



## EPC BEC303 Module 1 Guruprasad K N Notes

Electronic principles and circuits (Visvesvaraya Technological University)



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MODULE 1:Transistor Biasing

Biasing a transistor means applying DC voltages for keeping the transistor in the desired region of operation.

The purpose of biasing a transistor is setting particular values of voltages and currents for proper operation of transistor.

These value of voltage and current define the point at which transistor operates and this point is known as quiescent point or Q-point.

Voltage Divider Bias (VDB)

Figure below shows voltage divider bias circuit. It is most widely used biasing circuit.

The name voltage divider comes from the fact that, the base circuit contains a voltage divider  $R_1$  and  $R_2$  across the supply voltage  $V_{CC}$ .

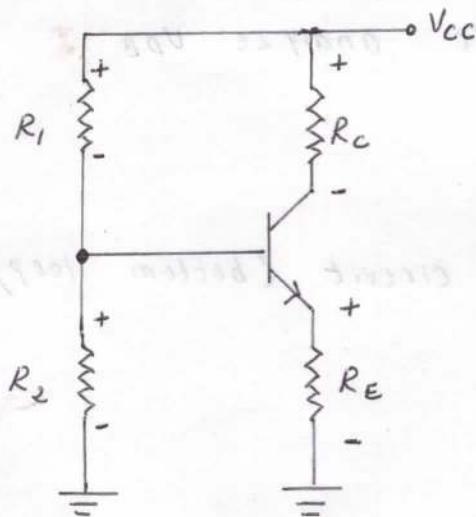


Fig : Voltage divider Bias

Simplified analysis

In any VDB circuit, the base current is much smaller than the current through the voltage divider.

Hence base current has negligible effect on the voltage divider.

Therefore the equivalent circuit can be drawn by opening the connection between voltage divider and base.

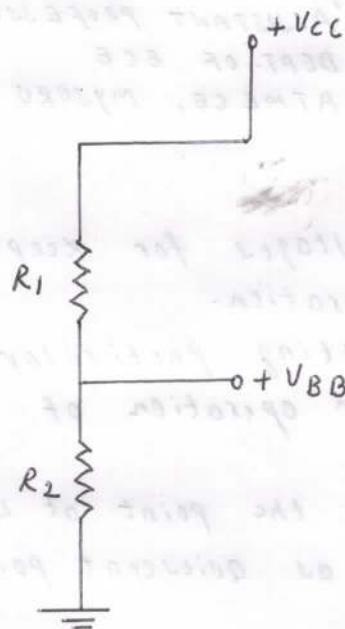
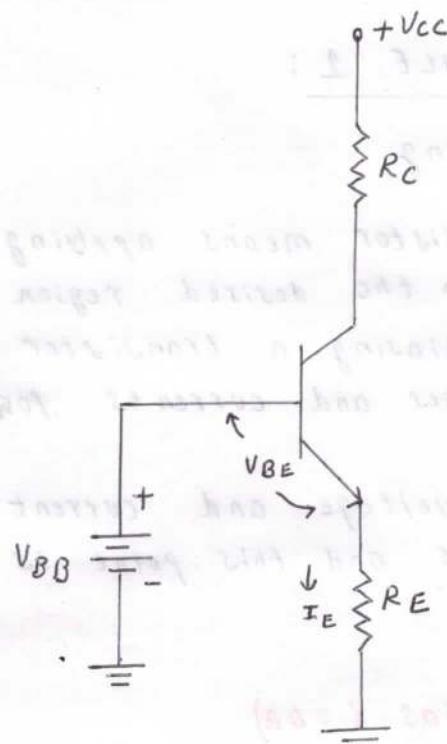


Fig: voltage divider



The output of the voltage divider is given by

$$V_{BB} = V_{CC} \cdot \frac{R_2}{R_1 + R_2}$$

Fig: simplified circuit

Following are the equations used to analyze  $V_{DB}$

$$1. \quad V_{BB} = V_{CC} \cdot \frac{R_2}{R_1 + R_2}$$

2. Applying KVL to simplified circuit (bottom loop)

$$-V_{BB} + V_{BE} + I_E R_E = 0$$

$$-V_{BB} + V_{BE} + V_E = 0$$

$$V_E = V_{BB} - V_{BE}$$

$$3. \quad \text{Emitter current} \quad I_E = \frac{V_E}{R_E}$$

$$4. \quad \text{Collector current} \quad I_C \approx I_E$$

$$5. \quad \text{Collector voltage} \quad V_C = V_{CC} - I_C R_C$$

$$6. \quad \text{Collector Emitter voltage} \quad V_{CE} = V_C - V_E$$

Following are the steps in the analysis.

1. Calculate the base voltage  $V_{BB}$  out of the voltage divider

$$V_{BB} = \frac{V_{CC} \cdot R_2}{R_1 + R_2}$$

2. Subtract 0.7 V to get emitter voltage (use 0.3 V for germanium)

Applying KVL to simplified circuit

$$-V_{BB} + V_{BE} + I_E R_E = 0$$

$$-V_{BB} + V_{BE} + V_E = 0$$

$$V_E = V_{BB} - V_{BE}$$

3. Divide by the emitter resistance to get the emitter current.

$$I_E = \frac{V_E}{R_E}$$

4. Assume that the collector current is approximately equal to the emitter current.

$$I_C \approx I_E$$

5. Calculate the collector-to-ground voltage by subtracting the voltage across the collector resistor from the collector supply voltage.

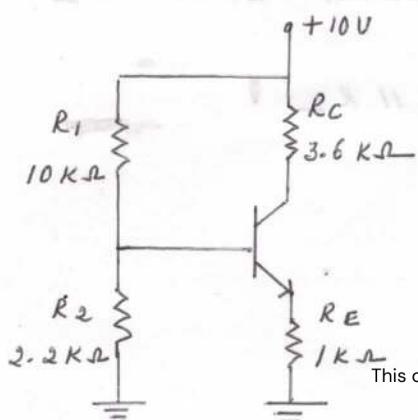
$$V_C = V_{CC} - I_C R_C$$

6. Calculate the collector-emitter voltage by subtracting the emitter voltage from the collector voltage.

$$V_{CE} = V_C - V_E$$

### Problem 1.1

What is the collector-emitter voltage in below figure.



Solution:

$$\text{Given : } V_{CC} = 10V$$

$$R_1 = 10k\Omega$$

$$R_2 = 2.2k\Omega$$

$$R_C = 3.6k\Omega$$

$$V_{BB} = \frac{V_{CC} \cdot R_2}{R_1 + R_2} = \frac{2.2K \cdot 10V}{10K + 2.2K} = 1.8V$$

$$V_E = V_{BB} - V_{BE} = 1.8V - 0.7V = 1.1V$$

The emitter current  $I_E = \frac{V_E}{R_E} = \frac{1.1}{1K} = 1.1mA$

$$I_C = I_E = 1.1mA$$

$$V_C = V_{CC} - I_C R_C = 10 - (1.1mA \times 3.6K) = 6.04V$$

$$V_{CE} = 6.04 - 1.1 = 4.94V$$

practice problem : change the power supply voltage from 10V to 15V and solve for  $V_{CE}$ .

Answer :  $V_{CE} = 0.8V$

### Accurate VDB Analysis

What is well-designed VDB circuit?

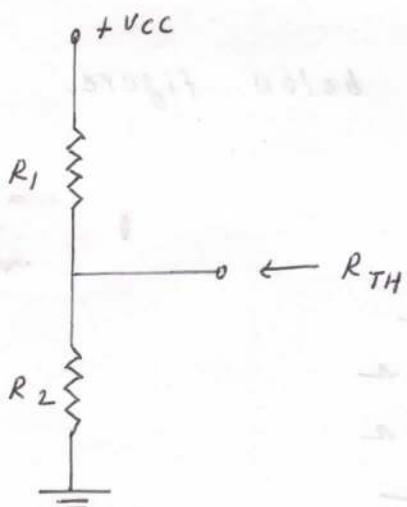
Ans: It is the circuit in which the voltage divider appears stiff to the input resistance of the base.

#### stiff voltage source

When the source resistance is at least 100 times smaller than the load resistance, then source resistance can be ignored. Any source that satisfies this condition is called a stiff voltage source.

i.e., stiff voltage source :  $R_s < 0.01 R_L$

#### Thevenin resistance



since  $R_H$  is parallel with  $R_2$

$$R_{TH} = R_1 || R_2$$

Fig : Thevenin's resistance

A more accurate analysis can be done by including the Thevenin's resistance in the simplified circuit.

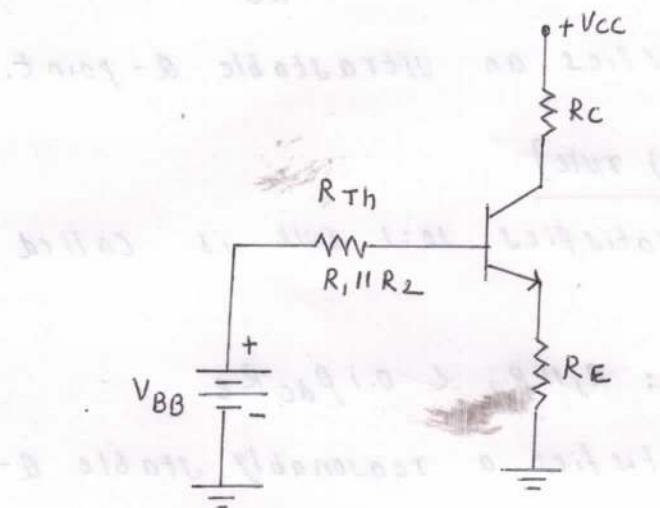
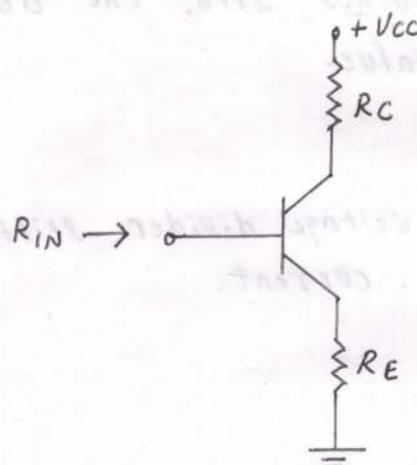


Fig : Equivalent circuit

#### Load Resistance

The voltage divider has to supply the base current in above equivalent circuit. The voltage divider sees a load resistance of  $R_{IN}$  as shown in below figure.



For a voltage divider to appear stiff to the base, it has to satisfy the 100:1 rule :

$$R_S < 0.01 R_L$$

i.e.

$$R_1 || R_2 < 0.01 R_{IN} \rightarrow (1)$$

#### Stiff Voltage divider (100:1 rule)

We have  $I_C = \beta I_B$ , which means the collector current is  $\beta$  times greater than base current.

Since  $I_C \approx I_E$ , it implies, the emitter current is also  $\beta$  times greater than base current.

When seen from the base side of the transistor, the emitter resistance appears to be  $\beta$  times greater.

i.e.  $R_{IN} = \beta_{dc} R_E$

Substituting in the above equation in eqn (1)

$$\text{Stiff voltage divider: } R_1 \parallel R_2 < 0.01 \beta_{dc} R_E$$

Note: This 100:1 rule satisfies an ultrastable Q-point.

- Firm voltage divider [10:1 rule]

Any voltage divider that satisfies 10:1 rule is called a firm voltage divider.

$$\text{Firm voltage divider: } R_1 \parallel R_2 < 0.1 \beta_{dc} R_E$$

Note: This 10:1 rule satisfies a reasonably stable Q-point.

- A closer approximation

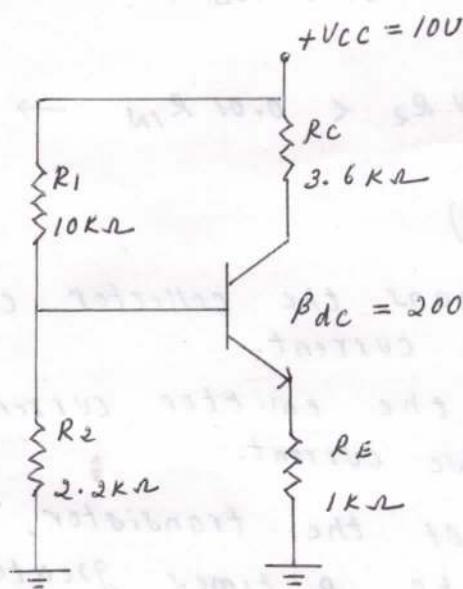
More accurate value for the emitter current is given by

$$I_E = \frac{V_{BB} - V_{BE}}{R_E + (R_1 \parallel R_2) / \beta_{dc}}$$

Note: As the term  $R_1 \parallel R_2 / \beta_{dc}$  approaches zero, the above equation simplifies to the stiff value.

### problem 1.2

Consider the circuit shown below. Is the voltage divider stiff? Calculate more accurate value of emitter current.



solution :

Using  $\approx 100:1$  rule

$$\text{stiff voltage divider: } R_1 \parallel R_2 < 0.01 \beta_{dc}$$

Thevenin resistance is given by

$$\begin{aligned} R_1 \parallel R_2 &= 10 \text{ k}\Omega \parallel 2.2 \text{ k}\Omega \\ &= \frac{(10 \text{ K})(2.2 \text{ K})}{10 \text{ K} + 2.2 \text{ K}} \end{aligned}$$

$$R_1 \parallel R_2 = 1.8 \text{ k}\Omega$$

The input resistance of the base is

$$\beta_{dc} R_E = (200) (1 \text{ k}\Omega) = 200 \text{ k}\Omega$$

$$0.01 \beta_{dc} R_E = 0.01 \times 200 \text{ k} = 2 \text{ k}\Omega$$

since  $1.8 \text{ k}\Omega < 2 \text{ k}\Omega$ , the voltage divider is stiff.  
 $(R_1 || R_2) < (0.01 \beta_{dc} R_E)$

Accurate value of emitter current is given by

$$I_E = \frac{V_{BB} - V_{BE}}{R_E + (R_1 || R_2)/\beta_{dc}} = \frac{1.8 - 0.7}{1 \text{ k}\Omega + (1.8 \text{ k}/200)} = \frac{1.1 \text{ V}}{1 \text{ k}\Omega + 9 \text{ k}\Omega}$$

$$I_E = 1.09 \text{ mA}$$

### V<sub>DB</sub> load line and Q-point

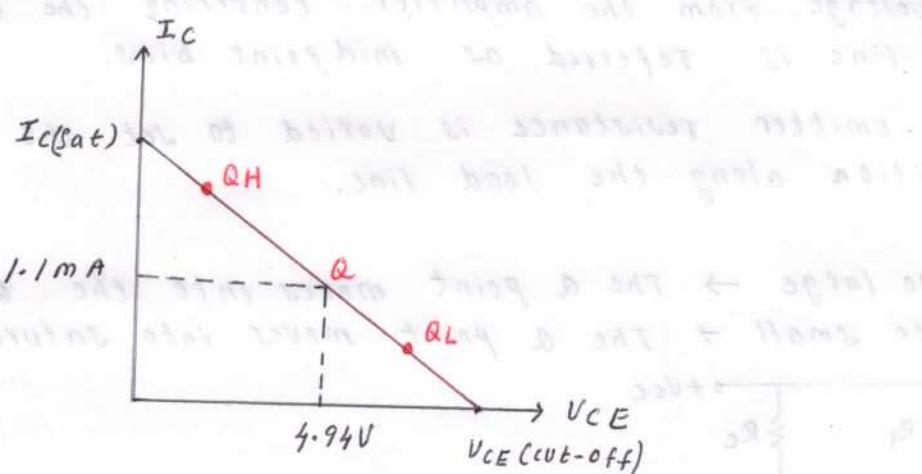
Q-point:

The Q-point is a steady-state DC voltage and current level at which the transistor operates. It determines the amount of power the transistor will use and the level of amplification it will provide. It is also called operating point.

Q-point: ( $I_C$ ,  $V_{CE}$ )

From problem 1.1  $I_C = 1.1 \text{ mA}$  and  $V_{CE} = 4.94 \text{ V}$

In the below figure, above points are shown as Q.



Let us assume, the emitter resistance is changed to  $2.2 \text{ k}\Omega$   
 i.e.  $R_E = 2.2 \text{ k}\Omega$

$$\text{Emitter current } I_E = \frac{V_E}{R_E}$$

$$I_E = \frac{1.1V}{2.2k\Omega} = 0.5 \text{ mA}$$

since  $I_E \approx I_C \Rightarrow I_C = 0.5 \text{ mA}$

$$V_C = V_{CC} - I_C R_C$$

$$= 10 - (0.5 \text{ mA} \times 3.6 \text{ k}\Omega)$$

$$V_C = 8.2 \text{ V}$$

$$V_{CE} = V_C - V_E = 8.2 - 1.1 = 7.1 \text{ V}$$

Therefore the new coordinates will be  $Q_L : (0.5 \text{ mA}, 7.1 \text{ V})$   
 $(I_C, V_{CE})$

Now, let us consider, the emitter resistance is decreased to  $510 \Omega$   
 i.e.  $R_E = 510 \Omega$

$$I_E = \frac{V_E}{R_E} = \frac{1.1V}{510\Omega} = 2.15 \text{ mA}$$

$$\text{since } I_C \approx I_E \Rightarrow I_C = 2.15 \text{ mA}$$

$$V_C = V_{CC} - I_C R_C = 10 - (2.15 \text{ mA} \times 510) = 2.26 \text{ V}$$

$$V_{CE} = V_C - V_E = 2.26 - 1.1 = 1.16 \text{ V}$$

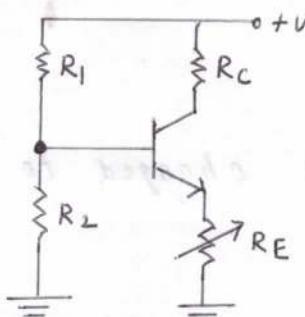
Now the  $\alpha$ -point shifts to new position at  $Q_H$   
 with coordinates  $Q_H : (2.15 \text{ mA}, 1.16 \text{ V})$

### $\alpha$ -point in middle of load line

Centering the  $\alpha$ -point on a transistor load line allows maximum AC output voltage from the amplifier. Centering the  $\alpha$ -point on the DC load line is referred as midpoint bias.

Normally the emitter resistance is varied to set the  $\alpha$ -point at any position along the load line.

If  $R_E$  is too large  $\rightarrow$  The  $\alpha$ -point moves into the  $\alpha$  cut-off point.  
 If  $R_E$  is too small  $\rightarrow$  The  $\alpha$ -point moves into saturation.



VDB Design Guideline

Emitter voltage is approximately one-tenth of the supply voltage.

$$V_E = 0.1 V_{CC}$$

Calculate  $R_E = \frac{V_E}{I_E}$

Collector resistance

$$R_C = 4 R_E$$

$$V_{CE} = 50\% V_{CC}$$

$$V_{RE} = 10\% V_{CC}$$

$$V_{RC} = 40\% V_{CC}$$

Using 100:1 rule

$$R_{TH} \leq 0.01 \beta_{dc} R_E$$

Usually  $R_2$  is smaller than  $R_1$

$$R_2 \leq 0.01 \beta_{dc} R_E$$

Using proportion

$$\frac{R_1}{R_2} = \frac{V_1}{V_2}$$

$$R_1 = \frac{V_1}{V_2} \cdot R_2$$

problem 1.3

For the circuit shown below, design the resistor values to meet these specifications.

$$V_{CC} = 10V \quad V_{CE} @ \text{midpoint} \quad I_C = 10mA \quad \beta_{dc} = 100-300$$

SOLUTION:

$$V_E = 0.1 V_{CC}$$

$$V_E = (0.1)(10) = 1V$$

$$R_E = \frac{V_E}{I_E} = \frac{1V}{10m} = 100\Omega$$

$$R_C = 4 R_E = (4)(100) = 400\Omega$$

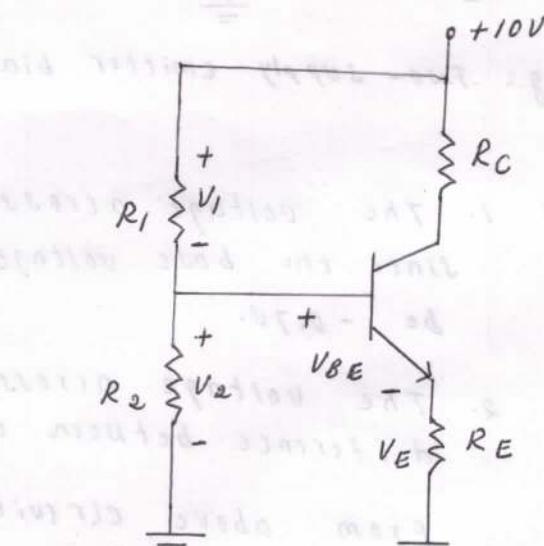
$$R_2 \leq 0.01 \beta_{dc} R_E \\ \leq (0.01)(100)(100) = 100\Omega$$

$$R_1 = \frac{V_1}{V_2} R_2$$

$$\text{Applying KVL} \quad -V_2 + V_{BE} + V_E = 0$$

$$V_2 = V_{BE} + V_E = 0.7 + 1V = 1.7V$$

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Also,

$$-V_{CC} + V_1 + V_2 = 0$$

$$V_1 = V_{CC} - V_2 = 10 - 1.7 = 8.3 \text{ V}$$

$$R_1 = \frac{8.3}{1.7} \times 100$$

$$R_1 = 488 \Omega$$

### TNO supply emitter bias

Some electronic equipment has a power supply that produces both positive and negative supply voltages.

Figure below shows a transistor circuit with two power supplies. The negative supply forward biases the emitter diode. The positive supply reverse biases the collector diode.

Since the circuit is derived from emitter bias, the circuit is also known as TNO-supply emitter bias (TSEB).

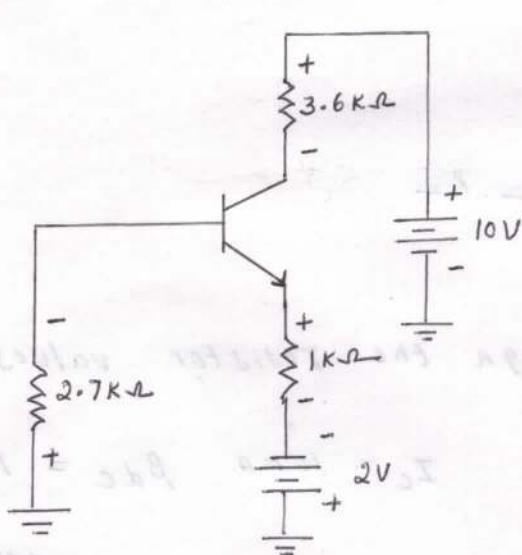


Fig: TNO-supply emitter bias

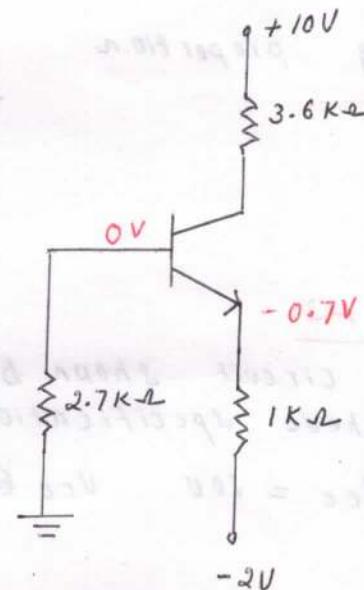


Fig: Redrawn TSEB circuit

Note: 1. The voltage across the emitter diode is 0.7V. Since the base voltage is 0V, the emitter voltage must be -0.7V.

2. The voltage across the emitter resistor equals the difference between the two voltages.

From above circuit diagram:

$$V_{RE} = (-0.7) - (-2V) = 1.3V$$

(Subtract more negative value from more positive value)

Design steps:

Using 100:1 rule,  $R_B < 0.01 \beta_{dc} R_E$   
 $V_B \approx 0$

$$I_E = \frac{V_{EE} - 0.7V}{R_E}$$

$$V_C = V_{CC} - I_C R_C$$

$$V_{CE} = V_C - V_E = V_C - (0.7)V = V_C + 0.7V$$

problem 1.4

for the circuit shown below, find collector-emitter voltage.

solution:

$$V_{RE} = -0.7V - (-2V) = 1.3V$$

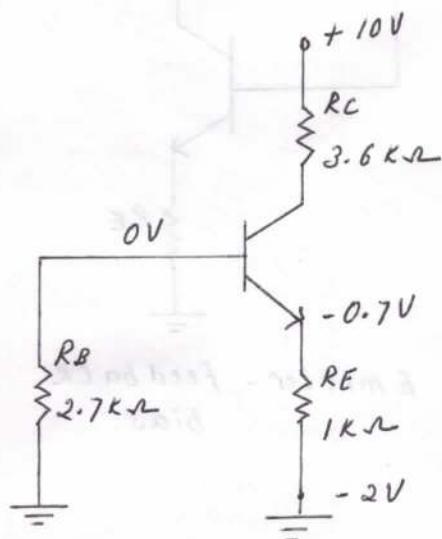
$$I_E = \frac{V_{RE}}{R_E} = \frac{1.3}{1K} = 1.3mA$$

$$V_C = V_{CC} - I_C R_C \\ = 10 - (1.3mA \times 3.6K\Omega)$$

$$V_C = 5.32V$$

$$V_{CE} = V_C - V_E = 5.32 - (-0.7)$$

$$V_{CE} = 6.02V$$

problem 1.5

for the circuit shown in above fig. (problem 1.4), what is the collector voltage, if the emitter resistor is increased to  $1.8K\Omega$ ?

solution:

$$V_{RE} = 1.3V$$

$$I_E = \frac{V_{RE}}{R_E} = \frac{1.3}{1.8K} = 0.722mA$$

The collector voltage is

$$V_C = V_{CC} - I_C R_C \\ = 10 - (0.722mA \times 3.6K\Omega)$$

$$V_C = 7.4V$$

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Other types of Bias (circuits to stabilize the Q-point)1. Emitter - Feedback Bias

Emitter - Feedback bias is used to overcome the disadvantage of base-bias circuit.

Disadvantage of base-bias circuit: The Q-point moves all over the load line with transistor replacement and temperature change.

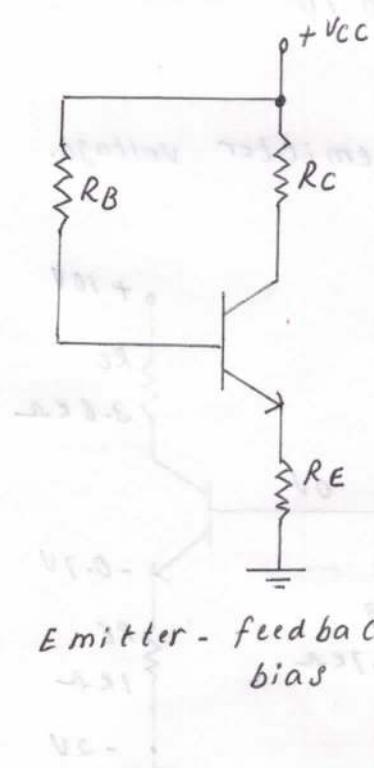


Fig: Emitter - feedback bias

The emitter resistor has been added to the circuit.

The circuit is called feedback because the change in emitter voltage is being fed back to the base circuit.

Equations for analyzing the emitter-feedback bias.

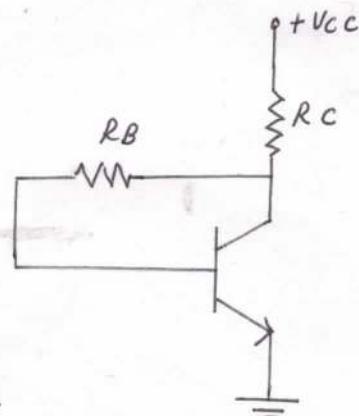
$$I_E = \frac{V_{CC} - V_{BE}}{R_E + R_B/\beta_{DC}} \rightarrow (1)$$

$$V_E = I_E R_E$$

$$V_B = V_E + 0.7V$$

$$V_C = V_{CC} - I_C R_C$$

In eqn (1), if  $R_E > R_B/\beta_{DC}$ , then the circuit will be insensitive to variation in  $\beta_{DC}$ .

2. Collector - Feedback Bias (Self-bias)

Equations for analyzing collector - feedback bias.

$$I_E = \frac{V_{CC} - V_{BE}}{R_C + R_B/\beta_{DC}}$$

$$V_B = 0.7V$$

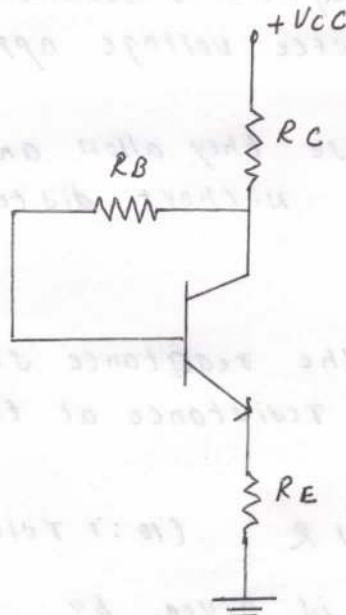
$$V_C = V_{CC} - I_C R_C$$

The Q-point is set near the middle of the load line by using the base resistance of  $R_B = \beta_{DC} R_C$ .

Note : Collector - feedback bias is more effective than the emitter - feedback in stabilizing the Q-point.

### 3. Collector - and Emitter - Feedback Bias

This circuit uses both emitter and collector feedback to improve the operation.



Equations for analyzing

$$I_E = \frac{V_{CC} - V_{BE}}{R_C + R_E + R_B / \beta_{DC}}$$

$$V_E = I_E R_E$$

$$V_B = V_E + 0.7V$$

$$V_C = V_{CC} - I_C R_C$$

### BJT AC MODELS

After the transistor has been biased with the Q-point near the middle of the load line, a small AC voltage can be coupled into the base. This will produce an AC collector voltage.

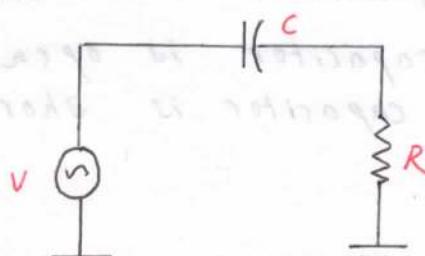
The AC collector voltage is amplified version of the AC base voltage.

### Base biased amplifier

#### Coupling capacitor

A coupling capacitor is a two terminal electrical device which couples or transmits the AC signal to the resistor.

Figure below shows an AC voltage source connected to a capacitor and a resistor.



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$$X_C = \frac{1}{2\pi f C}$$

- Since the capacitive reactance is inversely proportional to frequency, the capacitor effectively blocks DC voltage and transmits AC voltage.
- When the frequency is very high, the capacitive reactance is much smaller and all the AC source voltage appears across the resistor.
- Coupling capacitors are important because they allow an ac signal to couple into an amplifier without disturbing its Q-point.

For a coupling capacitor to work properly, the reactance should be at least 10 times smaller than the resistance at the lowest frequency of operation.

$$\text{i.e. Good coupling : } X_C < 0.1 R \quad (\text{10:1 rule})$$

The magnitude of impedance of fig(a) is given by

$$Z = \sqrt{R^2 + X_C^2}$$

Using 10:1 rule

$$Z = \sqrt{R^2 + (0.1)R^2} = \sqrt{R^2 + 0.01R^2} = \sqrt{1.01R^2} = 1.005R$$

$$Z = R$$

Fig (a) can be replaced by its equivalent circuit as shown in below fig (b)

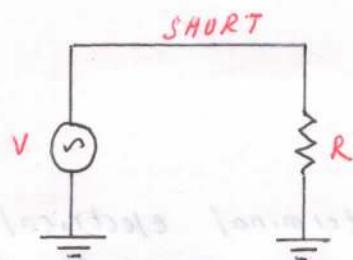


Fig (b) capacitors is an AC short

Following are the two approximations for a capacitor

1. For DC analysis, the capacitor is open
2. For AC analysis, the capacitor is short.

If  $R = 2\text{ k}\Omega$ , and the frequency range is from 20 Hz to 200 kHz, find the value of  $C$  needed to act as a good coupling capacitor.

Solution: Using 10:1 rule,

$$X_C < 0.01R \text{ at } 20 \text{ Hz}$$

$$X_C < 0.01 \times 2\text{k} \text{ at } 20 \text{ Hz}$$

$$X_C < 200\Omega \text{ at } 20 \text{ Hz}$$

We have

$$X_C = \frac{1}{2\pi f C} \Rightarrow C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi \times 20 \text{ Hz} \times 200\Omega}$$

$$C = 39.8 \mu\text{F}$$

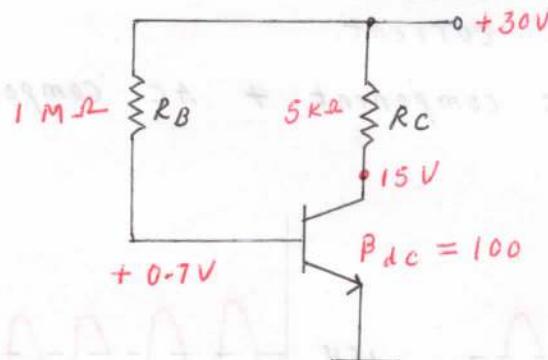
### practice problem

Find the value of  $C$  when the lowest frequency is 1 kHz and  $R$  is 1.6 k $\Omega$ .

$$\text{Ans: } C = 1 \mu\text{F}$$

### DC circuit

Fig. below shows a base-biased circuit.



Base current

$$I_B = \frac{30\text{V}}{1\text{M}} = 30 \mu\text{A}$$

Assume  $\beta = 100$

$$I_C = \beta I_B = 100 \times 30 \mu\text{A} \\ = 3 \text{ mA}$$

$$V_C = V_{CC} - I_C R_C$$

$$V_C = 30 - (3 \text{ mA} \times 5\text{k}\Omega) = 15\text{V}$$

Therefore the Q-point is located at (3 mA, 15 V)

### Amplifying circuit

Figure below shows a base-biased amplifier.

1. A coupling capacitor is used between an AC source and the base.

→ since the coupling capacitor is open to DC current, the DC base current remains same without the capacitor and AC voltage

2. A coupling capacitor is used between the collector and the load resistor.

→ since the capacitor is open to DC, the DC collector voltage remains same with or without capacitor and load resistance.

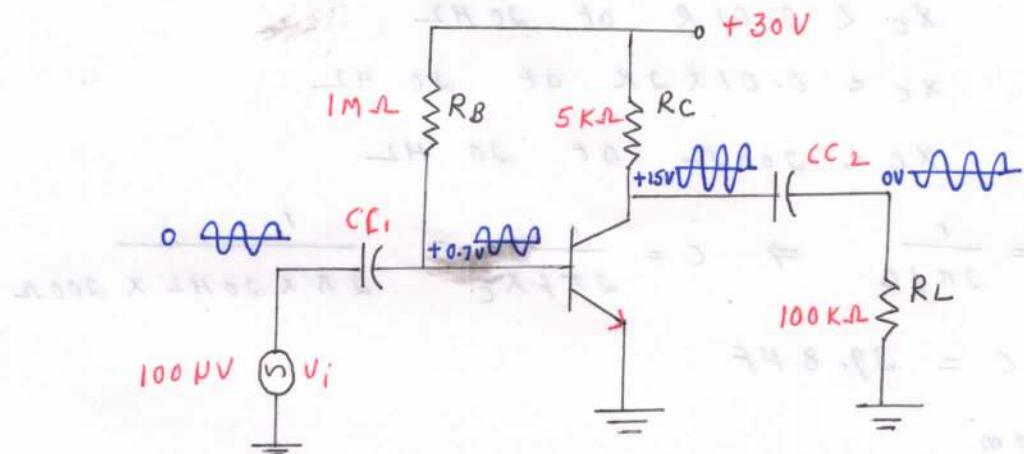


Fig: Base-biased amplifier

3. The coupling capacitors prevent the AC source and load resistance from changing the Q-point.

4. Since the coupling capacitor is an AC short, all the AC source voltage appears between the base and the ground.

5. The AC voltage produces an AC base current that is added to the existing dc base current.

$$\text{i.e. Total base current} = \text{DC component} + \text{AC component}$$

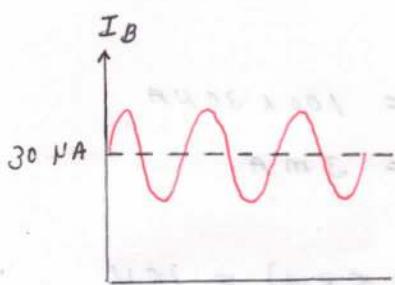


Fig: Base current

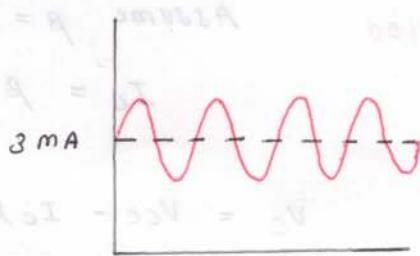


Fig: Collector current

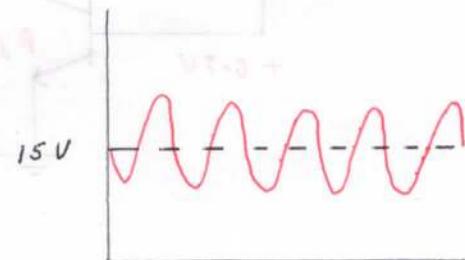


Fig: Collector voltage

### voltage gain

It is defined as the ac output voltage divided by AC input voltage.

$$A_v = \frac{V_{out}}{V_{in}}$$

$$\text{AC input voltage } V_{in} = \frac{V_{out}}{A_v}$$

$$\text{AC output voltage } V_{out} = A_v \cdot V_{in}$$

Problem 1.7

A base biased amplifier has an AC load voltage of 50 mV with an AC input voltage of 100 μV. Find the voltage gain.

Solution: Given

$$V_{out} = 50 \text{ mV}$$

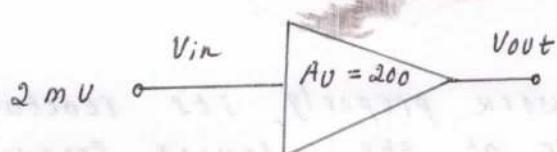
$$V_{in} = 100 \mu\text{V}$$

$$A_V = \frac{V_{out}}{V_{in}}$$

$$A_V = \frac{50 \text{ m}}{100 \mu\text{V}} = 500$$

problem 1.8

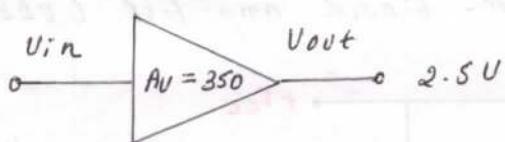
Calculate the output voltage for the circuit shown below



Solution:  $V_{out} = A_V V_{in} = (200) (2 \text{ mV}) = 400 \text{ mV}$

problem 1.9

Find the input voltage for the circuit shown below



Solution:

$$V_{in} = \frac{V_{out}}{A_V} = \frac{2.5}{350} = 7.14 \text{ mV}$$

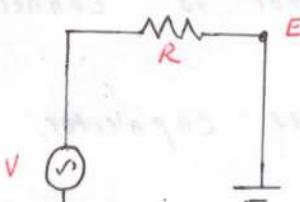
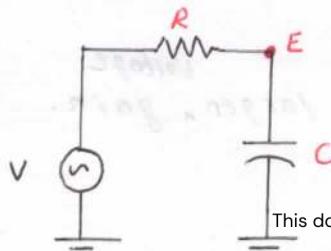
Emitter-Biased Amplifier

The base-biased amplifier has an unstable Q-point. For this reason, it is not used much as an amplifier.

Instead, an emitter-biased amplifier with its stable Q-point is preferred.

Bypass Capacitor

A bypass capacitor is similar to a coupling capacitor because it appears open to DC and shorted to AC. BUT it is not used to couple a signal between two points. Instead, it is used to create an AC ground.



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Fig(a) shows an AC voltage source connected to a resistor and a capacitor. The resistance  $R$  represents Thevenin's resistance.

When the frequency is high, the capacitive reactance is much smaller than the resistance, and the point E is shorted to ground as shown in fig (b)

The capacitor is called a bypass capacitor because it bypasses or shorts point E to the ground. A bypass capacitor creates an AC ground in an amplifier without disturbing the Q-point.

For a bypass capacitor to work properly, its reactance must be smaller than the resistance at the lowest frequency of the AC source.

$$\text{Good bypassing : } X_C < 0.1 R$$

### Amplifying circuit (using VOB circuit)

Figure below shows emitter-biased amplifier (VOB amplifier).

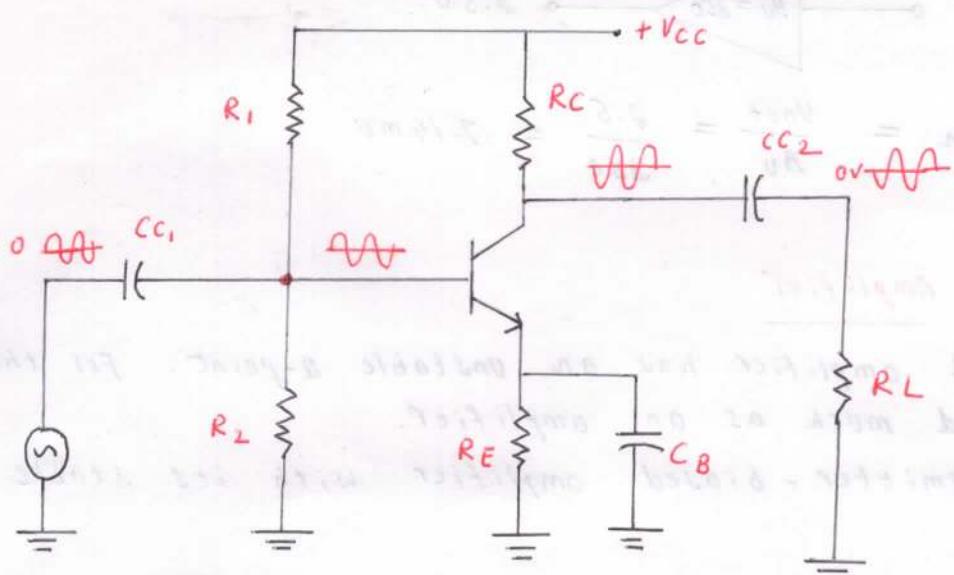


Fig: VOB amplifier with waveforms

In the circuit shown above two coupling capacitors are connected.  
 1) Between source and the base  
 2) Between the collector and the load resistance.

A bypass capacitor is connected between the emitter and the ground.

With the bypass capacitor, we get much larger gain.

problem 1.10

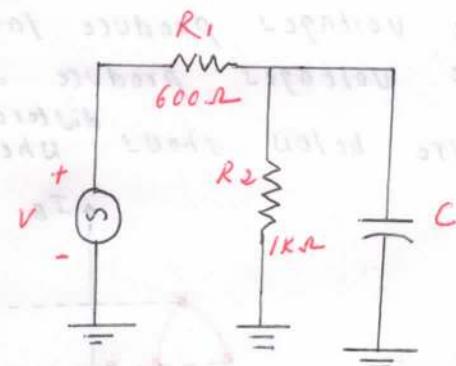
For the circuit shown below, the input frequency of  $V$  is 1 kHz. What value of  $C$  is needed to effectively short point  $E$  to ground?

Solution: Given:

$$R_1 = 600\Omega$$

$$R_2 = 1k\Omega$$

$$R_{TH} = R_1 \parallel R_2 = 600 \parallel 1k = 375\Omega$$



$$X_C < 0.1 R$$

$$X_C < 0.1 \times 375$$

$$X_C < 37.5\Omega$$

$$\text{We have } X_C = \frac{1}{2\pi f C}$$

$$C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi \times 1\text{kHz} \times 37.5}$$

$$C = 4.2 \mu F$$

practice problem

In the above figure, find the value of  $C$  needed if  $R$  is  $50\Omega$

$$\text{Ans: } C = 33 \mu F$$

Amplifying circuit (TSEB circuit)

Figure below shows two supply emitter bias. The circuit has two coupling capacitors and an emitter bypass capacitor.

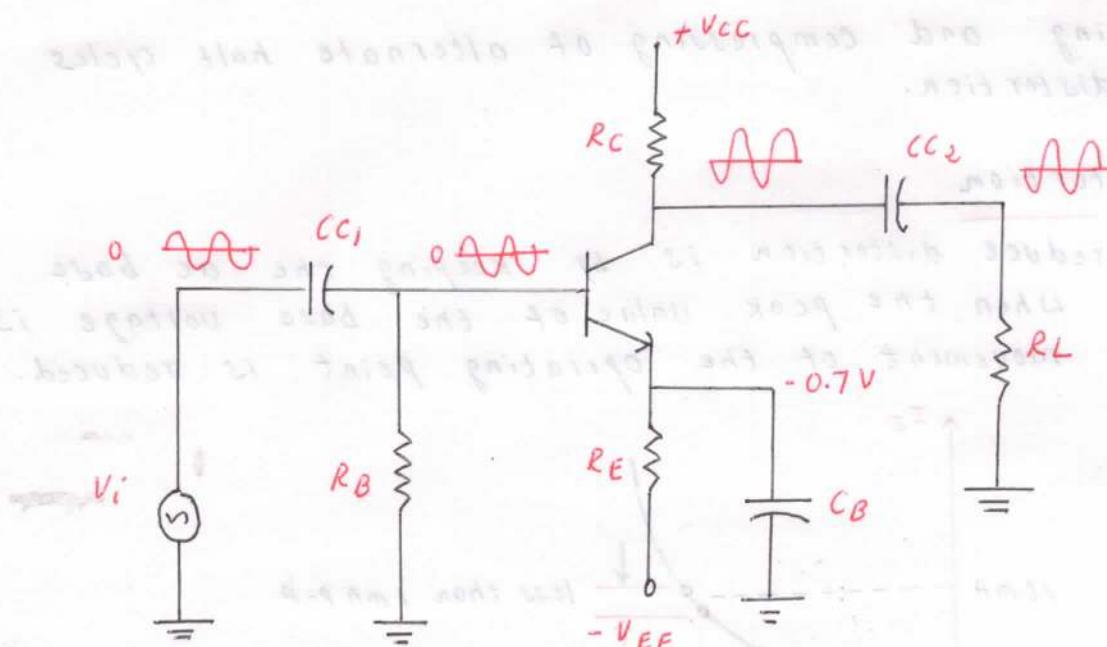


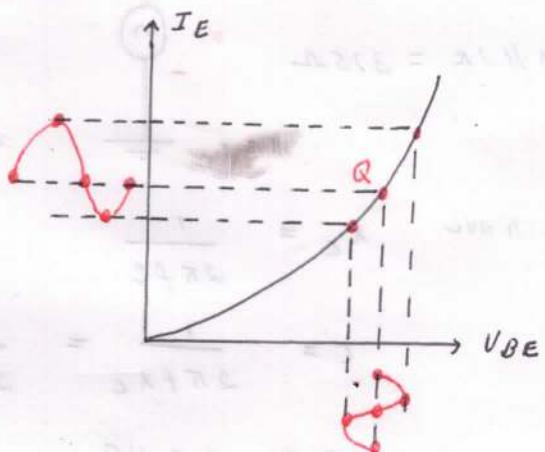
Fig: TSEB amplifier with waveforms

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small signal operation

The size of the AC voltage determines how far the instantaneous point moves away from the Q-point. Large AC base voltages produce large variations, whereas small AC base voltages produce small variations.

Figure below shows distortion when the signal is too large.



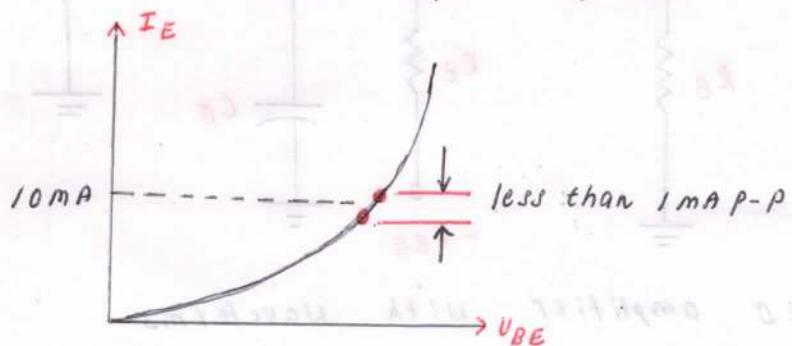
When the voltage increases to its positive peak, the operating point moves from Q to upper point. When the voltage decreases to its negative peak, the operating point moves from Q to the lower point.

The AC emitter current is not a perfect replica of the AC base voltage because of the curvature of the graph. Since the graph is curved upward, the positive half cycle of ac emitter current is stretched and the negative half cycle is compressed.

This stretching and compressing of alternate half cycles is called distortion.

Reducing Distortion

One way to reduce distortion is by keeping the ac base voltage small. When the peak value of the base voltage is reduced, the movement of the operating point is reduced.



Note: The smaller the variation, the less the curvature in the graph. If the signal is small enough, the graph appears to be linear.

### The 10% rule

The total emitter current consists of a dc component and an ac component.

$$I_E = I_{EQ} + i_e \quad \text{where } I_E = \text{the total emitter current}$$

$$I_{EQ} = \text{the dc emitter current}$$

$$i_e = \text{the ac emitter current}$$

To minimize the distortion, the peak-to-peak value of  $i_e$  must be small compared to  $I_{EQ}$ .

$$\text{Small signal: } i_e (\text{P-P}) < 0.1 I_{EQ}$$

### problem 1-11

Find the maximum small signal emitter current if  $-V_{EE} = -2V$  and  $R_E = 1k\Omega$ .

Solution  $I_{EQ} = \frac{V_{EE} - V_{BE}}{R_E} = \frac{2 - 0.7}{1k\Omega} = 1.3 \text{ mA}$

small signal emitter current  $i_e (\text{P-P})$  is given by

$$i_e (\text{P-P}) < 0.1 I_{EQ}$$

$$i_e (\text{P-P}) < (0.1) (1.3 \text{ mA})$$

$$i_e (\text{P-P}) = 130 \mu\text{A P-P}$$

### practice problem:

For the above problem, change  $R_E$  to  $1.5 k\Omega$  and calculate the maximum small signal emitter current.

$$\text{Ans: } i_e (\text{P-P}) = 86.7 \mu\text{A P-P}$$

### current gain

#### Notations:

##### 1. DC Quantities

$I_E$ ,  $I_C$ , and  $I_B$  for the dc currents

$V_E$ ,  $V_C$ , and  $V_B$  for the dc voltages

$V_{BE}$ ,  $V_{CE}$ , and  $V_{CB}$  for the dc voltages between terminals

2. AC quantitiesi<sub>e</sub>, i<sub>c</sub>, and i<sub>b</sub> for ac currentsv<sub>e</sub>, v<sub>c</sub>, and v<sub>b</sub> for the ac voltagesv<sub>be</sub>, v<sub>ce</sub>, and v<sub>cb</sub> for the ac voltages between terminals.DC current gain

$$\beta_{dc} = \frac{I_c}{I_b}$$

Note: The dc current gain depends on the location of the Q-point.

AC current gain

$$\beta = \frac{i_c}{i_b}$$

AC Resistance of the Emitter diode

The total emitter current is given by

$$I_E = I_{Ea} + i_e \rightarrow \begin{matrix} \text{AC emitter current} \\ \downarrow \\ \text{DC emitter current} \end{matrix}$$

The total base-emitter voltage is given by

$$V_{BE} = V_{BEQ} + v_{be} \rightarrow \begin{matrix} \text{AC base-emitter voltage} \\ \downarrow \\ \text{DC base-emitter voltage} \end{matrix}$$

The AC emitter resistance is given by

$$r_e' = \frac{v_{be}}{i_e}$$

problem 1-12

Find the AC resistance of the emitter diode, if AC base-emitter voltage is 5 mVpp and an AC emitter current is 100 nA p-p.

solution

$$\text{Given: } v_{be} = 5 \text{ mV} \\ i_e = 100 \text{ nA}$$

$$r_e' = \frac{v_{be}}{i_e} \\ = \frac{5 \text{ mV}}{100 \text{ nA}} = 50 \Omega$$

Formula for AC Emitter Resistance

$$r_e' = \frac{25 \text{ mV}}{I_E}$$

$$\frac{KT}{q} \Rightarrow K = 1.380 \times 10^{-23} \text{ J/K}$$

$$T = 300 \text{ K}$$

$$q = 1.602 \times 10^{-19} \text{ C}$$

i.e. The ac resistance of the emitter diode equals 25 mV divided by the DC emitter current.

$$\frac{KT}{q} \approx 25 \text{ mV}$$

The Transistor Model

To analyze the AC operation of a transistor amplifier, an AC equivalent circuit for a transistor is needed. The model simulates how it behaves when the AC signal is present.

The T Model

This model is also known as Ebers-Moll Model.

Here the emitter diode of a transistor acts like an AC resistance  $r_e'$ , and the collector diode acts like a current source  $i_c$ .

Since the model looks like a T on its side, the equivalent circuit is also called the T-Model.

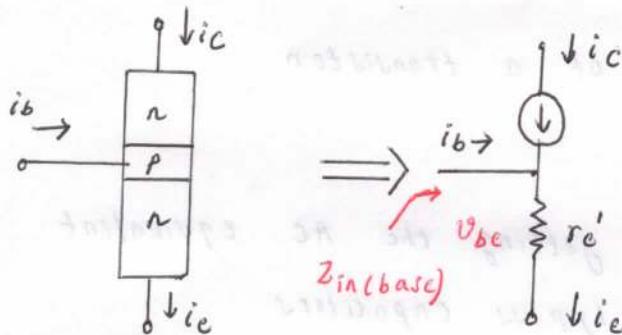


Fig: T Model of a transistor

When an AC input signal is applied to a transistor amplifier, an AC base-emitter voltage  $v_{be}$  is developed across the emitter diode. This produces an AC base current  $i_b$ .

The input impedance is given by

$$Z_{in(base)} = \frac{v_{be}}{i_b} \rightarrow (1)$$

Applying Ohm's law to the emitter diode

$$v_{be} = i_e r_e' \rightarrow (2)$$

Substituting eqn(2) in eqn (1)

$$Z_{in(base)} = \frac{v_{be}}{i_b} = \frac{i_e r_e'}{i_b}$$

since  $i_e \approx i_c$

$$Z_{in(base)} = \frac{i_c r_e'}{i_b} = \beta r_e'$$

Above equation indicates, the input impedance of the base is equal to the ac current gain times the ac resistance of the emitter diode.

### The $\pi$ Model

Figure below shows the  $\pi$  Model of a transistor. The  $\pi$  model is easier to use than the  $T$  model.

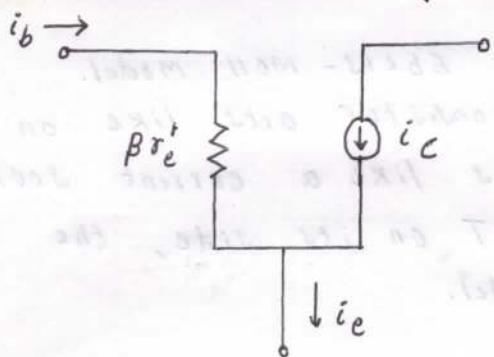


Fig:  $\pi$  model of a transistor

### Analyzing an Amplifier

Following are the steps in getting the AC equivalent circuit

1. Short all coupling and bypass capacitors
2. visualize all dc supply voltages as AC grounds
3. Replace the transistor by its  $\pi$  or  $T$  model
4. Draw the AC equivalent circuit.

### 1. Base-Biased Amplifier

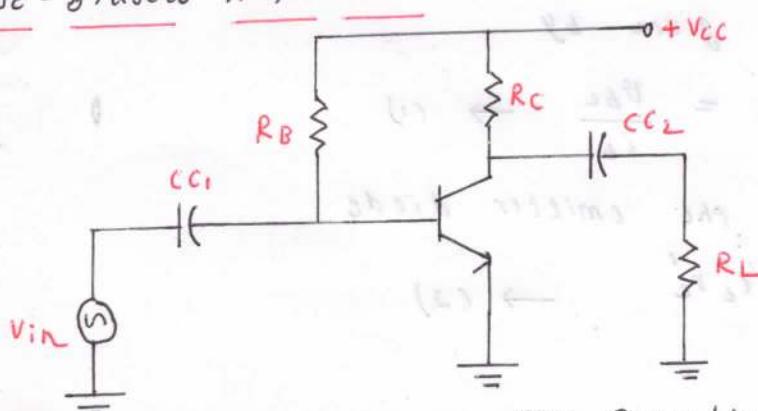


Fig: Base-biased amplifier

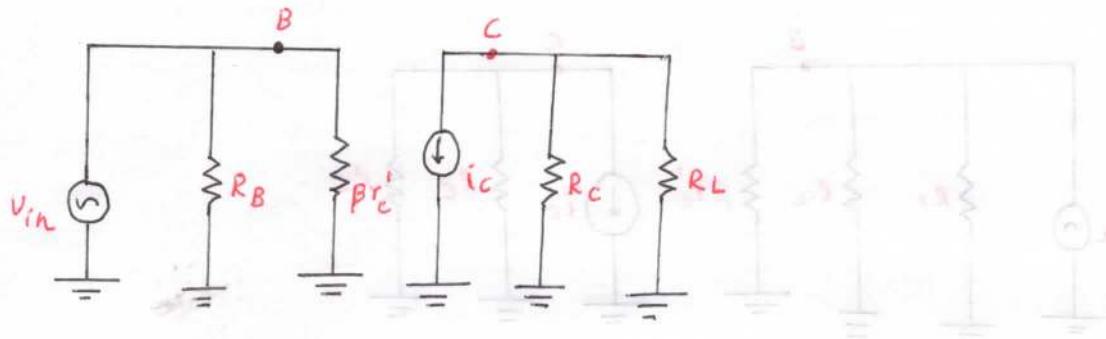
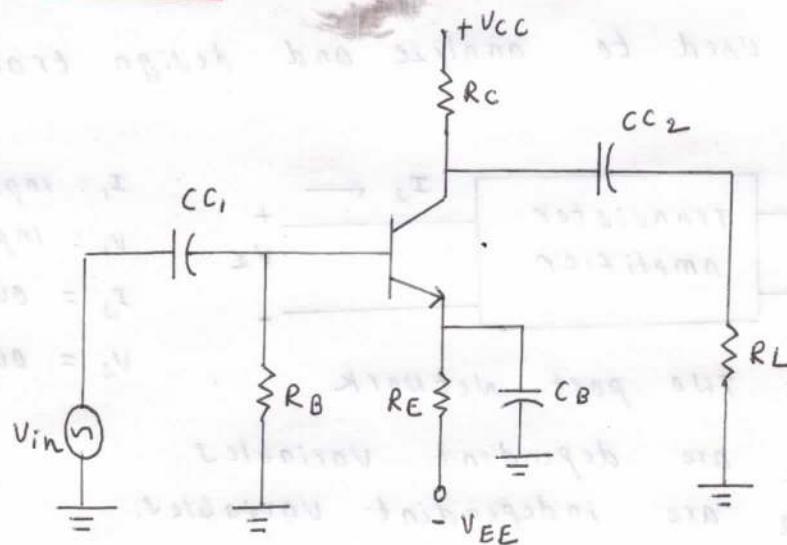


Fig (b) AC EQUIVALENT CIRCUIT

2. TSEB Amplifier

Fig(a) : TSEB Amplifier

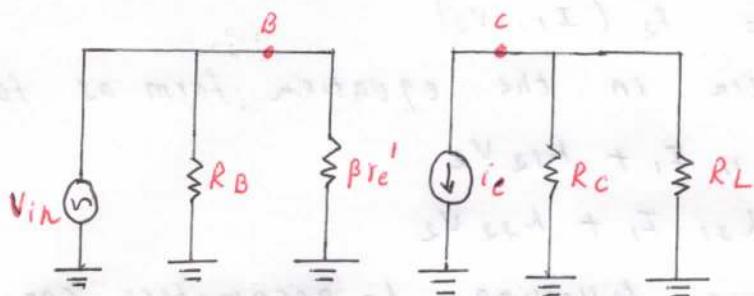
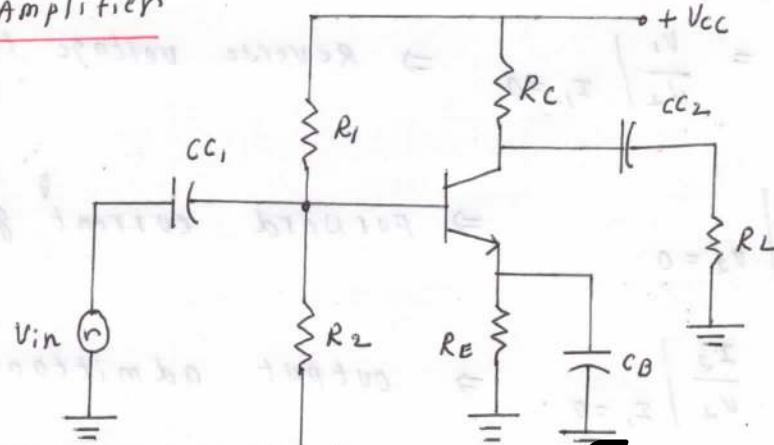


Fig (b) AC EQUIVALENT CIRCUIT

3. VDB Amplifier

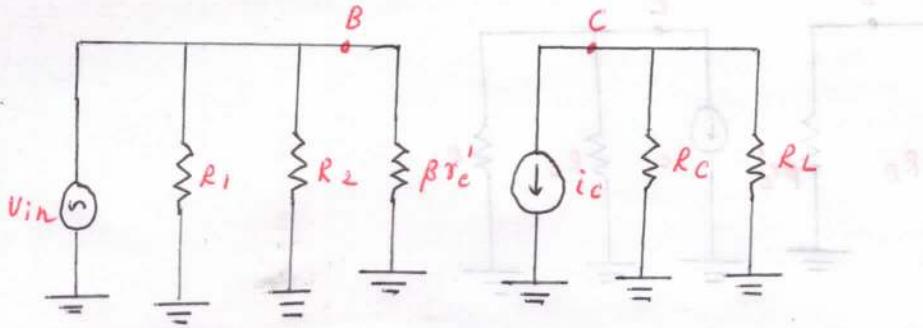


Fig (b) AC equivalent circuit

H Parameters

H parameters are used to analyze and design transistor circuits.

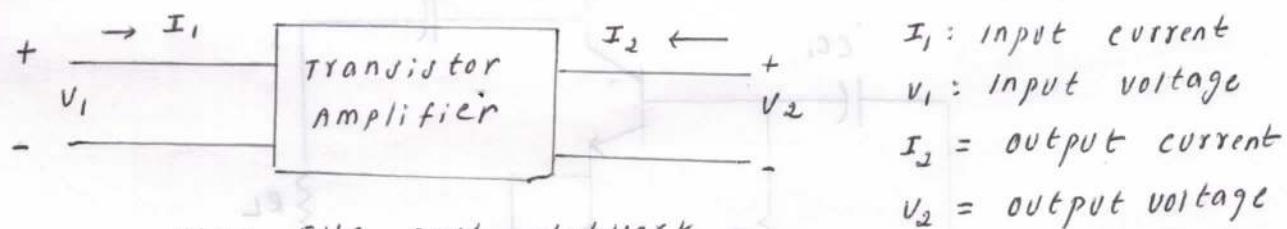


Fig: TWO port Network

Here  $V_1$  and  $I_2$  are dependent variables and  $I_1$  and  $V_2$  are independent variables.

$$V_1 = f_1(I_1, V_2)$$

$$I_2 = f_2(I_1, V_2)$$

This can be written in the equation form as follows

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

Using above equations, following h-parameters can be defined.

$$h_{11} = h_{ie} = \left. \frac{V_1}{I_1} \right|_{V_2=0} \Rightarrow \text{input resistance}$$

$$h_{12} = h_{re} = \left. \frac{V_1}{V_2} \right|_{I_1=0} \Rightarrow \text{Reverse voltage transfer ratio}$$

$$h_{21} = h_{fe} = \left. \frac{I_2}{I_1} \right|_{V_2=0} \Rightarrow \text{Forward current gain}$$

$$h_{22} = h_{oe} = \left. \frac{I_2}{V_2} \right|_{I_1=0} \Rightarrow \text{output admittance.}$$

Here  $h_{ie}$ ,  $h_{re}$ ,  $h_{fe}$ , and  $h_{oe}$  are called h-parameters.

Relation between R and H parameters

$$\beta = h_{fe}$$

Minimum  $h_{fe}$  is 100, Maximum  $h_{fe}$  is 400.

$$\sigma_e' = \frac{h_{ie}}{h_{fe}}$$

MAXIMUM value of  $h_{ie}$  is 10 k $\Omega$   
MAXIMUM value of  $h_{fe}$  is 400

VOLTAGE AMPLIFIERSVoltage gainDerived from the  $\pi$  model

consider the CE amplifier shown in below figure.

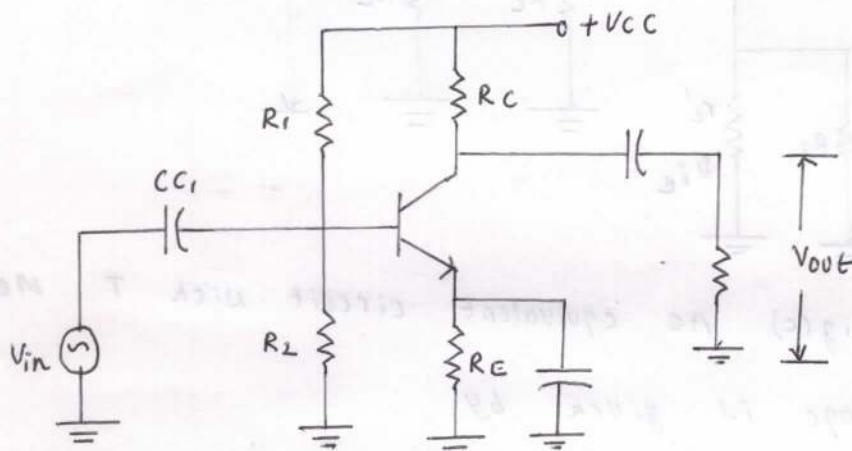


Fig (a) CE Amplifier

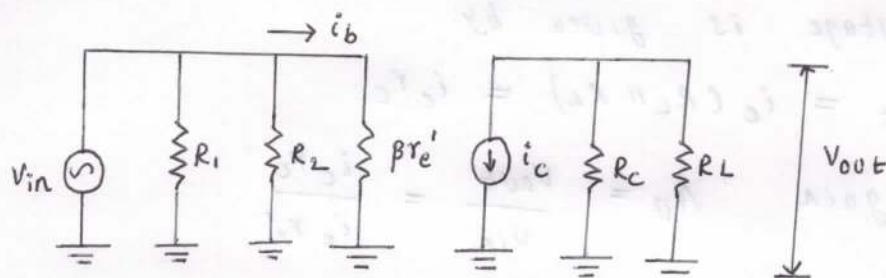


Fig (b) AC Equivalent circuit with  $\pi$  Model

From fig (b), the AC base current  $i_b$  flows through the input impedance of the base  $R_{be}'$ .

Using Ohm's law

$$V_{in} = i_b R_{be}' \rightarrow (1)$$

in the collector circuit, the AC output voltage will be

$$V_{out} = i_c R_C \times R_L / (R_C + R_L) \rightarrow (2)$$

Voltage gain

$$A_V = \frac{V_{out}}{V_{in}} = \frac{\beta i_b (R_C || R_L)}{i_b r_e'}$$

$$A_V = \frac{R_C || R_L}{r_e'}$$

AC collector resistance is given by  $r_c = R_C || R_L$ 

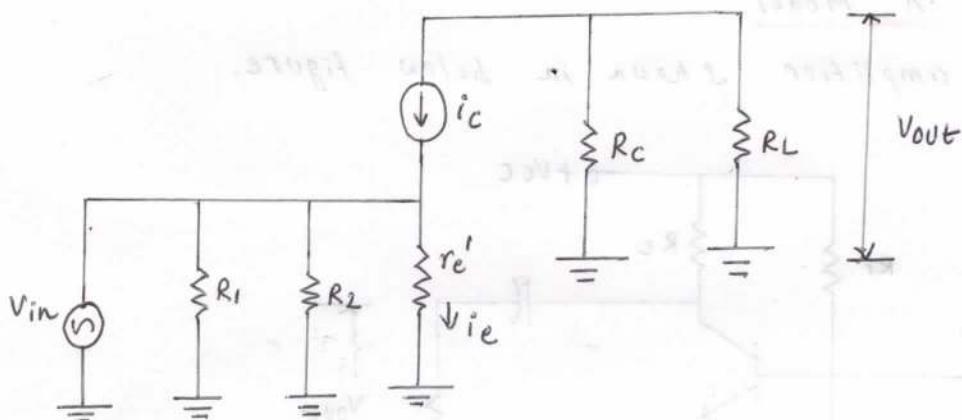
Then

$$A_V = \frac{r_c}{r_e'}$$

Derived from T Model

of fig(a)

Figure below shows AC equivalent circuit with T model.



Fig(c) AC equivalent circuit with T model

The input voltage is given by

$$V_{in} = i_e r_e'$$

The output voltage is given by

$$V_{out} = i_c (R_C || R_L) = i_c r_c$$

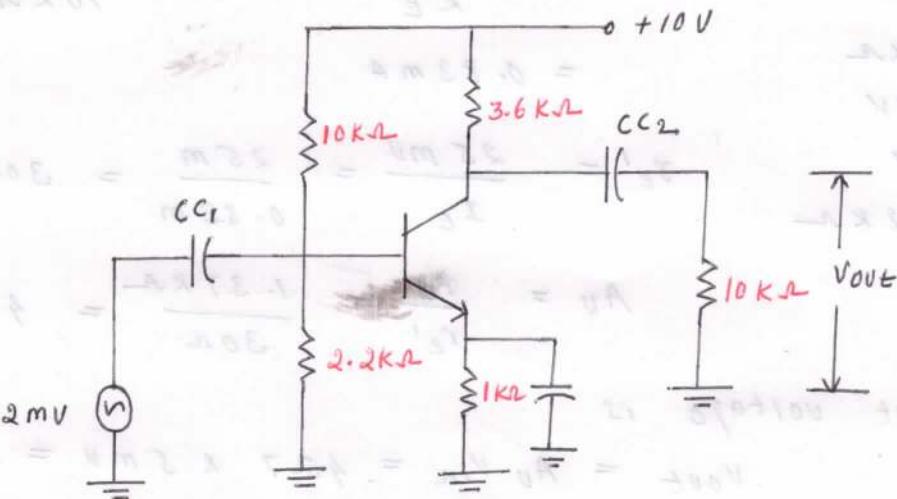
The voltage gain  $A_V = \frac{V_{out}}{V_{in}} = \frac{i_c r_c}{i_e r_e'}$ since  $i_c \approx i_e$ 

$$A_V = \frac{i_c r_c}{i_c r_e'} = \frac{r_c}{r_e'}$$

$$A_V = \frac{r_c}{r_e'}$$

Problem 1.13

For the circuit shown below, find the voltage gain and the output voltage across the load resistor.  
Given  $I_E = 1.1 \text{ mA}$ .

Solution:

The AC collector resistance is

Given:

$$r_C = R_C \parallel R_L = 3.6 \text{ k}\Omega \parallel 10 \text{ k}\Omega = 2.65 \text{ k}\Omega$$

$V_{in} = 2 \text{ mV}$

$R_1 = 10 \text{ k}\Omega$

$R_2 = 2.2 \text{ k}\Omega$

$R_C = 3.6 \text{ k}\Omega$

$R_E = 1 \text{ k}\Omega$

$R_L = 10 \text{ k}\Omega$

$$r_e' = \frac{25 \text{ mV}}{I_E} = \frac{25 \text{ m}}{1.1 \text{ mA}} = 22.7 \Omega$$

$$A_V = \frac{r_C}{r_e'} = \frac{2.65 \text{ k}}{22.7} = 117$$

The output voltage is

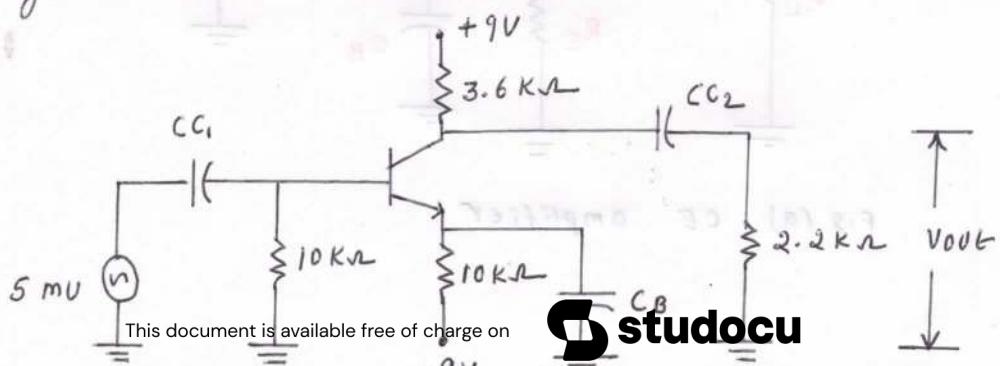
$$V_{out} = A_V V_{in} = 117 \times 2 \text{ mV} = 234 \text{ mV}$$

Practice problem

For the above figure, change  $R_L$  to  $6.8 \text{ k}\Omega$  and find  $A_V$

$$\text{Ans: } A_V = 104$$

Problem 1.14 For the circuit shown below, what is the voltage gain? Also calculate the voltage across the load resistor



SolutionGiven:  $V_{in} = 5 \text{ mV}$ 

$R_B = 10 \text{ k}\Omega$

$R_C = 3.6 \text{ k}\Omega$

$R_E = 10 \text{ k}\Omega$

$-V_{EE} = -9 \text{ V}$

$V_{cc} = 9 \text{ V}$

$R_L = 2.2 \text{ k}\Omega$

$r_C = R_C \parallel R_L = 3.6 \text{ k}\Omega \parallel 2.2 \text{ k}\Omega$

$= 1.37 \text{ k}\Omega$

$I_E = \frac{V_{EE} - V_{BE}}{R_E} = \frac{9 \text{ V} - 0.7 \text{ V}}{10 \text{ k}\Omega}$

$= 0.83 \text{ mA}$

$r_{e'} = \frac{25 \text{ mV}}{I_E} = \frac{25 \text{ m}}{0.83 \text{ m}} = 30 \Omega$

$A_v = \frac{r_C}{r_{e'}} = \frac{1.37 \text{ k}\Omega}{30 \Omega} = 45.7$

The output voltage is

$V_{out} = A_v V_{in} = 45.7 \times 5 \text{ mV} = 228 \text{ mV}$

practice problem:For the above problem, change  $R_E$  to  $8.2 \text{ k}\Omega$  and calculate the new output voltage  $V_{out}$ .

Ans:  $V_{out} = 277 \text{ mV}$

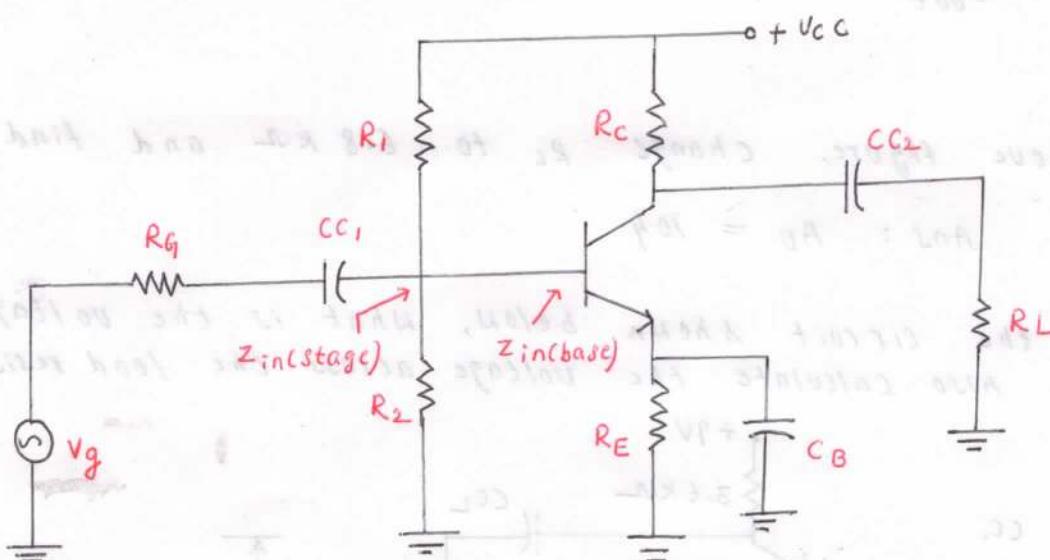
The loading effect of input impedanceConsider the CE amplifier shown in below figure. The AC voltage source  $V_g$  has an internal resistance of  $R_G$ . (The subscript 'g' stands for generator which means source)

Fig (a) CE amplifier

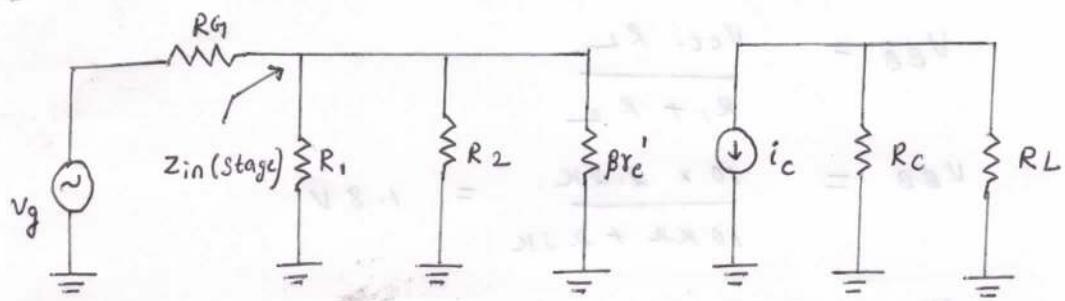


Fig: AC equivalent circuit

The AC generator has to drive the input impedance of the stage  $Z_{in(stage)}$ . The input impedance includes the effects of the biasing resistors  $R_1$  and  $R_2$ , in parallel with the input impedance of the base  $Z_{in(base)}$ .

$$Z_{in(stage)} = R_1 \parallel R_2 \parallel \beta r_e'$$

### Equation for input voltage

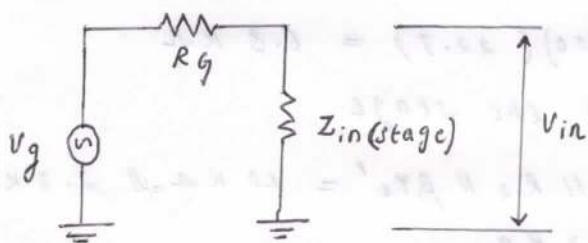


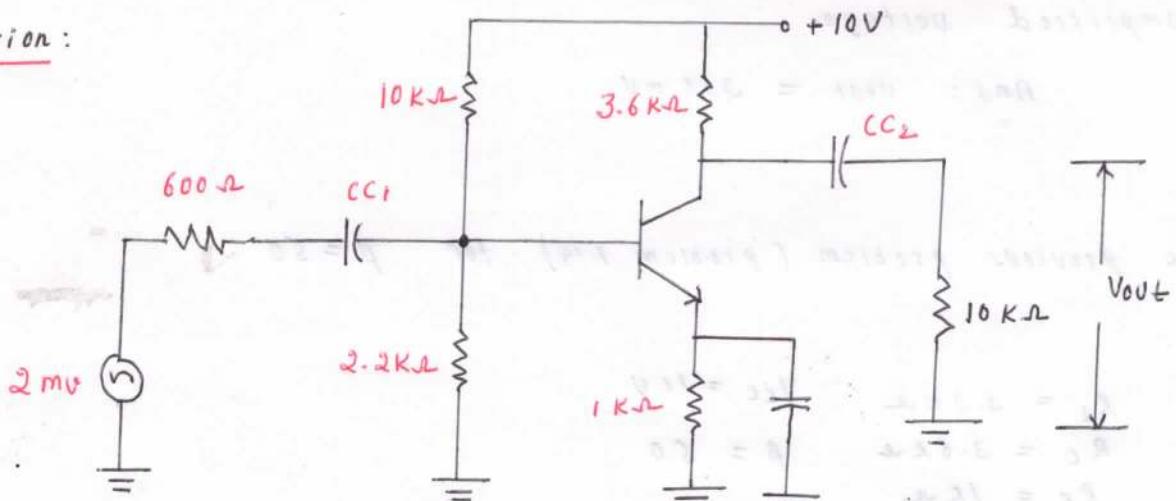
Fig: Effect of input impedance

$$V_{in} = \frac{V_g \cdot Z_{in(stage)}}{R_G + Z_{in(stage)}} \quad (\text{using voltage-divider theorem})$$

### problem 1.14

For the figure shown below, the AC generator has an internal resistance of  $600\Omega$ . What is the output voltage if  $\beta = 300$ ?

Solution:



Given :

$$V_i = 2 \text{ mV}$$

$$R_g = 600 \Omega$$

$$R_1 = 10 \text{ k}\Omega$$

$$R_2 = 2.2 \text{ k}\Omega$$

$$R_C = 3.6 \text{ k}\Omega$$

$$R_E = 1 \text{ k}\Omega$$

$$R_L = 10 \text{ k}\Omega$$

$$V_{CC} = 10 \text{ V}$$

$$\beta = 300$$

$$V_{BB} = \frac{V_{CC} \cdot R_2}{R_1 + R_2}$$

$$V_{BB} = \frac{10 \times 2.2 \text{ k}}{10 \text{ k}\Omega + 2.2 \text{ k}} = 1.8 \text{ V}$$

$$V_E = V_{BB} - V_{BE} = 1.8 - 0.7 = 1.1 \text{ V}$$

$$I_E = \frac{V_E}{R_E} = \frac{1.1 \text{ V}}{1 \text{ k}\Omega} = 1.1 \text{ mA}$$

$$r_e' = \frac{25 \text{ mV}}{I_E} = \frac{25 \text{ m}}{1.1 \text{ m}} = 22.7 \Omega$$

$$r_C = R_C \parallel R_L = 3.6 \text{ k}\Omega \parallel 10 \text{ k}\Omega = 2.65 \text{ k}\Omega$$

$$A_V = \frac{r_C}{r_e'} = \frac{2.65 \text{ k}\Omega}{22.7} = 117$$

$$Z_{in(base)} = \beta r_e' = (300)(22.7) = 6.8 \text{ k}\Omega$$

The input impedance of the stage

$$Z_{in(stage)} = R_1 \parallel R_2 \parallel \beta r_e' = 10 \text{ k}\Omega \parallel 2.2 \text{ k}\Omega \parallel 6.8 \text{ k}\Omega \\ = 1.42 \text{ k}\Omega$$

Input voltage

$$V_{in} = \frac{Z_{in(stage)} \cdot V_i}{R_g + Z_{in(stage)}} = \frac{2 \text{ m} \times 1.42 \text{ k}}{600 + 1.42 \text{ k}} = 1.41 \text{ mV}$$

$$V_{out} = A_V V_{in} = 117 \times 1.41 \text{ mV} = 165 \text{ mV}$$

Practice problem

change the  $R_g$  value of above problem to  $50 \Omega$  and solve for the new amplified voltage.

$$\text{Ans: } V_{out} = 226 \text{ mV}$$

problem 1-15

Repeat the previous problem (problem 1-14) for  $\beta = 50$

Solution :Given :

$$V_i = 2 \text{ mV}$$

$$R_g = 600 \Omega$$

$$R_1 = 10 \text{ k}\Omega$$

$$R_2 = 2.2 \text{ k}\Omega$$

$$R_C = 3.6 \text{ k}\Omega$$

$$R_E = 1 \text{ k}\Omega$$

$$R_L = 10 \text{ k}\Omega$$

$$V_{CC} = 10 \text{ V}$$

$$\beta = 50$$

$$V_{BB} = \frac{V_{CC} \cdot R_2}{R_1 + R_2} = 1.8V$$

$$V_E = V_{BB} - V_{BE} = 1.1V$$

$$I_E = \frac{V_E}{R_E} = 1.1mA$$

$$r_e' = \frac{25mV}{I_E} = 22.7\Omega$$

$$r_C = \frac{I_E}{R_C \parallel R_L} = 2.65k\Omega$$

$$A_V = \frac{r_C}{r_e'} = 117$$

$$Z_{in}(base) = \beta r_e' = (50)(22.7\Omega) = 1.14k\Omega$$

The input impedance

$$Z_{in(stage)} = R_1 \parallel R_2 \parallel \beta r_e' = 10k \parallel 2.2k \parallel 1.14k = 698\Omega$$

Input voltage  $V_{in} = \frac{Z_{in(stage)} \cdot V_g}{R_g + Z_{in(stage)}} = \frac{698.2mV}{600 + 698\Omega} = 1.08mV$

$$V_{out} = A_V V_{in} = (117)(1.08m) = 126mV$$

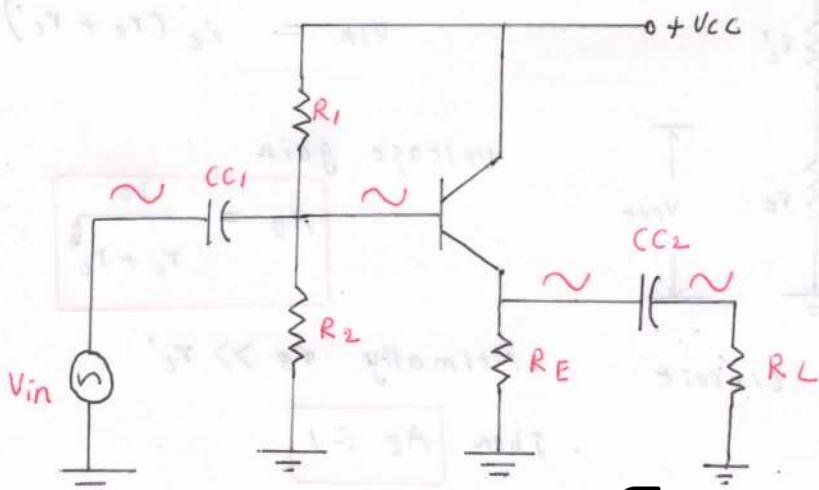
### Practice problem

Change  $\beta$  value to 400 and calculate the output voltage.

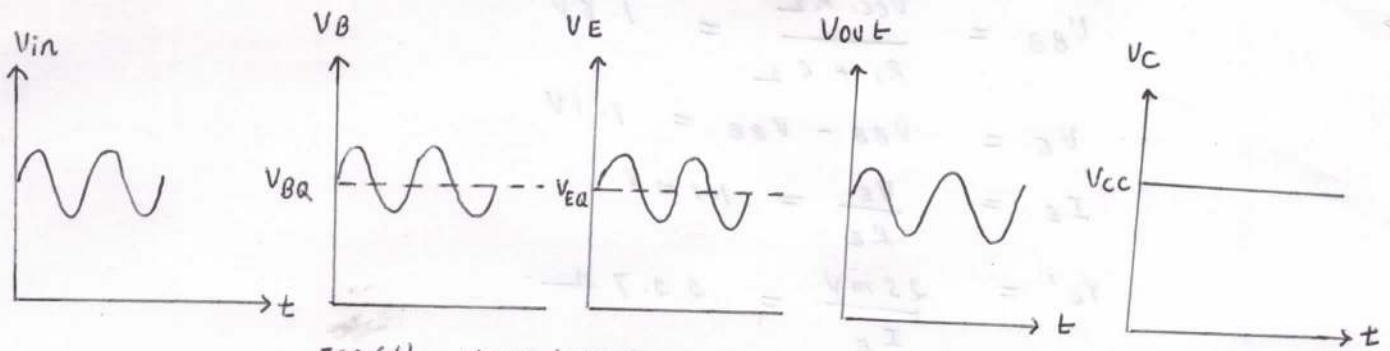
Ans:  $V_{out} = 167mV$

### CC Amplifier (Emitter follower)

Emitter follower is the circuit in which the output voltage signal at the emitter is approximately equal to the voltage signal input on the base.  
It is also called common-collector (cc) amplifier.



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Note :

Fig(b) Waveforms

1. The output and input voltage are approximately equal and they are in phase with each other.
2. The circuit is called an emitter follower since the output voltage follows the input voltage.
3. since there is no collector resistor, the total voltage between the collector and the ground equals the supply voltage.
4. The emitter follower uses negative feedback
5. The voltage gain is ultrastable and has maximum value of 1.
6. The distortion is almost non-existent.

AC Emitter Resistance

$$\delta_e = R_E \parallel R_L$$

Voltage gain

Consider the AC equivalent circuit of the emitter follower (using T Model) shown below:

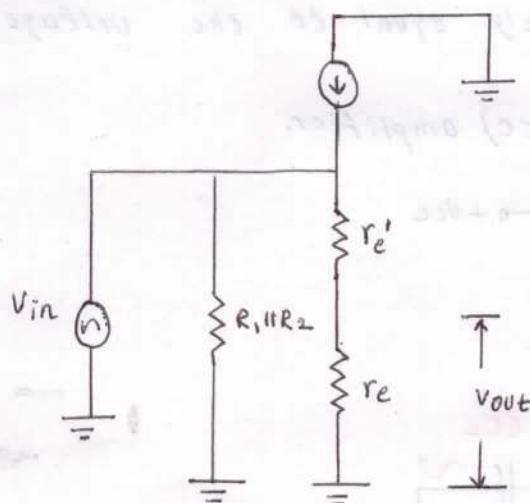


Fig: AC equivalent circuit

From the figure,

$$v_{out} = i_e r_e$$

$$v_{in} = i_e (r_{e'} + r_e)$$

Voltage gain

$$A_v = \frac{r_e}{r_e + r_e'}$$

Normally  $r_e \gg r_e'$ Then  $A_v \approx 1$

*input impedance of the base*

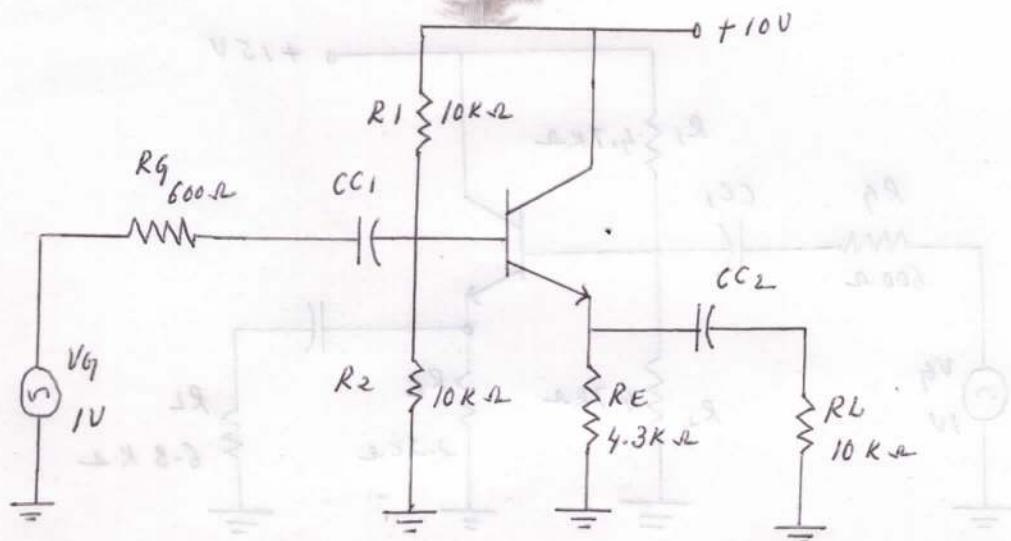
$$Z_{in}(\text{base}) = \beta(r_e + r'_e)$$

*input impedance of the stage*

$$Z_{in}(\text{stage}) = R_1 \parallel R_2 \parallel \beta(r_e + r'_e)$$

### problem 1.16

what is the input impedance of the base in the below shown figure? what is the input impedance of the stage? Given  $\beta = 200$ .



### Solution

$$R_g = 600\Omega$$

$$V_g = 1V$$

$$R_1 = 10k\Omega$$

$$R_2 = 10k\Omega$$

$$R_E = 4.3k\Omega$$

AC Emitter resistance

$$V_{CC} = 10V$$

$$R_L = 10k\Omega$$

$$V_{BB} = \frac{V_{CC} \cdot R_L}{R_1 + R_2} = \frac{10 \cdot 10k}{10k + 10k} = 5V$$

$$V_E = V_{BB} - V_{BE} = 5 - 0.7 = 4.3V$$

$$I_E = \frac{V_E}{R_E} = \frac{4.3}{4.3k} = 1mA$$

$$r'_e = \frac{25m}{I_E} = \frac{25mV}{1mA} = 25\Omega$$

The external AC emitter resistance is

$$r_e = R_E \parallel R_L = 4.3k\Omega \parallel 10k\Omega = 3k\Omega$$

$$Z_{in}(\text{base}) = \beta(r_e + r'_e) = 200(3k\Omega + 25\Omega) = 605k\Omega$$

$$\begin{aligned} Z_{in}(\text{stage}) &= R_1 \parallel R_2 \parallel \beta(r_e + r'_e) \\ &= 10k\Omega \parallel 10k\Omega \parallel 605k\Omega \end{aligned}$$

$$Z_{in}(\text{stage}) = 4.96k\Omega$$

Practice problem:

Find the input impedance of the base and the stage using the above figure, if  $\beta = 100$ .

$$\text{Ans: } Z_{in(\text{base})} = 303 \text{ k}\Omega$$

$$Z_{in(\text{stage})} = 4.92 \text{ k}\Omega$$

Problem 1.17

What is the voltage gain of the emitter follower shown in below figure? If  $\beta = 150$ , what is the AC load voltage?

Solution

Given:

$$V_g = 1V$$

$$R_g = 600\Omega$$

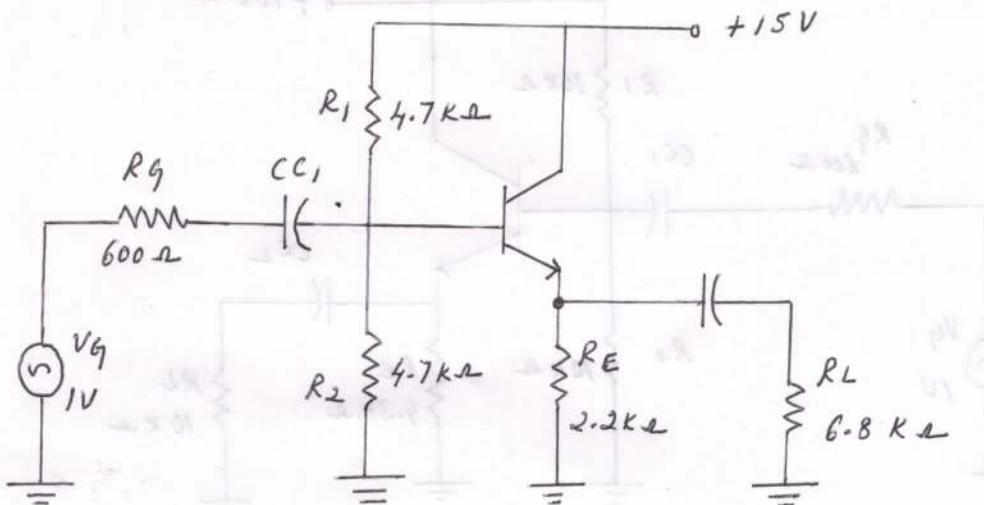
$$R_1 = 4.7 \text{ k}\Omega$$

$$R_2 = 4.7 \text{ k}\Omega$$

$$R_E = 2.2 \text{ k}\Omega$$

$$R_L = 6.8 \text{ k}\Omega$$

$$V_{CC} = 15V$$



$$V_{BB} = \frac{V_{CC} \cdot R_2}{R_1 + R_2} = \frac{15 \times 4.7 \text{ k}\Omega}{4.7 \text{ k} + 4.7 \text{ k}} = 7.5 \text{ V}$$

$$V_E = V_{BB} - V_{BE} = 7.5 - 0.7 = 6.8 \text{ V}$$

$$I_E = \frac{V_E}{R_E} = \frac{6.8}{2.2 \text{ k}} = 3.09 \text{ mA}$$

$$\tau_e' = \frac{25 \text{ mV}}{I_E} = \frac{25 \text{ mV}}{3.09 \text{ mA}} = 8.09 \text{ }\Omega$$

$$\tau_e = R_E \parallel R_L = 2.2 \text{ k} \parallel 6.8 \text{ k} = 1.66 \text{ k}\Omega$$

$$A_V = \frac{\tau_e}{\tau_e + \tau_e'} = \frac{1.66 \text{ k}}{1.66 \text{ k} + 8.09 \text{ }\Omega} = 0.995$$

$$Z_{in(\text{base})} = \beta (\tau_e + \tau_e') = 150 (1.66 \text{ k} + 8.09 \text{ }\Omega) = 250 \text{ k}\Omega$$

$$Z_{in(\text{stage})} = R_1 \parallel R_2 \parallel \beta (\tau_e + \tau_e') = 2.35 \text{ k}\Omega$$

$$V_{in} = \frac{V_g \cdot Z_{in(\text{stage})}}{R_g + Z_{in(\text{stage})}} = \frac{1V \times 2.35 \text{ k}}{600 + 2.35 \text{ k}} = 0.797 \text{ V}$$

$$V_{out} = A_V \cdot V_{in} = 0.995 \times 0.797 = 0.793 \text{ V}$$

Output impedance

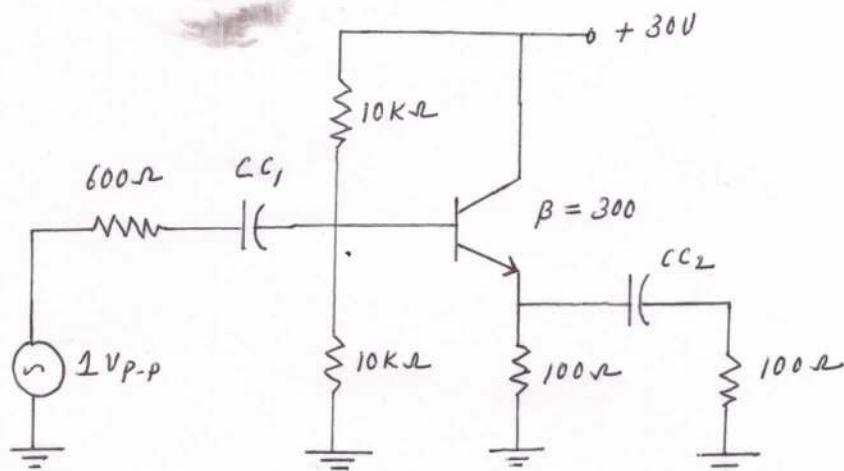
$$Z_{out} = R_E \parallel \left( r_e' + \frac{R_G \parallel R_1 \parallel R_2}{\beta} \right)$$

OR

$$Z_{out} = \frac{R_G}{\beta} \quad (\text{approximate formula})$$

problem 1.18

Calculate the output impedance of fig shown below.

Solution

$$\text{Given: } V_{CC} = 30V$$

$$R_G = 600\Omega$$

$$V_g = 1V_{p-p}$$

$$R_1 = 10K\Omega$$

$$R_2 = 10K\Omega$$

$$\beta = 300$$

$$R_E = 100\Omega$$

$$R_L = 100\Omega$$

$$V_{BB} = \frac{V_{CC} \cdot R_2}{R_1 + R_2} = 15V$$

$$V_E = V_{BB} - V_{BE} = 15 - 0.7 = 14.3V$$

$$I_E = \frac{V_E}{R_E} = \frac{14.3}{100\Omega} = 143mA$$

$$r_e' = \frac{25mV}{I_E} = \frac{25mV}{143mA} = 0.174\Omega$$

$$\text{output impedance } Z_{out} = R_E \parallel \left[ r_e' + \frac{R_G \parallel R_1 \parallel R_2}{\beta} \right]$$

$$= 100 \parallel \left[ 0.174 + \frac{600 \parallel 10K \parallel 10K}{300} \right]$$

$$Z_{out} = 1.91\Omega$$