Zeppelin Universität

Chair of Empirical Finance and Econometrics

Measuring the information content of VIX volatility in comparison to historic volatility

Humboldt Project

Name:	Sophia Charlotte Gläser
Student number:	15202284
Programme:	Corporate Management and Economics
Term:	Fall Semester 2018
Examinor:	Prof. Dr. Franziska Peter
Due date:	31.01.2019

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1 Introduction: The Importance of Volatility Measurement

1.1 Why Volatility matters: Volatility as Key Input to Option Pricing Models and Risk Measures

Distributional characteristics of asset returns are of high interest for the financial sector. They are for example key input to the pricing of financial instruments like derivatives, or to risk measures, such as the Value-at-Risk. Moreover they give information on the risk-return trade-off, which is a central question in portfolio allocation and managerial decision making. Of particular interest is the asset's return volatility, being the most dominant time-varying distribution characteristic. (**andersen2003**).
Risk is important because.. . The concept of risk is closely linked to that of volatility, as the second moment characterstic of asset return distributions. Many pricing models measure risk through volatility, which thus influences the expected return (**harvey1992**). Moreover many risk-measure, such as the Value-at-Risk are very closely related to volatility. (Alternative)
Volatility "seeks to capture the strength of the (unexpected) return variation over a given period of time" (Torben G. Andersen, Bollerslev and Diebold 2001, p.7)
As volatility is not directly observable, it has to be estimated. During the last years, considerable research has been devoted to the question, how volatility can be measured or estimated. Two prominent categories of approaches are on the one hand time series models using historic volatility, and on the other hand implied models using option price data[1]. In the recent years, Black-and-Scholes implied volatility measurement gained popularity, this approach uses the forward-looking nature of option prices. Options are contracts, giving the holder the right to either buy (call option), or sell (put option), an underlying asset, at a specified date in the future for a certain price (John et al. 2006). Assuming rational agents/expectation, the market uses all available information to form

[1]There are, of course, various other methods, such as nonparametric methods or neural networks based models (Jiang, George J & Tian 2003), however they shall not be discussed here

it's expectation about future price movements and thus about volatility. Assuming furthermore that the market is efficient (meaning as Eugene Fama defined it, that prices reflect all available information), the market's estimate of future volatility is the best possible forecast possible, given the current information (Bent Jesper Christensen and Hansen 2002). Due to this forward looking component of option contracts, option prices indirectly contain the market participants' expectations of the underlying asset's future movements. A widely used model to price this option contracts is the Black-and-Scholes-Merton model, which uses the option's volatility as an input factor. By using observed option prices as the input and solving for volatility, it is possible to obtain a volatility measure that is widely believed to be "informationally superior to the historic volatiltiy of the underlying asset" (Jiang, George J & Tian 2003, p.1305).

Early studies found implied volatility to be a biased forecast of realized volatility, not containing significantly more information than historic volatility. More recent studies however presented evidence that there is important information contained in option prices, that adds to the efficiency of volatility forecasting when implied volatility is included (ibid.). A reason for this discrepancy in results could be that early studies did not consider several data and methodological problems, such as long enough time series, a possible regime shift around the crash in 1987 and the use of non-overlapping samples (ibid.). Bent J Christensen and Prabhala for example took this into account and found that implied volatility outperforms historic volatility. All in all Jiang, George J & Tian summarize, that collectively "these studies present evidence that implied volatility a more efficient forecast for future volatility than historic volatility" (p.1306).

1.2 Weaknesses of Existing Models: VIX Introduced by CBOE

Even though BS implied volatility is found to be the overall more efficient forecast of realized volatility compared to historic volatility (ibid.), the BS implied volatility has some specification problems. Firstly, BS implied volatility focuses on at-the-money options. The advantage is, that at-the-money options are the once most actively traded and thus the most liquid ones. However this focus fails to include information contained

in other options. Moreover, volatility estimation with the BS model, includes the same assumptions as are made in the BS model itself. Thus tests based on the BS equation are actually joined tests of market efficiency (as market efficiency is assumped to use option prices for volatility estimation, as mentioned above) and the BS model, and therefore suffer from a model misspecification error (Jiang, George J & Tian 2003).
That is why during the last years, implied volatility indexes which are not based on a pricing assumption gained popularity. One of these model-free implied volatility indexes is the VIX from CBOE.

- power of volatility models lies in out-of-sample forecasting power
- so far BS implied volatility models had the best out of sampling forecasting power, but they have several problems (most importantly joint hypothesis problem)

Volatility is for example both a key input factor to risk measures (such as Value-at-risk), or pricing of derivative securities, which both again are crucial for financial decision making. As volatility can not be observed as directly as price can, it has to be both estimated and forecasted. There are multiple ways to forecast volatility. The strenght of a volatility model however lies in it's out-of-sample forecasting power (Poon and Granger 2003).
Volatility measures play an important role for financial market stability.
Stylized facts of financial market data suggests that return distributions are not i.i.d., meaning that the variance of returns over a long horizon can not be derived from a single observed period (ibid.).

2 Selected volatility concepts and models of volatility measurement

This section presents first some stylized facts of financial data, and gives both an introduction to the different ways to estimate volatility. By pointing out the advantages and disadvantages of the concepts and models and their fit to the stylized facts, the variables and model used for this paper shall be introduced. This leads to the conclusion, that HAR-RV is the model that should be used for this work.

To measure volatility, one can separate between parametric and non-parametric methods, where parametric models are both discrete and continuous time methods. For an encompassing overview, please see Torben G. Andersen, Bollerslev and Diebold.

2.1 The Return Process and Stylized Facts of Financial Data

As mentioned in the introduction, the challenge when measuring volatility is, that there are various definitions of asset volatility and for stocks the return volatility is not directly observable (**tsay2005**). This problem evolves from the fact that we can only observe one realization of the stochastic process generation stock prices, and even though stocks are traded and thus have market prices which could be used for volatility measurement, there is no continuous data available and it is always only possible to estimate volatility for a given period of time.
There are however several approaches that should be introduced here. To start with, the definition of the simple gross return:

$$1 + R_t = \frac{P_t}{P_{t-1}} \tag{1}$$

In the continuous-time setting, we use continuously compounded returns, given by

$$r_t = ln(1 + R_t) = ln(\frac{P_t}{P_t} + \frac{P_{t-1} - P_t}{P_t}) = ln\frac{P_t}{P_{t-1}} = p_t - p_{t-1}$$
$$with\ p_t = ln(P_t)$$

Torben G. Andersen, Bollerslev and Diebold found three stylized facts for the spot exchange rate market. First even though raw returns have a leptocurtic distribution, the returns standardized by realized volatility are approximately Gaussian. Second, the distribution of realized volatility of returns itself is right skewed, the one of the logarithms of realized volatility however are also approximately Gaussian. Third, the long-run dynamics of realized logarithmic volatilities are well approximated by a fractionally-integrated long-memory process.

Financial asset return volatility is time-varying, not only across time-periods, but also across asset classes, assets and countries (Torben G. Andersen, Bollerslev and Diebold 2001).

This stylized facts motivate the use of the HAR-RV model.

- strong persistence of the autocorrelation of square and absolute returns (Jiang, George J & Tian 2003)
- return distributions exhibits both fat tails and tail crossover (ibid.)
-

2.2 Concepts and Models using Historic Volatility

2.2.1 Volatility Concept - Realized Volatility

In order to measure the information content of the VIX implied volatility our model needs a dependent variable. There are multiple different volatility concepts, which can serve the measuring and modelling volatility and according to Torben G. Andersen, Bollerslev and Diebold can be grouped in (i) the notional volatility corresponding to the ex-post sample-path return variability over a fixed time interval, (ii) the ex-ante expected volatility over a fixed time interval or the (iii) the instantaneous volatility corresponding to the strength of the volatility process at a point in time. For this paper, given the dataset of actual return observations, one can compute the ex-post realized volatility.

It can be shown, that under some assumptions, realized volatility as the sum of squared high frequency returns, can be used to approximate the quadratic variation process which is the variation in a continuous time setting. This approach mainly building on the work of Torben G. Andersen, Bollerslev and Diebold und [noch jemanden finden] shall be briefly reproduced here.

To begin with, it should be assumed that we have a continuous-time no-arbitrage setting. By definition, return volatility aims to capture the strength of the unexpected return variation (the component of a price change as opposed to an expected price movement) over time. To identify the unexpected component can be done with discrete time assumption, by specifying the conditional mean return using for example an asset pricing model. In the continuous time setting this requires the decomposition of the return process in an expected and innovation component.

A comfortable feature of the continuously compounded returns, is that they are time additive, meaning

$$r(t,h) = r(t) - r(t-h) \; with \; 0 \leq h \leq t \leq T.$$

Assuming, that the asset prices are positive and finite, both price and return are defined in the interval [0,T] and as a consequence, r(t) has only countable many jump points in [0,T].

Assuming furthermore that the return process is a càdlàg process, that there are no arbitrage opportunities and frictions and that the expected return is finite, then the log-price process must constitute a semi-martingale. This leads to the following decomposition of the instantaneous return, into an expected return component and a martingale innovation

$$r(t) = p(t) - p(0) = \mu(t) + M(t) = \mu(t) + M^c(t) + M^j(t)$$

where $\mu(t)$ is a predictable and finite variation process, $M(t)$ is a local martingale which may be further decomposed into $M^c(t)$, a continuous sample path, infinite variation local martingale component, and $M^j(t)$, a compensated jump martingale.

Unfortunately, instantaneous returns can not be observed, thus this decomposition has to be transferred to the discrete interval setting. This is slightly complex, and for this work shall only be constituted, that that in discrete time there are two distinct terms in the return innovation instead of one, however one of these terms is a martingale component, too, and this is the dominant part.

2.2.2 Volatility Measurement - HAR-RV Model

Also **andersen2003** point out the advantage of using high-frequency returns is not only that they help predicting again high-frequency returns, but also that they contain information for longer horizons, such as monthly or quarterly.

2.3 Implied volatility

2.3.1 The General Idea of Implied Volatility

- explain basic idea of BS implied volatility
- advantages of BS implied volatility: forward-looking nature of option prices
- disadvantages of BS implied volatility: joint hypothesis problem due to underlying pricing assumption (is a joint test of market efficiency and underlying pricing assumption), use only at-the-money options and fail to incorporate information,..

Disadvantages of Black and Scholes: Black and Scholes uses only at-the-money option and thus fails to incorporate information (Jiang, George J & Tian 2003). Black and Scholes are joint tests of market efficiency and the B-S model, thus studies are subject to model misspecification errors (ibid.).

2.3.2 VIX and Model-Free Implied Volatility

- explain basic idea of model-free implied volatility
- advantages of model-free implied volatility: solved joint hypothesis problem (direct test of market efficiency), can incorporate not only at-the-money options,..
- the VIX as the model-free implied volatility estimate from the Cboe

Primilary described and derived by Britten-Jones and Neuberger. Instead of being based on a specific option pricing model, it is derived entirely from no-arbitrage conditions. After that some papers did various corrections, such as Jiang, George J & Tian extended the model so that is not derived under diffusion assumptions and generalized it to processes including random jumps. Two advantage of the model-free option implied volatility, are firstly that it has no pricing assumption and thus constitutes a direct test of the option market's informational efficiency, and not a joined test of market efficiency and an assumed option pricing model. Secondly it incorporates information from options across different strike prices.

3 Methodology and Data

3.1 Methodology: Linear Regression and HAR-RV model

- Reg1: without VIX
  - Reg1a: regress realized volatility on historic volatility using simple linear regression
  - Reg1b: regress realized volatility on historic volatility using HAR-RV model
- Reg2: with VIX
  - Reg2b: regress realized volatility on historic volatility using simple linear regression
  - Reg2b: regress realized volatility on historic volatility using HAR-RV model

$$\sigma_{t,x}^{RV} = \alpha_{x,t} + \beta_{x,t}^{HV}\sigma_{x,t}^{HV} \tag{2}$$

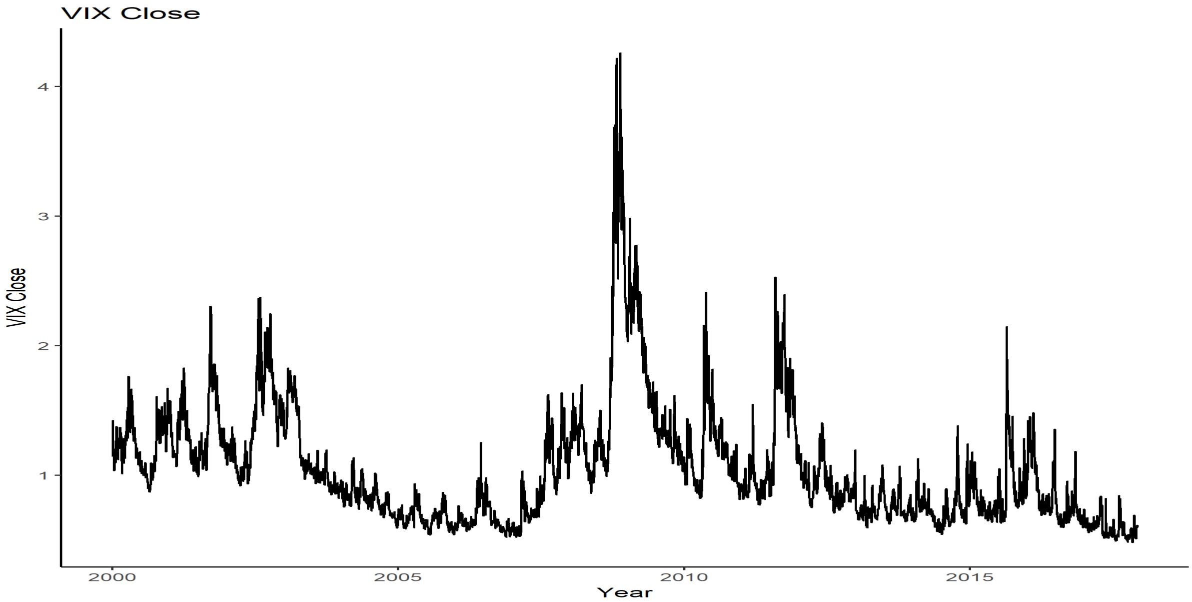
$$\sigma_{x,t}^{RV} = \alpha_{x,t} + \beta_{x,t}^{HV,d}\sigma_{x,t}^{HV,d} + \beta_{x,t}^{HV,w}\sigma_{x,t}^{HV,w} + \beta_{x,t}^{HV,m}\sigma_{x,t}^{HV,m} \tag{3}$$

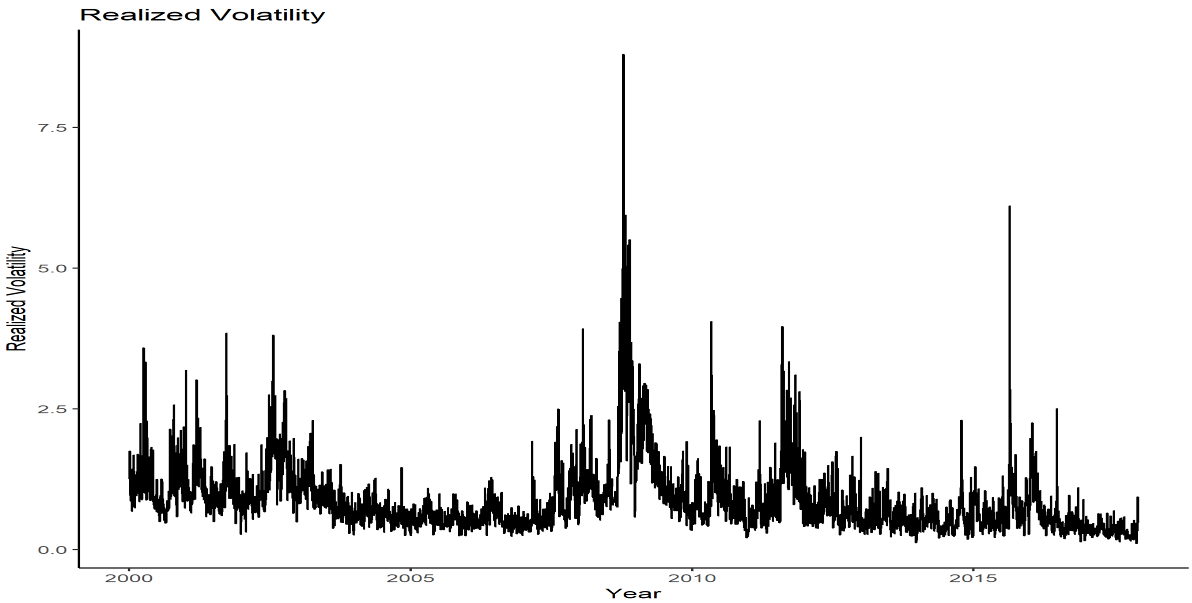
$$\sigma_{x,t}^{RV} = \alpha_{x,t} + \beta_{x,t}^{HV}\sigma_{x,t}^{HV} + \beta_{x,t}^{VIX} + \sigma_{x,t}^{VIX} \tag{4}$$

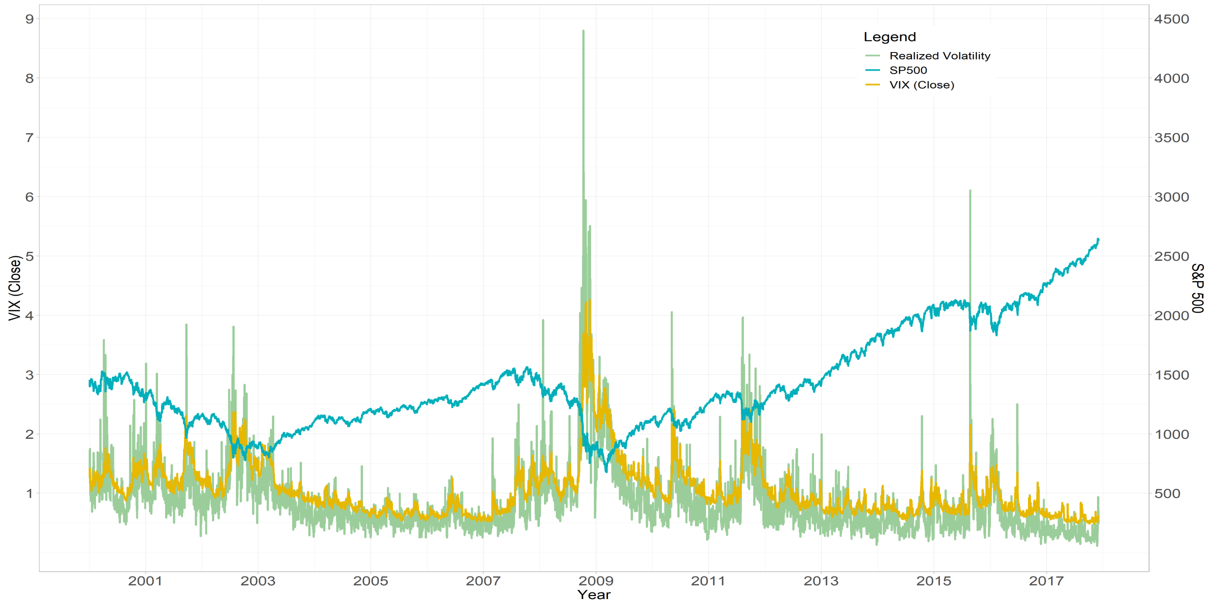
$$\sigma_{x,t}^{RV} = \alpha_{x,t} + \beta_{x,t}^{HV,d}\sigma_{x,t}^{HV,d} + \beta_{x,t}^{HV,w}\sigma_{x,t}^{HV,w} + \beta_{x,t}^{HV,m}\sigma_{x,t}^{HV,m} + \beta_{x,t}^{VIX} + \sigma_{x,t}^{VIX} \tag{5}$$

3.2 Data and Calculation of Input Factors

Graphics


VIX Close
VIX Close
4
3
2
1
2000
2005
2010
2015
Year


Realized Volatility
Realized Volatility
7.5
5.0
2.5
0.0
2000
2005
2010
2015
Year



Measure for daily return variability should be realized volatility, as Torben G. Andersen, Bollerslev and Diebold suggest, that under suitable conditions it provides an unbiased estimator of the return volatility.

- S&P 500 index data on daily basis
- sampling period: 2000 - 2018
- realized volatility: daily realized volatility of S&P 500, calculated using 5 minute returns, retrieved from
- model-free implied volatility: VIX index data
- historic volatility: lagged realized volatility, for HAR-RV model use the average over the time period used to forecast

4 Results

	AIC	BIC
OLS	2819.78	2851.77
OLS with VIX	1921.53	1959.92
log OLS	1862.08	1894.07
log OLS with VIX	1156.30	1194.68
HAR-RV	-56620.66	-56588.65

Figure 1: Level-level regression

	Historic	Historic and VIX
Intercept	4.49^{***}	-23.03^{***}
	(0.99)	(1.25)
RV_t^d	0.36^{***}	0.23^{***}
	(0.02)	(0.02)
RV_t^w	0.39^{***}	0.26^{***}
	(0.03)	(0.03)
RV_t^m	0.19^{***}	-0.29^{***}
	(0.02)	(0.03)
VIX		88.70^{***}
		(2.81)
R^2	0.71	0.76
Adj. R^2	0.71	0.76
Num. obs.	4434	4434
RMSE	33.23	30.03

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

Figure 2: log-log regression

	Historic	Historic and VIX
Intercept	0.19^{***}	2.68^{***}
	(0.04)	(0.10)
RV_t^d	0.34^{***}	0.24^{***}
	(0.02)	(0.02)
RV_t^w	0.41^{***}	0.28^{***}
	(0.03)	(0.03)
RV_t^m	0.20^{***}	-0.13^{***}
	(0.02)	(0.02)
VIX		0.80^{***}
		(0.03)
R^2	0.73	0.77
Adj. R^2	0.73	0.77
Num. obs.	4435	4435
RMSE	0.30	0.28

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

5 Discussion

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