



# **Digital Image Processing**

**Lecture #6**

**Ming-Sui (Amy) Lee**

# Announcement

## ■ Class Information

### ○ The following schedule

|       |           |       |                   |
|-------|-----------|-------|-------------------|
| 03/23 | Lecture 5 | 05/11 | <b>proposal</b>   |
| 03/30 | Lecture 6 | 05/18 | Lecture 9         |
| 04/06 | Lecture 7 | 05/25 | Lecture 10        |
| 04/13 | RealSense | 06/01 | Lecture 11        |
| 04/20 | midterm   | 06/08 | <b>Demo</b>       |
| 04/27 | RealSense | 06/15 | <b>Demo</b>       |
| 05/04 | Lecture 8 | 06/22 | Final Package Due |

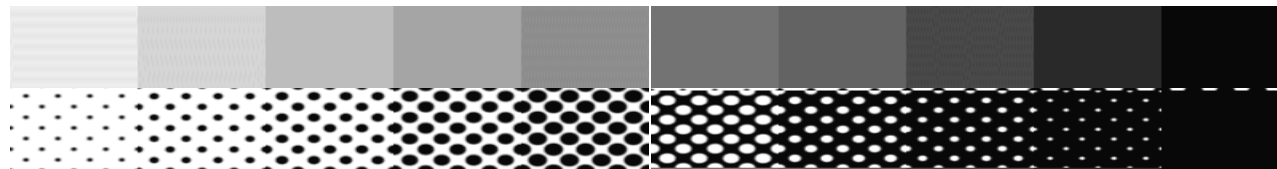


# Digital Halftoning

# Digital Halftoning

## ■ Goal

- Render the illusion of a continuous-tone image based on two-tone (half-tone) display



## ○ Applications

### ■ Computer hardcopies

- Laser printers/dot-matrix printers/color printers
- Fax machine

## ○ Implementation

### ■ Thresholding at $1/2$ ?

# Digital Halftoning



Gray-level image



Half-toned images



# Digital Halftoning

## ■ Color Printer

Continuous Image



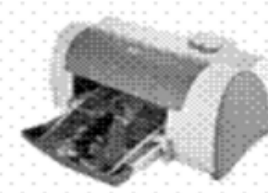
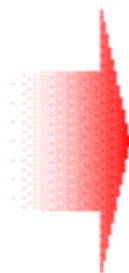
Binary Image



CMY channel



Black channel



# Digital Halftoning

- Basic idea

- Spatial modulation

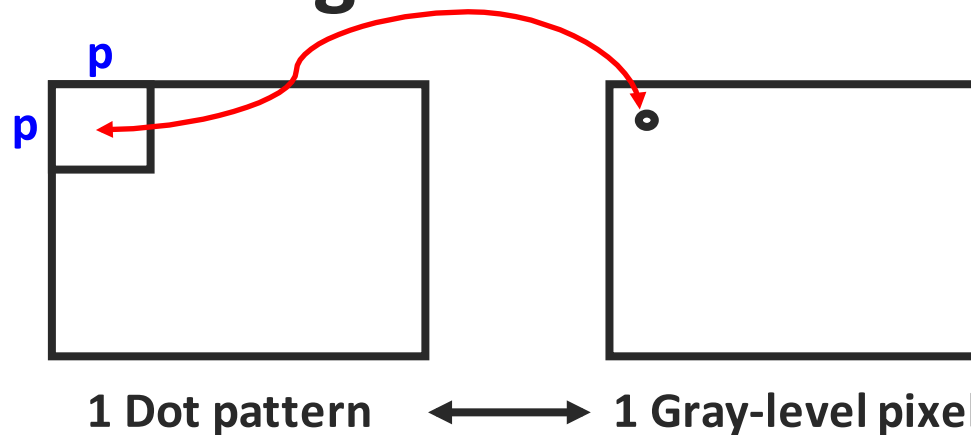
- Gray-level ↔ black/white
- Darker area ↔ denser black points per area
- Whiter area ↔ sparser black points per area

- Three approaches

- Patterning
- Dithering
- Error Diffusion

# Digital Halftoning

## ■ Patterning



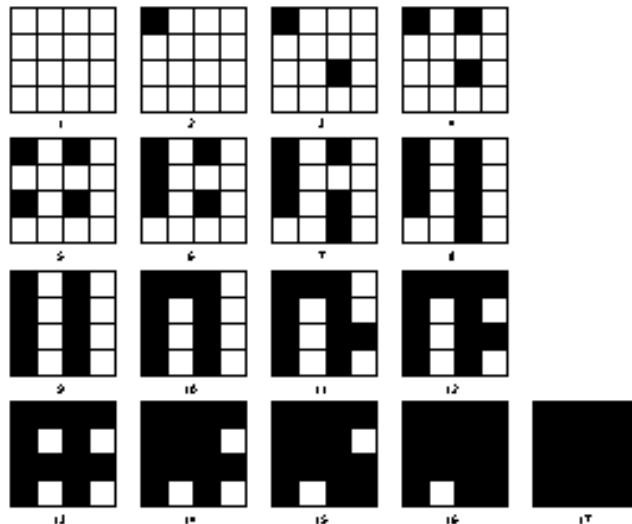
If  $p=4$

→ 16 binary pixels

→ 17 levels (0~16)

→ 256 gray levels

→ Quantization



Rylander's recursive  
patterning matrices



# Digital Halftoning

## ■ Patterning

### ○ Four steps

- Read in the given grey-level image
- Quantization
- Design the patterning table
- Map each pixel to its corresponding pattern

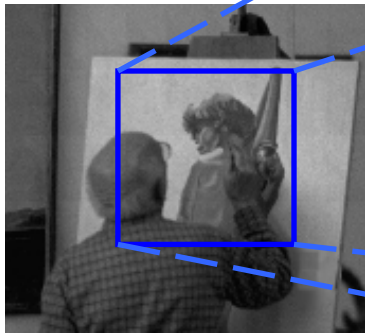
### ○ Simplest way

- Generates image with higher spatial resolution than the source image

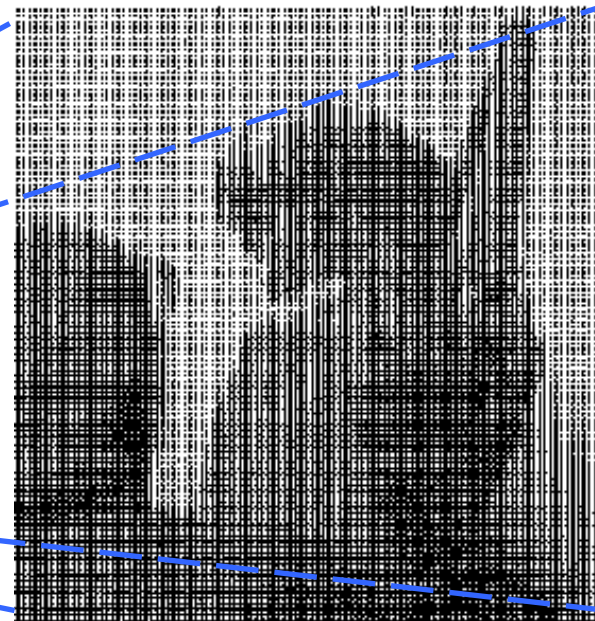
# Digital Halftoning

## ■ Patterning

### ○ Example



Original gray-level image



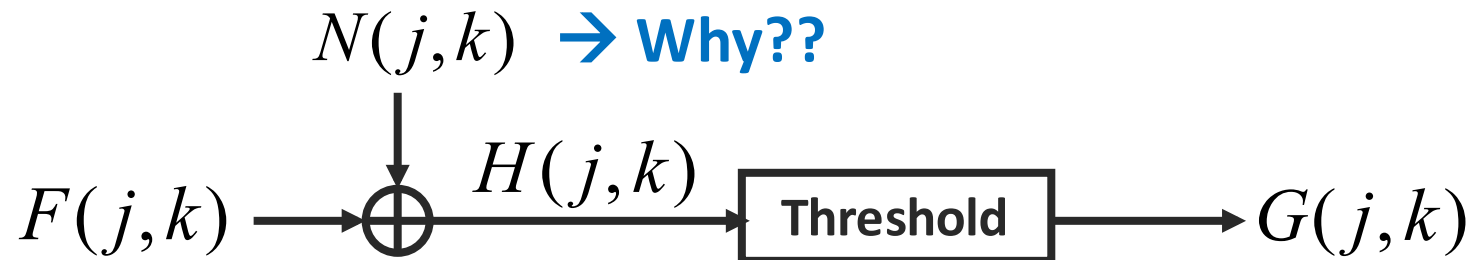
Half-toned image: patterning

# Digital Halftoning

## ■ Dithering

- Create an image with the same number of dots as the number of pixels in the source image

- Idea



# Digital Halftoning

## ■ Dithering

### ○ Why adding noise?

- Under fixed thresholding → taking MSB

- E.g. before and after adding noise

$\geq 128$

$< 127$

- To break the monotonicity of accumulated error in the area of constant (nearly constant) gray level
- White noise, pink noise, blue noise and green noise

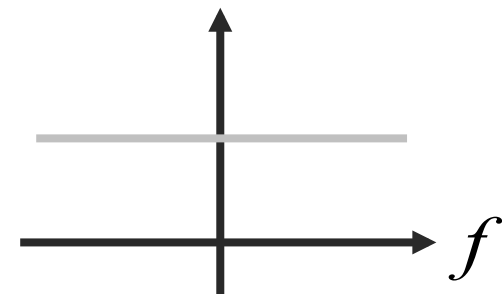
# Digital Halftoning

## ■ Dithering

### ○ Noise Type

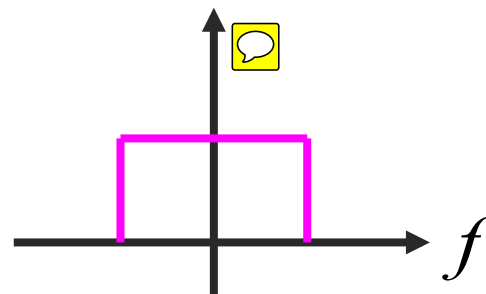
#### ■ Power spectral density

#### ■ White noise



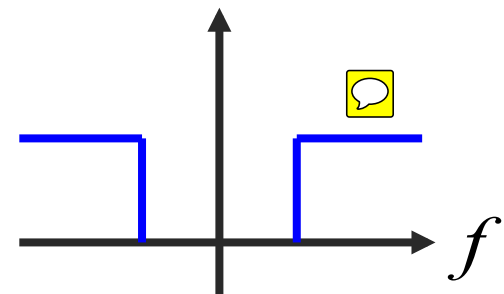
Grainy appearance

#### ■ Pink noise



Low-frequency noise

#### ■ Blue noise



High-frequency noise

### ○ Robert Ulichney, “Digital Halftoning”

■ <http://www.hpl.hp.com/people/u/>

# [ Digital Halftoning ]

## ■ Dithering

### ○ Adaptive thresholding

- Generate a **threshold matrix** according to a dither matrix
- Whenever the pixel value of the image is **greater** than the value in **the threshold matrix**, the pixel is turned on

### ○ Notes

- No randomness 
- **Region-to-region** mapping
- Recursive definition allowed



# [ Digital Halftoning ]

## ■ Dithering

### ○ Dither matrix

$$I_2(i, j) = \begin{bmatrix} 1 \rightarrow 2 \\ 3 \swarrow 0 \end{bmatrix}; \quad I_2(i, j) = \begin{bmatrix} 3 \nwarrow 1 \\ 0 \downarrow 2 \end{bmatrix}$$

- 0 → lowest threshold
- 3 → highest threshold

# Digital Halftoning

## ■ Dithering

- The general form of the NxN dither matrix

- $2 \times 2 \rightarrow 4 \times 4 \rightarrow 8 \times 8 \rightarrow 16 \times 16 \dots$

$$I_{2n}(i, j) = \begin{bmatrix} 4I_n(i, j) + 1 & 4I_n(i, j) + 2 \\ 4I_n(i, j) + 3 & 4I_n(i, j) + 0 \end{bmatrix}$$

- Eg. What is  $I_4(i, j)$  if  $I_2(i, j) = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$ ?

# Digital Halftoning

## ■ Dithering

- Determine the threshold matrix

$$T(i, j) = 255 \cdot \frac{I(i, j) + 0.5}{N^2}$$

## ■ Eg. N=4

$$I_4(i, j) = \begin{bmatrix} 5 & 9 & 6 & 10 \\ 13 & 1 & 14 & 2 \\ 7 & 11 & 4 & 8 \\ 15 & 3 & 12 & 0 \end{bmatrix}, \quad T_4(i, j) = ?$$

# [ Digital Halftoning ]

## ■ Dithering

Input image

|    |     |     |     |
|----|-----|-----|-----|
| 12 | 51  | 34  | 121 |
| 78 | 254 | 10  | 97  |
| 45 | 113 | 110 | 16  |
| 90 | 200 | 206 | 34  |

Repeated threshold matrix

|    |     |    |     |
|----|-----|----|-----|
| 0  | 60  | 0  | 60  |
| 45 | 110 | 45 | 110 |
| 0  | 60  | 0  | 60  |
| 45 | 110 | 45 | 110 |

Output image

|  |  |  |  |
|--|--|--|--|
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

Another repeated threshold matrix

|     |     |     |     |
|-----|-----|-----|-----|
| 128 | 128 | 128 | 128 |
| 128 | 128 | 128 | 128 |
| 128 | 128 | 128 | 128 |
| 128 | 128 | 128 | 128 |

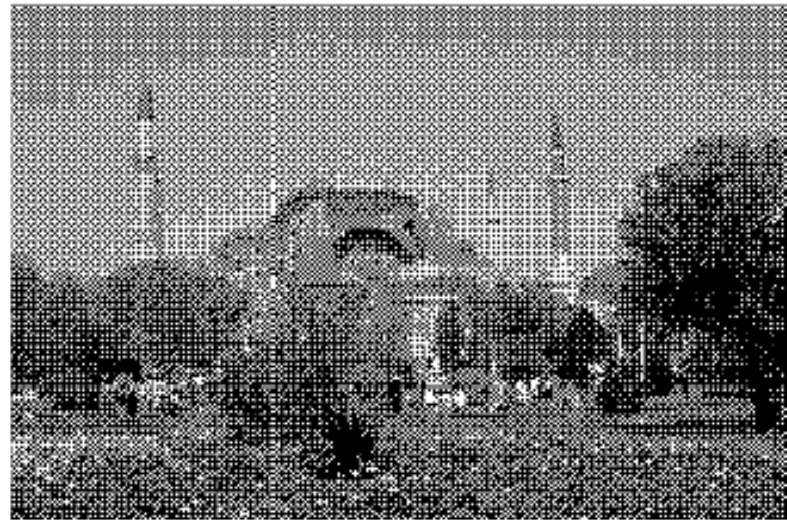
|  |  |  |  |
|--|--|--|--|
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

# [ Digital Halftoning ]

## ■ Experimental results



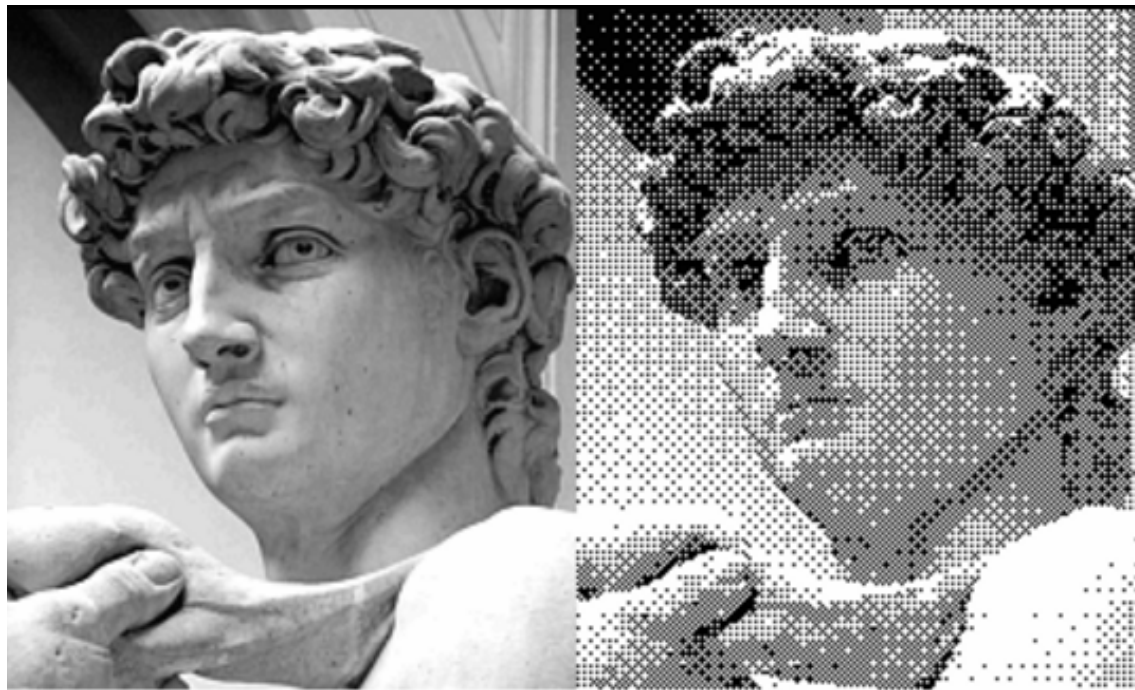
Original Image



Dithering

# [ Digital Halftoning ]

## ■ Experimental results



Original Image

Dithering

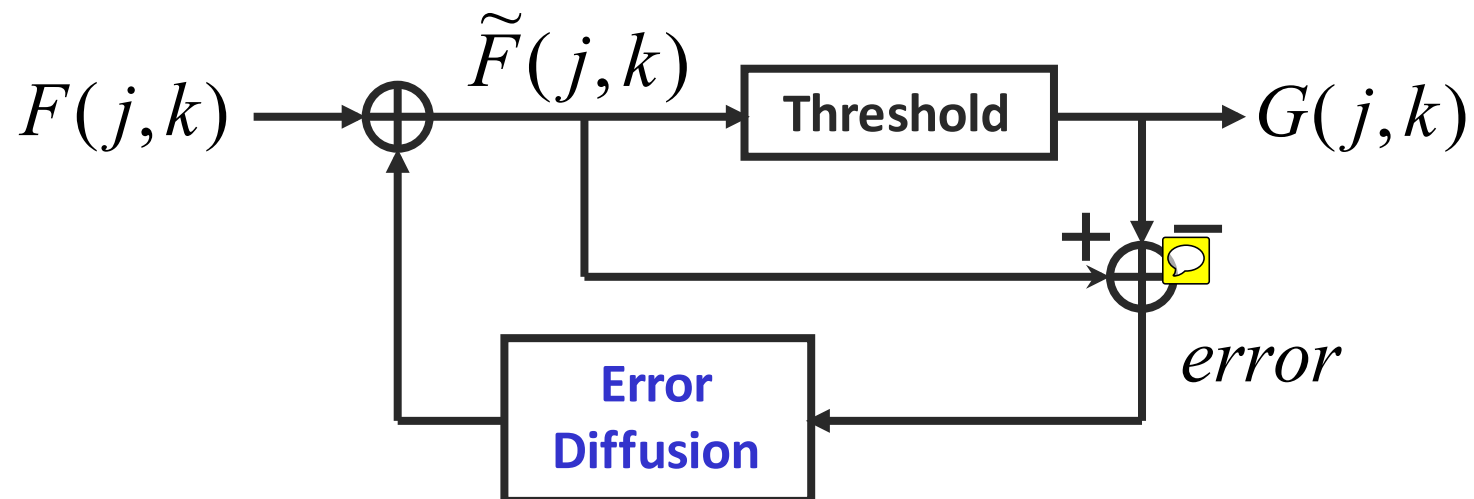


# Digital Halftoning

## Error diffusion

### 1975 Floyd & Steinberg

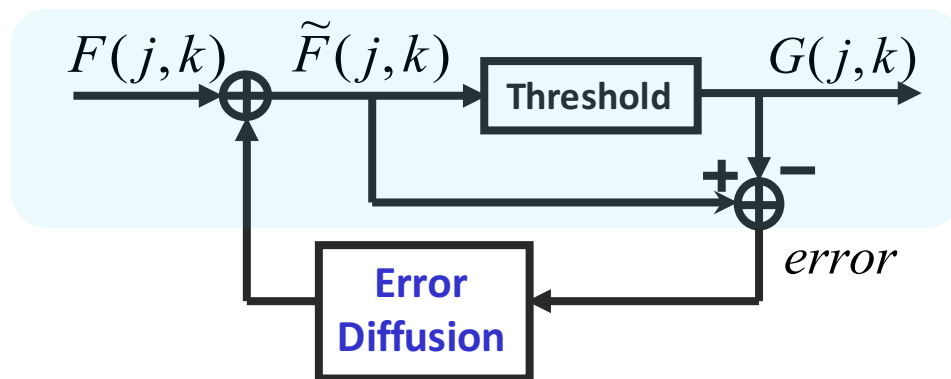
- A practical algorithm to implement blue noise dithering
- Framework



# Digital Halftoning

## ■ Error diffusion

- Normalize  $F(j,k)$  to lie between  $[0,1]$
- Set threshold=0.5
- Output image: 0 or 1



if  $\tilde{F}(j,k) \geq 0.5 \rightarrow G(j,k) = 1$   
if  $\tilde{F}(j,k) < 0.5 \rightarrow G(j,k) = 0$   
Define  $E(j,k) = \tilde{F}(j,k) - G(j,k)$

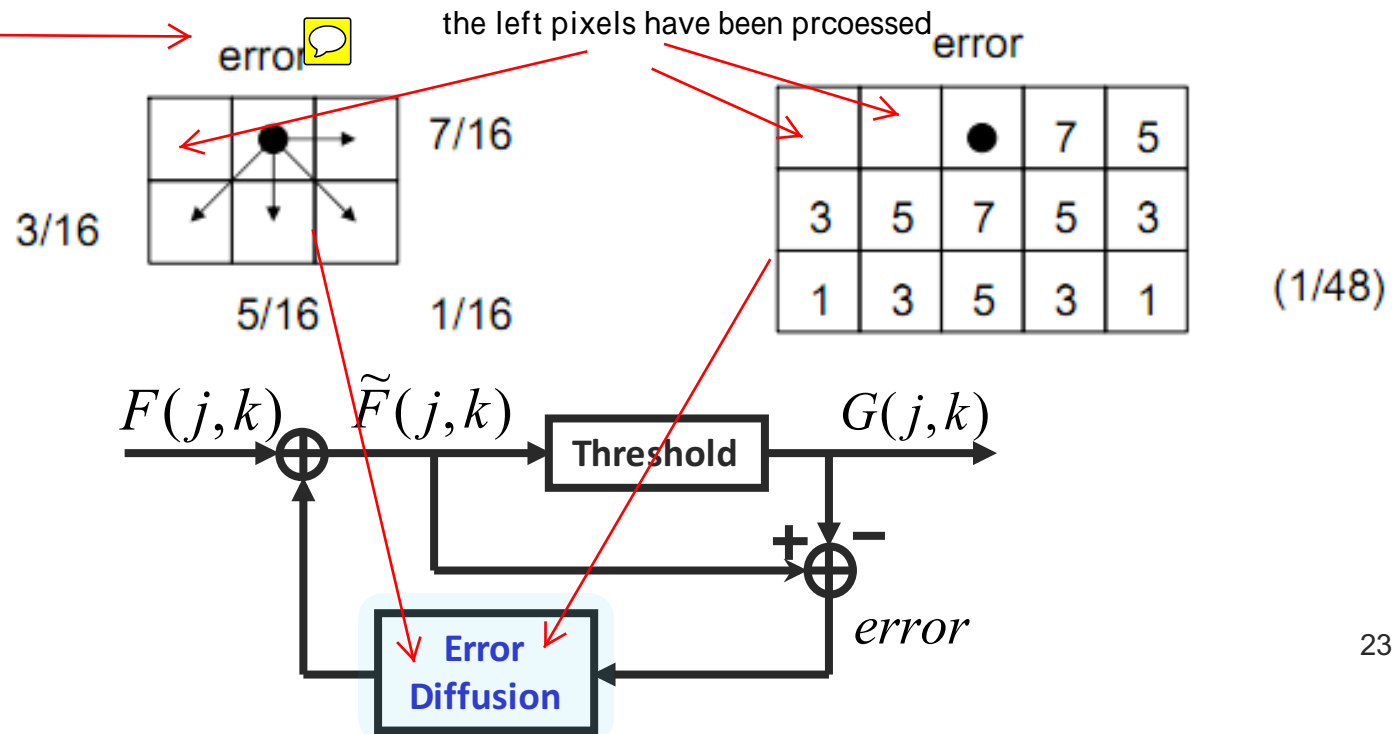
# Digital Halftoning

## ■ Error diffusion

### ○ Error diffusion filter masks

#### ■ 1975 Floyd Steinberg:

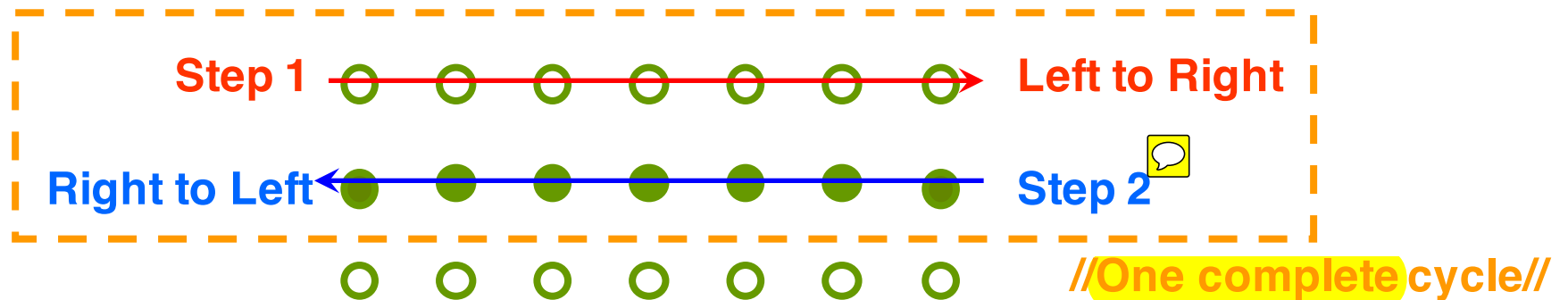
#### 1976 Jarvis et al:



# [ Digital Halftoning ]

## ■ Error diffusion

### ○ Error diffusion + serpentine scanning



$$\frac{1}{16} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 7 \\ 3 & 5 & 1 \end{pmatrix}$$

Left to Right

$$\frac{1}{16} \begin{pmatrix} 0 & 0 & 0 \\ 7 & 0 & 0 \\ 1 & 5 & 3 \end{pmatrix}$$

Right to Left

# [ Digital Halftoning ]

## ■ Experimental results



Original Image

Error Diffusion

# [ Digital Halftoning ]

## ■ Experimental results



Original Image

Floyd-Steinberg

Jarvis



# [ Digital Halftoning ]


## ■ Multi-scale Error diffusion

### ○ Several issues

#### ■ Region-to-region mapping

- Multi-resolution 

#### ■ Time series/causal error diffusion process

- Easy to implement
- Causality  appears to be artificial in images
- Is non-causal error diffusion possible?

#### ■ Quality metrics of half-toned images

# Digital Halftoning

## ■ Multi-scale Error diffusion

“A multiscale error diffusion technique for digital halftoning”

Ioannis Katsavounidis and C. –C. Jay Kuo

### ○ Problem set-up

■ Input image  $\rightarrow X(i, j) \in [0, 1]$

■ Output image  $\rightarrow B(i, j) \in \{0, 1\}$

■ Error image  $\rightarrow E(i, j) = X(i, j) - B(i, j)$

■ Intermediate stage  $\rightarrow$

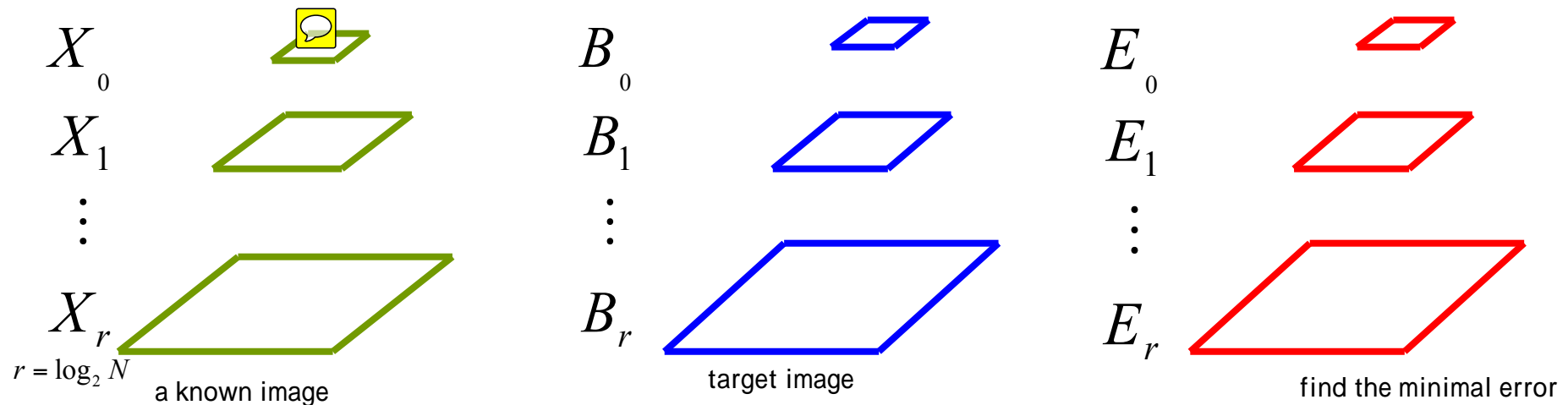
$$X_k(i_k, j_k), \quad 0 \leq k \leq r, \quad r = \log_2 N$$

$$X_k(i_k, j_k) = \sum_{i=0}^1 \sum_{j=0}^1 X_{k+1}(2i_k + i, 2j_k + j)$$

# Digital Halftoning

## Multi-scale Error diffusion

**input**  $X(i, j) \in [0, 1]$       **output**  $B(i, j) \in \{0, 1\}$       **error**  $E(i, j) = X(i, j) - B(i, j)$



$$X_k(i_k, j_k) = \sum_{i=0}^1 \sum_{j=0}^1 X_{k+1}(2i_k + i, 2j_k + j), \quad 0 \leq k \leq r$$

$$E_k(i_k, j_k) = X_k(i_k, j_k) - B_k(i_k, j_k), \quad 0 \leq k \leq r$$

**Goal: minimize the error pyramid in a certain way!**

# [ Digital Halftoning ]

## ■ Multi-scale Error diffusion

- multi-scale ↓
- //Step 1// Initialization
    - Set the entire output image pyramid to “0”
  - //Step 2// Dot assignment
    - Find the largest error from top to bottom level
    - 1 parent node distributes its dots (integer numbers) to 4 children
  - //Step 3// Error diffusion proces
    - $$\frac{1}{12} \begin{pmatrix} 1 & 2 & 1 \\ 2 & -12 & 2 \\ 1 & 2 & 1 \end{pmatrix}$$

**center**

$$\frac{1}{8} \begin{pmatrix} 0 & 0 & 0 \\ 2 & -8 & 2 \\ 1 & 2 & 1 \end{pmatrix}$$

**side**

$$\frac{1}{5} \begin{pmatrix} 0 & 0 & 0 \\ 0 & -5 & 2 \\ 0 & 2 & 1 \end{pmatrix}$$

**corner**
- error diffuse ↓

# Digital Halftoning

## ■ Multi-scale Error diffusion

### ○ Quality management

#### ■ MSE vector

$$MSEV = \begin{pmatrix} MSE_0 \\ MSE_1 \\ \vdots \\ MSE_r \end{pmatrix}$$

$$MSE_k = \frac{1}{N^2} \sum_{i=0}^{2^k-1} \sum_{j=0}^{2^k-1} E_k^2(i, j)$$

### ○ Notes

- Preserve **contrast of** the original image
- Does **not over-smooth** the image

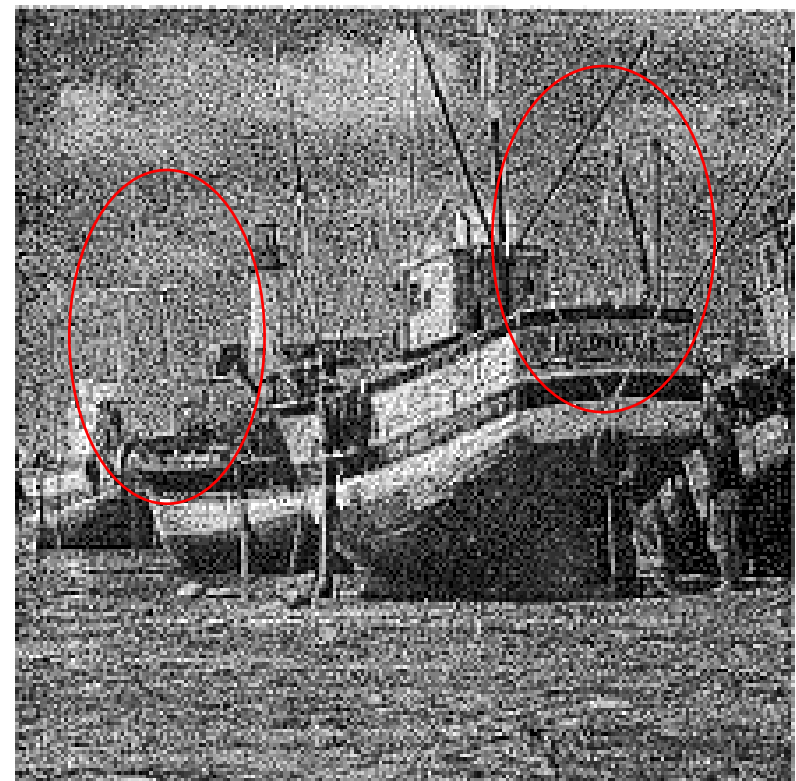
# [ Digital Halftoning ]

## ■ Experimental results

better detail and contrast



Error Diffusion

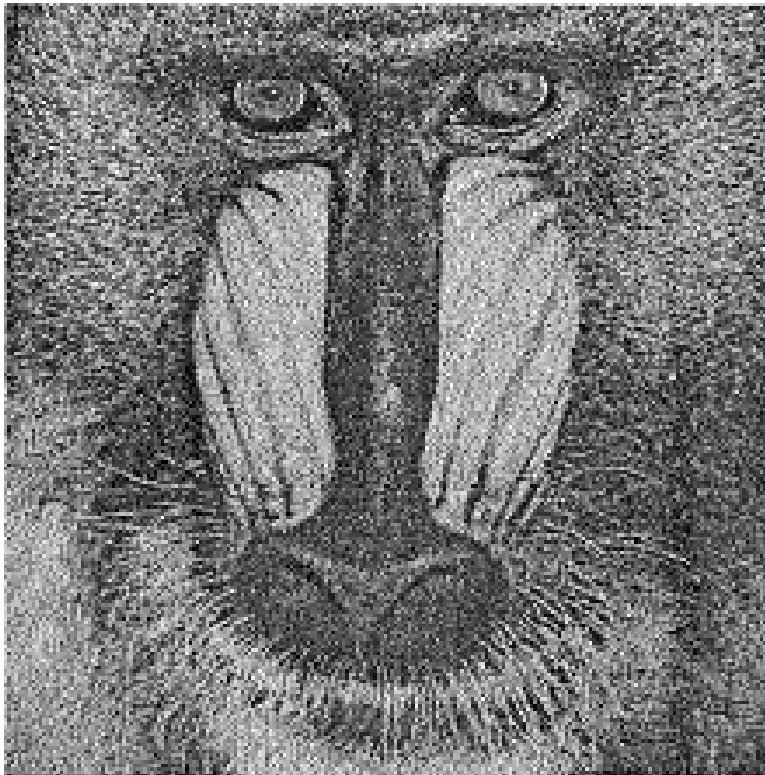


Multi-Scale Error Diffusion



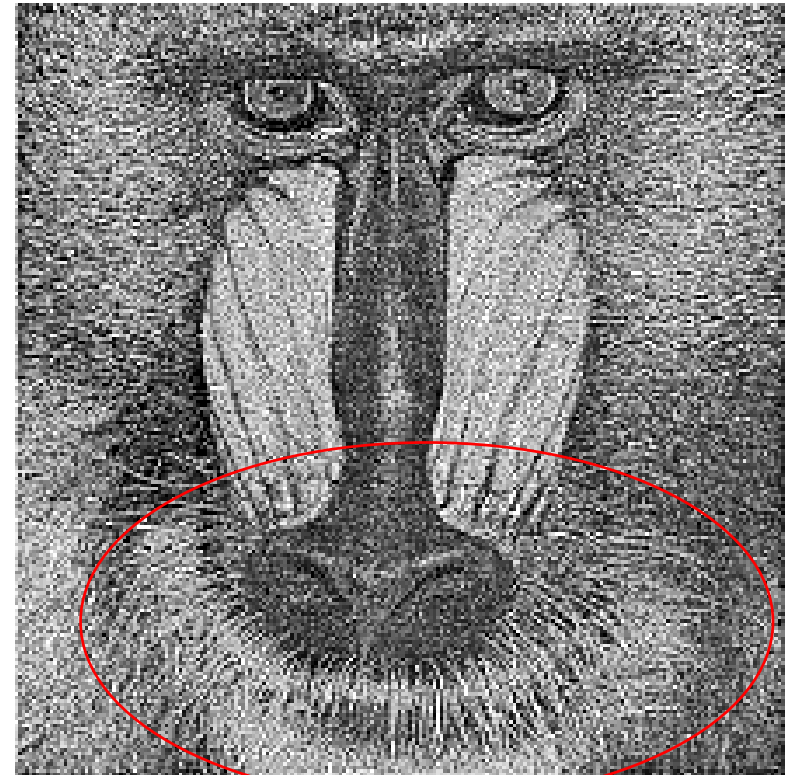
# [ Digital Halftoning ]

## ■ Experimental results



**Error Diffusion**

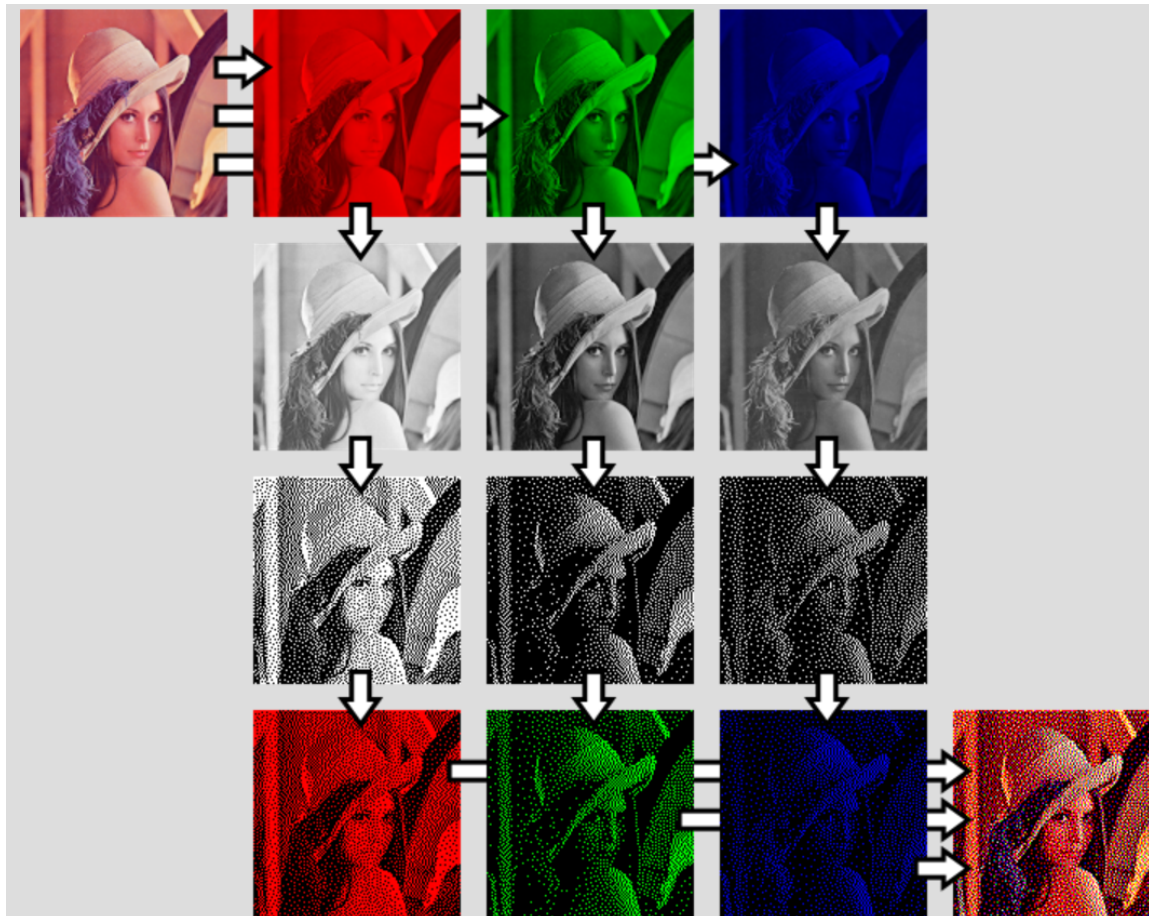
worse contrast and detail



**Multi-Scale Error Diffusion**

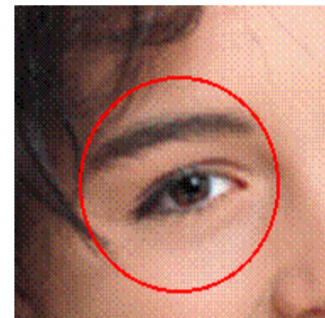
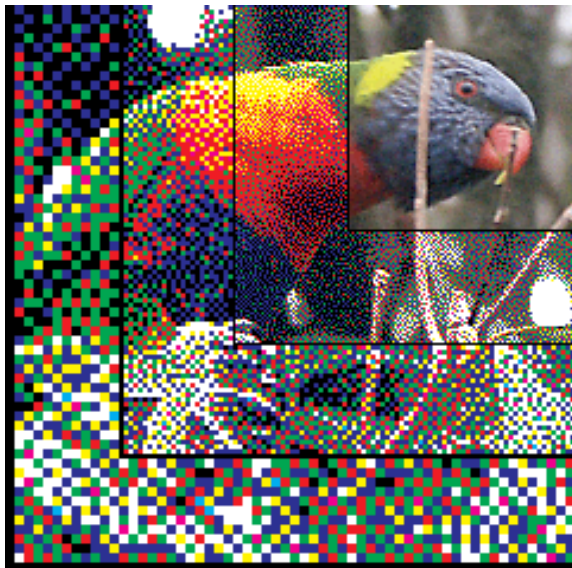
# Digital Halftoning

- **Color image** R,G,B is half-tone individually and then combined again.

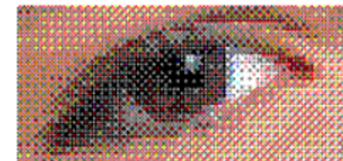


# [ Digital Halftoning ]

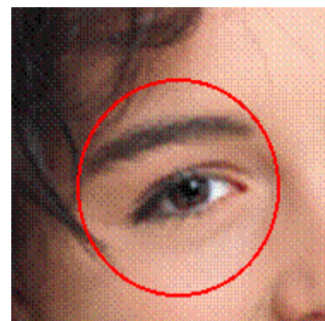
## ■ Examples



worse



**Dithering**



better



**Error Diffusion**



# [ Digital Halftoning ]

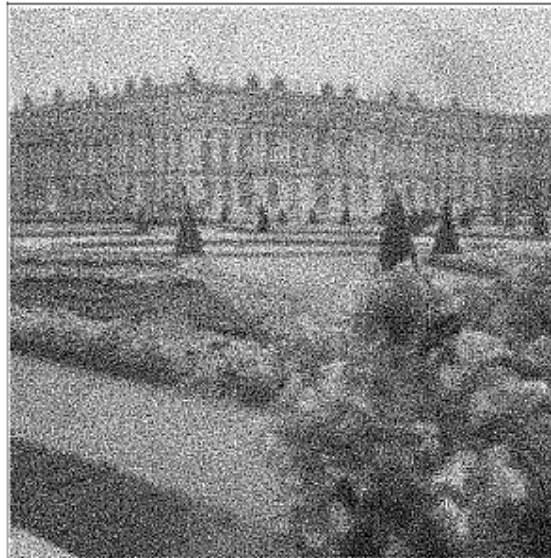
## ■ Application

### ○ Visual cryptography

“visual cryptography based on void-and-cluster halftoning technique” E. Myodo, S. Sakazawa and Y. Takishima



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