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Chair of Empirical Finance and Econometrics

**Measuring the information content of
VIX volatility in comparison to
historic volatility**

Humboldt Research Project Project

in

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Name:	Sophia Charlotte Gläser
Student number:	15202284
Programme:	Corporate Management and Economics
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Examinor:	Prof. Dr. Franziska Peter
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Glossary

BS Black-and-Scholes-Merton Model. 3, 4

Abstract

Abstract text

1 Introduction: The Importance of Volatility Measurement

1.1 Why Volatility matters: Volatility as Key Input to Option Pricing Models and Risk Measures

Distributional characteristics of asset returns are of high interest for the financial sector. They are for example key input to the pricing of financial instruments like derivatives, or to risk measures, such as the Value-at-Risk. Moreover they give information on the risk-return trade-off, which is a central question in portfolio allocation and managerial decision making. Of particular interest is the asset's return volatility, being the most dominant time-varying distribution characteristic. (Torben G Andersen et al. 2018).

Risk is important because... The concept of risk is closely linked to that of volatility, as the second moment characteristic of asset return distributions. Many pricing models measure risk through volatility, which thus influences the expected return (Harvey and Whaley 1992). Moreover many risk-measure, such as the Value-at-Risk are very closely related to volatility. (Alternative) Volatility “seeks to capture the strength of the (unexpected) return variation over a given period of time” (Torben G. Andersen, Bollerslev and Diebold 2001, p.7)

As volatility is not directly observable, it has to be estimated. During the last years, considerable research has been devoted to the question, how volatility can be measured or estimated. Two prominent categories of approaches are on the one hand time series models using historic volatility, and on the other hand implied models using option price data¹. In the recent years, Black-and-Scholes implied volatility measurement gained popularity, this approach uses the forward-looking nature of option prices. Options are contracts, giving the

¹There are, of course, various other methods, such as nonparametric methods or neural networks based models (Jiang, George J & Tian 2003), however they shall not be discussed here

holder the right to either buy (call option), or sell (put option), an underlying asset, at a specified date in the future for a certain price (John et al. 2006). Assuming rational agents/expectation, the market uses all available information to form its expectation about future price movements and thus about volatility. Assuming furthermore that the market is efficient (meaning as Eugene Fama defined it, that prices reflect all available information), the market's estimate of future volatility is the best possible forecast possible, given the current information (Bent Jesper Christensen and Hansen 2002). Due to this forward looking component of option contracts, option prices indirectly contain the market participants' expectations of the underlying asset's future movements. A widely used model to price this option contracts is the Black-and-Scholes-Merton model, which uses the option's volatility as an input factor. By using observed option prices as the input and solving for volatility, it is possible to obtain a volatility measure that is widely believed to be "informationally superior to the historic volatility of the underlying asset" (Jiang, George J & Tian 2003, p.1305).

Early studies found implied volatility to be a biased forecast of realized volatility, not containing significantly more information than historic volatility. More recent studies however presented evidence that there is important information contained in option prices, that adds to the efficiency of volatility forecasting when implied volatility is included (ibid.). A reason for this discrepancy in results could be that early studies did not consider several data and methodological problems, such as long enough time series, a possible regime shift around the crash in 1987 and the use of non-overlapping samples (ibid.). Bent J Christensen and Prabhala for example took this into account and found that implied volatility outperforms historic volatility. All in all Jiang, George J & Tian summarize, that collectively "these studies present evidence that implied volatility a more efficient forecast for future volatility than historic volatility" (p.1306).

1.2 Weaknesses of Existing Models: VIX Introduced by CBOE

Even though BS implied volatility is found to be the overall more efficient forecast of realized volatility compared to historic volatility (Jiang, George J & Tian 2003), the BS implied volatility has some specification problems. Firstly, BS implied volatility focuses on at-the-money options. The advantage is, that at-the-money options are the once most actively traded and thus the most liquid ones. However this focus fails to include information contained in other options. Moreover, volatility estimation with the BS model, includes the same assumptions as are made in the BS model itself. Thus tests based on the BS equation are actually joined tests of market efficiency (as market efficiency is assumed to use option prices for volatility estimation, as mentioned above) and the BS model, and therefore suffer from a model misspecification error (ibid.).

That is why during the last years, implied volatility indexes which are not based on a pricing assumption gained popularity. One of these model-free implied volatility indexes is the VIX from CBOE.

- power of volatility models lies in out-of-sample forecasting power
- so far BS implied volatility models had the best out of sampling forecasting power, but they have several problems (most importantly joint hypothesis problem)

2 Selected volatility concepts and models of volatility measurement

This section presents first some stylized facts of financial data, and gives an introduction to the different ways to estimate volatility. By pointing out the

advantages and disadvantages of the concepts and models and their fit to the stylized facts, the HAR-RV approach shall be motivated.

2.1 The Return Process and Stylized Facts of Financial Data

As mentioned in the introduction, the challenge when measuring volatility is, that stock return volatility is not directly observable (Tsay 2005). This problem evolves from the fact that we can only observe one realization of the underlying data generating process, and even though stocks are traded and thus have market prices which could be used for volatility measurement, there is no continuous data available and even for high-frequency data and extremely liquid markets microstructure effects and noise prevent getting close to a continuous sample path. It is thus only possible to estimate averages of discrete volatility for a given period of time. (Torben G. Andersen, Bollerslev and Diebold 2001). There are however several approaches that should be introduced here. To start with, the definition of the simple gross return is

$$1 + R_t = \frac{P_t}{P_{t-1}} \quad (1)$$

In the continuous-time setting, continuously compounded returns are used, which are given by

$$r_t = \ln(1 + R_t) = \ln\left(\frac{P_t}{P_{t-1}}\right) = \ln\frac{P_t}{P_{t-1}} = p_t - p_{t-1} \text{ with } p_t = \ln(P_t) \quad (2)$$

When observed over time, this asset returns show some distributional properties, often referred to as stylized facts of asset returns. Observed by many authors, only a few shall be mentioned here. Corsi for example mentions particularly the very strong persistence of autocorrelation of the square and absolute returns, which regularly poses challenges to econometric models. Moreover

return probability density functions are often leptocurtic with fat tails. As the time scale increases, the return distribution slowly converge to the normal distribution, but before convergence, the return distribution has different shapes depending on the time scale. Financial data also show evidence of scaling as described firstly by Mandelbrot, which is connected to the idea that patterns appear in different times, or that the distribution for returns has similar functional forms for various choices of the time interval.

Moreover Torben G. Andersen, Bollerslev and Diebold mentions that first, even though raw returns have a leptocurtic distribution, the returns standardized by realized volatility are approximately Gaussian. Second, the distribution of realized volatility of returns itself is right skewed, the one of the logarithms of realized volatility however are also approximately Gaussian. Third, the long-run dynamics of realized logarithmic volatilities are well approximated by a fractionally-integrated long-memory process. Other authors who mention stylized facts: (Jiang, George J & Tian 2003). Moreover, even though volatility is not directly observable, it has some properties that are commonly seen. Here the characteristics as presented by Tsay should be introduced. These are firstly volatility clusters, loosely speaking meaning that high volatility tends to be followed by high volatility, and the same is true for low volatility periods. Moreover, volatility jumps are rare. Third, volatility varies within a fixed range and does not diverge to infinity, which statistically means that volatility is often (weakly) stationary. Fourth, there is a leverage effect in volatility, meaning that it reacts differently to a big increase in price than to a drop.

2.2 Concepts and Models using Historic Volatility

2.2.1 Volatility Concept and Non-parametric ex-post Volatility Measurement - Realized Volatility

By definition “volatility seeks to capture the strength of the (unexpected) return variation over a given period of time” (Torben G. Andersen, Bollerslev

and Diebold 2001, p.7). However, there are multiple concepts and definitions of asset volatility. According to Torben G. Andersen, Bollerslev and Diebold the concepts can be grouped in (i) the *notional volatility* corresponding to the ex-post sample-path return variability over a fixed time interval, (ii) the ex-ante *expected volatility* over a fixed time interval or the (iii) the *instantaneous volatility* corresponding to the strength of the volatility process at a point in time. The aim of the HAR-RV model introduced later is to model volatility ex-post and then maybe use this approach to measure the information content of implied volatility and maybe also to build a forecast model. Thus the model needs some measure of latent volatility process, so that the performance can be evaluated.

It can be shown, that under some assumptions, realized volatility as the square root of the sum of squared high frequency returns, can be used to approximate the quadratic variation process which is the variation in a continuous time setting. This approach mainly building on the work of Torben G. Andersen, Bollerslev and Diebold und [noch jemanden finden] shall only be briefly introduced here.

To begin with, it should be assumed that we have a continuous-time no-arbitrage setting. As return volatility aims to capture the strength of the unexpected return variation, one needs to define the component of a price change as opposed to an expected price movement. This requires the decomposition of the return process in an expected and an innovation component. Torben G. Andersen, Bollerslev and Diebold show, that under certain assumptions the instantaneous return process can be decomposed into an expected return component, and a martingale innovation (in the discrete time setting this decomposition is more complex, for this paper it shall only be relevant that the martingale part is still the dominant contribution to the return variation over short intervals). As mentioned in the previous paragraph, volatility measures often focus on representing the average volatility over a discrete time, as a continuous record

of price data is not available, and even for liquid markets it is distorted by microstructure effects. Torben G. Andersen, Bollerslev and Diebold (2001) show that in order to measure the average variance², one can refer to the quadratic variation process of this martingale component, as the quadratic variation process represents the (cumulative) realized sample path variability of the martingale over any fixed time interval. To be precise, they define *notional variance* as the increment to the quadratic variation for the return series, measured ex-post.

Assuming that the mean of the return process is zero, taking the expected value of the notional variance and extending this concept slightly, one gets the *realized variance*, defined over the $[t - h, t]$, $0 < h \leq t \leq T$ time interval as

$$v^2(t, h; n) = \sum_{i=1}^n r(t - h + (i/n) \times h, h/n)^2. \quad (3)$$

Torben G. Andersen, Bollerslev and Diebold show not only that the realized variance is an *unbiased* estimator of ex-ante expected variance, or at least approximately unbiased when relaxing the zero mean assumption and taking a high sample frequency (their proposition 4). Moreover, Torben G. Andersen, Bollerslev and Diebold show that the realized variance is a *consistent* nonparametric measure of the notional variance for increasingly finely sampled returns over any fixed length interval (their proposition 5).

So in summary, the increment to the quadratic return variation and thus past variance can be consistently and well approximated through the accumulation of high-frequency squared returns. Taking the square root of the realized variance, one gets the *realized volatility*.

²actually Torben G. Andersen, Bollerslev and Diebold (2001) term this the *realized volatility*, but this paper will use the terminology of Corsi (2009), who uses the term realized volatility for the square-root of the integrated variance.

2.2.2 Volatility Model - HAR-RV Model

Having introduced the concept of notional volatility and its approximation by realized volatility, we now turn to volatility modelling/measurement. To measure volatility, one can separate between parametric and non-parametric methods. Whereas parametric methods try to measure the expected volatility making different assumptions about both the functional form and the variables in the information set available, non-parametric methods try to quantify notional volatility directly. The realized volatility is an example for a non-parametric methods. However, to estimate volatility ex-ante (and to potentially forecast later), this paper will refer to one type of the parametric methods based on the paper of Corsi (2009) termed the HAR-RV model.

In the last paragraph, volatility was measured as the average over discrete time. For the HAR-RV model this concept needs to be transferred to an instantaneous volatility measurement in continuous time. Moving to continuous time, the corresponding logarithmic price process is then given by Corsi (ibid.) as the following stochastic difference equation,

$$dp(t) = \mu(t)dt + \sigma(t)dW(t), \quad 0 \leq t \leq T \quad (4)$$

where $p(t)$ is the logarithm of the instantaneous price, μ is a finite variation stochastic process, $W(s)$ standard Brownian motion and σ a stochastic process independent of $W(s)$. For the volatility the difference is, that now the notional variance equals the *integrated variance (IV)*, which is the integral of the stochastic process. For a period of one day, this is represented by

$$IV_t^{(d)} = \int_{t-d}^t \sigma^2(w)dw. \quad (5)$$

Corsi (ibid.) then take the square-root of this integrated variance and term it *integrated volatility*. Torben G. Andersen, Bollerslev and Diebold (2001) show that also in this continuous-time setting the integrated variance equals the

notional variance, and thus can also be approximated with the sum of squared returns. This leads to the following notion of realized volatility over the one day interval

$$RV_t^{(d)} = \sqrt{\sum_{j=0}^{M-1} r_t^2 - j \times \Delta} \quad (6)$$

with $\Delta = 1d/M$ being the sampling frequency and $r_t^2 - j \times \Delta$ defined as the continuously compounded Δ -frequency returns.

This notion of volatility is combined with the *Heterogeneous Market Hypothesis* by Müller et al. (1993). This approach grounds on the fractal approach of Mandelbrot's 1983 paper, where (time series) objects are analyzed on different time scales and the obtained results are compared. The starting point of the argument is, that conventional time series analysis focusing on regularly spaced observations does not capture the real nature of the raw data, as the usual time choice for recording observations (e.g. a day) is arbitrary. Thus for example a process explaining monthly price changes could not be applied to daily data and vice versa. This fractal approach led to the heterogeneous market hypothesis. This hypothesis states that the market gives rise to heterogeneous trading behaviours as different market participants or components have different time horizons for their trading goals and for their consideration of past events. The time span has on the one side the high-frequency dealers such as market makers, in the middle some medium term dealers and on the other side the low-frequency dealers, such as central banks or commercial organizations. As the market is driven by these components, it can be described as "fractal". Müller et al. (ibid.) base this theory of different time horizons on multiple observations: Firstly, the decline of the return autocorrelation function is not exponential, as suggested for example by lower-order GARCH or ARCH models, but rather hyperbolic. Assuming, that each of the distinct components has an exponential decline with different time horizons, this in sum comes close to a hyperbolic

decline. Secondly, if market participants were homogeneous, volatility should be negatively correlated with market activity, as the price should converge to the “real value”. However, they are positively correlated, which might be explained by the fact that actors react and execute in different market situations (Müller et al. 1993). Other than to time scale, the heterogeneous market hypothesis can be applied to geographical location, degrees of risk aversion, institutional constraints or transactions costs. However, as in Corsi (2009) for this paper the time aspect should be relevant. Corsi (ibid.) refers to studies that measure volatility over different time horizons and observe that there is an asymmetric behaviour of volatility: “The volatility over longer time intervals has a stronger influence on volatility over shorter time intervals than conversely” (ibid., p.178). Thus a cascade pattern from low to high frequencies emerges. To formalize the model the latent partial volatility $\sigma_t^{(d)}$ is defined as the volatility generated by a certain market component (here daily). To account for short-term, medium-term and long-term traders, the time horizons of one day (d), one week (w) and one month (m) are considered. Each of this volatility components corresponds to a market component, that forms the expectation for the volatility of the next period based on both the observation of the current realized volatility according to the own time frame, and on the expectation of the one horizon longer volatility. This formalizes in a cascade model, written as

$$\sigma_{t+1d}^{(d)} = c + \beta^d RV_t^{(d)} + \beta^{(w)} RV_t^{(w)} + \beta^{(m)} RV_t^{(m)} + \tilde{w}_{t+1d}^{(d)}, \quad (7)$$

with $\tilde{w}_{t+1d}^{(d)}$ being the innovation term. Transferring this to a time series model, the ex-post approximation of latent volatility with realized volatility is used, which can be written as

$$\sigma_{t+1d}^{(d)} = RV_{t+1d}^{(d)} + w_{1+1d}^{(d)}. \quad (8)$$

where the $w_t^{(d)}$ now subsumes both the latent volatility measurement and estimation errors. Taking equations 7 and 8 together, we obtain the time series representation from the cascade model,

$$RV_{t+1d}^{(d)} = c + \beta^d RV_t^{(d)} + \beta^{(w)} RV_t^{(w)} + \beta^{(m)} RV_t^{(m)} + w_{t+1d}, \quad (9)$$

where $w_{t+1d} = \tilde{w}_{1+d}^{(d)} - w_{1+d}^{(d)}$. Using simulated data, Corsi (2009) shows, that the HAR-RV simulated returns and volatility reproduce the stylized facts mentioned above very well. The data has not only the excess of kurtosis, but also the tail cross-over, meaning that the fat tails get thinner as the aggregation level increases. Concerning the volatility memory, the simulated data is also able to reproduce the long memory of the empirical data. Moreover they use OLS and forecast to show [complete this section, saying that they also show that there is information content and forecasting ability].

2.3 Implied volatility

2.3.1 The General Idea of Implied Volatility

As mentioned, there are multiple ways to model and measure volatility. One approach is to use not only historic price data, but to augment the information set by incorporating options data and using their forward looking nature. This approach can be termed *implied volatility*, and can consist for example of a parametric volatility model for returns, accompanied by an asset pricing model and the option price information (Torben G. Andersen, Bollerslev and Diebold 2001). The intuition behind this is, that option prices can be seen as reflecting market participants' expectations of the future movements of the underlying asset. Assuming that the market is informationally efficient, as described by **fama1970**, and that the asset pricing model is correct, the implied volatility derived from this model should not only subsume all information contained in the models using historic volatility, but also be a more efficient forecast of

future volatility (Jiang, George J & Tian 2003).

One popular example of implied volatility is the *Black-and-Scholes implied volatility*, building on the Black-and-Scholes-Merton (BS) asset pricing model as presented by Black and Scholes (1973). One input for pricing options with the BS model is the volatility of the underlying asset. Having the derivative prices available on the market, it is possible to extract a value for the expected volatility, by inverting the theoretical asset pricing model. It is however important to note, that all of these procedures depend on the assumptions that are made in the asset pricing model (Torben G. Andersen, Bollerslev and Diebold 2001).

Many studies examined the information content of BS implied volatility and its forecasting ability for future volatility. Earlier studies find that implied volatility contains little additional information content compared to historical volatility and has no forecasting ability (Jiang, George J & Tian 2003). Canina and Figlewski (1993) for example found that implied volatility does not incorporate the information from historic volatility and constitute poor forecast of future volatility using S&P 500 options. In contrast, other studies support the evidence of implied volatility subsuming the information from historic volatility. These are for example Day and Lewis (1992), Lamoureux and Lastrapes (1993) or Jorion (1995). Day and Lewis (1992) examine information content from S&P 100 options between 1983 and 1989 and find that they have significant information content for realized volatility. With previous research leading ambiguous results, more recent research made several methodological and data corrections and found evidence supporting the hypothesis that implied volatility has predictive power for future volatility. These corrections included for example using longer time series extending the regime shift around the 1987 crash or adapting high-frequency asset returns to provide a more accurate view or use non-overlapping samples (Jiang, George J & Tian 2003). For example Bent J Christensen and Prabhala (1998) find that subsumes the information content of past volatility

and outperforms it in forecasting future volatility. They use a longer time series, take into account the 1987 regime shift and use non-overlapping samples (Jiang, George J & Tian 2003). Moreover Bent Jesper Christensen, Hansen and Prabhala (2001) found that under certain measurement errors such as overlapping samples statistical tests are no longer meaningful. Taking this measurement errors into account results in support of the hypothesis, that implied volatility subsumes all information contained in historic volatility. All in all, this insights provide a convincing argument why early studies found the forecast of implied volatility to be inefficient (Jiang, George J & Tian 2003). For a very comprehensive overview of the volatility research as was state of the art in 2003, please see S.-h. Poon and Granger (2003).

However, even though the recent evidence is in support of BS implied volatility, there are several disadvantages of using the implied volatility with an asset pricing model. One disadvantage is, that the BS implied volatility relies mostly on information from at-the-money options, which are generally the most actively traded ones. This however fails to incorporate information contained in other options. Moreover and more importantly, testing for the BS implied volatility always means testing market efficiency and the BS model jointly. Assumptions of the BS model include constant volatility, no transaction costs or taxes, no dividend before option maturity, no arbitrage, continuous trading, constant risk-free interest rate, divisible securities and no short sell (ibid.). As it is not possible in this setting to test for market efficiency or the pricing assumptions separately, this tests are subject to model misspecification errors (Jiang, George J & Tian 2003). Moreover most early paper's implied volatility ignores the early exercise opportunity and/or dividends (Blair, S. Poon and Taylor 2000). This is why an implied volatility that does not rely on an asset pricing model was introduced.

2.3.2 VIX and Model-Free Implied Volatility

The idea of model free implied volatility was introduced by Britten-Jones and Neuberger (2000) who show that the risk-neutral realized volatility can be derived from a set of options with matching expiration, thus extending the approach of Derman and Kani (1994) Dupire et al. (1994), Dupire (1997) and Rubinstein (1994) on implied distributions (Jiang, George J & Tian 2003). The idea of model free implied volatility is, that instead of specifying a process for the price of the underlying security and then to derive the option price as a function of this price process parameters, a complete set of option prices is taken as given and then as much information as possible is extracted out of the underlying price process (Britten-Jones and Neuberger 2000). To be more specific, Britten-Jones and Neuberger (ibid.) show in an approach resembling a binomial tree, that the probability of the stock price reaching any particular level and of a price move is determined by the initial set of option prices. This leads them to their proposition 1, that the expectation of the squared return, conditional on stock price and time, is determined by the initial option prices. Moreover, as this proposition one only infers a one-period forecast conditional on a stock price level, they propose a forecast over any multiperiod interval without conditioning, by showing that the risk-neutral expected sum of squared returns between two dates is given from the set of options expiring on these two dates (their proposition 2). This formula does not use any specific option pricing model to derive implied volatility and is only based on no-arbitrage conditions, therefore it solves the joint-hypothesis problem and test directly for market efficiency (Jiang, George J & Tian 2003).

Several papers used this result, examined whether the model free implied volatility subsumes the information from the historic and BS implied volatility, conducted forecasts using the model free implied volatility and extended the concept.

(write down papers that work with model free implied volatility, e.g. Jiang,

George J & Tian extended the model so that is not derived under diffusion assumptions and generalized it to processes including random jumps.)

One of the first implemented implied volatility index was the VIX from the CBOE. It is computed each trading day on a real-time basis, and to facilitate comparison it was calculated back to January 1986. It's introduction in 1993 had the intention to provide a benchmark for market- volatility in the short term, and a volatility index on which futures and options could be written and traded. At this time the VIX was based on the S&P 100 options, as at this time the S&P 100 options were the most actively traded in the U.S., which is critical to the usefulness of the VIX or any other implied volatility index. During the years the index option market changed and in 2003 the VIX adopted to this change. First, the S&P 500 option market superseded the S&P 100 options market as the most actively traded option market in the U.S., thus following 2003 the VIX was based on the S&P 500 options. Secondly, option index trading behavior changed. Whereas in the 1990s both call and put index options were equally important, over the years out-of-the money and at-the-money put gained popularity as they were bought by portfolio insurers. Thus the VIX also started to include out-of-the money options in its calculation, bringing another advantage that was not included in other early implied volatility indexes as mentioned above, which lacked this information content (Whaley 2008). Sometimes the VIX is called *investor fear gauge*, however it is important to notice that it measures, not causes market volatility. It is true, that using regression analysis Whaley (ibid.) found, that the rate of change in the VIX and S&P 500 is asymmetric, with the VIX reacting higher to a drop in the S&P 500 than its rise, which could be interpreted as a higher fear in the downside than excitement in an up-move. Nevertheless, this correlation must not express causality (ibid.). The VIX is constructed in the way that it eliminates mis-specification and "smile" effects thus making it an accurate measurement of implied volatility (Blair, S. Poon and Taylor 2000).

The formula for calculation is

$$VIX = \sqrt{\frac{2}{T} \sum_i \frac{\Delta K_i}{K_i^2} e^{RT} Q(K_i) - \frac{1}{T} \left(\frac{F}{K_0} - 1 \right)^2} \times 100 \quad (10)$$

with T being time to expiration, F the forward index level, K_0 the first strike below the forward index level, K_i the strike of the i^{th} out-of-the-money option, ΔK_i the interval between the strike prices, R the risk-free rate and $Q(K_i)$ the midpoint of bid-ask spread for each option with strike K_i . For the calculation first the options used for calculation are selected. The VIX always uses near- and next-term put and call options, with more than 23 and less than 37 days to expiration. The VIX will always reflect an interpolation of the volatility between these two option maturities, using the midpoint bid-ask price as the transaction price are subject to bid-ask bounde (S.-h. Poon and Granger 2003). Once a week the options to calculate the VIX roll over to new maturities. Then the volatility for both the near- and next-term options is calculated. Out of this two volatility, the 30-day-weighted average is calculated, taken as square root and multiplied by 100 to get the VIX (Exchange 2009). Various papers also tested the informational efficiency of the VIX. Blair, S. Poon and Taylor (2000) measured the information content with in-sample and out-of sample forecasts of the VIX (in the old version), using high-frequency data and find that the VIX subsumes nearly all the information contained in historical data. **hao2013** extend the VIX by deriving the VIX formulas under a risk-neutral valuation relationship, finding that this GARCH implied volatility is significantly lower than the VIX thus interpreting that the GARCH model can not capture the variance premium.

3 Methodology and Data

3.1 Data and Calculation of Input Factors

Also Torben G Andersen et al. point out the advantage of using high-frequency returns is not only that they help predicting again high-frequency returns, but also that they contain information for longer horizons, such as monthly or quarterly. Graphics

VIX is the daily implied volatility index, calculated from the annualized volatility as $VIX/\sqrt{252}$ as in Blair, S. Poon and Taylor (2000) and Whaley (2008).

For an analysis of the historic values of the VIX, see Whaley (ibid.) (chapter: historix, normal range).

Measure for daily return variability should be realized volatility, as Torben G. Andersen, Bollerslev and Diebold suggest, that under suitable conditions it provides an unbiased estimator of the return volatility.

- S&P 500 index data on daily basis
- sampling period: 2000 - 2018
- realized volatility: daily realized volatility of S&P 500, calculated using 5 minute returns, retrieved from
- model-free implied volatility: VIX index data
- historic volatility: lagged realized volatility, for HAR-RV model use the average over the time period used to forecast

3.2 Methodology: Linear Regression and HAR-RV model

- Reg1: without VIX
 - Reg1a: regress realized volatility on historic volatility using simple linear regression

- Reg1b: regress realized volatility on historic volatility using HAR-RV model
- Reg2: with VIX
 - Reg2b: regress realized volatility on historic volatility using simple linear regression
 - Reg2b: regress realized volatility on historic volatility using HAR-RV model

$$\sigma_{t,x}^{RV} = \alpha_{x,t} + \beta_{x,t}^{HV} \sigma_{x,t}^{HV} \quad (11)$$

$$\sigma_{x,t}^{RV} = \alpha_{x,t} + \beta_{x,t}^{HV,d} \sigma_{x,t}^{HV,d} + \beta_{x,t}^{HV,w} \sigma_{x,t}^{HV,w} + \beta_{x,t}^{HV,m} \sigma_{x,t}^{HV,m} \quad (12)$$

$$\sigma_{x,t}^{RV} = \alpha_{x,t} + \beta_{x,t}^{HV} \sigma_{x,t}^{HV} + \beta_{x,t}^{VIX} + \sigma_{x,t}^{VIX} \quad (13)$$

$$\sigma_{x,t}^{RV} = \alpha_{x,t} + \beta_{x,t}^{HV,d} \sigma_{x,t}^{HV,d} + \beta_{x,t}^{HV,w} \sigma_{x,t}^{HV,w} + \beta_{x,t}^{HV,m} \sigma_{x,t}^{HV,m} + \beta_{x,t}^{VIX} + \sigma_{x,t}^{VIX} \quad (14)$$

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- historic volatility: lagged realized volatility, for HAR-RV model use the average over the time period used to forecast

3.4 Limitations

As volatility is stochastic, the ex-ante estimation will not equal the return volatility, as it is a measurement over an aggregated discrete time period (Torben G. Andersen, Bollerslev and Diebold 2001). The VIX might be flawed, as [jiang2007](#) showed.

4 Results

Figure 1: Level-level regression

	Historic	Historic and VIX
Intercept	0.04*** (0.01)	-0.19*** (0.01)
RV_t^d	0.36*** (0.02)	0.25*** (0.02)
RV_t^w	0.39*** (0.03)	0.29*** (0.03)
RV_t^m	0.19*** (0.02)	-0.18*** (0.02)
VIX		0.62*** (0.02)
R ²	0.71	0.75
Adj. R ²	0.71	0.75
Num. obs.	4434	4434
RMSE	0.33	0.31

*** $p < 0.001$, ** $p < 0.01$, * $p < 0.05$

	AIC	BIC
OLS	2819.78	2851.77
OLS with VIX	2118.89	2157.27
log OLS	1862.08	1894.07
log OLS with VIX	1316.67	1355.05
HAR-RV	-56620.66	-56588.65

5 Discussion

Figure 2: log-log regression

	Historic	Historic and VIX
Intercept	0.19*** (0.04)	2.68*** (0.10)
RV_t^d	0.34*** (0.02)	0.24*** (0.02)
RV_t^w	0.41*** (0.03)	0.28*** (0.03)
RV_t^m	0.20*** (0.02)	-0.13*** (0.02)
VIX		0.80*** (0.03)
R^2	0.73	0.77
Adj. R^2	0.73	0.77
Num. obs.	4435	4435
RMSE	0.30	0.28

*** $p < 0.001$, ** $p < 0.01$, * $p < 0.05$

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