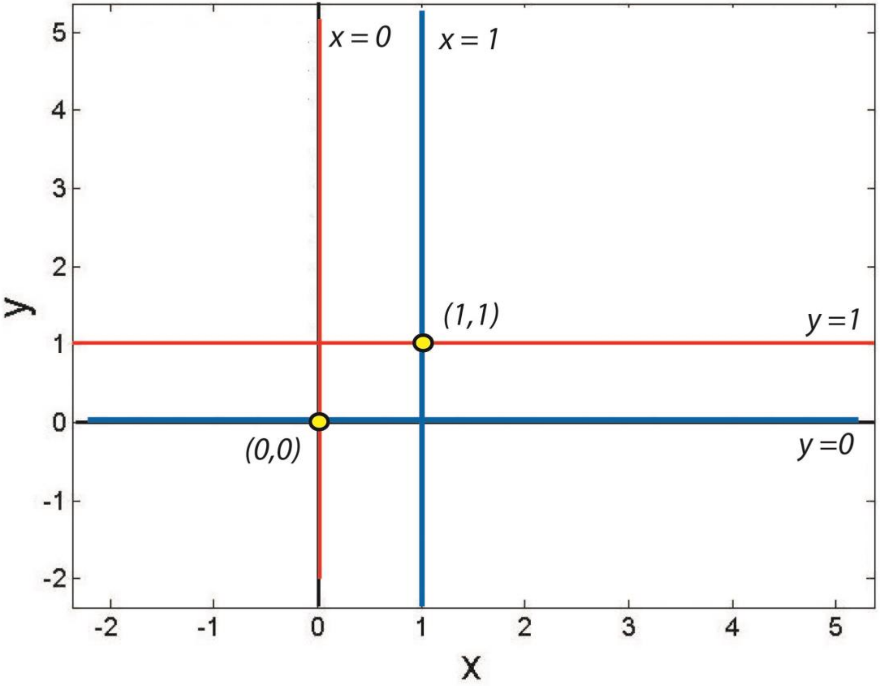

Phase Plane Analysis-I

Steady state analysis of a system of ODEs

$$\frac{dx}{dt} = k_1.x - k_2.x.y$$

$$\frac{dy}{dt} = k_2.x.y - k_3.y$$

At steady state, derivatives of all the dependent variables are zero

$$\frac{dx}{dt} = 0 \qquad \frac{dy}{dt} = 0$$

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$$k_1.x - k_2.x.y = 0$$

$$k_2.x.y - k_3.y = 0$$

$$k_1.x - k_2.x.y = 0$$

$$\Rightarrow x(k_1 - k_2 y) = 0$$

$$\therefore \quad x = 0 \text{ or } (k_1 - k_2 y) = 0$$

$$(k_1 - k_2 y) = 0$$

$$\Rightarrow y = \frac{k_1}{k_2}$$

$$k_2.x.y - k_3.y = 0$$

$$\text{When } x = 0$$

$$k_2.0.y - k_3.y = 0$$

$$\Rightarrow y = 0$$

$$\text{When } y = \frac{k_1}{k_2}$$

$$k_2.x.\frac{k_1}{k_2} - k_3.\frac{k_1}{k_2} = 0$$

$$\Rightarrow x.k_1 - k_3.\frac{k_1}{k_2} = 0$$

$$\Rightarrow x = k_3.\frac{k_1}{k_2}.\frac{1}{k_1} = \frac{k_3}{k_2}$$

The system has two steady states:

$x = 0$, $y = 0$

$x = k_3/k_2$, $y = k_1/k_2$

Analysis steady states graphically

$$\frac{dx}{dt} = k_1.x - k_2.x.y \qquad \frac{dy}{dt} = k_2.x.y - k_3.y$$

$$\text{let } k_1 = k_2 = k_3 = 1$$

At steady state,

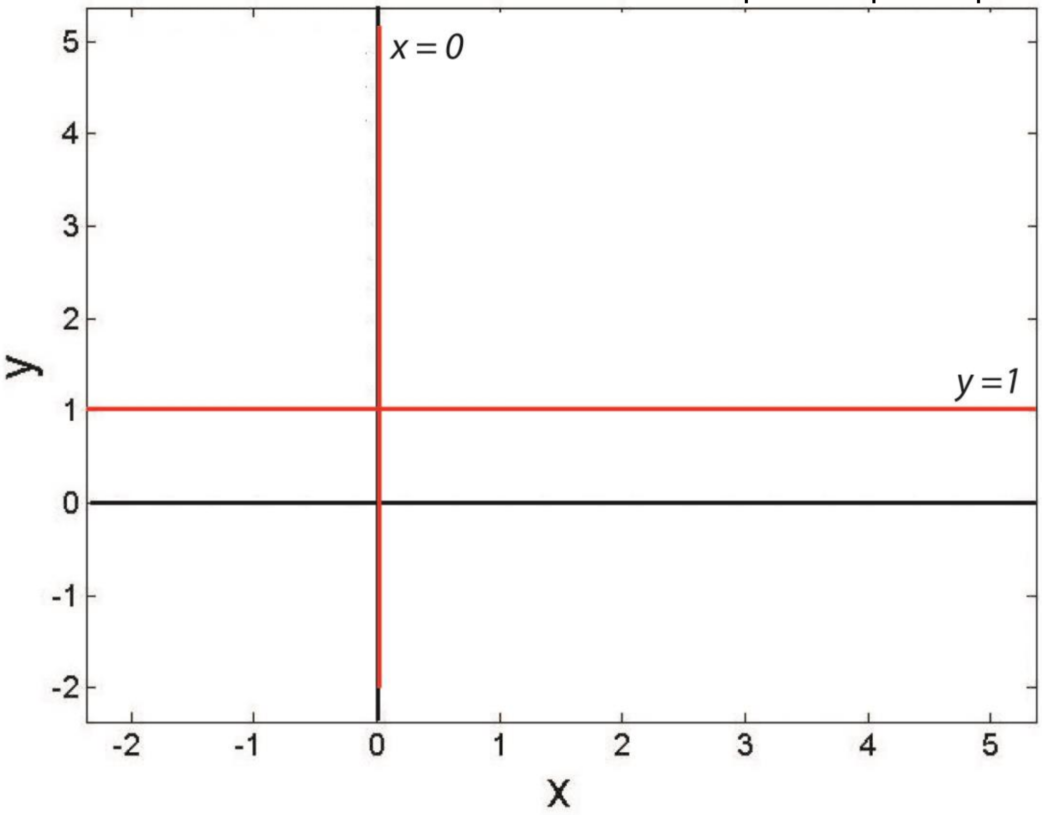
$$\frac{dx}{dt} = 0$$

$$\therefore\ x - x.y = 0$$

$$\Rightarrow x(1-y) = 0$$

$$\therefore\ x = 0 \text{ or } y = 1$$

Phase plane & phase plot



Analysis steady states graphically

$$\frac{dx}{dt} = k_1.x - k_2.x.y \qquad \frac{dy}{dt} = k_2.x.y - k_3.y$$

$$\text{let } k_1 = k_2 = k_3 = 1$$

At steady state,

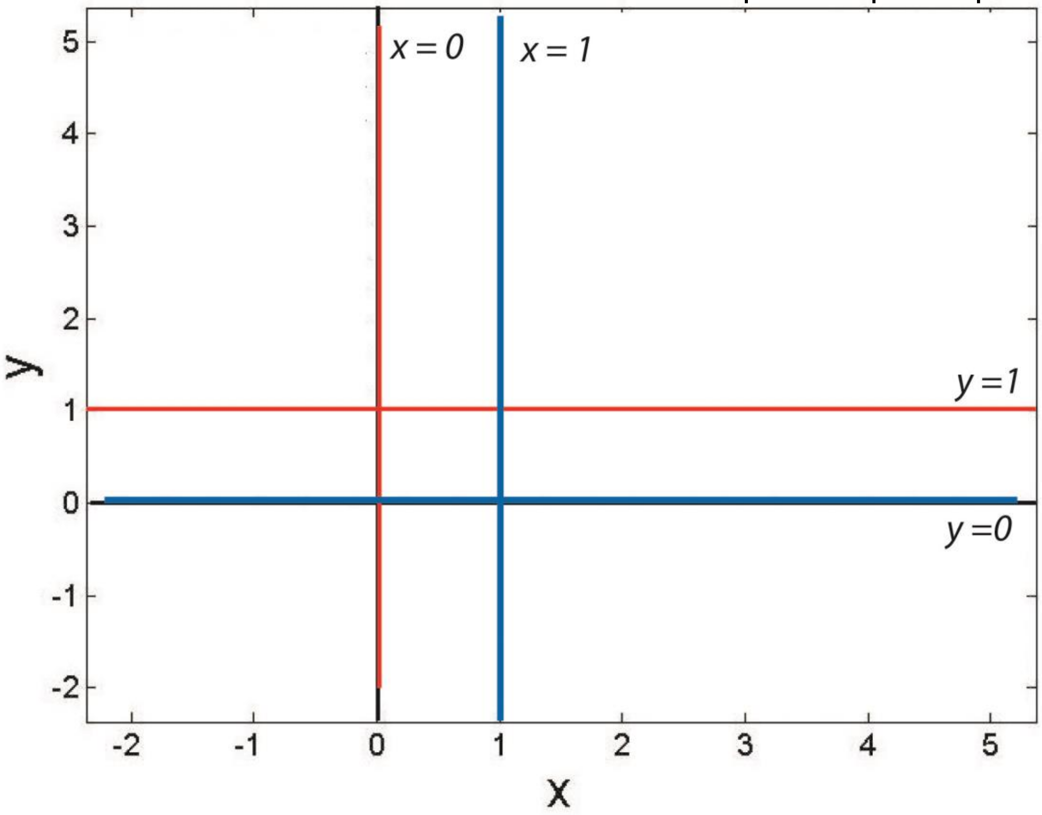
$$\frac{dy}{dt} = 0$$

$$\therefore \; x.y - y = 0$$

$$\Rightarrow y(x-1) = 0$$

$$\therefore \; x = 1 \text{ or } y = 0$$

Phase plane & phase plot



Analysis steady states graphically

$$\frac{dx}{dt} = 0$$

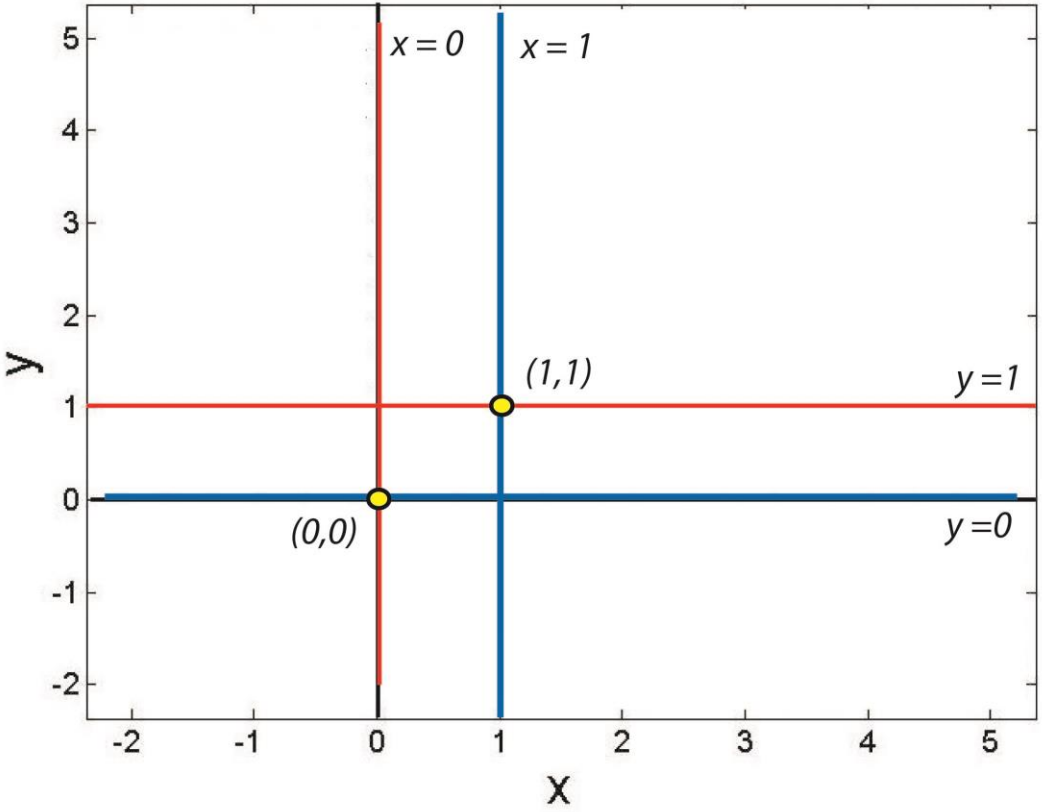
$x = 0 \text{ or } y = 1$ → *x* nullclines

$$\frac{dy}{dt} = 0$$

$x = 1 \text{ and } y = 0$ → *y* nullclines

Intersections of x and y nullclines are steady states

Phase plane & phase plot



Key points:

1. For a system of ODEs, at steady state, derivatives of all the dependent variables are zero
2. For a system of ODEs, the steady states can be identified by solving simultaneous equations.
3. Phase plane: with two axes representing two dependent variables
4. Phase plot: Graphical display of time evolution of two dependent variables in phase plane
5. Nullcline: a line or curve on a phase plane for which derivative of a dependent variable is zero.
6. Steady states: Intersections of nullclines of two dependent variables are steady states