Zeppelin Universität

Chair of Empirical Finance and Econometrics

Measuring the information content of VIX volatility in comparison to historic volatility

Humboldt Project

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1 Introduction: The Importance of Volatility Measurement

1.1 Why Volatility matters: Volatility as Key Input to Option Pricing Models and Risk Measures

Distributional characteristics of asset returns are of high interest for the financial sector. They are for example key input to the pricing of financial instruments like derivatives, or to risk measures, such as the Value-at-Risk. Moreover they give information on the risk-return trade-off, which is a central question in portfolio allocation and managerial decision making. Of particular interest is the asset's return volatility, being the most dominant time-varying distribution characteristic. (**andersen2003**).
Risk is important because.. . The concept of risk is closely linked to that of volatility, as the second moment characterstic of asset return distributions. Many pricing models measure risk through volatility, which thus influences the expected return (**harvey1992**). Moreover many risk-measure, such as the Value-at-Risk are very closely related to volatility. (Alternative)
Volatility "seeks to capture the strength of the (unexpected) return variation over a given period of time" (Torben G. Andersen, Bollerslev and Diebold 2001, p.7)
As volatility is not directly observable, it has to be estimated. During the last years, considerable research has been devoted to the question, how volatility can be measured or estimated. Two prominent categories of approaches are on the one hand time series models using historic volatility, and on the other hand implied models using option price data[1]. In the recent years, Black-and-Scholes implied volatility measurement gained popularity, this approach uses the forward-looking nature of option prices. Options are contracts, giving the holder the right to either buy (call option), or sell (put option), an underlying asset, at a specified date in the future for a certain price (John et al. 2006). Assuming rational agents/expectation, the market uses all available information to form

[1] There are, of course, various other methods, such as nonparametric methods or neural networks based models (Jiang, George J & Tian 2003), however they shall not be discussed here

it's expectation about future price movements and thus about volatility. Assuming furthermore that the market is efficient (meaning as Eugene Fama defined it, that prices reflect all available information), the market's estimate of future volatility is the best possible forecast possible, given the current information (Bent Jesper Christensen and Hansen 2002). Due to this forward looking component of option contracts, option prices indirectly contain the market participants' expectations of the underlying asset's future movements. A widely used model to price this option contracts is the Black-and-Scholes-Merton model, which uses the option's volatility as an input factor. By using observed option prices as the input and solving for volatility, it is possible to obtain a volatility measure that is widely believed to be "informationally superior to the historic volatiltiy of the underlying asset" (Jiang, George J & Tian 2003, p.1305).

Early studies found implied volatility to be a biased forecast of realized volatility, not containing significantly more information than historic volatility. More recent studies however presented evidence that there is important information contained in option prices, that adds to the efficiency of volatility forecasting when implied volatility is included (ibid.). A reason for this discrepancy in results could be that early studies did not consider several data and methodological problems, such as long enough time series, a possible regime shift around the crash in 1987 and the use of non-overlapping samples (ibid.). Bent J Christensen and Prabhala for example took this into account and found that implied volatility outperforms historic volatility. All in all Jiang, George J & Tian summarize, that collectively "these studies present evidence that implied volatility a more efficient forecast for future volatility than historic volatility" (p.1306).

1.2 Weaknesses of Existing Models: VIX Introduced by CBOE

Even though BS implied volatility is found to be the overall more efficient forecast of realized volatility compared to historic volatility (ibid.), the BS implied volatility has some specification problems. Firstly, BS implied volatility focuses on at-the-money options. The advantage is, that at-the-money options are the once most actively traded and thus the most liquid ones. However this focus fails to include information contained in other options. Moreover, volatility estimation with the BS model, includes the same assumptions as are made in the BS model itself. Thus tests based on the BS equation are actually joined tests of market efficiency (as market efficiency is assumped to use option prices for volatility estimation, as mentioned above) and the BS model, and therefore suffer from a model misspecification error (Jiang, George J & Tian 2003). That is why during the last years, implied volatility indexes which are not based on a pricing assumption gained popularity. One of these model-free implied volatility indexes is the VIX from CBOE.

- power of volatility models lies in out-of-sample forecasting power
- so far BS implied volatility models had the best out of sampling forecasting power, but they have several problems (most importantly joint hypothesis problem)

2 Selected volatility concepts and models of volatility measurement

This section presents first some stylized facts of financial data, and gives an introduction to the different ways to estimate volatility. By pointing out the advantages and disadvantages of the concepts and models and their fit to the stylized facts, the HAR-RV approach shall be motivated.

2.1 The Return Process and Stylized Facts of Financial Data

As mentioned in the introduction, the challenge when measuring volatility is, that stock return volatility is not directly observable (**tsay2005**). This problem evolves from the fact that we can only observe one realization of the underlying data generating process, and even though stocks are traded and thus have market prices which could be used for volatility measurement, there is no continuous data available and even for high-frequency data and extremely liquid markets microstructure effects and noise prevent getting close to a continuous sample path. It is thus only possible to estimate avearages of discrete volatility for a given period of time. (Torben G. Andersen, Bollerslev and Diebold 2001).

There are however several approaches that should be introduced here. To start with, the definition of the simple gross return is

$$1 + R_t = \frac{P_t}{P_{t-1}} \tag{1}$$

In the continuous-time setting, continuously compounded returns are used, which are given by

$$r_t = ln(1 + R_t) = ln(\frac{P_t}{P_t} + \frac{P_{t-1} - P_t}{P_t}) = ln\frac{P_t}{P_{t-1}} = p_t - p_{t-1} \ with \ p_t = ln(P_t) \tag{2}$$

When observed over time, this asset returns show some distributional properties, often referred to as stylized facts of asset returns. Observed by many authors, only a few shall be mentioned here. Corsi for example mentions particularly the very strong persistence of autocorrelation of the square and absolute returns, which regularly poses challenges to econometric models. Moreover return probability density functions are often leptocurtic with fat tails. As the time scale increases, the return distribution slowly converge to the normal distribution, but before convergence, the return distribution has different shapes depending on the time scale. Financial data also show evidence of scaling as described firstly by Mandelbrot, which is connected to the idea that patterns appear in different times, or that the distribution for returns has similar functional forms for various choices of the time interval.

Moreover Torben G. Andersen, Bollerslev and Diebold mentions that first, even though raw returns have a leptocurtic distribution, the returns standardized by realized volatility are approximately Gaussian. Second, the distribution of realized volatility of returns itself is right skewed, the one of the logarithms of realized volatility however are also approximately Gaussian. Third, the long-run dynamics of realized logarithmic volatilities are well approximated by a fractionally-integrated long-memory process. Other authors who mention stylized facts: (Jiang, George J & Tian 2003).

2.2 Concepts and Models using Historic Volatility

2.2.1 Volatility Concept and Non-parametric ex-post Volatility Measurement - Realized Volatility

By definition “volatility seeks to capture the strength of the (unexpected) return variation over a given period of time” (Torben G. Andersen, Bollerslev and Diebold 2001, p.7). However, there are multiple concepts and definitions of asset volatility. According to Torben G. Andersen, Bollerslev and Diebold the concepts can be grouped in (i) the *notional volatility* corresponding to the ex-post sample-path return variability over a fixed time interval, (ii) the ex-ante *expected volatility* over a fixed time interval or the (iii) the *instantaneous volatility* corresponding to the strength of the volatility process at a point in time. For this paper, given the dataset of actual return observations, one can compute the ex-post realized volatility.

It can be shown, that under some assumptions, realized volatility as the sum of squared high frequency returns, can be used to approximate the quadratic variation process which is the variation in a continuous time setting. This approach mainly building on the work of Torben G. Andersen, Bollerslev and Diebold und [noch jemanden finden] shall only be briefly introduced here.

To begin with, it should be assumed that we have a continuous-time no-arbitrage setting. As return volatility aims to capture the strength of the unexpected return variation, one needs to define the component of a price change as opposed to an expected price movement. In discrete time this can be done by specifying the conditional mean return using for example an asset pricing model. In the continuous time setting however it requires the decomposition of the return process in an expected and innovation component. Torben G. Andersen, Bollerslev and Diebold show, that under certain assumptions the log-price process must constitute a semi-martingale process, which allows for the decomposition of the instantaneous return process into an expected return component, and a martingale innovation. They show furthermore, that one can refer to the quadratic variation process of this martingale component as a volatility measure, as the quadratic variation process represents the (cumulative) realized sample path variability of the martingale over any fixed time interval. To be precise, they define *notional volatility* as the increment to the quadratic variation for the return series, measured ex-post.

Assuming that the mean of the return process is zero, taking the expected value of the notional volatility and extending this concept slightly, one gets the *realized volatility*, defined over the $[t-h, t], 0 < h \leq t \leq T$ time interval as

$$v^2(t, h; n) = \sum_{i=1}^{n} r(t - h + (i/n) \times h, h/n)^2 \tag{3}$$

Torben G. Andersen, Bollerslev and Diebold show not only that the realized volatility is an *unbiased* estimator of ex-ante expected volatility, or at least approximately unbiased when relaxing the zero mean assumption and taking a high sample frequency (their proposition 4). Moreover, Torben G. Andersen, Bollerslev and Diebold show that the realized volatility is a *consistent* nonparametric measure of the notional volatility for increasingly finely sampled returns over any fixed length interval (their proposition 5). So in summary, the increment to the quadratic return variation and thus past volatility can be consistently and well approximated through the accumulation of high-frequency squared returns.

For the purpose of this paper it shall only be said, that quadratic variation is... .

2.2.2 Volatility Model - HAR-RV Model

Having introduced the concept of notional volatility and it's approximation by realized volatility, we now turn to volatility modelling/measurement. To measure volatility, one can separate between parametric and non-parametric methods. Whereas parametric methods try to measure the expected volatility making different assumptions about both the functional form and the variables in the information set available, non-parametric methods try to quantify notional volatility directly. The realized volatility is an example for a non-parametric methods. However, to forecast or estimate volatility ex-ante, this paper will refer to one type of the parametric methods, termed the HAR-RV model.

As mentioned in the section above, the logreturn process can be decomposed in a predictable and finite variation process, and a local martingale. Corsi, does so by assuming the standard continuous time diffusion process:

$$dp_t = \mu(d)dt + \sigma_t dW_t \tag{4}$$

with $p(t)$ being the logarithm of the instantaneous price, $\mu(t)$ a cadlág finite variation process, $W(t)$ a standard Brownian motion, and $\sigma(t)$ a stochastic process independent of $W_{p,t}$.

As in Torben G. Andersen, Bollerslev and Diebold they approximate the instantaneous/notional variance with the sum of squared returns, but term not this variance as volatility, but it's square root. This terminology should also be used for the remainder of this paper, with the realized volatility for one trading day being then:

$$RV_t^{(d)} = \sqrt{\sum_{j=0}^{M-1} r_t^2 - j \times \Delta} \tag{5}$$

with $\Delta = 1d/M$ being the sampling frequency and $r_t^2 - j \times \Delta$ defined as the continuously compounded *Delta*-frequency returns.

Combining this notion of volatility with the Heterogeneous Market Hypothesis by **mueller1993**. This

Also **andersen2003** point out the advantage of using high-frequency returns is not only that they help predicting again high-frequency returns, but also that they contain information for longer horizons, such as monthly or quarterly.

2.3 Implied volatility

2.3.1 The General Idea of Implied Volatility

Torben G. Andersen, Bollerslev and Diebold define *implied volatility* as consisting of a parametric volatility model for returns, accompanied by an asset pricing model and an augmented information set, including also option prices. Having the derivative prices, it is possible to extract a value for the expected volatility, by inverting the theoretical asset pricing model. It is however important to note, that all of these procedures depend on the assumptions that are made in the asset pricing model (Torben G. Andersen, Bollerslev and Diebold 2001).

- explain basic idea of BS implied volatility
- advantages of BS implied volatility: forward-looking nature of option prices
- disadvantages of BS implied volatility: joint hypothesis problem due to underlying pricing assumption (is a joint test of market efficiency and underlying pricing assumption), use only at-the-money options and fail to incorporate information,..

Disadvantages of Black and Scholes: Black and Scholes uses only at-the-money option and thus fails to incorporate information (Jiang, George J & Tian 2003). Black and Scholes are joint tests of market efficiency and the B-S model, thus studies are subject to model misspecification errors (ibid.).

2.3.2 VIX and Model-Free Implied Volatility

- explain basic idea of model-free implied volatility
- advantages of model-free implied volatility: solved joint hypothesis problem (direct test of market efficiency), can incorporate not only at-the-money options,..
- the VIX as the model-free implied volatility estimate from the Cboe

Primilary described and derived by Britten-Jones and Neuberger. Instead of being based on a specific option pricing model, it is derived entirely from no-arbitrage conditions. After that some papers did various corrections, such as Jiang, George J & Tian extended the model so that is not derived under diffusion assumptions and generalized it to processes including random jumps. Two advantage of the model-free option implied volatility, are firstly that it has no pricing assumption and thus constitutes a direct test of the option market's informational efficiency, and not a joined test of market efficiency and an assumed option pricing model. Secondly it incorporates information from options across different strike prices.

3 Methodology and Data

3.1 Methodology: Linear Regression and HAR-RV model

- Reg1: without VIX
  - Reg1a: regress realized volatility on historic volatility using simple linear regression
  - Reg1b: regress realized volatility on historic volatility using HAR-RV model
- Reg2: with VIX
  - Reg2b: regress realized volatility on historic volatility using simple linear regression
  - Reg2b: regress realized volatility on historic volatility using HAR-RV model

$$\sigma^{RV}_{t,x} = \alpha_{x,t} + \beta^{HV}_{x,t}\sigma^{HV}_{x,t} \tag{6}$$

$$\sigma^{RV}_{x,t} = \alpha_{x,t} + \beta^{HV,d}_{x,t}\sigma^{HV,d}_{x,t} + \beta^{HV,w}_{x,t}\sigma^{HV,w}_{x,t} + \beta^{HV,m}_{x,t}\sigma^{HV,m}_{x,t} \tag{7}$$

$$\sigma^{RV}_{x,t} = \alpha_{x,t} + \beta^{HV}_{x,t}\sigma^{HV}_{x,t} + \beta^{VIX}_{x,t} + \sigma^{VIX}_{x,t} \tag{8}$$

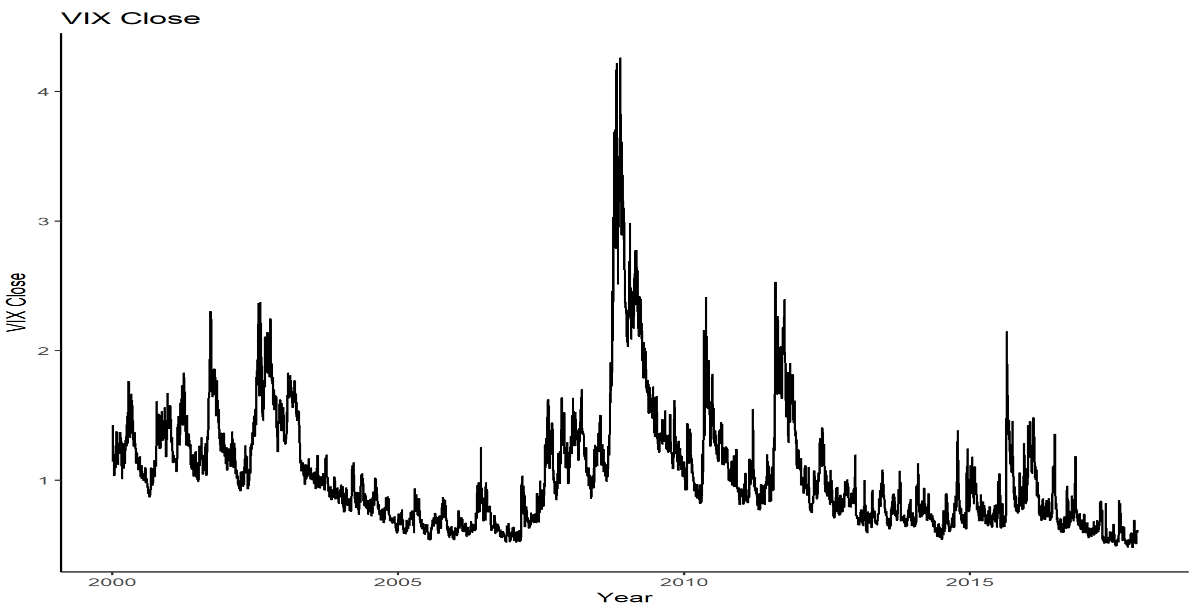
$$\sigma^{RV}_{x,t} = \alpha_{x,t} + \beta^{HV,d}_{x,t}\sigma^{HV,d}_{x,t} + \beta^{HV,w}_{x,t}\sigma^{HV,w}_{x,t} + \beta^{HV,m}_{x,t}\sigma^{HV,m}_{x,t} + \beta^{VIX}_{x,t} + \sigma^{VIX}_{x,t} \tag{9}$$

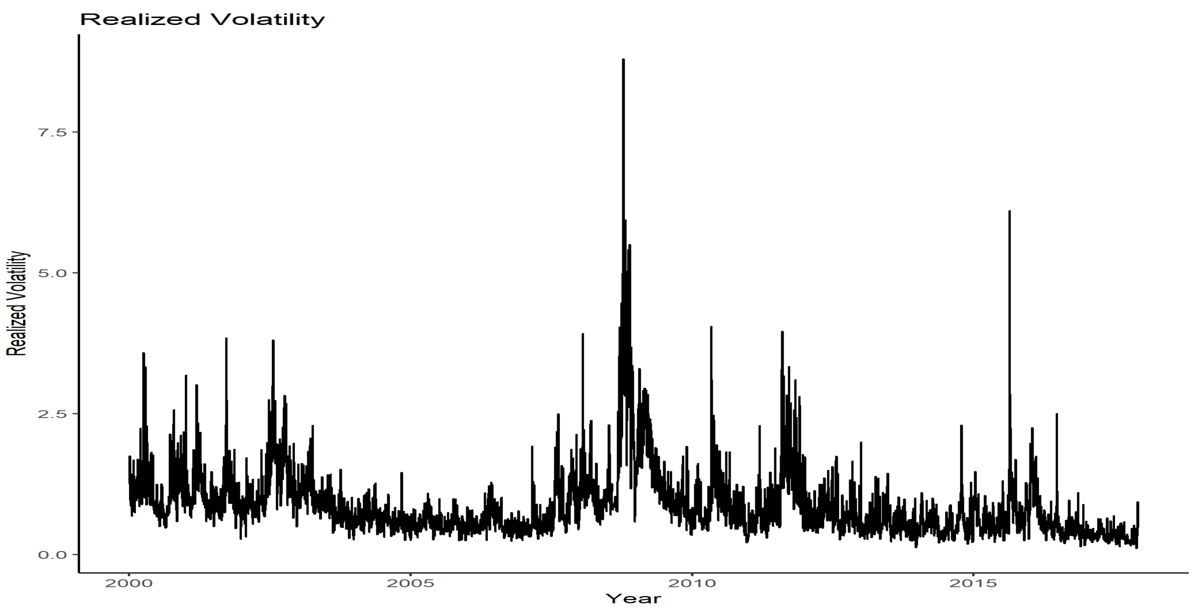
3.2 Data and Calculation of Input Factors

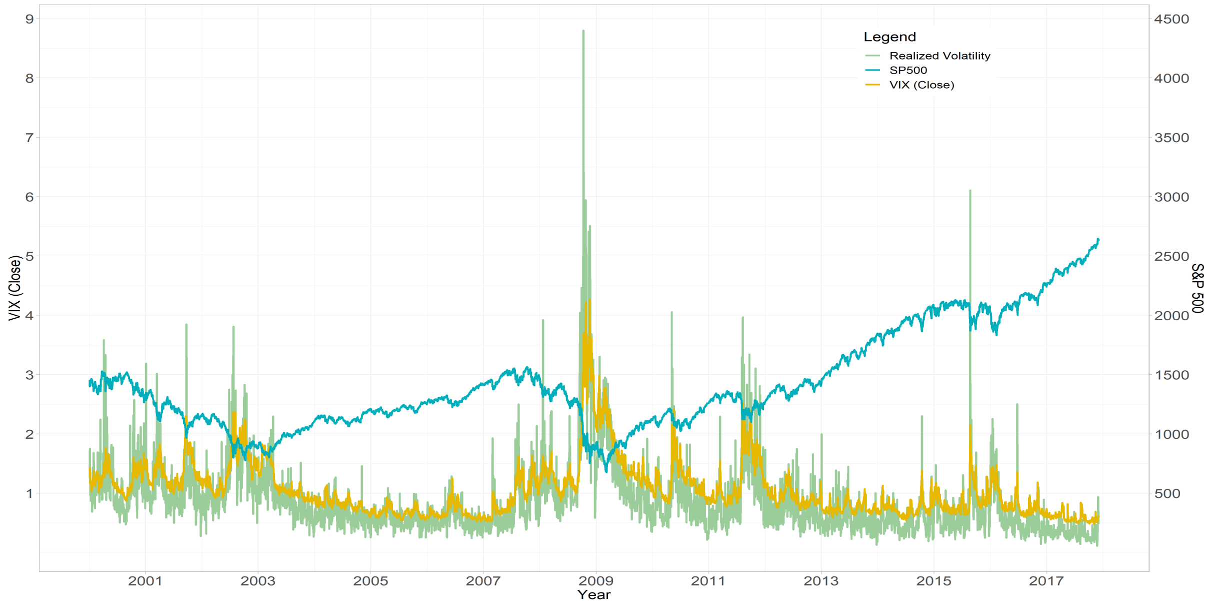
Graphics

Measure for daily return variability should be realized volatility, as Torben G. Andersen, Bollerslev and Diebold suggest, that under suitable conditions it provides an unbiased estimator of the return volatility.

- S&P 500 index data on daily basis
- sampling period: 2000 - 2018
- realized volatility: daily realized volatility of S&P 500, calculated using 5 minute returns, retrieved from


VIX Close
4
3
2
1
VIX Close
2000
2005
2010
2015
Year


Realized Volatility
7.5
5.0
2.5
0.0
Realized Volatility
2000
2005
2010
2015
Year



- model-free implied volatility: VIX index data
- historic volatility: lagged realized volatility, for HAR-RV model use the average over the time period used to forecast

3.3 Limitations

As volatility is stochastic, the ex-ante estimation will not equal the return volatility, as it is a measurement over an aggregated discrete time period (Torben G. Andersen, Bollerslev and Diebold 2001).

4 Results

	AIC	BIC
OLS	2819.78	2851.77
OLS with VIX	1921.53	1959.92
log OLS	1862.08	1894.07
log OLS with VIX	1156.30	1194.68
HAR-RV	-56620.66	-56588.65

5 Discussion

Figure 1: Level-level regression

	Historic	Historic and VIX
Intercept	4.49^{***}	-23.03^{***}
	(0.99)	(1.25)
RV_t^d	0.36^{***}	0.23^{***}
	(0.02)	(0.02)
RV_t^w	0.39^{***}	0.26^{***}
	(0.03)	(0.03)
RV_t^m	0.19^{***}	-0.29^{***}
	(0.02)	(0.03)
VIX		88.70^{***}
		(2.81)
R^2	0.71	0.76
Adj. R^2	0.71	0.76
Num. obs.	4434	4434
RMSE	33.23	30.03

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

Figure 2: log-log regression

	Historic	Historic and VIX
Intercept	0.19^{***}	2.68^{***}
	(0.04)	(0.10)
RV_t^d	0.34^{***}	0.24^{***}
	(0.02)	(0.02)
RV_t^w	0.41^{***}	0.28^{***}
	(0.03)	(0.03)
RV_t^m	0.20^{***}	-0.13^{***}
	(0.02)	(0.02)
VIX		0.80^{***}
		(0.03)
R^2	0.73	0.77
Adj. R^2	0.73	0.77
Num. obs.	4435	4435
RMSE	0.30	0.28

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

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