

# Game Theory & Strategy

## Basic Terms of Game Theory

What is a game?: A game is an

interaction between two or more players, where every player's outcome depends on the decision taken by the other players.

Game Theory: An analysis of how the players in a game behave while interacting with each other is called Game Theory.

## Basic Terms in Game Theory

- • **Players**: participants in the game.  
(e.g. Pepsi & Coca Cola)
- **Strategy**: is a plan that is used to achieve the desired objective.  
(e.g. Increase Price, Production)
- **Strategy Profile**: collection of a player's strategies is called the strategy profile  
(e.g. Increase or Decrease price)
- **Payoffs**: benefit that a player receives by adopting a particular strategy.  
generally denoted by a number. The set of payoffs shown in a tabulated form is called Payoff matrix.
- **Outcome**: Strategy chosen from a set of available strategies gives a payoff which is called the outcome of the strategy.

## Assumptions of Game Theory

- Every player is a rational agent
- Players are self-interested agents with a goal of maximising utility.
- A rational agent will choose a decision that will maximise their individual outcome.
  - Rationality can't be used in situations where the outcomes cannot be quantified
- Self-interested agents are individuals or firms that take decision with the sole motive of achieving their desired goals or to be in a better position than others, without any intention of harming other individuals or firms.
  - Utility refers to the satisfaction that an individual gets from the consumption of a set of goods or by performing some action

Utility Theory: states that a consumer strives to be in a position of maximum utility.

- Each self-interested agent has a utility function, which maps his/her utility to the different alternative strategies

Ordinal utility: It's not always possible to quantify utility. Hence relative terms are used.

Thus, Game theory involves rational, self-interested agents who act in a certain way to achieve their desired goals.

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- Motive of engaging in game theory is to maximize one's utility

→ What happens when two self-interested agents interact

### Game Theory Strategies

- Pure Strategy : When a player takes a decision independent of what the other player's move will be, it's called a pure strategy. (Rock, paper, scissors)
- Mixed Strategy : When a player tries to anticipate the opponent's move after a few turns and then assigns a probability to each possible strategy in order to choose the optimal one.

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### Dominant & Dominated strategies

Dominant : A dominant strategy is one that gives a player the best payoff irrespective of another player's move.

Dominated Strategy : is one that yields the worst outcome for a player out of all other strategies, irrespective of another player's move.



## Types of Games in Game Theory

- 1) Cooperative & Non-cooperative games
- 2) Sequential & Simultaneous games
- 3) Constant-sum, non-zero-sum, & zero-sum games
- 4) Symmetric & Asymmetric games

Also called

- Common Payoff games OR
- Pure Coordination games

### Cooperative game

Here, the agents have no conflicting interests; their sole challenge is to explicitly coordinate on an action that is maximally beneficial to all.

e.g: OPEC Cartel.

= Cooperative game

### Non-cooperative game

The players do not decide on a strategy mutually, but compete with each other to maximise their personal benefit.

Sequential Game: players play in a sequence & are aware of each other's moves before playing their own. e.g: Auctions

Simultaneous Games: All players play their strategies simultaneously, they play without knowing each other's moves. e.g: Tenders

Constant-sum: in which the sum of the payoffs remains constant even if the payoff for each individual player changes

Zero Sum game: is a type of constant sum game in which sum of payoffs is zero. Such that one player can gain only if other loses. Here gain of one player is equal to loss of other. e.g: Toss of a coin.

Non-zero sum: in which the sum of payoffs is not zero

Symmetric games: the strategy adopted by all the players is the same, such that their identities can be changed w/o any change in the payoff from the strategy. (Normally short term games)

Asymmetric games: The players don't choose identical strategies

Session → Do participants often take the optimal decisions?

Prisoner's dilemma: is a situation wherein two parties have to independently choose one of the two available decisions & each choice has a different pair of outcome for the two parties involved

		VISUAL	
		A	D
ARJUN	A	50k / 50k	20k / 80k
	D	80k / 20k	25k / 25k

Dominant strategy for Arjun = Accepting change  
as in either cases he pays lesser amount compared to other alternative

Dominant strategy for Visual = accept the change

Dominant strategy for Arjun = Accept the change

Dominated strategy = Except for dominant strategy payoff, all other quadrant's payoff are called dominated strategy

Nash Equilibrium is a stage where no player can benefit by unilaterally deviating from the present strategy

Maxmin & Minmax Strategy

Advertising here is dominant strategy for both

		QUINTAL	
		Adv	Not Adv
Advertiser	Adv	50 / 50	70 / 30
	Not Adv	40 / 60	55 / 45

If Arjun = Not Adv, then Adv Exm benefits from Adv

Dominant Strategy for Quintal = Advertise

Minmax strategy: used when firms want to minimise its opponent's max gain as a result of its own strategy

Maxmin strategy: used when firms want to maximise the minimum profit that they can gain by choosing a particular strategy



## Models of Game Theory

- The Bertrand Model
- The Cournot Model
- The Stackelberg Model

→ - homogeneous goods

- Each firm assumes that the prices of its competitors are fixed. The aim is to capture a large portion of market

- The price-setting process is continued by all the firms until they reach a point where no firm has an incentive to modify its prices further

- Nash equilibrium is achieved when the price is set equal to the marginal cost. This is because no firm has any incentive to deviate from its current state unilaterally


- If one firm lowers price, then entire consumer set goes to that firm.



Players in a market may adopt different models according to their purpose. The three main models of Game theory that can be applied by firms to maximize their profit in Business are:

1. The Bertrand models
2. The Cournot model, and
3. The Stackelberg model

Let's explain each of these in detail.



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**DIFFERENT MODELS OF GAME THEORY**

01

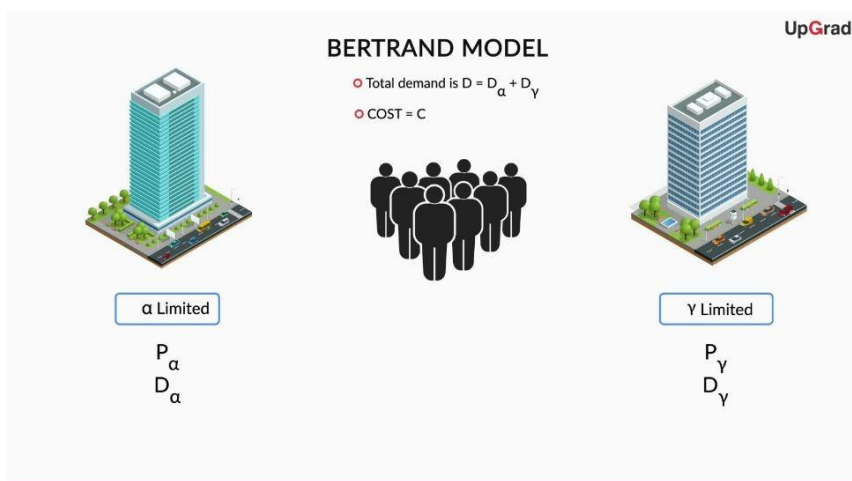
**THE BERTRAND MODEL**

- a. Each firm assumes that the prices of its competitors is fixed. The aim is to capture a large portion of the market
- b. The price-setting process is continued by all the firms until they reach a point where no firm has an incentive to modify its prices further
- c. The Nash equilibrium state is achieved when the price is set equal to the marginal cost. This is because no firm has any incentive to deviate from its current state unilaterally
- d. This is because if one firm lowers its price, then the entire consumer set goes to that firm, whereas the other firms suffer loss

The Bertrand model: This model was developed by Joseph Bertrand and is applicable to firms dealing in homogenous goods. Each firm assumes that the prices of its competitors are fixed, and so it has to set its price accordingly with the aim of grabbing the majority market share and profits.

The price-setting process is undertaken by all the firms and continues until the firms have reached a level where no firm has an incentive to modify their prices further and change their current position. Generally, the price set at this level is equal to the marginal cost (MC). This level is a Nash equilibrium state, as none of the firms has any incentive to unilaterally change their current state.

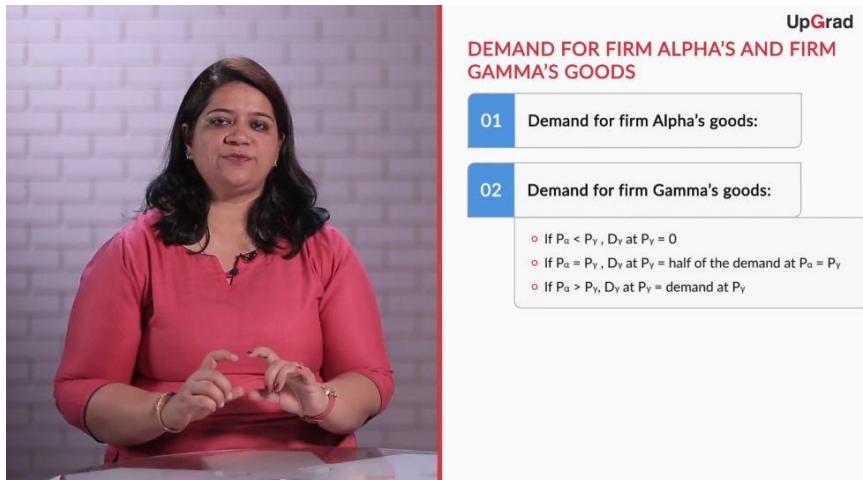
This happens because if one firm lowers its price, then the entire consumer set goes to him, as both the firms are selling homogenous goods. In reaction to this, the firm with higher prices also lowers its prices to that level such that the demand at that price gets divided between the two firms. This price modification continues until there is no incentive for any firm to lower its prices.



Now, let's understand how the optimum equilibrium price is set for both the firms. There are 2 cement manufacturing firms: Alpha and Gamma ltd. Let the price be  $P_{\alpha}$  and  $P_{\gamma}$ .



$D_\alpha$  and  $D_\gamma$  denote the demand for alpha and gamma, respectively.  $D_\alpha$  is dependent on both  $P_\alpha$  and  $P_\gamma$ , as both the firms are selling homogenous goods. Similarly,  $D_\gamma$  is dependent on both  $P_\alpha$  and  $P_\gamma$ . Total demand is  $D = D_\alpha + D_\gamma$ . Cost incurred both the firms is denoted by  $C$ .



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**DEMAND FOR FIRM ALPHA'S AND FIRM GAMMA'S GOODS**

- 01 Demand for firm Alpha's goods:
- 02 Demand for firm Gamma's goods:
  - If  $P_\alpha < P_\gamma$ ,  $D_\gamma$  at  $P_\gamma = 0$
  - If  $P_\alpha = P_\gamma$ ,  $D_\gamma$  at  $P_\gamma =$  half of the demand at  $P_\alpha = P_\gamma$
  - If  $P_\alpha > P_\gamma$ ,  $D_\gamma$  at  $P_\gamma =$  demand at  $P_\gamma$

The demand for firm Alpha's goods at several price points is as follows. If price of alpha is less than price of gamma, then the entire market demand would shift to alpha. If price of alpha is equal to price of gamma, then they'll be equal division of demand between alpha and gamma. If price of alpha is greater than price of gamma, then the entire demand would shift to gamma. Why? Because the consumers will buy from gamma.

Similarly, demand for firm gamma at seven price points is as follows. If price of alpha is less than price of gamma, then the entire demand would shift to alpha. If price of alpha is equal to price of gamma, then there will be division of demand between alpha and gamma. If price of alpha is more than price of gamma, then the entire demand would shift to gamma.




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**PROFIT FOR FIRM ALPHA**

- 01 Profit = Demand at  $P_\alpha$  \* (price - cost), if  $P_\alpha < P_\gamma$
- 02 Profit at  $P_\alpha =$  [half of the demand at price  $\alpha$  \* (price - cost)], if  $P_\alpha = P_\gamma$
- 03 Profit = 0, if  $P_\alpha > P_\gamma$

Profits at several price points for alpha limited are as follows. Demand at price alpha, if price of alpha is less than price gamma, then the entire demand would shift to alpha, and so, the profit would shift to alpha. If price of alpha is equal to price of gamma, then there'll be equal division of profit since the demand gets divided between the two firms since the firm is offering homogeneous products. Profit is zero if price of alpha is more than the price of gamma.





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### EQUILIBRIUM FOR FIRMS ALPHA AND GAMMA

**01** Equilibrium is achieved

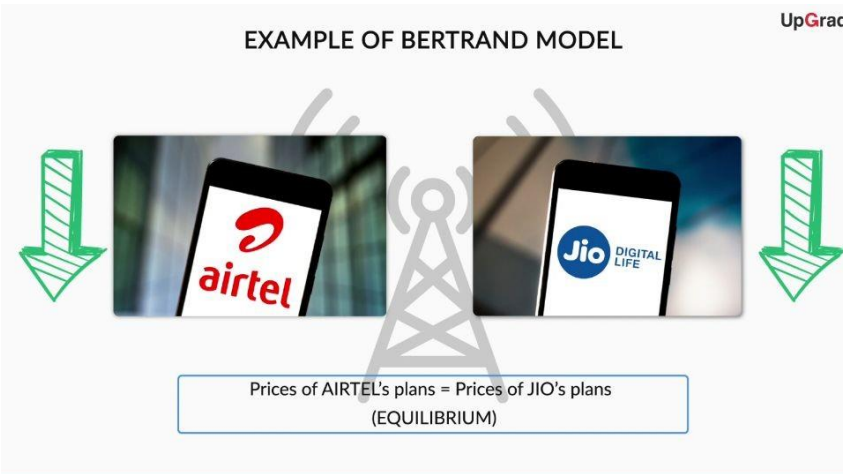
- When:  $P_\alpha = P_\gamma = MC$  (NASH EQUILIBRIUM STATE)
  - If  $P_\gamma > MC > P_\alpha$ , firm  $\alpha$  = negative profits, firm  $\gamma$  = 0 profits
  - If  $P_\gamma > P_\alpha > MC$ , firm  $\alpha$  = positive profits, firm  $\gamma$  = 0 profits
  - If  $P_\gamma = P_\alpha < MC$ , firm  $\alpha$  = firm  $\gamma$  = negative profits
  - If  $P_\gamma = P_\alpha > MC$ , firms  $\alpha$  and  $\gamma$  will each earn positive profits
  - If  $P_\gamma > P_\alpha = MC$ , firms  $\alpha$  and  $\gamma$  will each earn 0 profits, because if price = MC, then profit = 0

Equilibrium is achieved when  $P_\alpha = P_\gamma = MC$ . This state is called Nash equilibrium because:

- If  $P_\gamma > MC > P_\alpha$ , then firm alpha will earn negative profits and firm gamma will earn 0 profits.
- If  $P_\gamma > P_\alpha > MC$ , then firm alpha will earn positive profits and firm gamma will earn 0 profits.
- If  $P_\gamma = P_\alpha < MC$ , then both the firms alpha and gamma will earn negative profits.
- If  $P_\gamma = P_\alpha > MC$ , then both the firms alpha and gamma will earn positive profits.
- If  $P_\gamma > P_\alpha = MC$ , then both the firms alpha and gamma will earn 0 profits because if price = MC, then profit = 0

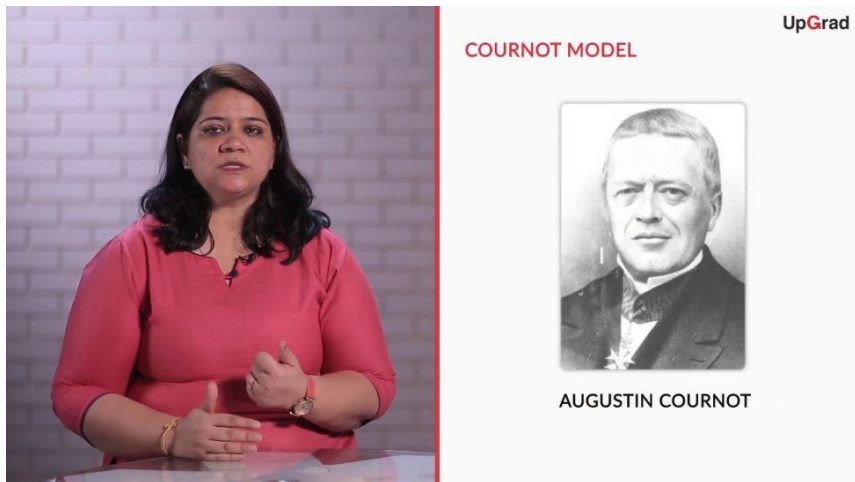
**EXAMPLE OF BERTRAND MODEL**

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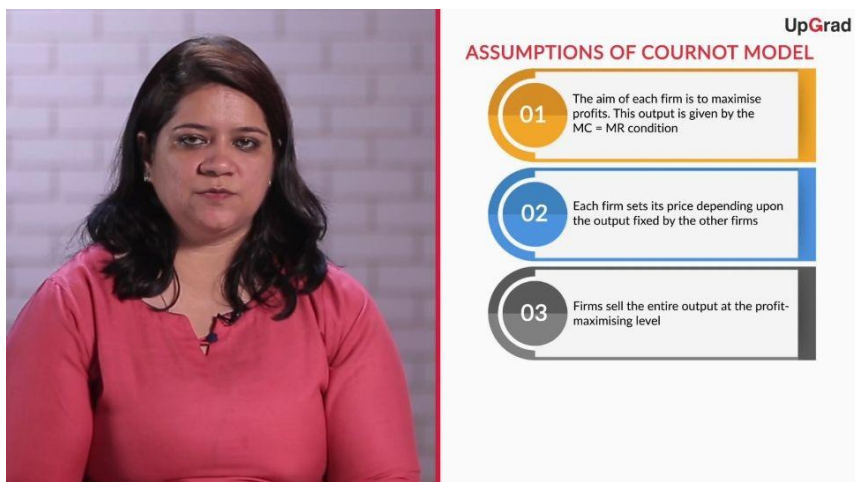


Prices of AIRTEL's plans = Prices of JIO's plans  
(EQUILIBRIUM)

The Indian telecom sector is a perfect example of the application of the Bertrand model of game theory. Let's consider the two major players- airtel and Jio, which produce homogeneous products and compete in terms of setting prices. If one firm lowers its prices, the other lowers its prices too. Both these firms are in equilibrium by setting similar prices such that neither has an incentive to deviate from its existing strategy and the market is not taken over by one player only.




The Cournot model: This model was developed by Augustin Cournot. It shows the interaction between 2 profit-maximizing players in a market, where both firms try to act independently as if they are single firms in the market. Each firm decides its output thinking that its rival firm has its output fixed and is not going to change it. This profit-maximizing output then dictates the price.



Some assumptions in this model are as follows:

- The aim of each firm is to set an output that will maximize its profits. This output is given by the  $MC = MR$  condition.
- Each firm has a linear, downward-sloping demand curve.
- Each firm sets its price depending upon the output fixed by the others under the assumption that the output will not change.
- Firms sell the entire output at the profit-maximizing level, i.e., at the price determined by their corresponding demand curves.





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**DIFFERENT MODELS OF GAME THEORY**

**01 THE BERTRAND MODEL**

**02 THE COURNOT MODEL**

- a. A firm first enters the market and tries to sell that quantity of output at which  $MC = MR$
- b. Firm B enters the market and sells the profit-maximising level of output
- c. Firm A then sets a price which captures the market (Currently held share of market + Unsatisfied portion of market)
- d. This process continues, such that share of firm A ↓ and that of firm B ↑ to the point where a certain portion of the market is left unsatisfied

Here, a firm first enters the market and tries to sell the level of output at which  $MC = MR$  at the corresponding price level. After that, firm B enters the market assuming that firm A's output is fixed and it (i.e., firm B) can capture the remaining unmet demand of the market.

With an aim to maximize its profits, it sells the profit-maximizing level of output corresponding to its demand curve. At this point, there is still some part of the market demand that is not fulfilled. So, firm A decides to set a price that allows it to capture that part of the market while selling it at the profit-maximizing level of output.

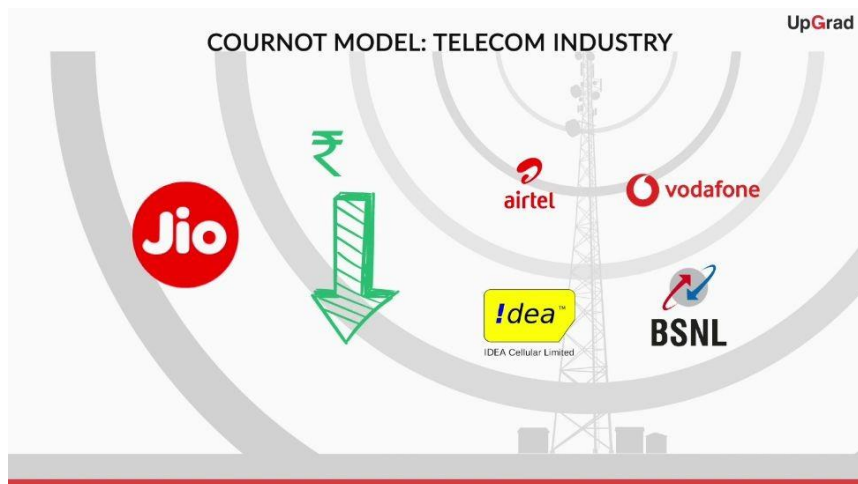
This process continues, and after each round, the share of firm A decreases and that of firm B increases, to the point where both the firms leave a portion of the market unsatisfied. Equilibrium is achieved at a point where both the firms earn equal profits and have no tendency to change their output levels.



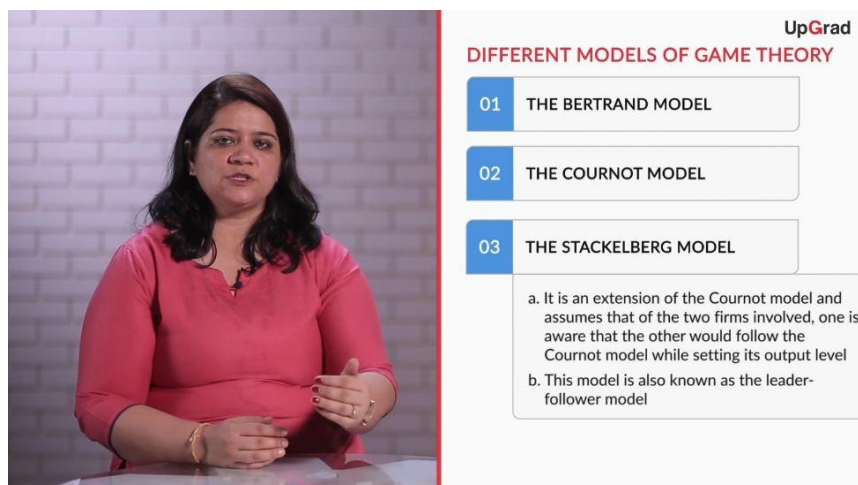
For example: Consider the time when data plans were launched in the Indian market.

They were exorbitantly priced and the market demand for data was very limited. As more and more players started offering these services, the cost of data started coming down until it became stabilized. All the firms had set the price which would help them service the profit maximizing output. However, even at this point the entire market demand was not being met as the prices were still high. At this point, the network industry was following the Cournot model.



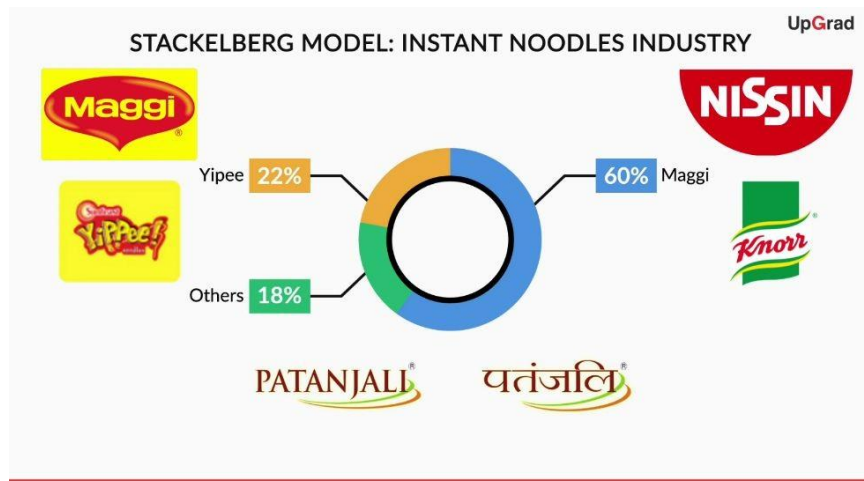


This unmet demand was realized when Jio launched data plans at very low prices. This forced all the market players to reduce their prices in order to retain their market share. It is then that the industry started following the Bertrand model.



The Stackelberg model: This model was developed by H.V. Steckelberg. It is an extension to the Cournot model and assumes that of the 2 firms involved, one firm (say firm A) knows that the other (say firm B) would follow the Cournot model while setting its output level. So, firm A anticipates the output level of firm B and incorporates this into its profit-maximizing function.

This model is also known as the leader-follower model, as here, one firm enters the market as a first entrant and becomes the leader, and the other follows it. The aim is to set an equilibrium profit-maximizing output. The leader firm gets the larger market share at equilibrium.



For example: When Nestle introduced Maggi in India in the year 1982, there were no other global brands in that segment in India. Later, a lot of brands followed it into the market, such as Nissin Top Ramen, ITC Yipee noodles, HUL's Knorr noodles, and Patanjali's noodles.

Nestle Maggi's market share is ~60%, followed by ITC's yipee with a 22% share, and the rest of the brands taking up the remaining portion of the market. An interesting fact to note here is that noodles are primarily known as Maggi in almost every Indian household. This shows how the firm with the first-mover advantage enjoys a larger market share as compared with the firms that follow.

