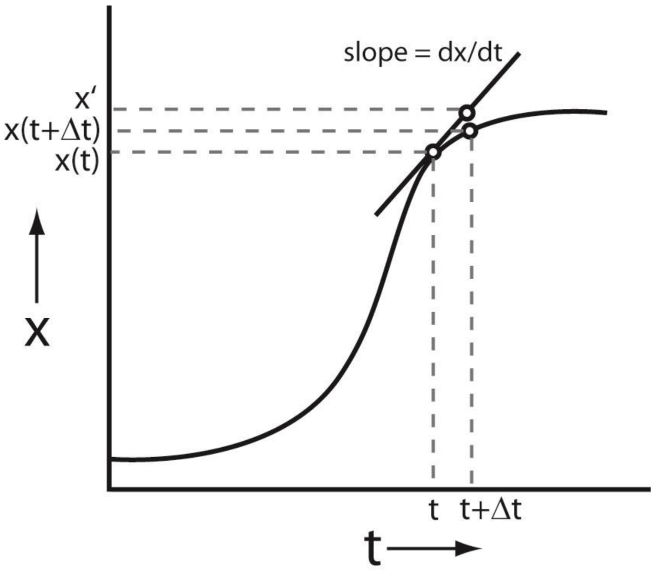


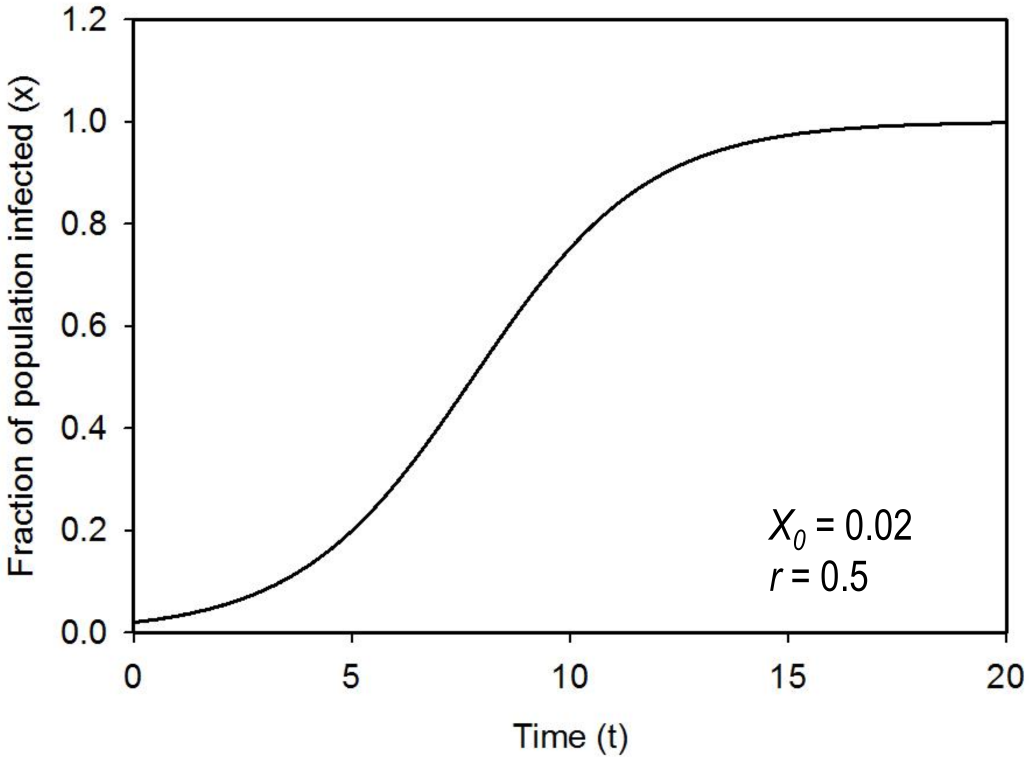
Numerical solution of ODE-1

Analytical method: Integrate the ODE

$$\frac{dx}{dt} = r(1-x)x$$

Integration, with initial conditions

$$x = \frac{1}{1+(\frac{1}{x_0}-1)e^{-rt}}$$



Limitations of analytical method:

Some differential equations are too complicated to be solved by analytical method.

Particularly for non-linear systems.

Large number of equations

System of coupled ODEs

Numerical methods:

1. Approximate but give workable result
2. We don't get functions, but get numerical values of dependent variables for each value of the independent variable. This can be used directly, like in plots.
3. Several algorithms are available
4. Can be executed by most programming languages.
5. Large number of plug-and-play software are available.

Euler method

$$\frac{dx}{dt} = r(1-x)x$$

$$\frac{x(t+\Delta t)-x(t)}{\Delta t} = r.(1-x).x$$

Do the approximation

$$\Rightarrow x(t+\Delta t) = x(t) + r.(1-x).x.\Delta t$$

$$\frac{dx}{dt} \approx \frac{\Delta x}{\Delta t} = \frac{x(t+\Delta t)-x(t)}{\Delta t}$$

@Biplab Bose, IIT Guwahati

Doing it numerically

$$\frac{dx}{dt} = r(1-x)x$$

$$x(t+\Delta t) = x(t) + r.(1-x).x.\Delta t$$

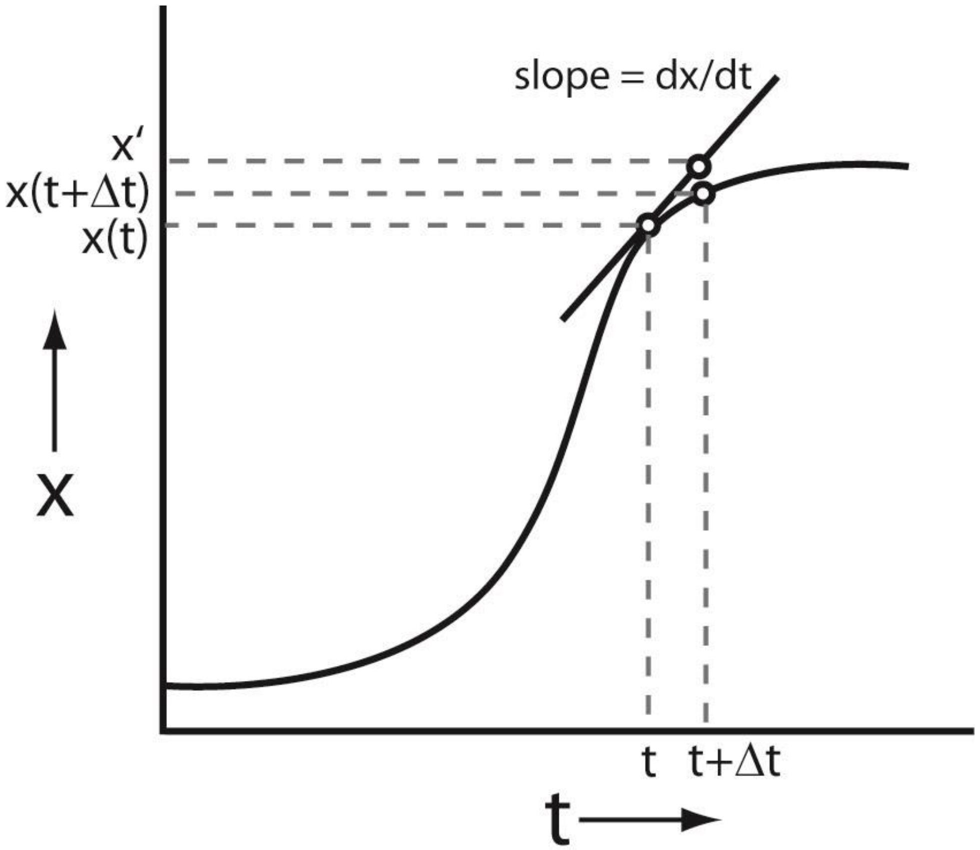
$x_0 = 0.02$ $r = 0.5$ $\Delta t = 0.1$

t	x(t)	r.(1-x).x.Δt	x(t+Δt)
0	0.02	0.00098	0.02098
0.1	0.02098	0.001027	0.022007
0.2	0.022007	0.001076	0.023083
0.3	0.023083	0.001128	0.024211
..	..	..	..
..	..	..	..
19.9	0.997793	0.00011	0.997903
20	0.997903	0.000105	0.998007

Executing numerical integration in MS Excel

Understanding numerical methods graphically

Estimate for next time point: Linear extrapolation using the slope estimated at present time point

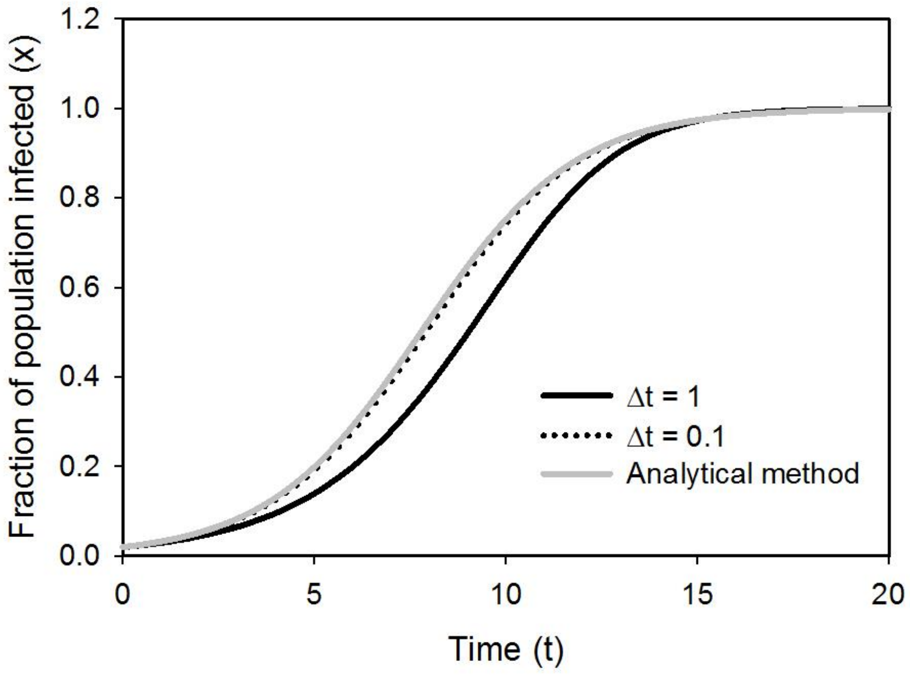


Effect of the size of the time step

Smaller time step gives more accurate result, but increases time for computation

$$\frac{dx}{dt} = r(1-x)x$$

$x_0 = 0.02$, $r = 0.5$



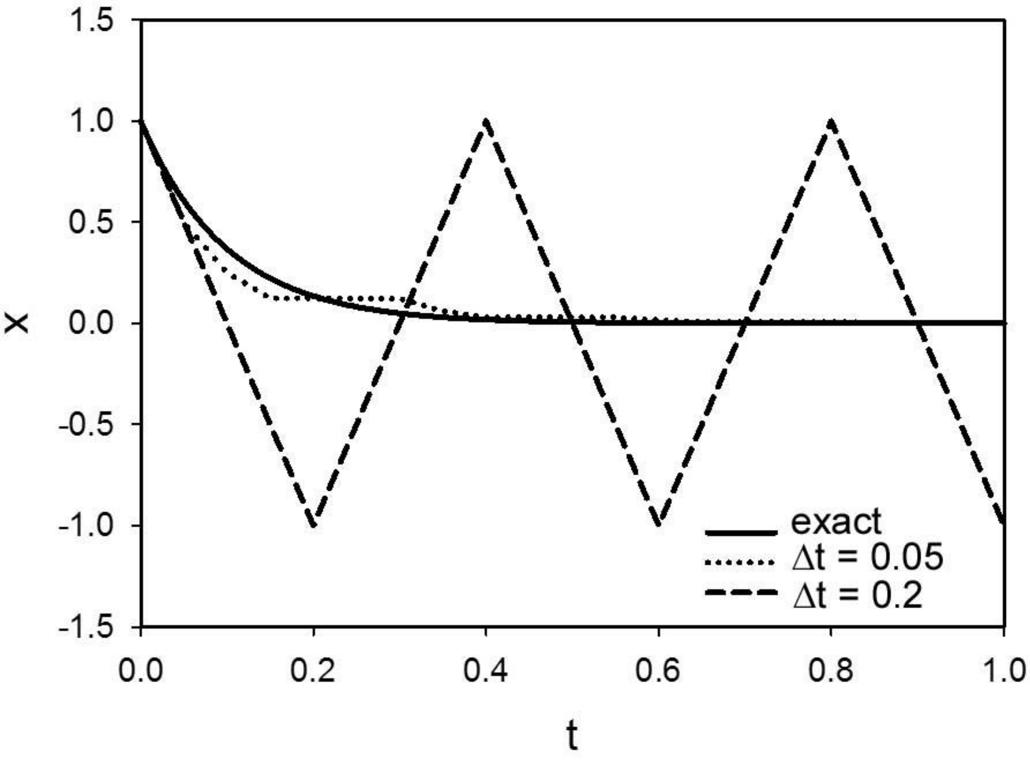
A stiffer system

Step size has drastic effect on result

The ODE: $$\frac{dx}{dt} = -10.x$$

The function: $$x = x_0.e^{-10.t}$$

Initial condition, $x_0 = 1$



Key points:

1. Numerical methods are approximate methods but powerful alternative to symbolic/analytical methods.
2. Dynamical models in biology are mostly analyzed by numerical methods
3. Large number of ready-made tools are available for numerical analysis of ODEs
4. Key concept: Do linear extrapolation using slope at a particular point to calculate the value at next time-step
5. Such linear extrapolation leads to error in estimate
6. This method is sensitive to size of time step. Smaller time step-size gives more accurate results but takes longer time to execute.
7. There are many advanced algorithms that work better than Euler method.