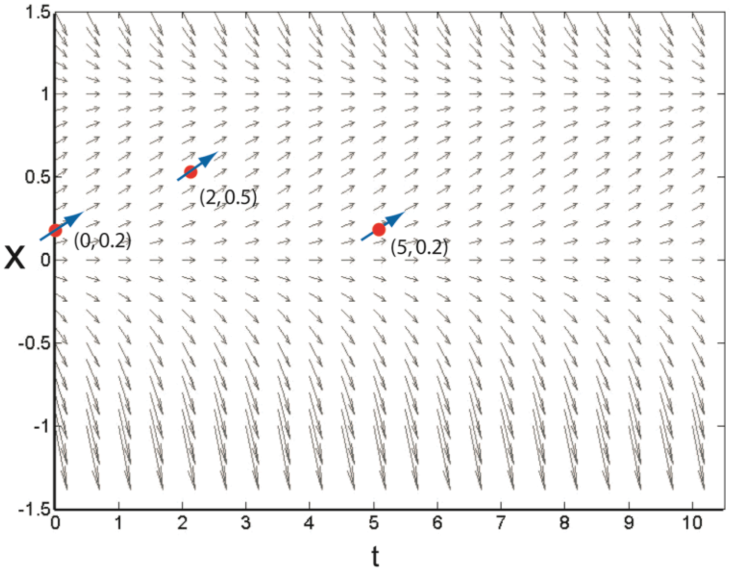


Understanding Steady State

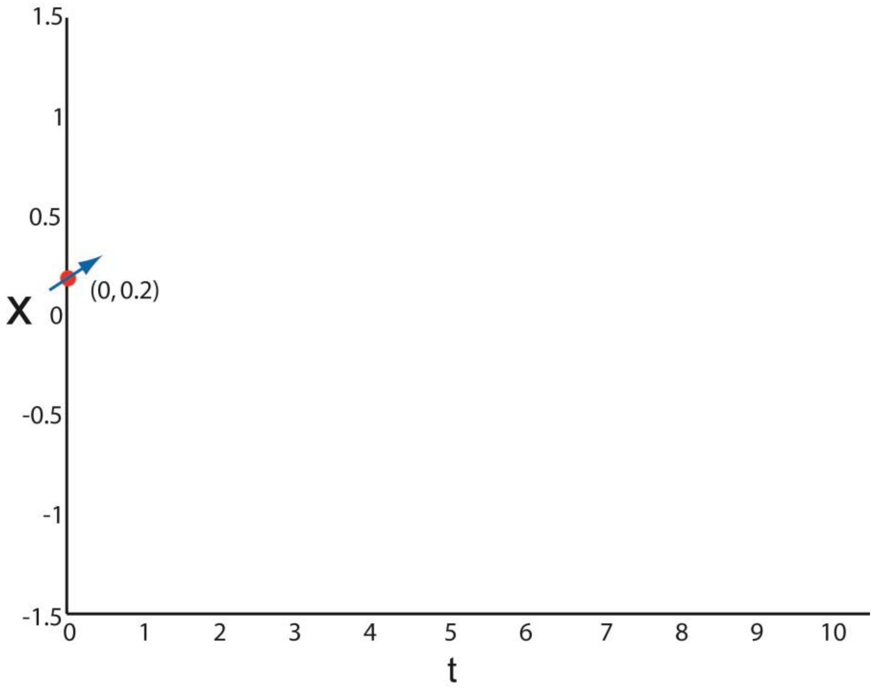
Direction field

$$\frac{dx}{dt} = r.x.(1-x)$$

Consider r = 1

$$\text{When } t = 0, x = 0.2$$

$$\frac{dx}{dt} = r.x.(1-x)$$
$$= 1 \times 0.2 \times (1-0.2) = 0.16$$



Slope of the arrow at (0, 0.2) is given by *dx/dt* at that point

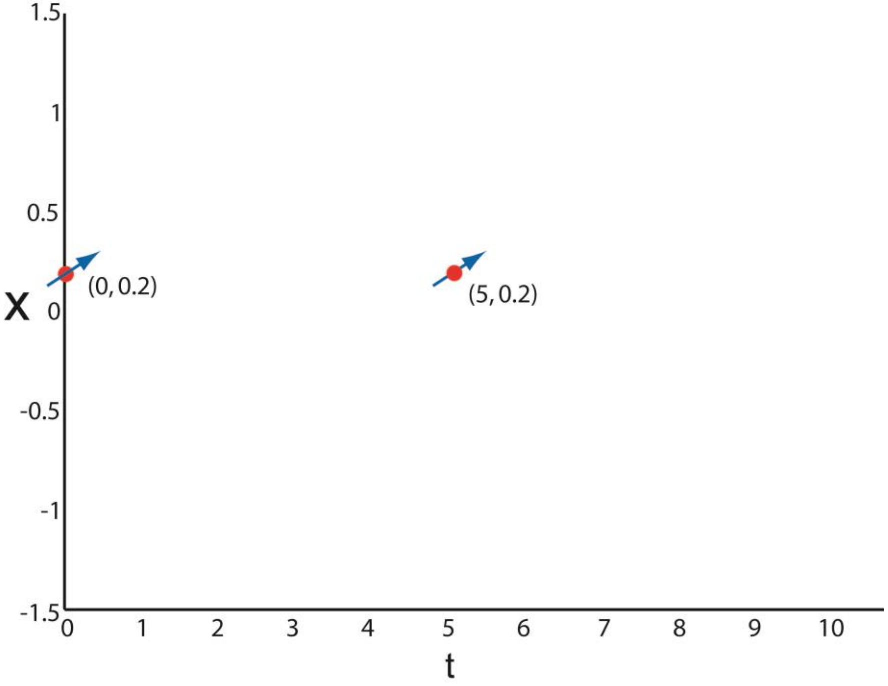
Direction field

$$\frac{dx}{dt} = r.x.(1-x)$$

Consider r = 1

$$\text{When } t = 5, x = 0.2$$

$$\frac{dx}{dt} = r.x.(1-x)$$
$$= 1\times 0.2\times(1-0.2) = 0.16$$



Slope of the arrow at (5, 0.2) is given by *dx/dt* at that point

Direction field

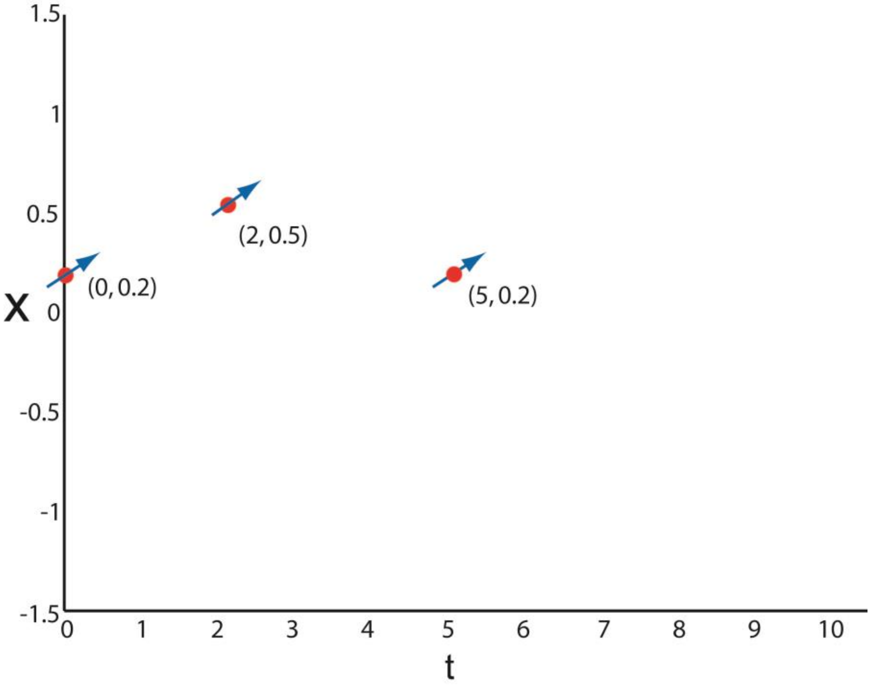
$$\frac{dx}{dt} = r.x.(1-x)$$

Consider r = 1

$$\text{When } t = 2, x = 0.5$$

$$\frac{dx}{dt} = r.x.(1-x)$$

$$= 1 \times 0.5 \times (1-0.5) = 0.25$$



Slope of the arrow at (2, 0.5) is given by *dx/dt* at that point

How to draw direction field

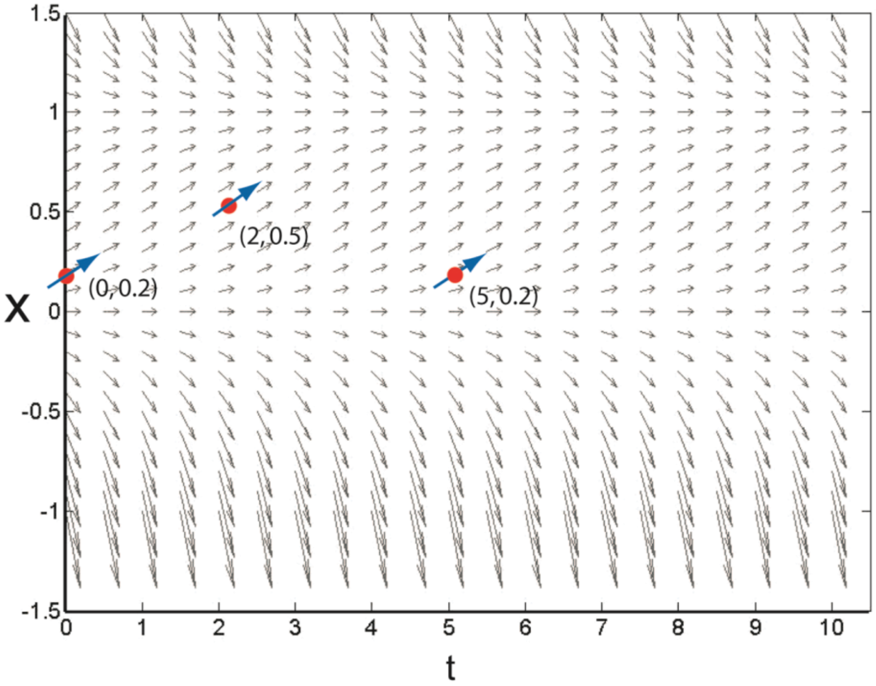
Take equidistance points and divide x-t space in grid.

At each grid point draw an arrow with slope = dx/dt at that point

Do it over the whole space

$$\frac{dx}{dt} = r.x.(1 - x)$$

Consider r = 1



Analyzing direction field

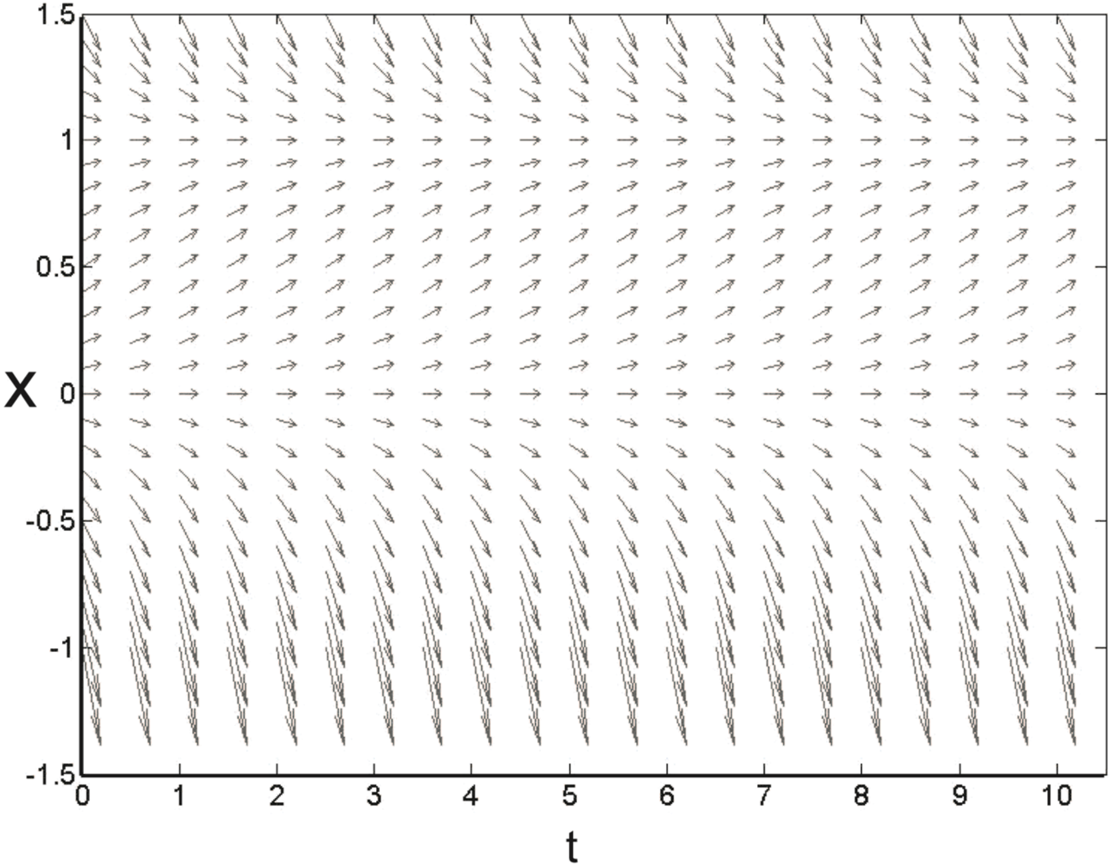
$$\frac{dx}{dt} = r.x.(1-x)$$

Consider r = 1

For *x* = *0* and *1*,

$$\frac{dx}{dt} = 0$$ and arrows are horizontal

Between 1 and 0, arrows are directing up

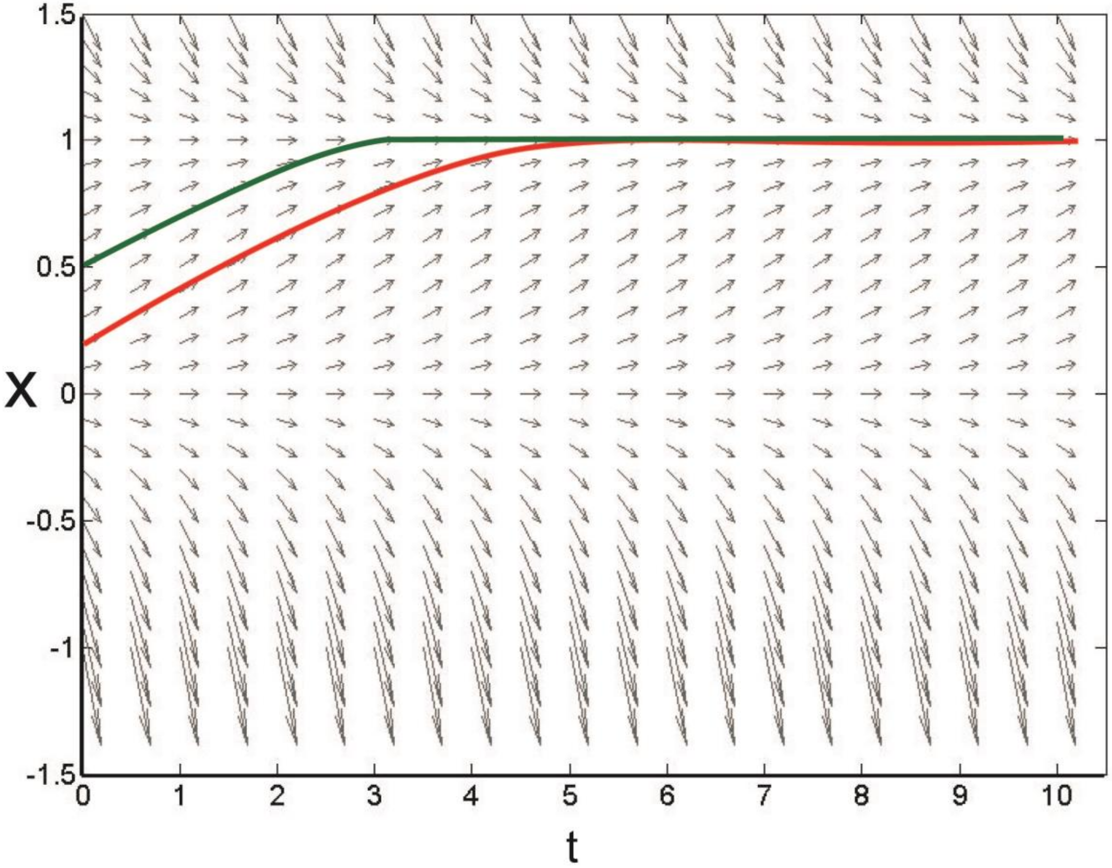


Analyzing direction field

Join successive arrows to generate integral curve

Integral curve shows how the dependent variable changes with independent variable.

We get similar curve by analytical and numerical method



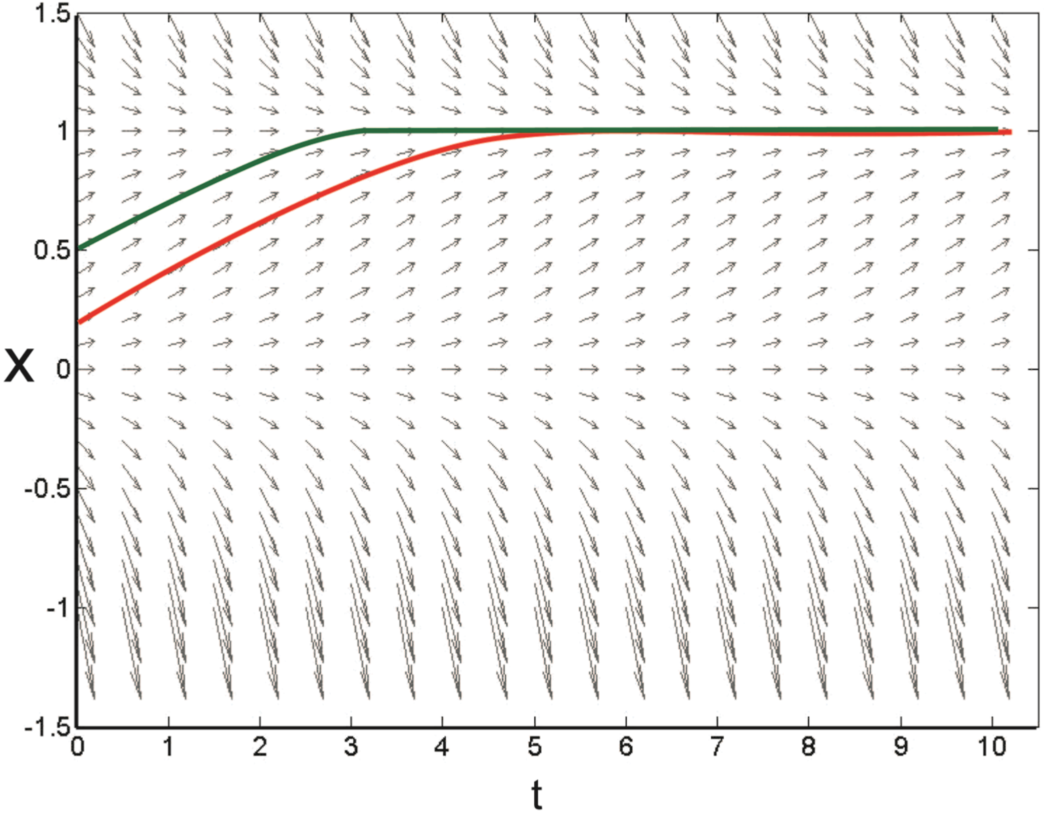
Understanding steady state

For $x = 0$ and 1,

$$\frac{dx}{dt} = 0$$ and arrows are horizontal

Once there, x does not changes with time

$x = 0$ and 1, are steady states of x



Identifying steady states

At steady state, dependent variable does not change with time and remains constant

For the ODE, $\frac{dx}{dt} = f(x,t)$

At steady state, $\frac{dx}{dt} = 0$

To find steady states, set

$$\frac{dx}{dt} = f(x,t) = 0$$
$$\Rightarrow f(x,t) = 0$$

Solve this relation algebraically to find steady state values of *x*

Calculating steady states

Logistic growth model

$$\frac{dx}{dt} = r.(1-\frac{x}{k}).x$$

$X_0 = 100$
$r = 0.05$ per min
$k = 10{,}000$

At steady state $\frac{dx}{dt} = 0$

$$\therefore \quad r.(1-\frac{x}{k}).x = 0$$

Therefore, $x = 0$ or $(1-\frac{x}{k}) = 0$

When, $(1-\frac{x}{k}) = 0$

$$\Rightarrow \frac{x}{k} = 1$$

$$\Rightarrow x = k$$

So, the population has two steady states, *x = 0* and *x = k = 10,000*.
These steady states are independent of rate constant for growth.

Key points:

1. Direction field: Arrows at regular grid points, having slope equal to the derivative of dependent variable.
2. Allows qualitative visualization of behavior of the dependent variable.
3. At a steady state the derivative of the dependent variable is equal to zero
4. At steady states the dependent variable doe not change with time and remain constant.
5. One can calculate steady states algebraically.
6. A system can have more than one steady states.