Skeleton Expansion In Conformal Field Theory

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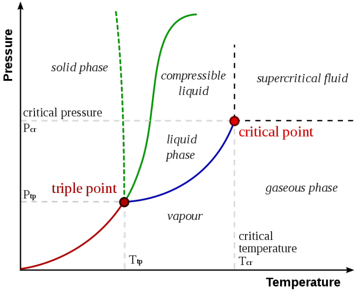

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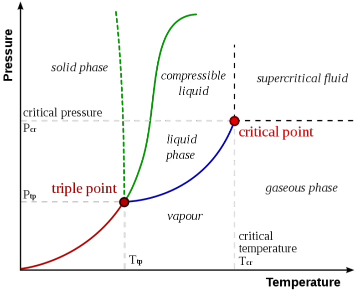

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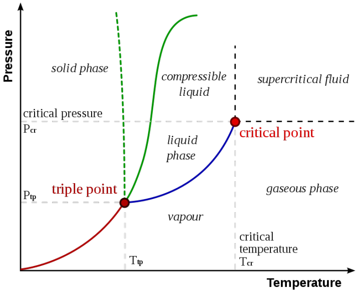

Phase transitions are characterised by *critical exponents*.

$$\chi \approx \frac{1}{(T - T_C)^\gamma}$$

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Here we use an improved method which bypasses the last two steps and reduce the number of diagrams, to calculate CFT data (which encodes the critical exponents) of ϕ^3 theory in $6-\epsilon$ dimension.[1]
We have rederived the result using Inversion Integral.

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Conformal Symmetry

Conformal Group

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- Special Conformal Transformation: $x'^{\mu} = \frac{x^{\mu} - (x\cdot x) b^{\mu}}{1 - 2(b\cdot x) + (b\cdot b)(x\cdot x)}$

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These transformations form a group (*Conformal Group*).

Conformal Invariance fixes the form of two and three point functions

[1] $x_{ij} = |x_i - x_j|$ and $\Delta_{ij,k} = \Delta_i + \Delta_j - \Delta_k$

Conformal Invariance fixes the form of two and three point functions

Two-point function

$$\langle \phi_i(x_1)\phi_j(x_2)\rangle = \frac{\delta_{ij}}{(x_{12}^2)^{\Delta_\phi}}$$

Three-point function

$$\langle \phi_i(x_1)\phi_j(x_2)\phi_k(x_3)\rangle = \frac{C_{ijk}}{(x_{12}^2)^{\frac{\Delta_{12,3}}{2}}(x_{23}^2)^{\frac{\Delta_{23,1}}{2}}(x_{13}^2)^{\frac{\Delta_{13,2}}{2}}}$$

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Conformal Symmetry does not completely fix four-point function.

$$\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle = \frac{\mathcal{A}(u,v)}{(x_{12}^2 x_{34}^2)^{\Delta_\phi}}$$

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where $\mathcal{A}(u,v)$ is an arbitrary function of the conformally invariant cross ratios

$$u = z\bar{z} = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} \quad v = (1-z)(1-\bar{z}) = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}$$

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Main Idea

Skeleton expansion systematically allow us to calculate n-point function.

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- Sum all 1PI diagrams with two external lines to get the exact propagator.
- Sum all 1PI diagrams with three external lines to obtain vertex of the theory.
- For four point function, only those 1PI diagrams are drawn where propagator and vertex correction is absent.

Conformal invariance trivialises the work in the first two steps.

[2] where $d = \sum_{i=1}^{3} \Delta_i$ and $\Delta_{12,3} = \Delta_1 + \Delta_2 - \Delta_3$

Skeleton Diagram

Adding Conformal Symmetry

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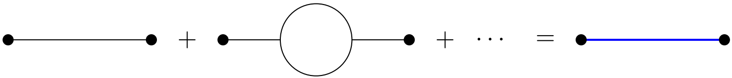

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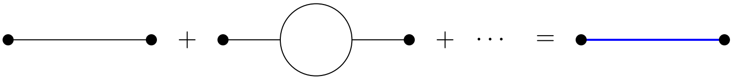
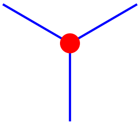


For the Vertex:

$$\langle\phi_1(x_1)\phi_2(x_2)\phi_3(x_3)\rangle$$



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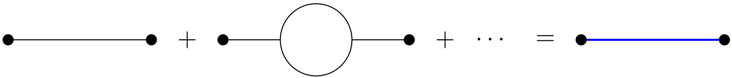
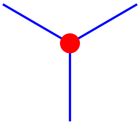


For the Vertex:

$$\langle \phi_1(x_1)\phi_2(x_2)\phi_3(x_3)\rangle = \int \left(\prod_{i=1}^{3} dy_i \langle \phi_i(x_i)\phi_i(y_i)\rangle\right) V(y_1, y_2, y_3)$$



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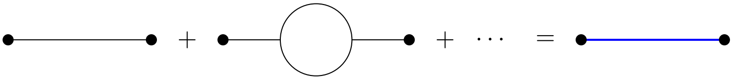

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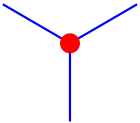

$$V(y_1, y_2, y_3) = \frac{g_{123}}{(y_{12}^2)^{\frac{d-\Delta_{12,3}}{2}} (y_{13}^2)^{\frac{d-\Delta_{13,2}}{2}} (y_{23}^2)^{\frac{d-\Delta_{23,1}}{2}}}$$ [2]

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$O(g^0)$

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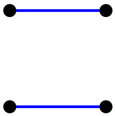
$$\frac{1}{(x_{12}^2 x_{34}^2)^{\Delta_\phi}} + \frac{1}{(x_{14}^2 x_{23}^2)^{\Delta_\phi}} + \frac{1}{(x_{13}^2 x_{24}^2)^{\Delta_\phi}}$$
$$= \frac{1}{(x_{12}^2 x_{34}^2)^{\Delta_\phi}} \left(1 + u^{\Delta_\phi} + (\frac{u}{v})^{\Delta_\phi}\right)$$



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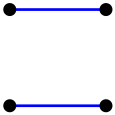

$$\mathcal{A}(u,v) = 1 + u^{\Delta_\phi} + \left(\frac{u}{v}\right)^{\Delta_\phi}$$

Calculations

$O(g^2)$

[0] Note that there is no divergence

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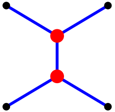
$$g_{\phi\phi\phi}^2 \int \frac{d^d x_5 \ldots d^d x_{10}}{\underbrace{(x_{15}^2 x_{27}^2 x_{36}^2 x_{49}^2)^{\Delta}(x_{810}^2)^{\Delta}}_{propagator}(\underbrace{x_{57}^2 x_{58}^2 x_{78}^2}_{vertex1}\underbrace{x_{69}^2 x_{610}^2 x_{910}^2}_{vertex2})^{\frac{d-\Delta}{2}}}$$



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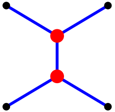

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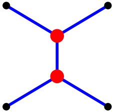

- All integrals except one can be done using Symanzik star-triangle formula.
- Remaining integral can be identified with the $\bar{D}$ function

$$\frac{C^2_{\phi\phi\phi}\Gamma(\Delta)}{\Gamma^4(\frac{\Delta}{2})\Gamma(\frac{d-2\Delta}{2})}u^{\frac{d-\Delta}{2}}\bar{D}_{\frac{d-\Delta}{2}\frac{d-\Delta}{2},\frac{\Delta}{2}\frac{\Delta}{2}}(u,v)$$

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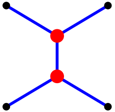

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- Simillarly other permutations.

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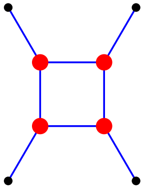
- Similar calculation.



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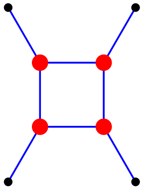

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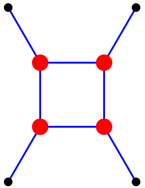
- Rest of the integrations can be done either using Parametric Integration or by obtaing a differential equation that the resultant function has to satisfy and solving it.



Operator Product Expansion

Operator Product Expansion

Operator action on a state

$$\phi_1(x)\phi_2(0)|0\rangle = \frac{1}{(x^2)^{\Delta_\phi}} \sum_{\mathcal{O}\, primaries} \lambda_{12\mathcal{O}} C_{\mathcal{O}}(x, \partial_y)\mathcal{O}(y)|_{y=0}|0\rangle$$

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Four-point function then can be written as,

$$\langle\phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle = \frac{1}{(x_{12}^2 x_{34}^2)^{\Delta_\phi}} \sum_{\mathcal{O}} \underbrace{\lambda_{12\mathcal{O}}\lambda_{34\mathcal{O}}}_{\text{OPE coefficient}} \underbrace{(\mathcal{C}_{\mathcal{O}}(x_{12}, \partial_y)\mathcal{C}_{\mathcal{O}}(x_{34}, \partial_z)\langle\mathcal{O}(y)\mathcal{O}(z)\rangle)}_{\text{Conformal Block}}$$

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These conformal blocks or conformal partial waves are fixed by conformal symmetry.

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We only need to determine the OPE coefficients.

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$$g_{0,\ell}(u,v) = \left(\frac{v-1}{2}\right)^{\ell} {}_2F_1\left(\frac{\Delta+\ell}{2}, \frac{\Delta+\ell}{2}; \Delta+\ell; 1-v\right)$$

$$g_{1,\ell}(u,v) = \frac{\Delta(\ell+\Delta)^2(\Delta-1)}{(\Delta+\ell-1)(\Delta+\ell+1)(\Delta+1-\frac{d}{2})} {}_2F_1\left(\frac{\Delta+2}{2}, \frac{\Delta+2}{2}; \Delta+2; 1-v\right)$$

Conformal Blocks

- A primary operator is characterised by its scaling dimension and its spin.
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Inversion Formula

[3] arXiv:1711.02031

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$$\hat{a}(\bar{h}) = \frac{2\bar{h}-1}{\pi^2}\int_0^1 dt d\bar{z} \frac{\bar{z}^{\bar{h}-2}(t(1-t))^{\bar{h}-1}}{(1-t\bar{z})^{\bar{h}}} \mathrm{dDisc}[\mathcal{A}(\bar{z})]$$

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The result matched with the previous calculations!

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Parametric Integration

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- Use Conformal Bootstrap methods.

First order calculations are done.

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- Use this method for other systems.
- Calculate higher orders in ϵ.
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- Comparing higher powers of u.

Operators other than twist four comes into picture

Thank You!!