Measuring the Information Content of VIX Volatility

Context: Humboldt Project

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Introduction

Motivation: Why this project? Why does Volatility matter?

- Risk measurement and the stability of the financial system
    - Volatility is closely related to risk
    - Volatility is crucial input to risk measures, such as the Value at Risk
- Volatility is used for..
    - .. the pricing of financial instruments, such as derivatives
    - .. the risk-return trade-off and therefore management decisions
- Forecast potential

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- In Finance, we are usually interested in the *conditional* standard deviation from the expected value of the underlying asset return (Tsay 2005)
- What causes asset price movement and thus volatility?
    - Assuming market efficiency (as introduced by Malkiel and Fama), stock prices incorporate all available information from the market
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The Problem: Why is it so hard to measure and forecast volatility?

- Volatility is not directly observable
  - We can estimate it for a given time period
  - The problem is, each period contains different information
- We can however observe stylized facts of volatility
  - Volatility clustering
  - Variation within a fixed range
  - Leverage effect: different reaction to price drop or increase

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Maybe a solution: How volatility has been calculated so far

- According to this stylized facts, we can use models that best capture the characteristics of volatility
- There are multiple approaches, examples are
    - Econometric models using historic volatility (e.g. ARCH)
    - Black-and-Scholes implied volatility
- Many papers argue, that Black-and Scholes implied volatility contains significant information for realized volatility (e.g. Jiang and Tian (2005))

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- Intuition behind Black-and-Scholes (BS) implied volatility
  - Option pricing model, that uses volatility as an input
  - + Reverse calculation and derive an *option implied volatility*
  - + Options contain the market's expectation of future stock price movement
- Problems with the BS implied volatility
  - – The BS model is mainly based on at-the-money options
  - – Most importantly the BS model has a pricing assumption
- Joint hypothesis problem

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Solving the joint hypothesis problem: Model-free implied volatility

- Model-free implied volatility is not based on any particular option pricing model
- One of the fist model-free implied volatility indices was the VIX from CBOE (CBOE 2009)
  - Measures the market's expectation of 30-day volatility
  - + Includes information from both at-the-money and out-of-money options
  - + No pricing assumption → direct test of the informational efficiency of the options market

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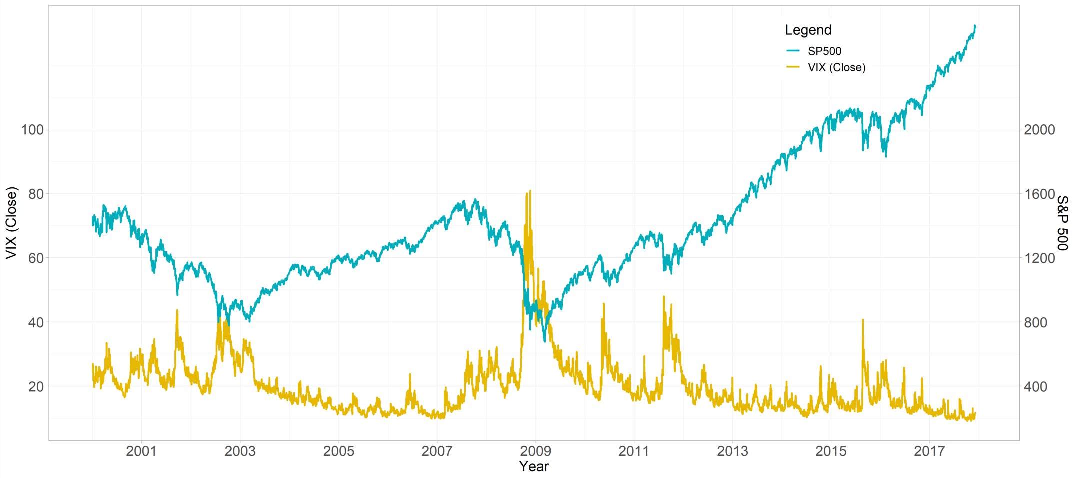
Data

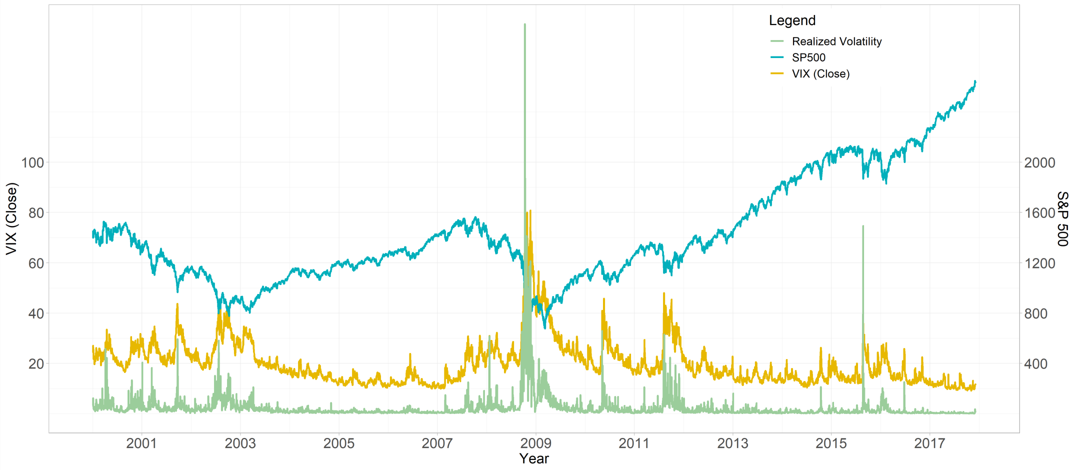
Volatility of S&P 500

- Components of Dataset
    - VIX, downloaded from CBOE
    - Realized Volatility, downloaded from Oxford Man Institute
    - S&P 500, downloaded from CBOE
- Sampling period January 2000 - December 2017

S&P 500 in Comparison to VIX



S&P 500 in Comparison to VIX and Realized Volatility



Method

My Method: Regression and HAR Model

Stepwise regression to compare information content:

$$\sigma_t = c + \beta^d RV_{t-1}^d + \beta^w RV_{t-1}^w + \beta^m RV_t^m \tag{1}$$

$$\sigma_t = c + \beta^d RV_{t-1}^d + \beta^w RV_{t-1}^w + \beta^m RV_t^m + \beta^{VIX} VIX_t^{VIX} \tag{2}$$

$+$ the same regressions as log-log model

with the weekly aggregation period being (monthly similar with 20 days):

$$RV_t^w = \frac{1}{5}(RV_t^d + RV_{t-1d}^d + RV_{t-2d}^d + RV_{t-3d}^d + RV_{t-4d}^d)$$

and Realized Volatility (RV) being: $RV_t^d = \sqrt{\sum_{m=1}^{M} r_{t,m}^2}$

σ = Volatility (sd), RV = Realized Volatility with M equally spaced intraday returns for a day t, with m = 1,..,M, here 10min returns

My Method: Regression and HAR Model

HAR-RV Model (according to Corsi (2009), but not implemented yet), idea:

$$\sigma_t = c + \beta^d RV^d_{t-1} + \beta^w RV^w_{t-1} + \beta^m RV^m_t + \tilde{w}^d_{t+1d} \tag{3}$$

$\tilde{w}^d_{t+1d}$ = innovation term

Results so far

Regression Results

Figure 1: level-level Model

	Historic	Historic with VIX
Intercept	0.00^{***}	-0.01^{***}
	(0.00)	(0.00)
RV_t^d	0.27^{***}	0.23^{***}
	(0.02)	(0.02)
RV_t^w	0.39^{***}	0.36^{***}
	(0.03)	(0.03)
RV_t^m	0.25^{***}	-0.04
	(0.03)	(0.03)
VIX		0.00^{***}
		(0.00)
R^2	0.54	0.57
Adj. R^2	0.54	0.57
Num. obs.	4436	4436
RMSE	0.02	0.02

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

Figure 2: log-log Model

	Historic	Historic with VIX
Intercept	-0.25^{***}	-7.85^{***}
	(0.05)	(0.28)
RV_t^d	0.34^{***}	0.24^{***}
	(0.02)	(0.02)
RV_t^w	0.40^{***}	0.28^{***}
	(0.03)	(0.03)
RV_t^m	0.21^{***}	-0.13^{***}
	(0.02)	(0.03)
VIX		1.60^{***}
		(0.06)
R^2	0.73	0.77
Adj. R^2	0.73	0.77
Num. obs.	4437	4437
RMSE	0.60	0.55

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

Possible Problems coming up

Next steps/Questions to solve

- Having gathered all this information about volatility measurement, what is the most accurate way to set up my regressions and my HAR-RV model?
- Next step: Bring current state of work to paper

References

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Appendix

The Black and Scholes Equation

price of an option over time:

$$\frac{\partial \mathrm{C}}{\partial \mathrm{t}} + \frac{1}{2}\sigma^2 \mathrm{S}^2 \frac{\partial^2 \mathrm{C}}{\partial \mathrm{C}^2} + \mathrm{rS}\frac{\partial \mathrm{C}}{\partial \mathrm{S}} = \mathrm{rC} \tag{4}$$

calculate the price of European call and put option:

$$\mathrm{C}(\mathrm{S},\mathrm{t}) = \mathrm{N}(\mathrm{d}_1)\mathrm{S} - \mathrm{N}(\mathrm{d}_2)\mathrm{K}\mathrm{e}^{-rt} \tag{5}$$

$$\mathrm{d}_1 = \frac{1}{\sigma\sqrt{\mathrm{t}}}\left[\ln\left(\frac{S}{K}\right) + t\left(r + \frac{\sigma^2}{2}\right)\right] \tag{6}$$

$$\mathrm{d}_2 = \frac{1}{\sigma\sqrt{\mathrm{t}}}\left[\ln\left(\frac{S}{K}\right) + t\left(r - \frac{\sigma^2}{2}\right)\right] \tag{7}$$

$$N(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} \mathrm{e}^{-\frac{1}{2}z^2} dz \tag{8}$$

C = Call option price

S = Current stock price

K = Strike price of the option

r = risk-free interest rate (a number between 0 and 1)

σ = volatility of the stocks return (a number between 0 and 1)

t = time to option maturity (in years)

N = normal cumulative distribution function

The VIX Equation

$$\sigma^2 = \frac{2}{T} \sum_i \frac{\Delta K_i}{K_i^2} e^{RT} Q(K_i) - \frac{1}{T} (\frac{F}{K_0} - 1)^2 \tag{9}$$

$VIX = \sigma * 100$, T = Time to expiration, F = Forward index level derived from index option prices, K_0 = first strike price below the forward index level, K_i = strike price of i^{th} out-of-the money option (call if $K_i > K_0$ and put if $K_i < K_0$), ΔK_i = interval between strike prices, R = Risk-free rate to expiration, $Q(K_i)$ = midpoint of bid-ask spread for each option with strike K_i

About the VIX

- 1993 CBOE introduced the first VIX, to measure the market's expectation for volatility, in 30-day period, based on S&P100 Index at-the-money options
- 2003 CBOE updated the VIX (together with Goldman Sachs)
  - now based on S&P500 Index (liquidity reasons)
- 2014 CBOE enhanced VIX
  - include also weekly options, thus most precisely match 30-day time frame
  - use SPX options with x [$23 < x < 36$] days to expiration to reflect an interpolation of two points
- Today, VIX is also a tradable asset (negative correlation to stock market returns makes it variable diversification)

Realized Volatility

$$RV_t^d = \sqrt{\sum_{m=1}^{M} r_{t,m}^2} \tag{10}$$

with r being:

$$R = \frac{P_t - P_{t-1}}{P_{t-1}} \Leftrightarrow 1 + R_t = \frac{P_t}{P_{t-1}} \tag{11}$$

with continuously compounded return (log return)

$$r_t = ln(1 + R_t) = ln\frac{P_t}{P_{t-1}} = p_t - p_{t-1} \tag{12}$$

RV = realized volatility, R = asset return, $r = ln(R)$, P = asset price, $p = ln(P)$

Why returns instead of asset prices?

- Returns are easier to handle than price series, because of statistical properties (e.g. time additivity)
- Asset return is a complete and scale-free summary of the investment opportunity (MacKinlay 1997 from Tsay (2005))