Measuring the Information Content of VIX Volatility

Context: Humboldt Project

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November 28, 2018

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Introduction

Motivation: Why this project? Why does Volatility matter?

- Risk measurement for the stability of the financial system
    - Volatility is closely related to risk
    - Volatility is crucial input to risk measures, such as the Value at Risk
- Volatility is used for..
    - .. the pricing of financial instruments, such as derivatives
    - .. the risk-return trade-off and therefore management decisions
- Forecast potential

More closely: What exactly is Volatility?

- In Finance, we are usually interested in the *conditional* standard deviation from the expected value of the underlying asset return (Tsay 2005)
- What causes asset price movement and thus volatility?
  - Assuming Market efficiency (as introduced by Malkiel and Fama), stock prices incorporate all available information from the market, because of competition and free entry
  - But that does not does not give us information about the volatility of stock prices

The Problem: Why is it so hard to measure and forecast volatility?

- Volatility is not directly observable
  - We can estimate it for a given time period
  - However, stock volatility consists of intraday and overnight volatility, each containing different information
- We can however observe some characteristics of volatility (stylized facts), such as
  - volatility clustering (periods of high volatility are followed by periods of high volatility, and similar for low volatility)
  - volatility is often stationary (it varies only within a fixed range)
  - leverage effect: volatility reacts differently to price drop or increase

Maybe a solution: How volatility has been calculated so far

- According to this stylized facts, we can use (econmetric) models that best capture the characteristics of volatility
- There is however multiple approaches to model volatility, examples are
    - Econometric models using historic volatility (e.g. ARCH)
    - Black-and-Scholes implied volatility
- Many papers argue, that Black-and Scholes implied volatility contains significant information for realized volatility (Jiang and Tian 2005)

Maybe a solution: Black-and-Scholes implied volatility

- Intuition behind Black-and-Scholes (BS) implied volatility
  - The BS model is an option pricng model, that uses volatility as an input
  + By using option prices from the market, it is possible to turn the calculation around and derive an *option implied volatility*
  + as options are contracts to buy an underlying in the future, then contain the market's expectation of future stock price movement
- There are however some problems with the BS implied volatility
  - The Black-and-Scholes model is mainly based on at-the-money options
  - Most importantly The Black-and-Scholes model makes an assumption about how the prices are formed (pricing assumption), and assumes that stock prices follow geometric Brownian motion)

➨ Joint hypothesis problem: Testing for BS implied volatility is always a test of *both* market effi ciency and the BS pricing assumptions

Solving the joint hypothesis problem: Model-free implied volatility

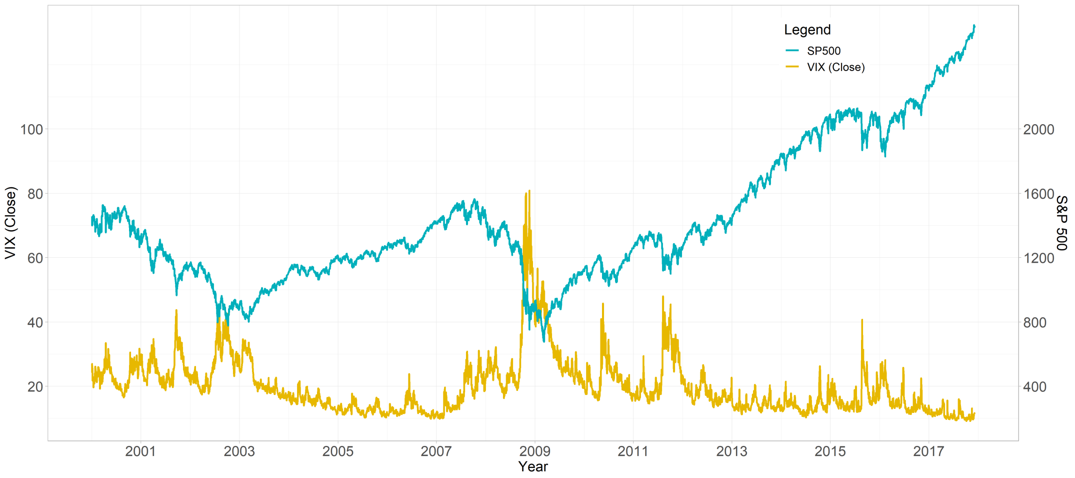
- For model-free implied volatility, no assumption is made regarding the underlying stochastic process, and it is not based on any particular option pricing model
  + The VIX includes information from both at-the-money and out-of-money options
  + As no pricing assumption is made, the model-free implied volatility provides a direct test of the informational efficiency of the options market
  -
- One of the fist model-free implied volatility indices was the VIX from CBOE (CBOE 2009)
  - VIX measures the market's expectation of 30-day volatility

Data

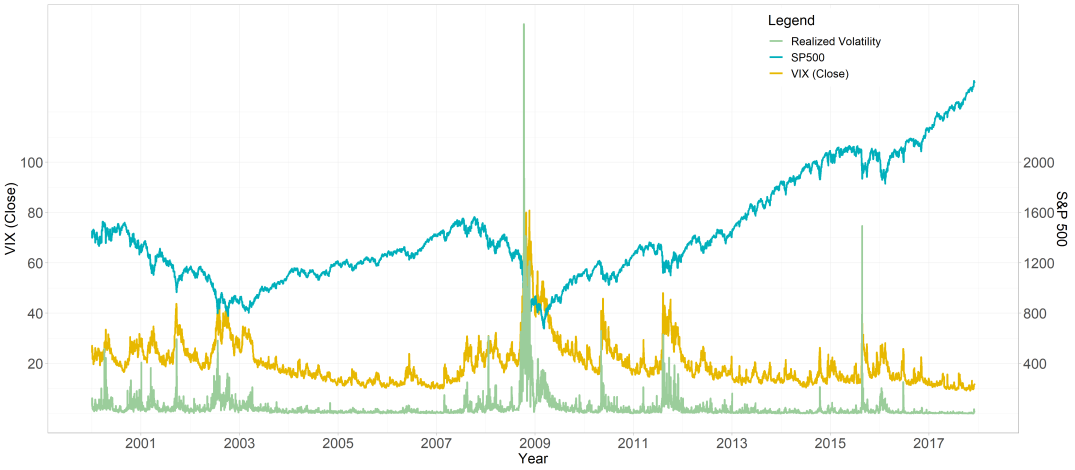
Volatility of S&P 500

- Components of Dataset
  - VIX, downloaded from CBOE
  - Realized Volatility, downloaded from Oxford Man Institute
  - S&P 500, downloaded from CBOE
- Sampling period January 2000 - December 2017

S&P 500 in Comparison to VIX



S&P 500 in Comparison to VIX and Realized Volatility



Method

My Method: Regression and HAR Model

Stepwise regressions, to compare information content:

$$\sigma_{t+1} = c + \beta^d RV_{t-1}^d + \beta^w RV_{t-1}^w + \beta^m RV_t^m \tag{1}$$

$$\sigma_{t+1} = c + \beta^d RV_{t-1}^d + \beta^w RV_{t-1}^w + \beta^m RV_t^m + \beta^m VIX_t^m \tag{2}$$

$+$ the same regressions as log-log model

with the weekly aggregation period being (monthly similar with 20 days):

$$\sigma_t^w = \frac{1}{5}(RV_t^d + RV_{t-1d}^d + RV_{t-2d}^d + RV_{t-3d}^d + RV_{t-4d}^d)$$

and Realized Variance (RV) being: $RV_t^d = \sqrt{\sum_{m=1}^{M} r_{t,m}^2}$

σ = Volatility (sd), RV = Realized Variance with M equally spaced intraday returns for a day t, with m = 1,..,M, here 10min returns

My Method: Regression and HAR Model

HAR-RV Model (not implemented yet)

$$\sigma_{t+1} = c + \beta^{d} RV^{d}_{t-1} + \beta^{w} RV^{w}_{t-1} + \beta^{m} RV^{m}_{t} + w^{\tilde{d}}_{t+1d} \tag{3}$$

$$\sigma_{t+1} = c + \beta^{d} RV^{d}_{t-1} + \beta^{w} RV^{w}_{t-1} + \beta^{m} RV^{m}_{t} + \beta^{m} VIX^{m}_{t} + \tilde{w}^{d}_{t+1d} \tag{4}$$

$\tilde{w}^{d}_{t+1d}$ = innovation

Results so far

Regression Results

Figure 1: level-level Model

	Historic	Historic with VIX
Intercept	0.00^{***}	-0.01^{***}
	(0.00)	(0.00)
RV_t^d	0.27^{***}	0.23^{***}
	(0.02)	(0.02)
RV_t^w	0.39^{***}	0.36^{***}
	(0.03)	(0.03)
RV_t^m	0.25^{***}	-0.04
	(0.03)	(0.03)
VIX		0.00^{***}
		(0.00)
R^2	0.54	0.57
Adj. R^2	0.54	0.57
Num. obs.	4436	4436
RMSE	0.02	0.02

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

Figure 2: log-log Model

	Historic	Historic with VIX
Intercept	-0.25^{***}	-7.85^{***}
	(0.05)	(0.28)
RV_t^d	0.34^{***}	0.24^{***}
	(0.02)	(0.02)
RV_t^w	0.40^{***}	0.28^{***}
	(0.03)	(0.03)
RV_t^m	0.21^{***}	-0.13^{***}
	(0.02)	(0.03)
VIX		1.60^{***}
		(0.06)
R^2	0.73	0.77
Adj. R^2	0.73	0.77
Num. obs.	4437	4437
RMSE	0.60	0.55

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

Possible Problems coming up

Next steps/Questions to solve

- Having gathered all this information about volatility measurement, what is the most accurate way to set up my regressions and my HAR-RV model?
- Next step: Bring current state of work to paper

References

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Appendix

The Black and Scholes Equation

price of an option over time:

$$\frac{\partial \mathrm{C}}{\partial \mathrm{t}} + \frac{1}{2}\sigma^2 \mathrm{S}^2 \frac{\partial^2 \mathrm{C}}{\partial \mathrm{C}^2} + \mathrm{rS}\frac{\partial \mathrm{C}}{\partial \mathrm{S}} = \mathrm{rC} \tag{5}$$

calculate the price of European call and put option:

$$\mathrm{C}(\mathrm{S},\mathrm{t}) = \mathrm{N}(\mathrm{d}_1)\mathrm{S} - \mathrm{N}(\mathrm{d}_2)\mathrm{K}\mathrm{e}^{-rt} \tag{6}$$

$$\mathrm{d}_1 = \frac{1}{\sigma\sqrt{\mathrm{t}}}\left[\ln\left(\frac{S}{K}\right) + t\left(r + \frac{\sigma^2}{2}\right)\right] \tag{7}$$

$$\mathrm{d}_2 = \frac{1}{\sigma\sqrt{\mathrm{t}}}\left[\ln\left(\frac{S}{K}\right) + t\left(r - \frac{\sigma^2}{2}\right)\right] \tag{8}$$

$$N(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} \mathrm{e}^{-\frac{1}{2}z^2} dz \tag{9}$$

C = Call option price

S = Current stock price

K = Strike price of the option

r = risk-free interest rate (a number between 0 and 1)

σ = volatility of the stocks return (a number between 0 and 1)

t = time to option maturity (in years)

N = normal cumulative distribution function

The VIX Equation

$$\sigma^2 = \frac{2}{T}\sum_i \frac{\Delta K_i}{K_i^2} e^{RT} Q(K_i) - \frac{1}{T}(\frac{F}{K_0} - 1)^2 \tag{10}$$