Measuring the Information Content of VIX Volatility

Context: Humboldt Project

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Introduction

Motivation: Why this project? Why does Volatility matter?

- For the stability of the financial system, precise risk measurement is of great importance
  - Volatility is closely related to risk
  - Volatility is crucial input to risk measures, such as the Value at Risk[1]
- Moreover volatility is used for..
  - .. the pricing of financial instruments, such as derivatives
  - .. the risk-return trade-off and therefore management decisions
- Finally, a good volatility estimate might be a valuable starting point for a forecast

[1] The Value at Risk is a quantile of the loss function, used for example by banks to estimate the amount of assets needed to cover possible losses. It estimates which loss is not going to be exceeded in a given time interval, for a given probability

More closely: What exactly is Volatility?

- In Finance, we are usually interested in the *conditional* standard deviation from the expected value of the underlying asset return (Tsay 2005)
- What causes asset price movement and thus volatility?
  - Assuming Market efficiency (as introduced by Malkiel and Fama), stock prices incorporate all available information from the market, because of competition and free entry
  - But that does not does not give us information about the volatility of stock prices

The Problem: Why is it so hard to measure and forecast volatility?

- Volatility is not directly observable
  - We can estimate it for a given time period
  - Problem is, stock volatility consists of intraday and overnight volatility, each containing different information
- We can however observe some characteristics of volatility (stylized facts), such as
  - Volatility clustering (periods of high volatility are followed by periods of high volatility, and similar for low volatility)
  - Volatility is often stationary (it varies only within a fixed range)
  - Leverage effect: volatility reacts differently to price drop or increase

Maybe a solution: How volatility has been calculated so far

- According to this stylized facts, we can use (econometric) models that best capture the characteristics of volatility
- There are however multiple approaches to model volatility, examples are
  - Econometric models using historic volatility (e.g. ARCH)
  - Black-and-Scholes implied volatility
- Many papers argue, that Black-and Scholes implied volatility contains significant information for realized volatility, (e.g. Jiang and Tian (2005))

Maybe a solution: Black-and-Scholes implied volatility

- Intuition behind Black-and-Scholes (BS) implied volatility
  - The BS model is an option pricing model, that uses volatility as an input
  - \+ By using option prices from the market, it is possible to reverse the calculation and derive an *option implied volatility*
  - \+ as options are contracts to buy an underlying in the future, then contain the market's expectation of future stock price movement
- There are however some problems with the BS implied volatility
  - – The BS model is mainly based on at-the-money options
  - – Most importantly the BS model makes an assumption about how the prices are formed (pricing assumption), and assumes that stock prices follow geometric Brownian motion)

➨ Joint hypothesis problem: Testing for BS implied volatility is always a test of *both* market efficiency and the BS pricing assumptions

Solving the joint hypothesis problem: Model-free implied volatility

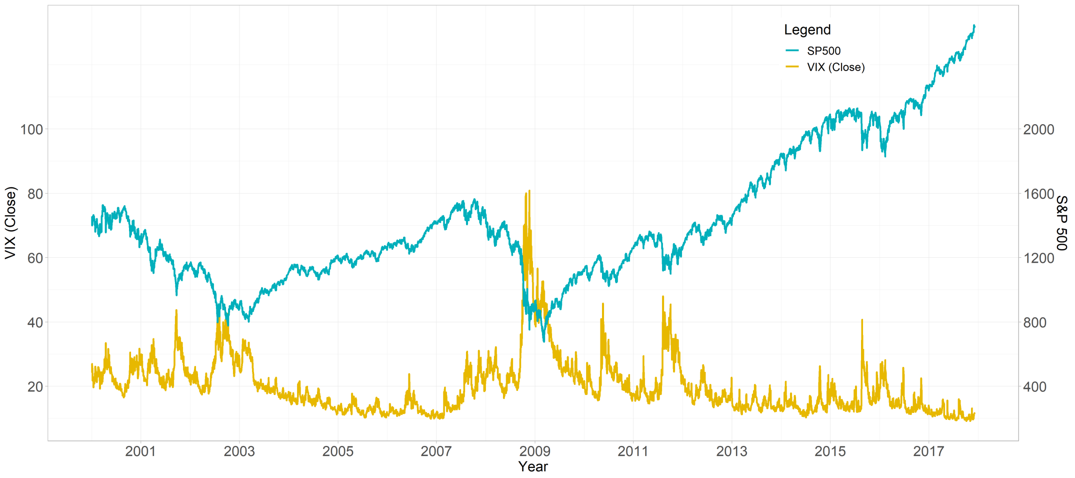
- For model-free implied volatility, no assumption is made regarding the underlying stochastic process, and it is not based on any particular option pricing model
  + The VIX includes information from both at-the-money and out-of-money options
  + As no pricing assumption is made, the model-free implied volatility provides a direct test of the informational efficiency of the options market
- One of the fist model-free implied volatility indices was the VIX from CBOE (CBOE 2009)
  - VIX measures the market's expectation of 30-day volatility

Data

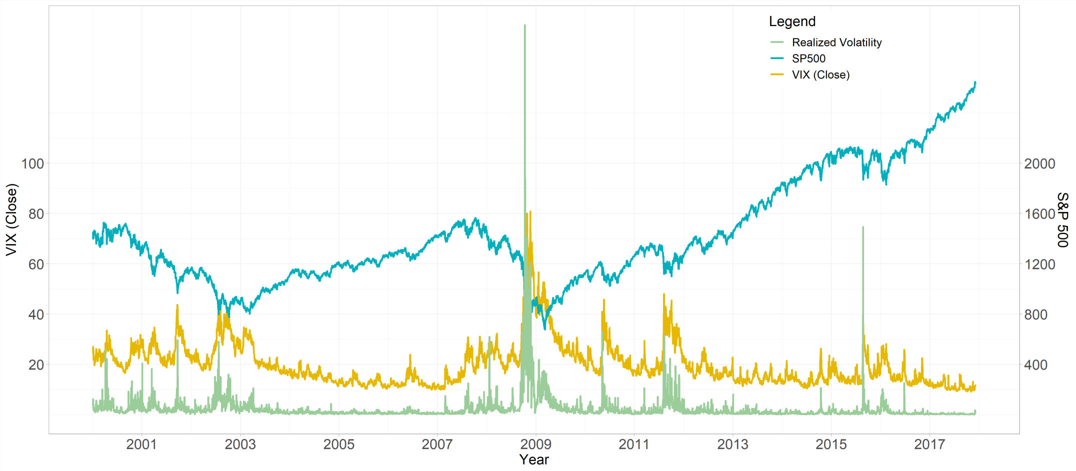
Volatility of S&P 500

- Components of Dataset
  - VIX, downloaded from CBOE
  - Realized Volatility, downloaded from Oxford Man Institute
  - S&P 500, downloaded from CBOE
- Sampling period January 2000 - December 2017

S&P 500 in Comparison to VIX



S&P 500 in Comparison to VIX and Realized Volatility



Method

My Method: Regression and HAR Model

Stepwise regression to compare information content:

$$\sigma_{t+1} = c + \beta^d RV_{t-1}^d + \beta^w RV_{t-1}^w + \beta^m RV_t^m \tag{1}$$

$$\sigma_{t+1} = c + \beta^d RV_{t-1}^d + \beta^w RV_{t-1}^w + \beta^m RV_t^m + \beta^{VIX} VIX_t^{VIX} \tag{2}$$

$+$ the same regressions as log-log model

with the weekly aggregation period being (monthly similar with 20 days):

$$\sigma_t^w = \frac{1}{5}(RV_t^d + RV_{t-1d}^d + RV_{t-2d}^d + RV_{t-3d}^d + RV_{t-4d}^d)$$

and Realized Variance (RV) being: $RV_t^d = \sqrt{\sum_{m=1}^{M} r_{t,m}^2}$

σ = Volatility (sd), RV = Realized Variance with M equally spaced intraday returns for a day t, with m = 1,..,M, here 10min returns

My Method: Regression and HAR Model

HAR-RV Model (according to Corsi (2009), but not implemented yet), idea:

$$\sigma_{t+1} = c + \beta^{d} RV_{t-1}^{d} + \beta^{w} RV_{t-1}^{w} + \beta^{m} RV_{t}^{m} + w_{t+1d}^{\tilde{d}} \tag{3}$$

$\tilde{w}_{t+1d}^{d}$ = innovation

Results so far

Regression Results

Figure 1: level-level Model

	Historic	Historic with VIX
Intercept	0.00^{***}	-0.01^{***}
	(0.00)	(0.00)
RV_t^d	0.27^{***}	0.23^{***}
	(0.02)	(0.02)
RV_t^w	0.39^{***}	0.36^{***}
	(0.03)	(0.03)
RV_t^m	0.25^{***}	-0.04
	(0.03)	(0.03)
VIX		0.00^{***}
		(0.00)
R^2	0.54	0.57
Adj. R^2	0.54	0.57
Num. obs.	4436	4436
RMSE	0.02	0.02

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

Figure 2: log-log Model

	Historic	Historic with VIX
Intercept	-0.25^{***}	-7.85^{***}
	(0.05)	(0.28)
RV_t^d	0.34^{***}	0.24^{***}
	(0.02)	(0.02)
RV_t^w	0.40^{***}	0.28^{***}
	(0.03)	(0.03)
RV_t^m	0.21^{***}	-0.13^{***}
	(0.02)	(0.03)
VIX		1.60^{***}
		(0.06)
R^2	0.73	0.77
Adj. R^2	0.73	0.77
Num. obs.	4437	4437
RMSE	0.60	0.55

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

Possible Problems coming up

Next steps/Questions to solve

- Having gathered all this information about volatility measurement, what is the most accurate way to set up my regressions and my HAR-RV model?
- Next step: Bring current state of work to paper

References

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Appendix

The Black and Scholes Equation

price of an option over time:

$$\frac{\partial C}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial C^2} + rS\frac{\partial C}{\partial S} = rC \tag{4}$$

calculate the price of European call and put option:

$$C(S, t) = N(d_1)S - N(d_2)Ke^{-rt} \tag{5}$$

$$d_1 = \frac{1}{\sigma\sqrt{t}}\left[\ln\left(\frac{S}{K}\right) + t\left(r + \frac{\sigma^2}{2}\right)\right] \tag{6}$$

$$d_2 = \frac{1}{\sigma\sqrt{t}}\left[\ln\left(\frac{S}{K}\right) + t\left(r - \frac{\sigma^2}{2}\right)\right] \tag{7}$$

$$N(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\frac{1}{2}z^2} dz \tag{8}$$

C = Call option price

S = Current stock price

K = Strike price of the option

r = risk-free interest rate (a number between 0 and 1)

σ = volatility of the stocks return (a number between 0 and 1)

t = time to option maturity (in years)

N = normal cumulative distribution function

The VIX Equation

$$\sigma^2 = \frac{2}{T} \sum_i \frac{\Delta K_i}{K_i^2} e^{RT} Q(K_i) - \frac{1}{T} (\frac{F}{K_0} - 1)^2 \tag{9}$$