SELF-GENERATING SENSORS

Self-generating sensors yield an electric signal from a measurand without requiring any electric supply. They offer alternative methods for measuring many common quantities—in particular, temperature, force, pressure, and acceleration. Furthermore, because they are based on reversible effects, these sensors can be used as actuators to obtain nonelectric outputs from electric signals.

This chapter also describes photovoltaic sensors and some sensors for chemical quantities (related to composition). Some effects described in this chapter can happen unexpectedly in circuits, thus becoming a source of interference. That is the case, for example, for thermoelectric voltages, for cable vibrations when they include piezoelectric materials, or for galvanic potentials at soldering points or electric contacts. We will describe the phenomena in sensors, but the same analysis applies to interference minimization.

6.1 THERMOELECTRIC SENSORS: THERMOCOUPLES

6.1.1 Reversible Thermoelectric Effects

Thermoelectric sensors are based on two effects that are reversible as contrasted with the irreversible Joule effect. They are the Peltier effect and the Thomson effect.

Historically, it was Thomas J. Seebeck who first discovered in 1822 that in a circuit with two dissimilar homogeneous metals A and B, having two junctions at different temperatures, an electric current arises (Figure 6.1). That is, there is a conversion from thermal to electric energy. If the circuit is opened, a thermoelectric electromotive force (emf) appears that depends only on the metals

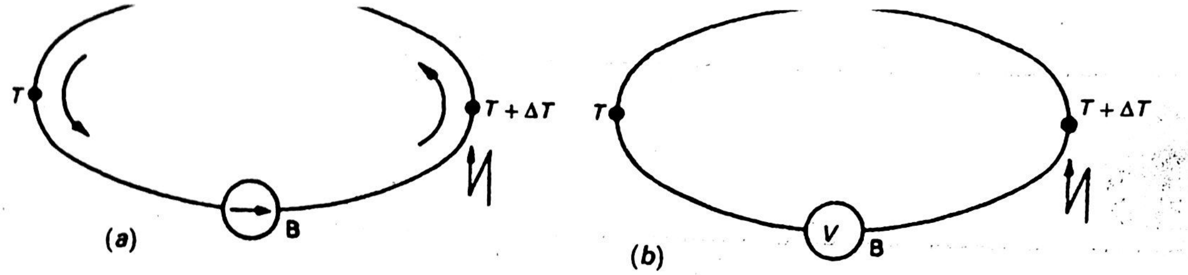


Figure 6.1 Seebeck effect in a thermocouple: (*a*) a current or (*b*) a potential difference appear when there are two metal junctions at different temperatures.

and on the junction temperatures. A pair of different metals with a fixed junction at a point or zone constitutes a *thermocouple.*

The relationship between the emf E_{AB} and the difference in temperature between both junctions T defines the Seebeck coefficient S_{AB},

$$S_{AB} = \frac{dE_{AB}}{dT} = S_A - S_B \tag{6.1}$$

where S_A and S_B are, respectively, the absolute thermoelectric power for A and B. S_{AB} is not in general constant but depends on T, usually increasing with T. It is important to realize that while the current flowing in the circuit depends on conductors' resistances, the emf does not depend on the resistivity, on the conductors' cross sections, or on temperature distribution or gradient. It depends only on the difference in temperature between both junctions and on the metals, provided that they are homogeneous. This emf is due to the Peltier and Thomson effects.

The Peltier effect, named to honor Jean C. A. Peltier, who discovered it in 1834, is the heating or cooling of a junction of two different metals when an electric current flows through it (Figure 6.2). When the current direction reverses, so does the heat flow. That is, if a junction heats (liberates heat), then when the current is reversed, it cools (absorbs heat), and if it cools, then when the current is reversed, it heats. This effect is reversible and does not depend on

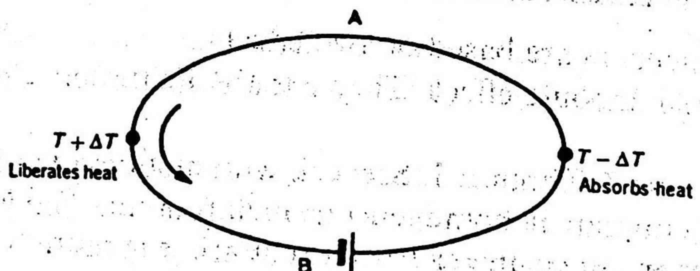


Figure 6.2 Peltier effect: When there is a current along a thermocouple circuit, one junction cools and the other warms.

the contact, namely, on the shape or dimensions of the conductors. It depends only on the junction composition and temperature. Furthermore, this dependence is linear and is described by the Peltier coefficient π_{AB}, sometimes called Peltier voltage because its unit is volts. π_{AB} is defined as the heat generated at the junction between A and B for each unit of (positive charge) flowing from B to A; that is,

$$dQ_P = \pm\pi_{AB} I\, dt \tag{6.2}$$

It can be shown [1] that for a junction at absolute temperature T we have

$$\pi_{AB}(T) = T \times (S_B - S_A) = -\pi_{BA}(T) \tag{6.3}$$

The fact that the amount of heat transferred per unit area at the junction is proportional to the current instead of its square makes this different from the Joule effect. In the Joule effect the heating depends on the square of the current and does not change when current direction reverses.

The Peltier effect is also independent of the origin of the current, which can thus even be thermoelectric as in Figure 6.1*a*. In this case the junctions reach a temperature different from that of the ambient, and this can be an error source as we will discuss later.

The Thomson effect, discovered by William Thomson (later Lord Kelvin) in 1847–1854, consists of heat absorption or liberation in a homogeneous conductor with a nonhomogeneous temperature when there is a current along it, as shown in Figure 6.3. The heat liberated is proportional to the current, not to its square, and therefore changes its sign for a reversed current. Heat is absorbed when charges flow from the colder to the hotter points, and it is liberated when they flow from the hotter to the colder one. In other words, heat is absorbed when charge and heat flow in opposite directions, and heat is liberated when they flow in the same direction.

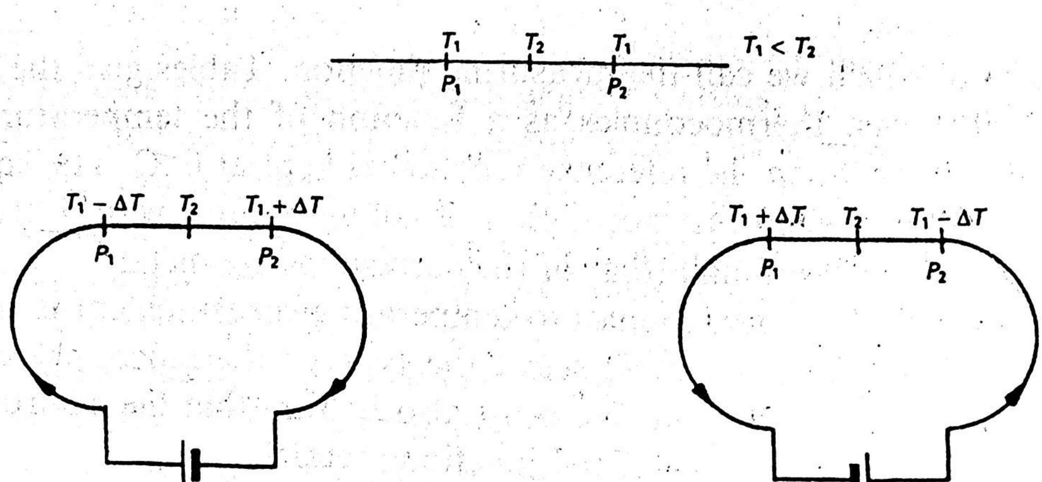


Figure 6.3 Thomson effect: When there is a current along a conductor with nonhomogeneous temperature, heat is absorbed or liberated.

The heat flux per unit volume q in a conductor of resistivity r with a longitudinal temperature gradient dT/dx, along which there is a current density i, is

$$q = i^2 r - i\sigma \frac{dT}{dx} \tag{6.4}$$

where σ is the Thomson coefficient. The first term on the right side describes the irreversible Joule effect, and the second term describes the reversible Thomson effect.

Going back to the circuit in Figure 6.1a, if the current is small enough to make the Joule effect negligible, we can consider only the reversible effects. Then the resulting thermoelectric power $(dE_{AB}/dT)\Delta T$ must equal the net thermal energy converted. In Figure 6.1a where one junction is at temperature $T + \Delta T$ and the other one is at T, the heat absorbed in the hot junction is $\pi_{AB}(T + \Delta T)$, while the heat liberated at the cool junction is $-\pi_{AB}(T)$. By the Thomson effect, there is an amount of heat $-\sigma_A \times \Delta T$ liberated along A while there is an amount of heat $\sigma_B \times \Delta T$ absorbed along B. The power balance is thus

$$\frac{dE_{AB}}{dT}\Delta T = \pi_{AB}(T + \Delta T) - \pi_{AB}(T) + (\sigma_B - \sigma_A) \times \Delta T \tag{6.5}$$

By dividing both sides by ΔT and taking limits when ΔT goes to zero, we have

$$\frac{dE_{AB}}{dT} = \frac{d\pi_{AB}}{dT} + \sigma_B - \sigma_A \tag{6.6}$$

This equation constitutes the basic theorem for thermoelectricity and shows that the Seebeck effect results from the Peltier and Thomson effects.

Equations (6.1) and (6.6) allow us to apply thermocouples to temperature measurement. A thermocouple circuit with a junction at constant temperature (reference junction) yields an emf that is a function of the temperature at the other junction, which we call the measuring junction. Tables give the voltages obtained with given thermocouples as a function of the temperature at the measuring junction when the reference junction is kept at 0 °C. The equivalent circuit for an ungrounded thermocouple is a voltage source with different output resistance at each terminal (that of the corresponding metal).

The application of thermocouples to temperature measurement is subject to several limitations. First, we must select the type of thermocouple so that it does not melt in our application. We must also be sure that the environment it is placed in does not attack any of the junction metals.

Second, we must keep the current along the thermocouple circuit very small. Otherwise, because the Peltier and Thomson effects are reversible, the temperatures of the conductors and particularly those of the junctions would differ from that of the environment because of the heat flow to and from the circuit. Depending on the intensity of the current, even the Joule effect could be considerable. All this would result in a temperature for the measuring junction different from the one we intend to measure, and also a reference temperature different from the assumed one, thus leading to serious errors. In addition, conductors must be homogeneous, so that caution is needed to prevent any mechanical or thermal stress during installation or operation—for example, because of aging caused by long exposure to large temperature gradients.

Another limitation is that one of the junctions must be kept at a fixed temperature if the temperature at the other junction is to be measured. Any change in that reference junction would result in a serious error because the output voltage is very small, typically from 6 μV/°C to 75 μV/°C. Furthermore, if the reference temperature is not close to the measured temperature, the output signal will have a relatively high constant value undergoing only very small changes due to the temperature changes we are interested in.

When high accuracy is desired, the nonlinearity of the relationship between the emf and the temperature may become important. An approximate formula valid for all thermocouples is

$$E_{AB} \approx C_1(T_1 - T_2) + C_2(T_1^2 - T_2^2) \tag{6.7}$$

where T_1 and T_2 are the respective absolute temperatures for each junction and C_1 and C_2 are constants that depend on materials A and B. From (6.7), we have

$$E_{AB} \approx (T_1 - T_2)[C_1 + C_2(T_1 + T_2)] \tag{6.8}$$

which shows that the emf depends not only on the temperature differences but also on their absolute value. The number of useful thermocouples available is limited because C_2 should be very small, thus reducing the possible choices. For copper-constantan, for example, $C_2 \approx 0.036$ μV/K^2. This nonlinearity may require a correction to be performed by the signal conditioner. All factors considered, thermocouples seldom achieve errors below 0.5 °C. Tolerance for same-type models can be up to several degrees Celsius.

In spite of the above limitations, thermocouples have many advantages and are by far the most frequently used sensors for temperature measurement. They have a very broad measurement range, as a group from −270 °C to 3000 °C, and each particular model has a broad range. They also display acceptable long-term stability and a high reliability. Furthermore, at low temperatures they have higher accuracy than RTDs. Their small size also yields a fast speed of response, on the order of milliseconds. They are also robust, simple, and easy to use, and very low cost models are available suitable for many applications. Because they do not need excitation, they do not have the self-heating problems suffered by RTDs, particularly in gas measurements. They also accept long connection wires.

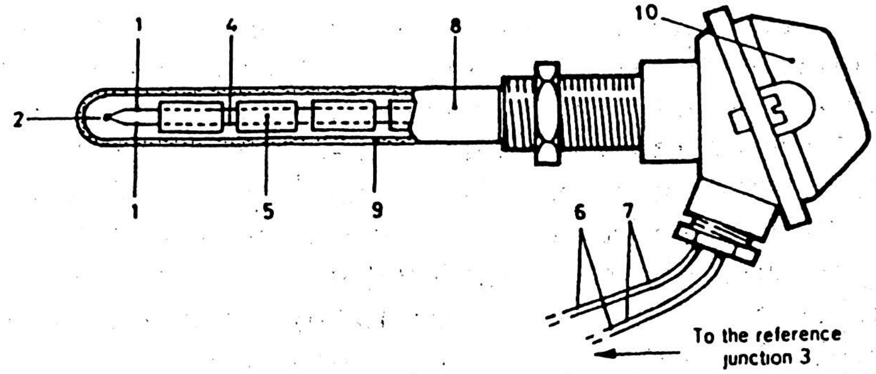


Figure 6.4 Industrial thermocouple with sheath. 1, conductors (different); 2, measurement junction; 3, reference junction; 4, bare thermocouple wires; 5, insulated thermocouple wires; 6, extension leads, of the same wire as that of the thermocouple; 7, compensation leads, different wire from that of the thermocouple but with small emf; 8, probe; 9, protection (external covering); 10, sheath head.

6.1.2 Common Thermocouples

In thermocouple junctions there is a simultaneous requirement for (a) a low-resistivity temperature coefficient, (b) resistance to becoming oxidized at high temperatures, in order to withstand the working environment, and (c) a linearity as high as possible.

Several particular alloys are used that fulfill all these requirements: $Ni_{90}Cr_{10}$ (chromel), $Cu_{57}Ni_{43}$ (constantan), $Ni_{94}Al_2Mn_3Si_1$ (alumel), and so forth. Environmental protection is obtained by a sheath, usually from stainless steel (Figure 6.4). Both speed of response and probe robustness depend on the thickness of the sheath. Both silicon and germanium display thermoelectric properties, but they have found greater application as cooling elements (Peltier elements) than as measurement thermocouples. Table 6.1 gives the characteristics for

TABLE 6.1 Characteristics of Some Common Thermocouples

ANSI Designation	Composition	Usual Range	Full-Range Output (mV)	Error (°C)
B	Pt(6 %)/rhodium–Pt(30 %)/rhodium	38 °C to 1800 °C	13.6	—
C	W(5 %)/rhenium–W-(26 %)/rhenium	0 °C to 2300 °C	37.0	—
E	Chromel–constantan	0 °C to 982 °C	75.0	±1.0
J	Iron–constantan	184 °C to 760 °C	43.0	±2.2
K	Chromel–alumel	−184 °C to 1260 °C	56.0	±2.2
N	Nicrosil (Ni–Cr–Si)–Nisil (Ni–Si–Mg)	−270 °C to 1300 °C	51.8	—
R	Pt(13 %)/rhodium–Pt	0 °C to 1593 °C	18.7	±1.5
S	Pt(10 %)/rhodium–Pt	0 °C to 1538 °C	16.0	±1.5
T	Copper–constantan	−184 °C to 400 °C	26.0	±1.0

some common thermocouples and their ANSI designation. Type C and N are not ANSI standards. There are also thin-film models for surface temperature measurement.

Type J thermocouples are versatile and have low cost. They withstand oxidizing and reducing environments. They are often used in open-air furnaces. Type K thermocouples are used in nonreducing environments and, in their measurement range, are better than types E, J, and T in oxidizing environments. Type T thermocouples resist corrosion; hence they are useful in high-humidity environments. Type E thermocouples have the highest sensitivity, and they withstand corrosion below 0 °C and in oxidizing environments. Type N thermocouples resist oxidation and are stable at high temperature. Thermocouples based on noble metals (types B, R, and S) are highly resistive to oxidation and corrosion.

Standard tables give the output voltage corresponding to different temperatures when the reference junction is at 0.00 °C. But this does not mean that a junction placed at 0.00 °C always gives a 0 V output for any thermocouple. This tabulation is only a matter of convenience arising from the fact that in order to measure the voltage generated by a junction, we cannot avoid introducing another junction. Therefore it is more convenient to speak of voltage differences between junctions at different temperatures than to consider the voltage of a single junction for each given temperature. For standardization purposes it has been agreed to take 0.00 °C as the reference temperature for the tables. Table 6.2 shows part of one of these tables [2]. Intermediate voltages or temperatures are obtained by linear interpolation.

Example 6.1 A J-type thermocouple circuit has one junction at 0 °C and the other at 45 °C. What is its open circuit emf?

In Table 6.2, at the intersection of the row corresponding to 40 (°C) and the column corresponding to 5 (°C) we read 2.321 mV.

Example 6.2 A given J-type thermocouple circuit with one junction at 0 °C generates a 5 mV output voltage. What is the temperature at the measuring junction?

At 96°C we have 5.050 mV. At 95 °C we have 4.996 mV. Therefore, the sensitivity in this range is 54 μV/°C, and the junction is at about 95.07 °C.

When interpreting this last result it is important to take into account the accuracy of each thermocouple type. For type J it is ±2.2 °C or 0.75 % (whichever gives the largest error). This means that in the result of the last example the uncertainty would be ±2 °C. This does not reduce the usefulness of tables given with 1 °C increments and interpolation because some applications need a high resolution but not necessarily a high accuracy.

Self-calibrating thermocouples [3] have improved accuracy. They include an encapsulated metal located near the junction. When the sensed temperature

TABLE 6.2 Part of the Voltage–Temperature Table for a Type J Thermocouple from 0 °C to 110 °C

Degrees	0	1	2	3	4	5	6	7	8	9	10
0	0.000	0.050	0.101	0.151	0.202	0.253	0.303	0.354	0.405	0.456	0.507
10	0.507	0.558	0.609	0.660	0.711	0.762	0.813	0.865	0.916	0.967	1.019
20	1.019	1.070	1.122	1.174	1.225	1.277	1.329	1.381	1.432	1.484	1.536
30	1.536	1.588	1.640	1.693	1.745	1.797	1.849	1.901	1.954	2.006	2.058
40	2.058	2.111	2.163	2.216	2.268	2.321	2.374	2.426	2.479	2.532	2.585
50	2.585	2.638	2.691	2.743	2.796	2.849	2.902	2.956	3.009	3.062	3.115
60	3.115	3.168	3.221	3.275	3.328	3.381	3.435	3.488	3.542	3.595	3.649
70	3.649	3.702	3.756	3.809	3.863	3.917	3.971	4.024	4.078	4.132	4.186
80	4.186	4.239	4.293	4.347	4.401	4.455	4.509	4.563	4.617	4.671	4.725
90	4.725	4.780	4.834	4.888	4.942	4.996	5.050	5.105	5.159	5.213	5.268
100	5.268	5.322	5.376	5.431	5.485	5.540	5.594	5.649	5.703	5.758	5.812

Note: The reference junction is assumed to be at 0 °C. Voltages are given in millivolts.

transcends the phase transition temperature of the encapsulated metal, the time–temperature record of the thermocouple reaches a plateau. By comparing the plateau temperature with the known phase transition temperature of the encapsulated metal, we perform a single-point calibration.

Systems with computation capability can use polynomials that approximate the values in the tables with accuracy dependent on their order. They all correspond to equations such as

$$T = a_0 + a_1 x + a_2 x^2 + \cdots \tag{6.9}$$

where x is the measured voltage. Table 6.3 gives the polynomial coefficients for different common thermocouples within a specified range and degree of approximation [2]. When the measurement range is very large, instead of using higher order polynomials it is better to divide the whole range into smaller temperature ranges and then use a lower order polynomial for each range.

Figure 6.5 shows different junction types available. Exposed junctions are used for static measurements or in noncorrosive gas flows where a fast response time is required. But they are fragile. Enclosed (ungrounded) junctions are intended for corrosive environments where there is the need for an electrical isolation of the thermocouple. The junction is enclosed by the sheath and is insulated from that by means of a good thermal conductor such as oil, mercury, or metallic powder. When a fast response is needed and a thick sheath is not required, then mineral insulators such as MgO, Al_2O_3, or BeO powders are

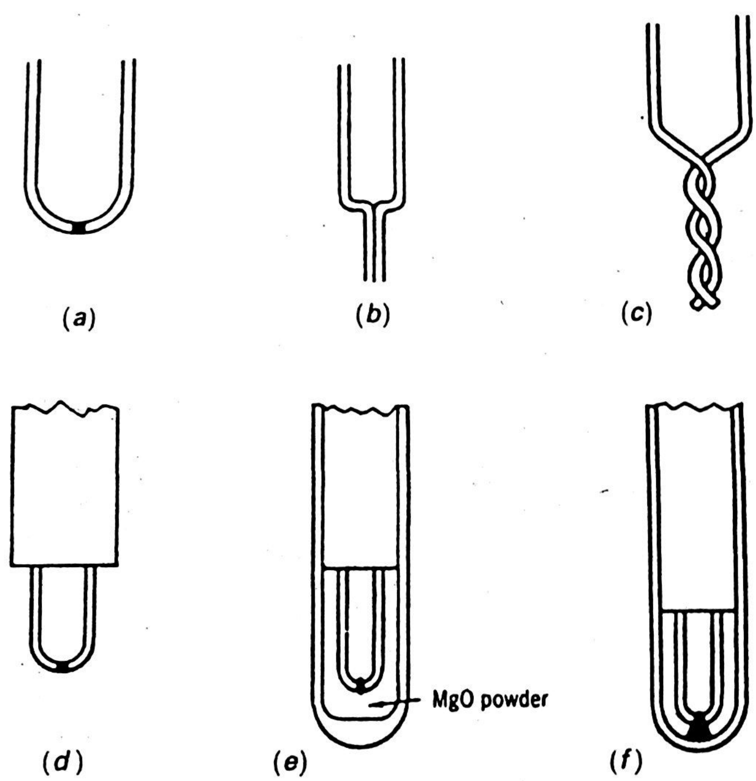


Figure 6.5 Different kinds of thermocouple junctions and their sheaths [4]: (*a*) butt-welded junction; (*b*) lap-welded junction; (*c*) twisted wire; (*d*) exposed thermocouple for fast response time; (*e*) enclosed thermocouple—electrical and ambient isolation; (*f*) grounded thermocouple soldered to the covering ...

TABLE 6.3 Polynomial Coefficients that Give the Approximate Temperature from the Output Voltage for Different Thermocouples According to (6.9)

Polynomial Coefficient	Type E −100 °C to 1000 °C	Type J 0 °C to 760 °C	Type K 0 °C to 1370 °C	Type R 0 °C to 1000 °C	Type S 0 °C to 1750 °C	Type T −160 °C to 400 °C
Accuracy	±0.5 °C	±0.1 °C	±0.7 °C	±0.5 °C	±1 °C	±0.5 °C
a_0	0.104967248	−0.048868252	0.226584602	0.263632917	0.927763167	0.100860910
a_1	17189.45282	19873.14503	24152.10900	179075.491	169526.5150	25727.94369
a_2	−282639.0850	−218614.5353	67233.4248	−48840341.37	−31568363.94	−767345.8295
a_3	12695339.5	11569199.78	2210340.682	1.90002E + 10	8990730663	780225595.81
a_4	−448703084.6	−264917531.4	−860963914.9	−4.82704E + 12	−1.63565E + 12	−9247486589
a_5	1.1086E + 10	2018441314	4.83506E + 10	7.62091E + 14	1.88027E + 14	6.97688E + 11
a_6	−1.76807E + 11		−1.18452E + 12	−7.20026E + 16	−1.37241E + 16	−2.6619E + 13
a_7	1.71842E + 12		1.38690E + 13	3.71496E + 18	6.17501E + 17	3.94078E + 14
a_8	−9.19278E + 12		−6.33708E + 13	−8.03104E + 19	−1.56105E + 19	
a_9	2.06132 E + 13				1.69535E + 20	

used. The final response will depend on the compactness of the insulator, and the maximal allowable temperature will also be different.

Grounded junctions suit the measurement of static temperatures or temperatures in flowing corrosive gases or liquids. They are also used in measurements performed under high pressures. The junction is soldered to the protective sheath so that the thermal response will be faster than when insulated. However, noisy grounds require ungrounded thermocouples.

6.1.3 Practical Thermocouple Laws

In addition to the advantages and disadvantages mentioned above, there are several experimental laws for temperature measurement using thermocouples that greatly simplify the analysis of thermocouple circuits.

6.1.3.1 Law of Homogeneous Circuits. It is not possible to maintain a thermoelectric current in a circuit formed by a single homogeneous metal by only applying heat, not even by changing the cross section of the conductor.

Figure 6.6 describes the meaning of this law. In Figure 6.6*a* the temperatures T_3 and T_4 do not alter the emf due to T_1 and T_2. In particular, if $T_1 = T_2$ and A or B are heated, there is no current. In other words, intermediate temperatures along a conductor do not alter the emf produced by a given temperature difference between junctions (Figure 6.6*b*). But this does not mean that if along a conductor there are different temperatures, then long extension wires identical to those of the thermocouple must be used. Instead of these, we can use compensation wires that are made from metals that do not display any appreciable emf and at the same time are cheaper than thermocouple wires. Nevertheless, they are four to five times more expensive than copper wires. Thermocouple wire coverings use standard colors.

6.1.3.2 Law of Intermediate Metals. The algebraic sum of all emfs in a circuit composed by several different metals remains zero as long as the entire circuit is at a uniform temperature. This implies that a meter can be inserted into the

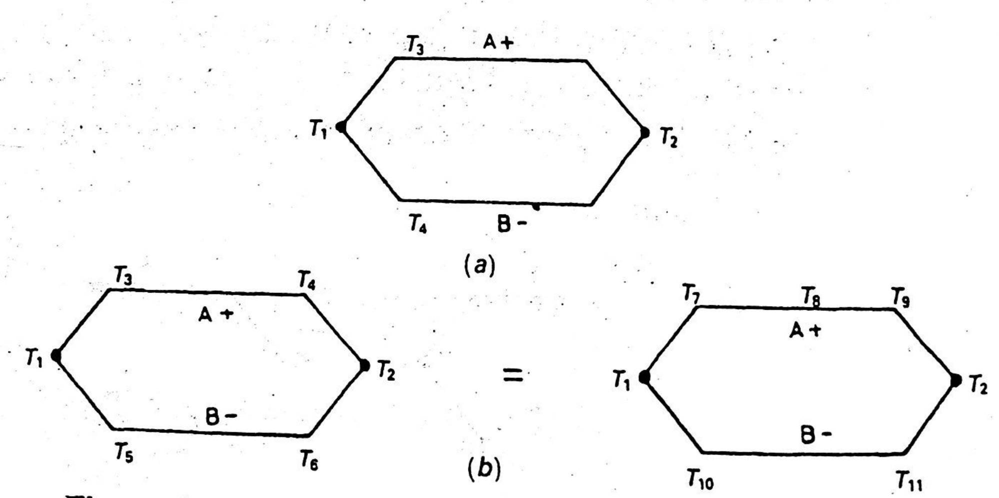


Figure 6.6 Homogeneous circuits law for thermocouples.

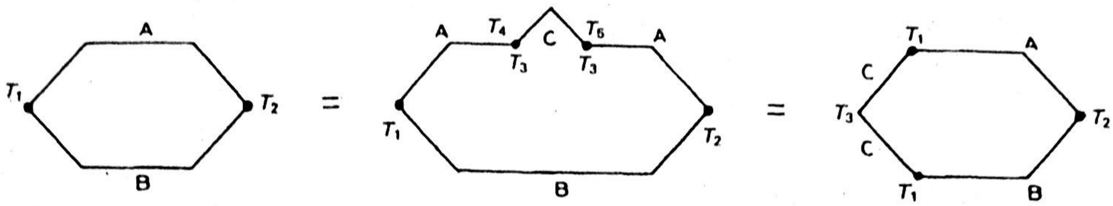


Figure 6.7 Intermediate metals law for thermocouple circuits.

TABLE 6.4 Thermoelectric Sensitivity of Different Metal and Alloy Pairs Common in Electric Circuits

Pair	S_{AB} (μV/K)
Alloy 180–nichrome	42
Au–kovar	25
Cu–Ag	0.3
Cu–Au	0.3
Cu–Cd/Sn	0.3
Cu–Cu	<0.2
Cu–CuO	1000
Cu–kovar	40
Cu–Pb/Sn	1–3

circuit without adding any errors, provided that the new junctions introduced are all at the same temperature, as indicated in Figure 6.7. The measuring instrument can be inserted at a point in a conductor or at a junction. Table 6.4 gives S_{AB} for different metal and alloy pairs common in electric circuits. Alloy 180 is the standard component lead alloy. Nichrome is used in wirewound resistors and strain gages. The Cu–Cu pair refers to copper with different purity grades. The Pb/Sn alloy refers to the common solder alloy, and the Cd/Sn alloy refers to a low-temperature solder alloy. Kovar is an alloy used in some IC pins. Because CuO/Cu yields a large emf, it is advisable to keep electric contacts clean.

A corollary of this law is that if the thermal relationship between each of two materials and a third one is known, then it is possible to deduce the relationship between the two first ones, as shown in Figure 6.8. Therefore it is not necessary to calibrate all the possible metal pairs in order to know the temperature cor-

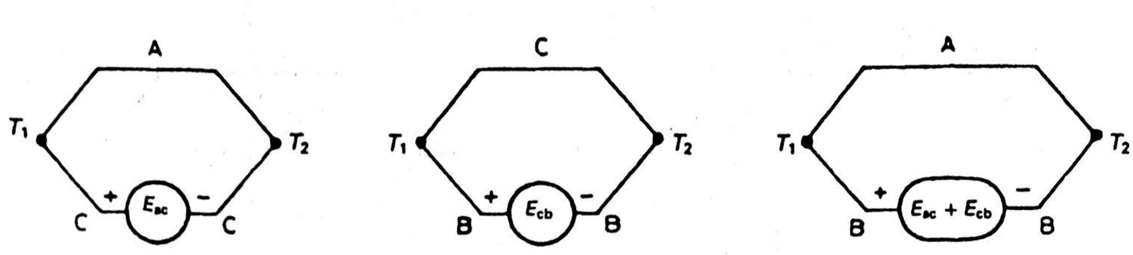


Figure 6.8 Corollary for intermediate metals law in thermocouple circuits.

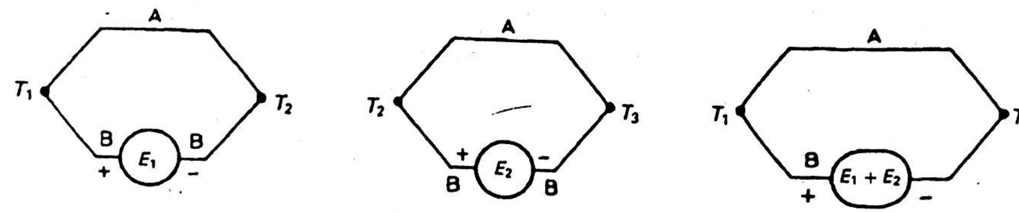


Figure 6.9 Intermediate temperature law for thermocouple circuits.

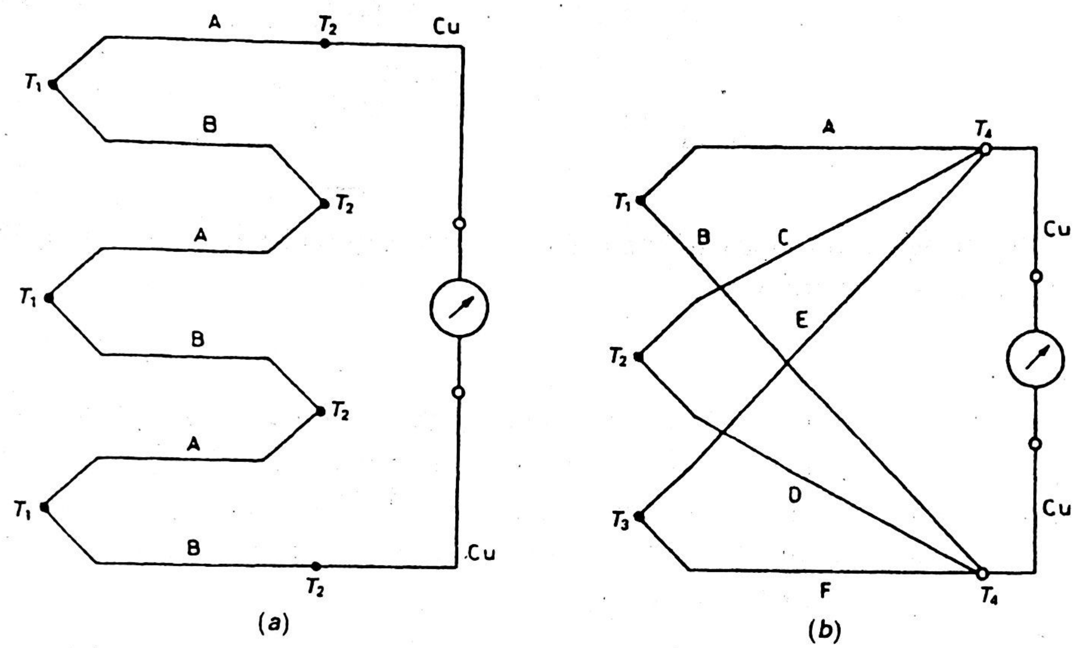


Figure 6.10 (a) Series (thermopile) and (b) parallel thermocouple connection.

responding to a given emf measured with a given pair. Rather, its behavior with respect a third material is enough. The reference metal is platinum.

6.1.3.3 Law of Successive or Intermediate Temperatures. If two homogeneous metals yield an emf E_1 when their junctions are at T_1 and T_2, and an emf E_2 when they are at T_2 and T_3, then the emf when the junctions are at T_1 and T_3 will be $E_1 + E_2$ (Figure 6.9). This means, for example, that it is not necessary for the reference junction to be at 0 °C. Any other reference temperature is also acceptable.

The preceding laws enable us to analyze circuits such as those in Figure 6.10. Case (a) shows several thermocouples connected in series, thus constituting a thermopile. It is straightforward to verify that this increases the sensitivity compared to the case where a single thermocouple is used. Case (b) shows a parallel connection, which yields the average temperature if all thermocouples are linear in the measurement range and have the same resistance.

6.1.4 Cold Junction Compensation in Thermocouple Circuits

In order to apply the Seebeck effect to temperature measurement, one junction must remain at a fixed reference temperature. Placing the

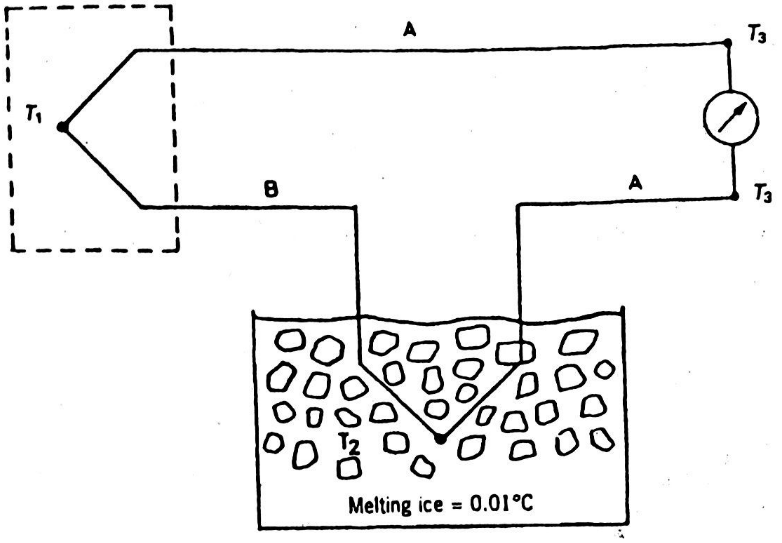


Figure 6.11 Temperature measurement using thermocouples with a junction held at a constant reference temperature.

into melting ice (Figure 6.11) is easy and highly accurate, but it requires frequent maintenance and has a high cost. It is also possible to keep the reference junction at a fixed temperature by means of a Peltier cooler or by means of a constant temperature oven. In any case a long length of one of the thermocouple metal wires must be used, thus increasing cost.

Figure 6.12 shows how to use a cheaper connecting wire (copper), but the need for a constant reference temperature is still expensive. When the expected range of variation for ambient temperature is smaller than the required resolution, we can just leave the reference junction exposed to the ambient. Otherwise, we can use the reference (or cold) junction temperature compensation method.

This consists of leaving the reference junction to undergo the ambient temperature fluctuations but at the same time measuring these by another temperature sensor placed near the reference junction. Then a voltage equal to that generated at the cold junction is subtracted from the one produced by the circuit, as shown in Figure 6.13. The bridge supply voltage must be highly stable and can be provided by a mercury cell or reference voltage generator (Section

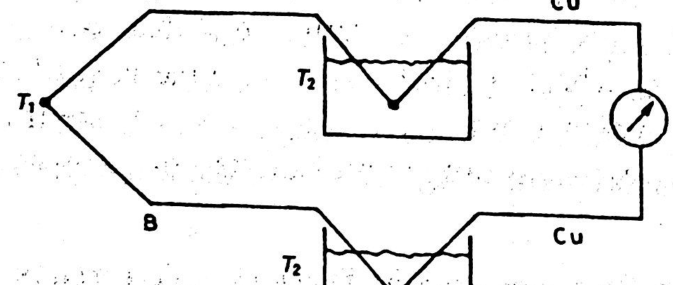


Figure 6.12 Temperature measurement using two junctions at constant temperature and common metal leads.

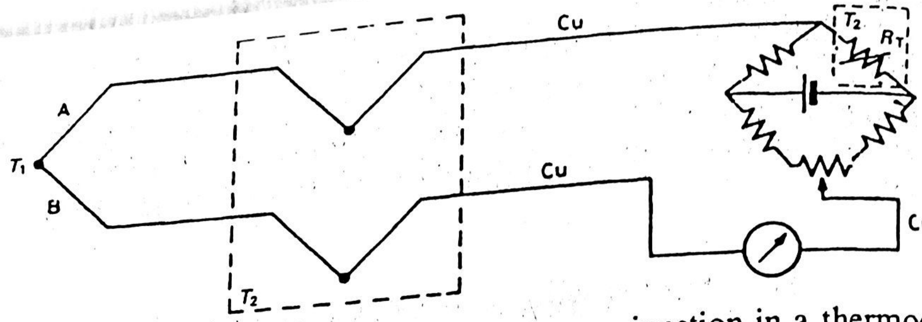
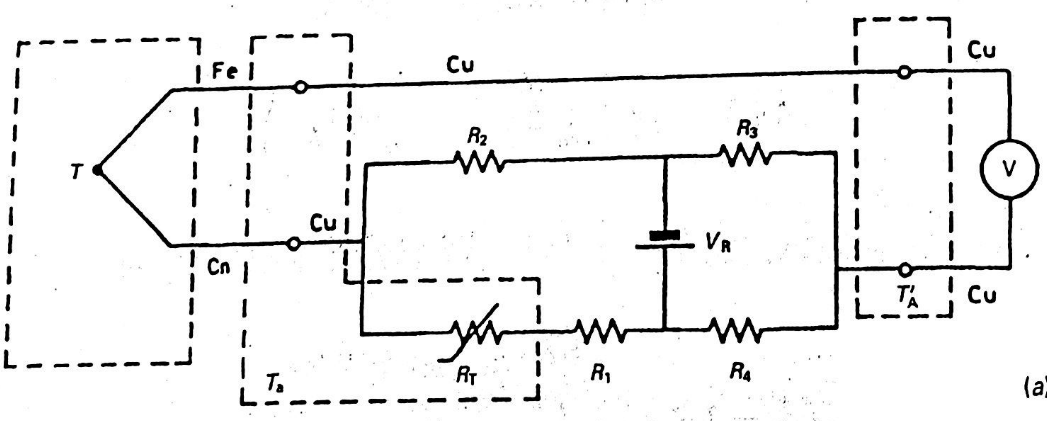


Figure 6.13 Electronic compensation for the reference junction in a thermocouple circuit. Ambient temperature fluctuations are measured by another sensor, and a voltage equal to that generated by the cold junction is subtracted from the output voltage.

3.4.5), for example. There are ICs that measure the ambient temperature and provide the compensation voltage for some specific thermocouples. The LT1025 works with types E, J, K, R, S, and T. The AD594/AD595 is an instrumentation amplifier and thermocouple cold junction compensator (for types J and K, respectively). The AD596/AD597 are monolithic set-point controllers that include the amplifier and cold junction compensation for, respectively, type J and K thermocouples.

Example 6.3 Figure E6.3*a* shows a circuit to measure a temperature by means of a J-type thermocouple and electronic compensation of the reference junction based on an NTC thermistor at ambient temperature. Design the circuit in



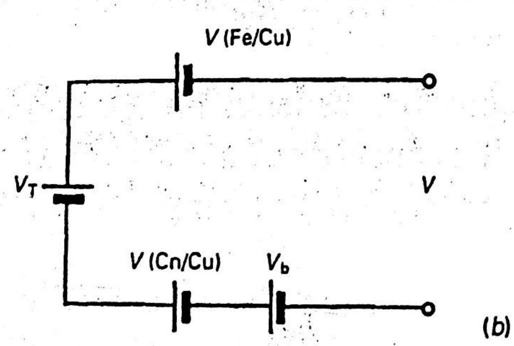


Figure E6.3 (*a*) Proposed circuit for cold junction compensation. (*b*) Equivalent circuit when lead and bridge resistance are much smaller than the input resistance of the voltmeter.

order to have compensation in the range from 10 °C to 40 °C with an NTC thermistor having $B = 3546$ K and resistance 10 kΩ at 25 °C.

If we assume that the current along thermocouple wires is very low because of the high input impedance offered by the voltmeter, then the equivalent circuit for the thermocouples is shown in Figure E6.3*b*. If we call the bridge output V_b, then we have

$$V_T - V(\text{Fe/Cu})|_{T_a} + V(\text{Cn/Cu})|_{T_a} + V_b = V$$

In order for the ambient temperature not to affect the measurement, we need

$$V = V_T$$

By applying the law of intermediate metals, we have

$$-V(\text{Fe/Cu})|_{T_a} + V(\text{Cn/Cu})|_{T_a} = -V(\text{Fe/Cn})|_{T_a} \approx -kT_a$$

where $k \approx 52$ μV/°C is the sensitivity for the J thermocouple in the range from 10 °C to 40 °C, assuming that it is constant (Table 6.2).

In principle, we are thus interested in having $V_b = kT_a$; that is,

$$\frac{dV_b}{dT} = k$$

and also $V_b(0\,°\text{C}) = 0$ V. The actual bridge output is

$$V_b = -V_R\left(\frac{R_1'}{R_1' + R_2} - \frac{R_4}{R_3 + R_4}\right)$$

where $R_1' = R_1 + R_0 \exp(B/T - B/T_0)$. The bridge sensitivity is

$$\frac{dV_b}{dT} = V_R \frac{\dfrac{BR_2}{T^2} R_0 e^{B(1/T - 1/T_0)}}{\left(R_1 + R_0 e^{B(1/T - 1/T_0)} + R_2\right)^2}$$

which, far from being constant, depends on the temperature. This means that the bridge is nonlinear. If as linearization criterion we chose to have the desired slope at the middle of the temperature range to be compensated (25 °C), then we have

$$V_R \frac{\dfrac{BR_2}{(298\text{ K})^2} R_0}{(R_1 + R_0 + R_2)^2} = k$$

Another condition that we can force is that at this same temperature the bridge output equals the voltage at the reference junction, namely about 1.3 mV (Table 6.2). Therefore

$$1.3 \text{ mV} = -V_R \left(\frac{R_1'}{R_1' + R_2} - \frac{R_4}{R_3 + R_4} \right)$$

In order for V_R to be stable, we can choose a mercury cell (1.35 V). In order to calculate the values for the resistors, we can choose R_2 and then the equation for the slope determines R_1 (or conversely) and the output at 25°C determines the ratio R_3/R_4. In the first case, for example, if $R_2 = 100\ \Omega$ we have

$$R_1 = 22{,}097\ \Omega$$

$$\frac{R_3}{R_4} = 2.15 \times 10^{-3}$$

For example, $R_1 = 22.1\ \text{k}\Omega$, $R_3 = 1\ \text{k}\Omega$, and $R_4 = 46.4\ \text{k}\Omega$.

6.2 PIEZOELECTRIC SENSORS

6.2.1 The Piezoelectric Effect

The piezoelectric effect is the appearance of an electric polarization in a material that strains under stress. It is a reversible effect. Therefore, when applying an electric voltage between two sides of a piezoelectric material, it strains. Both effects were discovered by Jacques and Pierre Curie in 1880–1881.

Piezoelectricity must not be confused with ferroelectricity, which is the property of having a spontaneous or induced electric dipole moment. Ferroelectricity was first discovered by J. Valasek in 1921 in Rochelle salt. All ferroelectric materials are piezoelectric, but the converse is not always true. Piezoelectricity is related to the crystalline (ionic) structure. Ferromagnetism is instead related to electron spin.

Piezoelectric equations describe the relationship between electric and mechanical quantities in a piezoelectric material. In Figure 6.14*a*, where two metal plates have been placed to form a capacitor, for a dielectric nonpiezoelectric material we have that an applied force F yields a strain S that, according to Hooke's law (Section 2.2), in the elastic range is

$$S = sT \tag{6.10}$$

where s is compliance, $1/s$ is Young's modulus, and T is the stress (F/A).

A potential difference applied between plates creates an electric field $\boldsymbol{E}$ and we have

$$\boldsymbol{D} = \epsilon \boldsymbol{E} = \epsilon_0 \boldsymbol{E} + \boldsymbol{P} \tag{6.11}$$

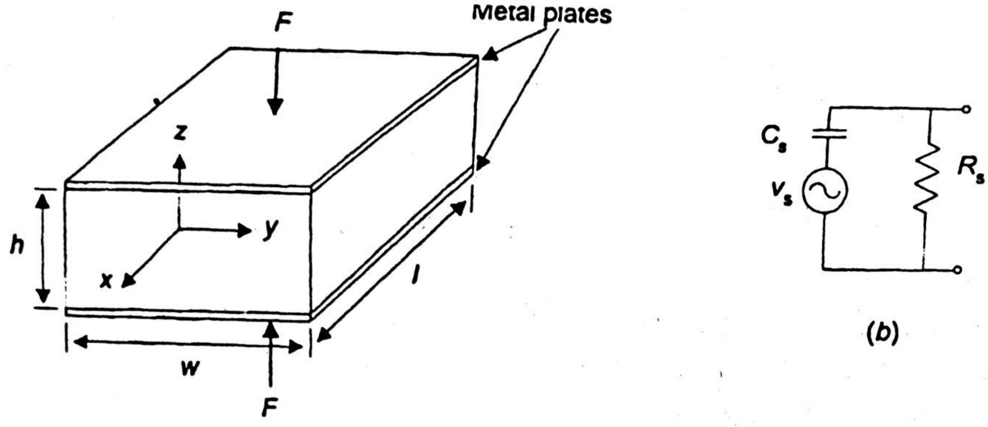


Figure 6.14 (*a*) Parameters used in piezoelectric equations. (*b*) Equivalent circuit for a piezoelectric sensor.

where $\boldsymbol{D}$ is the displacement vector (or electric flux density), ϵ is the dielectric constant, $\epsilon_0 = 8.85$ pF/m is the permittivity of vacuum, and $\boldsymbol{P}$ is the polarization vector.

For a unidimensional piezoelectric material with field, stress, strain, and polarization in the same direction, according to the principle of energy conservation, at low frequency we have

$$D = dT + \epsilon^T E \tag{6.12}$$

$$S = s^E T + d'E \tag{6.13}$$

where ϵ^T is the permittivity at constant stress and s^E is the compliance at constant electric field. Therefore, when compared to a nonpiezoelectric material, there is also a strain due to the electric field and an electric charge due to the mechanical stress (charges displaced inside the material induce opposite polarity surface charges on the plates).

When the surface area does not change under the applied stress (which is not true in polymers), then $d = d'$ [5]. d is the *piezoelectric charge coefficient* or piezoelectric constant, whose dimensions are coulombs divided by newtons [C/N].

Solving equation (6.12) for E yields

$$E = \frac{D}{\epsilon^T} - \frac{Td}{\epsilon^T} = \frac{D}{\epsilon^T} - gT \tag{6.14}$$

where $g = d/\epsilon^T$ is the *piezoelectric voltage coefficient.*

By solving (6.13) for T, we have

$$T = -\frac{d}{s^E}E + \frac{1}{s^E}S = c^E S - eE \tag{6.15}$$

where $e = d/s^E$ is the *piezoelectric stress coefficient.*

Example 6.4 For lead titanate we have in the principal direction $d = -44$ pC/N, $\epsilon^T = 600\epsilon_0$, $g = -8$ (mV/m)/(N/m^2), $e = -4.4$ C/m^2, and $s^E = 1/(100$ GPa).

For a cube with 1 cm sides, from (6.14) 1000 N ($\approx$ 100 kg) yields at open circuit ($D = 0$)

$$E = -\frac{dT}{\epsilon^T} = -82.8 \text{ kV/m}$$

that is, 828 V between two sides.

If 1 kV is applied between two sides, the resulting strain is

$$S = dE = -44 \times 10^{-7} = -4.4 \ \mu\varepsilon$$

and the elongation is

$$\Delta l = (1 \text{ cm}) \times 44 \times 10^{-7} = 44 \text{ nm}$$

The *electromechanical coupling coefficient* is the square root of the quotient between the energy available at the output and the stored energy, at frequencies well below that of mechanical resonance. Therefore, it is nondimensional. It can be shown that

$$k = \sqrt{\frac{d^2}{\epsilon^T s^E}} \qquad (6.16)$$

A three-dimensional crystalline solid can experience tension and also compression forces along the three coordinate axes, designated by the subscripts 1, 2, 3, and also torsion forces designated by the subscripts 4, 5, and 6 (Figure 6.15). Using this notation, when there is no piezoelectric effect we have

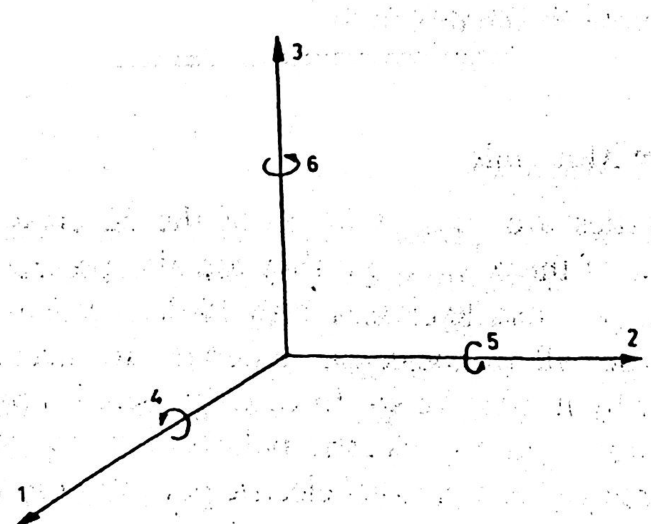


Figure 6.15 Meaning of the indices for the directions in a piezoelectric material.

$$[S_i] = [s_{ij}][T_j] \qquad i = 1,2,3 \qquad j = 1,\ldots,6 \tag{6.17}$$

$$[D_i] = [\epsilon_{ij}][E_j] \qquad i,j = 1,2,3 \tag{6.18}$$

When there is a piezoelectric effect, the piezoelectric equations are

$$[S_i] = [s_{ij}][T_j] + [d_{ik}][E_k] \tag{6.19}$$

$$[D_i] = [\epsilon_{lm}][E_m] + [d_{\mathrm{ln}}][T_n] \tag{6.20}$$

where $j,n = 1,\ldots,6$, and $i,k,l,m = 1,2,3$.

The coefficients d_{ij} are the piezoelectric constants, which relate the electric field in direction i to the deformation in direction j, and also the surface charge density in the direction perpendicular to i with the stress in direction j. It holds that $d_{ij} = d_{ji}$, and it also holds that $\epsilon_{lm} = 0$ whenever $l \neq m$.

Also we have

$$d_{ij} = \epsilon_i g_{ij} \tag{6.21}$$

The same notation applies to the subscripts of the coupling coefficient k.

Example 6.5 PXE 5 material (Philips) has the following specifications:

Piezoelectric Charge Constants	Piezoelectric Voltage Constants	Coupling Coefficient
$d_{33} = 384$ pC/N	$g_{33} = 24.2 \times 10^{-3}$ V·m/N	$k_{33} = 0.70$
$d_{31} = -169$ pC/N	$g_{33} = -10.7 \times 10^{-3}$ V·m/N	$k_{31} = 0.34$
$d_{15} = 515$ pC/N	$g_{15} = 32.5 \times 10^{-3}$ V·m/N	$k_{15} = 0.66$

This means, for example, that a torsion stress of 1 N/m^2 applied around the axis 2 (direction "5") induces a charge density of 515 pC/m^2 in two metal plates placed on the material in the direction 1.

6.2.2 Piezoelectric Materials

Piezoelectric properties are present in 20 of the 32 crystallographic classes, although only a few of them are used; they are also present in amorphous ferroelectric materials. Of those 20 classes, only 10 display ferroelectric properties.

Whatever the case, all piezoelectric materials are necessarily anisotropic. Figure 6.16 shows why it must be so. In case (a) there is central symmetry. An applied force does not yield any electric polarization. In case (b), on the contrary, an applied force yields a parallel electric polarization, while in case (c) an applied force yields a perpendicular polarization.

The natural piezoelectric materials most frequently used are quartz and

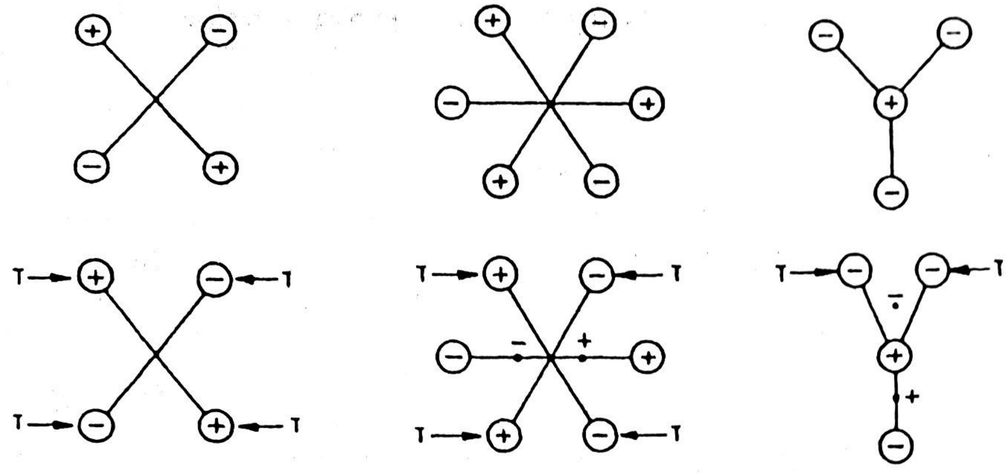


Figure 6.16 Effects of a mechanical stress on different molecules depending on their symmetry [5]: (*a*) When there is central symmetry, no electric polarization arises. (*b*) Polarization parallel to the effort. (*c*) Polarization perpendicular to the effort.

tourmaline. The synthetic materials more extensively used are not crystalline but ceramics. These are formed by many little tightly compacted monocrystals (about 1 μm in size). These ceramics are ferroelectrics, and to align the monocrystals in the same direction (i.e., to polarize them) they are subjected to a strong electric field during their fabrication. The applied field depends on the material thickness, but values of about 10 kV/cm are common at temperatures slightly above the Curie temperature (at higher temperatures they are too conductive). They are cooled in the maintained field. When the field is removed, the monocrystals cannot reorder randomly because of the mechanical stresses accumulated, so that a permanent electric polarization remains.

Piezoelectric ceramics display a high thermal and physical stability and can be manufactured in many different shapes and with a broad range of values for the properties of interest (dielectric constant, piezoelectric coefficient, Curie temperature, etc.). Their main shortcomings are the temperature sensitivity of their parameters and their susceptibility to aging (loss of piezoelectric properties) when they are close to their Curie temperature. The most commonly used ceramics are lead zirconate titanate (PZT), barium titanate, and lead niobate. Bimorphs consist of two ceramic plates glued together and with opposite polarization. If one end is clamped and a mechanical load is applied to the other, one plate elongates and the other shortens, thus generating two voltages of the same amplitude.

Some polymers lacking central symmetry also display piezoelectric properties with a value high enough to consider them for those applications where because of the size and shape required it would be impossible to use other solid materials. The most common is polyvinylidene fluoride (PVF_2 or PVDF), whose piezoelectric voltage coefficient is about four times that of quartz, and its copolymers. Electrodes are screen printed or vacuum deposited.

In order to improve the mechanical properties for piezoelectric sensors

TABLE 6.5 Some Properties for Common Piezoelectric Materials

Parameter Unit	Density (kg·m^{-3})	T_C (°C)	$\epsilon^T{}_{11}/\epsilon_0$	$\epsilon^T{}_{33}/\epsilon_0$	d (pC/N)	Resistivity (Ω·cm)
Quartz	2649	550	4.52	4.68	d_{11} 2.31 d_{14} 0.73	≈ 10^{14}
PZT	7500–7900	193–490	—	425–1900	d_{33} 80–593	≈ 10^{13}
PVDF (Kynar)	1780	—	—	12	d_{31} 23	≈ 10^{15}

zoelectric "composite" materials are used. They are heterogeneous systems consisting of two or more different phases, one of which at least shows piezoelectric properties. Table 6.5 lists the most important properties of some common piezoelectric materials.

6.2.3 Applications

The application of the piezoelectric effect to sense mechanical quantities based on (6.19) and (6.20) is restricted by several limitations. First, the electric resistance for piezoelectric materials is very high but never infinite (Figure 6.14*b*). Therefore, a constant stress initially generates (or, better, displaces) a charge that will slowly drain off as time passes. Hence, there is no dc response.

Example 6.6 A given piezoelectric sensor uses a PVDF strip measuring 10 cm by 10 cm and 52 μm thick, with deposited electrodes in the vertical direction (direction 3 in Figure 6.15). Calculate the voltage output when applying a 40 kg compression along direction 1. Determine the lowest frequency of a dynamic compression for an allowed amplitude error of 5%.

When stressing the film, electric charge ($Q_3 = D_3 A_3$) accumulates on the electrodes of area A_3 to yield a voltage $V_3 = Q_3/C_3$. From (6.19), the electric polarization is $D_3 = d_{31} T_1 = d_{31} F_1/A_1$. From Table 6.5, $d_{31} = 23$ pC/N and $\epsilon_{33}^T = 12\epsilon_0$. Therefore,

$$V_3 = d_{31}\frac{F_1}{A_1}A_3\frac{h}{\epsilon_{33}^T A_3} = d_{31}\frac{F_1}{lh}\frac{h}{\epsilon_{33}^T} = d_{31}\frac{F_1}{l\epsilon_{33}^T}$$

$$= \left(23\frac{\text{pC}}{\text{N}}\right)\frac{40 \times 9.8 \text{ N}}{(0.1 \text{ m})\left(12 \times 8.85\dfrac{\text{pF}}{\text{m}}\right)} = 849 \text{ V}$$

From Table 6.5, $\rho = 10$ TΩ·m. Therefore, the leakage resistance between electrodes is

$$R_3 = \rho\frac{h}{A_3} = (10^{13}\ \Omega \cdot \text{m})\frac{52\ \mu\text{m}}{0.1 \times 0.1 \text{ m}^2} = 52 \text{ G}\Omega$$

The capacitance between electrodes and the leakage resistance form a high-pass filter whose transfer function is

$$H(f) = \frac{j2\pi f R_3 C_3}{1 + j2\pi f R_3 C_3} = \frac{1}{1 - j\dfrac{f_c}{f}}$$

where $f_c = 1/(2\pi R_3 C_3)$. C_3 can be estimated as the capacitance of a parallel-plate capacitor,

$$C_3 = 12 \times 8.85 \frac{\text{pF}}{\text{m}} \times \frac{0.1 \times 0.1\ \text{m}^2}{56\ \mu\text{m}} = 20.4\ \text{nF}$$

An amplitude error below 5% imposes the condition

$$\frac{1}{\sqrt{1 + \left(\dfrac{f_c}{f}\right)^2}} > 0.95$$

$$f > 3.04 f_c = 3.04 \frac{1}{2\pi \times (52\ \text{G}\Omega) \times (20.4\ \text{nF})} = 0.5\ \text{mHz}$$

Piezoelectric sensors show a high resonant peak in their frequency response. This is because when a dynamic force is applied to them, the only damping source is the internal friction in the material. Thus we must always work at frequencies well below the mechanical resonant frequency, and the sensor output must be low-pass filtered to prevent amplifier saturation. Figure 6.17 shows the frequency response of a commercial piezoelectric accelerometer. The gain at the resonant frequency (35 kHz) is 20 times that in the 5 Hz to 7 kHz band, where the frequency response is flat within ±5%. Reference 6 describes a method to increase the useful range up to the resonant frequency. It is based on electromechanical feedback relying on the reversibility of the piezoelectric effect, thus providing damping to the otherwise undamped system.

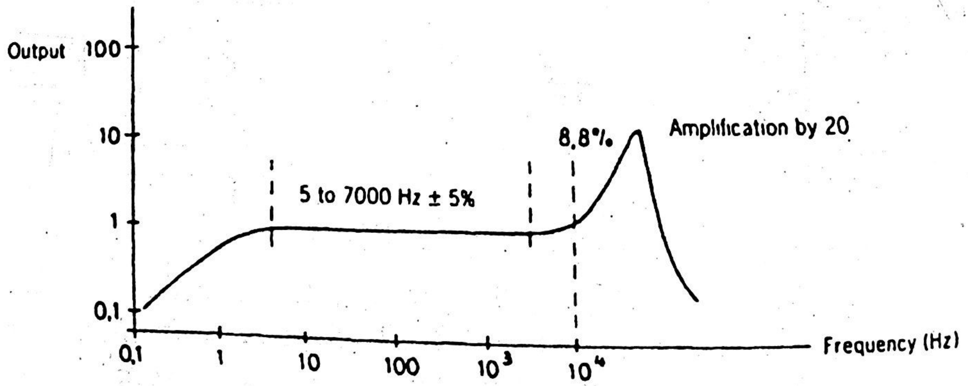


Figure 6.17 Frequency response for a piezoelectric accelerometer displaying a large resonance and lack of dc response.

The piezoelectric coefficients are temperature-sensitive. Furthermore, above the Curie temperature all materials lose their piezoelectric properties. That temperature is different for each material, and in some cases it is even lower than typical temperatures in industrial environments. Quartz is used up to 260 °C, tourmaline up to 700 °C, barium titanate up to 125 °C, and PVDF up to 135 °C. Some materials that display piezoelectric properties are hygroscopic and therefore are inappropriate for sensors.

Piezoelectric materials have a very high output impedance (small capacitance with a high leakage resistance) (Figure 6.14*b*). Therefore, in order to measure the signal generated we must use an electrometer (voltage mode) or charge amplifiers (charge mode) (Sections 7.2 and 7.3). Some sensors include an integrated amplifier, but this limits the temperature of operation to the range acceptable for the electronic components.

Piezoelectric sensors offer high sensitivity (more than one thousand times that of strain gages), usually at a low cost. They undergo deformations smaller than 1 μm, and this high mechanical stiffness makes them suitable for measuring effort variables (force, pressure). Equation (1.31) shows that a high stiffness results in a broad frequency range. Their small size (even less than 1 mm) and the possibility of manufacturing devices with unidirectional sensitivity are also properties of interest in many applications, particularly for vibration monitoring.

Figure 6.18 shows several simplified examples illustrating different possible

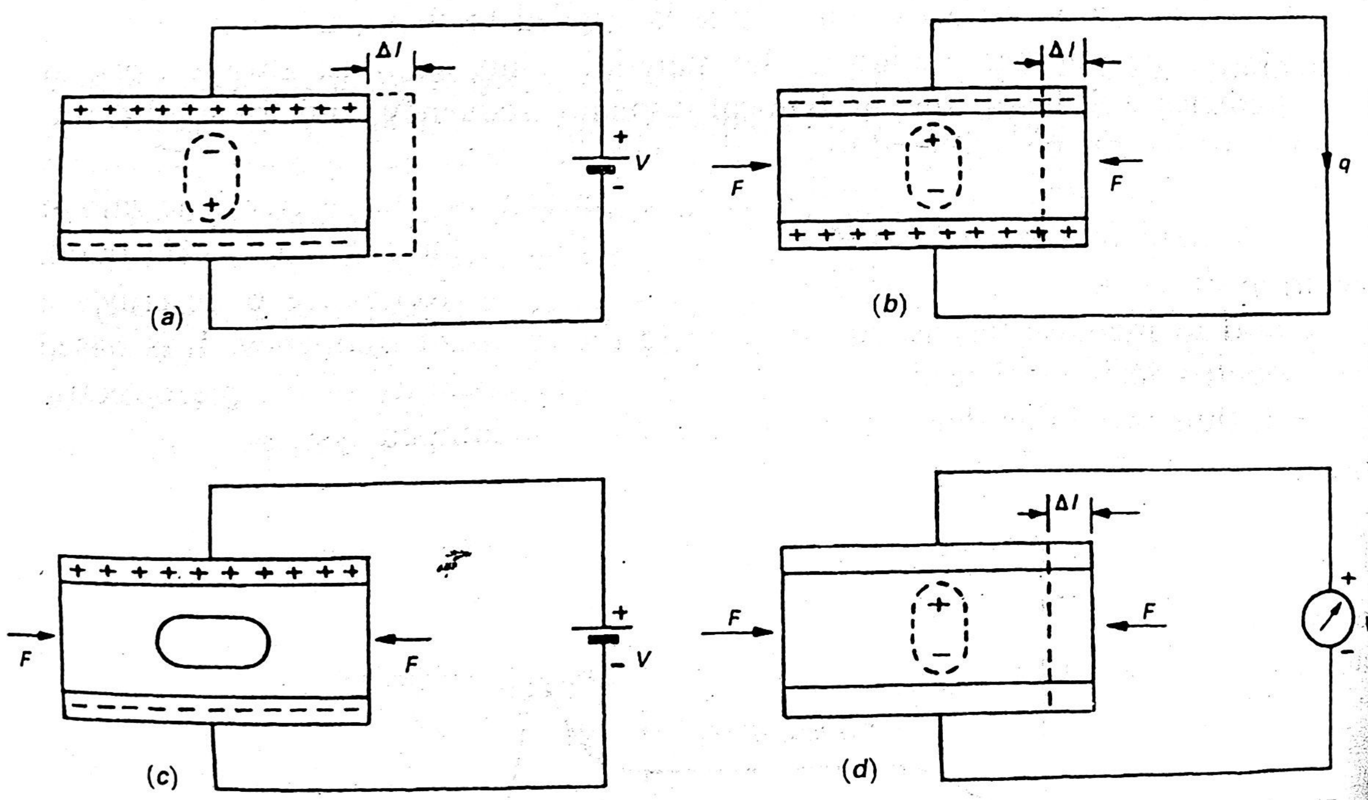


Figure 6.18 Several forms of applying the piezoelectric effect at low frequencies. In each case, one of the quantities is zero. (*a*) Null effort, $T = 0$; (*b*) null electric field, $E = 0$; (*c*) null strain, $S = 0$; (*d*) null charge density, $D = 0$. (From W. Welkowitz and ... copyright 1976. Reprinted by

low-frequency applications for the piezoelectric effect [7]. In case (a) no force is applied but only a voltage V. Therefore, a strain results. Given that $T = 0$, from (6.13) we have

$$S = dE \tag{6.22}$$

With the terminology shown in Figure 6.14, if the strain is in the longitudinal direction (x), we have

$$\frac{\Delta l}{l} = d\frac{V}{h} \tag{6.23}$$

From (6.12) we have

$$D = \epsilon^T E \tag{6.24}$$

that is, an electric polarization appears as in any normal capacitor. This arrangement is used for micropositioning—for example, of mirrors in lasers and of samples in scanning tunneling microscopes [8, 9].

In case (b) the metallic plates are short-circuited and a force F is applied. The result is that a polarization appears because electric charges migrate from one plate to the other. Given that $E = 0$, from (6.12) we have

$$D = dT \tag{6.25}$$

The charge obtained will be

$$q = Dlw = lwd\frac{F}{hw} = \frac{ld}{h}F \tag{6.26}$$

As in any solid body, a compression strain also results:

$$S = s^E T \tag{6.27}$$

This arrangement is applied to measure vibration, force, pressure, and deformation.

In case (c) the net deformation is zero because a force F is applied just to compensate for the field E due to the applied voltage. Therefore, we have $S = 0$ and from (6.13) we deduce

$$0 = s^E T + dE \tag{6.28}$$

and then

The charge induced at each plate can be calculated from (6.12) to be

$$D = \frac{q}{wl} = dT + \epsilon^T E = d\frac{F}{wh} + \epsilon^T \frac{V}{h} \tag{6.30}$$

$$q = V\frac{wl}{h}\left(\epsilon^T - \frac{d^2}{s^E}\right) \tag{6.31}$$

The factor enclosed by the parentheses is designated ϵ^S, and it shows that the dielectric constant decreases because of the piezoelectric effect.

For the open circuit, case (*d*), it is not possible to transfer any charge from one plate to the other (although there will always be a certain leakage through the voltmeter). Therefore, despite the applied force, we have $D = 0$. From (6.12) we deduce

$$0 = dT + \epsilon^T E \tag{6.32}$$

$$V = -\frac{dhT}{\epsilon^T} = -\frac{dFh}{wh\epsilon^T} = -\frac{dF}{w\epsilon^T} \tag{6.33}$$

The resulting strain will be

$$S = s^E T + dE = s^E \frac{F}{wh} + d\frac{V}{h} \tag{6.34}$$

$$\frac{\Delta l}{l} = \frac{F}{wh}\left(s^E - \frac{d^2}{\epsilon^T}\right) \tag{6.35}$$

The term inside the parentheses is now designated s^D; and it shows that because of the piezoelectric effect, the material stiffness increases. A hammer or cam striking a piezoelectric ceramic generates more than 20 kV. The resulting spark is used for lighting gas ranges or for ignition in small internal combustion engines.

The application of the arrangement in Figure 6.18*b* to the measurement of forces, pressures, and movements (using a mass–spring system) is straightforward, and it is very similar for the three quantities. Figure 6.19 shows an outline for the three types of sensors. This similarity makes these sensors sensitive to the three quantities, and therefore special designs are required that minimize interference. Figure 6.20 shows a pressure sensor compensated for acceleration by combining signals from the stressed diaphragm and an inertial mass. Table 6.6 gives some characteristics for two quartz pressure sensors. Piezoelectric pressure sensors are used for monitoring internal combustion engines and in hydrophones. Because they lack dc response, they do not suit load cells.

Table 6.7 lists some data for two quartz accelerometers with integral electronics. Piezoelectric sensors with integral electronics are more reliable than sensors with external electronics because the connector is less critical, which is

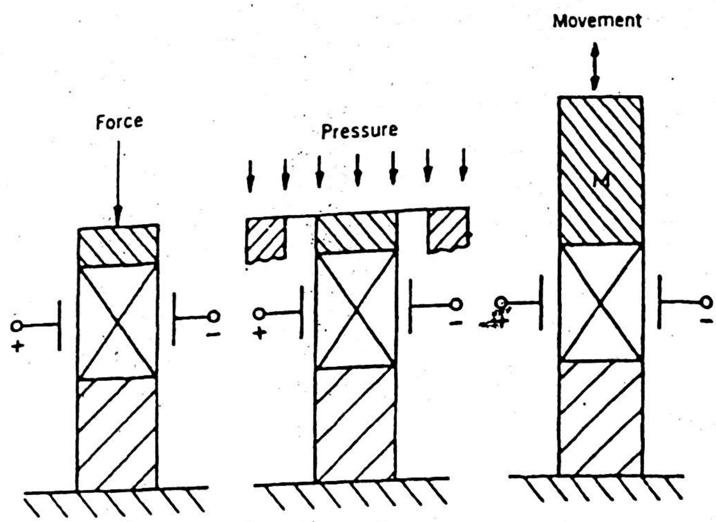


Figure 6.19 Force, pressure, and movement sensors based on piezoelectric elements (courtesy of PCB Piezotronics).

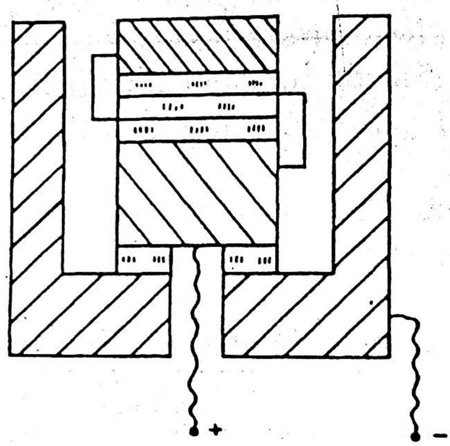


Figure 6.20 Piezoelectric pressure sensor with acceleration compensation by combining the signals from piezoelectric materials sensing both pressure and acceleration with that of one sensing only acceleration (courtesy of PCB Piezotronics).

important in shock and vibration monitoring. Piezoelectric accelerometers offer wider frequency bandwidth (0.1 Hz to 30 kHz), much lower power consumption, and higher shock survivability than micromachined accelerometers. However, they are inferior for static or very low frequency measurements. They are applied to machine monitoring, shock detection in shipment monitoring, impact detection, or drop testing, vehicle dynamics assessment and control, and structural dynamics analysis to detect response to load, fatigue, and resonance.

Polymer-based piezolectric sensors are applied to microphones, machine monitoring, leak detection in pipes and high-pressure vessels (which produce a characteristic sound), keyboards, coin sensors, occupancy sensing, and vehicle classification and counting in highways. They are becoming relevant in medical applications such as pacemaker rate adjustment according to acceleration, sleep disorder monitoring, blood pressure monitoring, and blood flow and respiratory sounds monitoring in ambulances [10].