# Measuring the Information Content of VIX Volatility

Context: Humboldt Project

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# Introduction

# Motivation: Why this project? Why does Volatility matter?

- Risk measurement and the stability of the financial system
    - Volatility is closely related to risk
    - Volatility is crucial input to risk measures, such as the Value at Risk
- Volatility is used for..
    - .. the pricing of financial instruments, such as derivatives
    - .. the risk-return trade-off and therefore management decisions
- Forecast potential

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# More closely: What exactly is Volatility?

- In Finance, we are usually interested in the *conditional* standard deviation from the expected value of the underlying asset return (Tsay 2005)
- What causes asset price movement and thus volatility?
    - Assuming market efficiency (as introduced by Malkiel and Fama), stock prices incorporate all available information from the market
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## The Problem: Why is it so hard to measure and forecast volatility?

- Volatility is not directly observable
  - We can estimate it for a given time period
  - The problem is, each period contains different information
- We can however observe stylized facts of volatility
  - Volatility clustering
  - Variation within a fixed range
  - Leverage effect: different reaction to price drop or increase

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# Maybe a solution: How volatility has been calculated so far

- According to this stylized facts, we can use models that best capture the characteristics of volatility
- There are multiple approaches, examples are
    - Econometric models using historic volatility (e.g. ARCH)
    - Black-and-Scholes implied volatility
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- Intuition behind Black-and-Scholes (BS) implied volatility
  - Option pricing model, that uses volatility as an input
  - \+ Reverse calculation and derive an *option implied volatility*
  - \+ Options contain the market's expectation of future stock price movement
- Problems with the BS implied volatility
  - – The BS model is mainly based on at-the-money options
  - – Most importantly the BS model has a pricing assumption
- Joint hypothesis problem

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- Model-free implied volatility is not based on any particular option pricing model
- One of the fist model-free implied volatility indices was the VIX from CBOE (CBOE 2009)
  - Measures the market's expectation of 30-day volatility
  - + Includes information from both at-the-money and out-of-money options
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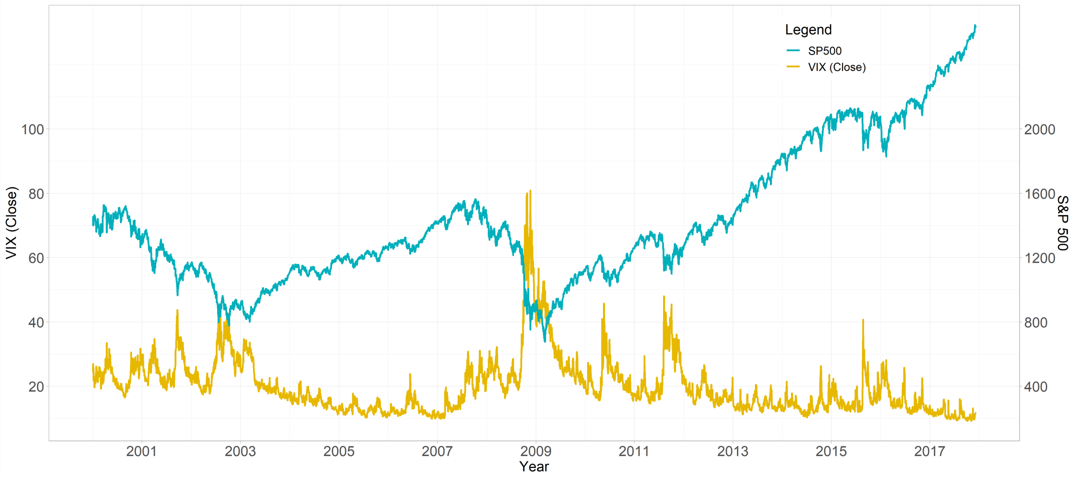
# Data

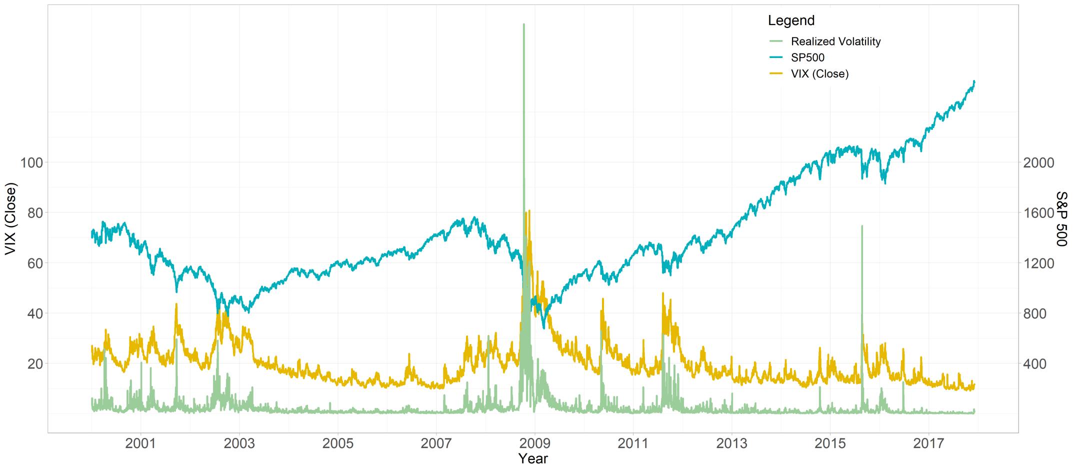
# Volatility of S&P 500

- Components of Dataset
  - VIX, downloaded from CBOE
  - Realized Volatility, downloaded from Oxford Man Institute
  - S&P 500, downloaded from CBOE
- Sampling period January 2000 - December 2017

# S&P 500 in Comparison to VIX



# S&P 500 in Comparison to VIX and Realized Volatility



# Method

# My Method: Regression and HAR Model

Stepwise regression to compare information content:

$$\sigma_{t+1} = c + \beta^d RV^d_{t-1} + \beta^w RV^w_{t-1} + \beta^m RV^m_t \tag{1}$$

$$\sigma_{t+1} = c + \beta^d RV^d_{t-1} + \beta^w RV^w_{t-1} + \beta^m RV^m_t + \beta^{VIX} VIX^{VIX}_t \tag{2}$$

$+$ the same regressions as log-log model

with the weekly aggregation period being (monthly similar with 20 days):

$$\sigma^w_t = \frac{1}{5}(RV^d_t + RV^d_{t-1d} + RV^d_{t-2d} + RV^d_{t-3d} + RV^d_{t-4d})$$

and Realized Variance (RV) being: $RV^d_t = \sqrt{\sum_{m=1}^{M} r^2_{t,m}}$

$\sigma$ = Volatility (sd), $RV$ = Realized Variance with M equally spaced intraday returns for a day t, with m = 1,..,M, here 10min returns

# My Method: Regression and HAR Model

HAR-RV Model (according to Corsi (2009), but not implemented yet), idea:

$$\sigma_{t+1} = c + \beta^d RV_{t-1}^d + \beta^w RV_{t-1}^w + \beta^m RV_t^m + \tilde{w}_{t+1d}^d \tag{3}$$

$\tilde{w}_{t+1d}^d$ = innovation term

# Results so far

# Regression Results

**Figure 1:** level-level Model

| | Historic | Historic with VIX |
|---|---|---|
| Intercept | $0.00^{***}$ | $-0.01^{***}$ |
| | (0.00) | (0.00) |
| $RV_t^d$ | $0.27^{***}$ | $0.23^{***}$ |
| | (0.02) | (0.02) |
| $RV_t^w$ | $0.39^{***}$ | $0.36^{***}$ |
| | (0.03) | (0.03) |
| $RV_t^m$ | $0.25^{***}$ | $-0.04$ |
| | (0.03) | (0.03) |
| VIX | | $0.00^{***}$ |
| | | (0.00) |
| $R^2$ | 0.54 | 0.57 |
| Adj. $R^2$ | 0.54 | 0.57 |
| Num. obs. | 4436 | 4436 |
| RMSE | 0.02 | 0.02 |

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

**Figure 2:** log-log Model

| | Historic | Historic with VIX |
|---|---|---|
| Intercept | $-0.25^{***}$ | $-7.85^{***}$ |
| | (0.05) | (0.28) |
| $RV_t^d$ | $0.34^{***}$ | $0.24^{***}$ |
| | (0.02) | (0.02) |
| $RV_t^w$ | $0.40^{***}$ | $0.28^{***}$ |
| | (0.03) | (0.03) |
| $RV_t^m$ | $0.21^{***}$ | $-0.13^{***}$ |
| | (0.02) | (0.03) |
| VIX | | $1.60^{***}$ |
| | | (0.06) |
| $R^2$ | 0.73 | 0.77 |
| Adj. $R^2$ | 0.73 | 0.77 |
| Num. obs. | 4437 | 4437 |
| RMSE | 0.60 | 0.55 |

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

# Possible Problems coming up

# Next steps/Questions to solve

- Having gathered all this information about volatility measurement, what is the most accurate way to set up my regressions and my HAR-RV model?
- Next step: Bring current state of work to paper

# References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# Appendix

# The Black and Scholes Equation

price of an option over time:

$$\frac{\partial \mathrm{C}}{\partial \mathrm{t}} + \frac{1}{2}\sigma^2 \mathrm{S}^2 \frac{\partial^2 \mathrm{C}}{\partial \mathrm{C}^2} + \mathrm{rS}\frac{\partial \mathrm{C}}{\partial \mathrm{S}} = \mathrm{rC} \tag{4}$$

calculate the price of European call and put option:

$$\mathrm{C(S,t)} = \mathrm{N(d_1)S} - \mathrm{N(d_2)Ke}^{-rt} \tag{5}$$

$$\mathrm{d_1} = \frac{1}{\sigma\sqrt{\mathrm{t}}}\left[\ln\left(\frac{S}{K}\right) + t\left(r + \frac{\sigma^2}{2}\right)\right] \tag{6}$$

$$\mathrm{d_2} = \frac{1}{\sigma\sqrt{\mathrm{t}}}\left[\ln\left(\frac{S}{K}\right) + t\left(r - \frac{\sigma^2}{2}\right)\right] \tag{7}$$

$$N(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} \mathrm{e}^{-\frac{1}{2}z^2} dz \tag{8}$$

C = Call option price

S = Current stock price

K = Strike price of the option

r = risk-free interest rate (a number between 0 and 1)

$\sigma$ = volatility of the stocks return (a number between 0 and 1)

t = time to option maturity (in years)

N = normal cumulative distribution function

# The VIX Equation

$$\sigma^2 = \frac{2}{T}\sum_i \frac{\Delta K_i}{K_i^2} e^{RT} Q(K_i) - \frac{1}{T}(\frac{F}{K_0} - 1)^2 \qquad (9)$$

$VIX = \sigma * 100$, $T$ = Time to expiration, $F$ = Forward index level derived from index option prices, $K_0$ = first strike price below the forward index level, $K_i$ = strike price of $i^{th}$ out-of-the money option (call if $K_i > K_0$ and put if $K_i < K_0$), $\Delta K_i$ = interval between strike prices, $R$ = Risk-free rate to expiration, $Q(K_i)$ = midpoint of bid-ask spread for each option with strike $K_i$

# About the VIX

- 1993 CBOE introduced the first VIX, to measure the market's expectation for volatility, in 30-day period, based on S&P100 Index at-the-money options
- 2003 CBOE updated the VIX (together with Goldman Sachs)
    - now based on S&P500 Index (liquidity reasons)
- 2014 CBOE enhanced VIX
    - include also weekly options, thus most precisely match 30-day time frame
    - use SPX options with x $[23 < x < 36]$ days to expiration to reflect an interpolation of two points
- Today, VIX is also a tradable asset (negative correlation to stock market returns makes it variable diversification)

# Realized Variance

$$RV_t^d = \sqrt{\sum_{m=1}^{M} r_{t,m}^2} \tag{10}$$

with r being:

$$R = \frac{P_t - P_{t-1}}{P_{t-1}} \Leftrightarrow 1 + R_t = \frac{P_t}{P_{t-1}} \tag{11}$$

with continuously compounded return (log return)

$$r_t = ln(1 + R_t) = ln\frac{P_t}{P_{t-1}} = p_t - p_{t-1} \tag{12}$$

$RV$ = realized volatility, $R$ = asset return, $r = ln(R)$, $P$ = asset price, $p = ln(P)$

Why returns instead of asset prices?

- Returns are easier to handle than price series, because of statistical properties (e.g. time additivity)
- Asset return is a complete and scale-free summary of the investment opportunity (MacKinlay 1997 from Tsay (2005))