3D Object representations

Graphical scenes can contain many different kinds of objects like trees, flowers, rocks, waters...etc. There is no one method that we can use to describe objects that will include all features of those different materials. To produce realistic display of scenes, we need to use representations that accurately model object characteristics.

- Simple Euclidean objects like polyhedrons and ellipsoids can be represented by polygon and quadric surfaces.
- Spline surface are useful for designing aircraft wings, gears and other engineering objects.
- Procedural methods and particle systems allow us to give accurate representation of clouds, clumps of grass, and other natural objects.
- Octree encodings are used to represent internal features of objects. Such as medical CT images.

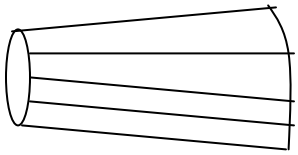
Representation schemes for solid objects are often divided into two broad categories:

1. Boundary representations
2. Space-partitioning representation

1. Boundary representations

describes a 3D object as a set of polygonal surfaces, separate the object interior from environment. Each 3D object is supposed to be formed its surface by collection of polygon facets and spline patches. Some of the boundary representation methods for 3D surface are:

1. Polygon Surfaces: It is he most common representation for 3D graphics object. In this representations, a 3D object is represented by a set of surfaces that enclose the object interior. Many graphics system use this method. Set of polygons are stored for object description. This simplifies and speeds up the surface rendering and display of object since all surfaces can be described with linear equations.



A 3D object represented by polygons

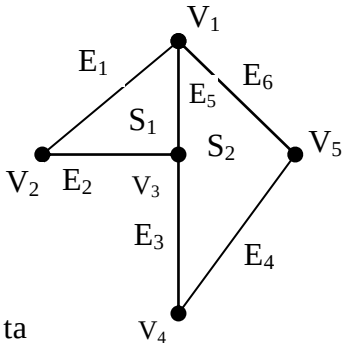
The polygon surface are common in design and solid-modeling applications, since wire frame display can be done quickly to give general indication of surface structure. Then realistic scenes are produced by interpolating shading patterns across polygon surface to illuminate.

Polygon Table: A polygon surface is specified with a set of vertex co-ordinates and associated attribute parameters. A convenient organization for storing geometric data is to create 3 lists:

- A vertex table
- An edge table
- A polygon surface table.

- Vertex table stores co-ordinates of each vertex in the object.
- The edge table stores the Edge information of each edge of polygon facets.
- The polygon surface table stores the surface information for each surface i.e. each surface is represented by edge lists of polygon.

Consider the surface contains polygonal facets as shown in figure (only two polygon are taken here)



→ S_1 and S_2 are two polygon surface that represent the boundary of some 3D object.

Foe storing geometric data, we can use following three ta

VERTEX TABLE
V_1: x_1,y_1,z_1
V_2: x_2,y_2,z_2
V_3: x_3,y_3,z_3
V_4: x_4,y_4,z_4
V_5: x_5,y_5,z_5

EDGE TABLE
E_1: V_1,V_2
E_2: V_2,V_3
E_3: V_3,V_4
E_4: V_4,V_5
E_5: V_1,V_3
E_6: V_5,V_1

POLYGON SURFACE TABLE
S_1: E_1,E_2,E_3
S_2: E_3,E_4,E_5,E_6

The object can be displayed efficiently by using data from tables and processing them for surface rendering and visible surface determination.

Plane Equation Method: Plane equation method is another method for representation the polygon surface for 3D object. The information about the spatial orientation of object is described by its individual surface, which is obtained by the vertex co-ordinates and the equation of each surface. The equation for a plane surface can be expressed in the form,

$$Ax + By + Cz + D = 0$$

Where (x,y,z) is any point on the plane, and A,B,C,D are constants describing the spatial properties of the plane. The values of A,B,C,D can be obtained by solving a set of three plane equations using co-ordinate values of 3 non collinear points on the plane.

Let (x_1,y_1,z_1), (x_2,y_3,z_2) and (x_3,y_3,z_3) are three such points on the plane, then-

$$Ax_1 + By_1 + Cz_1 + D = 0$$
$$Ax_2 + By_2 + Cz_2 + D = 0$$
$$Ax_3 + By_3 + Cz_3 + D = 0$$

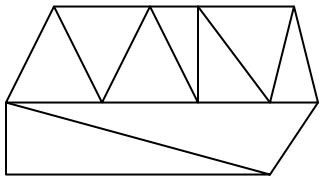
The solution of these equations can be obtained in determinant from using Cramer's rule as:-

$$A = \begin{vmatrix} 1 & y_1 & z_1 \\ 1 & y_2 & z_2 \\ 1 & y_3 & z_3 \end{vmatrix} \quad B = \begin{vmatrix} x_1 & 1 & z_1 \\ x_2 & 1 & z_2 \\ x_3 & 1 & z_3 \end{vmatrix} \quad C = \begin{vmatrix} x_1 & y_1 & 1 \\ x_1 & y_2 & 1 \\ x_1 & y_3 & 1 \end{vmatrix} \quad D = \begin{vmatrix} x_1 & y_1 & z_1 \\ x_1 & y_2 & z_2 \\ x_1 & y_3 & z_3 \end{vmatrix}$$

For any points (x,y,z)
If $Ax + By + Cz + D \neq 0$, then (x,y,z) is not on the plane.
If $Ax + By + Cz + D < 0$, then (x,y,z) is inside the plane i. e. invisible side
If $Ax + By + Cz + D > 0$, then (x,y,z) is lies out side the surface.

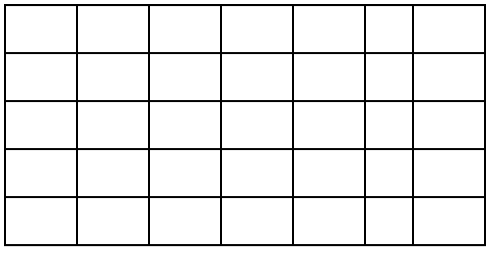
2. Polygon Meshes: A polygon mesh is collection of edges, vertices and polygons connected such that each edge is shared by at most two polygons. An edge connects two vertices and a polygon is a closed sequence of edges. An edge can be shared by two polygons and a vertex is shared by at least two edges.

When object surface is to be tiled, it is more convenient to specify the surface facets with a mesh function. One type of polygon mesh is triangle strip. This function produce n-2 connected triangles.



Triangular Mesh

Another similar function is the quadrilateral mesh, which generates a mesh of (n-1) by (m-1) quadrilaterals, given the co-ordinates for an $n \times m$ array of vertices.



6 by 8 vertices array , 35 element quadrilateral mesh

- If the surface of 3D object is planer, it is comfortable to represent surface with meshes.

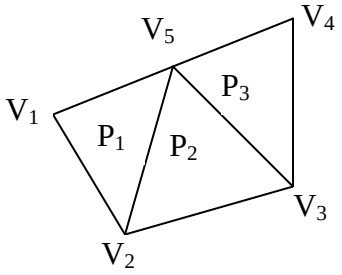
Representing polygon meshes

In explicit representation, each polygon is represented by a list of vertex co-ordinates.

$$P = ((x_1, y_1, z_1), (x_2, y_2, y_2), ..., (x_n, y_n, z_n))$$

The vertices are stored in order traveling around the polygon. There are edge between successive vertices in the list and between the last and first vertices.

- For a single polygon it is efficient but for polygon mesh it is not space efficient since no of vertices may duplicate.
- So another method is to define polygon with pointers to a vertex list. So each vertex is stored just once, in vertex list $V = \{v_1, v_2 ..... v_n\}$ A polygon is defined by list of indices (pointers) into the vertex list e.g. A polygon made up of vertices 3,5,7,10 in vertex list be represented as $P_1 = \{3,5,7,10\}$



- Representing polygon mesh with each polygon as vertex list.
  - $P_1 = \{v_1, v_2, v_5\}$
  - $P_2 = \{v_2, v_3, v_5\}$
  - $P_3 = \{v_3, v_4, v_5\}$

  Here most of the vertices are duplicated so it is not efficient.
- Representation with indexes into a vertex list

  $V = \{v_1, v_2, v_3, v_4, v_5\} = \{(x_1, y_1, z_1), ...(x_5, y_5, z_5)\}$

  $P_1 = \{1,2,3\}$

  $P_2 = \{2,3,5\}$

  $P_3 = \{3,4,5\}$

- **<u>Defining polygons by pointers to an edge list</u>**

In this method, we have vertex list V, represent the polygon as a list of pointers not to the vertex list but to an edge list. Each edge in edge list points to the two vertices in the vertex list. Also to one or two polygon, the edge belongs.

Hence we describe polygon as

$$P = (E_1, E_2, ...E_n)$$

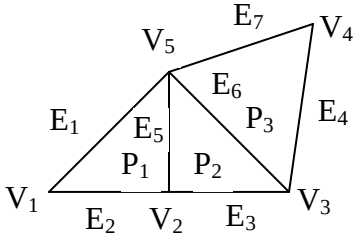
and an edge as

$$E = (V_1, V_2, P_1, P_2)$$

2Here if edge belongs to only one polygon, either

Then P_1 or P_2 is null.

For the mesh given below,



$$V = \{v_1, v_2, v_3, v_4, v_5\} = \{(x_1, y_1, z_1), ...(x_5, y_5, z_5)\}$$

$E_1 = (V_1, V_5, P_1, N)$
$E_2 = (V_1, V_2, P_1, N)$
$E_3 = (V_2, V_3, P_2, N)$
$E_4 = (V_3, V_4, P_3, N)$
$E_5 = (V_2, V_5, P_1, P_2)$
$E_6 = (V_3, V_5, P_1, P_3)$ Here N represents Null.
$E_7 = (V_4, V_5, P_3, N)$
$P_1 = (E_1, E_2, E_3)$
$P_2 = (E_3, E_6, E_5)$
$P_3 = (E_4, E_7, E_6)$

Polygon mesh defined with edge lists for each polygon.

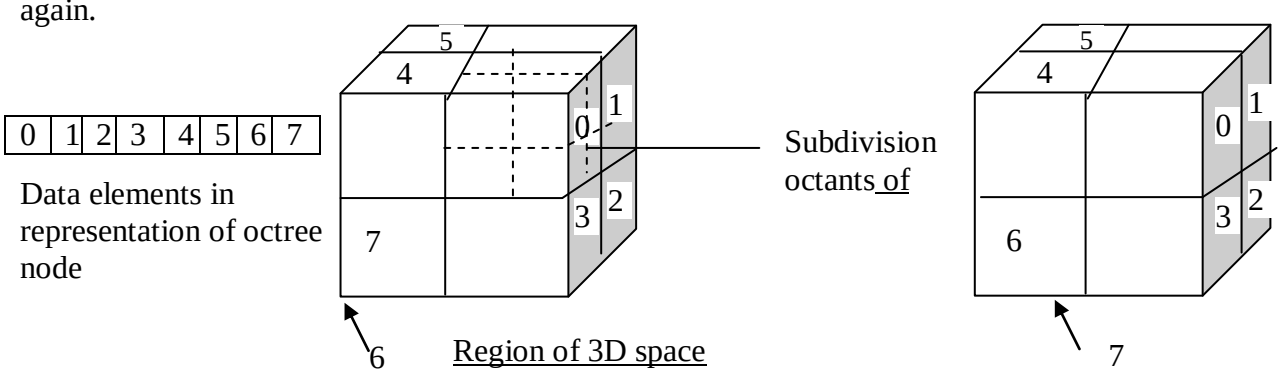
2. <u>Space-partitioning representation</u>

used to describe interior properties, by partitioning the spatial region, containing an object into a set of small, non overlapping, contiguous solids. e.g. 3D object as Octree representation.

<u>Octree Representation: (Solid-object representation):</u> This is the space-partitioning method for 3D solid object representation. This is hierarchical tree structures (octree) used to represent solid object in some graphical system. Medical imaging and other applications that require displays of object cross section commonly use this method. E.g.: CT-scan

It provides a convenient representation for storing information about object interiors.

An octree encoding scheme divides region of 3D space into ocatants and stores 8 data elements in each node of the tree. Individual elements are called volume element or vexels. When all voxels in an octant are of same type, this type value is stored in corresponding data elements. Any heterogeneous octants are subdivided into octants again.



3. Blobby Objects:

Some objects don't maintain a fixed shape but change their surface characteristics in certain motions or when proximity to another objects e.g molecular structures, water droplets, other liquid effects, melting objects, muscle shaped in human body etc. These objects can be described as exhibiting "blobbiness" and are referred as blobby objects.

Several models have been developed for representing blobby objects as distribution functions over a region of space. One way is to use Gaussian density function or bumps. A surface function is defined as:

f(x,y,z) = $\sum_k b_{k0} e^{-a_k r_k^2} - T = 0$

Where $r_k = \sqrt{x_k^2 + y_k^2 + z_k^2}$, T = Threshold and a and b are used to adjust amount of blobbness.

Other method for generating for generating blobby objects uses quadratic density function as:

$$f(x) = \begin{cases} b(1 - \frac{3r^3}{d^2}) & \text{if } 0 < r \le d/3 \\ \frac{3}{2} b(1 - \frac{r}{d})^2 & \text{if } d/3 < r \le d \\ 0 & \text{if } r > d \end{cases}$$

Advantages

- Can represent organic, blooby or liquid line structures
- Suitable for modeling natural phenomena like water, human body
- Surface properties cam be easily derived from mathematical equations.

Disadvantages:

- Requires expensive computation
- Requires special rendering engine
- Not supported by most graphics hardware

4. Spline Representation

A Spline is a flexible strips used to produce smooth curve through a designated set of points. A curve drawn with these set of points is spline curve. Spline curves are used to model 3D object surface shape smoothly.

Mathematically, spline are described as pice-wise cubic polynomial functions. In computer graphics, a spline surface can be described with two set of orthogonal spline curves. Spline is used in graphics application to design and digitalize drawings for storage in computer and to specify animation path. Typical CAD application for spline includes the design of automobile bodies, aircraft and spacecraft surface etc.

Interpolation and approximation spline

- Given the set of control points, the curve is said to interpolate the control point if it passes through each points.
- If the curve is fitted from the given control points such that it follows the path of control point without necessarily passing through the set of point, then it is said to approximate the set of control point.

Cubic spline:

It is most often used to set up path for object motions o tot provide a representation for an existing object or drawing. To design surface of 3D object any spline curve can be represented by piece-wise cubic spline.

Cubic polynomial offers a reasonable compromise between flexibility and speed of computation. Cubic spline requires less calculations with comparison to higher order polynomials and require less memory. Compared to lower order polynomial cubic spline are more flexible for modeling arbitrary curve shape.

Given a set of control points, cubic interpolation spines are obtained by fitting the input points with a picewise cubic polynomial curve that passes through every control points.

Suppose we have n+1 control points specified with co-ordinates.

$p_k = (x_k, y_k, z_k), \quad k = 0,1,2,3\ldots\ldots\ldots\ldots\ldots\ldots..n$

A cubic interpolation fit of those points is

We can describe the parametric cubic polynomial that is to be fitted between each pair of control points with the following set of parametric equations.

$x(u) = a_x u^3 + b_x u^2 + c_x u + d_x$
$y(u) = a_y u^3 + b_y u^2 + c_y u + d_y \qquad (0 \le u \le 1)$
$z(u) = a_z u^3 + b_z u^2 + c_z u + d_z$

There are three equivalent methods for specifying a particular spline representation.

1. Set of boundary conditions.

   For the parametric cubic polynomial for the x-coordinate along the path of spline section

   $x(u) = a_x U^3 + b_x U^2 + c_x U + d \qquad 0 \le U \le 1$

   Boundary condition for this curve be the set on the end point coordinate x(0) and x(1) and in the first derivatives at end points x'(0) and x'(1). These four boundary condition are sufficient to determine the four coefficient a_x, b_x, c_x, d_x.

2. From the boundary condition, we can obtain the characterizing matrix for spline. Then the parametric equation can be written as-

   $$x(u) = [\ u3\ \ u2\ \ u\ \ 1] \begin{bmatrix} a_x \\ b_x \\ c_x \\ d_x \end{bmatrix}$$

3. Blending function that determines how specified geometric constraints on the curve are combined to calculate position along the curve path.

   $$x(u) = \sum_{k=0}^{3} g_k . BF_k(u)$$

   Where g_k are the geometric constraint parameters such as control points co-ordinate and slope of the curve at control point. $BF_k(u)$ are the polynomial Blending functions.

Bezier curve and surface

This is spline approximation method, developed by the French Engineer Pierre Bezier for use in the design of automobile body. Beizer spline has a number of properties that make them highly useful and convenient for curve and surface design. They are easy to implement. For this reason, Bezier spline is widely available in various CAD systems.

In general Bezier curve can be fitted to any number of control points. The number of control points to be approximated and their relative position determine the degree of Bezier polynomial. The Bezier curve can be specified with boundary condition, with characterizing matrix or blending functions. But for general blending function specification is most convenient.

Suppose we have n+1 control points: $p_k(x_k,y_k,z_k)$, k = 0,1,2,3,4,…….n. These co-ordinate points can be blended to produce the following position vector p(u) which describes path of an approximating Bezier polynomial function p_0 and p_n.

$$p(u) = \sum_{k=0}^{n} p_k . BEZ_{k,n}(u), \qquad 0 \le u \le 1 \text{ ---------- } 1$$

The Bezier belending function $BEZ_{k,n}(u)$ are the Bernstein polynomial as,

$$BEZ_{k,n}(u) = c(n,k)u_k(1-u)^{n-k}$$

The vector equation (1) represents a set of three parametric equation for individual curve conditions.

i.e.
$$x(u) = \sum_{k=0}^{n} x_k . BEZ_{k,n}(u)$$

$$y(u) = \sum_{k=0}^{n} y_k . BEZ_{k,n}(u)$$

$$z(u) = \sum_{k=0}^{n} z_k . BEZ_{k,n}(u)$$

Bezier curve is a polynomial of degree one less than control points i.e. 3 points generate parabola, 4 points a cubic curve and so n.

Properties of Bezier Curve:

1. It always passs through initial and final control points. i.e $p(0) = p_0$ and $p(1)=p_n$.

2. Values of the parametric first derivatives of a Bezier curve at the end points can be calculated from control points as-
   $p'(0) = -np_0 + np_1$
   $p'(1) = -np_{n-1} + np_n$

3. The slope at the beginning of the curve is along the line joining the first two points and slope at the end of curve is along the line joining last two points.

4. Parametric second derivative at a Bezier curve at end points are-
   $p''(0) = n(n-1)[(p_2-p_1) – (p_1-p_0)]$
   $p''(1) = n(n-1)[(p_{n-2}-p_{n-1}) – (p_{n-1}-p_n)]$

Quadric Surface

Quadric Surface is one of the frequently used 3D objects surface representation. The quadric surface can be represented by a second degree polynomial. This includes:

1. Sphere: For the set of surface points (x,y,z) the spherical surface is represented as: $x^2+y^2+z^2 = r^2$, with radius r and centered at co-ordinate origin.

2. Ellipsoid: $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$, where (x,y,z) is the surface points and a,b,c are the radii on X,Y and Z directions respectively.

3. Elliptic parboiled: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=z$

4. Hyperbolic parboiled : $\frac{x^2}{a^2}-\frac{y^2}{b^2}=z$

5. Elliptic cone : $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=0$

6. Hyperboloid of one sheet: $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$

7. Hyperboloid of two sheet: $\frac{x^2}{a^2}-\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$