To - do -1:

[17]:
```
# 1
df = pd.read_csv("student.csv")
```

[20]:
```
# 2
print("First 5 rows: ")
print(df.head())
print("\nLast 5 rows: ")
print(df.tail())
```

```
First 5 rows:
   Math  Reading  Writing
0    48       68       63
1    62       81       72
2    79       80       78
3    76       83       79
4    59       64       62

Last 5 rows:
     Math  Reading  Writing
995    72       74       70
996    73       86       90
997    89       87       94
998    83       82       78
999    66       66       72
```

[22]:
```
# 3
print("Info: ")
df.info()
```

```
Info:
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 1000 entries, 0 to 999
Data columns (total 3 columns):
 #   Column   Non-Null Count  Dtype
---  ------   --------------  -----
 0   Math     1000 non-null   int64
 1   Reading  1000 non-null   int64
 2   Writing  1000 non-null   int64
dtypes: int64(3)
memory usage: 23.6 KB
```

[23]:
```
# 4
print("Descriptive info: ")
df.describe()
```

Descriptive info:

[23]:

	Math	Reading	Writing
count	1000.000000	1000.000000	1000.000000
mean	67.290000	69.872000	68.616000
std	15.085008	14.657027	15.241287
min	13.000000	19.000000	14.000000
25%	58.000000	60.750000	58.000000
50%	68.000000	70.000000	69.500000
75%	78.000000	81.000000	79.000000
max	100.000000	100.000000	100.000000

[24]:
```
# 5

features_X = df.drop(columns = ["Writing"]).values
label_Y = df["Writing"].values
```

To - Do - 2:

[25]:
```python
X = features_X.T
Y = label_Y

d = X.shape[0]
W = np.zeros((d, 1))

y_pred = (W.T @ X).T

print("X shape:", X.shape)     # (d, n)
print("W shape:", W.shape)        # (d, 1)
print("Y shape:", Y.shape)     # (n, 1)
print("Y_pred shape:", y_pred.shape)
```

```
X shape: (2, 1000)
W shape: (2, 1)
Y shape: (1000,)
Y_pred shape: (1000, 1)
```

To - Do - 3:

[ ]:
```python
def train_test_split(X, Y, test_size = 0.2, random_state=42):
    indices = np.arange(len(X))
    np.random.seed(random_state)
    np.random.shuffle(indices)

    split = int(test_size * len(X))

    train_indices = indices[split:]
    test_indices = indices[:split]

    return X[train_indices], X[test_indices], Y[train_indices], Y[test_indices]

x_train, x_test, y_train, y_test = train_test_split(features_X, label_Y, test_size = 0.2, random_state = 42)
```

To - Do - 4:

[27]:
```python
def cost_function(X, Y, W):
    """
    Calculates the Mean Square Error (MSE)

    Arguments:
    X: array-like, shape (n_samples, n_features)
        Feature Maxtrix
    Y: array-like, shape (n_samples,)
        True target values.
    W: array-like, shape (n_features, )
        Weight vector

    Returns:
    float
        The Mean Squared Error (MSE)
    """
    X  = np.array(X, dtype = float)
    Y = np.array(Y, dtype = float).reshape(-1, 1)
    W = np.array(W, dtype = float).reshape(-1, 1)

    n = len(Y)

    Y_predicted = X @ W
    error = Y_predicted - Y
    MSE = (1 / (2 * n)) * np.sum(error ** 2)

    return MSE
```

To - Do - 5:

```python
# Test case
X_test = np.array([[1, 2], [3, 4], [5, 6]])
Y_test = np.array([3, 7, 11])
W_test = np.array([1, 1])
cost = cost_function(X_test, Y_test, W_test)
if cost == 0:
    print("Proceed Further")
else:
    print("something went wrong: Reimplement a cost function")
print("Cost function output:", cost_function(X_test, Y_test, W_test))
```

```
Proceed Further
Cost function output: 0.0
```

To - Do - 6:

```python
def gradient_descent(X, Y, W, alpha, iterations):
    """
    Perform gradient descent to optimize the parameters of a linear regression model.

    Parameters:
        X (numpy.ndarray): Feature matrix (m x n).
        Y (numpy.ndarray): Target vector (m x 1).
        W (numpy.ndarray): Initial guess for parameters (n x 1).
        alpha (float): Learning rate.
        iterations (int): Number of iterations for gradient descent.

    Returns:
        tuple: A tuple containing the final optimized parameters (W_update) and the history of cost values.
            W_update (numpy.ndarray): Updated parameters (n x 1).
            cost_history (list): History of cost values over iterations.
    """
    X = np.array(X,dtype=float)
    Y= np.array(Y,dtype=float).reshape(-1,1)
    W= np.array(W,dtype=float).reshape(-1,1)

    m= len(Y)
    cost_history = []  # To store cost at each iteration
    W_update = W.copy()

    for iteration in range(iterations):
        # Step 1: Hypothesis values
        Y_pred = X @ W_update

        # Step 2: Difference between hypothesis and actual Y
        loss = Y_pred - Y

        # Step 3: Gradient calculation
        dw = (1/m) * (X.T @ loss)

        # Step 4: Update W
        W_update = W_update - alpha * dw

        # Step 5: Compute new cost
        cost = cost_function(X, Y, W_update)
        cost_history.append(cost)

        # # PRINT one line per iteration
        # print(f"Iteration {iteration+1}:")
        # print("  Weights:\n", W_update)
        # print("  Cost:", cost)
        # print("-" * 30)
    return W_update, cost_history
```

To - Do - 7:

```python
# Generate random test data
np.random.seed(0) # For reproducibility
X = np.random.rand(100, 3) # 100 samples, 3 features
Y = np.random.rand(100)
W = np.random.rand(3) # Initial guess for parameters
# Set hyperparameters
alpha = 0.01
iterations = 1000
# Test the gradient_descent function
final_params, cost_history = gradient_descent(X, Y, W, alpha, iterations)
# Print the final parameters and cost history
print("Final Parameters:", final_params)
print("Cost History:", cost_history)
```

```
Final Parameters: [[0.20551667]
 [0.54295081]
 [0.10388027]]
Cost History: [np.float64(0.10711197094660153), np.float64(0.10634880599939901), np.float64(0.10559826315680618), np.float64(0.10486012948320558), np.f
loat64(0.1041341956428534), np.float64(0.10342025583900626), np.float64(0.1027181077540776), np.float64(0.1020275524908062), np.float64(0.1013483945144
1932), np.float64(0.1006804415957737), np.float64(0.10002350475545868), np.float64(0.09937739820884378), np.float64(0.0987419393120561), np.float64(0.0
9811694850887098), np.float64(0.09750224927850094), np.float64(0.0968976680842672), np.float64(0.09630303432313951), np.float64(0.09571818027612913), n
p.float64(0.09514294105952065), np.float64(0.09457715457692842), np.float64(0.09402066147216397), np.float64(0.09347330508290017), np.float64(0.0929349
3139511913), np.float64(0.09240538899833017), np.float64(0.09188452904154543), np.float64(0.0913722051899995), np.float64(0.09086827358260123), np.floa
t64(0.09037259279010502), np.float64(0.08988502377398919), np.float64(0.08940542984603007), np.float64(0.08893367662855953), np.float64(0.0884696320153
9433), np.float64(0.08801316613342668), np.float64(0.08756415130486386), np.float64(0.08712246201010665), np.float64(0.08668797485125508), np.float64
(0.08626056851623207), np.float64(0.0858401237435128), np.float64(0.08542652328745133), np.float64(0.08501965188419301), np.float64(0.0846193962181636
1), np.float64(0.08422564488912489), np.float64(0.08383828837978766), np.float64(0.08345721902397185), np.float64(0.08308233097530582), np.float64(0.08
271352017645425), np.float64(0.08235068432886682), np.float64(0.08199372286303819), np.float64(0.08164253690927113), np.float64(0.08129702926893387), n
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772706855), np.float64(0.07933798841853089), np.float64(0.07902964196486459), np.float64(0.07872625310147845), np.float64(0.07842773845346056), np.floa
t64(0.07813401605495938), np.float64(0.0778450053253578), np.float64(0.07756062704584993), np.float64(0.07728080333641406), np.float64(0.07700545763317
516), np.float64(0.07673451466614989), np.float64(0.07646790043736813), np.float64(0.0762055421993645), np.float64(0.07594736843403344), np.float64(0.0
7569330883184208), np.float64(0.07544329427139429), np.float64(0.07519725679934074), np.float64(0.07495512961062821), np.float64(0.07471684702908328),
np.float64(0.07448234448832412), np.float64(0.0742515585129952), np.float64(0.07402442670031913), np.float64(0.07380088770196072), np.float64(0.0735808
812061975), np.float64(0.07336434792039191), np.float64(0.07315122955375962), np.float64(0.07294146880042968), np.float64(0.07273500932279067), np.floa
t64(0.07253179573511871), np.float64(0.07233177358748233), np.float64(0.0721348893499193), np.float64(0.0719410903968814), np.float64(0.071750324991941
82), np.float64(0.07156254227276149), np.float64(0.07137769223630935), np.float64(0.07119572572433286), np.float64(0.07101659440907385), np.float64(0.0
7084025077922625), np.float64(0.070666648126131), np.float64(0.07049574053020463), np.float64(0.07032748284759717), np.float64(0.07016183069707572), n
p.float64(0.06999874044712992), np.float64(0.06983816920329523), np.float64(0.06968007479569094), np.float64(0.06952441576676846), np.float64(0.0693711
5135926715), np.float64(0.06922024150437375), np.float64(0.06907164681008185), np.float64(0.06892532854974837), np.float64(0.0687812486508435), np.floa
t64(0.06863936968389096), np.float64(0.06849965485159508), np.float64(0.06836206797815196), np.float64(0.06822657349874124), np.float64(0.0680931364491
956), np.float64(0.067961722455845), np.float64(0.06783229772553254), np.float64(0.06770482903579932), np.float64(0.06757928372523506), np.float64(0.06
745562968399212), np.float64(0.0673338353444597), np.float64(0.06721386967209597), np.float64(0.06709570215641501), np.float64(0.06697930280212629), n
p.float64(0.06686464212042396), np.float64(0.06675169112042349), np.float64(0.0666404213007429), np.float64(0.06653080464122667), np.float64(0.06642281
359480932), np.float64(0.06631642107951677), np.float64(0.0662116004706028), np.float64(0.06610832559281864), np.float64(0.0660065707128131), np.float6
4(0.0659063105316614), np.float64(0.06580752017752023), np.float64(0.06571017519840698), np.float64(0.06561425155510119), np.float64(0.0655197256141658
7), np.float64(0.0654265741410871), np.float64(0.06533477429352925), np.float64(0.06524430361470465), np.float64(0.06515514002685512), np.float64(0.065
06726182484374), np.float64(0.06498064766985516), np.float64(0.06489527658320231), np.float64(0.06481112794023773), np.float64(0.06472818146436811), n
p.float64(0.0646464172211699), np.float64(0.06456581561260431), np.float64(0.06448635737133043), np.float64(0.0644080235551142), np.float64(0.064330795
54133217), np.float64(0.06425465502156798), np.float64(0.06417958399630046), np.float64(0.06410556476968135), np.float64(0.06403257994440142), np.float
64(0.0639606124166433), np.float64(0.06388964537111992), np.float64(0.06381966227619644), np.float64(0.06375064687909507), np.float64(0.063682583201180
77), np.float64(0.06361545553332656), np.float64(0.06354924843135755), np.float64(0.06348394671157163), np.float64(0.06341953544633615), np.float64(0.0
6335599995975896), np.float64(0.06329332582343267), np.float64(0.06323149885225086), np.float64(0.06317050510029514), np.float64(0.06311033085679155),
np.float64(0.06305096264213549), np.float64(0.06299238720398384), np.float64(0.0629345915134133), np.float64(0.06287756276114324), np.float64(0.0628212
```

To - Do - 8:

```python
def rmse(y, y_pred):
    """
    Calculates the Root Mean Squred Error (RMSE) between actual and predicted values.

    Arguments:
    y: array-like
        Array of actual (target) values.
    y_pred: array-like
        Array of predicted values.

    Returns:
    float
        The root mean squared error.
    """
    Y = np.array(y, dtype = float).flatten()
    Y_pred = np.array(y_pred, dtype = float).flatten()

    rmse = np.sqrt(np.mean((Y - Y_pred) ** 2))
    return rmse
```

To - Do - 9:

```
def r2(Y, Y_pred):
    """
    This function calculates the R Squared Error.

    Arguments:
    Y: array-like
        Array of actual (target) dependent values.
    Y_pred: array-like
        Array of predicted dependent values.

    Returns:
    float
        R Squared error.
    """
    Y = np.array(Y, dtype=float).flatten()
    Y_pred = np.array(Y_pred, dtype=float).flatten()

    mean_y = np.mean(Y)  # Mean of actual values

    # Total sum of squares
    ss_tot = np.sum((Y - mean_y) ** 2)

    # Sum of squared residuals
    ss_res = np.sum((Y - Y_pred) ** 2)

    # R squared
    r2_score = 1 - (ss_res / ss_tot)

    return r2_score
```

To - Do - 10:

```
# Main Function

# Step 1: Load the dataset
data = pd.read_csv('student.csv')

# Step 2: Split the data into features (X) and target (Y)
X = data[['Math', 'Reading']].values # Features: Math and Reading marks
Y = data['Writing'].values # Target: Writing marks

# Step 3: Split the data into training and test sets (80% train, 20% test)
X_train, X_test, Y_train, Y_test = train_test_split(X, Y, test_size=0.2, random_state=42)

# Step 4: Initialize weights (W) to zeros, learning rate and number of iterations
W = np.zeros(X_train.shape[1]) # Initialize weights
alpha = 0.00001 # Learning rate
iterations = 1000 # Number of iterations for gradient descent

# Step 5: Perform Gradient Descent
W_optimal, cost_history = gradient_descent(X_train, Y_train, W, alpha, iterations)

# Step 6: Make predictions on the test set
Y_pred = np.dot(X_test, W_optimal)

# Step 7: Evaluate the model using RMSE and R-Squared
model_rmse = rmse(Y_test, Y_pred)
model_r2 = r2(Y_test, Y_pred)

# Step 8: Output the results
print("Final Weights:", W_optimal)
print("Cost History (First 10 iterations):", cost_history[:10])
print("RMSE on Test Set:", model_rmse)
print("R-Squared on Test Set:", model_r2)
```

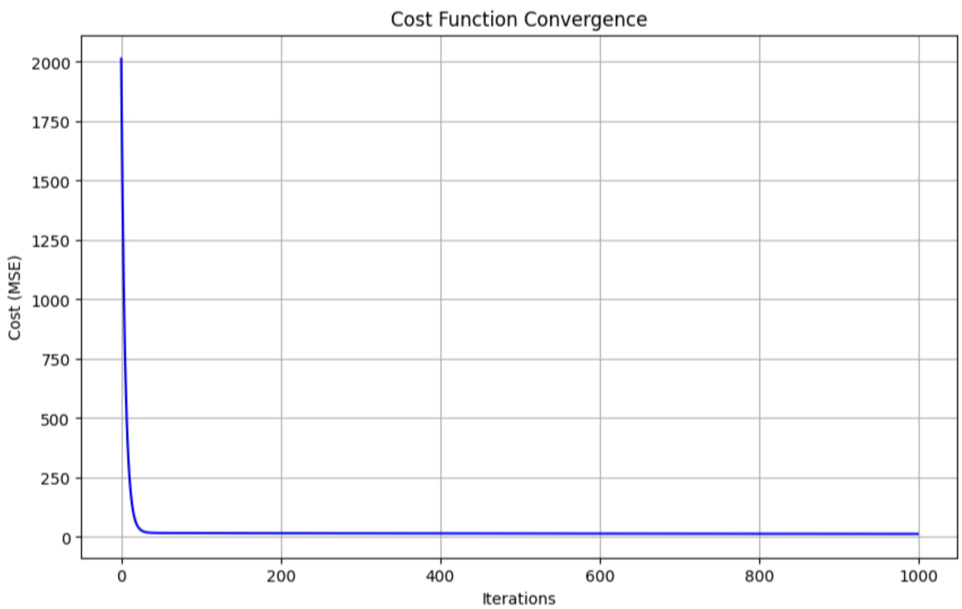
```
Final Weights: [[0.34811659]
 [0.64614558]]
Cost History (First 10 iterations): [np.float64(2013.165570783755), np.float64(1640.2868325996924), np.float64(1337.0619994901588), np.float64(1090.479
4892850578), np.float64(889.9583270083234), np.float64(726.8940993009545), np.float64(594.2897260808594), np.float64(486.4552052951635), np.float64(39
8.7634463599484), np.float64(327.4517147324688)]
RMSE on Test Set: 5.2798239764188635
R-Squared on Test Set: 0.8886354462786421
```

1. Did your Model Overfitt, Underfitts, or performance is acceptable.

```
import matplotlib.pyplot as plt

def plot_cost(cost_history):
    plt.figure(figsize=(10, 6))
    plt.plot(range(len(cost_history)), cost_history, color='blue')
    plt.title('Cost Function Convergence')
    plt.xlabel('Iterations')
    plt.ylabel('Cost (MSE)')
    plt.grid(True)
    plt.show()

plot_cost(cost_history)
```



The model performs well, as the cost function decreases smoothly, which shows that the model is learning properly. The RMSE value of around 5.28 indicates that the prediction error is low. Similarly, the R-squared value of about 0.89 means that the model can explain most of the variation in writing marks. Therefore, the model does not underfit or overfit the data and gives reliable predictions.

2. Experiment with different value of learning rate, making it higher and lower, observe the result.

```
learning_rates = [0.000001, 0.00001, 0.0001]

for alpha in learning_rates:
    print("\nLearning rate:", alpha)

    W = np.zeros(X_train.shape[1])
    iterations = 1000

    W_optimal, cost_history = gradient_descent(
        X_train, Y_train, W, alpha, iterations
    )

    Y_pred = X_test @ W_optimal

    print("  Final Cost:", cost_history[-1])
    print("  RMSE:", rmse(Y_test, Y_pred))
    print("  R²:", r2(Y_test, Y_pred))
```

```
Learning rate: 1e-06
  Final Cost: 16.535602355147176
  RMSE: 5.856694748793876
  R²: 0.8629707528684534

Learning rate: 1e-05
  Final Cost: 13.150619992105618
  RMSE: 5.2798239764188635
  R²: 0.8886354462786421

Learning rate: 0.0001
  Final Cost: 10.26076310841341
  RMSE: 4.792607360540954
  R²: 0.908240340333986
```

According to the data above, as the learning rate increases from 0.000001 to 0.0001, the model shows better convergence, with a decrease in the final cost and RMSE and an increase in the R-squared value. This indicates that a very small learning rate leads to slow convergence, while a moderate learning rate helps the model learn faster and more effectively. However, if the learning rate becomes too large, the model may start to diverge instead of converging.