

Causal Inference

Vanessa Didelez and Robin Evans

BIPS, University of Bremen (Germany), and University of Oxford (UK)

August 2023 APTS — Glasgow

Part 3a: Estimating a Causal Effect

(of a Point Treatment)

Basic Setting



A = binary (point-) treatment

Y = some (numeric) outcome
 (not survival / duration — that's special)

X = sufficient adjustment set of pre-treatment covariates

Basic Setting



A = binary (point-) treatment

Y = some (numeric) outcome
 (not survival / duration — that's special)

X = sufficient adjustment set of pre-treatment covariates

Keeping it simple to focus on essentials!

General reference:

Goetghebeur, E, le Cessie, S, De Stavola, B, Moodie, EE, Waernbaum, I. Formulating causal questions and principled statistical answers. Statistics in Medicine. 2020; 39: 4922–4948.

(Total) Causal Effect



Will focus on: Average causal effect

$$ACE = E(Y|\mathsf{do}(A=1)) - E(Y|\mathsf{do}(A=0))$$

or, with potential outcomes

$$ACE = E(Y(1) - Y(0)) = E(Y(1)) - E(Y(0))$$

aka: average treatment (ATE) or total causal effect (TCE), etc.

Key Assumptions

Consistency



Consistency Assumption:

If we observe $A^i=a$ then $Y^i=Y^i(a)$ (for individual i)

i.e. the outcome we observe under the actual treatment is the potential outcome had the treatment been *set* to what it actually was.

Violated, e.g., if manipulation of A not well defined or so 'invasive' that observational setting not informative.

Example: A is 'BMI' — how to manipulate BMI itself?

Often: if violated, need more elaborate model; e.g. intervene in physical activity to change BMI.

Consistency



Under consistency, and e.g. for binary A, can write

$$Y^{i} = Y(1)^{i} A^{i} + Y(0)^{i} (1 - A^{i})$$

Note:

consistency implicit in graphical / $do(\cdot)$ approaches \rightarrow invariance (at distributional level)

No-Interference



Common assumption: no-interference:

Vector $\mathbf{a} =$ treatment assignments for n units, then $Y^i(\mathbf{a}) = Y^i(a^i)$,

i.e. PO does not depend on treatment other units received.

Violation: e.g. vaccines, social networks.

Stable unit-treatment value (SUTVA):

consistency + no-interference.



Assumption of cond. exchangeability or no unmeasured confounding (or random treatment assignment, ignorability or ...):



Assumption of cond. exchangeability or no unmeasured confounding (or random treatment assignment, ignorability or ...):

Let *X* be (subset of) measured pre-treatment covariates, then

$$Y(a) \perp \!\!\! \perp A \mid X$$



Assumption of cond. exchangeability or no unmeasured confounding (or random treatment assignment, ignorability or ...):

Let *X* be (subset of) measured pre-treatment covariates, then

$$Y(a) \perp \!\!\! \perp A \mid X$$

Interpretation: within values of X, can consider A like randomised wrt outcome.



Assumption of cond. exchangeability or no unmeasured confounding (or random treatment assignment, ignorability or ...):

Let *X* be (subset of) measured pre-treatment covariates, then

$$Y(a) \perp \!\!\! \perp A \mid X$$

Interpretation: within values of X, can consider A like randomised wrt outcome.

Denote: *X* is *sufficient* to adjust for confounding;

or 'valid adjustment set'.

Conditional Exchangeability with do(·)



Assumption of **cond. exchangeability** or **no unmeasured confounding** & **'consistency'** with do—notation:

$$p(y \mid x; \mathsf{do}(A = a)) = p(y \mid x, a)$$

Interpretation: within values of X, whether A=a obtained by intervention or observation makes no difference wrt. distribution of Y.

Useful software for querying DAGs: DAGitty (Textor et al, 2016)



What makes *X* pre-treatment covariates?

 \Rightarrow must be known not to be affected by intervention in treatment A!



What makes *X* pre-treatment covariates?

 \Rightarrow must be known not to be affected by intervention in treatment A!

Sufficient: X prior in time to A — but not necessary.



What makes *X* pre-treatment covariates?

 \Rightarrow must be known not to be affected by intervention in treatment A!

Sufficient: X prior in time to A — but not necessary.

Often: X and A contemp. & share themselves common causes through past history, e.g. patient's medical history.



What makes *X* pre–treatment covariates?

 \Rightarrow must be known not to be affected by intervention in treatment A!

Sufficient: X prior in time to A — but not necessary.

Often: X and A contemp. & share themselves common causes through past history, e.g. patient's medical history.

Graphically: X non-descendants of A.

(Overview: methods for causal covariate selection see Witte & Didelez, 2018:BiomJ)

Positivity Assumption checking overlap



Often, methods for effect estimation require

Assumption of positivity:

$$p(a \mid x) > 0$$
 for all $a, x \quad (p(x) > 0)$

Interpretation: for all (suff.) covariate values, it must be possible that a subject receives any value of treatment.

In practice: often empirically (nearly) violated — modifications and adaptations of methods exist sometimes.



 Target trial: can / should define eligibility criteria so that positivity is satisfied — requires domain knowledge



- Target trial: can / should define eligibility criteria so that positivity is satisfied — requires domain knowledge
- (Lack of) positivity can be evaluated empirically (look at p(a|x) or p(x|a)) high-dim X becomes challenging Methods exist to characterise 'area of overlap'



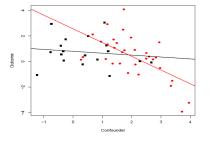
- Target trial: can / should define eligibility criteria so that positivity is satisfied — requires domain knowledge
- (Lack of) positivity can be evaluated empirically (look at p(a|x) or p(x|a)) high-dim X becomes challenging Methods exist to characterise 'area of overlap'
- Do not include superfluous variables in X, especially: strong predictors of A that do not affect Y — can lead to apparent violations of positivity

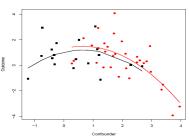


- Target trial: can / should define eligibility criteria so that positivity is satisfied — requires domain knowledge
- (Lack of) positivity can be evaluated empirically (look at p(a|x) or p(x|a)) high-dim X becomes challenging Methods exist to characterise 'area of overlap'
- Do not include superfluous variables in X, especially: strong predictors of A that do not affect Y — can lead to apparent violations of positivity
- Regression-based approaches may mask lack of positivity as fitted models allow extrapolation.

Positivity Assumption Extrapolation









We consider p(y | do(A = a)) or equivalently p(Y(a)).

$$p(Y(a)) \stackrel{(i)}{=} \sum_{x} p(Y(a)|x) p(x) \stackrel{(ii)}{=} \sum_{x} p(Y(a)|a,x) p(x)$$

$$\dots \stackrel{(iii)}{=} \sum_{x} p(y|a,x) p(x)$$



We consider p(y | do(A = a)) or equivalently p(Y(a)).

With the above assumptions:

$$p(Y(a)) \stackrel{(i)}{=} \sum_{x} p(Y(a)|x) p(x) \stackrel{(ii)}{=} \sum_{x} p(Y(a)|a,x) p(x)$$

$$\dots \stackrel{(iii)}{=} \sum_{x} p(y|a,x) p(x)$$

(i) probability calculus



We consider p(y | do(A = a)) or equivalently p(Y(a)).

$$p(Y(a)) \stackrel{(i)}{=} \sum_{x} p(Y(a)|x)p(x) \stackrel{(ii)}{=} \sum_{x} p(Y(a)|a,x)p(x)$$

$$\dots \stackrel{(iii)}{=} \sum_{x} p(y|a,x)p(x)$$

- (i) probability calculus
- (ii) valid adjustment set



We consider p(y | do(A = a)) or equivalently p(Y(a)).

$$p(Y(a)) \stackrel{(i)}{=} \sum_{x} p(Y(a)|x)p(x) \stackrel{(ii)}{=} \sum_{x} p(Y(a)|a,x)p(x)$$

$$\dots \stackrel{(iii)}{=} \sum_{x} p(y|a,x)p(x)$$

- (i) probability calculus
- (ii) valid adjustment set
- (iii) causal consistency & positivity



We consider p(y | do(A = a)) or equivalently p(Y(a)).

$$p(Y(a)) \stackrel{(i)}{=} \sum_{x} p(Y(a)|x)p(x) \stackrel{(ii)}{=} \sum_{x} p(Y(a)|a,x)p(x)$$

$$\dots \stackrel{(iii)}{=} \sum_{x} p(y|a,x)p(x)$$

- (i) probability calculus
- (ii) valid adjustment set
- (iii) causal consistency & positivity
- (iv) no-interference was needed for well-definedness of causal effect

Checking Assumptions?



- Consistency / no-interference: domain knowledge, study design
- No-unmeasured-confounding: compare analysis of observational data with actual randomised trial — often not possible; triangulation, e.g. negative controls etc.
- Positivity
 - basic: boxplot of each variable in *X* by treatment group;
 - advanced: fit model for $\pi(x) = P(A=1|X=x)$ the **propensity score** obtain fitted values $\hat{\pi}^i = \hat{P}(A=1|X=x^i)$ for each unit i and check for near zero/one in treated and control group, respectively.

Propensity Score: Checking Positivity



Example:

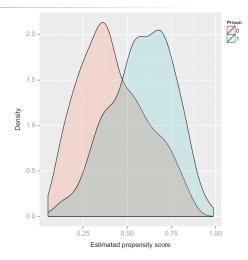
n=1022 offenders sentenced to either probation A=0or prison A=1; X=17 covariates;

Y = recidivism (yes/no);

 \Rightarrow reasonable overlap.

Example taken from Guo et al. (2016))

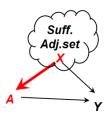
Covariate balance: R package cobalt

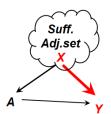


Methods



Methods use either treatment model p(a|x) (propensity score) or outcome model p(y|a,x) or both





Principles:

- regression (+standardisation),
- inverse-probability weighting (IPTW),
- stratification / matching,
- Hybrid: doubly-robust estimation (double-ML)

Standardisation



aka: G-Formula

(Robins, 1986)

Reminder: if X is sufficient set of covariates

$$E(Y \mid \mathsf{do}(A = a)) = \sum_{x} E(Y \mid a, x) p(x)$$

An obvious way to use this is:

- fit flexible regression model for $E(Y|a,x;\beta)$ to data to avoid 'g-null paradox'! (Evans & Didelez, 2023:JRSSB)
- average over empirical X-distribution: $\sum_i E(Y|a, x^i; \hat{\beta})/n$
- e.g. with R package stdReg (Sjølander and Dahlqwist, 2017)

Standardisation — Example



```
Y = 'low birth weight' (binary); X = 'mother smokes' (binary),
C = \{\text{'age'}, \text{'race'}\}
                                                      (Siolander, 2016)
> fit2 <- glm(formula=lbw~(smoker+race+age)^2,
    family="binomial", data=clslowbwt)flexible outcome model</pre>
 > fit.std <- stdGlm(fit=fit2, data=clslowbwt, X="smoker",
    clusters="id")
                                          standardised means
                                      control / treatment groups
 > summary(fit.std)
   Estimate Std. Error lower 95 upper 95
   0.279 0.0406 0.199 0.358
 1 0.407 0.0555 0.298 0.516
 > summary(fit.std, contrast="difference", reference=0)
   Estimate Std. Error lower 95 upper 95 ostimated ACE
                                                 difference, i.e.
```

Using Standardisation



Why not just look at regression model E(Y|A,X)?

- Consider: marginal versus conditional causal effect and collapsible versus non-collapsible parameters;
- Logistic regression / odds ratios not collapsible.
- If set of sufficient covariates X not unique, cond. effects may depend on choice of X (e.g. COR), but not marginal ones.
- Marginal E(Y | do(A = a)) corresponds to randomised trial where covariates X can be /are ignored.

Note: Consistency of effect estimation relies on correctly specified model for p(y|a,x).

Using Standardisation

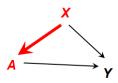


- R package stdReg (Sjølander and Dahlqwist, 2017)
- The method is special case of G-formula for sequential treatments (Robins, 1986).
- Population effect E(Y | do(A = a)) depends on *distribution* p(x) of covariates in target population
 - \Rightarrow not necessarily the same in different populations (e.g. age distribution). If p(y|a,x) regarded as 'stable' across populations, then can just replace $\hat{p}(x)$ in the above by different covariate distribution for different populations (e.g. UK versus USA covariate distribution).

Treatment Modelling Approaches



The following methods are all based on models for A given X instead of modelling Y given X.



From Standardisation to MSM



The functionals

$$\sum_x p(y|a,x)p(x) \quad \text{ or } \quad \sum_x E(Y|a,x)p(x)$$

might be 'awkward', especially if A continuous, E(Y|a,x) non-linear with interactions, or X high dimensional and/or partly continuous.

- \Rightarrow Parameterise E(Y | do(A = a)) itself?!
- ⇒ Marginal structural models (MSM)

Marginal Structural Models



(Hernán et al, 2001)

MSM: semiparametric model for

$$p(y | \operatorname{do}(A = a))$$
 or more typically $E(Y | \operatorname{do}(A = a))$

e.g. linear, logistic, CoxPH, loglinear, probit etc.

Marginal: refers to time-varying covariates → later

Structural: model under intervention in A (not observational)

Note: term 'structural' is used in many different ways — here it always refers to modelling the <u>underlying causal</u> relationships.



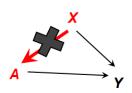
Idea is based on:

$$\begin{aligned} p(y \mid \mathsf{do}(A = a)) \\ &= \sum_{x} p(y \mid a, x) p(x) \end{aligned}$$



Idea is based on:

$$\begin{aligned} p(y \,|\, \mathsf{do}(A = a)) \\ &= \sum_x p(y|a, x) p(x) \\ &= \sum_x \frac{p(y, a, x)}{p(a|x)} \end{aligned}$$





Idea is based on:

$$p(y \mid \mathsf{do}(A = a)) = \sum_{x} p(y|a, x) p(x) = \sum_{x} \frac{p(y, a, x)}{p(a|x)}$$

 \Rightarrow fit model for E(Y | do(A = a)) with weights $w^i = p(a^i | x^i)^{-1}$



Idea is based on:

$$p(y | do(A = a)) = \sum_{x} p(y|a, x)p(x) = \sum_{x} \frac{p(y, a, x)}{p(a|x)}$$

- \Rightarrow fit model for E(Y | do(A = a)) with weights $w^i = p(a^i | x^i)^{-1}$
- \Rightarrow creates 'pseudo sample' where X is balanced



Idea is based on:

$$p(y | do(A = a)) = \sum_{x} p(y|a, x)p(x) = \sum_{x} \frac{p(y, a, x)}{p(a|x)}$$

- \Rightarrow fit model for E(Y | do(A = a)) with weights $w^i = p(a^i | x^i)^{-1}$
- \Rightarrow creates 'pseudo sample' where X is balanced
- \Rightarrow unbiased estimating equations for parameters of E(Y | do(A = a)).

Here, $w^i = p(a^i|x^i)^{-1}$ is the inverse of the probability that individual i receives 'treatment' a^i given they have covariates x^i .

IPTW Estimator





Simple situation: binary exposure A; define $\pi(x) = P(A = 1|X = x)$.

Can show (under our assumptions):

$$E\left(\frac{A}{\pi(X)}Y\right) = E(Y \mid \operatorname{do}(A=1))$$

and similarly

$$E\left(\frac{1-A}{1-\pi(X)}Y\right) = E(Y \mid \mathsf{do}(A=0))$$

Proof: iterated conditional expectation (exercise!)

→ see 'Horvitz–Thompson' principle

IPTW Estimator



With model $\pi(X; \alpha) \Rightarrow \text{plug-in } \pi(X; \hat{\alpha})$

IPTW yields consistent estimator for ACE:

- if $\pi(X; \alpha)$ correctly specified;
- can obtain sandwich standard errors or bootstrap, or theoretical asymptotical standard errors.

IPTW — Implementation



Easy to implement with standard software for regression models by specifying weights: first, obtain weights, then fit chosen model.

Note: default standard errors ignore variability in (estimated!) weights ⇒ sandwich st.error (or: bootstrap) ²⁹

Notes on IPTW



- Consistent when both, models for E(Y | do(A = a)) and $\pi(x)$ correctly specified.
- If $p(a|x) \approx 0 \Rightarrow$ large weights \Rightarrow use 'stabilised' weights, e.g. $w = \tilde{p}(a)/p(a|x)$, where $\tilde{p}(a)$ some distribution for $A \Rightarrow$ more efficient estimators.
- Check assumptions: in weighted population, observed covariates must be 'balanced' — e.g. package cobalt for balance plots.

Checking Assumptions: Balance



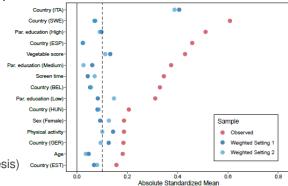
Example: causal effect of 'fibre intake' on children's BMI; large adjustment set (country, parental edu, vege-score, etc.)

Plot: std.mean-diff (abs)

treated / untreated

red: unweighted

blue: weighted



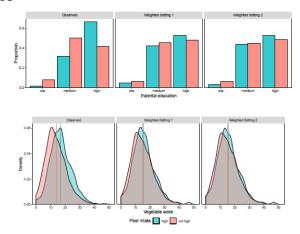
(Nöhren, 2021:MAthesis)

Absolute Standardized Mean Differences

Checking Assumptions: Balance



Example(ctd.), checking balance of whole distribution of covariates



Notes on IPTW



- MSMs with IPTW mostly used in longitudinal situations / sequential treatments with time-varying confounding ⇒ 'marginal' over time-dependent confounders / covariates.
- IPTW especially useful when study design (or other) supplies background knowledge to model weights p(a|x).
- Software: R package ipw for longitudinal data and correct standard errors
- Problem: estimation of weights $p(a|x)^{-1}$ not obvious, but possible, when A continuous.

Continuous Treatments?



Wanted: E(Y(a)) or $E(Y \mid do(a))$ as a function of a

- assume a (semi-)parametric model;
- but non-parametric methods exist

(e.g. Kennedy et al, 2017:JRSSB)

Issues:

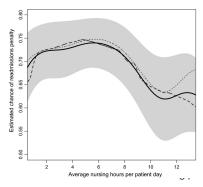
- positivity?
- meaningful interventions?

Example: in hospitals,

A = nurse staffing hours per day

Y =excess re-admission

dashed = regr.std dotted = IPTW full = "double-robust"



Propensity Score (PS)



(Rosenbaum & Rubin, 1983)

Have used
$$\pi(x) = P(A = 1|X = x)$$

⇒ propensity score.

Note: $\pi := \pi(X)$ is random variable.

MSM: used $\pi(X)$ for weighting.

But: can also use $\pi(X)$ for adjustment-type approaches, due to it being a balancing score...

Propensity / Balancing Score



(Still assuming: X sufficient set of covariates; A binary.)

Use of propensity scores (vs. IPTW) is based on

$$A \perp \!\!\! \perp X \mid \pi$$
 i.e. π balances X

Hence (with properties of X):

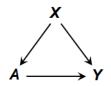
$$Y(a) \perp \!\!\! \perp A \mid \pi$$

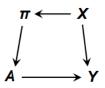
making π a minimal sufficient reduction of X (univariate $\in [0,1]$).

Propensity Score — Graphically



Propensity score $\pi:=\pi(X)=P(A=1|X)$ satisfies these conditional independencies:





Left: assumption of X being sufficient set of covariates.

Right: π is deterministic function of X and $A \perp \!\!\! \perp X \mid \pi$.



- Estimate propensity score $\hat{\pi}$ with model for $\pi(X; \alpha)$.
- Required: correctly specified model $\pi(X; \alpha)$. Non-parametric approaches: random forests etc.
- Note: predictive quality of $\pi(X;\alpha)$ for A not important because need X to be (A,Y)-confounders, not nec. strong predictors of A.

In fact: strong A-predictors \Rightarrow bias amplification (Pearl, 2011).

- Check balancing property (various diagnostics).
- Check positivity / overlap ⇒ if necessary: prune!



Methods for using PS (other than weighting):

• PS **stratification**: divide into strata (often quintiles) and fit p(y|a) to each stratum separately.

(Strata specific effects can reveal effect modification.)

Then weighted average to obtain overall population effect.



Methods for using PS (other than weighting):

- PS stratification: divide into strata (often quintiles) and fit p(y|a) to each stratum separately.
 (Strata specific effects can reveal effect modification.)
 Then weighted average to obtain overall population effect.
- Alternative: matching on propensity score, i.e. match each treated with k untreated with similar propensity score this estimates ETT!



Methods for using PS (other than weighting):

- PS stratification: divide into strata (often quintiles) and fit p(y|a) to each stratum separately.
 (Strata specific effects can reveal effect modification.)
 Then weighted average to obtain overall population effect.
- Alternative: matching on propensity score, i.e. match each treated with k untreated with similar propensity score this estimates ETT!
- Sometimes: **PS adjustment** specify model for $p(y|a,\pi)$ and fit with $\hat{\pi}$ plugged in.



Methods for using PS (other than weighting):

- PS stratification: divide into strata (often quintiles) and fit p(y|a) to each stratum separately.
 (Strata specific effects can reveal effect modification.)
 Then weighted average to obtain overall population effect.
- Alternative: matching on propensity score, i.e. match each treated with k untreated with similar propensity score this estimates ETT!
- Sometimes: **PS adjustment** specify model for $p(y|a,\pi)$ and fit with $\hat{\pi}$ plugged in.
- Extrapolation is automatically avoided.

Survival of Cancer Patients Example



US National Cancer Institute's SEER data base; observational study. Covariates: year of diagnosis, tumor size, geogr. registry, race, marital status

Propensity score	Treatment	No.	5-Year-Surv.	Difference
1st quintile	Α	56	85.6%	
	В	1008	86.7%	-1.1%
2nd quintile	Α	106	82.8%	
	В	964	83.4%	-0.6%
3rd quintile	Α	193	85.2%	
·	В	866	88.8%	-3.6%
4th quintile	Α	289	88.7%	
·	В	978	87.3%	1.4%
top quintile	Α	462	89.0%	
	В	604	88.5%	0.5%

Overall estimated (weighted average) ACE = -0.68.

From strata specific results: slight suggestion that treatment B is better for those who are more likely to receive it.

Notes on Propensity Score



- Best with binary treatment / exposure.
- PS stratification consistent if $\pi(X;\alpha)$ correctly specified, but can be markedly biased due to residual confounding within strata possible.
- Consistency can be achieved by increasing number of strata when sample size is 'large' or by additional modelling of E(Y|X = x; do(A = a)) within strata.
- PS popular especially for matching: π is 'one-dimensional reduction' of covariates but at cost of first modelling / estimating $\pi = p(a|x;\alpha)$.
- PS matching / stratification not really suitable for sequential treatments.

41

Notes on Propensity Score



- Danger: modelling π(X; α) may focus on strong predictors
 of A ⇒ can amplify bias! ⇒ selection of X as adjustment
 set should be separate process from fitting π(X; α).
- Interpretation of PS sometimes regarded as difficult compared to actual covariate values.
- Simulations suggest that IPTW use of π superior to stratification. (Lunceford & Davidian, 2004)
- Critique of PS matching: King & Nielsen (2019)

Estimating Causal Effects



Summary (no unobs. conf.)

Given sufficient obs. confounders X (& positivity):

- Traditional: regression adjustment or...
- ... standardise to obtain population effect (g-formula in time-varying context) – underused in practice
- or MSMs fitted by IPTW easy to use, also with time-varying data – but can be inefficient
- propensity score methods (stratification / matching) overused?
- Combination of above approaches leads to doubly robust estimation procedures
- Should always check positivity / overlap!

Thank You!

www.leibniz-bips.de/en

Contact
Vanessa Didelez
Leibniz Institute for Prevention Research
and Epidemiology – BIPS
Achterstraße 30
D-28359 Bremen
didelez@leibniz-bips.de

