Understanding Octupole vibration in Ca^{48} and A Toy Model

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TDHF

Basics

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- Schrodinger equation is solved iteratively in a mean field

TDHF

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- Schrodinger equation is solved iteratively in a mean field
- Time-dependent potential for vibration

TDHF

Basics

- Schrodinger equation is solved iteratively in a mean field
- Time-dependent potential for vibration
- Linear regime: only 1p1h states contribute

Ca^{48} Octupole vibration

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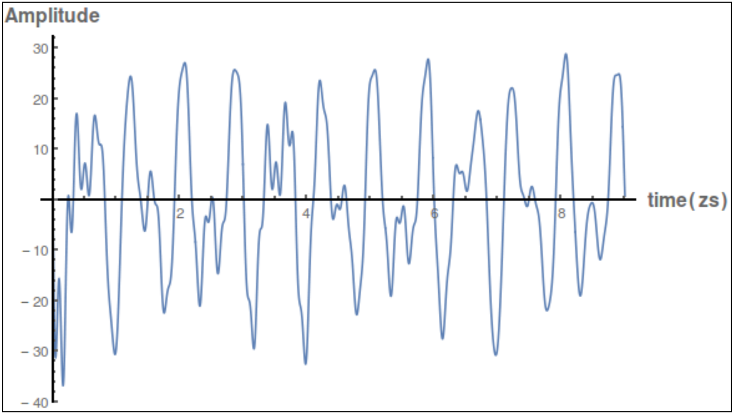
- Octupole moment

Ca^{48} Octupole vibration

- Octupole moment

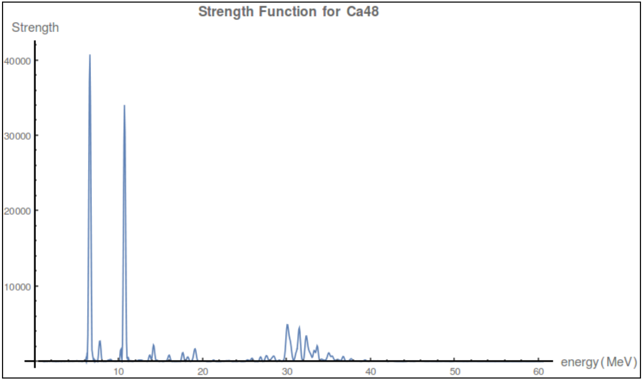


Ca^{48} Octupole vibration

- Strength function

Ca^{48} Octupole vibration

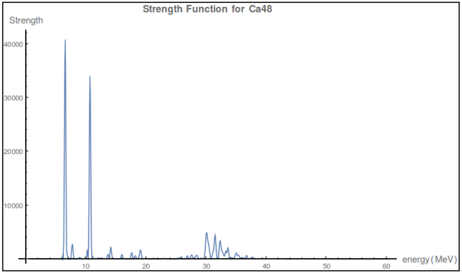
- Strength function



Ca^{48} Octupole vibration

- Strength function

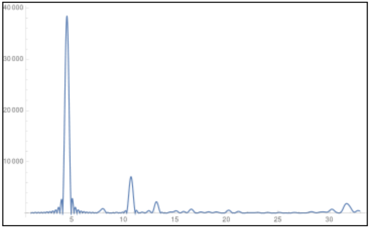
Peaks : 6MeV and 10MeV



Background

Digging deeper

Strength function for Ca^{40}

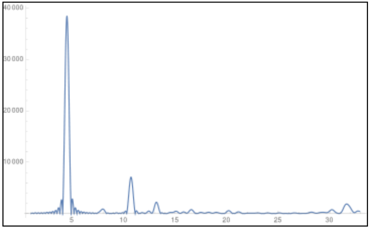
1 

[1] All energies are in MeV

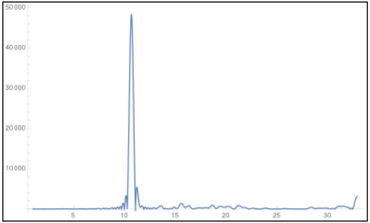
Background

Digging deeper

Ca^{40}



Strength function for Ni^{56}



[1]

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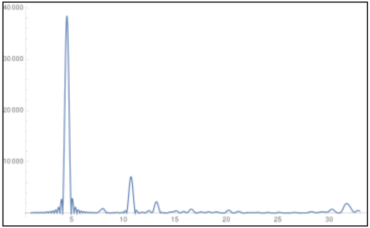
[1] All energies are in MeV

Background

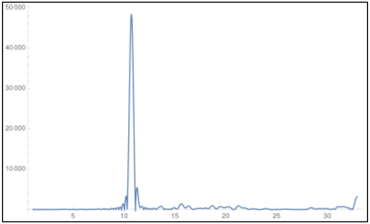
Digging deeper

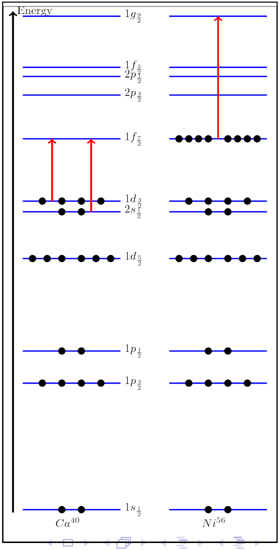
Only 3^{-} states contribute

Ca^{40}



Ni^{56}:





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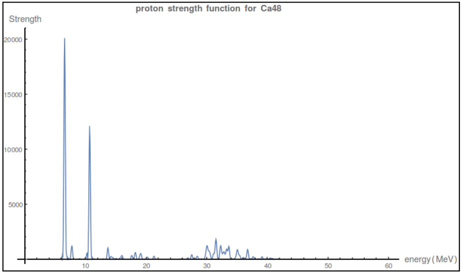
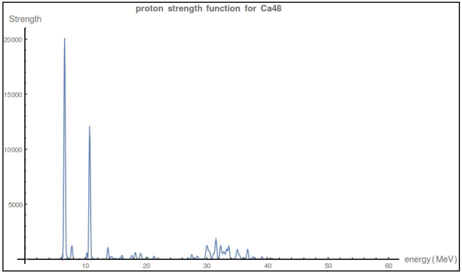
Neutron and Proton transition densities for Ca^{48}

Strength function for proton

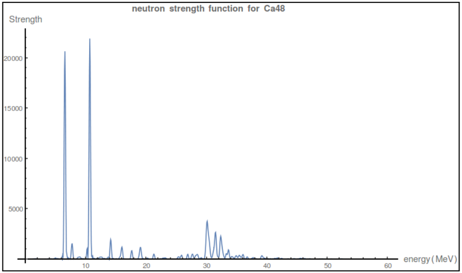


Neutron and Proton transition densities for Ca^{48}

Strength function for proton

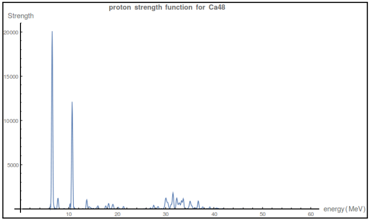


Strength function for neutron

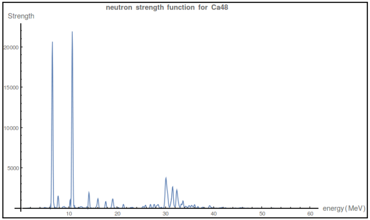


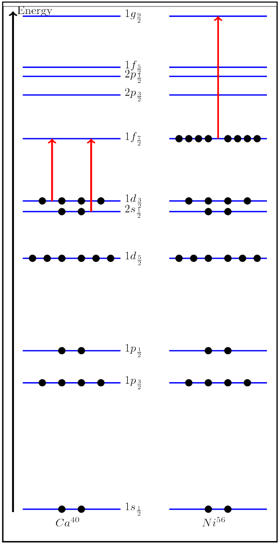
Neutron and Proton transition densities for Ca^{48}

Proton



Neutron





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RPA

- Full hamiltonian with residual interaction is taken into account

$$H = H_{MF} + V_{res}$$

RPA

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- Matrix representation of the full hamiltonian in 1p1h basis is diagonalised

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- Excited states are written as a linear combination of all 1p1h states.

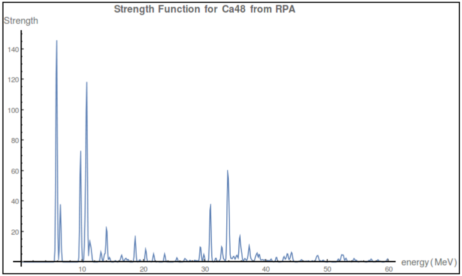
$$|\nu\rangle = \sum_{1p1h} C_{1p1h} |1p1h\rangle$$

RPA

- Full hamiltonian with residual interaction is taken into account
- Matrix representation of the full hamiltonian in 1p1h basis is diagonalised
- Excited states are written as a linear combination of all 1p1h states.
- particle-hole correlation

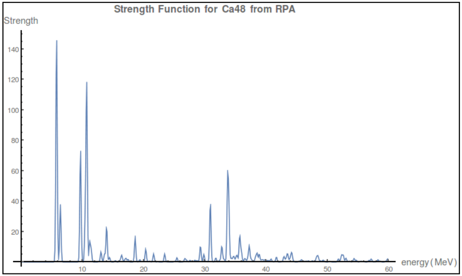
$$|\nu\rangle = \sum_{1p1h} C_{1p1h}|1p1h\rangle$$

strength function

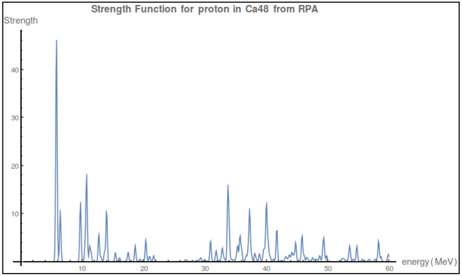


RPA

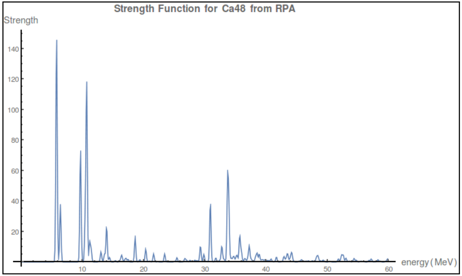
Calculations

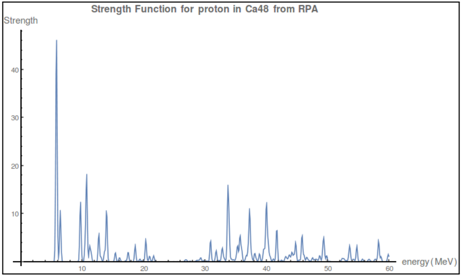


Proton:

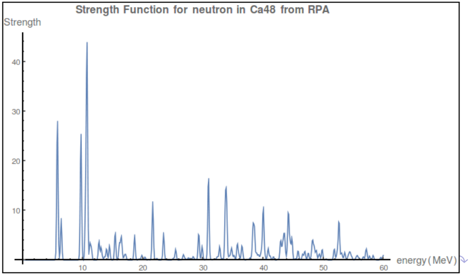


RPA
Calculations





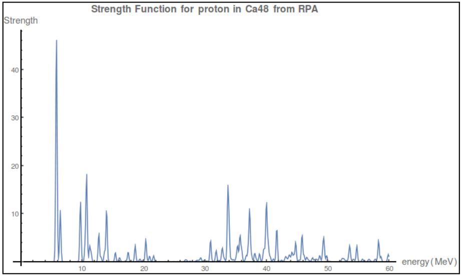
Neutron:

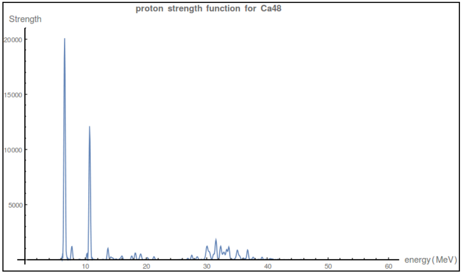


TDHF vs RPA

Comparison

Proton





TDHF vs RPA

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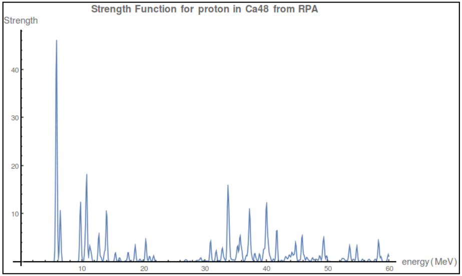
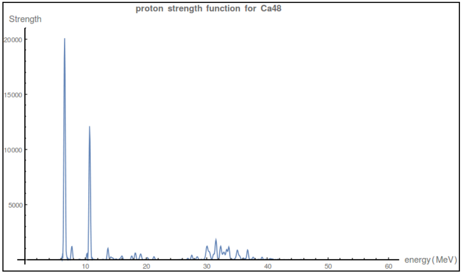
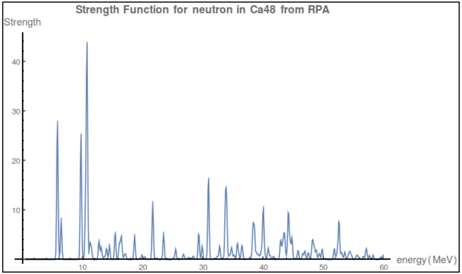
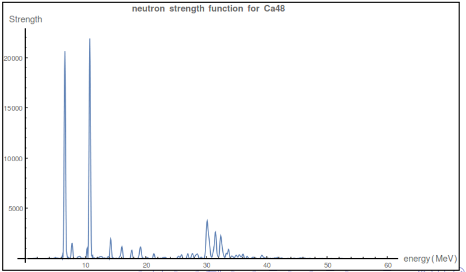




Neutron





RPA

1p1h contribution

RPA

1p1h contribution

- Random Phase Approximation $Q_\nu^\dagger = \sum_{mi} \left(X_{mi}^\nu \mathbf{a}_m^\dagger \mathbf{a}_i - Y_{mi}^\nu \mathbf{a}_i^\dagger \mathbf{a}_m \right)$

RPA

1p1h contribution

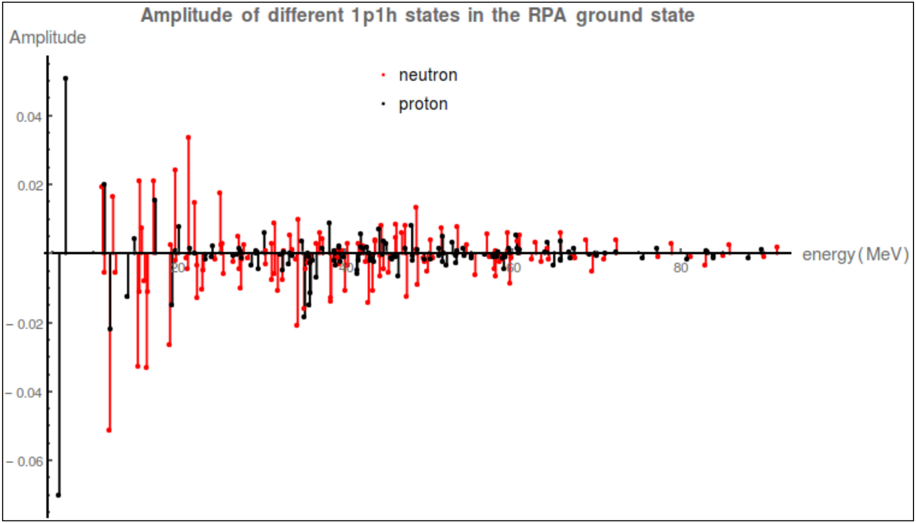
- Random Phase Approximation
- Tamm-Dancoff Approximation

$$Q_\nu^\dagger = \sum_{mi} X_{mi}^\nu \mathbf{a}_m^\dagger \mathbf{a}_i$$

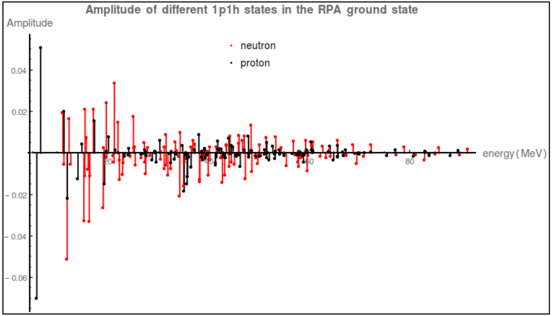
RPA

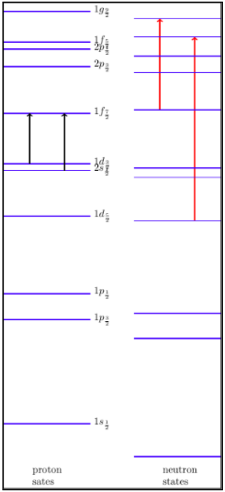
1p1h contribution

Tamm-Dancoff Approximation:

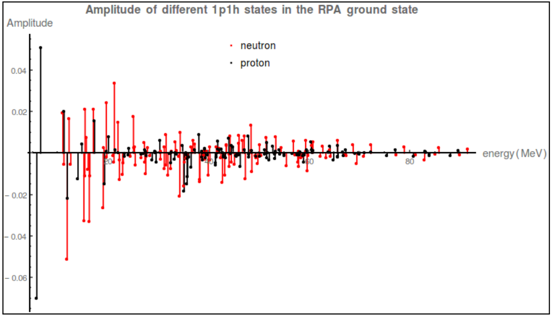


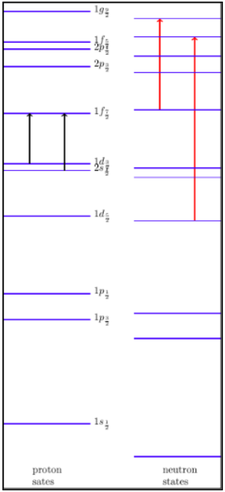
Tamm-Dancoff Approximation





Tamm-Dancoff Approximation





- $\pi(1f_{\frac{7}{2}})(1d_{\frac{3}{2}})^{-1}$ 5.12%
- $\pi(1f_{\frac{7}{2}})(2s_{\frac{1}{2}})^{-1}$ 3.7%
- $\nu(1g_{\frac{9}{2}})(1f_{\frac{7}{2}})^{-1}$ 3.75%
- $\nu(1f_{\frac{5}{2}})(1d_{\frac{5}{2}})^{-1}$ 2.45%

Table of content

The problem

- Two nucleus approaching each other

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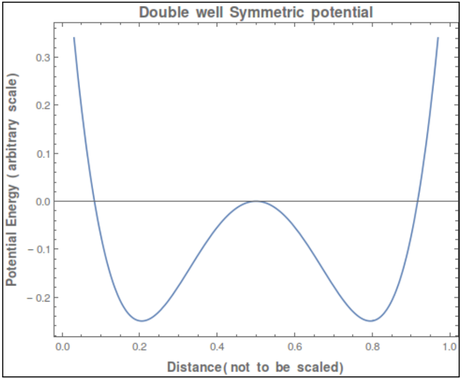
- An alpha particle exchange

The problem

- Two nucleus approaching each other
- An alpha particle exchange
- Recurrance time

Toy Model

- Symmetric Potential Well



Toy Model

- Symmetric Potential Well
- Two eigenstate $(|+\rangle and |-\rangle)$

Hamiltonian

$$\begin{pmatrix} 0 & 0 \\ 0 & \epsilon \end{pmatrix}$$

Toy Model

- Symmetric Potential Well
- Two eigenstate ($|+\rangle and |-\rangle$)
- Left and right state (45 degree rotation) $|L\rangle$ and $|R\rangle$.

Hamiltonian

$$\begin{pmatrix} \epsilon/2 & -\epsilon/2 \\ -\epsilon/2 & \epsilon/2 \end{pmatrix}$$

Toy Model

- Symmetric Potential Well
- Two eigenstate ($|+\rangle and |-\rangle$)
- Left and right state (45 degree rotation) $|L\rangle$ and $|R\rangle$.
- Two internal states $|0\rangle$ and $|1\rangle$.

Hamiltonian

$$\begin{pmatrix} \frac{\epsilon}{2} & 0 & -\frac{\epsilon}{2} & 0 \\ 0 & \frac{\epsilon}{2}+\delta & 0 & -\frac{\epsilon}{2}+\delta \\ -\frac{\epsilon}{2} & 0 & \frac{\epsilon}{2} & 0 \\ 0 & -\frac{\epsilon}{2}+\delta & 0 & \frac{\epsilon}{2}+\delta \end{pmatrix}$$

Toy Model

- Symmetric Potential Well
- Two eigenstate ($|+\rangle and |-\rangle$)
- Left and right state (45 degree rotation) $|L\rangle$ and $|R\rangle$.
- Two internal states $|0\rangle$ and $|1\rangle$.
- Coupling V

Hamiltonian

$$\begin{pmatrix} \frac{\epsilon}{2} & 0 & -\frac{\epsilon}{2} & 0 \\ 0 & \frac{\epsilon}{2}+\delta & 0 & -\frac{\epsilon}{2}+\delta \\ -\frac{\epsilon}{2} & 0 & \frac{\epsilon}{2} & V \\ 0 & -\frac{\epsilon}{2}+\delta & V & \frac{\epsilon}{2}+\delta \end{pmatrix}$$

Evolution

- Initial state $|\Psi(t=0)\rangle = |L0\rangle$

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Evolution

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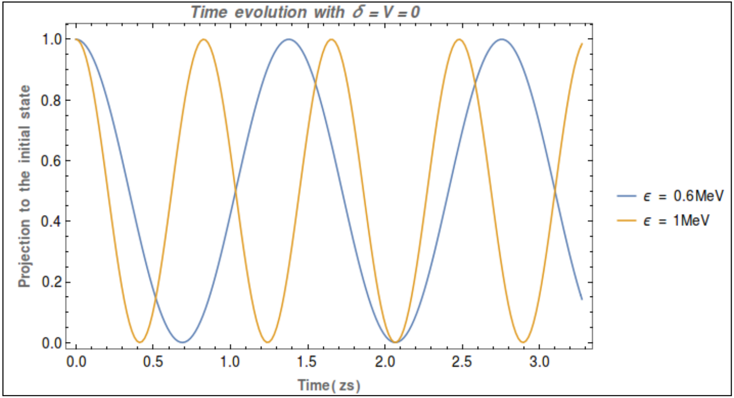
- $|\Psi(t)\rangle = e^{-i\hat{H}t}|\Psi(0)\rangle$
- Projection: $|\langle\Psi(0)|\Psi(t)\rangle|^2$

Evolution

Projection: $|\langle\Psi(0)|\Psi(t)\rangle|^2$

$\delta = V = 0$

K



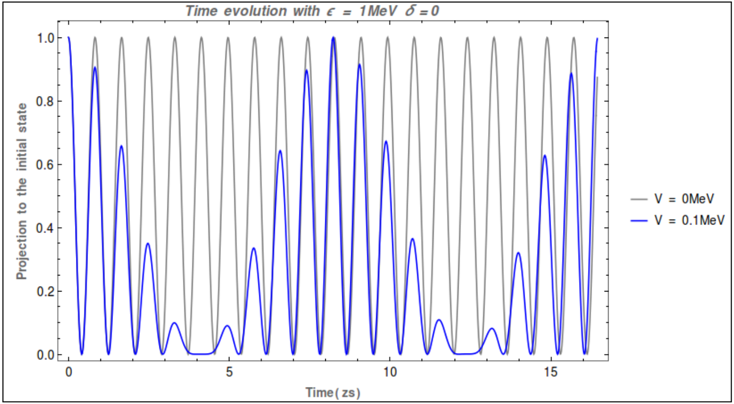
$T \propto \frac{1}{\epsilon}$

Evolution

Projection: $|\langle\Psi(0)|\Psi(t)\rangle|^2$

$\delta = 0$ small V

K



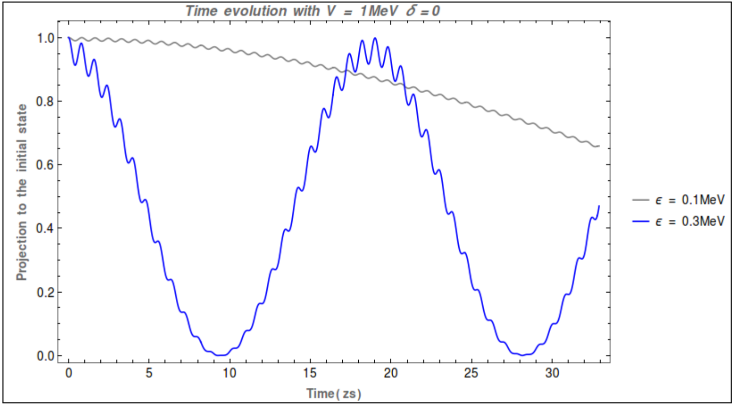
An envelop due to introduction of coupling

Evolution

Projection: $|\langle\Psi(0)|\Psi(t)\rangle|^2$

$\delta = 0$, large V

K

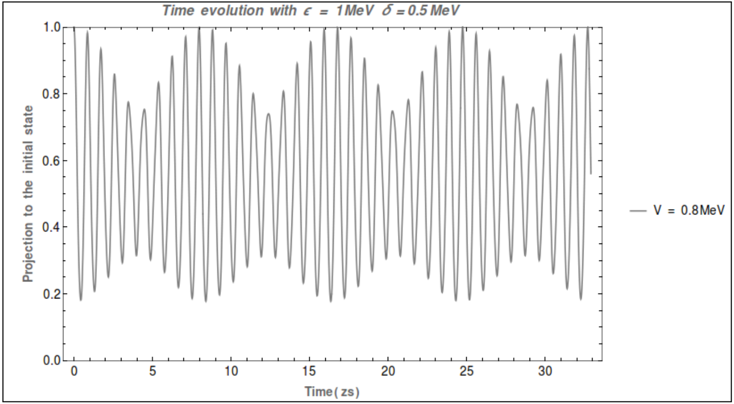


Evolution

Projection: $|\langle\Psi(0)|\Psi(t)\rangle|^2$

Everything non-zero

K

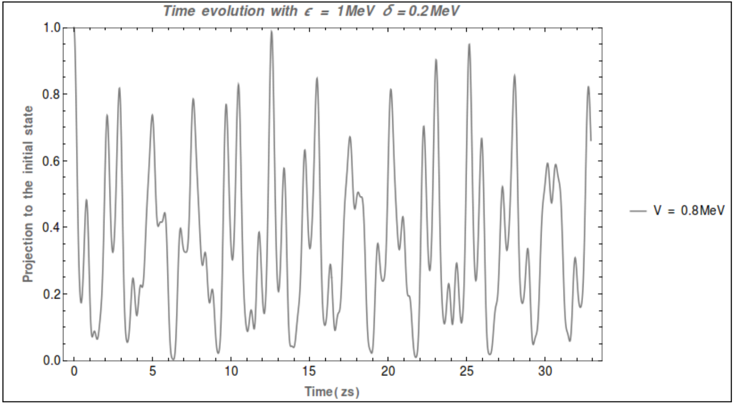


Evolution

Projection: $|\langle\Psi(0)|\Psi(t)\rangle|^2$

Everything non-zero

K



LCM Energy

Alternative way to obtain Recurrance time

- Diagonalise the Hamiltonian

- Eigenvalues: E_i
- Eigenvectors: v_i

LCM Energy

Alternative way to obtain Recurrance time

- Diagonalise the Hamiltonian
- Any state can be written as a linear combination of eigenstate.

$$|\Psi\rangle = \sum_{i=1}^{4} c_i |v_i\rangle$$

LCM Energy

Alternative way to obtain Recurrance time

- Diagonalise the Hamiltonian
- Any state can be written as a linear combination of eigenstate.
- Time evolution of the state

$$|\Psi(t)\rangle = \sum_{i=1}^{4} c_i e^{-iE_i t}|v_i\rangle$$

LCM Energy

Alternative way to obtain Recurrance time

- Diagonalise the Hamiltonian
- Any state can be written as a linear combination of eigenstate.
- Time evolution of the state
- Note: Each of the time dependent term repeats when $E_i t = 2\pi n_i, n \in \mathbb{Z}$.

$$|\Psi(t)\rangle = \sum_{i=1}^{4} c_i e^{-iE_i t} |v_i\rangle$$

LCM Energy

Alternative way to obtain Recurrance time

- Diagonalise the Hamiltonian
- Any state can be written as a linear combination of eigenstate.
- Time evolution of the state
- Note: Each of the time dependent term repeats when $E_i t = 2\pi n_i, n \in \mathbb{Z}$.
- So recurrance time is LCM$(\frac{2\pi}{E_i})$.

$$|\Psi(t)\rangle = \sum_{i=1}^{4} c_i e^{-iE_i t} |v_i\rangle$$

Future Directions

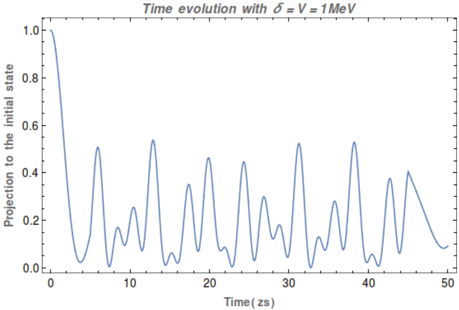
Unfinished Business

- Change-epsilon(time-dependent)

Future Directions

Unfinished Business

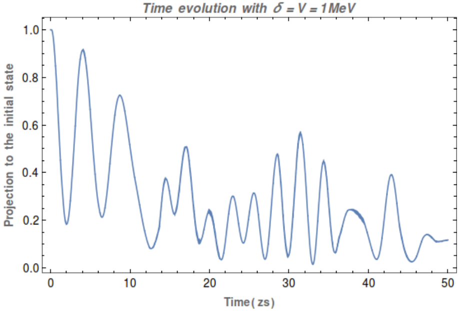
- Change-epsilon(time-dependent)
  - Step



Future Directions

Unfinished Business

- Change-epsilon(time-dependent)
    - Step
    - Linear



Future Directions

Unfinished Business

- Change-epsilon(time-dependent)
    - Step
    - Linear
- Introduce more internal states

$$\begin{pmatrix} \frac{\epsilon}{2} & 0 & 0 & -\frac{\epsilon}{2} & 0 & 0 \\ 0 & \frac{\epsilon}{2} & 0 & 0 & -\frac{\epsilon}{2} & 0 \\ 0 & 0 & \frac{\epsilon}{2}+\delta & 0 & 0 & \delta-\frac{\epsilon}{2} \\ -\frac{\epsilon}{2} & 0 & 0 & \frac{\epsilon}{2} & 0 & 0 \\ 0 & -\frac{\epsilon}{2} & 0 & 0 & \frac{\epsilon}{2} & 0 \\ 0 & 0 & \delta-\frac{\epsilon}{2} & 0 & 0 & \frac{\epsilon}{2}+\delta \end{pmatrix}$$

- Change-epsilon(time-dependent)
    - Step
    - Linear
- Introduce more internal states
    - Coupling with only ground state

$$\begin{pmatrix} \frac{\epsilon}{2} & 0 & 0 & -\frac{\epsilon}{2} & 0 & 0 \\ 0 & \frac{\epsilon}{2} & 0 & 0 & -\frac{\epsilon}{2} & 0 \\ 0 & 0 & \frac{\epsilon}{2}+\delta & 0 & 0 & \delta-\frac{\epsilon}{2} \\ -\frac{\epsilon}{2} & 0 & 0 & \frac{\epsilon}{2} & V & V \\ 0 & -\frac{\epsilon}{2} & 0 & V & \frac{\epsilon}{2} & 0 \\ 0 & 0 & \delta-\frac{\epsilon}{2} & V & 0 & \frac{\epsilon}{2}+\delta \end{pmatrix}$$

$$V' = \sqrt{2}V$$

- Change-epsilon(time-dependent)
  - Step
  - Linear
- Introduce more internal states
  - Coupling with only ground state
  - Coupling between excited states

$$\begin{pmatrix} \frac{\epsilon}{2} & 0 & 0 & -\frac{\epsilon}{2} & 0 & 0 \\ 0 & \frac{\epsilon}{2} & 0 & 0 & -\frac{\epsilon}{2} & 0 \\ 0 & 0 & \frac{\epsilon}{2}+\delta & 0 & 0 & \delta-\frac{\epsilon}{2} \\ -\frac{\epsilon}{2} & 0 & 0 & \frac{\epsilon}{2} & V_1 & V_2 \\ 0 & -\frac{\epsilon}{2} & 0 & V_1 & \frac{\epsilon}{2} & V_3 \\ 0 & 0 & \delta-\frac{\epsilon}{2} & V_2 & V_3 & \frac{\epsilon}{2}+\delta \end{pmatrix}$$

Thank you!!