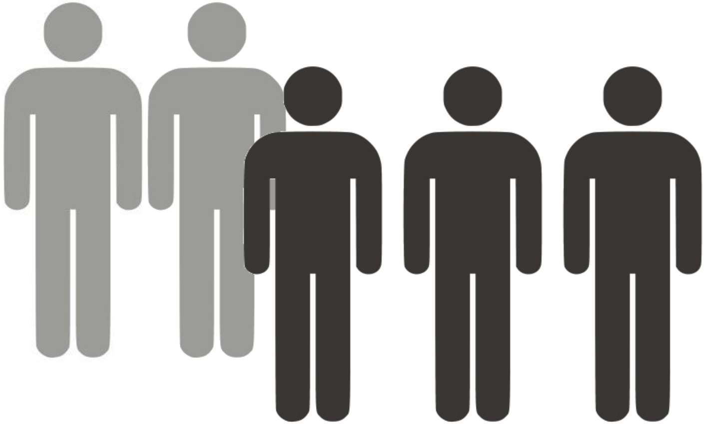


# Modeling the spread of infectious disease

# Modeling spread of infectious disease

$$\text{Infected} + \text{Normal} \xrightarrow{r} 2.\ \text{Infected}$$

Assumptions:

a) The disease spreads only when an infected person comes in contact with an uninfected one

b) Every body can come in contact with every one

c) The total population is very large

d) The total population remains constant over time (no death or birth)

e) No one gets cured

## Creating an ODE-based model

$$\text{Infected} + \text{Normal} \xrightarrow{r} 2.\ \text{Infected}$$

*x* = fraction of the population that is infected
*(1-x)* = fraction of the population uninfected

$$\frac{dx}{dt} = r(1-x)x$$

*r* = rate constant for spread of the infection

## Questions to be answered

The phenomena: $\text{Infected} + \text{Normal} \xrightarrow{r} 2.\ \text{Infected}$

The model: $$\frac{dx}{dt} = r(1-x)x$$

Q1. Let at *time* = *0*, fraction of the population infected be $x_0$.
What will be size of the infected population at time *t*?

Q2. Create *time* vs *x* plot to show the dynamics of spread of infection.

# Integrate to get the answers

$$\frac{dx}{dt} = r(1-x)x$$

$$x = f(t)$$

$$\int_{x_0}^{x} \frac{dx}{x(1-x)} = r\int_0^t dt,$$

$$\Rightarrow \int_{x_0}^{x} \frac{dx}{x} + \int_{x_0}^{x} \frac{dx}{(1-x)} = r\int_0^t dt,$$

$$\Rightarrow \left[\ln x\right]_{x_0}^{x} - \left[\ln(1-x)\right]_{x_0}^{x} = rt,$$

$$\Rightarrow \ln\frac{x}{x_0} - \ln\frac{(1-x)}{(1-x_0)} = rt,$$

$$\Rightarrow \frac{x(1-x_0)}{x_0(1-x)} = e^{rt},$$

$$\Rightarrow x = \frac{1}{1+(\frac{1}{x_0}-1)e^{-rt}}$$

## Question 1

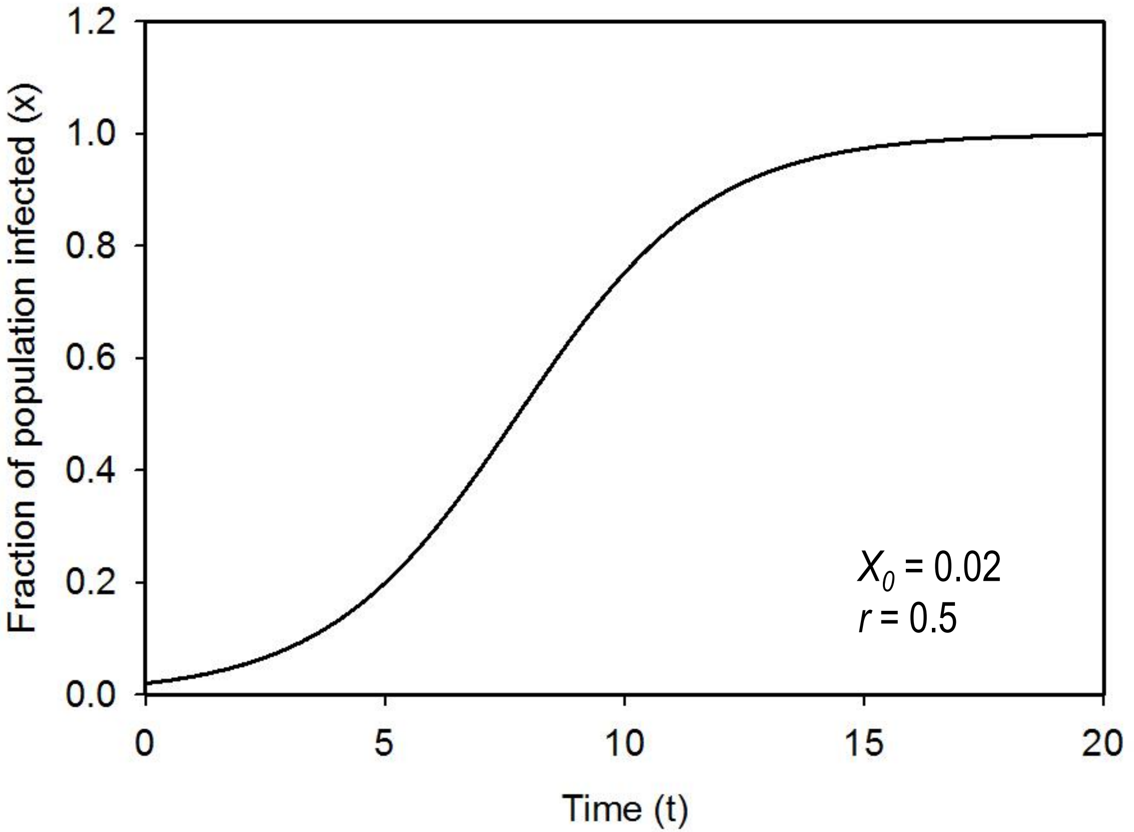
Starting with $x_0$ infected population, what will be size of the infected population at time $t$?

$$\frac{dx}{dt} = r(1-x)x \xrightarrow{\text{Integration, with initial conditions}} x = \frac{1}{1+(\frac{1}{x_0}-1)e^{-rt}}$$

## Question 2

$$\frac{dx}{dt} = r(1-x)x$$

$$x = \frac{1}{1+(\frac{1}{x_0}-1)e^{-rt}}$$



## Key points:

1. Make assumptions to simplify the problem
2. Key assumptions for ODE model: Homogeneity & large system size
3. If possible consider conservation of variables
4. Represent the rate of the process by an ODE
5. Integrate the ODE to get the function describing dynamics of the process
6. Answer questions using this function