Homework#2

September 27, 2021

```
[1]: import numpy as np
import pandas as pd

import bokeh.plotting as bp
from bokeh.models import tools as bmt, ColumnDataSource

from bokeh.io import output_notebook, export_png
output_notebook()
```

```
[2]: def initialize_fig(title: str = 'your_title',
                        x_axis_label: str = 'x_axis_label',
                        x_axis_type: str = 'auto',
                        y_axis_label: str = 'y_axis_label',
                        y_axis_type: str = 'auto',
                        tools: str = 'pan,wheel_zoom,box_zoom,reset',
                        tooltips: list = [],
                        formatters: dict = {},
                        plot_height: int = 300,
                        fig_sizing_mode: str = 'scale_width',
                        ) -> bp.figure:
         # bokeh style
         TOOLS = tools
         hover_tool = bmt.HoverTool(tooltips=tooltips, formatters=formatters)
         fig = bp.figure(title=title,
                         x_axis_label=x_axis_label,
                         x_axis_type=x_axis_type,
                         y_axis_label=y_axis_label,
                         y_axis_type=y_axis_type,
                         plot_height=plot_height,
                         tools=TOOLS,
         fig.add_tools(hover_tool)
         fig.sizing_mode = fig_sizing_mode
         return fig
```

0.0.1 Question 1

Compare the Brooks and Corey, Campbell and van Genuchten $k(\)$ – analytic relationship (i.e. moisture conductivity) for a silty clay loam. Plot the data and briefly discuss the differences. Use $_r=0.057$. For the van Genuchten parameters: Percent by Weight Finer Than Indicated Diameter (mm)

Soil	50	19	9.5	4.76	2.00	0.420	0.074	0.020	0.005	0.002
1.1	100	100	100	100	100	100	97	79	45	16
1.2	100	100	98	94	70	19	15	8	3	2
1.3	93	91	88	85	69	44	40	27	13	6
1.4	100	100	100	100	100	97	92	75	47	31

```
[3]: # soil df setup
              soil_mm_int = [50., 19., 9.5, 4.76, 2., 0.420, 0.074, 0.020, 0.005, 0.002]
              \# soil\_mm = ['50.', '19.', '9.5', '4.76', '2.', '0.420', '0.074', '0.020', '0.074', '0.020', '0.074', '0.020', '0.074', '0.020', '0.074', '0.020', '0.074', '0.020', '0.074', '0.020', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074', '0.074'
               →005', '0.002']
              soil_1 = [ 100., 100., 100., 100., 100., 100., 97., 79.,
                                                                                                                                                                                                                                45.
                                  16.]
               ⇔ ,
              soil_2 = [ 100., 100., 98., 94., 70., 19., 15., 8.,
                                                                                                                                                                                                                                          3.
               ⇔ ,
                                    2.]
              soil_3 = [ 93., 91., 88., 85., 69.,
                                                                                                                                                           44., 40., 27.,
                                                                                                                                                                                                                                          13.
               \hookrightarrow ,
                                    6.]
              soil_4 = [ 100., 100., 100., 100., 100., 97., 92., 75.,
                                                                                                                                                                                                                                           47.
               ↔, 31.]
              df_1 = pd.DataFrame(data=soil_1, index=soil_mm_int, columns=['sample_percent'])
              df_1.index.name = 'soil_mm'
              df_2 = pd.DataFrame(data=soil_2, index=soil_mm_int, columns=['sample_percent'])
              df_2.index.name = 'soil_mm'
              df 3 = pd.DataFrame(data=soil 3, index=soil mm int, columns=['sample percent'])
              df_3.index.name = 'soil_mm'
              df_4 = pd.DataFrame(data=soil_4, index=soil_mm_int, columns=['sample_percent'])
              df 4.index.name = 'soil_mm'
              # df4.index = df4.index.map(str)
```

```
# soil 2
    soil_2_2mm = df_2[df_2.index == 2.].sample_percent.values[0]
    df_2['lt_2mm_percent'] = df_2[df_2.index \le 2.].apply(lambda x: round(x / L))
     \rightarrowsoil_2_2mm * 100, 2))
    # soil 3
    soil_3_2mm = df_3[df_3.index == 2.].sample_percent.values[0]
    \rightarrowsoil_3_2mm * 100, 2))
    # soil 4
    soil_4_2mm = df_4[df_4.index == 2.].sample_percent.values[0]
    \rightarrowsoil_4_2mm * 100, 2))
[5]: fig = initialize fig(title = 'Diameter (mm) Vs Percentage finer (%)',
                        x_axis_label = 'Diameter (mm) [Log-scale]',
                        y_axis_label = 'Percentage finer (%)',
                        x_axis_type = 'log',
                        tooltips = [
                            ('dimeter(mm)', '$x'),
                            ('% finer' , '$y'),
                       plot_height = 300,
    fig.title.text_font_size = '15pt'
    fig.xaxis.axis_label_text_font_size = '15pt'
    fig.yaxis.axis_label_text_font_size = '15pt'
    fig.circle(df_1.index, df_1.sample_percent, fill_color='red', size=10, ___
     →legend_label='soil 1 (sample percent)')
    fig.line(df_1.index, df_1.sample_percent, line_width=3, line_color='red',u
     →legend_label=f'soil 1 (sample percent)')
    fig.circle(df_1.index, df_1.lt_2mm_percent, fill_color='maroon', size=10, ___
     →legend_label='soil 1 (finer percent)')
    fig.line(df_1.index, df_1.lt_2mm_percent, line_width=3, line_color='maroon', u
     →legend_label=f'soil 1 (finer percent)')
    fig.circle(df_2.index, df_2.sample_percent, fill_color='green', size=10, __
     →legend_label='soil 2 (sample percent)')
    fig.line(df_2.index, df_2.sample_percent, line_width=3, line_color='green', u
```

fig.circle(df_2.index, df_2.lt_2mm_percent, fill_color='lime', size=10,u

→legend_label=f'soil 2 (sample percent)')

→legend_label='soil 2 (finer percent)')

```
fig.line(df_2.index, df_2.lt_2mm_percent, line_width=3, line_color='lime',_
     →legend_label=f'soil 2 (finer percent)')
     fig.circle(df_3.index, df_3.sample_percent, fill_color='blue', size=10,__
      →legend_label='soil 3 (sample percent)')
     fig.line(df_3.index, df_3.sample_percent, line_width=3, line_color='blue', ___
     →legend_label=f'soil 3 (sample percent)')
     fig.circle(df_3.index, df_3.lt_2mm_percent, fill_color='navy', size=10, __
     →legend_label='soil 3 (finer percent)')
     fig.line(df_3.index, df_3.lt_2mm_percent, line_width=3, line_color='navy', __
     →legend_label=f'soil 3 (finer percent)')
     fig.circle(df_4.index, df_4.sample_percent, fill_color='purple', size=10,
     →legend_label='soil 4 (sample percent)')
     fig.line(df_4.index, df_4.sample_percent, line_width=3, line_color='purple', __
     →legend_label=f'soil 4 (sample percent)')
     fig.circle(df_4.index, df_4.lt_2mm_percent, fill_color='fuchsia', size=10,__
     →legend_label='soil 4 (finer percent)')
     fig.line(df_4.index, df_4.lt_2mm_percent, line_width=3, line_color='fuchsia',__
     →legend_label=f'soil 4 (finer percent)')
     fig.legend.location = 'top left'
     fig.legend.click_policy='hide'
     bp.show(fig)
[6]: # Texture calculation
     # soil 1
     x1 = df_1.iloc[5:].index.values.tolist()
     y1 = df_1.iloc[5:].sample_percent.values.tolist()
     coefficients1, _ = np.polynomial.polynomial.polyfit(x1, y1, 1, full=True)
     coefficients1 = coefficients1[::-1]
     m1, c1 = coefficients1[0], coefficients1[1]
     p1 05 = m1 * 0.05 + c1
     # soil 2
     x2 = df 2.iloc[1:].index.values.tolist()
     y2 = df_2.iloc[1:].sample_percent.values.tolist()
     coefficients2, _ = np.polynomial.polynomial.polyfit(x2, y2, 1, full=True)
     coefficients2 = coefficients2[::-1]
     m2, c2 = coefficients2[0], coefficients2[1]
     p2_05 = m2 * 0.05 + c2
     # soil 3
```

x3 = df_3.index.values.tolist()

```
y3 = df_3.sample_percent.values.tolist()
     coefficients3, _ = np.polynomial.polynomial.polyfit(x3, y3, 1, full=True)
     coefficients3 = coefficients3[::-1]
     m3, c3 = coefficients3[0], coefficients3[1]
     p3_05 = m3 * 0.05 + c3
     # soil 3
     x4 = df_4.iloc[4:].index.values.tolist()
     y4 = df_4.iloc[4:].sample_percent.values.tolist()
     coefficients4, _ = np.polynomial.polynomial.polyfit(x4, y4, 1, full=True)
     coefficients4 = coefficients4[::-1]
     m4, c4 = coefficients4[0], coefficients4[1]
     p4_05 = m4 * 0.05 + c4
     print(f'p1_05: {p1_05}')
     print(f'p2_05: {p2_05}')
     print(f'p3_05: {p3_05}')
     print(f'p4_05: {p4_05}')
     # since these values are significantly off just by looking at the table.
     # Thus to avoid significant difference, we will just guess the value
     s1 \text{ sample } 05 = 89.
     s2\_sample\_05 = 12.
     s3 \text{ sample } 05 = 34.
     s4\_sample\_05 = 85.
     s1_lt_05
                 = 89.
                = 16.43
     s2_lt_05
     s3_lt_05
                = 49.85
     s4_lt_05
                 = 85.
     df_1.loc[0.05] = [s1_sample_05, s1_lt_05]
     df_2.loc[0.05] = [s2\_sample_05, s2\_lt_05]
     df_3.loc[0.05] = [s3_sample_05, s3_lt_05]
     df_4.loc[0.05] = [s4_sample_05, s4_lt_05]
    p1_05: 60.66839664793997
    p2_05: 24.384433872847595
    p3_05: 44.15098690890004
    p4_05: 66.12465271160718
[7]: df 1 = df 1.sort index(ascending=False)
     df_2 = df_2.sort_index(ascending=False)
     df 3 = df 3.sort index(ascending=False)
     df_4 = df_4.sort_index(ascending=False)
[8]: df_2
```

```
[8]:
             sample_percent lt_2mm_percent
     soil_mm
     50.000
                      100.0
                                       NaN
     19.000
                      100.0
                                       NaN
     9.500
                       98.0
                                       NaN
     4.760
                       94.0
                                       NaN
     2.000
                       70.0
                                    100.00
                                     27.14
     0.420
                       19.0
     0.074
                       15.0
                                     21.43
     0.050
                       12.0
                                     16.43
     0.020
                        8.0
                                     11.43
     0.005
                        3.0
                                      4.29
     0.002
                        2.0
                                      2.86
[9]: soil_texture =
                          ['gravel(d>2mm)', 'sand(0.05<=d<2mm)', 'silt(0.002<=d<0.
     \hookrightarrow05mm)', 'clay(d<0.002mm)']
     # total sample
     soil 1 sample =
                          [ 0.,
                                              11.,
                                                                    73.,
      soil_2_sample =
                                               58.,
                               30.,
                                                                    10.,
      \hookrightarrow
                 2.]
     soil_3_sample =
                               31.,
                                               35.,
                                                                    28.,
                  6.]
     soil_4_sample =
                                               15.,
                               0.,
                                                                    54.,
                 31. ]
     # lt 2 mm sample
     soil_1_lt_2 =
                          [ np.nan,
                                              11.,
                                                                   73.,
                 16. T
                          [ np.nan,
                                    83.56,
     soil_2_lt_2
                                                                  13.58,
                 2.86
     soil 3 lt 2 =
                          [ np.nan,
                                            50.15,
                                                                  41.15,
      → 8.70 ]
                                              15.,
     soil_4_lt_2 =
                          [ np.nan,
                                                                    54.,
                  31. ]
[10]: # Looking at the figure 7.5 to classify the sand
     print(f'The soil 1.1 is silt Loam.')
     print(f'The soil 1.2 is Loamy Sand.')
     print(f'The soil 1.3 is sandy Loam.')
     print(f'The soil 1.4 is Silty Clay Loam.')
     The soil 1.1 is silt Loam.
     The soil 1.2 is Loamy Sand.
     The soil 1.3 is sandy Loam.
```

The soil 1.4 is Silty Clay Loam.

```
[]:
[]:
[]:
```

Question 2 0.0.2

Compare the Brooks and Corey, Campbell and van Genuchten k() - analytic relationship (i.e. moisture conductivity) for a silty clay loam. Plot the data and briefly discuss the differences. Use $_{\rm r}=0.057$. For the van Genuchten parameters: n=

```
1.24 \text{ (vg)*} = *
 []:
[11]: n = 1.24 # parameter that depends on pore-size distribution
      porosity = 0.477 # porosity
      r = 0.057 \# residual water content
      Kh = 1.7e-4 # cm/s saturated hydraulic conductivity
      b = 7.75 # parameter that depends upon pore size distribution
[12]: # index is the
      df = pd.DataFrame(index = np.arange(0., 1.05, .025), columns = ['BC', 'C', L
[13]: df['*'] = (df.index - r) / (porosity - r)
```

```
[14]: df
```

```
[14]:
                     BC
                              С
                                    VG
         0.000
                   {\tt NaN}
                           {\tt NaN}
                                  NaN -0.135714
         0.025
                   {\tt NaN}
                           {\tt NaN}
                                   NaN -0.076190
         0.050
                   {\tt NaN}
                           {\tt NaN}
                                   NaN -0.016667
         0.075
                           {\tt NaN}
                                   NaN 0.042857
                   {\tt NaN}
         0.100
                           NaN
                   {\tt NaN}
                                   {\tt NaN}
                                         0.102381
         0.125
                   {\tt NaN}
                           NaN
                                   {\tt NaN}
                                         0.161905
         0.150
                   {\tt NaN}
                           {\tt NaN}
                                   {\tt NaN}
                                         0.221429
         0.175
                           {\tt NaN}
                                   {\tt NaN}
                                         0.280952
                   {\tt NaN}
         0.200
                   {\tt NaN}
                           {\tt NaN}
                                   NaN 0.340476
         0.225
                   {\tt NaN}
                           {\tt NaN}
                                   {\tt NaN}
                                         0.400000
         0.250
                   {\tt NaN}
                           NaN
                                   {\tt NaN}
                                         0.459524
         0.275
                           {\tt NaN}
                                          0.519048
                   {\tt NaN}
                                   {\tt NaN}
         0.300
                           {\tt NaN}
                                         0.578571
                   {\tt NaN}
                                   {\tt NaN}
         0.325
                   \mathtt{NaN}
                           NaN
                                   {\tt NaN}
                                          0.638095
         0.350
                   {\tt NaN}
                           NaN
                                   {\tt NaN}
                                          0.697619
                           {\tt NaN}
```

0.375

0.400

 \mathtt{NaN}

 ${\tt NaN}$

NaN

 ${\tt NaN}$

NaN

0.757143

```
0.450
              NaN
                    NaN
                          NaN
                               0.935714
      0.475
              NaN
                    NaN
                          NaN
                                0.995238
      0.500
              NaN
                    NaN
                          NaN
                                1.054762
      0.525
              NaN
                    NaN
                          NaN
                                1.114286
      0.550
                    NaN
                          NaN
              NaN
                                1.173810
      0.575
              NaN
                    NaN
                          NaN
                                1.233333
      0.600
              NaN
                    NaN
                          NaN
                                1.292857
      0.625
                                1.352381
              {\tt NaN}
                    NaN
                          NaN
      0.650
              NaN
                    NaN
                          NaN
                                1.411905
      0.675
              NaN
                    NaN
                          NaN
                                1.471429
      0.700
              NaN
                    NaN
                                1.530952
                          {\tt NaN}
      0.725
              NaN
                    NaN
                          NaN
                                1.590476
      0.750
              NaN
                    NaN
                          NaN
                                1.650000
      0.775
              NaN
                    NaN
                          NaN
                                1.709524
      0.800
              NaN
                    NaN
                          NaN
                                1.769048
      0.825
              NaN
                    NaN
                          NaN
                                1.828571
      0.850
              NaN
                    NaN
                          NaN
                                1.888095
      0.875
              NaN
                    NaN
                          NaN
                                1.947619
      0.900
              NaN
                    NaN
                          NaN
                                2.007143
      0.925
              NaN
                    NaN
                          {\tt NaN}
                               2.066667
      0.950
              NaN
                    NaN
                          {\tt NaN}
                               2.126190
      0.975
              NaN
                    NaN
                          {\tt NaN}
                               2.185714
      1.000
              NaN
                    NaN
                          \tt NaN
                                2.245238
      1.025
              NaN
                    NaN
                          {\tt NaN}
                               2.304762
[15]:
      df['BC'] = (df['*']**(2. * b + 3)) * Kh
[16]:
      df
[16]:
                          BC
                                 C
                                     VG
      0.000
                         NaN
                              NaN
                                    NaN -0.135714
      0.025
                         NaN
                              NaN
                                    NaN -0.076190
      0.050
                         NaN
                              NaN
                                    NaN -0.016667
      0.075
              8.372948e-30
                              NaN
                                    NaN
                                          0.042857
      0.100
              8.308134e-23
                              NaN
                                    NaN
                                          0.102381
      0.125
              3.997164e-19
                                          0.161905
                              NaN
                                    NaN
      0.150
               1.310001e-16
                              NaN
                                    NaN
                                          0.221429
      0.175
               1.071900e-14
                              NaN
                                    NaN
                                          0.280952
      0.200
               3.750194e-13
                              NaN
                                    NaN
                                          0.340476
      0.225
              7.388542e-12
                              NaN
                                    NaN
                                          0.400000
      0.250
              9.619384e-11
                              NaN
                                    NaN
                                          0.459524
      0.275
                                    NaN
              9.157611e-10
                              NaN
                                          0.519048
      0.300
              6.824221e-09
                              NaN
                                    {\tt NaN}
                                          0.578571
      0.325
                                    NaN
              4.176676e-08
                              NaN
                                          0.638095
      0.350
                                    NaN
              2.174643e-07
                              NaN
                                          0.697619
      0.375
              9.890967e-07
                              NaN
                                    NaN
                                          0.757143
```

0.876190

0.425

NaN

NaN

NaN

```
0.425
              1.474082e-05
                              NaN
                                   NaN
                                         0.876190
      0.450
              4.972780e-05
                              NaN
                                   NaN
                                         0.935714
      0.475
              1.556318e-04
                              NaN
                                   NaN
                                         0.995238
      0.500
              4.558332e-04
                              NaN
                                   NaN
                                         1.054762
      0.525
              1.258593e-03
                                   NaN
                              NaN
                                         1.114286
      0.550
              3.296135e-03
                              NaN
                                   NaN
                                         1.173810
      0.575
              8.230716e-03
                              NaN
                                   NaN
                                         1.233333
      0.600
              1.968493e-02
                                         1.292857
                              NaN
                                   NaN
      0.625
              4.526695e-02
                              NaN
                                   NaN
                                         1.352381
      0.650
              1.004266e-01
                              NaN
                                   NaN
                                         1.411905
              2.155876e-01
      0.675
                              NaN
                                   NaN
                                         1.471429
      0.700
              4.489935e-01
                              NaN
                                   NaN
                                         1.530952
      0.725
              9.092883e-01
                              NaN
                                   NaN
                                         1.590476
      0.750
              1.794328e+00
                              NaN
                                   NaN
                                         1.650000
      0.775
              3.456522e+00
                              NaN
                                   NaN
                                         1.709524
      0.800
              6.510736e+00
                              NaN
                                   NaN
                                         1.769048
      0.825
              1.200935e+01
                              NaN
                                   NaN
                                         1.828571
      0.850
              2.172156e+01
                              NaN
                                   NaN
                                         1.888095
      0.875
              3.857209e+01
                              NaN
                                   NaN
                                         1.947619
      0.900
              6.732048e+01
                              NaN
                                   NaN
                                         2.007143
      0.925
              1.155984e+02
                              NaN
                                   {\tt NaN}
                                         2.066667
      0.950
              1.954740e+02
                              NaN
                                   {\tt NaN}
                                         2.126190
      0.975
              3.257817e+02
                              NaN
                                   {\tt NaN}
                                         2.185714
      1.000
              5.355541e+02
                              NaN
                                   {\tt NaN}
                                         2.245238
      1.025
              8.690228e+02
                              NaN
                                   NaN
                                         2.304762
[17]: df['C'] = (df.index / porosity)**(2. * b + 3) * Kh
[18]:
      df
                                          С
[18]:
                         BC
                                               VG
                                             NaN -0.135714
      0.000
                        NaN
                              0.000000e+00
      0.025
                        NaN
                              3.465366e-28
                                             NaN -0.076190
      0.050
                        NaN
                              1.284707e-22
                                             NaN -0.016667
      0.075
              8.372948e-30
                              2.325371e-19
                                             NaN
                                                   0.042857
              8.308134e-23
                              4.762762e-17
                                                   0.102381
      0.100
                                             NaN
      0.125
              3.997164e-19
                              2.955930e-15
                                             NaN
                                                   0.161905
      0.150
              1.310001e-16
                              8.620793e-14
                                             NaN
                                                   0.221429
              1.071900e-14
                              1.493010e-12
      0.175
                                             NaN
                                                   0.280952
      0.200
              3.750194e-13
                              1.765687e-11
                                             NaN
                                                   0.340476
      0.225
              7.388542e-12
                              1.560398e-10
                                             {\tt NaN}
                                                   0.400000
      0.250
              9.619384e-11
                              1.095845e-09
                                             {\tt NaN}
                                                   0.459524
      0.275
              9.157611e-10
                              6.390190e-09
                                             {\tt NaN}
                                                   0.519048
      0.300
              6.824221e-09
                              3.195966e-08
                                             {\tt NaN}
                                                   0.578571
      0.325
              4.176676e-08
                              1.405080e-07
                                             {\tt NaN}
                                                   0.638095
      0.350
              2.174643e-07
                              5.535001e-07
                                             {\tt NaN}
                                                   0.697619
```

0.400

4.011237e-06

NaN

NaN

```
0.400 4.011237e-06
                           6.545891e-06
                                          {\tt NaN}
                                               0.816667
      0.425
             1.474082e-05
                           2.009343e-05
                                          NaN
                                               0.876190
      0.450
            4.972780e-05
                            5.784828e-05
                                          {\tt NaN}
                                               0.935714
      0.475
            1.556318e-04
                           1.572862e-04
                                          NaN
                                               0.995238
      0.500 4.558332e-04
                           4.062599e-04
                                          NaN
                                               1.054762
      0.525
            1.258593e-03
                           1.001858e-03
                                          {\tt NaN}
                                               1.114286
      0.550 3.296135e-03
                           2.369020e-03
                                          {\tt NaN}
                                               1.173810
      0.575 8.230716e-03
                           5.391555e-03
                                               1.233333
                                          {\tt NaN}
      0.600
            1.968493e-02
                           1.184833e-02
                                          NaN
                                               1.292857
      0.625
             4.526695e-02
                           2.521385e-02
                                          {\tt NaN}
                                               1.352381
      0.650 1.004266e-01 5.209020e-02
                                          {\tt NaN}
                                              1.411905
      0.675
            2.155876e-01 1.047077e-01
                                          {\tt NaN}
                                               1.471429
      0.700 4.489935e-01
                           2.051978e-01
                                          {\tt NaN}
                                              1.530952
      0.725 9.092883e-01
                           3.927462e-01
                                              1.590476
                                          {\tt NaN}
      0.750
            1.794328e+00
                           7.353469e-01
                                          {\tt NaN}
                                              1.650000
      0.775
             3.456522e+00
                           1.348778e+00
                                          {\tt NaN}
                                               1.709524
      0.800
             6.510736e+00
                           2.426742e+00
                                          NaN
                                               1.769048
      0.825
            1.200935e+01 4.288022e+00
                                          {\tt NaN}
                                              1.828571
      0.850
                           7.449190e+00
             2.172156e+01
                                          NaN
                                               1.888095
      0.875
            3.857209e+01
                           1.273526e+01
                                          {\tt NaN}
                                              1.947619
      0.900
             6.732048e+01 2.144595e+01
                                          NaN 2.007143
      0.925
            1.155984e+02 3.560254e+01 NaN
                                               2.066667
      0.950
            1.954740e+02 5.831034e+01 NaN
                                               2.126190
      0.975 3.257817e+02
                           9.428537e+01
                                          NaN
                                               2.185714
      1.000 5.355541e+02 1.506118e+02 NaN
                                               2.245238
      1.025 8.690228e+02 2.378211e+02 NaN
                                               2.304762
[19]: # Since vg* = *
      df[' vg*'] = df[' *']
[20]: df['VG_1'] = df['vg*']**(1/2)
      df['VG_2'] = 1 - (df['vg*'])**(n/(n-1))
      df['VG_3'] = (df['VG_2'])**((n - 1)/n)
      df['VG_4'] = (1 - df['VG_3'])**2
      \# \ df['VG'] = df['\ vg*']**0.5 * (1 - (1 - df['\ vg*']**(n/(n-1)))**((n - 1)/n))**2_{\square}
      →* Kh
      df['VG'] = df['VG_1'] * df['VG_4'] * Kh
      df = df.drop(columns=['VG_1', 'VG_2', 'VG_3', 'VG_4'], axis=1)
      # df = df.dropna()
[21]: df
[21]:
                       BC
                                       C
                                                    VG
                                                                       vg*
      0.000
                      NaN
                            0.000000e+00
                                                   NaN -0.135714 -0.135714
                                                   NaN -0.076190 -0.076190
      0.025
                      NaN
                            3.465366e-28
```

0.375 9.890967e-07

1.983524e-06

 ${\tt NaN}$

```
0.075
             8.372948e-30
                            2.325371e-19
                                           9.644496e-21
                                                          0.042857
                                                                     0.042857
      0.100
             8.308134e-23
                            4.762762e-17
                                           1.206160e-16
                                                          0.102381
                                                                     0.102381
      0.125
             3.997164e-19
                            2.955930e-15
                                           1.728633e-14
                                                          0.161905
                                                                     0.161905
      0.150
             1.310001e-16
                            8.620793e-14
                                           5.138910e-13
                                                          0.221429
                                                                     0.221429
      0.175
             1.071900e-14
                            1.493010e-12
                                           6.782330e-12
                                                          0.280952
                                                                     0.280952
      0.200
             3.750194e-13
                            1.765687e-11
                                           5.448901e-11
                                                          0.340476
                                                                     0.340476
      0.225
             7.388542e-12
                            1.560398e-10
                                           3.134012e-10
                                                          0.400000
                                                                     0.400000
      0.250
             9.619384e-11
                                           1.419215e-09
                            1.095845e-09
                                                          0.459524
                                                                     0.459524
      0.275
             9.157611e-10
                            6.390190e-09
                                           5.379781e-09
                                                          0.519048
                                                                     0.519048
      0.300
             6.824221e-09
                            3.195966e-08
                                           1.781551e-08
                                                          0.578571
                                                                     0.578571
      0.325
             4.176676e-08
                            1.405080e-07
                                           5.322302e-08
                                                          0.638095
                                                                     0.638095
      0.350
             2.174643e-07
                            5.535001e-07
                                           1.472770e-07
                                                          0.697619
                                                                     0.697619
      0.375
             9.890967e-07
                            1.983524e-06
                                           3.868570e-07
                                                          0.757143
                                                                     0.757143
      0.400
             4.011237e-06
                            6.545891e-06
                                           9.912848e-07
                                                          0.816667
                                                                     0.816667
      0.425
             1.474082e-05
                            2.009343e-05
                                           2.578740e-06
                                                          0.876190
                                                                     0.876190
      0.450
             4.972780e-05
                            5.784828e-05
                                           7.442989e-06
                                                          0.935714
                                                                     0.935714
      0.475
             1.556318e-04
                            1.572862e-04
                                           4.458991e-05
                                                          0.995238
                                                                     0.995238
      0.500
             4.558332e-04
                            4.062599e-04
                                                     NaN
                                                          1.054762
                                                                     1.054762
      0.525
             1.258593e-03
                            1.001858e-03
                                                     NaN
                                                          1.114286
                                                                     1.114286
      0.550
             3.296135e-03
                            2.369020e-03
                                                     NaN
                                                          1.173810
                                                                     1.173810
      0.575
             8.230716e-03
                            5.391555e-03
                                                     NaN
                                                          1.233333
                                                                     1.233333
      0.600
             1.968493e-02
                            1.184833e-02
                                                     NaN
                                                          1.292857
                                                                     1.292857
      0.625
             4.526695e-02
                            2.521385e-02
                                                     NaN
                                                          1.352381
                                                                     1.352381
      0.650
             1.004266e-01
                            5.209020e-02
                                                     NaN
                                                          1.411905
                                                                     1.411905
      0.675
             2.155876e-01
                            1.047077e-01
                                                     NaN
                                                          1.471429
                                                                     1.471429
                            2.051978e-01
      0.700
             4.489935e-01
                                                     NaN
                                                          1.530952
                                                                     1.530952
      0.725
             9.092883e-01
                            3.927462e-01
                                                     NaN
                                                          1.590476
                                                                     1.590476
      0.750
             1.794328e+00
                            7.353469e-01
                                                     NaN
                                                          1.650000
                                                                     1.650000
      0.775
             3.456522e+00
                            1.348778e+00
                                                          1.709524
                                                                     1.709524
                                                     NaN
      0.800
                                                          1.769048
             6.510736e+00
                            2.426742e+00
                                                     NaN
                                                                     1.769048
      0.825
             1.200935e+01
                            4.288022e+00
                                                     NaN
                                                          1.828571
                                                                     1.828571
      0.850
             2.172156e+01
                            7.449190e+00
                                                     NaN
                                                          1.888095
                                                                     1.888095
      0.875
             3.857209e+01
                            1.273526e+01
                                                     NaN
                                                          1.947619
                                                                     1.947619
      0.900
             6.732048e+01
                            2.144595e+01
                                                     NaN
                                                          2.007143
                                                                     2.007143
      0.925
             1.155984e+02
                            3.560254e+01
                                                     NaN
                                                          2.066667
                                                                     2.066667
      0.950
             1.954740e+02
                            5.831034e+01
                                                     NaN
                                                          2.126190
                                                                     2.126190
      0.975
             3.257817e+02
                            9.428537e+01
                                                     NaN
                                                          2.185714
                                                                     2.185714
      1.000
             5.355541e+02
                            1.506118e+02
                                                     NaN
                                                          2.245238
                                                                     2.245238
      1.025
             8.690228e+02
                            2.378211e+02
                                                     NaN
                                                          2.304762
                                                                     2.304762
[22]:
      df = df.dropna()
[23]:
      df
[23]:
                        BC
                                        С
                                                      VG
                                                                         vg*
      0.075
             8.372948e-30
                            2.325371e-19
                                           9.644496e-21
                                                         0.042857
                                                                     0.042857
```

NaN -0.016667 -0.016667

1.284707e-22

NaN

```
0.100 \quad 8.308134e-23 \quad 4.762762e-17 \quad 1.206160e-16 \quad 0.102381 \quad 0.102381
      0.125 3.997164e-19 2.955930e-15 1.728633e-14 0.161905 0.161905
      0.150 1.310001e-16 8.620793e-14 5.138910e-13 0.221429 0.221429
      0.175 \quad 1.071900e - 14 \quad 1.493010e - 12 \quad 6.782330e - 12 \quad 0.280952 \quad 0.280952
      0.200 \quad 3.750194 e - 13 \quad 1.765687 e - 11 \quad 5.448901 e - 11 \quad 0.340476 \quad 0.340476
      0.225 \quad 7.388542e-12 \quad 1.560398e-10 \quad 3.134012e-10 \quad 0.400000 \quad 0.400000
      0.250 \quad 9.619384e-11 \quad 1.095845e-09 \quad 1.419215e-09 \quad 0.459524 \quad 0.459524
      0.275 9.157611e-10 6.390190e-09 5.379781e-09 0.519048 0.519048
      0.300 6.824221e-09 3.195966e-08 1.781551e-08 0.578571 0.578571
      0.325 4.176676e-08 1.405080e-07
                                             5.322302e-08 0.638095 0.638095
      0.350 \quad 2.174643e-07 \quad 5.535001e-07 \quad 1.472770e-07 \quad 0.697619 \quad 0.697619
      0.375 \quad 9.890967e-07 \quad 1.983524e-06 \quad 3.868570e-07 \quad 0.757143 \quad 0.757143
      0.400 4.011237e-06 6.545891e-06 9.912848e-07 0.816667 0.816667
      0.425 \quad 1.474082e - 05 \quad 2.009343e - 05 \quad 2.578740e - 06 \quad 0.876190 \quad 0.876190
      0.450 4.972780e-05 5.784828e-05 7.442989e-06 0.935714 0.935714
      0.475 \quad 1.556318 \\ e^{-04} \quad 1.572862 \\ e^{-04} \quad 4.458991 \\ e^{-05} \quad 0.995238 \quad 0.995238
[24]: | fig = initialize fig(title = 'Volumertic water content () Vs Hydraulic,
       x_axis_label = 'Volumertic water content ()',
                              y_axis_label = 'Hydraulic Conductivity Log scale Kh()',
                              y_axis_type = 'log',
                              tooltips = [
                                  ('', '$x'),
                                                '$y'),
                                  ('Kh()',
                              ],
                             plot_height = 300,
      fig.title.text_font_size = '15pt'
      fig.xaxis.axis_label_text_font_size = '15pt'
      fig.yaxis.axis_label_text_font_size = '15pt'
      fig.circle(df.index, df.BC, fill_color='red', size=10, legend_label=f'Brooks &L

Corev¹)
      fig.line(df.index, df.BC, line width=3, line color='red', legend label=f'Brooks,
       fig.circle(df.index, df.C, fill_color='blue', size=10,_
       →legend_label=f'Campbell')
      fig.line(df.index, df.C, line_width=3, line_color='blue', __
       →legend label=f'Campbell')
      fig.circle(df.index, df.VG, fill_color='green', size=10, legend_label=f'Vanu
```

Genuchten¹)

```
fig.line(df.index, df.VG, line_width=3, line_color='green', legend_label=f'Van_
Genuchten')

fig.legend.location = 'top_left'
fig.legend.click_policy='hide'
bp.show(fig)
```

The graph above shows the relation between the Volumetric Water Content () plotted in the X-axis and the Hydraulic Conductivity Kh() plotted in the Y-axis. The Kh() is plotted in log scale. This shows the general relationship where for a soil, the unsaturated hydraulic conductivity is low, and it increases nonlinearly to its saturated value as water content increases to saturation. The Kh() is calcualted from 3 methods. Brooks and Corey shows significant difference (several orders of magnitude) with the other two at the lower volumetric water content, but the Kh() gets closer as we move towards the higher saturation.

```
[]:
```

0.0.3 Question 3

```
[25]: # For various soils
       _{\text{sand}} = 0.395 \# porosity
       _{\text{silt\_loam}} = 0.485
      loam = 0.451
       _{clay} = 0.482
       _abs_sand = 12.1 # absolute pressure head
       _abs_silt_loam = 78.6
       _{abs}loam = 47.8
      _{abs\_clay} = 40.5
      Kh_sand = 1.76e-2 # saturated hydraulic conductivity
      Kh_silt_loam = 7.20e-4
      Kh_{loam} = 6.95e-4
      Kh_clay = 1.28e-4
      b_sand = 4.05 # parameter that depends upon pore size distribution
      b_silt_loam = 5.30
      b_{loam} = 5.39
      b_{clay} = 11.40
      # assume
      r = 0.05 \# residual water content
      # HC: Hydraulic conductivity
```

```
# PH: Pressure head
# index is the
df = pd.DataFrame(index = np.arange(0.01, 0.5, .01), columns=[
    '*_Sand', '*_Silt_Loam', '*_Loam', '*_Clay',
    'HC_Sand', 'HC_Silt_Loam', 'HC_Loam', 'HC_Clay',
    'PH_Sand', 'PH_Silt_Loam', 'PH_Loam', 'PH_Clay',
])
df['*_Sand'] = (df.index - r) / (_sand - r)
df['*_Silt_Loam'] = (df.index - r) / (_silt_loam - r)
df['*\_Loam'] = (df.index - r) / (\_loam - r)
df['*_Clay'] = (df.index - r) / (_clay - r)
df['HC_Sand'] = (df['*_Sand']**(2. * b_sand + 3.)) * Kh_sand
df['HC_Silt_Loam'] = (df['*_Silt_Loam']**(2. * b_silt_loam + 3.)) *__
→Kh_silt_loam
df['HC Loam'] = (df['* Loam']**(2. * b loam + 3.)) * Kh loam
df['HC_Clay'] = (df['*_Clay']**(2. * b_clay + 3.)) * Kh_clay
df['PH_Sand'] = np.where(df['*_Sand'] < 1, _abs_sand * (1 /_

    df['*_Sand'])**b_sand, _abs_sand)
df['PH_Silt_Loam'] = np.where(df['*_Silt_Loam'] < 1, _abs_silt_loam * (1 /_

→df['*_Silt_Loam'])**b_silt_loam, _abs_silt_loam)
df['PH_Loam'] = np.where(df['*_Loam'] < 1, _abs_loam * (1 /_</pre>

    df['*_Loam'])**b_loam, _abs_loam)
df['PH_Clay'] = np.where(df['*_Clay'] < 1, _abs_clay * (1 /_

df['*_Clay'])**b_clay, _abs_clay)
```

[26]: df

```
HC_Sand HC_Silt_Loam \
[26]:
            * Sand * Silt Loam
                                 * Loam
                                          *_Clay
     0.01 -0.115942
                       -0.091954 -0.099751 -0.092593
                                                           NaN
                                                                         NaN
     0.02 -0.086957
                       -0.068966 -0.074813 -0.069444
                                                            NaN
                                                                         NaN
     0.03 -0.057971
                       -0.045977 -0.049875 -0.046296
                                                           NaN
                                                                         NaN
     0.04 -0.028986
                       -0.022989 -0.024938 -0.023148
                                                           NaN
                                                                         NaN
     0.05 0.000000
                       0.000000 0.000000 0.000000
                                                   0.000000e+00
                                                                0.000000e+00
     0.06 0.028986
                        0.022989 0.024938 0.023148
                                                   1.498721e-19
                                                                3.748682e-26
     0.07 0.057971
                       0.045977 0.049875 0.046296
                                                   3.289680e-16
                                                                4.654645e-22
     0.08 0.086957
                       0.068966 0.074813 0.069444
                                                   2.963239e-14
                                                               1.155388e-19
     0.09 0.115942
                        0.091954 0.099751 0.092593
                                                   7.220822e-13 5.779556e-18
     0.10 0.144928
                        0.114943 0.124688 0.115741
                                                   8.595829e-12 1.201906e-16
     0.11 0.173913
                        0.137931 0.149626 0.138889
                                                   6.504285e-11
                                                                1.434616e-15
     0.12 0.202899
                        0.160920 0.174564 0.162037
                                                   3.600054e-10
                                                                1.167379e-14
     0.13 0.231884
                        0.183908 0.199501 0.185185
                                                   1.584965e-09
                                                                7.176330e-14
     0.14 0.260870
                        0.206897 0.224439 0.208333
                                                   5.858853e-09
                                                                3.561039e-13
     0.15 0.289855
                        0.229885 0.249377 0.231481 1.886778e-08 1.492377e-12
     0.16 0.318841
```

```
0.17
      0.347826
                     0.275862
                                0.299252
                                           0.277778
                                                      1.427685e-07
                                                                     1.781327e-11
0.18
                     0.298851
                                0.324190
      0.376812
                                           0.300926
                                                      3.471298e-07
                                                                     5.290640e-11
0.19
      0.405797
                     0.321839
                                0.349127
                                           0.324074
                                                      7.902089e-07
                                                                     1.449506e-10
0.20
      0.434783
                     0.344828
                                0.374065
                                           0.347222
                                                                     3.704416e-10
                                                      1.699549e-06
                                0.399002
0.21
      0.463768
                     0.367816
                                           0.370370
                                                      3.478985e-06
                                                                     8.910670e-10
0.22
                                0.423940
                                           0.393519
                                                                     2.032282e-09
      0.492754
                     0.390805
                                                      6.818734e-06
0.23
      0.521739
                     0.413793
                                0.448878
                                           0.416667
                                                      1.286013e-05
                                                                     4.421653e-09
0.24
      0.550725
                     0.436782
                                0.473815
                                           0.439815
                                                      2.343612e-05
                                                                     9.224176e-09
0.25
                     0.459770
                                0.498753
                                           0.462963
      0.579710
                                                      4.141462e-05
                                                                     1.853047e-08
0.26
      0.608696
                     0.482759
                                0.523691
                                           0.486111
                                                      7.117949e-05
                                                                     3.597999e-08
0.27
      0.637681
                     0.505747
                                0.548628
                                           0.509259
                                                      1.192923e-04
                                                                     6.773717e-08
                                0.573566
0.28
      0.666667
                     0.528736
                                           0.532407
                                                      1.953888e-04
                                                                     1.239873e-07
0.29
      0.695652
                     0.551724
                                0.598504
                                           0.555556
                                                      3.133758e-04
                                                                     2.211829e-07
0.30
      0.724638
                     0.574713
                                0.623441
                                           0.578704
                                                      4.930090e-04
                                                                     3.853565e-07
0.31
                                0.648379
                                                                     6.569255e-07
      0.753623
                     0.597701
                                           0.601852
                                                      7.619472e-04
0.32
      0.782609
                     0.620690
                                0.673317
                                           0.625000
                                                      1.158400e-03
                                                                     1.097554e-06
0.33
                                0.698254
      0.811594
                     0.643678
                                           0.648148
                                                      1.734502e-03
                                                                     1.799815e-06
0.34
      0.840580
                     0.666667
                                0.723192
                                           0.671296
                                                      2.560581e-03
                                                                     2.900624e-06
0.35
      0.869565
                     0.689655
                                0.748130
                                           0.694444
                                                      3.730497e-03
                                                                     4.599680e-06
0.36
      0.898551
                     0.712644
                                0.773067
                                           0.717593
                                                      5.368284e-03
                                                                     7.184518e-06
                                                      7.636344e-03
0.37
      0.927536
                     0.735632
                                0.798005
                                           0.740741
                                                                     1.106416e-05
0.38
      0.956522
                     0.758621
                                0.822943
                                           0.763889
                                                      1.074547e-02
                                                                     1.681389e-05
0.39
      0.985507
                     0.781609
                                0.847880
                                           0.787037
                                                      1.496707e-02
                                                                     2.523434e-05
                     0.804598
                                                                     3.742864e-05
0.40
      1.014493
                                0.872818
                                           0.810185
                                                      2.064791e-02
0.41
      1.043478
                     0.827586
                                0.897756
                                           0.833333
                                                      2.822789e-02
                                                                     5.490256e-05
0.42
      1.072464
                     0.850575
                                0.922693
                                           0.856481
                                                      3.826131e-02
                                                                     7.969334e-05
                     0.873563
0.43
      1.101449
                                0.947631
                                           0.879630
                                                      5.144209e-02
                                                                     1.145343e-04
0.44
      1.130435
                     0.896552
                                0.972569
                                           0.902778
                                                      6.863377e-02
                                                                     1.630637e-04
0.45
      1.159420
                     0.919540
                                0.997506
                                           0.925926
                                                      9.090476e-02
                                                                     2.300883e-04
                     0.942529
                                1.022444
                                           0.949074
                                                                     3.219134e-04
0.46
      1.188406
                                                      1.195698e-01
      1.217391
0.47
                     0.965517
                                1.047382
                                           0.972222
                                                      1.562384e-01
                                                                     4.467545e-04
0.48
      1.246377
                     0.988506
                                1.072319
                                           0.995370
                                                      2.028713e-01
                                                                     6.152471e-04
0.49
      1.275362
                     1.011494
                                1.097257
                                           1.018519
                                                      2.618457e-01
                                                                     8.410755e-04
            HC_Loam
                           HC_Clay
                                          PH_Sand
                                                    PH_Silt_Loam
                                                                        PH_Loam
                                                                                  \
0.01
                NaN
                               NaN
                                              NaN
                                                                            NaN
                                                             NaN
0.02
                NaN
                               NaN
                                              NaN
                                                             NaN
                                                                            NaN
0.03
                NaN
                               NaN
                                              NaN
                                                             NaN
                                                                            NaN
0.04
                NaN
                               NaN
                                              NaN
                                                             NaN
                                                                            NaN
0.05
      0.000000e+00
                     0.000000e+00
                                              inf
                                                             inf
                                                                            inf
0.06
      5.631905e-26
                     8.160735e-47
                                    2.046222e+07
                                                    3.796782e+10
                                                                   2.091106e+10
0.07
      7.922253e-22
                     4.767637e-39
                                     1.235325e+06
                                                    9.637328e+08
                                                                   4.986828e+08
0.08
      2.115370e-19
                     1.665167e-34
                                    2.391176e+05
                                                    1.123758e+08
                                                                   5.606509e+07
0.09
      1.114402e-17
                                    7.457783e+04
                     2.785333e-31
                                                    2.446232e+07
                                                                   1.189249e+07
0.10
      2.412470e-16
                     8.813642e-29
                                    3.020815e+04
                                                    7.496777e+06
                                                                   3.572134e+06
0.11
      2.975634e-15
                     9.728179e-27
                                     1.443578e+04
                                                    2.852423e+06
                                                                   1.337029e+06
0.12
      2.489467e-14
                     5.191092e-25
                                    7.732246e+03
                                                    1.260074e+06
                                                                   5.825023e+05
```

```
0.13
      1.567600e-13
                     1.627238e-23
                                   4.502340e+03
                                                  6.209243e+05
                                                                 2.836097e+05
0.14
      7.945425e-13
                     3.397708e-22
                                   2.794284e+03
                                                  3.326062e+05
                                                                 1.503173e+05
0.15
      3.393559e-12
                     5.149076e-21
                                    1.823697e+03
                                                  1.902898e+05
                                                                 8.518754e+04
0.16
      1.261963e-11
                     6.020888e-20
                                   1.239688e+03
                                                  1.148245e+05
                                                                 5.096470e+04
0.17
      4.185746e-11
                     5.683364e-19
                                   8.715025e+02
                                                  7.240274e+04
                                                                 3.188520e+04
0.18
      1.261231e-10
                     4.481958e-18
                                   6.302054e+02
                                                  4.737136e+04
                                                                 2.071197e+04
0.19
      3.501867e-10
                     3.032722e-17
                                   4.668036e+02
                                                  3.198431e+04
                                                                 1.389140e+04
0.20
      9.061350e-10
                     1.798390e-16
                                   3.530068e+02
                                                  2.218870e+04
                                                                 9.577324e+03
0.21
      2.205102e-09
                     9.506592e-16
                                   2.718108e+02
                                                  1.576085e+04
                                                                 6.763467e+03
0.22
      5.084421e-09
                     4.542673e-15
                                   2.126349e+02
                                                  1.142974e+04
                                                                 4.878164e+03
0.23
      1.117662e-08
                     1.984997e-14
                                   1.686938e+02
                                                  8.442505e+03
                                                                 3.584737e+03
0.24
      2.354398e-08
                     8.008888e-14
                                                  6.339011e+03
                                                                 2.678515e+03
                                   1.355191e+02
0.25
      4.773629e-08
                     3.008176e-13
                                   1.100985e+02
                                                  4.830106e+03
                                                                 2.031535e+03
0.26
      9.350552e-08
                     1.059222e-12
                                   9.035757e+01
                                                  3.729523e+03
                                                                 1.561759e+03
0.27
      1.775170e-07
                     3.517503e-12
                                   7.484122e+01
                                                  2.914576e+03
                                                                 1.215396e+03
0.28
      3.275404e-07
                     1.107408e-11
                                   6.251079e+01
                                                  2.302810e+03
                                                                 9.564517e+02
0.29
                                                                 7.603919e+02
      5.887979e-07
                     3.320315e-11
                                   5.261350e+01
                                                  1.837791e+03
0.30
      1.033401e-06
                     9.518785e-11
                                   4.459598e+01
                                                  1.480245e+03
                                                                 6.102101e+02
0.31
      1.774141e-06
                     2.618434e-10
                                   3.804614e+01
                                                  1.202422e+03
                                                                 4.939349e+02
0.32
      2.984338e-06
                     6.932892e-10
                                   3.265348e+01
                                                  9.844364e+02
                                                                 4.030189e+02
0.33
      4.925984e-06
                     1.771767e-09
                                   2.818141e+01
                                                  8.118542e+02
                                                                 3.312793e+02
                     4.381246e-09
0.34
      7.989135e-06
                                   2.444786e+01
                                                  6.740719e+02
                                                                 2.741895e+02
      1.274636e-05
                                                  5.632134e+02
                                                                 2.283981e+02
0.35
                     1.050649e-08
                                   2.131138e+01
0.36
      2.002717e-05
                     2.448280e-08
                                    1.866113e+01
                                                  4.733677e+02
                                                                 1.913976e+02
0.37
      3.101859e-05
                     5.553909e-08
                                    1.640950e+01
                                                  4.000559e+02
                                                                 1.612938e+02
0.38
      4.739990e-05
                     1.228539e-07
                                    1.448676e+01
                                                  3.398535e+02
                                                                 1.366425e+02
      7.152121e-05
                                                  2.901198e+02
0.39
                     2.653905e-07
                                   1.283699e+01
                                                                 1.163335e+02
0.40
      1.066382e-04
                     5.606410e-07
                                    1.210000e+01
                                                  2.488026e+02
                                                                 9.950601e+01
                                                                 8.548808e+01
0.41
      1.572186e-04
                     1.159668e-06
                                    1.210000e+01
                                                  2.142952e+02
0.42
      2.293375e-04
                     2.351439e-06
                                   1.210000e+01
                                                  1.853303e+02
                                                                 7.375115e+01
0.43
      3.311870e-04
                     4.678925e-06
                                    1.210000e+01
                                                  1.609024e+02
                                                                 6.387670e+01
0.44
      4.737243e-04
                     9.145262e-06
                                    1.210000e+01
                                                  1.402081e+02
                                                                 5.553130e+01
0.45
      6.714938e-04
                     1.757426e-05
                                    1.210000e+01
                                                  1.226020e+02
                                                                 4.844765e+01
0.46
      9.436629e-04
                     3.323172e-05
                                   1.210000e+01
                                                  1.075626e+02
                                                                 4.780000e+01
0.47
      1.315317e-03
                     6.188152e-05
                                   1.210000e+01
                                                  9.466607e+01
                                                                 4.780000e+01
0.48
      1.819075e-03
                     1.135574e-04
                                   1.210000e+01
                                                  8.356661e+01
                                                                 4.780000e+01
                                   1.210000e+01
0.49
      2.497084e-03
                     2.054984e-04
                                                  7.860000e+01
                                                                 4.780000e+01
           PH_Clay
0.01
               NaN
0.02
               NaN
0.03
               NaN
0.04
               NaN
0.05
                inf
0.06
      1.786363e+20
0.07
      6.610401e+16
0.08
      6.498116e+14
```

```
0.09 2.446165e+13
     0.10 1.921815e+12
     0.11 2.404615e+11
     0.12 4.148127e+10
     0.13 9.051986e+09
     0.14 2.363770e+09
     0.15 7.111640e+08
     0.16 2.399347e+08
     0.17 8.898228e+07
     0.18 3.572856e+07
     0.19 1.535006e+07
     0.20 6.990840e+06
     0.21 3.349669e+06
     0.22 1.678232e+06
     0.23 8.747082e+05
     0.24 4.722573e+05
     0.25 2.631648e+05
     0.26 1.508932e+05
     0.27 8.878737e+04
     0.28 5.348998e+04
     0.29 3.292771e+04
     0.30 2.067539e+04
     0.31 1.322128e+04
     0.32 8.598503e+03
     0.33 5.680260e+03
     0.34 3.807436e+03
     0.35 2.586946e+03
     0.36 1.780108e+03
     0.37 1.239538e+03
     0.38 8.727921e+02
     0.39 6.210264e+02
     0.40 4.462662e+02
     0.41 3.236840e+02
     0.42 2.368482e+02
     0.43 1.747579e+02
     0.44 1.299670e+02
     0.45 9.738360e+01
     0.46 7.349093e+01
     0.47 5.583774e+01
     0.48 4.270014e+01
     0.49 4.050000e+01
[27]: # / / vs
     fig = initialize_fig(title = '| |- plot for various soil',
                          x_axis_label = 'Volumertic water content ()',
                          y_axis_label = 'Pressure Head | |()',
                          y_axis_type = 'log',
```

```
tooltips = [
                         ('', '$x'),
                         ('||()', '$y'),
                    plot_height = 300,
fig.title.text_font_size = '15pt'
fig.xaxis.axis label text font size = '15pt'
fig.yaxis.axis_label_text_font_size = '15pt'
fig.circle(df.index, df.PH_Sand, fill_color='red', size=10,_
→legend_label=f'Sand')
fig.line(df.index, df.PH_Sand, line_width=3, line_color='red',__
→legend_label=f'Sand')
fig.circle(df.index, df.PH_Silt_Loam, fill_color='blue', size=10,u
→legend_label=f'Silt Loam')
fig.line(df.index, df.PH Silt_Loam, line_width=3, line_color='blue',
→legend_label=f'Silt Loam')
fig.circle(df.index, df.PH_Loam, fill_color='green', size=10,
 →legend label=f'Loam')
fig.line(df.index, df.PH_Loam, line_width=3, line_color='green',_
→legend_label=f'Loam')
fig.circle(df.index, df.PH_Clay, fill_color='fuchsia', size=10,__
→legend_label=f'Clay')
fig.line(df.index, df.PH_Clay, line_width=3, line_color='fuchsia',__
→legend_label=f'Clay')
fig.legend.location = 'top_right'
fig.legend.click_policy='hide'
bp.show(fig)
```

The Pressure Head $| \ | \ ()$ and Volumertic water content () relation is highly non-linear as can be seen from above graph. Note that the Y-axis plots the absolute value. So a high absolute pressure head means more tension in the soil. For soils, as the porosity increases, the pressure head also increases. For example, the clay has very high pressure head compared to the sand. At very high tension, the curve is nearly vertical and reflects the residual water content held in the soil against all forces. As the water content in the soil increases, the tension also decreases and becomes zero (0) when the water content equals the porosity (saturation). Also can be noticed that as the water content increases the difference in the tension for various soil also decreases.

```
y_axis_type = 'log',
                     tooltips = [
                         ('',
                                   '$x'),
                         ('Kh()', '$y'),
                     ],
                     plot_height = 300,
fig.title.text font size = '15pt'
fig.xaxis.axis label text font size = '15pt'
fig.yaxis.axis label text font size = '15pt'
fig.circle(df.index, df.HC_Sand, fill_color='red', size=10,_
→legend label=f'Sand')
fig.line(df.index, df.HC_Sand, line_width=3, line_color='red',__
 →legend label=f'Sand')
fig.circle(df.index, df.HC_Silt_Loam, fill_color='blue', size=10,u
→legend_label=f'Silt Loam')
fig.line(df.index, df.HC_Silt_Loam, line_width=3, line_color='blue',u
→legend_label=f'Silt Loam')
fig.circle(df.index, df.HC Loam, fill color='green', size=10,
→legend_label=f'Loam')
fig.line(df.index, df.HC_Loam, line_width=3, line_color='green',__
→legend_label=f'Loam')
fig.circle(df.index, df.HC_Clay, fill_color='fuchsia', size=10,__
 →legend label=f'Clay')
fig.line(df.index, df.HC_Clay, line_width=3, line_color='fuchsia', __
→legend_label=f'Clay')
fig.legend.location = 'top left'
fig.legend.click_policy='hide'
bp.show(fig)
```

The graph above shows the relation between the Volumetric Water Content () plotted in the X-axis and the Hydraulic Conductivity Kh() plotted in the Y-axis. The Kh() is plotted in log scale. This shows the general relationship where for a soil, the unsaturated hydraulic conductivity is low, and it increases nonlinearly to its saturated value as water content increases to saturation. For soil, as the porosity increases the hydraulic conductivity decreases as it becomes increasingly difficult for water to flow through highly porous soil. Thus the clay which is highly porous soil has much less conductivity than sand, and they differ by several orders of magnitude, but the Kh() gets closer as we move towards the higher saturation.

Question 4 The 50-year return period, one-hour design storm for a location in Alabama is, is 4.9 inches. For a clay-loam soil, apply the Green and Ampt model to estimate the infiltration and runoff resulting from this storm. Assume that the soil has a moisture content of 0.2 at the beginning of the storm. Compare your results with an initial moisture content of $_i=0.10$. Compare your results with an intensity of 2 in/hr. How would your results look if the soil was a sandy loam instead? Plot F(t) vs t and f(t) vs t for each scenario. Discuss your results in a paragraph. Ks = 0.1 cm/hr $_s=0.44$ =-28.4cm

```
[29]: i = 4.9 * 2.54 # cm
      a = 0.20
      ns = 0.44
      delta_ = ns - a
      # for clay-loam soil
      _{clay_{loam}} = 0.44
      _clay_loam = 28.4 # cm # absolute pressure head
      Ks_clay_loam = 0.1 # cm/h # saturated hydraulic conductivity
      b_clay_loam = 8.52 # parameter that depends upon pore size distribution
      F_tp = (delta_ * _clay_loam) / (1. + (i / Ks_clay_loam)) # cm total mass at_
      \rightarrow ponding
      tp = F_tp / i # hour
      dt = 1. - tp # 1-hour design storm
      print(f'time of ponding: {tp} hr')
      print(f'Mass at ponding: {F_tp} cm')
      # rewriting green and ampt model eqn in simpler form
      # _ + + * ln((_( + )+ )/(_ + ))
# m + * ln(c)
      df_ga_clay_loam = pd.DataFrame(index = np.arange(0, 1.5, .01), columns = ['m', _
      \hookrightarrow 'c', 'GA', 'delta', 'f_t', 't'])
      df_ga_clay_loam.index.name = 'F_tp_dt'
      df_ga_clay_loam['m'] = F_tp + (Ks_clay_loam * dt)
      df_ga_clay_loam['c'] = (df_ga_clay_loam.index + (delta_ * _clay_loam)) / (F_tp_
      →+ (delta * clay loam))
      df_ga_clay_loam['GA'] = df_ga_clay_loam.m + delta_ * _clay_loam * np.
       →log(df_ga_clay_loam.c)
      df_ga_clay_loam['delta'] = df_ga_clay_loam.GA - df_ga_clay_loam.index
      df_ga_clay_loam['f_t'] = Ks_clay_loam * (1. + ( (delta_ * _clay_loam) /__
      →df_ga_clay_loam.index ))
      df_ga_clay_loam['f_t'] = np.where(df_ga_clay_loam.index <= F_tp, i,_

df_ga_clay_loam['f_t'])
      # cannot apply this as the infiltration rate is changing over time after time_
       \rightarrow of ponding
```

```
# df_ga_clay_loam['t'] = df_ga_clay_loam.index / df_ga_clay_loam.f_t

# instead use the formula in slide 16

df_ga_clay_loam['t1'] = (df_ga_clay_loam.index - F_tp) / Ks_clay_loam

df_ga_clay_loam['t2'] = _clay_loam * delta_ / Ks_clay_loam

df_ga_clay_loam['t3'] = (df_ga_clay_loam.index + (_clay_loam * delta_)) /__

\( \times (F_tp + (_clay_loam * delta_)) \)

df_ga_clay_loam['t'] = df_ga_clay_loam['t1'] - df_ga_clay_loam['t2'] * np.

\( \times \) log(df_ga_clay_loam['t3']) + tp

df_ga_clay_loam = df_ga_clay_loam.drop(columns=['t1', 't2', 't3'], axis=1)
```

time of ponding: 0.004365103060621213 hr Mass at ponding: 0.054328072692491625 cm

```
[30]: df_ga_clay_loam
```

```
[30]:
                                        GA
                                               delta
                                                            f_t
                                                                        t
                     m
                               С
     F_tp_dt
     0.00
              0.153892  0.992092  0.099779  0.099779  12.446000  0.002211
     0.01
              0.153892 0.993548 0.109772 0.099772
                                                      12.446000 0.002285
     0.02
              0.153892  0.995003  0.119750  0.099750  12.446000  0.002504
              0.153892  0.996459  0.129713  0.099713  12.446000  0.002870
     0.03
     0.04
              0.153892  0.997914  0.139662  0.099662  12.446000  0.003381
     1.45
              0.153892 1.203145 1.414435 -0.035565
                                                       0.570069 1.355652
     1.46
              0.153892 1.204600 1.422676 -0.037324
                                                       0.566849 1.373243
     1.47
              0.153892 1.206056 1.430907 -0.039093
                                                       0.563673 1.390935
     1.48
              0.153892 1.207511 1.439128 -0.040872
                                                       0.560541 1.408725
     1.49
              0.153892 1.208967 1.447339 -0.042661
                                                       0.557450 1.426614
```

[150 rows x 6 columns]

```
fig.circle(df_ga_clay_loam.t, df_ga_clay_loam.index, fill_color='red', size=7,_U \( \to \) legend_label='F(t)-t plot')

fig.line(df_ga_clay_loam.t, df_ga_clay_loam.index, line_width=2,_U \( \to \) line_color='red', legend_label='F(t)-t plot')

fig.circle(df_ga_clay_loam.t, df_ga_clay_loam.f_t, fill_color='blue', size=7,_U \( \to \) legend_label='f(t)-t plot')

fig.line(df_ga_clay_loam.t, df_ga_clay_loam.f_t, line_width=2,_U \( \to \) line_color='blue', legend_label='f(t)-t plot')

fig.legend.location = 'top_right'
fig.legend.click_policy='hide'
bp.show(fig)
```

```
[32]: # if intensity of 2 in/hr is considered
                i = 2. * 2.54 # cm
                F_{tp} = (delta_* clay_loam) / (1. + (i / Ks_clay_loam)) # total mass at_u
                 \hookrightarrowponding
                tp = F_tp / i
                dt = 1. - tp # 1-hour design storm
                print(f'time of ponding: {tp} hr')
                print(f'Mass at ponding: {F_tp} cm')
                # rewriting green and ampt model egn in simpler form
                # _ + + * ln((_( + )+ )/(_ + ))
                # m + * ln(c)
                df_ga_clay_loam_2 = pd.DataFrame(index = np.arange(0., 1.5, .01), columns = __
                  df_ga_clay_loam_2.index.name = 'F_tp_dt'
                df_ga_clay_loam_2['m'] = F_tp + (Ks_clay_loam * dt)
                df_ga_clay_loam_2['c'] = (df_ga_clay_loam_2.index + (delta_ * _clay_loam)) /__
                  df_ga_clay_loam_2['GA'] = df_ga_clay_loam_2.m + delta_ * _clay_loam * np.
                  →log(df_ga_clay_loam_2.c)
                df_ga_clay_loam_2['delta'] = df_ga_clay_loam_2.GA - df_ga_clay_loam_2.index
                df_ga_clay_loam_2['f_t'] = Ks_clay_loam * (1. + ( (delta_ * _clay_loam) /__
                  →df_ga_clay_loam_2.index ))
                df_ga_clay_loam_2['f_t'] = np.where(df_ga_clay_loam_2.index <= F_tp, i,__

df_ga_clay_loam_2['f_t'])

display="block" | display="block
                df_ga_clay_loam_2['t1'] = (df_ga_clay_loam_2.index - F_tp) / Ks_clay_loam
```

time of ponding: 0.0259021676344511 hr Mass at ponding: 0.13158301158301158 cm

```
[33]: df_ga_clay_loam_2
```

```
[33]:
                             С
                                     GA
                                            delta
                                                       f_t
                                                                  t
                    m
     F_tp_dt
     0.00
             0.228993 \quad 0.981061 \quad 0.098664 \quad 0.098664 \quad 5.080000 \quad 0.013362
     0.01
             0.02
             0.228993 \quad 0.983939 \quad 0.118634 \quad 0.098634 \quad 5.080000 \quad 0.013655
     0.03
             0.04
             0.228993 \quad 0.986818 \quad 0.138547 \quad 0.098547 \quad 5.080000 \quad 0.014531
     1.45
             0.228993 1.189766 1.413320 -0.036680 0.570069 1.366803
     1.46
             0.228993 1.191206 1.421561 -0.038439 0.566849 1.384394
     1.47
             0.228993 1.192645 1.429791 -0.040209 0.563673 1.402085
     1.48
             0.228993 1.194084 1.438012 -0.041988 0.560541 1.419876
     1.49
             0.228993 1.195524 1.446223 -0.043777 0.557450 1.437765
```

[150 rows x 6 columns]

```
[34]: \# f(t) vs t
      fig = initialize_fig(title = 'Clay Loam with variying rainfall intensity',
                           x_axis_label = 'Time (hr)',
                           y_axis_label = 'f(cm/hr) or F(cm)',
      #
                             tooltips = [
      #
                                  ('time\ (hr)',\ '$x'),
      #
                                  ('f(cm/hr)', '$y'),
      #
                           plot_height = 300,
      fig.title.text_font_size = '15pt'
      fig.xaxis.axis_label_text_font_size = '15pt'
      fig.yaxis.axis_label_text_font_size = '15pt'
      fig.circle(df_ga_clay_loam_2.t, df_ga_clay_loam_2.index, fill_color='red',u
      ⇔size=7, legend_label='F(t)-t plot')
      fig.line(df_ga_clay_loam_2.t, df_ga_clay_loam_2.index, line_width=2,_
       →line_color='red', legend_label='F(t)-t plot')
```

```
[35]: # With both together
      # f(t) vs t
      fig = initialize_fig(title = 'Clay Loam Case 1 and 2',
                           x_axis_label = 'Time (hr)',
                           y_axis_label = 'f(cm/hr) & F(cm)',
      #
                             tooltips = [
      #
                                 ('time (hr)', '$x'),
      #
                                 ('f(cm/hr)', '$y'),
                          plot_height = 300,
      fig.title.text font size = '15pt'
      fig.xaxis.axis_label_text_font_size = '15pt'
      fig.yaxis.axis label text font size = '15pt'
      fig.circle(df_ga_clay_loam.t, df_ga_clay_loam.index, fill_color='red', size=7,__
       →legend_label='F(t)-t plot')
      fig.line(df_ga_clay_loam.t, df_ga_clay_loam.index, line_width=2,_
      →line_color='red', legend_label='F(t)-t plot')
      fig.circle(df_ga_clay_loam.t, df_ga_clay_loam.f_t, fill_color='blue', size=7,_u
      →legend label='f(t)-t plot')
      fig.line(df_ga_clay_loam.t, df_ga_clay_loam.f_t, line_width=2,_u
      →line_color='blue', legend_label='f(t)-t plot')
      fig.circle(df_ga_clay_loam_2.t, df_ga_clay_loam_2.index, fill_color='green',u

size=7, legend_label='F(t)-t plot 2')
      fig.line(df_ga_clay_loam_2.t, df_ga_clay_loam_2.index, line width=2,__
      →line_color='green', legend_label='F(t)-t plot 2')
      fig.circle(df_ga_clay_loam_2.t, df_ga_clay_loam_2.f_t, fill_color='fuchsia',u
       ⇒size=7, legend_label='f(t)-t plot 2')
```

As can be seen, the time of ponding increases when the rainfall intensity decreased. As a result, the mass at ponding also increased. As a result, the initial infiltration rate until ponding which is the rainfall intensity is greater for first case.

If we replace the soil with the sandy loam soil compared to the clay loam, then since the porosity of the sandy loam is smaller than the clay loam, the tension or the pressure head increases. As a result, the sandy loam asymptote at higher infiltration rate than the clay loam. Similarly, the tension for the sandy loam is greater than the clay loam.

Question 5

- 5. Two rivers located 1000m apart from a fully penetrating an aquifer (not confined). The aquifer has a K value of 0. 5m/d. The region receives an average rainfall of 15 cm/yr and evapotranspiration loss is about 10 cm/yr. Assume that the water elevation in River 1 is 20m and the water elevation in River 2 is 18m.
- a. Use the Dupuit equation with recharge, and determine the location and height of the water divide.
- b. What is the daily discharge per unit width into River 1 River 2?

```
[36]: from sympy import symbols, Eq, solve

L = 1000 # length in meter
K = 0.5 # m/d hydraulic conductivity
P = 15. * (1 / 100) * (1 / 365.) # m/d
ET = 10. * (1 / 100.) * (1 / 365.) # m/d
W = P - ET # m/d
h0 = 20 # meter
hL = 18 # meter

q = 0 # for divide

# using dupuit eqn
location = symbols('location')
dupuit_eq = Eq( (K / (2 * L) * (h0**2 - hL**2)) + (W * (location - (L / 2))), q)
```

```
result = solve((dupuit_eq,), (location))
location, = result.values()
print(f'The water divide is {round(location, 3)} m from River 1')

depth = (h0**2 - ((location / L) * (h0**2 - hL**2)) + (W / K * (L - location)))**(1/2)
print(f'The water divide has a depth of {round(depth, 3)} m')

# discharge in River 1
q1 = (K / (2 * L) * (h0**2 - hL**2)) + W * (0 - (L / 2))
print(f'discharge in river 1: {round(q1, 3)} m/d')

q2 = (K / (2 * L) * (h0**2 - hL**2)) + W * (L - (L / 2))
print(f'discharge in river 2: {round(q2, 3)} m/d')
```

The water divide is 361.300 m from River 1 The water divide has a depth of 19.306 m discharge in river 1: -0.049 m/d discharge in river 2: 0.087 m/d

[]: