

Causal Learning for Data Science

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Part 2: Estimating a Causal Effect

Basic Setting



X = binary (point-) treatment

Y =some (numeric) outcome

(not survival / duration — that's special)

C =sufficient adjustment set of pre-treatment covariates

Keeping it simple to focus on principles!

REFERENCE: Goetghebeur, E, le Cessie, S, De Stavola, B, Moodie, EE, Waernbaum, I. Formulating causal questions and principled statistical answers. Statistics in Medicine. 2020; 39: 4922–4948. 2

Defining '(Point-)Treatment'



Oscar-winners live longer

X = binary (point-) treatment

Well-defined?

Beware of **immortal-time bias**:

X = 'did patient ever receive drug ABC? (yes/no)'

→ not a point treatment!

Target trial:

define unique time of eligibility and treatment assignment!

(Total) Causal Effects



In words

Total marginal (or population) effect:

what is the overall effect of intervening in X on Y?

Target trial: randomise X, regression of Y on X

Contrast setting do(X=1) versus setting do(X=0) by some well-defined (but possibly hypothetical) intervention.

(Total) Causal Effects



Can consider subgroups

Total conditional (or subgroup) effect:

what is the overall effect of intervening in X on Y within a subgroup, e.g. women aged 50-60?

Target trial: restrict to subgroup, randomise X; regress Y on X.

Note: subgroups relevant if we expect effect heterogeneity

⇒ nothing to do with confounding!

Finding such subgroups: active research

(Total) Causal Effect



Will focus on:

Formally: average causal effect

$$ACE = E(Y|\mathsf{do}(X=1)) - E(Y|\mathsf{do}(X=0))$$

or, with potential responses

$$ACE = E(Y(1) - Y(0)) = E(Y(1)) - E(Y(0))$$

aka: average treatment (ATE) or total causal effect (TCE), etc.

Key Assumptions

Consistency



Consistency Assumption:

If we observe $X^i = x$ then $Y^i = Y^i(x)$ (for individual i)

i.e. the outcome we observe under the observed treatment is the potential response had the treatment been *set* to what it was observed to be.

Violated if manipulation of X not well defined or so 'invasive' that observational setting not informative.

Example: X is 'BMI' — how to manipulate BMI itself?

Often: if violated, need more elaborate model and suitably detailed data.

Consistency



Under consistency and binary X:

$$Y^{i} = Y(1)^{i}X^{i} + Y(0)^{i}(1 - X^{i})$$

Note:

consistency implicit in graphical / $do(\cdot)$ approaches \rightarrow invariance

No-Interference



Common assumption: no-interference:

Vector $\mathbf{x} =$ treatment values for **all** n units, then $Y^i(\mathbf{x}) = Y^i(x^i)$, i.e. PR does not depend on treatment other units received.

Violation: e.g. vaccines, social networks.

Stable unit-treatment value (SUTVA):

consistency + no-interference.

No-Unmeasured-Confounding



Assumption of no unmeasured confounding:

(aka: random treatment assignment, or cond. exchangeability, ignorability, or \ldots)

Set ${\cal C}$ of observed (measured) pre–treatment covariates exists such that

$$Y(x) \perp \!\!\! \perp X \mid C$$

for all x to be considered as treatment values

Interpretation: within values of C, can consider X like

randomised wrt Y

Denote: *C* is *sufficient* to adjust (control) for confounding;

or 'valid adjustment set'

Pre-Treatment Covariates?



What makes *C* pre–treatment covariates?

 \Rightarrow must be known not to be affected by intervention in treatment X!

Sufficient: C prior in time to X — but not necessary.

Often: C and X contemp. & share themselves common causes through past history, e.g. patient's medical history.

Graphically: C non-descendants of X.

(Overview: methods for causal covariate selection

see Witte & Didelez, 2018)

Positivity Assumption (Overlap)



All methods for effect estimation essentially require

Assumption of positivity:

$$p(x \mid c) > 0$$
 for all $x, c \quad (p(c) > 0)$

Interpretation: for all possible confounder values, it must be possible that a subject receives any value of treatment.

Positivity Assumption — Notes

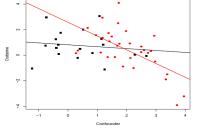


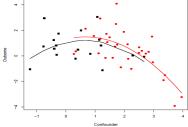
- Positivity can be violated either by coincidence (small sample size), or **structurally**: certain combinations of C and X may not make sense!
- Target trial: can / should define eligibility criteria so that positivity is satisfied — requires domain knowledge
- (Lack of) positivity can be evaluated empirically (look at p(x|c) or p(c|x)) high-dim C becomes challenging Methods exist to characterise 'area of overlap' (Oberst et al, 2020)
- Do not include superfluous variables in C, especially: strong predictors of X that do not affect Y — can lead to apparent lack of positivity despite not being a problem

Positivity — Extrapolation



Regression-based approaches (based on fitting p(y|x,c)) may **mask** lack of positivity as regession models **extrapolate** \Rightarrow can lead to vastly different causal effect estimates





Checking Assumptions?



- Consistency / no-interference: domain knowledge, study design
- No-unmeasured-confounding: compare analysis of observational data with actual randomised trial — Example: HRT-controversy
 Also: negative controls and similar designs
- Positivity (overlap) check:
 - basic: boxplot of each variable in C by treatment group;
 - advanced: consider **propensity score**, i.e. assess P(X=1|C=c) obtain fitted values $\hat{p}^i=\hat{P}(X=1|C=c^i)$ for each unit i, check \hat{p}^i near zero in treated / controls, respectively.

Propensity Score: Checking Positivity

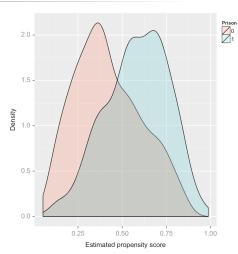


Example:

n=1022 offenders sentenced to either probation X=0or prison X=1; C=17 covariates; Y= recidivism (yes/no);

 \Rightarrow reasonable overlap.

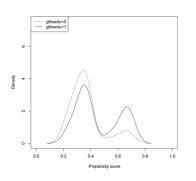
(Example taken from Guo et al., 2016)

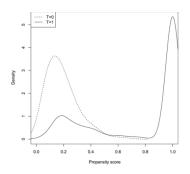


Propensity Score: Checking Positivity



Which is good / bad overlap?



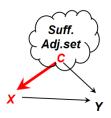


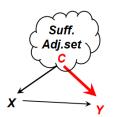
From: Brumback (2021, book)

Methods of Estimation



Based on treatment model p(x|c) (propensity score) or outcome model p(y|x,c) or both





Principles:

- regression + standardisation,
- inverse-probability weighting (IPTW),
- stratification / matching,
- hybrid: doubly-robust estimation (double-ML)

Regression + Standardisation



aka: G-Formula (Robins, 1986)

Reminder: if *C* is sufficient set of covariates

$$E(Y \mid \mathsf{do}(X = x)) = \sum_{c} E(Y \mid x, c) p(c)$$

An obvious way to use this is:

- fit flexible regression model for $E(Y|x,c;\beta)$ to data
- average over empirical C-distribution: $\sum_i E(Y|x,c^i;\hat{\beta})/n$
- R package stdReg (Sjolander and Dahlqwist, 2021)

Standardisation — **Example**



```
Y = \text{`low birth weight' (binary); } X = \text{`mother smokes' (binary),}  C = \{\text{`age', `race'}\}  (Sjolander, 2016)
```

> fit2 <- glm(formula=lbw~(smoker+race+age)^2,
family="binomial", data=clslowbwt)flexible outcome model</pre>

Standardisation — Example



```
Y = 'low birth weight' (binary); X = 'mother smokes' (binary),
C = \{\text{'age'}, \text{'race'}\}
                                                         (Sjolander, 2016)
> fit2 <- glm(formula=lbw~(smoker+race+age)^2,
    family="binomial", data=clslowbwt)flexible outcome model</pre>
 > fit.std <- stdGlm(fit=fit2, data=clslowbwt, X="smoker",</pre>
    clusters="id")
                                             standardised means
                                         control / treatment groups
 > summary(fit.std)
   Estimate Std. Error lower 95 upper 95
       0.279 0.0406 0.199 0.358
   0.407 0.0555 0.298 0.516
```

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   0.279 0.0406 0.199 0.358
 1 0.407 0.0555 0.298 0.516
 > summary(fit.std, contrast="difference", reference=0)
   Estimate Std. Error lower 95 upper 95 ostimated ACE
                                                 difference, i.e.
```

Regression, but Why Standardisation?



Why not just look at coefficient ('effect') of X in a regression model for E(Y|X,C)?

- Marginal effect sensible summary also with arbitrary interactions / complex models
- Contrast of marginal E(Y | do(X = x)) corresponds to randomised trial where covariates C can be / are ignored
- Further issue: non-collapsibility! logistic regression / odds ratios not collapsible.
 - If set of sufficient covariates C not unique, cond. effects may depend on choice of C, but not marginal ones.

Regression + Standardisation



- Consistency (asy. unbiasedness) of estimation relies on correctly specified model for p(y|x,c).
- Danger of extrapolation: it can happen that the regression relation p(y|x,c) is determined primarily by treated subjects in one region of C and control subjects in another...
- ... should not happen under positivity must be checked!

Regression + Standardisation



- The method is special case of G-formula for sequential treatments (Robins, 1986).
- Population effect E(Y | do(X = x)) depends on *distribution* p(c) of covariates target population
 - \Rightarrow not necessarily the same in different populations (e.g. age distribution). If p(y|x,c) regarded as 'stable' across populations, then can just replace $\hat{p}(c)$ in the above by different covariate distribution for different populations (e.g. UK versus USA covariate distribution).
 - \Rightarrow 'Transportability'

Partial Dependency Plots



(Zhao & Hastie, 2021)

- Close relation between Friedman's partial dependence plot (PDP) for visualising black-box prediction methods and back-door adjustment / standardisation
- Under the causal assumptions, can interpret PDP like ACE even for continuous treatment \approx standardising over adjustment set C
- But: positivity hard to justify for entire range of a continuous treatment...
- PDP as basis for estimation of total causal effect very unstable and erratic asymptotic behaviour
 - ⇒ double-machine-learning! (later)

Partial Dependency Plots

Znibniz

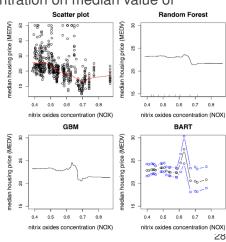
(Zhao & Hastie, 2021)

Effect of nitrix oxides concentration on median value of

owner-occupied homes

'adjusted' for: crime rate, prop. residential/industrial zones, av. # of rooms per dwelling, age of the houses, distance to city / highways, pupil-teacher ratio, % of blacks and % of lower class

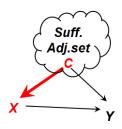
— valid adjustment set?



Treatment Modelling Approaches



The following methods are all based on models for X given C instead of modelling Y given C.



Marginal Structural Models (MSMs)



The adjustment formula

$$\sum_{c} p(y|x,c)p(c) \quad \text{ or } \quad \sum_{c} E(Y|x,c)p(c)$$

might be 'awkward', E(Y|x,c) non–linear with interactions, or C high dimensional and/or partly continuous.

- \Rightarrow Parameterise E(Y | do(X = x)) itself?!
- ⇒ Marginal structural models (MSM)
- ⇒ fitted by inverse probability of treatment weighting (IPTW)

Marginal Structural Models



(Hernán et al, 2001)

MSM: semiparametric model for

$$p(y | \operatorname{do}(X = x))$$
 or more typically $E(Y | \operatorname{do}(X = x))$

e.g. linear, logistic, CoxPH, loglinear, probit etc.

Marginal: refers to time-varying covariates \rightarrow not covered *Structural:* model under intervention in X (not observational)

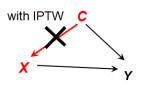
Note: term 'structural' is used in many different ways — here it always refers to modelling the <u>underlying causal</u> relationships.

Inverse Probability Weighting (IPTW)



Idea is based on:

$$\begin{aligned} p(y \mid \mathsf{do}(X = x)) \\ &= \sum_{c} p(y | x, c) p(c) \\ &= \sum_{c} \frac{p(y, x, c)}{p(x \mid c)} \end{aligned}$$



In words: the population is re-weighted so that X becomes independent of \mathcal{C} .

Covariate balance check:

success of re-weighting can be assessed empirically in data!

Inverse Probability Weighting (IPTW)



Idea is based on:

$$p(y | do(X = x)) = \sum_{c} p(y|x, c)p(c) = \sum_{c} \frac{p(y, x, c)}{p(x|c)}$$

- \Rightarrow fit MSM with individuals' weights $w^i = p(x^i|c^i)^{-1}$
- \Rightarrow creates 'pseudo sample' in which C is not confounding
- \Rightarrow unbiased estimating equations for parameters of MSM $E(Y | do(X = x); \beta)$

Here, $w^i = p(x^i|c^i)^{-1}$ is the inverse of the probability that individual i receives 'treatment' x^i given they have covariates c^i .

IPTW Estimator



Define $\pi(c) = P(X = 1 | C = c)$ — propensity score.

Can show (under our assumptions):

$$E\left(\frac{X}{\pi(C)}Y\right) = E(Y\mid \operatorname{do}(X=1))$$

and similarly

$$E\left(\frac{1-X}{1-\pi(C)}Y\right) = E(Y \mid \mathsf{do}(X=0))$$

Proof: iterated conditional expectation.

IPTW Estimator



With model $\pi(C; \alpha) \Rightarrow \text{plug-in } \pi(C; \hat{\alpha})$

IPTW yields consistent estimator for ACE

- if $\pi(C; \alpha)$ correctly specified
- can obtain sandwich standard errors or bootstrap, or theoretical asymptotical standard errors
- IPTW often large variance, wide CIs reflects lack of information in areas with 'extreme weights'
- ... extreme weights indicate possible near violation of positivitiy
- Solution: restriction of relevant population and / or truncation of weights (e.g. at 99%-percentile).

MSM / IPTW — Implementation



Easy to implement with standard software for regression models by specifying weights

Note: default standard errors ignore variability in (estimated!) weights

Notes on IPTW



- Consistent when both, models for $E(Y | \operatorname{do}(X = x))$ and $\pi(c)$ correctly specified...
- ... avoids modelling of Y-C relation
- Extension: 'overlap weights' new estimand with most weights on subpopulations with higher overlap
- IPTW especially useful when study design (or other) supplies background knowledge to model weights p(x|c).
- Problem: estimation of weights $p(x|c)^{-1}$ not obvious when X continuous.

Notes on IPTW

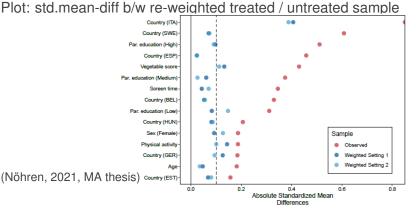


- MSM mimics a model for Y given X in the situation of a trial where X was randomised
- Even if X actually randomised, there may be changes to treatment status over time (non-adherence) — this then becomes a problem of time-dependent treatment
- MSMs with IPTW mostly used in longitudinal situations / time-dependent / sequential treatments with time-varying confounding
 - \Rightarrow 'marginal' over time-dep. confounders / covariates.

Checking Assumptions: Balance



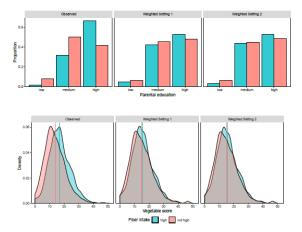
Example: causal effect of 'fibre intake' on children's BMI; large adjustment set (country, parental edu, vege-score, etc.)



Checking Assumptions: Balance



Example(ctd.), checking balance of whole distribution of covariates



Double Robustness (DR)



(Robins & Rotnitzky, 2001)

- Regression-standardisation relies on correct outcome model
- MSM / IPTW relies on correct treatment model
- Danger (especially with high-dim C): models will be 'misspecified'
 - ⇒ want to fit them data-adaptively
 - ⇒ known to yield unstable (irregular) effect estimators!
 - ⇒ Better: double-machine learning of causal effects based on doubly-robust estimation.

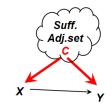
Double Robustness (DR)



(Robins & Rotnitzky, 2001)

Can find class of augmented IPTW (AIP(T)W) estimators:

consistent for ACE if



- either model $\pi(C;\alpha) = p(X=1 \mid C;\alpha)$ correctly specified or
- model for $p(y \mid x, c; \beta)$ correctly specified;
- but one of them can be wrong.

AIPTW Estimator



Basic Idea

Still binary exposure XFor *arbitrary* functions m(C) and $\pi(C)$, define

$$\hat{\mu}_1 = m(C) + \frac{X}{\pi(C)}(Y - m(C)) = \frac{X}{\pi(C)}Y + \left[1 - \frac{X}{\pi(C)}\right]m(C)$$

Property (under our assumptions): if

either
$$m(C) = E(Y \mid X = 1, C)$$
 or $\pi(c) = P(X = 1 | C = c)$, then

$$E(\hat{\mu}_1) = E(Y \mid \mathsf{do}(X=1))$$

Analogously for $\hat{\mu}_0$ and $E(Y \mid \text{do}(X=0))$.

AIPTW Comments



- Kang and Shafer (2007) find: AIPTW with parametric models can be quite bad if both models slightly misspecified → much research on improvements
- NEW: Statistical properties of doubly-robust estimators allow the use of machine learning for treatment and outcome models
 - ⇒ DR minimises slow convergence rates / overfitting typical for machine learning with sample-splitting or cross-fitting
- AIPW R package (Zhong et al., 2021) or npcausal R package (Kennedy, 2021)

Super Learner??



- To fit m(C) and $\pi(c)$ can use data-adaptive methods developed for prediction!
 - AIPW package uses the Super Learner
 - ... an ensemble method allowing combination of several prediction algorithms into one
 - ... uses k-fold cross-validation to build the optimal weighted combination of predictions from a library of candidate algorithms
 - choice of library quite important (active research)
- double machine learning methods avoid strong modeling assumptions
 - ... and can still achieve optimal \sqrt{n} rate of convergence for causal effect estimation under *some* conditions

Propensity Score (PS) — Other Usage



(Rosenbaum & Rubin, 1983)

Have used
$$\pi(c) = P(X = 1 | C = c)$$

 \Rightarrow also known as **propensity score**.

Note: $\pi := \pi(C)$ is random variable.

MSM: used $\pi(C)$ for weighting.

But: can also use $\pi(C)$ for adjustment-type approaches, due to it being a balancing score...

Propensity / Balancing Score



(Still assuming: C sufficient set of covariates; X binary.)

Use of propensity scores (vs. IPTW) is based on

$$X \perp\!\!\!\perp C \mid \pi$$
 i.e. π balances C

Hence (with properties of *C*):

$$Y(x) \perp \!\!\! \perp X \mid \pi$$

making π a minimal sufficient reduction of C (univariate $\in [0,1]$).

Propensity Score — Graphically



Propensity score $\pi:=\pi(C)=P(X=1|C)$ satisfies these conditional independencies:





Left: assumption of C being sufficient set of covariates.

Right: π is deterministic function of C and $X \perp \!\!\! \perp C \mid \pi$.

Propensity Score in Practice



- Estimate propensity score $\hat{\pi}$ with model for $\pi(C; \alpha)$.
- Required: correctly specified model $\pi(C; \alpha)$. Non-parametric approaches: random forests etc.
- Note: predictive quality of π(C; α) for X not important because need C to be (X, Y)-confounders, not nec. strong predictors of X.

In fact: strong X-predictors \Rightarrow bias amplification (Pearl, 2011).

- Check balancing property (see IPTW).
- Check positivity / overlap ⇒ if necessary: restrict / prune!

Propensity Score in Practice



Methods for using PS (other than weighting):

- PS stratification: divide into strata (often quintiles) and fit p(y|x) to each stratum separately.
 (Strata specific effects can reveal effect modification.)
 Then weighted average to obtain overall population effect.
- Alternative: matching on propensity score, i.e. match each treated with k untreated with similar propensity score this estimates effect of treatment on the treated (ETT)!
- Sometimes: **PS adjustment** specify model for $p(y|x,\pi)$ and fit with $\hat{\pi}$ plugged in.
- Extrapolation is automatically avoided.

Survival of Cancer Patients Example



US National Cancer Institute's SEER data base; observational study. Covariates: year of diagnosis, tumor size, geogr. registry, race, marital status

Propensity score	Treatment	No.	5-Year-Surv.	Difference
1st quintile	Α	56	85.6%	
	В	1008	86.7%	-1.1%
2nd quintile	Α	106	82.8%	
	В	964	83.4%	-0.6%
3rd quintile	Α	193	85.2%	
	В	866	88.8%	-3.6%
4th quintile	Α	289	88.7%	
	В	978	87.3%	1.4%
top quintile	Α	462	89.0%	
	В	604	88.5%	0.5%

Overall estimated (weighted average) ACE = -0.68.

From strata specific results: slight suggestion that treatment B is better for those who are more likely to receive it.

Notes on Propensity Score



- Best with binary treatment / exposure.
- PS stratification consistent if $\pi(C;\alpha)$ correctly specified, but can be markedly biased due to residual confounding within strata possible.
- Consistency can be achieved by increasing number of strata when sample size is 'large' or by additional modelling of $E(Y|C=c;\operatorname{do}(X=x))$ within strata.
- PS popular especially for matching: π is 'one-dimensional reduction' of covariates but at cost of first modelling / estimating $\pi = p(x|c;\alpha)$.
- PS matching / stratification not really suitable for sequential treatments.

Notes on Propensity Score



- Danger: modelling $\pi(C; \alpha)$ may focus on strong predictors of $X \Rightarrow$ can amplify bias! \Rightarrow selection of C as adjustment set should be separate process from fitting $\pi(C; \alpha)$.
- Interpretation of PS-analyses sometimes regarded as difficult compared to actual covariate values.
- Simulations suggest that IPTW with π superior to stratification. (Lunceford & Davidian, 2004)
- Critique of PS-matching: King & Nielsen (working paper)

Estimating Causal Effects



Summary (no unobs. conf.)

Given suff. adjustment set C (& other structural asspts):

- Traditional: regression adjustment to be supplemented by...
- ... standardisation to obtain population effect (g-formula in time-varying context) – underused in practice
- or MSMs fitted by IPTW easy to use, also with time-varying data – but can be inefficient
- propensity score methods (stratification / matching) overused?
- Combination leads to *doubly robust* estimation procedures
 promising new methods use double-machine learning
- Always check positivity/overlap & balance with all methods!

Validation?



- Causal (counterfactual) conclusions from observational data cannot be validated on that same data!
 - Can check balance on observed confounders, but more important for unobserved factors
 - Need: experimental validation on different / new (ideally experimental) data → some example tomorrow
- Recent studies: compare randomised trials with real-world (observational) studies
- ⇒ Often: evidence that much bias is due to inappropriate analysese, more than to lack of randomisation

Validation?



- Helpful: compare very different methods for estimating the same estimand
 - ⇒ if not in agreement some assumptions are violated
- Sometimes: different study designs can be used to check if same conclusion is obtained
 - Natural experiments, pragmatic trials ...
- Negative controls: similar exposure or similar outcome with same source of confounding but known zero-effect => assess unobserved confounding
- Instrumental variables topic of its own...

Quantitative Bias Analysis



- Can investigate:
 - "How much would our conclusions change if there was an unobserved confounder with certain properties"
 - ⇒ Sensitivity / bias analysis (Lash et al, book)
- Formal approaches: based on Bayesian models (Greenland, Handbook Epidemiology chapter!)
- Ad-hoc method: E-value (Ding & Vanderweele)
 "How strongly must an unobserved confounder be associated with X and Y to explain away the causal effect (in the worst case)?"

Extensions / Outlook



- Multiple treatments (X has more than two levels), continuous treatments (positivity?)
- Different interventions: nudging / shift-interventions
- Different estimands: effect of treatment on the treated, (in)direct effects, 'principal-stratum' effect etc.
- Different outcomes: survival / time-to-event (censoring), multivariate outcomes
- Sequential / time-dependent treatments (dealing with 'switching', 'when-to-start?')
 - → time-dependent confounding!
- Effect heterogeneity, individualised / adaptive / optimal treatments — (optimal) dynamic treatments