Digital Image Processing

Lecture #7

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Announcement

- **Class Information**
  - **The following schedule**

03/23	Lecture 5	05/11	proposal
03/30	Lecture 6	05/18	Lecture 9
04/06	Lecture 7	05/25	Lecture 10
04/13	RealSense	06/01	Lecture 11
04/20	midterm	06/08	Demo
04/27	RealSense	06/15	Demo
05/04	Lecture 8	06/22	Final Package Due

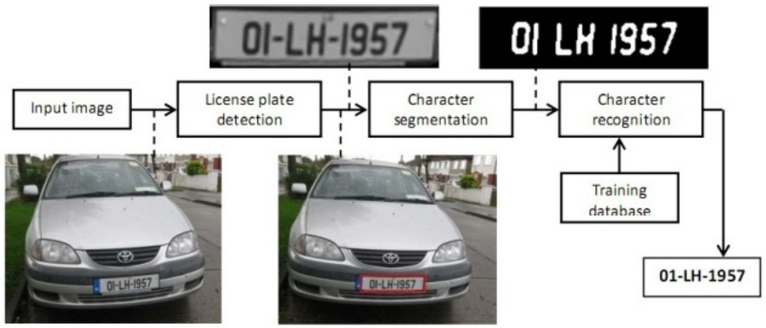
Announcement

- Please form a team with 2 to 3 students
- Email TA the member list by Apr. 5, 2016
- The team will work together on RealSense homework assignment and term project

Shape Analysis & Document Processing

Shape Analysis & Document Processing

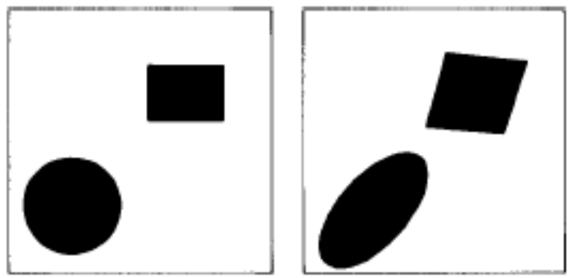
- **OCR – Optical Character Recognition**
  - Extract features from characters (A~Z) and numerals (0~9) and other special symbols (*&^%$#)



http://www4.hcmut.edu.vn/~bmthanh/research.html

Shape Analysis & Document Processing

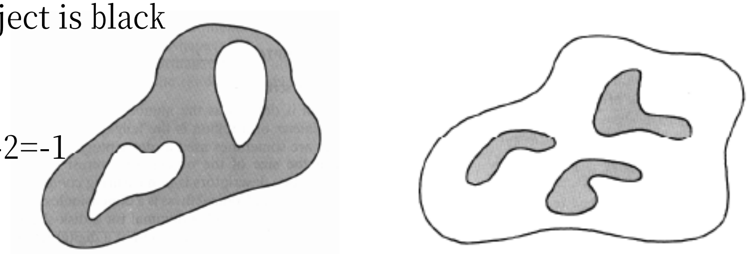
- **Topological Attributes**
  - May design your own attributes
  - Properties are invariant under the rubber-sheet transformation



Shape Analysis & Document Processing

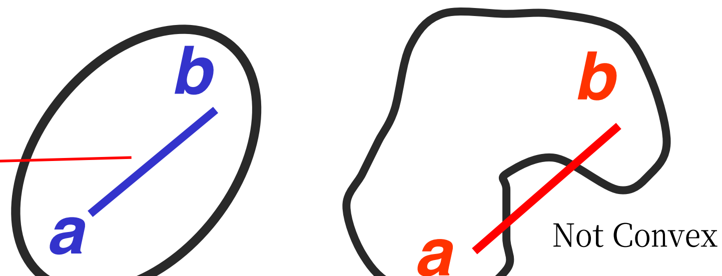
- **Topological Attributes**
  - **C: number of connected object components**
  - **H: number of holes**
  - **E: Euler number E=C-H**

if object is black
C=1
H=2
E=1-2=-1



  - **Convex Set**
    - **An object C is convex if**

$$\forall a,b \in C$$
$$\Rightarrow ta+(1-t)b \in C, \quad 0 \le t \le 1$$



Shape Analysis & Document Processing

- Convex Hull & Convex Deficiency
  - Convex Hull
    - The convex hull of a set is the smallest convex set that contains the set
  - Convex Deficiency
    - The set of points within the convex hull but not in the object form the convex deficiency
    - Divide subsets of convex deficiency into two types
      - Lake and Bay
      - L: number of lakes
      - B: number of bays

(a) Object (b) Convex hull (c) Bays and lake

Shape Analysis & Document Processing

- **Examples**

C: number of connected object components

H: number of holes

E: Euler number E=C-H

Convex Hull? Convex Deficiency: Lake? Bay?

c=2
h=0
E=2-0=2

H i ? Q @ *a*

C=1
H=0

g B *m* a D H

Lake=0,
Bay=4

- **Geometrical Properties**

p-norm

  - **Distance**
    - **Euclidean distance** $d_E = \left[(j_1 - j_2)^2 + (k_1 - k_2)^2 \right]^{1/2}$ 2-norm
    - **Magnitude distance** $d_M = |j_1 - j_2| + |k_1 - k_2|$ 1-norm
    - **Maximum value distance** $d_X = MAX\{ |j_1 - j_2|, |k_1 - k_2| \}$ infinity norm
  - **Perimeter**
    - The number of "sides" which separate pixels with different values
  - **Area**
    - Total number of pixels with F(j,k)=1
    - The "enclosed area" is the total number of pixels with F(j,k)=0 or 1 within the outer perimeter of an object

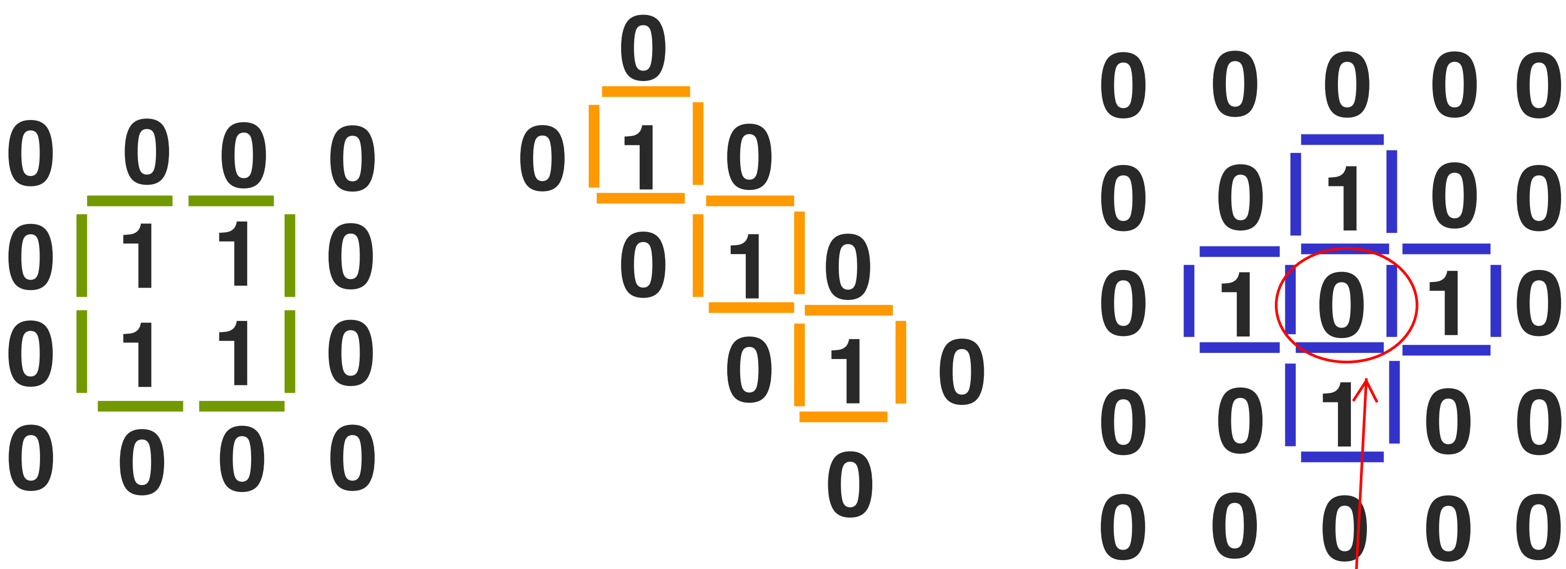
Shape Analysis & Document Processing

- **Examples**

- Area? [Total number of pixels with F(j,k)=1]

- Perimeter? [The number of “sides” which separate pixels with different values]

- Enclosed Area? [the total number of pixels with F(j,k)=0 or 1 within the outer perimeter of an object]



A=4; P=8; EA=4

A=3; P=12; EA=3

A=4; P=16; EA=5

Shape Analysis & Document Processing

- Geometrical Properties
  - Relative measure
    - Scaling-invariant
    - Normalized area/perimeter
    - Normalized w.r.t. the bounding box

w.r.t == with respect to

  - Computation of several attributes with local patterns
    - Bit Quads
      - Let n{Q} represent the count of the number of matches between image pixels and pattern Q

$$Q = 1 \Rightarrow n\{Q\} = Area$$

a perimeter is to separate 0 and 1

$$Q = [0 \quad 1] or [1 \quad 0] or \begin{bmatrix} 1 \\ 0 \end{bmatrix} or \begin{bmatrix} 0 \\ 1 \end{bmatrix} \Rightarrow n\{0 \quad 1\} + n\{1 \quad 0\} + n\begin{Bmatrix} 1 \\ 0 \end{Bmatrix} + n\begin{Bmatrix} 0 \\ 1 \end{Bmatrix} = Perimeter$$

Shape Analysis & Document Processing

- **Geometrical Properties**
  - **Bit Quads (Gray's algorithm)**
    - **A systematic way to compute geometric attributes based on local pattern matching**

// Bit quad patterns //

$$Q_0 \quad \begin{array}{|c|c|}\hline 0 & 0 \\ \hline 0 & 0 \\ \hline \end{array}$$

$$Q_1 \quad \begin{array}{|c|c|}\hline 1 & 0 \\ \hline 0 & 0 \\ \hline \end{array} \; \begin{array}{|c|c|}\hline 0 & 1 \\ \hline 0 & 0 \\ \hline \end{array} \; \begin{array}{|c|c|}\hline 0 & 0 \\ \hline 0 & 1 \\ \hline \end{array} \; \begin{array}{|c|c|}\hline 0 & 0 \\ \hline 1 & 0 \\ \hline \end{array}$$

$$Q_2 \quad \begin{array}{|c|c|}\hline 1 & 1 \\ \hline 0 & 0 \\ \hline \end{array} \; \begin{array}{|c|c|}\hline 0 & 1 \\ \hline 0 & 1 \\ \hline \end{array} \; \begin{array}{|c|c|}\hline 0 & 0 \\ \hline 1 & 1 \\ \hline \end{array} \; \begin{array}{|c|c|}\hline 1 & 0 \\ \hline 1 & 0 \\ \hline \end{array}$$

$$Q_3 \quad \begin{array}{|c|c|}\hline 1 & 1 \\ \hline 0 & 1 \\ \hline \end{array} \; \begin{array}{|c|c|}\hline 0 & 1 \\ \hline 1 & 1 \\ \hline \end{array} \; \begin{array}{|c|c|}\hline 1 & 0 \\ \hline 1 & 1 \\ \hline \end{array} \; \begin{array}{|c|c|}\hline 1 & 1 \\ \hline 1 & 0 \\ \hline \end{array}$$

$$Q_4 \quad \begin{array}{|c|c|}\hline 1 & 1 \\ \hline 1 & 1 \\ \hline \end{array}$$

$$Q_D \quad \begin{array}{|c|c|}\hline 1 & 0 \\ \hline 0 & 1 \\ \hline \end{array} \; \begin{array}{|c|c|}\hline 0 & 1 \\ \hline 1 & 0 \\ \hline \end{array}$$

$$A = \frac{1}{4}\left[n\{Q_1\} + 2n\{Q_2\} + 3n\{Q_3\} + 4n\{Q_4\} + 2n\{Q_D\}\right]$$

$$P = n\{Q_1\} + n\{Q_2\} + n\{Q_3\} + 2n\{Q_D\}$$

- **Example**

0	0	0	0
0	1	1	0
0	1	1	0
0	0	0	0

$$A = \frac{1}{4}\left[n\{Q_1\} + 2n\{Q_2\} + 3n\{Q_3\} + 4n\{Q_4\} + 2n\{Q_D\}\right]$$ A=4

$$P = n\{Q_1\} + n\{Q_2\} + n\{Q_3\} + 2n\{Q_D\}$$ P=8

Q_0

0	0
0	0

Q_1

1	0	0	1	0	0	0	0
0	0	0	0	0	1	1	0

Q_2

1	1	0	1	0	0	1	0
0	0	0	1	1	1	1	0

Q_3

1	1	0	1	1	0	1	1
0	1	1	1	1	1	1	0

Q_4

1	1
1	1

Q_D

1	0	0	1
0	1	1	0

Shape Analysis & Document Processing

- **Geometrical Properties**
  - **Bit Quads (Duda's algorithm)**
    - **More accurate to represent the area of a continuous object that has been coarsely discretized than Gray's**
    - **2x2 patterns**

$$A = \frac{1}{4} n\{Q_1\} + \frac{1}{2} n\{Q_2\} + \frac{7}{8} n\{Q_3\} + n\{Q_4\} + \frac{3}{4} n\{Q_D\}$$

$$P = n\{Q_2\} + \frac{1}{\sqrt{2}} \left[n\{Q_1\} + n\{Q_3\} + 2n\{Q_D\} \right]$$

Shape Analysis & Document Processing

- **Geometrical Properties**
  - **Bit Quads**
    - **Easy to determine the "Euler number" of an image**
  - **Euler Number (Gray's)**
    - **Four-connectivity**

$$E = \frac{1}{4}\left[n\{Q_1\} - n\{Q_3\} + 2n\{Q_D\} \right]$$

    - **Eight-connectivity**

$$E = \frac{1}{4}\left[n\{Q_1\} - n\{Q_3\} - 2n\{Q_D\} \right]$$

e.g.

	0			
0	1	0		
	0	1	0	
		0	1	0
			0	

  - **Note// We are not able to compute the number of connected components *C* and the number of holes *H* (*E=C-H*) separately by local neighborhood computation**

Shape Analysis & Document Processing

- **Examples**

Q_0: $\begin{bmatrix}0&0\\0&0\end{bmatrix}$

Q_1: $\begin{bmatrix}1&0\\0&0\end{bmatrix}$ $\begin{bmatrix}0&1\\0&0\end{bmatrix}$ $\begin{bmatrix}0&0\\0&1\end{bmatrix}$ $\begin{bmatrix}0&0\\1&0\end{bmatrix}$

Q_2: $\begin{bmatrix}1&1\\0&0\end{bmatrix}$ $\begin{bmatrix}0&1\\0&1\end{bmatrix}$ $\begin{bmatrix}0&0\\1&1\end{bmatrix}$ $\begin{bmatrix}1&0\\1&0\end{bmatrix}$

Q_3: $\begin{bmatrix}1&1\\0&1\end{bmatrix}$ $\begin{bmatrix}0&1\\1&1\end{bmatrix}$ $\begin{bmatrix}1&0\\1&1\end{bmatrix}$ $\begin{bmatrix}1&1\\1&0\end{bmatrix}$

Q_4: $\begin{bmatrix}1&1\\1&1\end{bmatrix}$

Q_D: $\begin{bmatrix}1&0\\0&1\end{bmatrix}$ $\begin{bmatrix}0&1\\1&0\end{bmatrix}$

0	0	0	0	0
0	1	1	1	0
0	1	0	1	0
0	1	1	1	0
0	0	0	0	0

Four-connectivity

$$E = \frac{1}{4}\left[n\{\overset{8}{Q_1}\} - n\{\overset{0}{Q_3}\} + 2n\{\overset{2}{Q_D}\}\right]$$

n=1

0	0	0	0	0
0	0	1	0	0
0	1	0	1	0
0	0	1	0	0
0	0	0	0	0

Eight-connectivity

$$E = \frac{1}{4}\left[n\{Q_1\} - n\{Q_3\} - 2n\{Q_D\}\right]$$

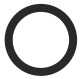
n=1

Shape Analysis & Document Processing

- **Other Attributes and Properties**
  - **Symmetry property**
    - **Horizontally symmetric / vertically symmetric**
  - **Circularity (thinness ratio)**

$$C_0 = \frac{4\pi A_0}{(P_0)^2}$$

- **E.g.**



$$A_0 = \pi r^2; \quad P_0 = 2\pi r \Rightarrow C_0 = \frac{4\pi \pi r^2}{(2\pi r)^2} = \frac{4\pi^2 r^2}{4\pi^2 r^2} = 1$$

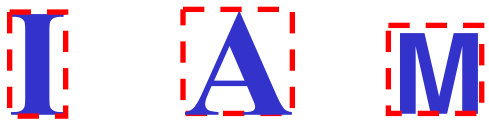
$$A_0 = a^2; \quad P_0 = 4a \Rightarrow C_0 = \frac{4\pi a^2}{(4a)^2} = \frac{\pi}{4} \approx 0.8$$

$b >> a$

$$A_0 = ab; \quad P_0 = 2(a+b) \Rightarrow C_0 = \frac{4\pi ab}{(2(a+b))^2} = \frac{\pi ab}{a^2 + b^2 + 2ab} \approx \frac{\pi a}{b}$$

Shape Analysis & Document Processing

- Other attributes and properties
  - Width and height
    - Bounding box



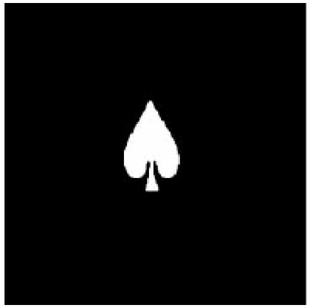
- Width ratio: $b/(a+b)$

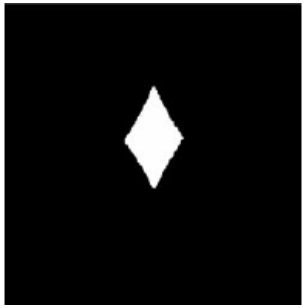
- Height ratio: $a/(a+b)$

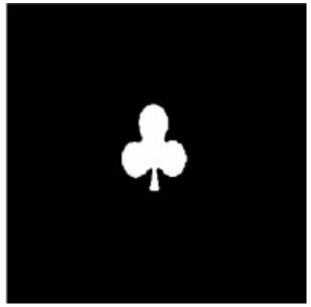
    - An image with many components but fewer holes
      - Euler number may be an approximation of # of components
      - Average area: $A_A = \dfrac{A_0}{E}$
      - Average perimeter: $P_A = \dfrac{P_0}{E}$
    - Thin objects (typewritten or script characters)
      - Average length $\approx L_A = \dfrac{P_A}{2}$
      - Average width $\approx W_A = \dfrac{2A_A}{P_A}$

Shape Analysis & Document Processing

- **Examples**



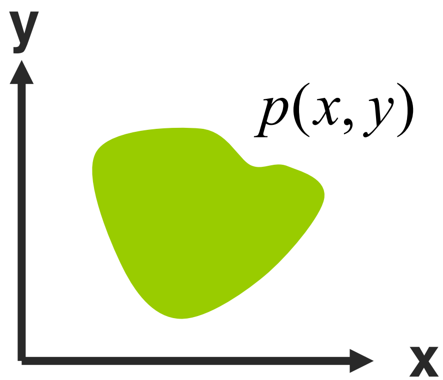




Attribute	spade	heart	diamond	club
Outer perimeter	652	512	548	668
Enclosed area	8421	8681	8562	8820
Average area	8421	8681	8562	8820
Average perimeter	652	512	548	668
Average length	326	256	274	334
Average width	25.8	33.9	31.3	26.4
Circularity	0.25	0.42	0.36	0.25

Shape Analysis & Document Processing

- **Other attributes and properties**
  - **Spatial moments**
    - **Treat the object shape as a pdf,** $p(x,y)$
    - **For a joint pdf,** $p(x,y)$**, its** $(m,n)^{th}$ **moment is defined as**



$$M(m,n) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^m y^n p(x,y)\,dx\,dy$$

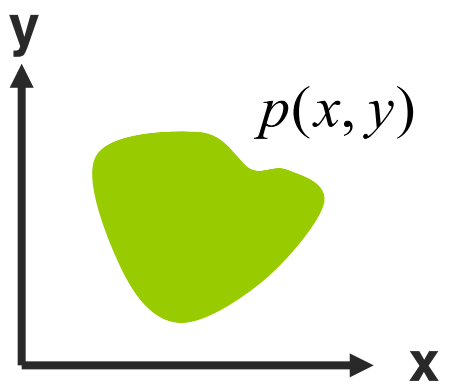
$$M(0,0) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p(x,y)\,dx\,dy = A;$$

marginal mean

$$M(1,0) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xp(x,y)\,dx\,dy = \eta_x;\ M(0,1) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} yp(x,y)\,dx\,dy = \eta_y$$

Shape Analysis & Document Processing

- **Other attributes and properties**
  - **Spatial moments**
    - **Usually, the central moments are more interesting since they are invariant under translation (shift-invariant)**

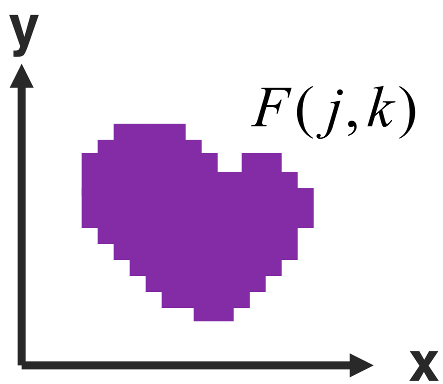


$$U(m,n) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(x-\eta_x)^m (y-\eta_y)^n \, p(x,y)\,dx\,dy$$

where η_x and η_y are marginal means of $p(x, y)$

Shape Analysis & Document Processing

- **Other attributes and properties**
  - **Discrete Image Spatial Moments**
    - The $(m,n)^{th}$ spatial geometric moment is defined as



$$M(m,n) = \frac{1}{J^m K^n} \sum_{j=1}^{J} \sum_{k=1}^{K} (x_j)^m (y_k)^n F(j,k)$$

$$M(0,0) = \sum_{j=1}^{J} \sum_{k=1}^{K} F(j,k)$$ < Image surface >

$$M(1,0) = \frac{1}{J} \sum_{j=1}^{J} \sum_{k=1}^{K} x_j F(j,k); \quad M(0,1) = \frac{1}{K} \sum_{j=1}^{J} \sum_{k=1}^{K} y_k F(j,k)$$

Shape Analysis & Document Processing

- **Other attributes and properties**
  - **Spatial moments**
    - **Examples**
    - **Table 18.3-1 (O&S p. 635)**

Image	$M(0,0)$	$M(1,0)$	$M(0,1)$	$M(2,0)$	$M(1,1)$	$M(0,2)$	$M(3,0)$	$M(2,1)$	$M(1,2)$	$M(0,3)$
Spade	8,219.98	4,013.75	4,281.28	1,976.12	2,089.86	2,263.11	980.81	1,028.31	1,104.36	1,213.73
Rotated spade	8,215.99	4,186.39	3,968.30	2,149.35	2,021.65	1,949.89	1,111.69	1,038.04	993.20	973.53
Heart	8,616.79	4,283.65	4,341.36	2,145.90	2,158.40	2,223.79	1,083.06	1,081.72	1,105.73	1,156.35
Rotated heart	8,613.79	4,276.28	4,337.90	2,149.18	2,143.52	2,211.15	1,092.92	1,071.95	1,008.05	1,140.43
Magnified heart	34,523.13	17,130.64	17,442.91	8,762.68	8,658.34	9,402.25	4,608.05	4,442.37	4,669.42	5,318.58
Minified heart	2,104.97	1,047.38	1,059.44	522.14	527.16	535.38	260.78	262.82	266.41	271.61
Diamond	8,561.82	4,349.00	4,704.71	2,222.43	2,390.10	2,627.42	1,142.44	1,221.53	1,334.97	1,490.26
Rotated diamond	8,562.82	4,294.89	4,324.09	2,196.40	2,168.00	2,196.97	1,143.83	1,108.30	1,101.11	1,122.93
Club	8,781.71	4,323.54	4,500.10	2,150.47	2,215.32	2,344.02	1,080.29	1,101.21	1,153.76	1,241.04
Rotated club	8,787.71	4,363.23	4,220.96	2,196.08	2,103.88	2,057.66	1,120.12	1,062.39	1,028.90	1,017.60
Ellipse	8,721.74	4,326.93	4,377.78	2,175.86	2,189.76	2,226.61	1,108.47	1,109.92	1,122.62	1,146.97

Shape Analysis & Document Processing

- **Other attributes and properties**
  - **Row moment of inertia**

$$\mu'_{20} = \mu_{20}/\mu_{00} = M_{20}/M_{00} - \bar{x}^2$$

  - **Column moment of inertia**

$$\mu'_{02} = \mu_{02}/\mu_{00} = M_{02}/M_{00} - \bar{y}^2$$

  - **Row-column cross moment of inertia**

$$\mu'_{11} = \mu_{11}/\mu_{00} = M_{20}/M_{00} - \bar{x}\bar{y}$$

Shape Analysis & Document Processing

- **Other attributes and properties**
  - **Covariance Matrix**

$$U = \mathrm{cov}[I(x,y)] = \begin{bmatrix} \mu'_{20} & \mu'_{11} \\ \mu'_{11} & \mu'_{02} \end{bmatrix}$$

- **Perform SVD of the covariance matrix** $E^T U E = \Lambda$

The columns of $E = \begin{bmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{bmatrix}$ **are the eigenvectors of U and** $\Lambda = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$

Shape Analysis & Document Processing

- **Other attributes and properties**
  - **Eigenvalues can be derived explicitly**

$$\lambda_1 = \frac{1}{2}[\mu'_{20} + \mu'_{02}] + \frac{1}{2}\left[(\mu'_{20})^2 + (\mu'_{02})^2 - 2\mu'_{20}\mu'_{02} + 4(\mu'_{11})^2\right]^{1/2}$$

$$\lambda_2 = \frac{1}{2}[\mu'_{20} + \mu'_{02}] + \frac{1}{2}\left[(\mu'_{20})^2 + (\mu'_{02})^2 - 2\mu'_{20}\mu'_{02} + 4(\mu'_{11})^2\right]^{1/2}$$

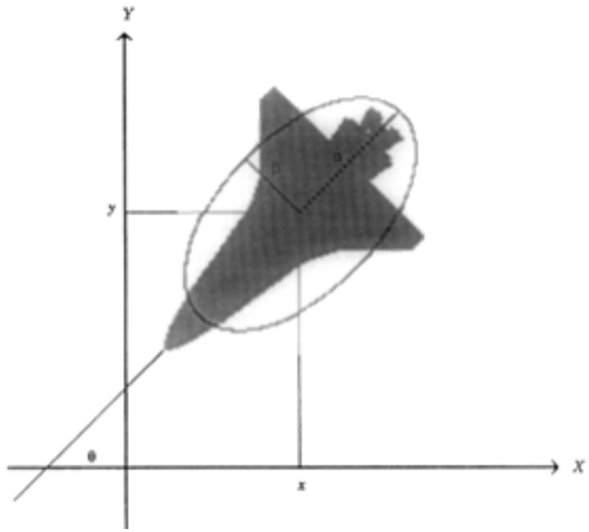
  - **Let** $\lambda_M = MAX\{\lambda_1, \lambda_2\}$ **and** $\lambda_N = MIN\{\lambda_1, \lambda_2\}$
    The eigenvalue ratio is λ_N / λ_M
  - **The orientation is** $\theta = \frac{1}{2}\arctan\left\{\frac{2\mu_{11}}{\mu_{20} - \mu_{02}}\right\}$

Shape Analysis & Document Processing

- **Other attributes and properties**
  - **The orientation is**

$$\theta = \frac{1}{2}\arctan\left\{\frac{2\mu_{11}}{\mu_{20} - \mu_{02}}\right\}$$

  - **Eclipse defined by 2 eigenvectors and orientation angle ϑ**



Shape Analysis & Document Processing

- **Other attributes and properties**
  - **Table 18.3-3**

Image	Largest Eigenvalue	Smallest Eigenvalue	Orientation (radians)	Eigenvalue Ratio = λ_N / λ_M
Spade	33.286	16.215	–0.153	0.487
Rotated spade	33.223	16.200	–1.549	0.488
Heart	36.508	16.376	1.561	0.449
Rotated heart	36.421	16.400	–0.794	0.450
Magnified heart	589.190	262.290	1.562	0.445
Minified heart	2.165	0.984	1.560	0.454
Diamond	42.189	13.334	1.560	0.316
Rotated diamond	42.223	13.341	–0.030	0.316
Club	37.982	21.831	–1.556	0.575
Rotated club	38.073	21.831	0.802	0.573
Ellipse	47.149	11.324	0.785	0.240

rotation invariant

Shape Analysis & Document Processing

- **Seven invariant moments**

$$\phi_1 = \eta_{20} + \eta_{02}$$

$$\phi_2 = (\eta_{20} - \eta_{02})^2 + 4\eta_{11}{}^2$$

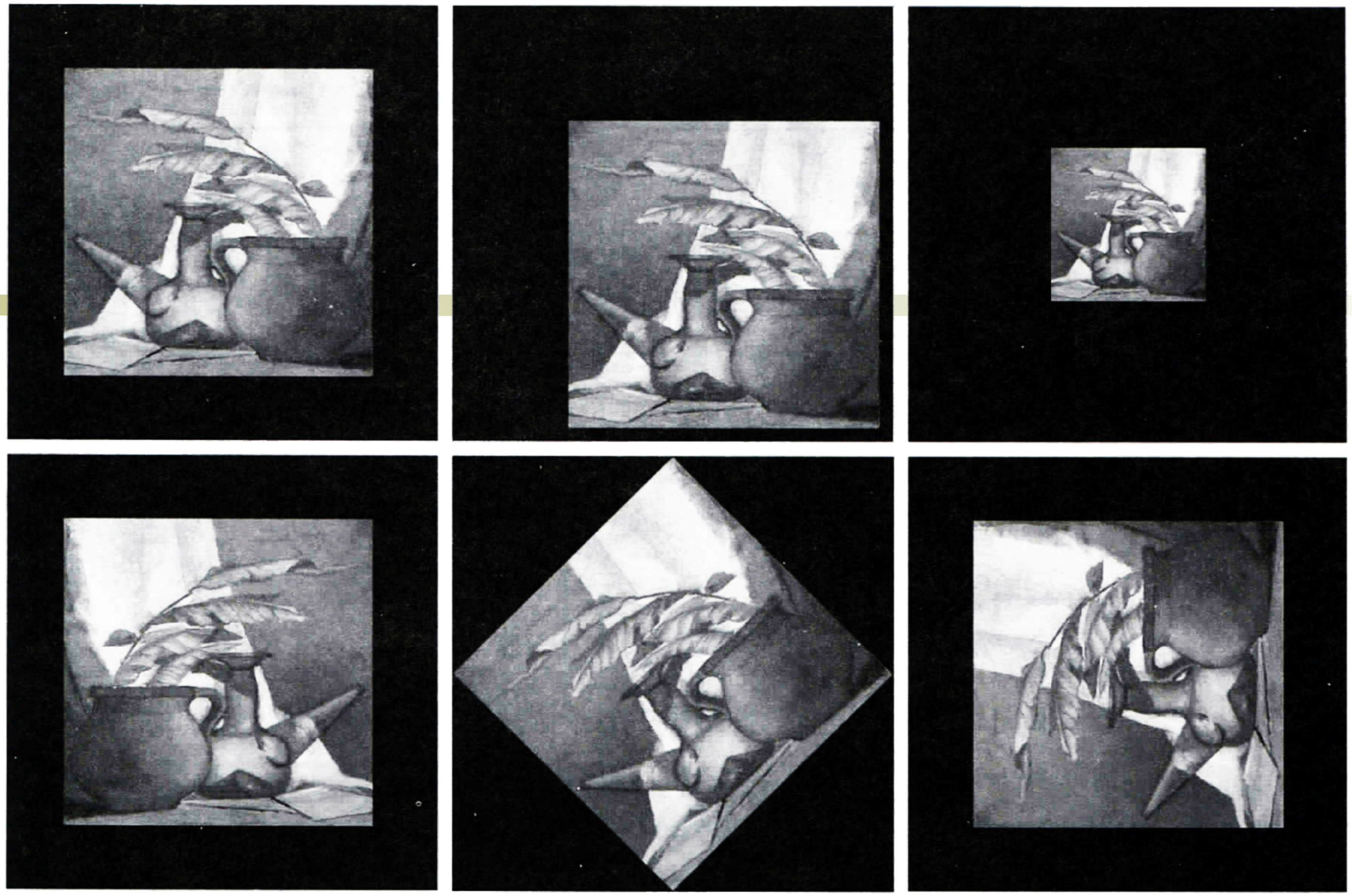
$$\phi_3 = (\eta_{30} - 3\eta_{12})^2 + (3\eta_{21} - \eta_{03})^2$$

$$\phi_4 = (\eta_{30} + \eta_{12})^2 + (\eta_{21} + \eta_{03})^2$$

$$\begin{aligned}\phi_5 = {} & (\eta_{30} - 3\eta_{12})(\eta_{30} + \eta_{12})\left[(\eta_{30} + \eta_{12})^2 - 3(\eta_{21} + \eta_{03})^2\right] \\ & + (3\eta_{21} - \eta_{03})(\eta_{21} + \eta_{03})\left[3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2\right]\end{aligned}$$

$$\begin{aligned}\phi_6 = {} & (\eta_{20} - \eta_{02})\left[(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2\right] \\ & + 4\eta_{11}{}^2(\eta_{30} + \eta_{12})(\eta_{21} + \eta_{03})\end{aligned}$$

$$\begin{aligned}\phi_7 = {} & (3\eta_{21} - \eta_{03})(\eta_{30} + \eta_{12})\left[(\eta_{30} + \eta_{12})^2 - 3(\eta_{21} + \eta_{03})^2\right] \\ & + (3\eta_{12} - \eta_{30})(\eta_{21} + \eta_{03})\left[3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2\right]\end{aligned}$$

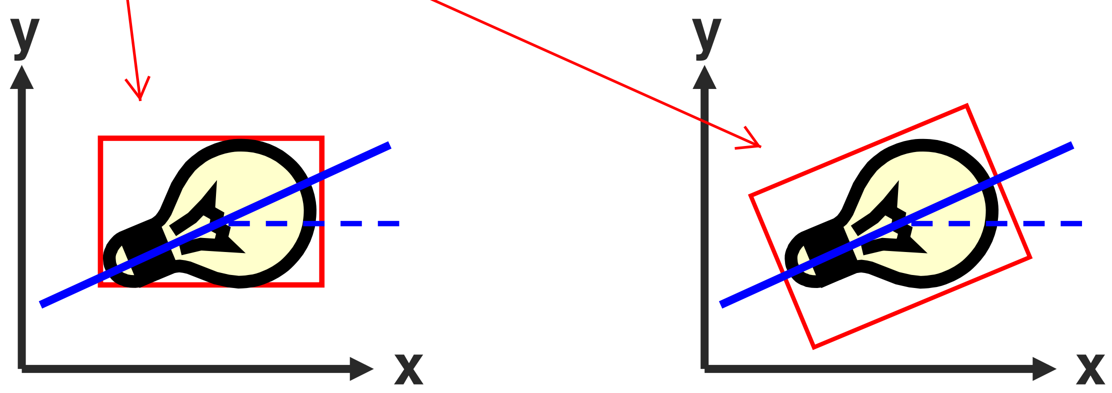


translation, scaling, rotation are all invariant

Invariant Moment	Φ_1	Φ_2	Φ_3	Φ_4	Φ_5	Φ_6	Φ_7
Original image	2.8662	7.1265	10.4109	10.3742	21.3674	13.9417	-20.7809
Shift	2.8662	7.1265	10.4109	10.3742	21.3674	13.9417	-20.7809
Half size	2.8664	7.1267	10.4107	10.3719	21.3924	13.9383	-20.7724
Mirrow	2.8662	7.1265	10.4109	10.3742	21.3674	13.9417	20.7809
Rotate 45°	2.8661	7.1266	10.4115	10.3742	21.3663	13.9417	-20.7813
Rotate 90°	2.8662	7.1265	10.4109	10.3742	21.3674	13.9417	-20.7809

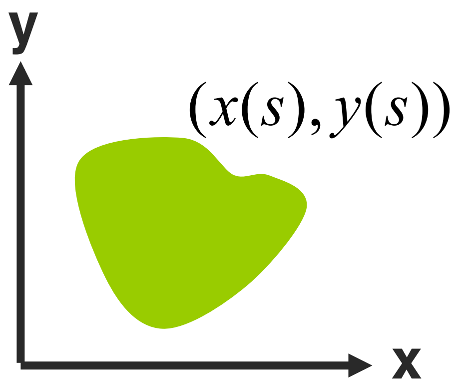
Shape Analysis & Document Processing

- **Other attributes and properties**
  - **Shape Orientation Descriptors**
    - **Trace the edge points along the contour**
    - **The direction of connected neighbors (clock-wise or counter-clockwise)**
    - **Image-oriented bounding box**
    - **Object-oriented bounding box**
      - **Height/width/area/ratio/min v.s. max radius/radius angle/radius ratio**



Shape Analysis & Document Processing

- **Other attributes and properties**



  - **Fourier Descriptors**
    - Polar coordinates: z(s)=x(s)+iy(s)
    - Total length = L → x(s+L)=x(s); y(s+L)=y(s)
    - Time, s → parameter of a parameterized curve
    - Fourier series expansion
    - Apply Fourier analysis to x(s) and y(s)

  - **Wavelet Descriptors**
    - Total length = L → x(s+L)=x(s); y(s+L)=y(s)
    - Time, s → parameter of a parameterized curve
    - Apply wavelet transform to x(s) and y(s)

Shape Analysis & Document Processing

- **Attributes/Features**

// Training data //

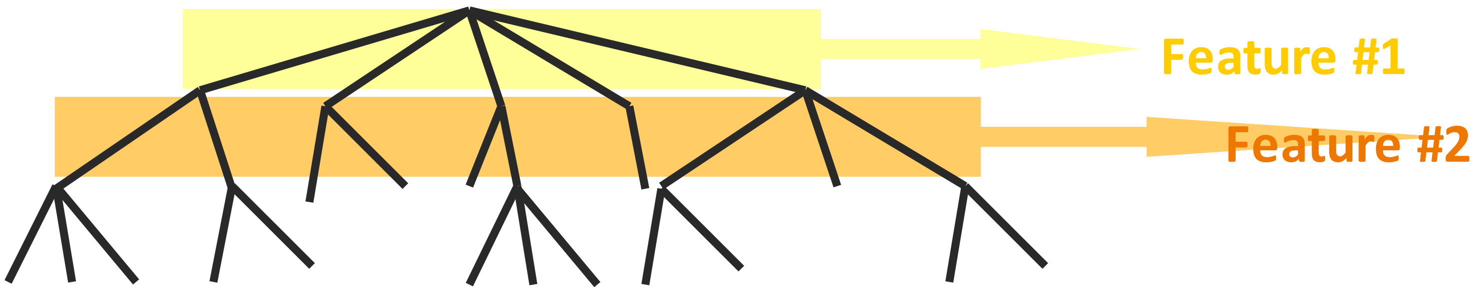
symbol	index	E	C	L	…	…	…
A	1						
B	2						
…	…						

// Test data //

input	E	C	L	…	…	…
…						

Shape Analysis & Document Processing

- Identify a set of features
  - Parallel classification
    - Form a feature vector
    - Consider them simultaneously
  - Sequential classification
    - Apply one feature at a time



    - A leaf represents one object