Measuring the Information Content of VIX Volatility

Context: Humboldt Project

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November 27, 2018

Table of Contents

Introduction

Motivation: Why this project? Why does Volatility matter?

- For the stability of the financial system, precise risk measurement is of great importance
  - Volatility is closely related to risk
  - it is crucial input to risk measures, such as the Value at Risk[1]
- Moreover volatility is used for..
  - .. the pricing of financial instruments, such as derivatives
  - .. the risk-return trade-off and therefore management decisions

[1] The Value at Risk is a quantile of the loss function, used for example by banks to estimate the amount of assets needed to cover possible losses. It estimates which loss is not going to be exceeded in a given time interval, for a given probability

More closely: What exactly is Volatility?

- In Finance, we are usually interested in the *conditional* standard deviation from the expected value of the underlying asset return (Tsay 2005)
- What causes asset price movement and thus volatility?
    - Assuming Market efficiency (as introduced by Malkiel and Fama), stock prices incorporate all available information from the market, because of competition and free entry
    - But that does not does not give us information about the distribution of stock prices

The Problem: Why is it so hard to measure and forecast volatility?

- Volatility is not directly observable
  - We can estimate it for a given time period
  - However, stock volatility consists of intraday and overnight volatility, each containing different information
- But we observe some characteristics about volatility, and can thus use econometric models, that best “copy” these stylized facts
  - ..

Maybe a solution: How volatility has been calculated so far

- According to the observed characteristics (and making some assumptions), we can use (econometric) models to estimate volatility
    - Econometric models using historic volatility (e.g. ARCH)
    - Black-and-Scholes implied volatility
    - ...
- So far Black-and Scholes implied volatility has proven to contain significant information for realized volatility

Black-and-Scholes implied volatility

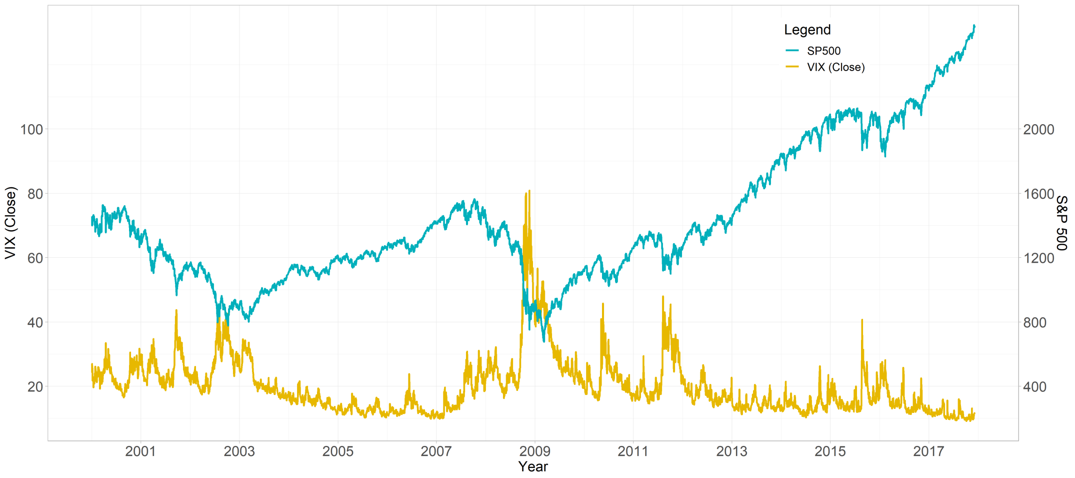
- Intuition behind Black-and-Scholes (BS) implied volatility
  - The BS model is a model, that uses volatility to price options
  - By using option prices from the market, it is possible to turn the calculation around and derive an *option implied volatility*
- there are however some problems with the BS implied volatility
  - The Black-and-Scholes model is mainly based on at-the-money options (fails to incorporate information contained in others)
  - Most imprtantly The Black-and-Scholes model makes pricing assumptions (e.g. that stock prices follow geometric Brownian motion)
- $\rightarrow$ Joint hypothesis problem
  - Testing for BS implied volatility is always a test of *both* market efficiency and the BS pricing assumptions
  - So far market efficiency per se is not testable

Solving the joint hypothesis problem: Model-free implied volatility

- to correct for the model specification problems with the BS implied volatility
- one of the fist model-free implied volatility indices was the VIX from CBOE
- the VIX uses options

Data

Volatility of S&P 500



Method

- Regression of realized volatility on historic volatility

Results so far

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Possible Problems coming up

Questions currently to solve

- Having gathered all this information about volatility measurement, what is the most accurate way to set up my regression?

Sources

References

Malkiel, Burton G and Eugene F Fama (1970). "Efficient capital markets: A review of theory and empirical work". In: *The journal of Finance* 25.2, pp. 383–417.

Tsay, Ruey S (2005). *Analysis of financial time series*. Vol. 543. John Wiley & Sons.

Appendix

The Black and Scholes Equation

price of an option over time:

$$\frac{\partial \mathrm{C}}{\partial \mathrm{t}} + \frac{1}{2}\sigma^2 \mathrm{S}^2 \frac{\partial^2 \mathrm{C}}{\partial \mathrm{C}^2} + \mathrm{rS}\frac{\partial \mathrm{C}}{\partial \mathrm{S}} \ = \mathrm{rC} \qquad (1)$$

calculate the price of European call and put option:

$$\mathrm{C}(\mathrm{S},\mathrm{t}) = \mathrm{N}(\mathrm{d}_1)\mathrm{S} - \mathrm{N}(\mathrm{d}_2)\mathrm{K}\mathrm{e}^{-rt} \qquad (2)$$

$$\mathrm{d}_1 = \frac{1}{\sigma\sqrt{\mathrm{t}}}\left[\ln\left(\frac{S}{K}\right) + t\left(r + \frac{\sigma^2}{2}\right)\right] \qquad (3)$$

$$\mathrm{d}_2 = \frac{1}{\sigma\sqrt{\mathrm{t}}}\left[\ln\left(\frac{S}{K}\right) + t\left(r - \frac{\sigma^2}{2}\right)\right] \qquad (4)$$

$$N(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} \mathrm{e}^{-\frac{1}{2}z^2} dz \qquad (5)$$

C = Call option price

S = Current stock price

K = Strike price of the option

r = risk-free interest rate (a number between 0 and 1)

σ = volatility of the stocks return (a number between 0 and 1)

t = time to option maturity (in years)

N = normal cumulative distribution function

The VIX Equation