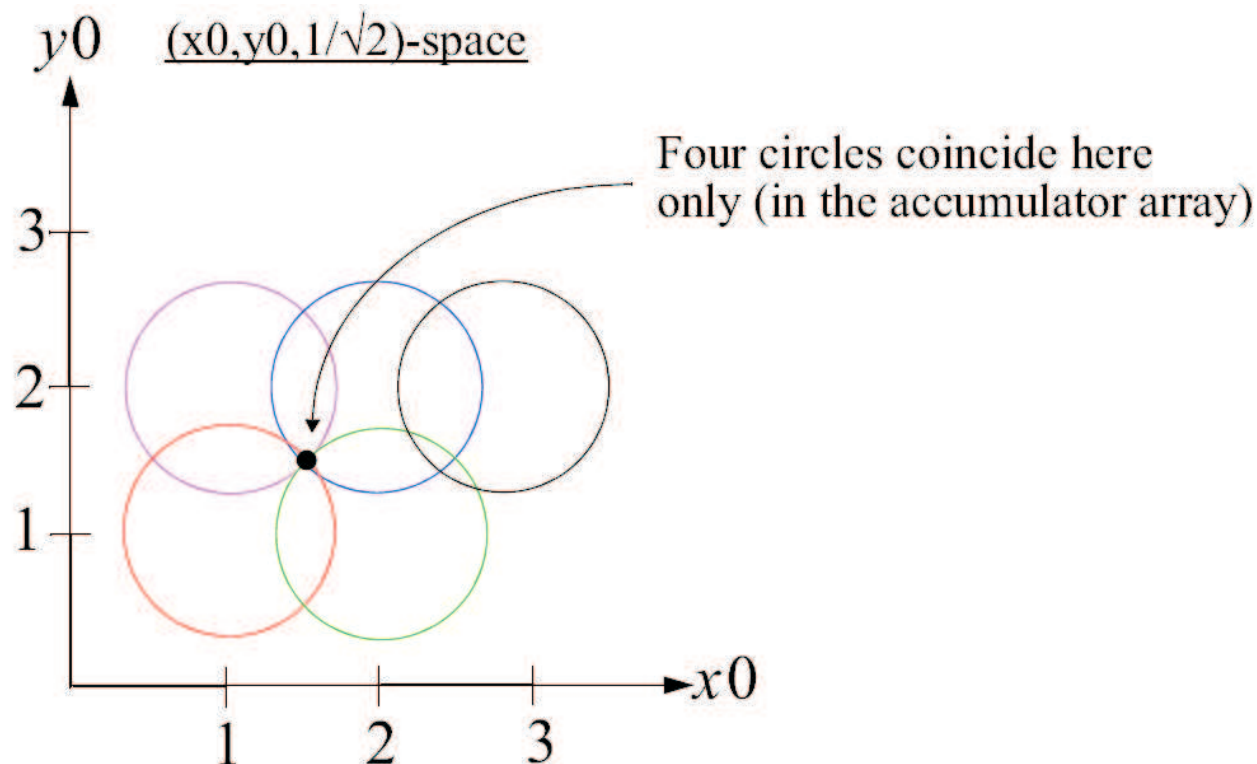
Summing Up

- Consider the following three questions;
  - What do i need to work on?
  - What have I learnt today?
  - What was the main point left unanswered?
- Write your answers in the journal with the lecture number at the top of the page.