MODULE 3

TRANSIENT BEHAVIOR AND INITIAL CONDITIONS

A network in which branch current & node voltages are not changing wrt. time is said to be in steady state.

→ Even if the voltages & currents in the ckt. have constant amplitude & frequency throughout the time interval it is said to be steady state.

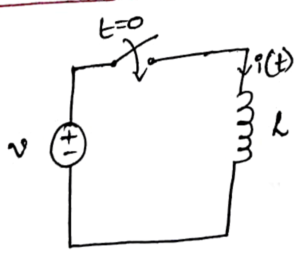
→ The time interval required for a ckt. to switch from one condition to another by changing applied voltage (or) by changing branch currents & voltages from one value to new value is called transition period.

→ The response (or) o/p. of n/w. during transition period is called transient response of n/w.

→ If the n/w. condition is not disturbed after transition period, then the n/w. attains steady state at infinite time.

* Initial & final conditions in elements :-

i) The inductor :-



The switch is closed at $t=0$. Hence $t=0^-$ corresponds to the instant when the switch is just open & $t=0^+$ corresponds to the instant when the switch is just closed.

∴ The current through inductor is,

$$i = \frac{1}{L}\int_{-\infty}^{t} v\,dt$$

$$i = \frac{1}{L}\int_{-\infty}^{0^-} v\,dt + \frac{1}{L}\int_{0^-}^{t} v\,dt$$

$$i(t) = i(0^-) + \frac{1}{L}\int_{0^-}^{t} v\,dt$$

put $t = 0^+$ on both sides, we get

$$i(0^+) = i(0^-) + \frac{1}{L}\int_{0^-}^{0^+} v\,dt$$

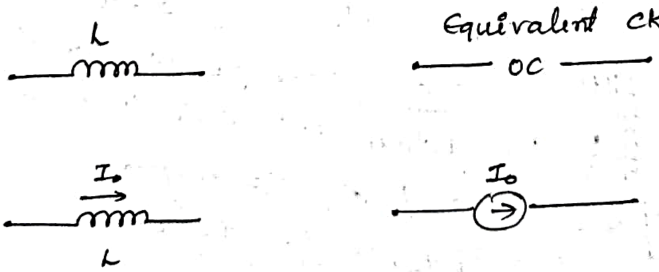
$$\boxed{i(0^+) = i(0^-)}$$

From above eqn. it is clear that the current in an inductor cannot change instantaneously.

* If $i(0^-) = 0$, then $i(0^+) = 0$. This means at $t = 0^+$, inductor acts as an open circuit, irrespective of the voltage across terminals.

* If $i(0^-) = I_0$, then $i(0^+) = I_0$, then the inductor can be thought of as a current source of 'I_0' A ∴ the equivalent ckt. is shown below,

Equivalent ckt. at $t=0^+$



The initial condition equivalent ckt. of an inductor

The final condition equivalent ckt. of an inductor is derived from basic relationship,

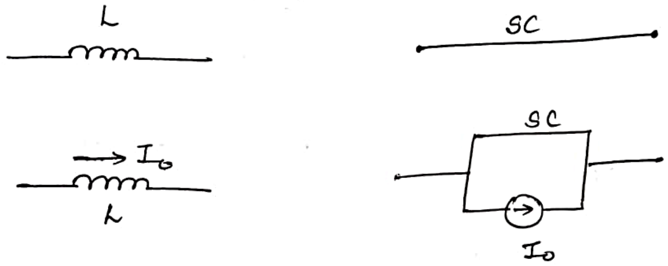
$$v = L \frac{di}{dt}$$

Under steady state condition, $\frac{di}{dt} = 0$

$\therefore v = 0$

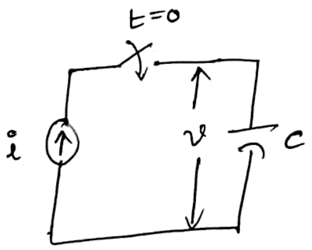
Hence 'L' acts as short ckt. at $t = \infty$ [final (or) steady state]

Equivalent ckt. at $t = \infty$



The final condition equivalent ckt. of an inductor

ii) The capacitor :-



The voltage across capacitor is given by,

$$v = \frac{1}{C} \int_{-\infty}^{t} i \, d\tau$$

$$v(t) = \frac{1}{C} \int_{-\infty}^{0^-} i \, d\tau + \frac{1}{C} \int_{0^-}^{t} i \, d\tau$$

$$v(t) = v(0^-) + \frac{1}{C} \int_{0^-}^{t} i \, d\tau.$$

At $t = 0^+$:

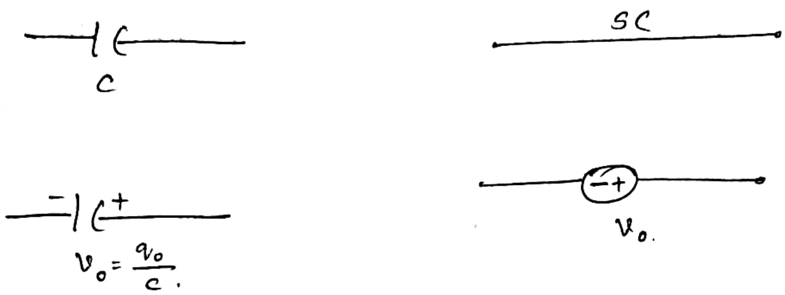
$$v(0^+) = v(0^-) + \frac{1}{C} \int_{0^-}^{0^+} i \, d\tau$$

$$\boxed{v(0^+) = v(0^-)}$$

$\therefore$ Vtg. across a capacitor cannot change instantaneously

* If $v(0^-) = 0$, then $v(0^+) = 0$, then at $t = 0^+$, capacitor acts as short ckt.
* If $v(0^-) = \frac{q_0}{c}$, then $v(0^+) = \frac{V_0}{c}$, then capacitor acts as voltage source of 'V_0' V.

Equivalent ckt at $t = 0^+$



Initial condition equivalent ckts of a capacitor

∴ The current through capacitor is given by,

$$i = C\frac{dv}{dt}$$

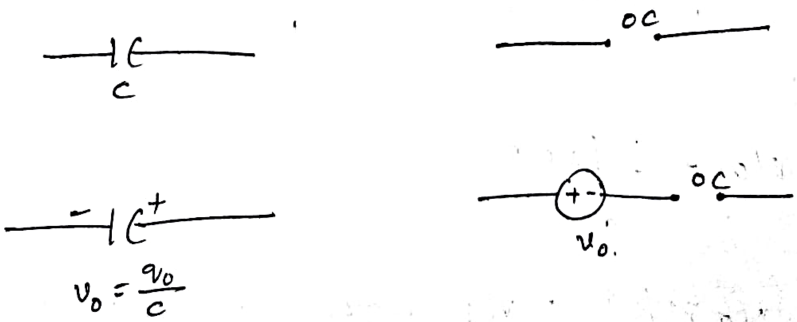
Under steady state condition,

$$\frac{dv}{dt} = 0$$

∴ At $t = \infty$, $i = 0$

i.e. At $t = \infty$ (or) steady state, capacitor acts as an open ckt.

Equivalent ckt. at $t = \infty$



Final condition equivalent ckt. of a capacitor

iii) The resistor :-

$v = Ri$

Current through ~~resistor~~ will change instantaneously if voltage changes instantaneously. Similarly, voltage will change instantaneously if current changes instantaneously.

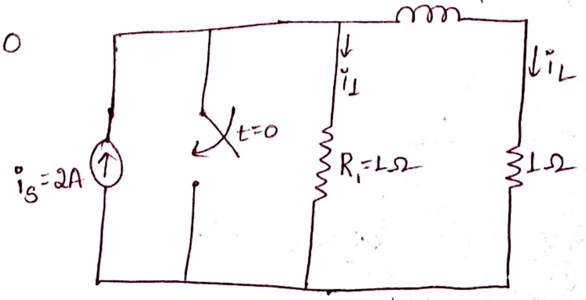
* Procedure for evaluating initial conditions:-

→ At first, the initial values of currents & voltages are found & then solve for the derivatives

→ For finding initial values of currents & voltages of the original n/w. at $t = 0^+$ an equivalent n/w. is constructed using following rules:

1. Replace all inductors with open ckt. or with current sources having the value of current flowing at $t = 0^+$
2. Replace all capacitors with short ckt. or with an vtg. srcs. of value $V_0 = \frac{q_0}{C}$ if there is an initial charge.
3. Resistors are left in the n/w. without any change.

1. Find $i_1(0^+)$ & $i_L(0^+)$. The ckt. is in steady state for $t < 0$



At $t=0^+$:

$$0.15\frac{dV_2(0^+)}{dt} + 0.05\frac{d^2v_2(0^+)}{dt^2} = -0.1e^{-(0^+)} = -0.1$$

$$(0.15\times 2) + 0.05\frac{d^2v_2(0^+)}{dt^2} = -0.1$$

$$\therefore \boxed{\frac{d^2v_2(0^+)}{dt^2} = -8\,V/sec^2}$$

Diff. (2) with 't'

$$0.15\frac{d^2v_2}{dt^2} + 0.05\frac{d^3v_2}{dt^3} = 0.1e^{-t}$$

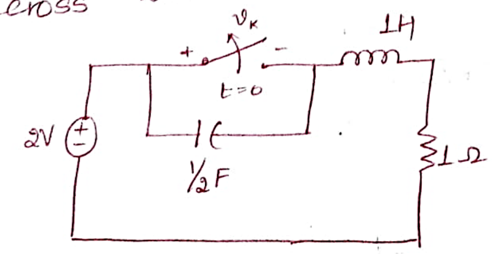
At $t=0^+$:

$$0.15\frac{d^2v_2(0^+)}{dt^2} + 0.05\frac{d^3v_2(0^+)}{dt^3} = 0.1e^{-(0^+)} = 0.1$$

$$0.15(-8) + 0.05\frac{d^3v_2(0^+)}{dt^3} = 0.1$$

$$\boxed{\frac{d^3v_2(0^+)}{dt^3} = 26\,V/sec^3}$$

9 The ckt. is in steady state with switch 'K' close. At $t=0$, the switch is opened. Determine the vtg. across the switch v_K & $\frac{dv_K}{dt}$ at $t=0^+$.



At $t=0^+$: sw → open

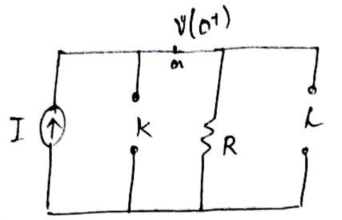


fig. (1)

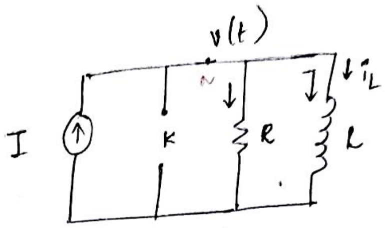


fig.(2)

$\therefore\ v(0^+) = I \times R = 2 \times 200$

$$\boxed{v(0^+) = 400V}$$

At $t \geq 0^+$, KCL at 'a'

$$I = \frac{v(t)}{R} + \frac{1}{L}\int_{0^+}^{t} v(\tau)d\tau \longrightarrow (1)$$

$$\left[\cancel{I = \frac{v(0^+)}{R}} + \cancel{\frac{1}{L}\int_{0^+}^{t} v(0^+)}\right]$$

Diff. (1) wrt 't'

$$0 = \frac{1}{R}\frac{dv}{dt} + \frac{1}{L}v(t) \longrightarrow (2)$$

At $t=0^+$,

$$0 = \frac{1}{R}\frac{dv(0^+)}{dt} + \frac{1}{L}v(0^+)$$

$$0 = \frac{1}{200} \cdot \frac{dv(0^+)}{dt} + \frac{400}{1}$$

$$\therefore \boxed{\frac{dv(0^+)}{dt} = 8 \times 10^4 V/sec.}$$

Diff. (2) wrt 't'

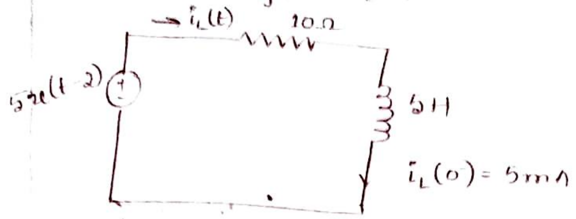
$$0 = \frac{1}{R}\frac{d^2v}{dt^2} + \frac{1}{L}\frac{dv}{dt}$$

At $t=0^+$,

$$0 = \frac{1}{200} \cdot \frac{d^2v(0^+)}{dt^2} + \frac{1}{1} \cdot (8 \times 10^4).$$

$$\therefore \boxed{\frac{d^2v(0^+)}{dt^2} = 16 \times 10^6 V/sec^2}$$

1 For the figure shown, (a) write the differential equation for the inductor current $i_L(t)$.
(b) Find $I_L(s)$ the Laplace transform of $i_L(t)$.
(c) Solve for $i_L(t)$



⇒ Apply KVL, $5u(t-2) - 10i_L(t) - 5\frac{di_L(t)}{dt} = 0$

$$\therefore 5u(t-2) = 10i_L(t) + 5\frac{di_L(t)}{dt}$$

Apply L.T.

$$\frac{5e^{-2s}}{s} = 10I_L(s) + 5\left[sI_L(s) - i_L(0^-)\right] \qquad u(t-a) \xrightarrow{L.T} \frac{e^{-as}}{s}$$

$$\hookrightarrow = [10+5s]I_L(s) - 5i_L(0^-)$$

$$\frac{5e^{-2s}}{s} + 5\times5\times10^{-3} = (10+5s)I_L(s)$$

$$\therefore I_L(s) = \frac{\cancel{5}\left[\frac{e^{-2s}+5\times10^{-3}s}{s}\right]}{\cancel{5}[s+2]} = \frac{e^{-2s}+5\times10^{-3}s}{s[s+2]}$$

$$I_L(s) = \frac{e^{-2s}}{s[s+2]} + \frac{5\times10^{-3}}{s+2} = e^{-2s}\left[\frac{A}{s} + \frac{B}{s+2}\right] + \frac{5\times10^{-3}}{s+2}$$

$$\therefore 1 = A(s+2) + Bs.$$

Let $s=0$; $1 = A(2)$; $A = 1/2$

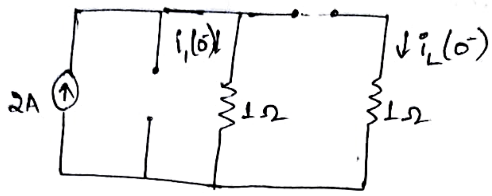
Let $s=-2$; $1 = 0 + B(-2)$; $B = -1/2$

$$\therefore I_L(s) = e^{-2s}\left[\frac{1/2}{s} - \frac{1/2}{s+2}\right] + \frac{5\times10^{-3}}{s+2}$$

$\therefore$ Taking Inverse L.T.

c) $$i_L(t) = \frac{1}{2}u(t-2) - \frac{1}{2}e^{-2(t-2)}\cdot u(t-2) + 5\times10^{-3}e^{-2t}u(t).$$

→ Soln: At $t = 0^-$; switch is open & ckt is in steady state & 'L' acts as short ckt



Using current division,

$$i_L(0^-) = \frac{1 \times 2}{1+1} = 1A$$

Since the current in inductor will not change instantaneously,

$$i_L(0^-) = i_L(0^+) = 1A$$

At $t=0^-$, $i_1(0^-) = 2 - i_L(0^-)$

$= 2 - 1$

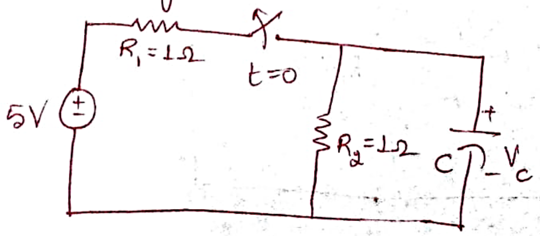
$i_1(0^-) = 1A$

At $t=0^+$: switch is closed, the voltage drop across R_1 is zero

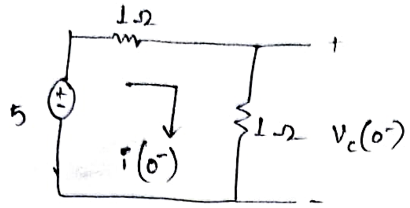
$\therefore$ $i_1(0^+) = 0A$

$\therefore$ $\boxed{i_1(0^+) = 0A ;\ i_L(0^+) = 1A}$

2. Find $v_c(0^+)$. Assume that switch was in closed state for long time.



→ Soln:- At $t=0^-$; Switch is closed ; C = open ckt



$$i(0^-) = \frac{5}{1+1} = \frac{5}{2} = 2.5A$$

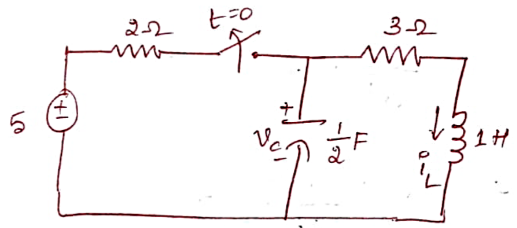
$$V_c(0^-) = R_2 i(0^-) = 1 \times 2.5$$

$$V_c(0^-) = 2.5V$$

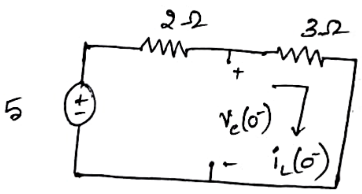
Since voltage across capacitor will not change instantaneously.

$$\boxed{V_c(0^-) = V_c(0^+) = 2.5V}$$

3. Find $i_L(0^+)$ & $V_c(0^+)$. The ckt. is in steady state with the switch in closed condition



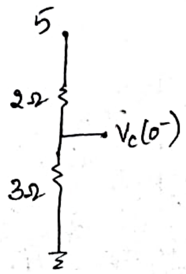
→ Soln:- At $t=0^-$, Switch is closed, C = open ckt
L = short ckt



$$\therefore i_L(0^-) = \frac{5}{2+3} = 1A$$

$$\therefore V_c(0^-) = \frac{3 \times 5}{2+3}$$

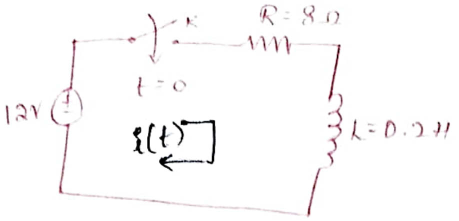
$$V_c(0^-) = 3V$$



$$\therefore \boxed{\begin{aligned} i_L(0^-) &= i_L(0^+) = 1A \\ V_c(0^-) &= V_c(0^+) = 3V. \end{aligned}}$$

4. In the given ckt. 'K' is closed at t=0 with zero current in the inductor. Find the values of

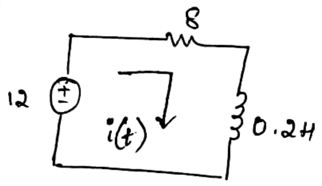
$i, \frac{di}{dt}, \frac{d^2i}{dt^2}$ at $t=0^+$, if $R=8\Omega$ & $L=0.2H$



→ Solⁿ:- At $t=0^-$, switch is open,

$$\therefore \boxed{i(0^-) = 0A = i(0^+)}$$ {∵ current through 'L'}.

At $t=0^+$, switch is closed.



$$Ri + L\frac{di}{dt} = 12$$

$$8i + 0.2\frac{di}{dt} = 12 \longrightarrow (1)$$

At $t=0^+$, $$8i(0^+) + 0.2\frac{di(0^+)}{dt} = 12$$

$$0.2\frac{di(0^+)}{dt} = 12 - \cancel{8(0)}^{\,0}$$

$$\frac{di(0^+)}{dt} = \frac{12}{0.2}$$

$$\therefore \boxed{\frac{di(0^+)}{dt} = 60A/sec}$$

Diff. eqn. (1) wrt 't'

$$8\frac{di}{dt} + 0.2\frac{d^2i}{dt^2} = 0$$

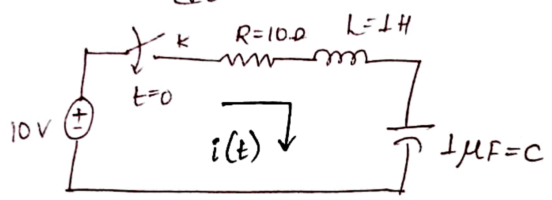
At $t=0^+$: $$8\frac{di(0^+)}{dt} + 0.2\frac{d^2i(0^+)}{dt^2} = 0$$

$$0.2\,\frac{d^2 i(0^+)}{dt^2} = -8 \times 60 = -480$$

$$\frac{d^2 i(0^+)}{dt^2} = \frac{-480}{0.2}$$

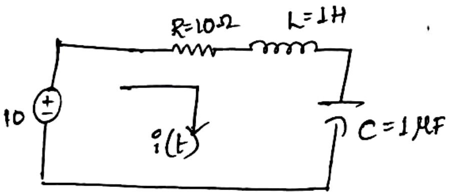
$$\boxed{\frac{d^2 i(0^+)}{dt^2} = -2400\,A/sec^2}$$

5. In the n/w., the switch is closed at $t=0$, find $i, \frac{di}{dt}, \frac{d^2 i}{dt^2}$ at $t=0^+$.



⇒ Sol^n: At $t=0^-$; switch is open, $i(0^-) = 0A = i(0^+)$.

At $t=0^+$: Switch is closed



$$10 = R\,i(t) + L\frac{di}{dt} + \frac{1}{C}\int_{0^+}^{t} i\,dt \rightarrow (1)$$

$$10 = 10\,i(t) + 1\frac{di}{dt} + \frac{1}{10^{-6}}\int_{0^+}^{t} i\,dt$$

At $t=0^+$:

$$10 = \cancel{10\,i(0^+)}^{\,0} + 1\frac{di(0^+)}{dt} + 10^6 \cancel{\int_{0^+}^{t} i(0^+)\,dt}^{\,0}$$

$$10 = \frac{di(0^+)}{dt} \;;\; \boxed{\frac{di(0^+)}{dt} = 10\,A/sec}$$

Diff (1) wrt 't'

$$0 = R\frac{di}{dt} + L\frac{d^2 i}{dt^2} + \frac{1}{C} i$$

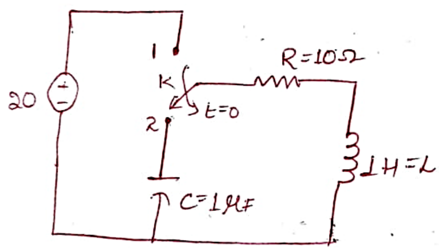
At $t=0^+$.

$$R\frac{di(0^+)}{dt} + L\frac{d^2i(0^+)}{dt^2} + \cancel{\frac{i(0^+)}{C}}^{\,0} = 0$$

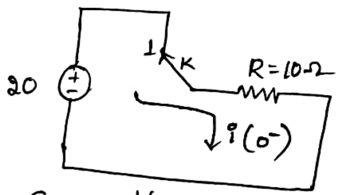
$$10\times 10 + 1\cdot\frac{d^2i(0^+)}{dt^2} = 0$$

$$\boxed{\frac{d^2i(0^+)}{dt^2} = -100\,A/sec^2}$$

6. The switch 'K' is changed from position 1 to position 2 at $t=0$. Steady state condition having been reached at position 1. Find the values of $i, \frac{di}{dt}, \frac{d^2i}{dt^2}$ at $t=0^+$



Soln: At $t=0^-$: position 1 → steady state, L = short ckt, C = open ckt.



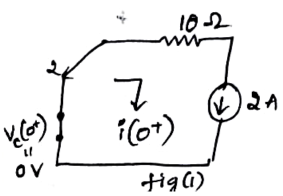
$$i(0^-) = \frac{20}{10} = 2$$

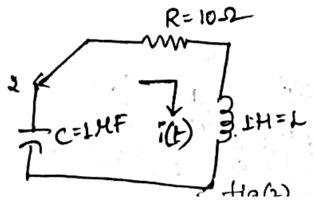
$$\therefore \boxed{i(0^-) = i(0^+) = 2A}$$

Since there is no initial charge in the capacitor.

$$\therefore \boxed{v_c(0^-) = 0V = v_c(0^+)}$$

At $t=0^+$: position 2.





Apply KVL to fig.(2).

$$Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int_{0^+}^{t} i(\tau)d\tau = 0 \longrightarrow (1)$$

$$Ri(t) + L\frac{di(t)}{dt} + v_c(t) = 0 \longrightarrow (2)$$

At $t=0^+$:

$$R\,i(0^+) + L\frac{di(0^+)}{dt} + \cancel{v_c(0^+)}^{\,0} = 0.$$

$$10 \times 2 + 1 \cdot \frac{di(0^+)}{dt} = 0$$

$$\boxed{\frac{di(0^+)}{dt} = -20\,A/sec.}$$

Diff. eqn. (1) wrt. 't'

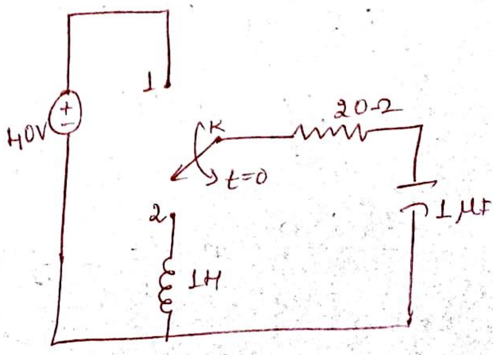
$$R\frac{di(t)}{dt} + L\frac{d^2i(t)}{dt^2} + \frac{1}{C}i(t) = 0$$

At $t=0^+$: $$R\frac{di(0^+)}{dt} + L\frac{d^2i(0^+)}{dt^2} + \frac{i(0^+)}{C} = 0$$

$$10\times(-20) + 1 \times \frac{d^2i(0^+)}{dt^2} + \frac{2}{10^{-6}} = 0$$

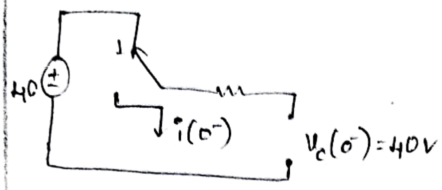
$$\therefore \boxed{\frac{d^2i(0^+)}{dt^2} \simeq -2\times10^{-6}\,A/sec^2}$$

7. The switch is moved from position 1 to position 2 at $t=0$. The steady state has been reached before switching. Calculate i, $\frac{di}{dt}$ & $\frac{d^2i}{dt^2}$ at $t=0^+$



Sol^n: At $t=0^-$: Switch $\rightarrow$ position 1 $\rightarrow$ steady state

$L \rightarrow$ short ckt ; $C \rightarrow$ open ckt.



$$\therefore \boxed{\begin{array}{l} V_c(0^-) = 40V = V_c(0^+) \\ i(0^-) = 0A = i(0^+) \end{array}}$$

At $t \geq 0^+$: Switch $\rightarrow$ position 2

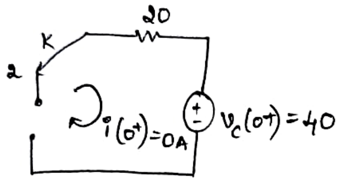


fig. (1)

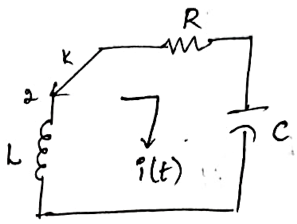


fig. (2)

Apply KVL to fig. (2).

$$Ri(t) + \frac{1}{C}\int_{0^+}^{t} i(t)\,dt + L\frac{di(t)}{dt} = 0 \longrightarrow (1)$$

At $t=0^+$:

$$\cancel{Ri(0^+)}^{0} + V_c(0^+) + L\frac{di(0^+)}{dt} = 0$$

$$40 + 1 \cdot \frac{di(0^+)}{dt} = 0.$$

$$\boxed{\frac{di(0^+)}{dt} = -40A/sec}$$

Diff. eq^n. (1) wrt 't'

$$R\frac{di}{dt} + \frac{1}{C} i(t) + L\frac{d^2 i(t)}{dt^2} = 0$$

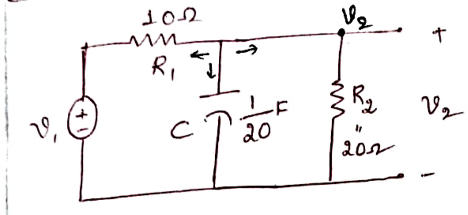
At $t=0^+$:

$$R\frac{di(0^+)}{dt} + \frac{1}{C}\cancel{i(0^+)}^{0} + L\frac{d^2 i(0^+)}{dt^2} = 0$$

$$20(-40) + 1 \cdot \frac{d^2 i(0^+)}{dt^2} = 0$$

$$\boxed{\frac{d^2 i(0^+)}{dt^2} = +800 \text{ A/sec}^2}$$

8. *** In the n/w shown, $v_1(t) = e^{-t}$ for $t \geq 0$ & zero for all $t < 0$. If the capacitor is initially uncharged, determine the value of $\frac{d^2 v_2}{dt^2}$ & $\frac{d^3 v_2}{dt^3}$ at $t = 0^+$.



⇒ Soln:- Since the capacitor is initially uncharged.

$$v_2(0^+) = 0.$$

Apply KCL.

$$\frac{v_2 - v_1}{R_1} + C \cdot \frac{dv_2}{dt} + \frac{v_2}{R_2} = 0.$$

$$\left[\frac{1}{R_1} + \frac{1}{R_2}\right] v_2 + C\frac{dv_2}{dt} = \frac{v_1}{R_1}$$

$$0.15 v_2 + 0.05 \frac{dv_2(0^+)}{dt} = 0.1 e^{-t} \longrightarrow (1)$$

At $t = 0^+$:

$$0.15 \cancel{v_2(0^+)}^{\,0} + 0.05 \frac{dv_2(0^+)}{dt} = 0.1 e^{-(0^+)} = 0.1.$$

$$0.05 \frac{dv_2(0^+)}{dt} = 0.1$$

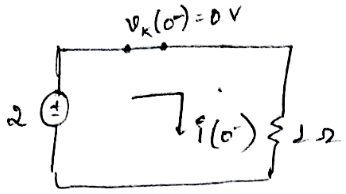
$$\therefore \boxed{\frac{dv_2(0^+)}{dt} = 2 \text{ V/sec}}$$

Diff. (1) w.r.t. 't'

$$0.15 \frac{dv_2}{dt} + 0.05 \frac{d^2 v_2}{dt^2} = -0.1 e^{-t} \longrightarrow (2)$$

⇒ Sol^n: At $t=0^-$: Switch → closed → steady state

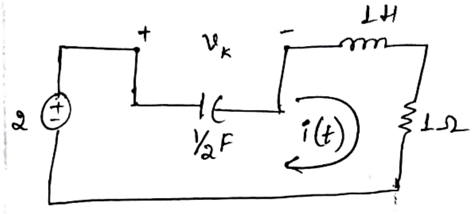
$L \to$ short ckt, $C \to$ open ckt



$i(0^-) = \frac{2}{1} = 2A = i(0^+)$

$\therefore \boxed{v_K(0^-) = 0V = v_K(0^+)}$

At $t=0^+$:



$i_c(t) = C \cdot \frac{dv_K}{dt}$

At $t=0^+$,

$i_c(0^+) = C \frac{dv_K(0^+)}{dt}$

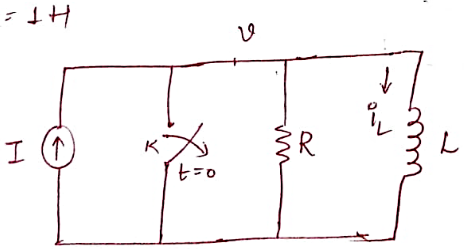
$\frac{2}{C} = \frac{dv_K(0^+)}{dt}$

$\therefore \frac{dv_K(0^+)}{dt} = \frac{2}{1/2} = 4\,V/sec$

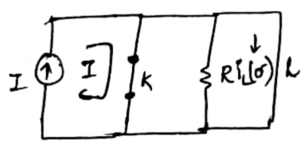
$\cancel{2 = v_K + 1\frac{di}{dt} + 1 \times 2}$

$0 = \frac{dv_K}{dt} + \cancel{\frac{d^2 i}{dt^2} + R\frac{di}{dt}}$

10. The switch 'K' is opened at $t=0$. At $t=0^+$ solve for the values of v, dv/dt & $\frac{d^2v}{dt^2}$ if $I = 2A$, $R = 200\Omega$ & $L = 1H$

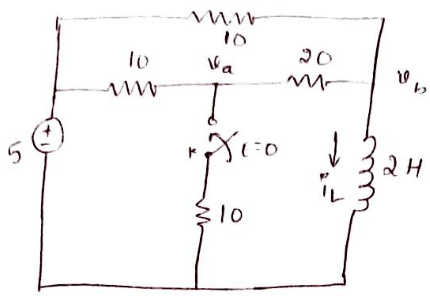


⇒ Sol^n:- At $t=0^-$: SW → closed, $L \to$ short ckt.

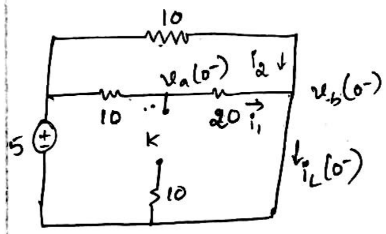


$\boxed{i_L(0^-) = 0A = i_L(0^+)}$

11 In the ckt, a steady state is reached with switch 'k' open. At $t=0$, the switch is closed. For element values given determine the values of $v_a(0^-)$ & $v_a(0^+)$



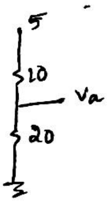
Sol^n:- At $t=0^-$; SW → open → steady state, L → s.c.



$i_L = i_1 + i_2$

$$i_L(0^-) = \frac{5}{30} + \frac{5}{10} = \frac{2}{3}\ A = i_L(0^+)$$

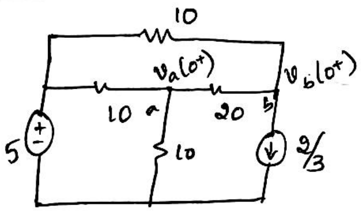
Using vtg. division.



$$v_a = \frac{20 \times 5}{30}$$

$$\boxed{v_a(0^-) = \frac{10}{3}\ V.}$$

At $t=0^+$:-



KCL at 'a'

$$\frac{v_a - 5}{10} + \frac{v_a(0^+)}{10} + \frac{v_a(0^+) - v_b(0^+)}{20}$$

$$\left[\frac{1}{10} + \frac{1}{10} + \frac{1}{20}\right] v_a(0^+) - \frac{v_b(0^+)}{20} = \frac{5}{10} \rightarrow (1)$$

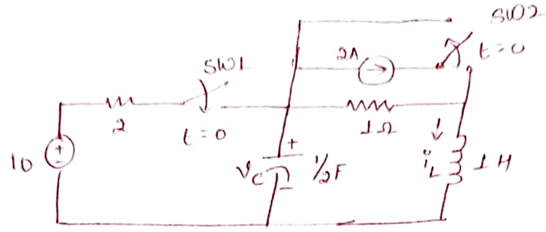
KCL at 'b'

$$\frac{2}{3} + \frac{v_b(0^+) - 5}{10} + \frac{v_b(0^+) - v_a(0^+)}{20} = 0$$

$$-\frac{v_a(0^+)}{20} + \left[\frac{1}{10} + \frac{1}{20}\right] v_b(0^+) = \frac{5}{10} - \frac{2}{3} \longrightarrow (2)$$

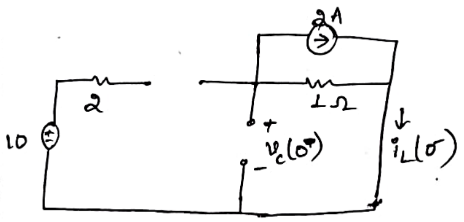
$$\therefore \boxed{v_a(0^+) = \frac{40}{21}\ V}$$

*12. Find $i_L(0^+)$, $v_C(0^+)$, $\frac{dv_C(0^+)}{dt}$, $\frac{di_L(0^+)}{dt}$. Assume that switch 1 has been opened & switch 2 has been closed for a long time & steady state prevail at $t=0^-$

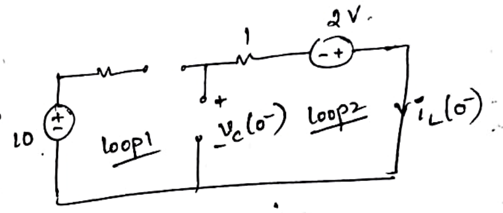


⇒ Sol^n:- At $t=0^-$: SW1 → open ; SW2 → closed.

L → S.C ; C → O.C.



→



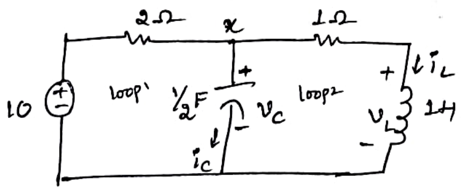
$$\therefore \boxed{i_L(0^-) = 0A = i_L(0^+)}$$

Apply KVL to loop2,

$$v_C(0^-) - i_L(0^-) + 2 = 0$$

$$\boxed{v_C(0^-) = -2V = v_C(0^+)}$$

At $t \geq 0^+$:- SW1 → closed, SW2 – open



Apply KVL to loop2,

$$v_C(0^+) + i_L(0^+) - v_L(0^+) = 0$$

$$-2 - 0 - v_L(0^+) = 0$$

$$v_L(0^+) = L\frac{di_L(0^+)}{dt} = -2.$$

$$\therefore \boxed{\frac{di_L(0^+)}{dt} = -2 A/sec}$$

Apply KCL at x

$$\frac{v_C - 10}{2} + i_C + i_L = 0$$

At $t = 0^+$, $\frac{v_C(0^+)}{2} - 5 + i_C(0^+) + i_L(0^+) = 0$; $\frac{-2}{2} - 5 + i_C(0^+) + 0 = 0$

$-6 + i_C(0^+) = 0.$

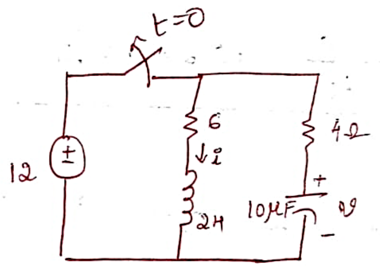
$\therefore \; i_C(0^+) = 6A.$

$\therefore \; C \cdot \frac{dv_C(0^+)}{dt} = i_C(0^+) = 6.$

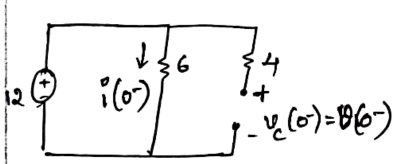
$\therefore \; \frac{dv_C(0^+)}{dt} = \frac{6}{1/2}$

$$\boxed{\frac{dv_C(0^+)}{dt} = 12V/sec}$$

*13 Find
i) $i(0^+)$ & $v(0^+)$
ii) $\frac{di(0^+)}{dt}$ & $\frac{dv(0^+)}{dt}$
iii) $i(\infty)$ & $v(\infty)$



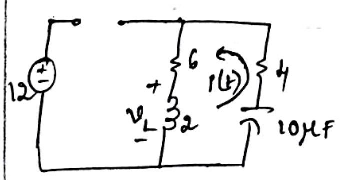
→ Sol^n:- At $t = 0^-$: sw → closed → steady state, $L \to$ S.C ; $C \to$ O.C



$i(0^-) = \frac{12}{6} = 2A = i(0^+)$

$v(0^-) = 12V = v(0^+)$ doubt

At $t \geq 0^+$:- Sw → open.



Apply KVL,

$v_L + 6i(t) + 4i(t) - v = 0$

At $t = 0^+$,

$v_L(0^+) + 10i(0^+) - v(0^+) = 0.$

$v_L(0^+) + 20 - 12 = 0$; $\therefore \; v_L(0^+) = -8V$

$$\therefore \quad v_L = L\frac{di}{dt}$$

$$v_L(0^+) = L\frac{di(0^+)}{dt}$$

$$\frac{di(0^+)}{dt} = \frac{-8}{10}$$

$$\boxed{\frac{di(0^+)}{dt} = -8/10 \text{ A/sec}}$$

$$v_L = L\frac{di}{dt}$$

$$= 2\times(-4)$$

$$v_L = -8V$$

$$\frac{di(0^+)}{dt} = \frac{v_L(0^+)}{L} = \frac{-8}{2}$$

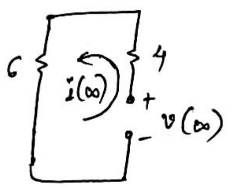
$$= -4 \text{ A/sec}$$

III|y. $i_C = C\frac{dv}{dt}$

$$\frac{dv(0^+)}{dt} = \frac{i_C(0^+)}{C} = \frac{+i(0^+)}{C} = \frac{+2}{10\times10^{-6}}$$

$$\boxed{\frac{dv(0^+)}{dt} = +0.2\times10^{6} \text{ V/sec}}$$

(c) At $t \to \infty$, sw $\to$ open & ckt attain steady state



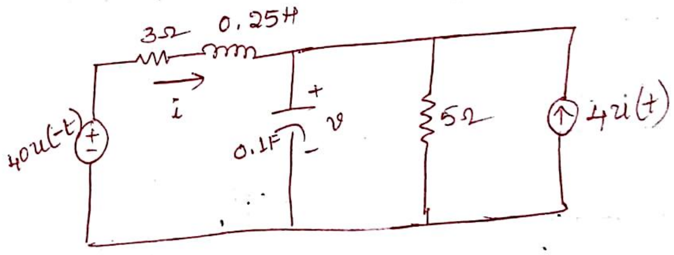
$$\boxed{\begin{aligned} i(\infty) &= 0 \\ v(\infty) &= 0 \end{aligned}}$$

Ex 4 Find,

a) $v(0^+)$ & $i(0^+)$

b) $\frac{dv(0^+)}{dt}$ & $\frac{di(0^+)}{dt}$

c) $v(\infty)$ & $i(\infty)$

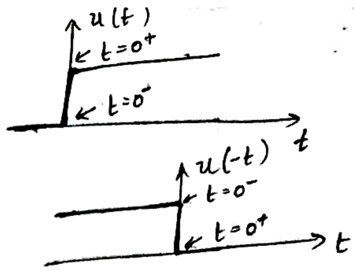


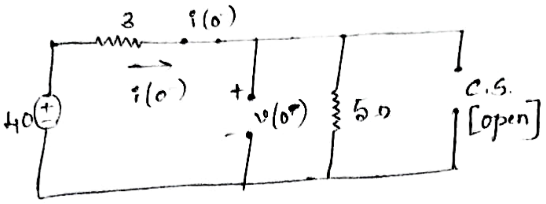
→ Soln:-

$$u(t) = \begin{cases} 1 & t \geq 0^+ \\ 0 & t \leq 0^- \end{cases}$$

$$u(-t) = \begin{cases} 1 & t \leq 0^+ \\ 0 & t \geq 0^- \end{cases}$$

At $t=0^-$: Independent C.S is open because $u(t)=0$ at $t=0^-$ & the ckt is in steady state

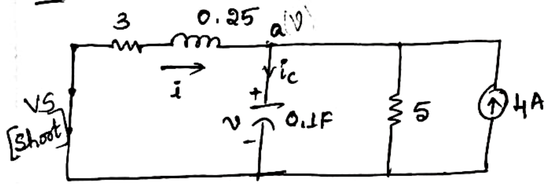




(a) $i(0^-) = \dfrac{40}{3+5} = 5A = i(0^+)$

$v(0^-) = 5\,i(0^-) = 5 \times 5 = 25V = v(0^+)$

(b) At $t \geq 0^+$: $u(-t)=0$ [VS = 0 {short ckt}]



Apply KCL at node a,

$$4 + i = C\frac{dv}{dt} + \frac{v}{5}$$

At $t=0^+$: $4 + i(0^+) = C\dfrac{dv(0^+)}{dt} + \dfrac{v(0^+)}{5}$

$$4 + 5 = 0.1\frac{dv(0^+)}{dt} + \frac{25}{5}$$

$$\frac{9-5}{0.1} = \frac{dv(0^+)}{dt} \quad ; \quad \boxed{\frac{dv(0^+)}{dt} = 40V/sec}$$

Apply KVL to loop1.

$$3i + 0.25\frac{di}{dt} + v = 0$$

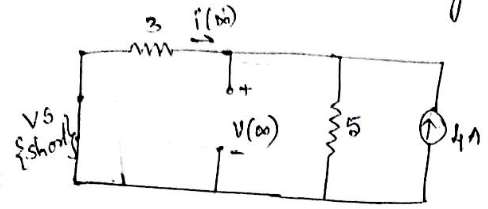
At $t=0^+$. $3i(0^+) + 0.25\dfrac{di(0^+)}{dt} + v(0^+) = 0$

$$3 \times 5 + 0.25\frac{di(0^+)}{dt} + 25 = 0$$

$$\therefore \boxed{\frac{di(0^+)}{dt} = -160\,A/sec}$$

(c) $t \rightarrow \infty$; ckt $\rightarrow$ steady state

$u(-t) = 0$ at $t = \infty$; ∴ $V_S = 0$ [si

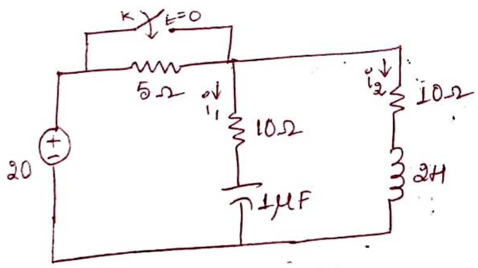


$$i(\infty) = -\left[\frac{5 \times 4}{3+5}\right] = -2.5A$$

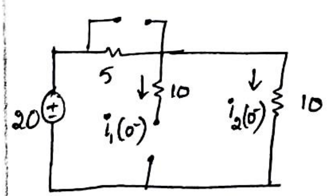
$$v(\infty) = \left[i(\infty) + 4\right]5 = \left[-2.5 + 4\right]5$$

$$\boxed{v(\infty) = 7.5V}$$

15. In the ckt., steady-state is reached with switch 'K' open. The switch is closed at $t=0$. Find i_1, i_2, $\frac{di_1}{dt}$ & $\frac{di_2}{dt}$ at $t=0^+$.



⇒ Soln: At $t=0^-$; $K \rightarrow$ open $\rightarrow$ steady state; $L \rightarrow SC$; $C \rightarrow O.C.$



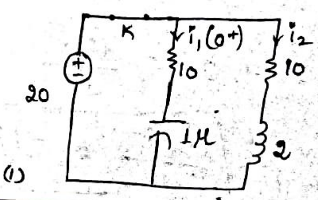
$$i_2(0^-) = \frac{20}{15} = 1.33A = i_2(0^+)$$

$$V_c(0^-) = 10 \times i_2(0^-) = 10 \times 1.33$$

$$V_c(0^-) = 13.3V = V_c(0^+)$$

$$i_1(0^-) = 0A$$

At $t \geq 0^+$: $K \rightarrow$ closed



Apply KVL to loop1.

$$20 = 10i_1 + \frac{1}{C}\int_{0^+}^{t} i_1(\tau)\,d\tau \rightarrow (1)$$

diff. eqn (1) wrt 't'

$$10\frac{di_1}{dt} + \frac{1}{C} i_1 = 0$$

At $t=0^+$, $10\frac{di_1(0^+)}{dt} + \frac{i_1(0^+)}{C} = 0$

But $i_1(0^+) = \frac{20-13.3}{10} = 0.67A$

$$\frac{di_1(0^+)}{dt} = \frac{-0.67}{10^{-6} \times 10}$$

$$\boxed{\frac{di_1(0^+)}{dt} = -0.67\times 10^5 \ A/sec}$$

Apply KVL to the path $20V \rightarrow K \rightarrow 10\Omega \rightarrow 2H$, we get,

$$20 = 10 i_2 + 2\frac{di_2}{dt}$$

At $t=0^+$: $20 = 10 i_2(0^+) + 2\frac{di_2(0^+)}{dt}$

$$\frac{20-10(1.33)}{2} = \frac{di_2(0^+)}{dt} \ ; \quad \boxed{\frac{di_2(0^+)}{dt} = 3.35 A/sec}$$

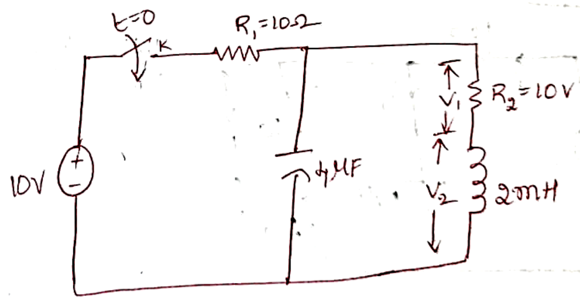
16 The switch 'K' is closed at $t=0$. Find:

i) v_1 & v_2 at $t=0^+$

ii) v_1 & v_2 at $t=\infty$

iii) $\frac{dv_1}{dt}$ & $\frac{dv_2}{dt}$ at $t=0^+$

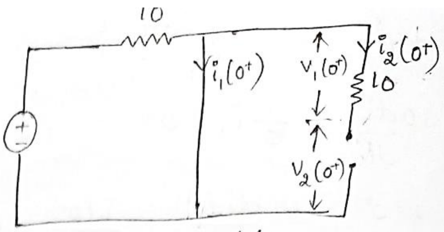
iv) $\frac{d^2v_1}{dt^2}$ at $t=0^+$



Soln:- a) At $t=0^-$: $K \rightarrow$ open, ckt. is not active & all the initial conditions are zero.

At $t=0^+$: $K \rightarrow$ closed, $L \rightarrow$ O.C. ; $C \rightarrow$ S.C.

$i_1(0^+) = \frac{10}{10} = 1A$



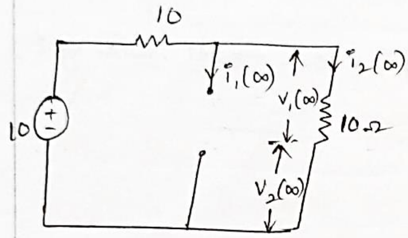
$\boxed{v_1(0^+) = 0}$; $i_2(0^+) = 0$

Apply KVL to $10V \to 10\Omega \to 10\Omega \to 2mH$, yields

$$10 - 10\,i_1(0^+) - 10\,\cancel{i_2(0^+)}^{\,0} - v_2(0^+) = 0$$

$$10 - 10 - v_2(0^+) = 0 \quad ; \quad \boxed{v_2(0^+) = 0V}$$

b) At $t \to \infty$: $k \to$ closed $\xrightarrow{ckt}$ steady state. $L \to S.C$; $C \to OC$



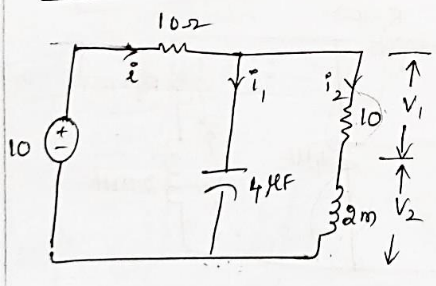
$i_1(\infty) = 0A$

$i_2(\infty) = \frac{10}{20} = 0.5A$

$V_1(\infty) = 10\,i_2(\infty) = 10 \times 0.5$

$$\boxed{\begin{aligned} V_1(\infty) &= 5V \\ V_2(\infty) &= 0V \end{aligned}}$$

c) $t \geq 0^+$:



$$i_2 = \frac{1}{L}\int_{0^+}^{t} v_2(\tau)\,d\tau = \frac{v_1(t)}{R_2}$$

diff. wrt. 't'

$$\frac{v_2}{L} = \frac{1}{R_2} \cdot \frac{dv_1}{dt}$$

At $t = 0^+$, $\frac{dv_1(0^+)}{dt} = \frac{R_2}{L}\, v_2(0^+)$

$$\frac{dv_1(0^+)}{dt} = 0V/sec$$

Apply KVL to the path $10V \rightarrow K \rightarrow 10\Omega \rightarrow 4\mu F$

$$-10 + 10i + \frac{1}{C}\int_{0^+}^{t} [i(\tau) - i_2(\tau)] d\tau = 0$$

diff. wrt. 't'

$$10\frac{di}{dt} + \frac{1}{C}[i - i_2] = 0$$

At $t = 0^+$: $\frac{di(0^+)}{dt} = \frac{i_2(0^+) - i(0^+)}{C \times 10}$

$i(0^+) = i_1(0^+) + i_2(0^+)$

$= 1 + 0$

$\Rightarrow = 1A.$

$\Rightarrow = -25000 A/sec$

Apply KVL to $10V \rightarrow K \rightarrow 10\Omega \rightarrow 10\Omega \rightarrow 2mH$

v_1

$-10 + 10i + 10i_2 + v_2 = 0$ $\quad$ $+10 - 10i - 10i_2 - v_2 = 0$

$10i + v_1 + \cancel{v_2} = 10$

diff. wrt 't'

$$10\frac{di}{dt} + \frac{dv_1}{dt} + \frac{dv_2}{dt} = 0$$

At $t = 0^+$: $10\frac{di(0^+)}{dt} + \cancel{\frac{dv_1(0^+)}{dt}}^{\,0} + \frac{dv_2(0^+)}{dt} = 0$

$$10(-25000) + 0 + \frac{dv_2(0^+)}{dt} = 0$$

$$\frac{dv_2(0^+)}{dt} = 25 \times 10^4 \, V/sec$$

d) $\frac{1}{L}\int_{0^+}^{t} v_2(\tau) d\tau = \frac{v_1}{10}$

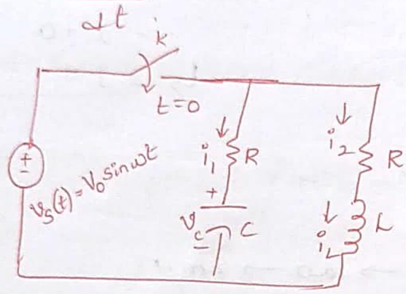
diff. wrt 't' twice,

$$\frac{1}{L}\frac{dv_2}{dt} = \frac{1}{10}\frac{d^2v_1}{dt^2}$$

At $t=0^+$: $\frac{1}{L}\frac{dv_2(0^+)}{dt} = \frac{1}{10}\frac{d^2v_1(0^+)}{dt^2}$

$$\frac{d^2v_1(0^+)}{dt^2} = 125\times10^7 \ V/sec^2$$

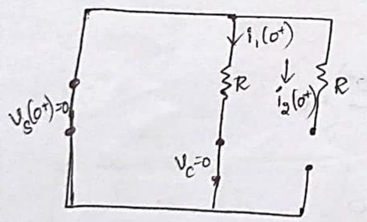
17. The switch 'K' is closed at $t=0$. Find (a) $\frac{di_1(0^+)}{dt}$ (b) $\frac{di_2(0^+)}{dt}$.



→ Soln:- At $t=0^-$; $K\longrightarrow$ open ; ckt is inactive.

$\therefore\ i_L(0^-)=i_2(0^-)=0A$; $v_C(0^-)=0V$; $i_2(0^+)=0=i_L(0^+)$

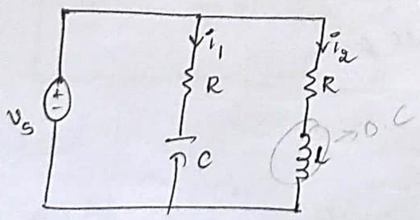
At $t=0^+$:



$i_1(0^+)=0$

$i_2(0^+)=0$.

At $t\geq0^+$:



$$V_0\sin\omega t = Ri_1 + \frac{1}{C}\int_{0^+}^{t} i_1\,dt$$

diff wrt 't'

$$V_0\omega\cos\omega t = \frac{Rdi_1}{dt} + \frac{i_1}{C}.$$

At $t=0^+$

$$\omega V_0(1) = R\frac{di_1(0^+)}{dt} + \cancel{\frac{i_1(0^+)}{C}}^{\,0}$$

$$\boxed{\frac{di_1(0^+)}{dt} = \frac{V_0 \omega}{R} \text{ A/sec}}$$

Also,

$$V_0 \sin\omega t = i_2 R + L\frac{di_2}{dt}$$

At $t=0^+$

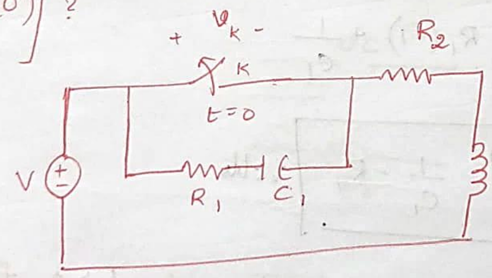
$$V_0 \cancel{(0)}^{0} = i_2\cancel{(0^+)}^{0} R + L\frac{di_2(0^+)}{dt}$$

$$\boxed{\frac{di_2(0^+)}{dt} = 0 \text{ A/sec.}}$$

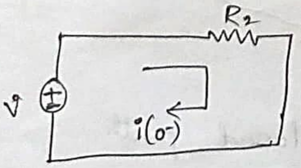
18. In the n/w, the switch K is opened at $t=0$ after the n/w has attained steady state with the switch closed.

(a) Find the expression for v_K at $t=0^+$

(b) If the parameters are adjusted such that $i(0^+)=1$ & $\frac{di(0^+)}{dt}=-1$, what is the value of the derivative of the vtg. across the switch at $t=0^+$, $\left[\frac{dv_K}{dt}(0^+)\right]$?



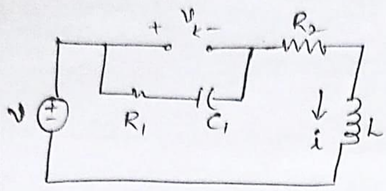
⇒ Sol^n: (a) At $t=0^-$: K → closed; ckt. → steady state



$$i(0^-) = \frac{V}{R_2}$$

$$V_c(0^-) = 0\text{ V} = V_c(0^+)$$

At $t \geq 0^+$: K → open.



$$v_K = R_1 i + \frac{1}{C_1}\int_{0^+}^{t} i(t)\,dt$$

$$v_K = R_1 i + v_C(t)$$

At $t=0^+$; $v_K(0^+) = R_1 i(0^+) + \cancel{v_C(0^+)}^{\,0}$

$$v_K(0^+) = R_1 \frac{V}{R_2} \text{ volts}$$

(b)

$$v_K = R_1 i + \frac{1}{C_1}\int_{0^+}^{t} i(\tau)\,d\tau$$

diff. wrt 't'

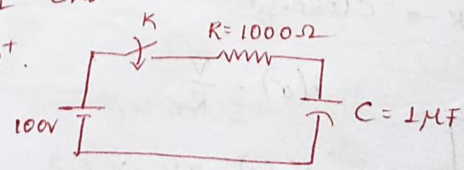
$$\frac{dv_K}{dt} = R_1\frac{di}{dt} + \frac{i(t)}{C_1}$$

At $t=0^+$; $\frac{dv_K(0^+)}{dt} = R_1\frac{di(0^+)}{dt} + \frac{i(0^+)}{C_1}$

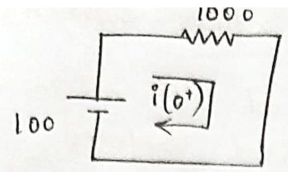
$$\frac{dv_K(0^+)}{dt} = R_1(-1) + \frac{1}{C_1}$$

$$\therefore \boxed{\frac{dv_K(0^+)}{dt} = \frac{1}{C_1} - R_1} \text{ volts}$$

19 For the given ckt. K is closed at $t>0$, find $i, \frac{di}{dt}, \frac{d^2i}{dt^2}$ at $t=0^+$.

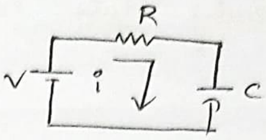


⇒ Sol^n:- At $t=0^+$; capacitor acts as closed path.



$i(0^+) = \frac{100\cancel{0}}{1000} = 0.10A$

Apply KVL to loop1



$V = Ri + \frac{1}{C}\int i\,dt \rightarrow (1)$

Diff. (1) wrt t.

$\therefore 0 = R\frac{di}{dt} + \frac{1}{C}\cdot i \rightarrow (2)$

At $t = 0^+$

$0 = R\frac{di(0^+)}{dt} + \frac{1 \cdot i(0^+)}{C}$

$\therefore R\frac{di(0^+)}{dt} = \frac{-0.1}{10^{-6}}$

$\therefore \boxed{\frac{di(0^+)}{dt} = -100A/sec}$

Diff. (2) wrt 't'

$0 = R\frac{d^2i}{dt^2} + \frac{1}{C}\frac{di}{dt}$

At $t = 0^+$

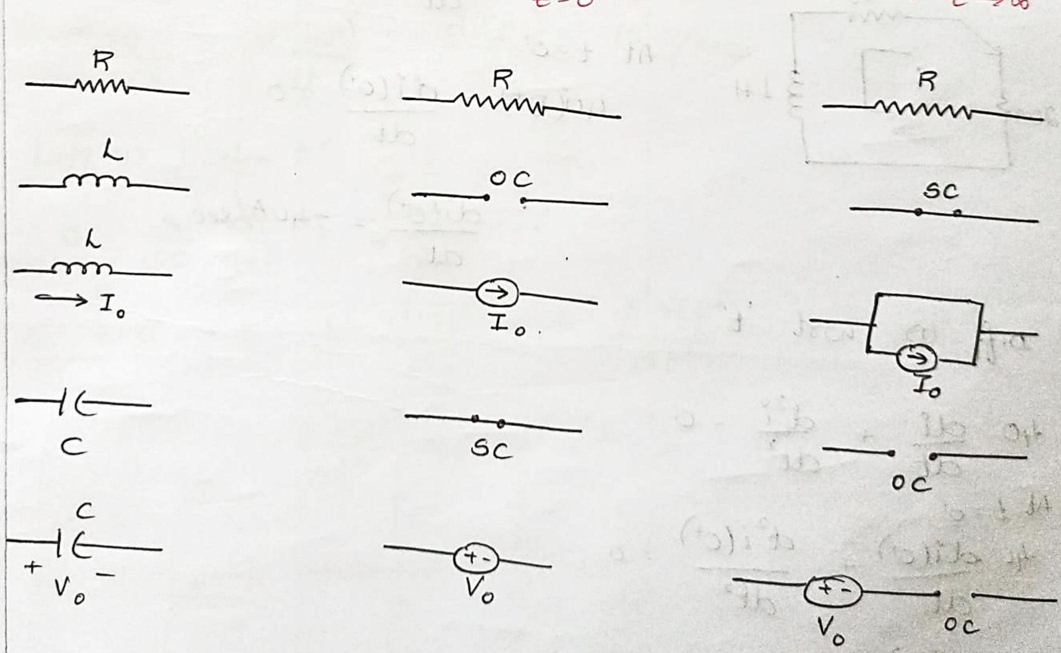
$0 = R\frac{d^2i(0^+)}{dt^2} + \frac{1}{C}\frac{di(0^+)}{dt}$

$\frac{d^2i(0^+)}{dt^2} = \frac{-1}{RC}\frac{di(0^+)}{dt}$

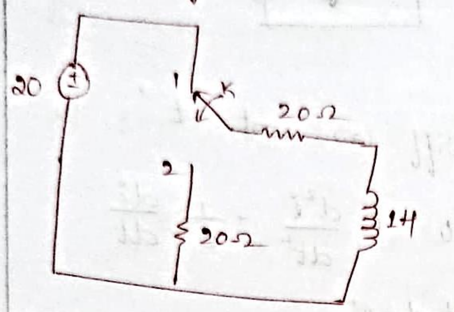
$\hookrightarrow = 100k\ A/sec^2$

* Summary:-

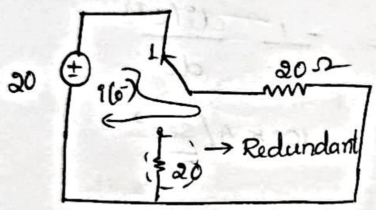
Elements | After excitation $t=0^+$ | Steady state $t \rightarrow \infty$



Q. Determine i, $\frac{di}{dt}$, $\frac{d^2i}{dt^2}$ at $t = 0^+$ when switch 'K' is moved from position '1' to '2' at $t = 0$ in the n/w, steady state having reached before switching.



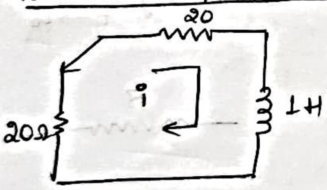
Soln: At K in position '1': ckt is in steady state, L is short circuited.



$$i(0^-) = \frac{V}{R} = \frac{20}{20} = 1A$$

$$\therefore i(0^-) = 1A = i(0^+).$$

At K in position 2.



$$40i + 1 \cdot \frac{di}{dt} = 0 \rightarrow (1)$$

At $t = 0^+$

$$40i(0^+) + \frac{di(0^+)}{dt} = 0$$

$$\therefore \frac{di(0^+)}{dt} = -40 A/sec$$

Diff. (1) wrt 't'

$$40 \cdot \frac{di}{dt} + \frac{d^2i}{dt^2} = 0$$

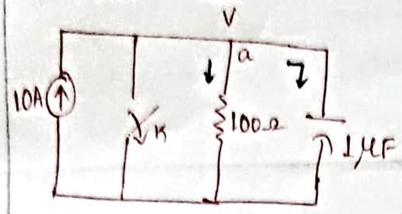
At $t = 0^+$

$$40 \frac{di(0^+)}{dt} + \frac{d^2i(0^+)}{dt^2} = 0$$

$$\therefore \frac{d^2i(0^+)}{dt^2} = -40 \cdot (-40)$$

$$= 1600 A/sec^2$$

21. In the ckt. given below 'K' is opened at $t=0$, find $V, \frac{dV}{dt}, \frac{d^2V}{dt^2}$ at $t=0^+$



sol^n: At $t=0^-$, K is closed & current flows through the switch ignoring 'R' & 'C'. Hence vtg. drop across capacitor is '0'. $V_c(0^-)=0$

$\therefore V_c(0^-) = 0 = V_c(0^+)$, $= V(0^-) = V(0^+)$

At $t=0$, K is opened.

Applying KCL at node 'a'

$$10 = \frac{V}{100} + C \cdot \frac{dV}{dt} \rightarrow (1)$$

At $t=0^+$

$$10 = \cancel{\frac{V(0^+)}{100}}^{\,0} + C \cdot \frac{dV(0^+)}{dt}$$

$$\therefore \frac{10}{10^{-6}} = \frac{dV(0^+)}{dt}; \quad \frac{dV(0^+)}{dt} = 10^7 \text{ V/sec}$$

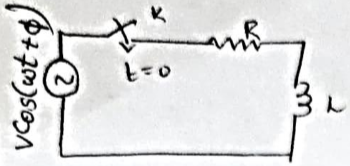
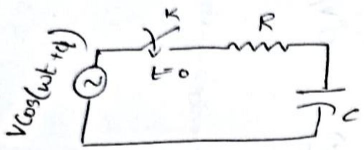
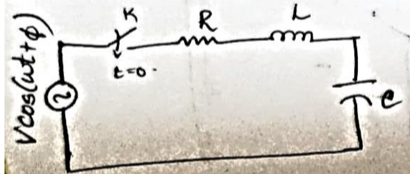
Diff (1) wrt 't'

$$0 = \frac{1}{100}\frac{dV}{dt} + C\frac{d^2V}{dt^2}$$

At $t=0^+$, $$0 = \frac{1}{100}\frac{dV(0^+)}{dt} + 10^{-6}\frac{d^2V(0^+)}{dt^2}$$

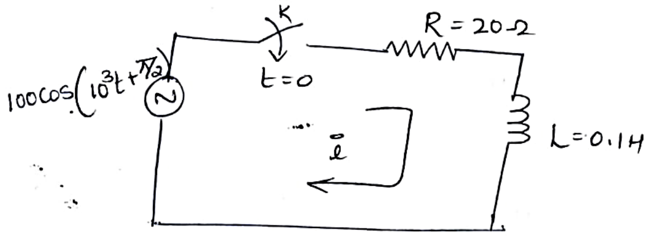
$$\therefore \frac{d^2V(0^+)}{dt^2} = -10^{11} \text{ V/sec}^2$$

* Summary of AC excitation :-

Type of circuit	Solution
1] RL circuit 	$i = \underbrace{e^{-\frac{Rt}{L}}\left[\frac{-V}{R^2+(\omega L)^2}\cos\left(\phi - \tan^{-1}\frac{\omega L}{R}\right)\right]}_{i_{CF}} + \underbrace{\frac{V}{\sqrt{R^2+(\omega L)^2}}\left[\cos\omega t + \phi - \tan^{-1}\left(\frac{\omega L}{R}\right)\right]}_{i_{PS}}$
2] RC circuit 	$i = \underbrace{e^{-t/RC}\left[\frac{V}{R}\cos\phi - \frac{V}{\sqrt{R^2+(1/\omega C)^2}}\cos\left(\phi + \tan^{-1}\left(\frac{1}{\omega RC}\right)\right)\right]}_{i_{CF}} + \underbrace{\frac{V}{\sqrt{R^2+(1/\omega C)^2}}\cos\left(\omega t + \phi + \tan^{-1}\left(1/\omega RC\right)\right)}_{i_{PS}}$
3] RLC circuit 	$i_{PI} = \frac{V}{\sqrt{R^2+\left(\frac{1}{\omega C}-\omega L\right)^2}}\cos\left[\omega t + \phi + \tan^{-1}\frac{\left(\frac{1}{\omega C}-\omega L\right)}{R}\right]$ i_{CF} :- C.E. $P^2 + \frac{R}{L}P + \frac{1}{LC} = 0$ [K_1 & K_2 are roots] 1] Roots are real & Unequal:- $i_{CF} = K_3 e^{(K_1+K_2)t} + K_4 e^{(K_1-K_2)t}$.

AC excitation Problems

1) In the circuit shown, determine complete solution for current when switch 'K' is closed at $t=0$. Applied voltage is $V(t)$ which is given as $100\cos(10^3 t + \pi/2)$.



Applying KVL, $20i + 0.1\frac{di}{dt} = 100\cos(10^3 t + \pi/2)$

$$\therefore \frac{di}{dt} + 200i = 1000\cos(10^3 t + \pi/2).$$

$\therefore$ It is in the form of Non-homogeneous eqn.

$$\therefore i = i_{CF} + i_{PI}.$$

$$\therefore i_{CF} = K.e^{-a_1/a_0 t} = K.e^{-200t}.$$

$$\therefore i_{PI} = \frac{V}{\sqrt{R^2+(\omega L)^2}}\cos\left(\omega t + \phi - \tan^{-1}\frac{\omega L}{R}\right).$$

$\therefore \omega = 1000, \phi = \pi/2, V = 100, L = 0.1, R = 20.$

$$\therefore i_{PI} = \frac{100}{\sqrt{(20)^2+(1000\times 0.1)^2}}\cos\left[1000t + \pi/2 - \tan^{-1}\left(\frac{1000\times 0.1}{20}\right)\right]$$

$$= 0.9805\cos(1000t + \pi/2 - 78.69^\circ)$$

$$\therefore \boxed{i = Ke^{-200t} + 0.9805\cos(1000t + \pi/2 - 78.6^\circ)}$$

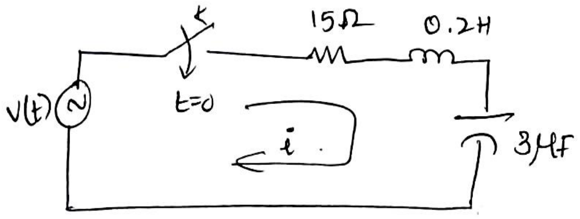
To find K, we use initial condition. At $t=0$, the current flowing through inductor is '0'

$$0 = Ke^0 + 0.9805\cos(0 + \pi/2 - 78.6^\circ)$$

$$\therefore K = -0.98\cos(\pi/2 - 78.6^\circ).$$

$$i = -0.98\cos(\pi/2 - 78.6^\circ)\, e^{-200t} + 0.98\cos(1000t + \pi/2 - 78.6^\circ)\,A$$

2] Determine the complete solution for the current, when switch is closed at $t=0$. Applied vtg is $V(t) = 400\cos(500t + \pi/4)$



→
$$0.2\frac{di}{dt} + 15i + \frac{1}{3\times10^{-6}}\int i\,dt = -400\cos(500t + \pi/4) \rightarrow (*)$$

diff. wrt. 't'

$$0.2\frac{d^2i}{dt^2} + 15\frac{di}{dt} + \frac{i}{3\times10^{-6}} = -400\times500\sin(500t+\pi/4)$$

$$\frac{d^2i}{dt^2} + 75\frac{di}{dt} + 16.67\times10^5 i = -2\times10^5\sin(\omega t + \pi/4).$$

This eqn. can be written as -

$$(P^2 + 75P + 16.67\times10^5)\,i = -2\times10^5\sin(\omega t + \pi/4)$$

$\therefore$ Roots of the eqn. is, $P_1 = -37.5 + j1290 \Rightarrow \begin{cases} K_1 = -37.5 \\ K_2 = 1290. \end{cases}$

$P_2 = -37.5 - j1290$

when roots are complex conjugate, $i_{CF} = e^{K_1 t}(K_3\cos K_2 t + K_4 \sin K_2 t)$

$\therefore$ The CF is, $i_{CF} = e^{-37.5t}(K_3\cos1290t + K_4\sin1290t)$.

$\therefore$ The PS is, $i_{PI} = \dfrac{V}{\sqrt{R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2}} \cos\left[\omega t + \phi + \tan^{-1}\left(\dfrac{\frac{1}{\omega RC} - \frac{\omega L}{R}}{}\right)\right]$

$V = 400V$, $R = 15$, $L = 0.2H$, $\omega = 500\,rad/sec$, $C = 3\times10^{-6}F$

$\therefore i_{PI} = 0.705\cos(500t + \pi/4 + 88.48^\circ)$

$\therefore \quad i = i_{CF} + i_{PI}$

$$i = e^{-37.5t}\left[K_3 \cos 1290t + K_4 \sin 1290t\right] + 0.705\cos\left(500t + \pi/4 + 88.48^\circ\right). \rightarrow (1)$$

To find K_1 & K_2:-

At $t=0$, $i=0$:

$$0 = e^{0}\left[K_3 \cos 0\right] + 0.705\cos(133.48^\circ)$$

$$K_3 = -0.705\cos(133.48) = 0.485$$

diff. (1) wrt 't'.

$$\frac{di}{dt} = e^{-37.5t}\left[-1290K_3 \sin 1290t + 1290K_4 \cos 1290t\right] - 37.5e^{-37.5t}\left[K_3 \cos 1290t + K_4 \sin 1290t\right] - 0.705 \times 500 \sin\left[500t + \pi/4 + 88.48\right] \rightarrow (2).$$

But at $t=0$ in (*),

$$15 \times 0 + 0.2\frac{di(0^+)}{dt} = 400\cos(0+\pi/4).$$

$$\frac{di(0^+)}{dt} = 1414 A/sec.$$

Sub. $\frac{di}{dt}$ in (2)

$$1414 = e^{0}\left[0 + 1290K_4\right] - 37.5 \times e^{0} \times K_3 - 0.705 \times 500 \times \sin(133.5^\circ)$$

$$1414 = 1290K_4 - 37.5K_3 - 255.6$$

$$\boxed{K_4 = 1.3}$$

$\therefore$ The complete soln. is,

$$i = e^{-37.5t}\left[0.485\cos 1290t + 1.3\sin 1290t\right] + 0.705\cos(500t + 133.5^\circ)$$