



Digital Image Processing

Lecture #6

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Announcement

■ Class Information

○ The following schedule

03/23	Lecture 5	05/11	proposal
03/30	Lecture 6	05/18	Lecture 9
04/06	Lecture 7	05/25	Lecture 10
04/13	RealSense	06/01	Lecture 11
04/20	midterm	06/08	Demo
04/27	RealSense	06/15	Demo
05/04	Lecture 8	06/22	Final Package Due

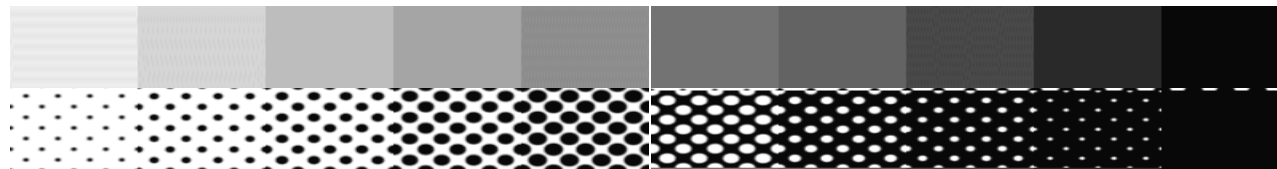


Digital Halftoning

Digital Halftoning

■ Goal

- Render the illusion of a continuous-tone image based on two-tone (half-tone) display



○ Applications

■ Computer hardcopies

- Laser printers/dot-matrix printers/color printers
- Fax machine

○ Implementation

■ Thresholding at $1/2$?

Digital Halftoning



Gray-level image



Half-toned images

Digital Halftoning

■ Color Printer

Continuous Image



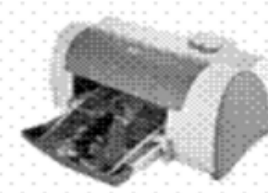
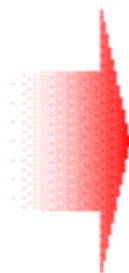
Binary Image



CMY channel



Black channel



Digital Halftoning

- Basic idea

- Spatial modulation

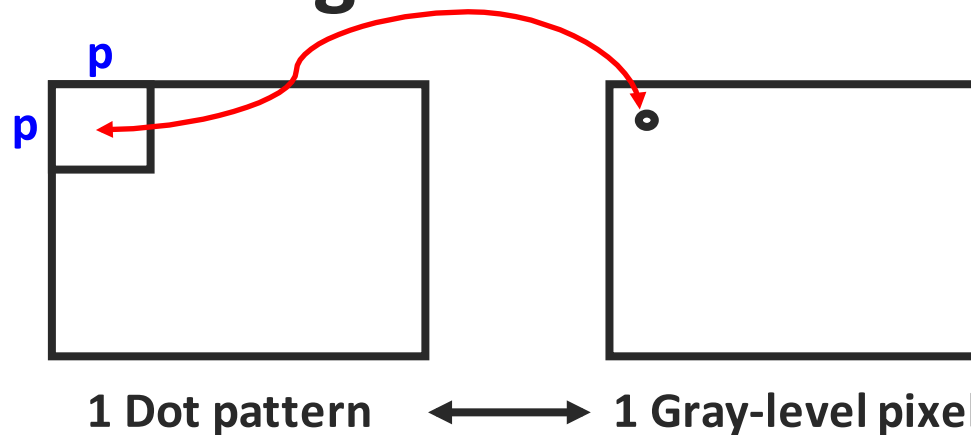
- | | | |
|---------------|---|-------------------------------|
| ■ Gray-level | ↔ | black/white |
| ■ Darker area | ↔ | denser black points per area |
| ■ Whiter area | ↔ | sparser black points per area |

- Three approaches

- Patterning
- Dithering
- Error Diffusion

Digital Halftoning

■ Patterning



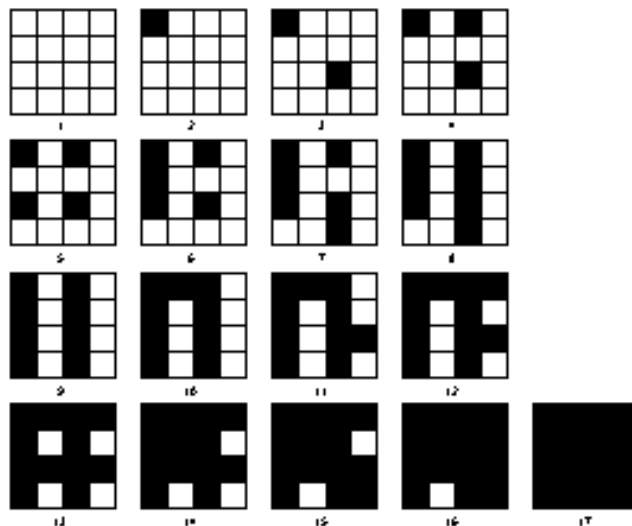
If $p=4$

→ 16 binary pixels

→ 17 levels (0~16)

→ 256 gray levels

→ Quantization



Rylander's recursive
patterning matrices

[Digital Halftoning]

■ Patterning

○ Four steps

- Read in the given grey-level image
- Quantization
- Design the patterning table
- Map each pixel to its corresponding pattern

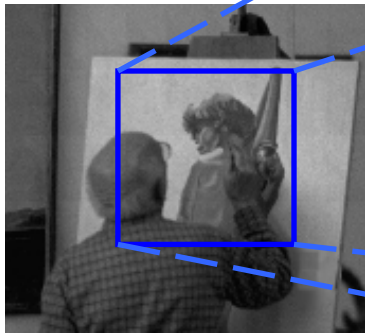
○ Simplest way

- Generates image with higher spatial resolution than the source image

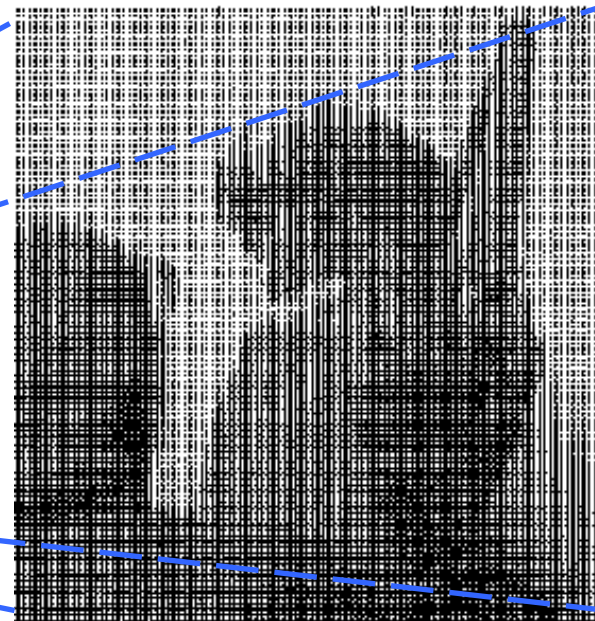
Digital Halftoning

■ Patterning

○ Example



Original gray-level image



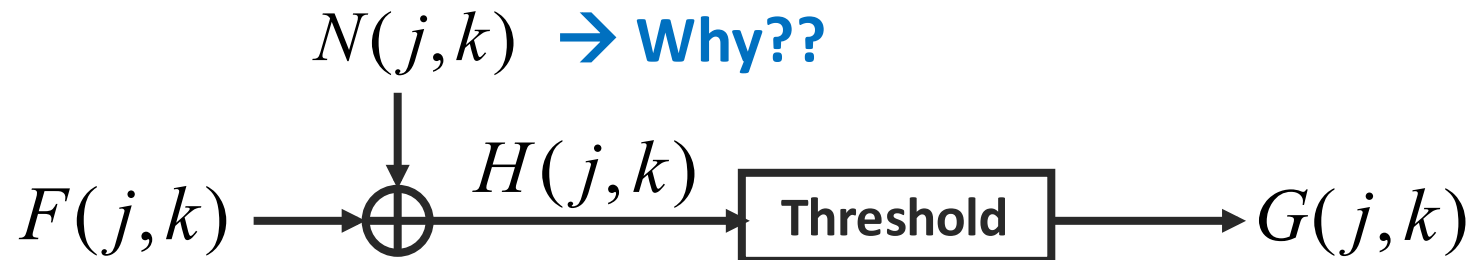
Half-toned image: patterning

Digital Halftoning

■ Dithering

- Create an image with the same number of dots as the number of pixels in the source image

- Idea



Digital Halftoning

■ Dithering

○ Why adding noise?

■ Under fixed thresholding → taking MSB

○ E.g. before and after adding noise



■ To break the monotonicity of accumulated error in the area of constant (nearly constant) gray level

■ White noise, pink noise, blue noise and green noise

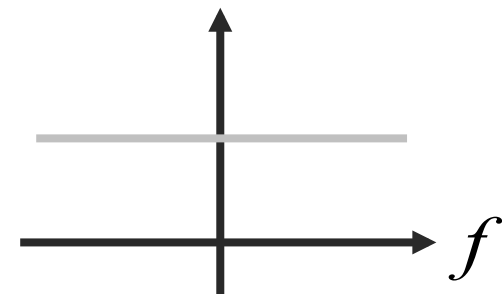
Digital Halftoning

■ Dithering

○ Noise Type

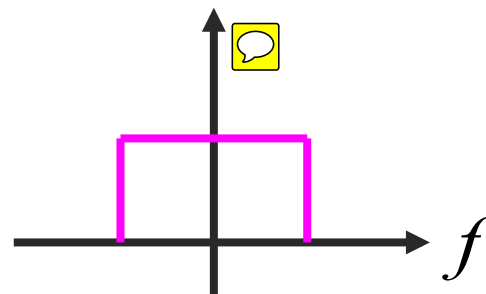
■ Power spectral density

■ White noise



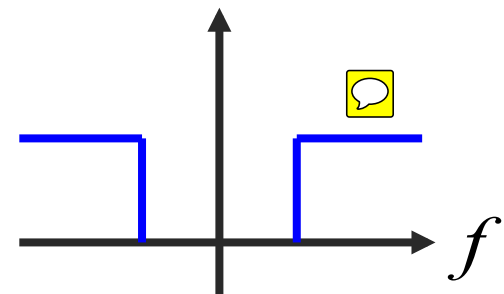
Grainy appearance

■ Pink noise



Low-frequency noise

■ Blue noise



High-frequency noise

○ Robert Ulichney, “Digital Halftoning”

■ <http://www.hpl.hp.com/people/u/>

Digital Halftoning

■ Dithering

○ Adaptive thresholding

- Generate a threshold matrix according to a dither matrix
- Whenever the pixel value of the image is greater than the value in the threshold matrix, the pixel is turned on

○ Notes

- No randomness 
- **Region-to-region** mapping
- Recursive definition allowed

[Digital Halftoning]

■ Dithering

○ Dither matrix

$$I_2(i, j) = \begin{bmatrix} 1 \rightarrow 2 \\ 3 \swarrow 0 \end{bmatrix}; \quad I_2(i, j) = \begin{bmatrix} 3 \nwarrow 1 \\ 0 \downarrow 2 \end{bmatrix}$$

- 0 → lowest threshold
- 3 → highest threshold

[Digital Halftoning]

■ Dithering

- The general form of the NxN dither matrix

- $2 \times 2 \rightarrow 4 \times 4 \rightarrow 8 \times 8 \rightarrow 16 \times 16 \dots$

$$I_{2n}(i, j) = \begin{bmatrix} 4I_n(i, j) + 1 & 4I_n(i, j) + 2 \\ 4I_n(i, j) + 3 & 4I_n(i, j) + 0 \end{bmatrix}$$

- Eg. What is $I_4(i, j)$ if $I_2(i, j) = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$?

Digital Halftoning

■ Dithering

- Determine the threshold matrix

$$T(i, j) = 255 \cdot \frac{I(i, j) + 0.5}{N^2}$$

■ Eg. N=4

$$I_4(i, j) = \begin{bmatrix} 5 & 9 & 6 & 10 \\ 13 & 1 & 14 & 2 \\ 7 & 11 & 4 & 8 \\ 15 & 3 & 12 & 0 \end{bmatrix}, \quad T_4(i, j) = ?$$

Digital Halftoning

■ Dithering

Input image

12	51	34	121
78	254	10	97
45	113	110	16
90	200	206	34

Repeated threshold matrix

0	60	0	60
45	110	45	110
0	60	0	60
45	110	45	110

Output image

Another repeated threshold matrix

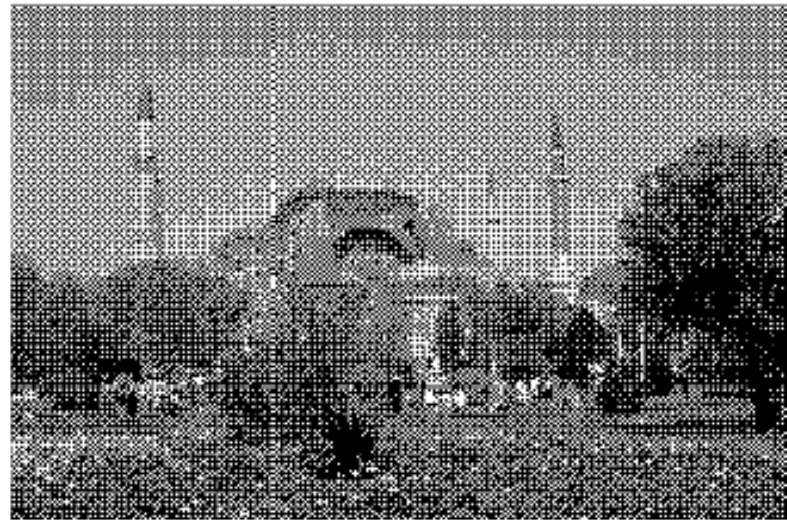
128	128	128	128
128	128	128	128
128	128	128	128
128	128	128	128

[Digital Halftoning]

■ Experimental results



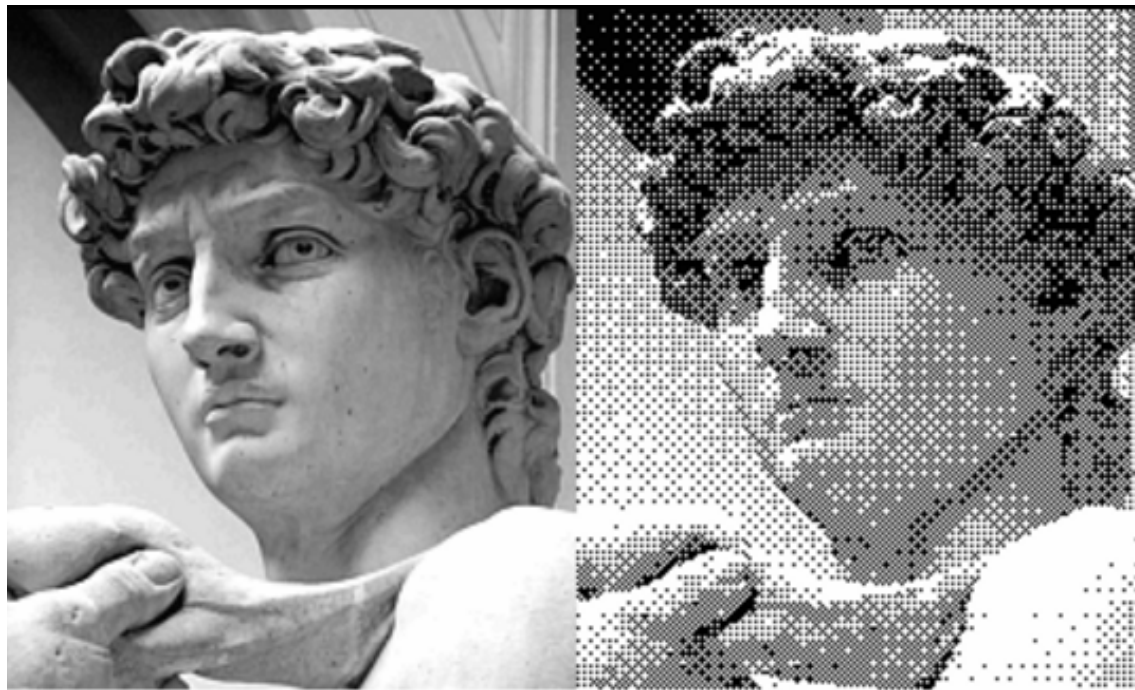
Original Image



Dithering

[Digital Halftoning]

■ Experimental results



Original Image

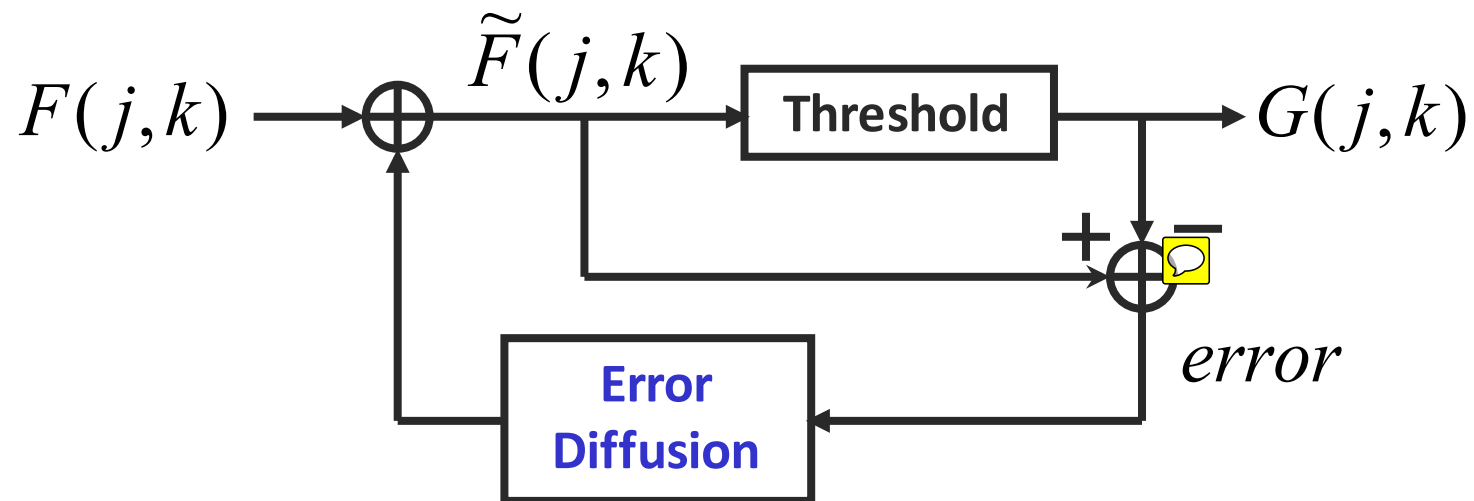
Dithering

Digital Halftoning

■ Error diffusion

○ 1975 Floyd & Steinberg

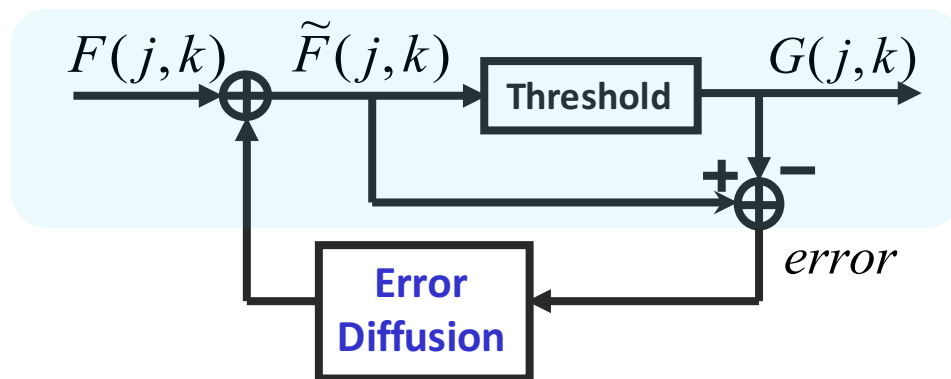
- A practical algorithm to implement **blue noise** dithering
- Framework



[Digital Halftoning]

■ Error diffusion

- Normalize $F(j,k)$ to lie between $[0,1]$
- Set threshold=0.5
- Output image: 0 or 1



if $\tilde{F}(j,k) \geq 0.5 \rightarrow G(j,k) = 1$
if $\tilde{F}(j,k) < 0.5 \rightarrow G(j,k) = 0$
Define $E(j,k) = \tilde{F}(j,k) - G(j,k)$

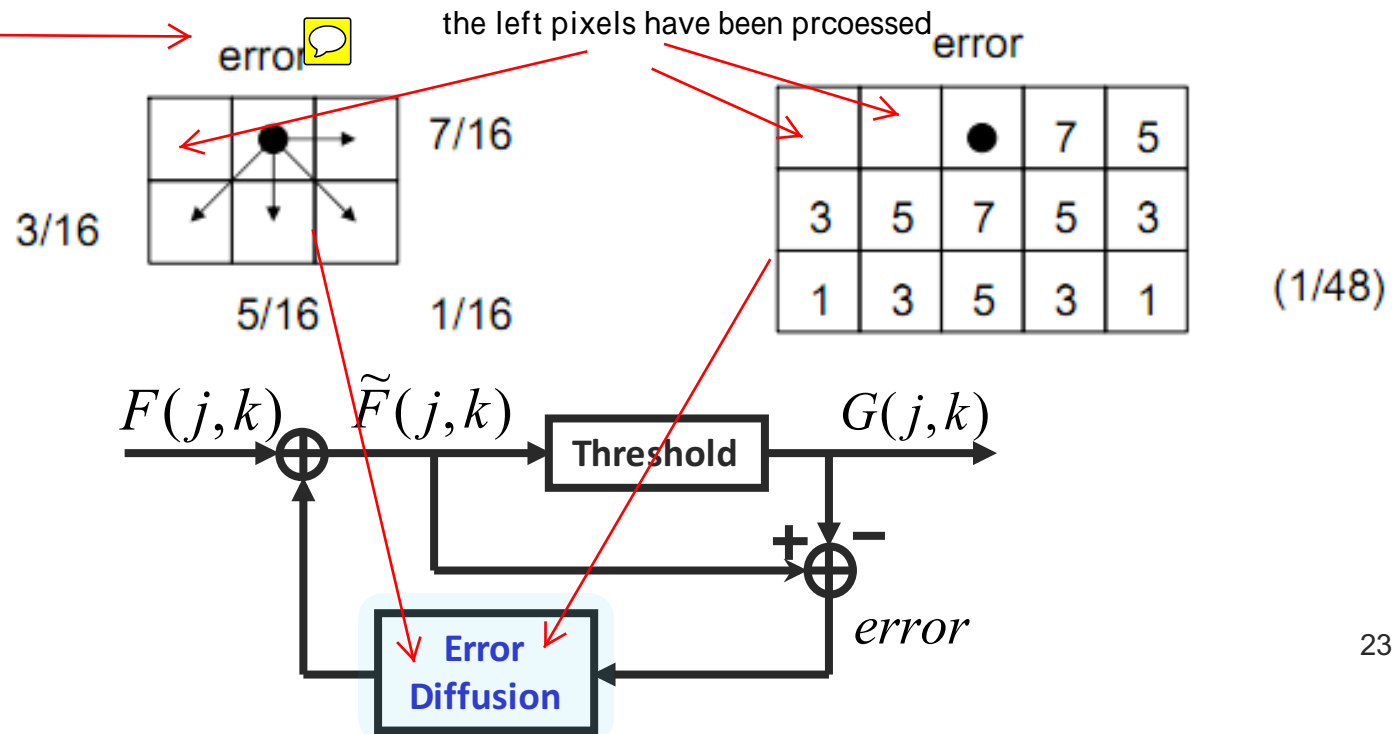
Digital Halftoning

■ Error diffusion

○ Error diffusion filter masks

■ 1975 Floyd Steinberg:

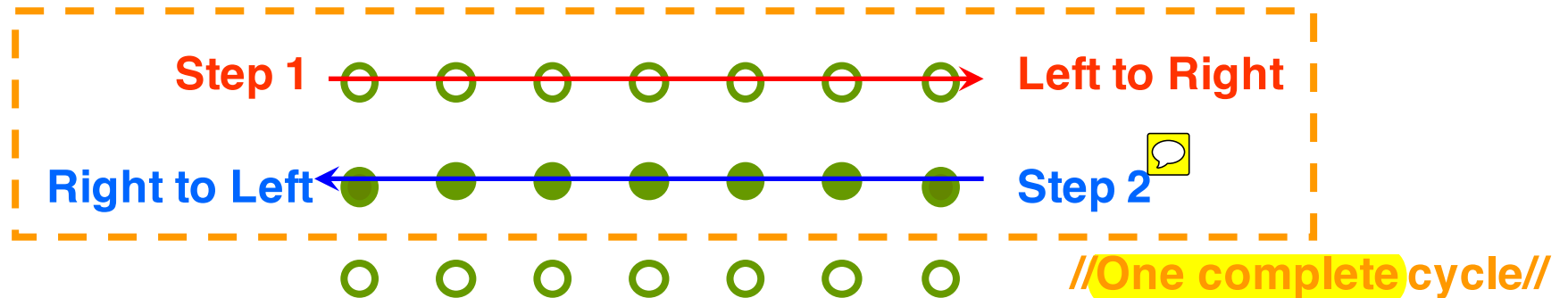
1976 Jarvis et al:



Digital Halftoning

■ Error diffusion

○ Error diffusion + serpentine scanning



$$\frac{1}{16} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 7 \\ 3 & 5 & 1 \end{pmatrix}$$

Left to Right

$$\frac{1}{16} \begin{pmatrix} 0 & 0 & 0 \\ 7 & 0 & 0 \\ 1 & 5 & 3 \end{pmatrix}$$

Right to Left

[Digital Halftoning]

■ Experimental results



Original Image

Error Diffusion

[Digital Halftoning]

■ Experimental results



Original Image

Floyd-Steinberg

Jarvis

Digital Halftoning


■ Multi-scale Error diffusion

○ Several issues

■ Region-to-region mapping

- Multi-resolution 

■ Time series/causal error diffusion process

- Easy to implement
- Causality  appears to be artificial in images
- Is non-causal error diffusion possible?

■ Quality metrics of half-toned images

Digital Halftoning

■ Multi-scale Error diffusion

“A multiscale error diffusion technique for digital halftoning”

Ioannis Katsavounidis and C. –C. Jay Kuo

○ Problem set-up

■ Input image $\rightarrow X(i, j) \in [0, 1]$

■ Output image $\rightarrow B(i, j) \in \{0, 1\}$

■ Error image $\rightarrow E(i, j) = X(i, j) - B(i, j)$

■ Intermediate stage \rightarrow

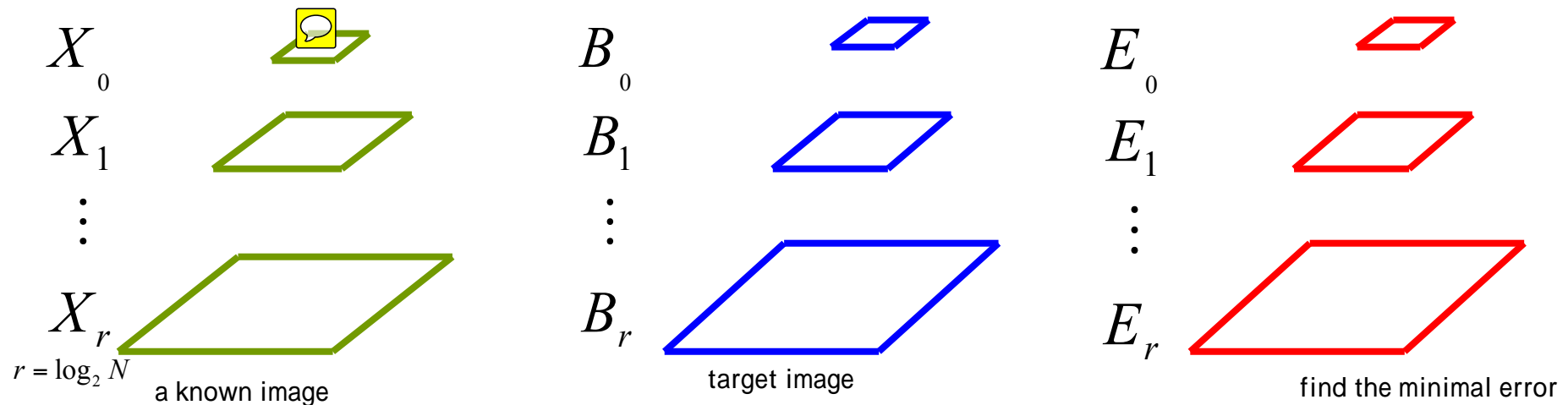
$$X_k(i_k, j_k), \quad 0 \leq k \leq r, \quad r = \log_2 N$$

$$X_k(i_k, j_k) = \sum_{i=0}^1 \sum_{j=0}^1 X_{k+1}(2i_k + i, 2j_k + j)$$

Digital Halftoning

Multi-scale Error diffusion

input $X(i, j) \in [0, 1]$ **output** $B(i, j) \in \{0, 1\}$ **error** $E(i, j) = X(i, j) - B(i, j)$



$$X_k(i_k, j_k) = \sum_{i=0}^1 \sum_{j=0}^1 X_{k+1}(2i_k + i, 2j_k + j), \quad 0 \leq k \leq r$$

$$E_k(i_k, j_k) = X_k(i_k, j_k) - B_k(i_k, j_k), \quad 0 \leq k \leq r$$

Goal: minimize the error pyramid in a certain way!

[Digital Halftoning]

■ Multi-scale Error diffusion

- multi-scale ↓
- //Step 1// Initialization
 - Set the entire output image pyramid to “0”
 - //Step 2// Dot assignment
 - Find the largest error from top to bottom level
 - 1 parent node distributes its dots (integer numbers) to 4 children
 - //Step 3// Error diffusion proces
 - - $\frac{1}{12} \begin{pmatrix} 1 & 2 & 1 \\ 2 & -12 & 2 \\ 1 & 2 & 1 \end{pmatrix}$
center
 - $\frac{1}{8} \begin{pmatrix} 0 & 0 & 0 \\ 2 & -8 & 2 \\ 1 & 2 & 1 \end{pmatrix}$
side
 - $\frac{1}{5} \begin{pmatrix} 0 & 0 & 0 \\ 0 & -5 & 2 \\ 0 & 2 & 1 \end{pmatrix}$
corner
- error diffuse ↓

Digital Halftoning

- Multi-scale Error diffusion

- Quality management

- MSE vector

$$MSEV = \begin{pmatrix} MSE_0 \\ MSE_1 \\ \vdots \\ MSE_r \end{pmatrix} \quad MSE_k = \frac{1}{N^2} \sum_{i=0}^{2^k-1} \sum_{j=0}^{2^k-1} E_k^2(i, j)$$

- Notes

- Preserve **contrast of** the original image
 - Does **not over-smooth** the image 

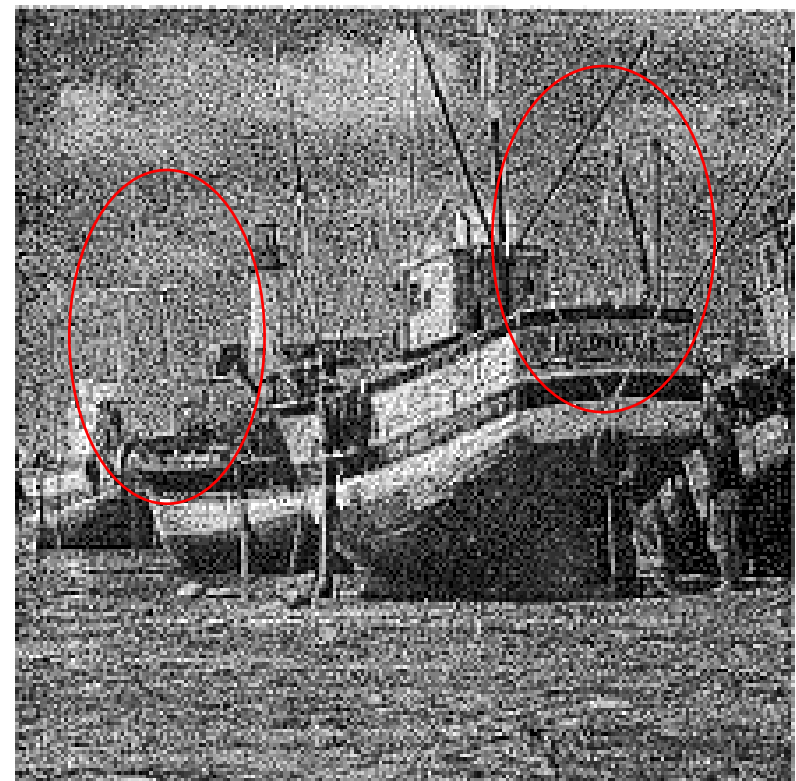
[Digital Halftoning]

■ Experimental results

better detail and contrast



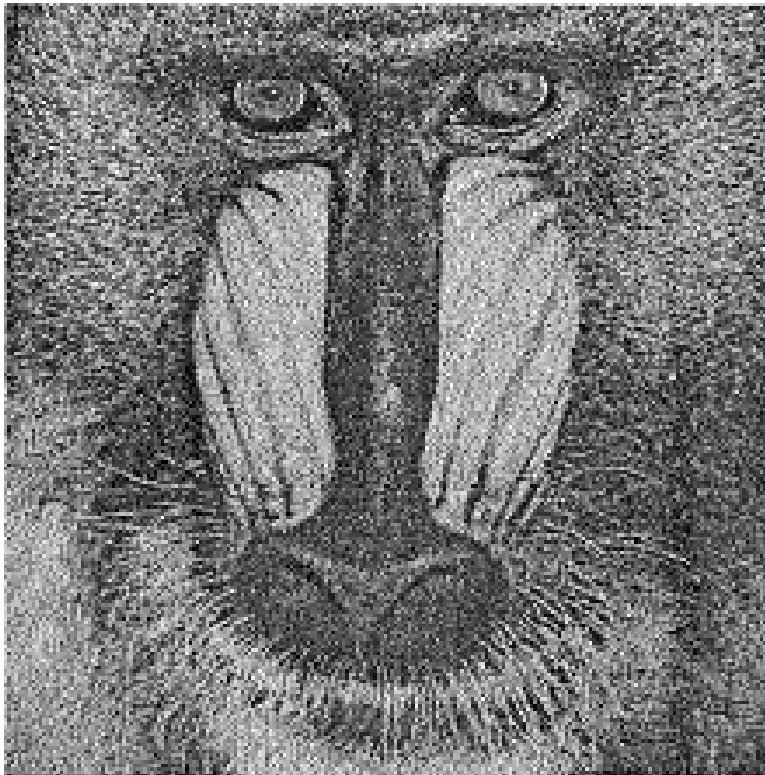
Error Diffusion



Multi-Scale Error Diffusion

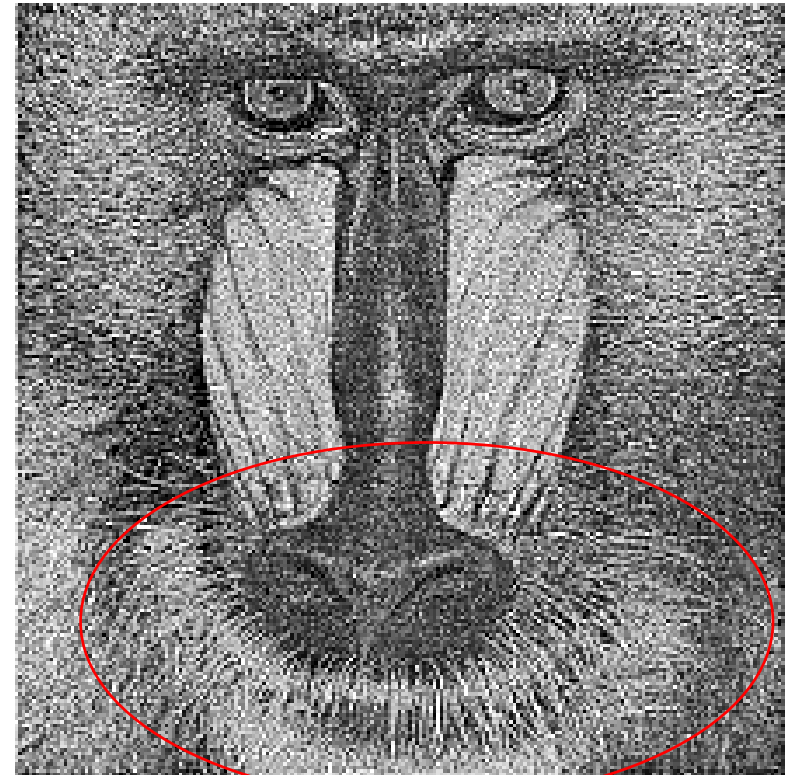
[Digital Halftoning]

■ Experimental results



Error Diffusion

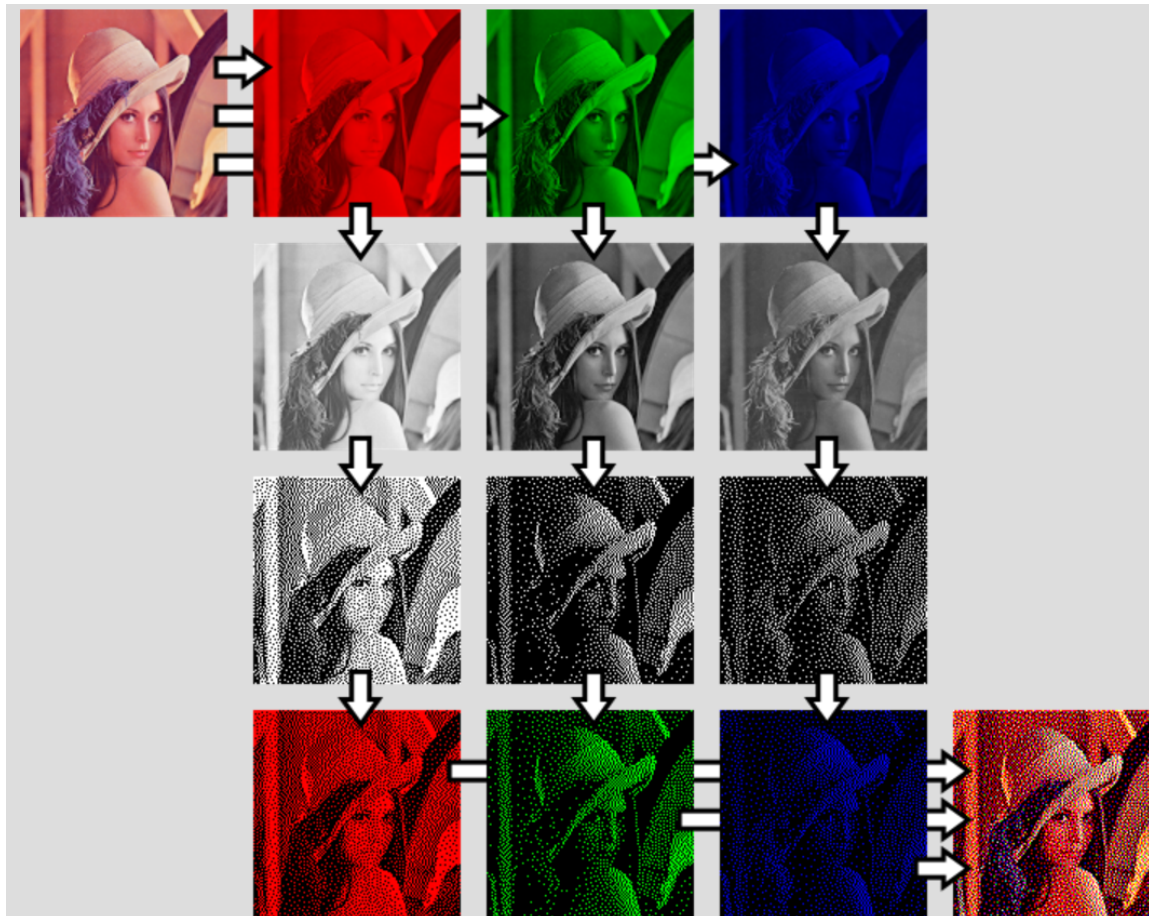
worse contrast and detail



Multi-Scale Error Diffusion

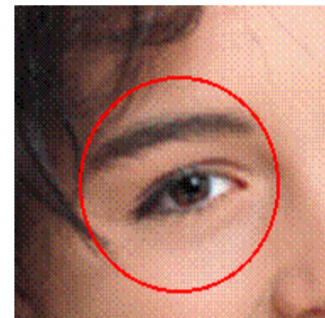
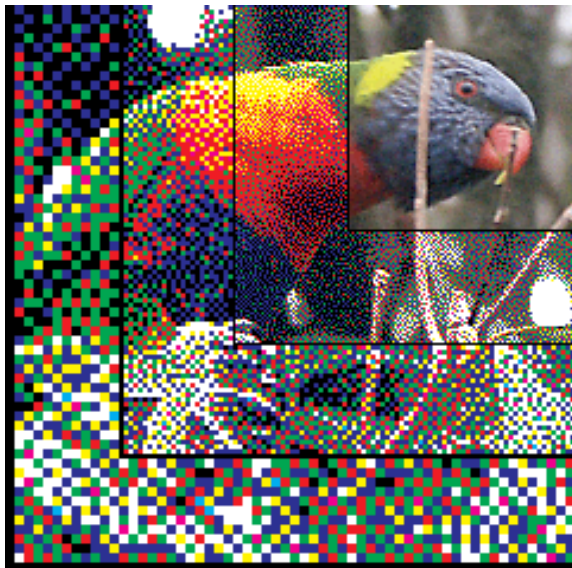
Digital Halftoning

- **Color image** R,G,B is half-tone individually and then combined again.

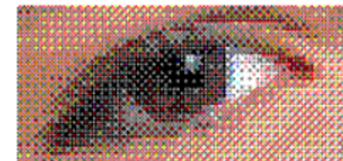


[Digital Halftoning]

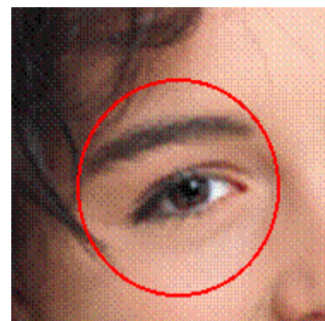
■ Examples



worse



Dithering



better



Error Diffusion

[Digital Halftoning]

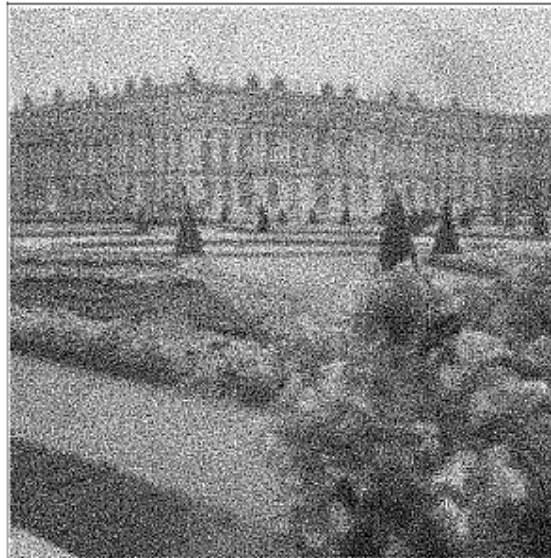
■ Application

○ Visual cryptography

“visual cryptography based on void-and-cluster halftoning technique” E. Myodo, S. Sakazawa and Y. Takishima



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