CSCE 590-1: From Data to Decisions with Open Data: A Practical Introduction to AI

Lecture 17: Reasoning and Decisions Under Uncertainty

PROF. BIPLAV SRIVASTAVA, AI INSTITUTE

11TH MAR, 2021

Carolinian Creed: "I will practice personal and academic integrity."

Organization of Lecture 17

- Introduction Segment
  - Recap/ Discussion of Lecture 16

- Main Segment
  - Kind of uncertainties
  - Probability/ Bayesian methods
  - Utility theory: choosing from possible outcomes
    - What is the best decision possible: Maximize Expectation
  - Choosing actions

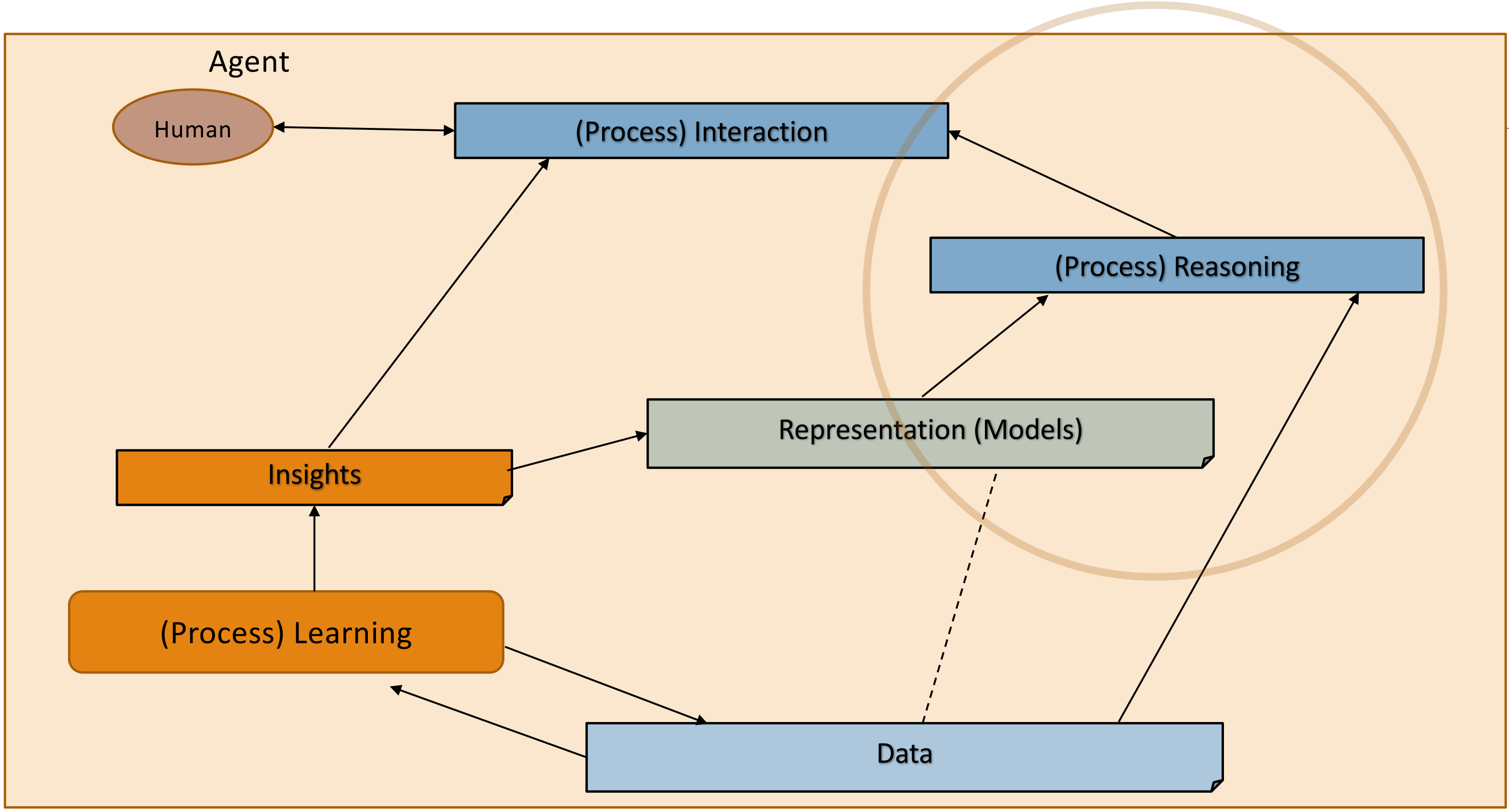
- Concluding Segment
  - About Next Lecture – Lecture 18
  - Ask me anything

Introduction Segment

Recap of Lecture 16

- Continued discussion on
  - Search
  - Sequential decision making: Planning
- Explored optimal decisions
  - Domain-specific optimal methods
  - A* for general search

Relationship Between AI Processes



Example Situation – Course Selection

- A person wants to pass an academic program in two majors: A and B
- There are three subjects: A, B and C, each with three levels (*1, *2, *3). There are thus 9 courses: A1, A2, A3, B1, B2, B3, C1, C2, C3
- To graduate, at least one course at beginner (*1) level is needed in major(s) of choice(s), and two courses at intermediate levels (*2) are needed
- **Uncertainty**: *A course is not offered in a semester, student needs to repeat a course, the student wants to change the major subject or commit to it after intermediate level*
- Answer questions
  - Q1: How many minimum courses does the person have to take ?
  - Q2: Can a person graduate in 2 majors studying 3 courses only?
  - …

Main Segment

Forms of Uncertainty

- Uncertain knowledge, caused by
  - Incomplete knowledge
  - Incorrect knowledge
- Uncertain actions, caused by
  - Physics of the domain
  - External events

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Alternative approaches to represent
- Degree of belief: Probability. The sentence still is true or false
- Degree of truth: Fuzzy logic

Language	Ontological Commitment (What exists in the world)	Epistemological Commitment (What an agent believes about facts)
Propositional logic	facts	true/false/unknown
First-order logic	facts, objects, relations	true/false/unknown
Temporal logic	facts, objects, relations, times	true/false/unknown
Probability theory	facts	degree of belief 0…1
Fuzzy logic	degree of truth	degree of belief 0…1

Credits:
- Russell & Norvig, AI - A Modern Approach
- Deepak Khemani - A First Course in AI

Forms of Uncertainty

- Uncertain knowledge, caused by
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Use Probability Theory
Infer using probabilities

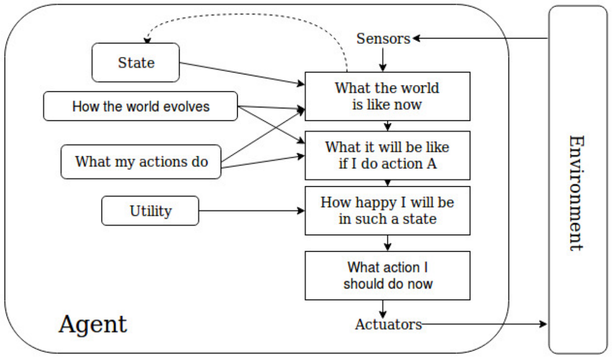
Decision Processes = create
situational policies (state-action based)

Decision-theoretic Agent

Probability theory: degree of belief in sentences

- Summarizes the uncertainty t

Utility theory: represent and reason with preferences



function DT-AGENT(*percept*) **returns** an *action*
static: a set probabilistic beliefs about the state of the world

calculate updated probabilities for current state based on
available evidence including current percept and previous action
calculate outcome probabilities for actions,
given action descriptions and probabilities of current states
select *action* with highest expected utility
given probabilities of outcomes and utility information
return *action*

Source: Russell & Norvig, AI - A Modern Approach

Review - Basic Probability

Axioms of Probability Theory

- All probabilities between 0 and 1

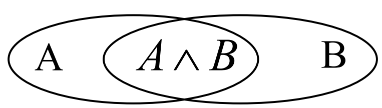
$$0 \leq P(A) \leq 1$$

- True proposition has probability 1, false has probability 0.

  P(true) = 1   P(false) = 0.

- The probability of disjunction is:

$$P(A \vee B) = P(A) + P(B) - P(A \wedge B)$$



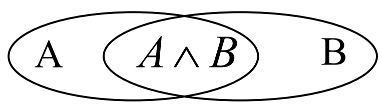
Slide adapted from: Ray Mooney's AI Course

Conditional Probability

- P(*A* | *B*) is the probability of *A* given *B*
- Assumes that *B* is all and only information known.

Defined by:

$$P(A \mid B) = \frac{P(A \wedge B)}{P(B)}$$



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Independence

A and *B* are *independent* iff:

$$P(A \mid B) = P(A) \qquad\qquad P(B \mid A) = P(B)$$

Therefore, if *A* and *B* are independent:

$$P(A \mid B) = \frac{P(A \wedge B)}{P(B)} = P(A)$$

$$P(A \wedge B) = P(A)P(B)$$

Slide adapted from: Ray Mooney's AI Course

Bayes Theorem

$$P(H \mid E) = \frac{P(E \mid H)P(H)}{P(E)}$$

H: Hypothesis, E: Evidence

Simple proof from definition of conditional probability:

$$P(H \mid E) = \frac{P(H \wedge E)}{P(E)} \qquad P(E \mid H) = \frac{P(H \wedge E)}{P(H)}$$

(Definitions of cond. prob.)

Then: $$P(H \mid E) = \frac{P(E \mid H)P(H)}{P(E)}$$

Slide adapted from: Ray Mooney's AI Course

Joint Probability Distribution

The **joint probability distribution** for a set of N random variables, $X_1,\ldots,X_n$ gives the probability of every combination of values

!A1

	A3	!A3
A2	0.20	0.02
!A2	0.02	0.01

A1

	A3	!A3
A2	0.05	0.30
!A2	0.20	0.20

The probability of any possible conjunctions can be calculated by summing the appropriate subset of values from the joint distribution.

$P(A2 \wedge A3) = 0.20 + 0.05 = 0.25$

$P(A3) = 0.2 + 0.02 + 0.05 + 0.2 = 0.47$

Importance of Joint Probability Distribution

- The probability of any possible conjunctions can be calculated from the joint distribution

> **Basic inferencing procedure with probability**
> 1) Compute joint distribution
> 2) Compute any desired conditional probability using the joint distribution

- But estimating "joint" is infeasible for most practical situations
- In many situations, conditional dependence/ independence occurs naturally in the domain which can be exploited for compact representation and efficient calculation

Exercise and Code

Probabilistic Reasoning

- Pomegranate library

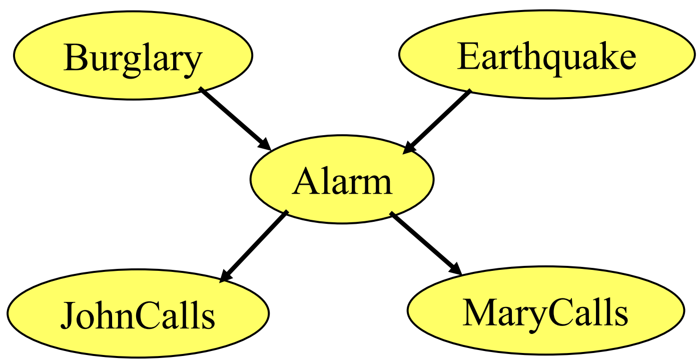
- Game: TV show – Monty Python
  - One Line Demo: http://math.ucsd.edu/~crypto/Monty/monty.html

- Sample code https://github.com/biplav-s/course-d2d-ai/blob/main/sample-code/l17-uncertainty/DecisionMaking%20Uncertainty.ipynb

Bayesian Networks

Directed Acyclic Graph (DAG)
- Nodes are random variables
- Edges indicate causal influences

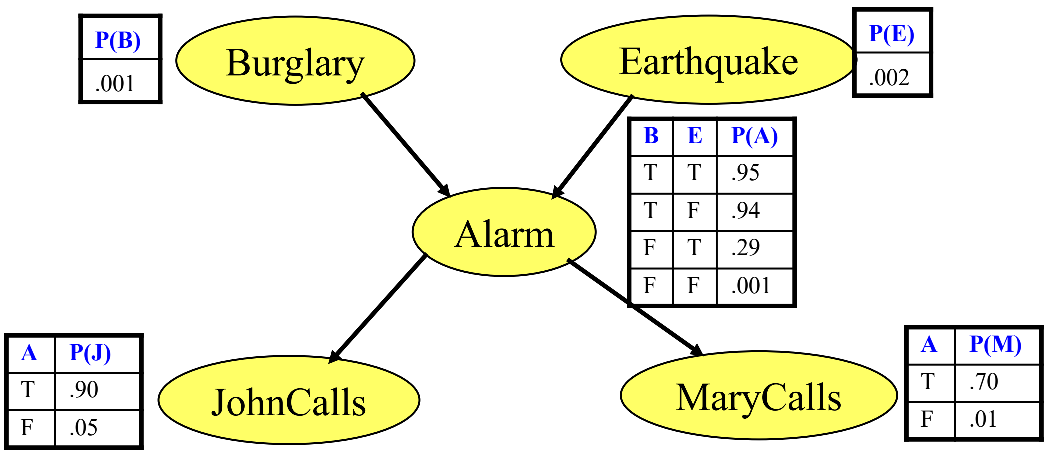
$$P(x_1, x_2, \ldots x_n) = \prod_{i=1}^{n} P(x_i \mid \text{Parents}(X_i))$$



Slide adapted from: Ray Mooney's AI Course
Example from Russell & Norvig AI book

Conditional Probability Tables

- Each node has a **conditional probability table** (**CPT**) that gives the probability of each of its values given every possible combination of values for its parents (conditioning case).
- Roots (sources) of the DAG that have no parents are given prior probabilities.



P(B)
.001

P(E)
.002

B	E	P(A)
T	T	.95
T	F	.94
F	T	.29
F	F	.001

A	P(J)
T	.90
F	.05

A	P(M)
T	.70
F	.01

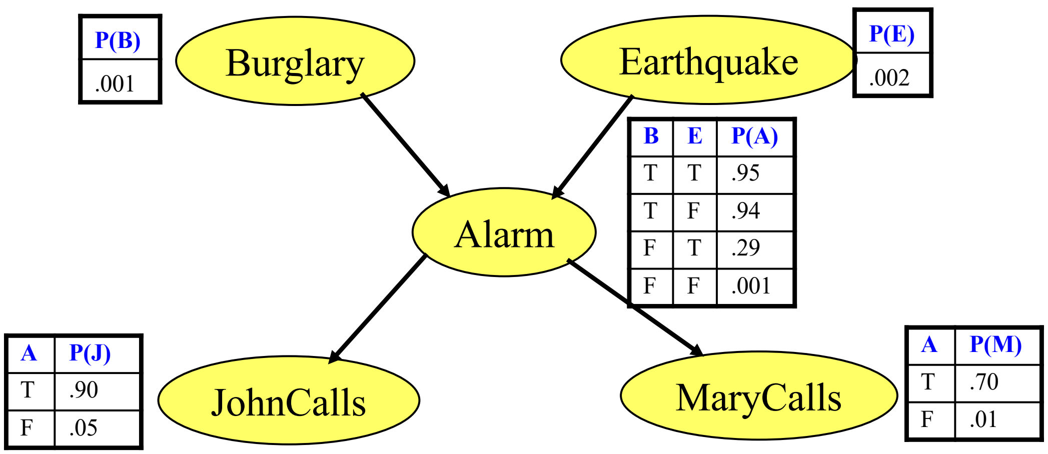
Slide adapted from: Ray Mooney's AI Course
Example from Russell & Norvig AI book

Conditional Probability Tables

- Probability that John and Mary call on hearing an alarm but there is no burglary or earthquake

= P (J |A) * P (M|A) * P(A| !B ∧ !E) * (!B) * (!E)

= 0.9 * 0.7 * 0.001 * 0.999 * 0.998 = 0.00062



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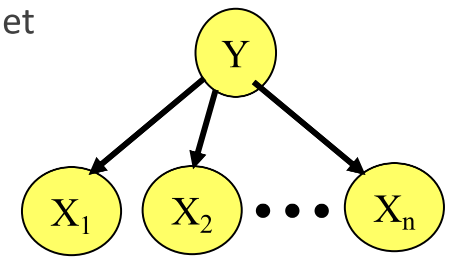
Example from Russell & Norvig AI book

Naïve Bayes as a Bayes Net

Naïve Bayes is a simple Bayes Net



Priors P(Y) and conditionals P(X_i|Y) for Naïve Bayes provide CPTs for the network.

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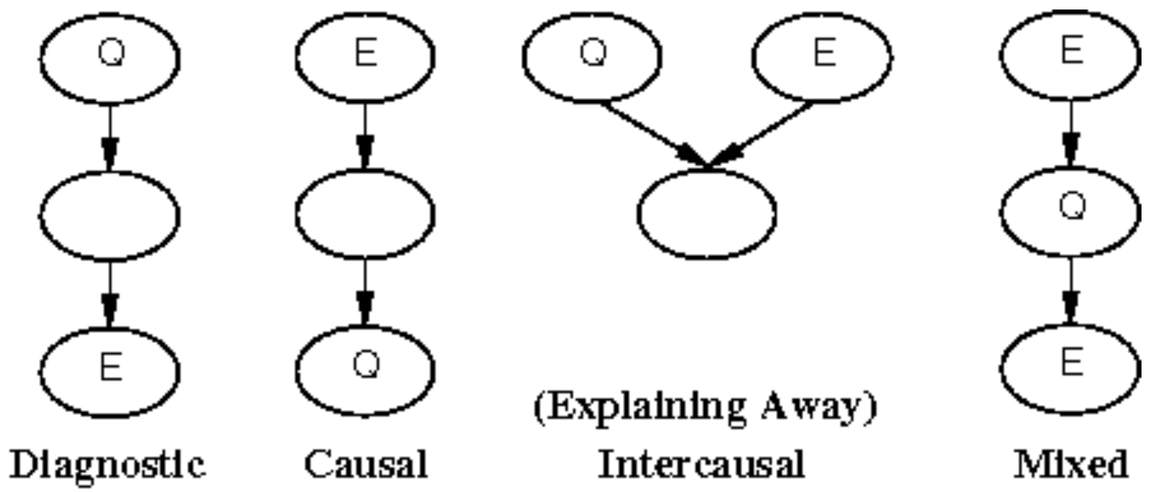
Types of Inference



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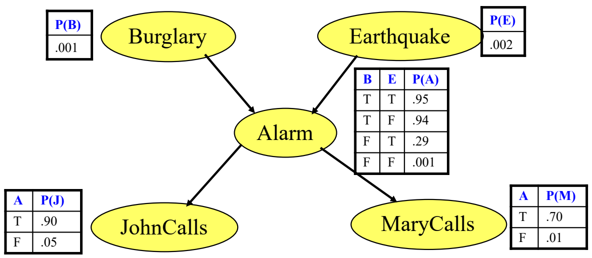
Sample Inferences

Diagnostic (evidential, abductive): From effect to cause.

- P(Burglary | JohnCalls) = 0.016
- P(Burglary | JohnCalls ∧ MaryCalls) = 0.29
- P(Alarm | JohnCalls ∧ MaryCalls) = 0.76
- P(Earthquake | JohnCalls ∧ MaryCalls) = 0.18



Causal (predictive): From cause to effect

- P(JohnCalls | Burglary) = 0.86
- P(MaryCalls | Burglary) = 0.67

Intercausal (explaining away): Between causes of a common effect.

- P(Burglary | Alarm) = 0.376
- P(Burglary | Alarm ∧ Earthquake) = 0.003

Mixed: Two or more of the above combined

- (diagnostic and causal) P(Alarm | JohnCalls ∧ ¬Earthquake) = 0.03
- (diagnostic and intercausal) P(Burglary | JohnCalls ∧ ¬Earthquake) = 0.017

Slide adapted from: Ray Mooney's AI Course

Exercise and Code

Probabilistic and Bayesian Methods

- From Book: AI – A Modern Approach, https://github.com/aimacode/aima-python/blob/master/probability.ipynb

Source: Russell & Norvig, AI: A Modern Approach

Utility Theory: What Choice to Make?

Expected Utility

- An action A has outcomes $Result_i(A)$, where i ranges over possible outcomes
- Probability of outcome
  - $P(Result_i(A) \mid Do(A), E)$ where E is the current evidence
- Expected utility
  - $EU(A|E) = \Sigma\, P(Result_i(A) \mid E, Do(A)) * U(Result_i(A))$
- Maximum expected utility (MEU) principle
  - A rational agent should choose an action that maximizes the agent's expected utility

Constraints on Utility Functions

- Decision scenarios are called lotteries. Simplest lottery L has two possible outcomes—state A with probability p, and state B with the remaining probability (1-p).

L=[p, A; (1-p), B]

- Agent can have preferences
  - A > B : A preferred over B
  - A < B : B preferred over A
  - A ~ B : agent indifferent to either state

Axioms of Utility Theory

ORDERABILITY

◇ **Orderability**: Given any two states, a rational agent must either prefer one to the other or else rate the two as equally preferable. That is, an agent should know what it wants.

$$(A \succ B) \vee (B \succ A) \vee (A \sim 5)$$

TRANSITIVITY

◇ **Transitivity:** Given any three states, if an agent prefers A to B and prefers B to C, then the agent must prefer A to C.

$$(A \succ B) \text{ A } (B \succ C) \;\Rightarrow\; (A \succ C)$$

CONTINUITY

0 **Continuity:** If some state B is between A and C in preference, then there is some probability p for which the rational agent will be indifferent between getting B for sure and the lottery that yields A with probability p and C with probability $1 - p$.

$$A \succ B \succ C \;\Rightarrow\; \exists p\ [p, A;\ 1 - p, C] \sim B$$

SUBSTITUTABILITY

◇ **Substitutability:** If an agent is indifferent between two lotteries, A and B, then the agent is indifferent between two more complex lotteries that are the same except that B is substituted for A in one of them. This holds regardless of the probabilities and the other outcome(s) in the lotteries.

$$A \sim B \;\Rightarrow\; [p, A;\ 1 - p, C] \sim [p, B; 1 - p, C]$$

MONOTONICITY

◇ **Monotonicity:** Suppose there are two lotteries that have the same two outcomes, A and B. If an agent prefers A to B, then the agent must prefer the lottery that has a higher probability for A (and vice versa).

$$A \succ B \;\Rightarrow\; (p \geq q \Leftrightarrow [p, A;\ 1 - p, B] \succsim [q, A;\ 1 - q, B])$$

DECOMPOSABILITY

◇ **Decomposability**: Compound lotteries can be reduced to simpler ones using the laws of probability. This has been called the "no fun in gambling" rule because it says that an agent should not prefer (or disprefer) one lottery just because it has more choice points than another.[2]

$$[p, A;\ 1 - p, [q, B;\ 1 - q, C]] \sim [p, A; (1 - p)q, B;\ (1 - p)(1 - q), C]$$

Source: Russell & Norvig, AI: A Modern Approach

What It Means to Obey Axioms of Utility

- Utility principle
  - U(A) > U(B) ⇔ A > B
  - U(A) = U(B) ⇔ A ~ B

- Maximum Expected Utility principle
  - $U(p1, S1; \ldots; pn, Sn] = \Sigma\, p_i\, U(S_i)$

Source: Russell & Norvig, AI: A Modern Approach

Humans Do Now Always Follow Utility Theory

- Subjects in this experiment are given a choice between lotteries A and B:
  - Comparison scenario 1
    - A : 80% chance of $4000
    - B : 100% chance of $3000
  - Comparison scenario 2
    - C : 20% chance of $4000
    - D : 25% chance of $3000

Tversky and Kahneman (1982) experiment

- The majority of survey respondents choose B over A and C over D.
  - Comparison scenario 1:
    - A: 0.8 * 4000 + 0.2 * 0 = **3200**
    - B: 3000
  - Comparison scenario 2:
    - C: 0. 2* 4000 + 0.8 * 0 = **800**
    - D: 0.25 * 3000 + 0.75 * 0 = 750

Consistent utility: A over B and C over D.

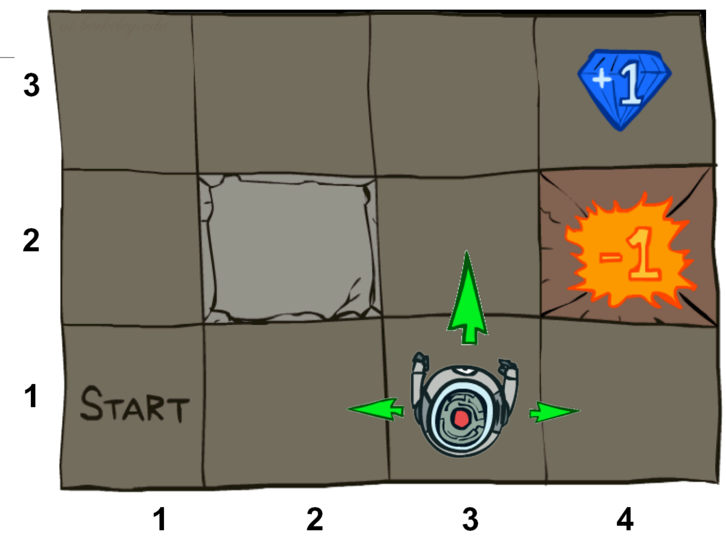
Source: Russell & Norvig, AI: A Modern Approach

Uncertainty in Action

- Example in course domain
  - *Student became sick*
  - *Student needs to repeat a course*
  - *The course was cancelled (other agents)*

Synthetic Example: Grid World

- A maze-like problem
  - The agent lives in a grid
  - Walls block the agent's path
- Noisy movement: actions do not always go as planned
  - 80% of the time, the action North takes the agent North (if there is no wall there)
  - 10% of the time, North takes the agent West; 10% East
  - If there is a wall in the direction the agent would have been taken, the agent stays put
- The agent receives rewards each time step
  - Small "living" reward each step (can be negative)
  - Big rewards come at the end (good or bad)
- Goal: maximize sum of rewards

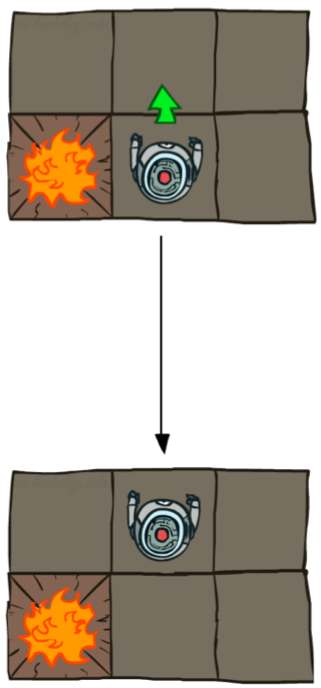


Slide adapted from: Dan Klein and Pieter Abbeel's AI lecture
Original example in Russell & Norvig's AI book

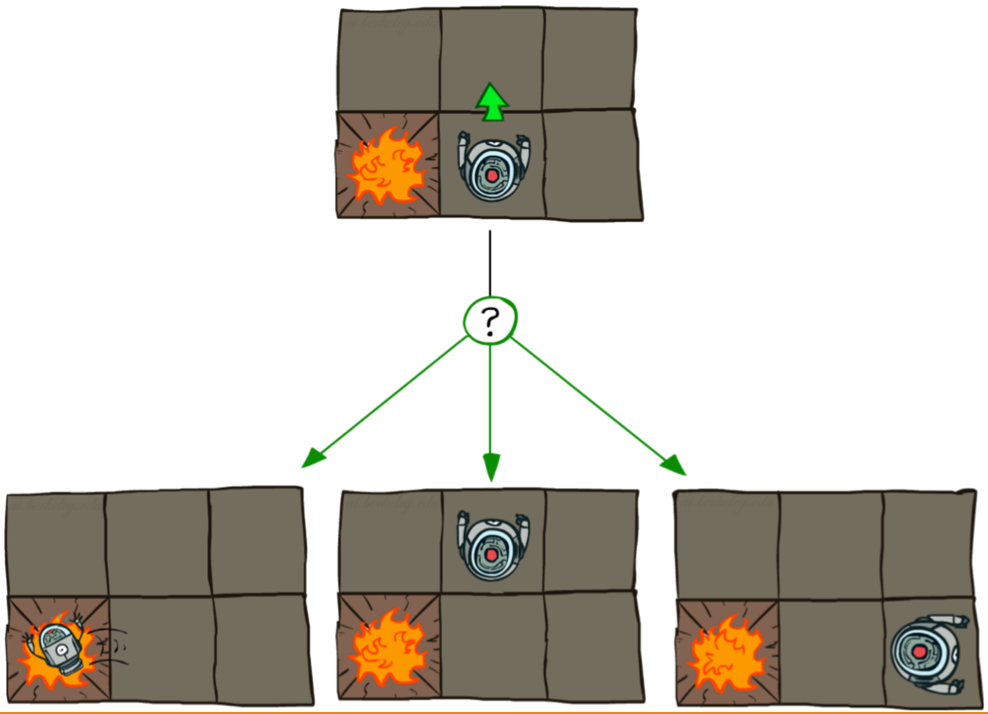
Grid World Actions

Slide adapted from: Dan Klein and Pieter Abbeel's AI lecture
Original example in Russell & Norvig's AI book

Deterministic Grid World



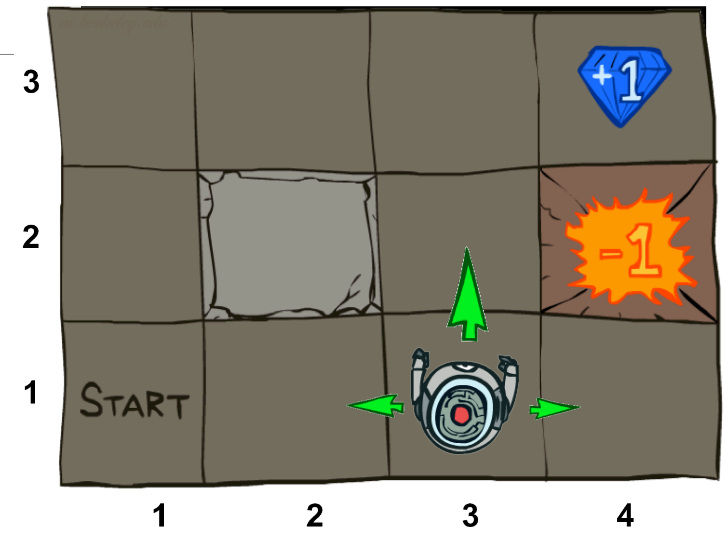
Stochastic Grid World



Markov Decision Processes

An MDP is defined by:

- A set of states $s \in S$
- A set of actions $a \in A$
- A transition function T(s, a, s')
  - Probability that a from s leads to s', i.e., P(s'| s, a)
  - Also called the model or the dynamics
- A reward function R(s, a, s')
  - Sometimes just R(s) or R(s')
- A start state
- Maybe a terminal state



MDPs are non-deterministic search problems

Slide adapted from: Dan Klein and Pieter Abbeel's AI lecture
Original example in Russell & Norvig's AI book

[Demo – gridworld manual intro (L8D1)]

Markovian Assumption

"Markov" generally means that given the present state, the future and the past are independent

For Markov decision processes, "Markov" means action outcomes depend only on the current state

$$P(S_{t+1} = s' | S_t = s_t, A_t = a_t, S_{t-1} = s_{t-1}, A_{t-1}, \dots S_0 = s_0)$$

$$=$$

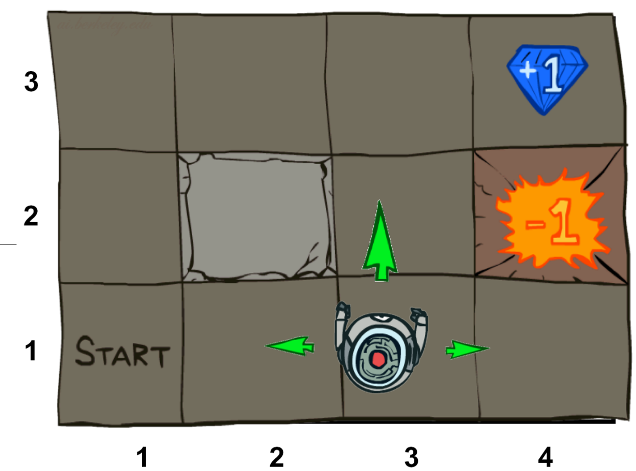
$$P(S_{t+1} = s' | S_t = s_t, A_t = a_t)$$

Andrey Markov
(1856-1922)

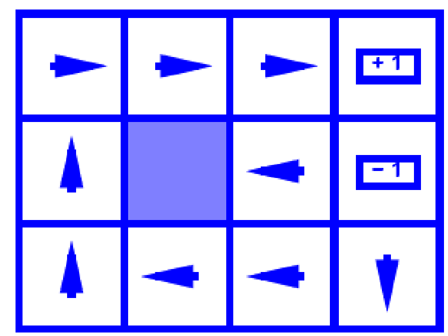
Slide adapted from: Dan Klein and Pieter Abbeel's AI lecture

Output: Policies

- In deterministic single-agent search problems, we have a plan, or sequence of actions, from start to a goal



- For MDPs, we want an optimal policy $\pi^*: S \rightarrow A$
  - A policy π gives an action for each state
  - An optimal policy is one that maximizes expected utility if followed



R(s) = -0.01

Slide adapted from: Dan Klein and Pieter Abbeel's AI lecture

Exercise and Code

- MDP Solution Methods
  - From Book: AI – A Modern Approach, https://github.com/aimacode/aima-python/blob/master/mdp.ipynb

Source: Russell & Norvig, AI: A Modern Approach

Lecture 16: Concluding Comments

- Kind of uncertainties
- What is the best decision possible: Maximize Expectation
- Some methods
  - Bayesian methods
  - Utility theory
  - MDPs

Concluding Segment

Upcoming Classes

15	**Mar 4 (Th)**	**Reasoning and Search**	**Semester - Midpoint**
16	Mar 9 (Tu)	Agent – Optimization	
17	Mar 11 (Th)	Agent – Handling Uncertain World	
18	Mar 16 (Tu)	Agent – Learning	
19	Mar 18 (Th)	Text: Data Prep (NLP)	Quiz 3
20	Mar 23 (Tu)	Text: Analysis - Supervised (NLP)_	
21	Mar 25 (Th)	Review, Paper presentations, Discussion	
22	Mar 30 (Tu)	Text: Advanced – Summarization, Sentiment	
23	Apr 1 (Th)	Text: Visualization, Explanation	
24	Apr 6 (Tu)	Multimodal Agents: Structured+Text: Examples	
25	Apr 8 (Th)	Case Study 1: Water	Quiz 4
26	Apr 13 (Tu)	Case Study 2: Finance	

Focus on Integrated Agent Behavior (Lectures 17, 18)

Reduce focus on Case-studies (1 per domain)

About Next Lecture – Lecture 18

Lecture 18: Agents That Learn Over Time

- Reinforcement Learning
- Bayesian Optimization