# MODULE 2

## NETWORK THEOREMS

Mesh & nodal analysis are general analysis which can be used for detailed analysis of a given n/w. In many application it may be necessary to find current & voltage in particular branch, in such cases n/w. theorems are used.

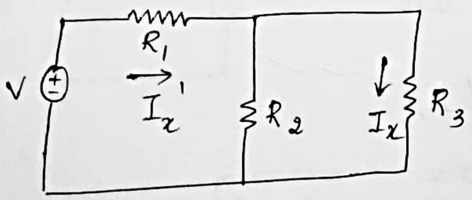
### 1. RECIPROCITY THEOREM :-

This theorem is applicable only to single src n/w.

**Statement :-** In a single src linear bilateral n/w the ratio of response to excitation remains the same when the positions of src. & response are interchanged.

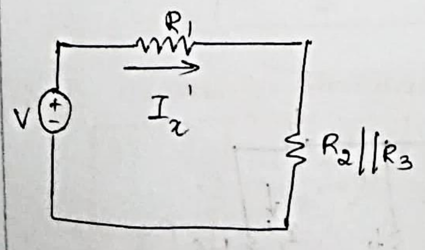
**Proof:-** consider the linear n/w. shown below,

Let the response be '$I_x$'




$$I_x = \frac{R_2 \times I_x'}{R_2 + R_3}$$

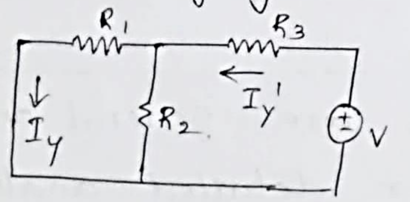
But, $I_x' = \dfrac{V}{R_1 + (R_2 \| R_3)}$




$$\therefore I_x = \frac{R_2}{R_2 + R_3} \cdot \frac{V}{R_1 + \dfrac{R_2 R_3}{R_2 + R_3}}$$

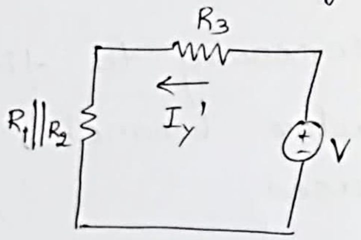
$$\hookrightarrow = \frac{R_2 V}{R_1 R_2 + R_1 R_3 + R_2 R_3} \quad \rightarrow (1)$$

Interchanging the positions of src & response




* NOTE:-
Independent V.S. should be replaced by short ckt. & C.S. by open ckt.

$$I_y = \frac{R_2 \cdot I_y'}{R_1 + R_2}$$



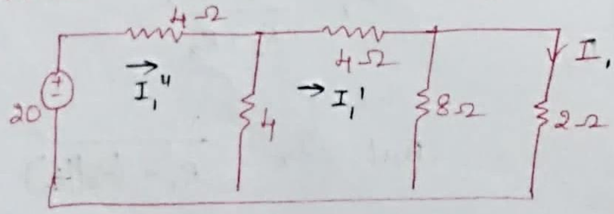

But $I_y' = \dfrac{V}{R_3 + (R_1 \| R_2)}$

$$\therefore I_y = \frac{R_2}{R_1 + R_2} \cdot \frac{V}{R_3 + \frac{R_1 R_2}{R_1 + R_2}}$$

$$I_y = \frac{R_2 V}{R_1 R_2 + R_1 R_3 + R_2 R_3} \longrightarrow (2)$$

$\therefore$ From (1) & (2) $\boxed{I_x = I_y}$

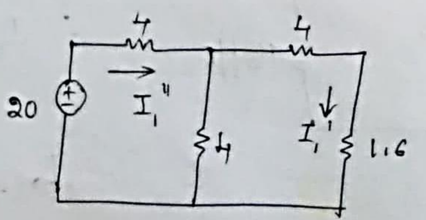
1. Find current through $2\Omega$



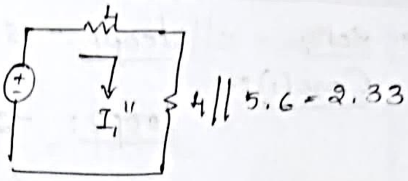

→ Case 1:

$$I_1 = \frac{8 \times I_1'}{10} \longrightarrow (1)$$

$8 \| 2 = 1.6$



$$\therefore \; I_1' = \frac{4}{7.6} \times I_1''$$



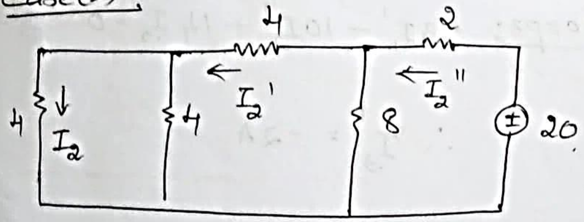

$$\therefore \; I_1'' = \frac{20}{4 + 2.33}$$

$$\Rightarrow \; = 3.159\,A$$

$$\therefore \; I_1' = \frac{4 \times 3.159}{7.6} = 1.3162\,A$$

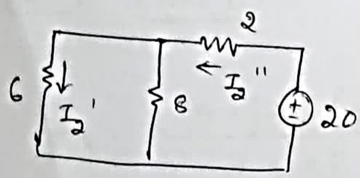
$$I_1 = \frac{8 \times 1.3162}{10} = 1.053\,A$$

Case (2):



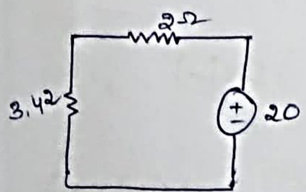

$$I_2 = \frac{4}{8} \times I_2'$$

$$4 \| 4 = 2\Omega$$




$$I_2' = \frac{8 \times I_2''}{8+6}$$

$$6 \| 8 = 3.42$$



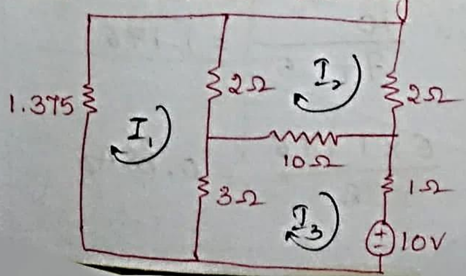

$$\therefore \; I_2'' = \frac{20}{2 + 3.42} = 3.69\,A$$

$$\therefore \; I_2' = \frac{8 \times 3.69}{8+6} = 2.10\,A$$

$$I_2 = \frac{4 \times 2.10}{8} = 1.05\,A$$

$$\therefore \; \boxed{I_1 = I_2 = 1.05\,A}$$

(2) Find current through 1.375Ω.



→ Soln:-

Case(1):-

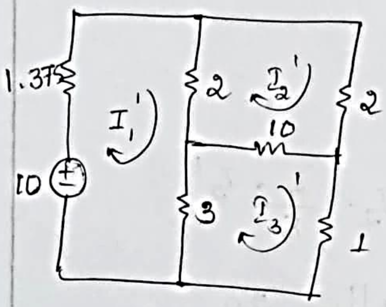
loop1: $6.375I_1 - 2I_2 - 3I_3 = 0 \rightarrow (1)$

loop2: $-2I_1 + 14I_2 - 10I_3 = 0 \rightarrow (2)$

loop3: $-3I_1 - 10I_2 + 14I_3 = -10 \rightarrow (3)$

$\therefore \quad I_1 = -2A$

Case(2):-




loop1: $6.375I_1' - 2I_2' - 3I_3' = 10 \rightarrow (1)$

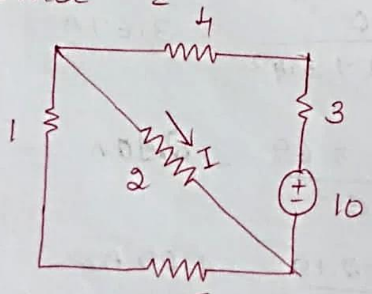
loop2: $-2I_1' + 14I_2' - 10I_3' = 0 \rightarrow (2)$

loop3: $-3I_1' - 10I_2' + 14I_3' = 0 \rightarrow (3)$

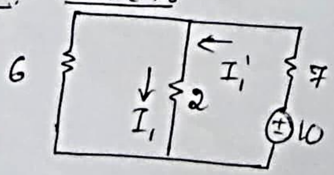
$\therefore \quad I_3' = -2A$

$\therefore \boxed{I_1 = I_3' = -2A}$

3. Find '$I$'




→ Soln:- Case(1):



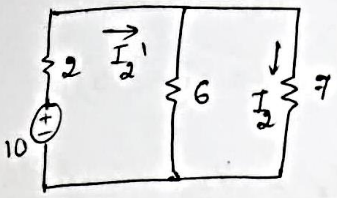

$I_1 = \dfrac{6 \times I_1'}{8}$

$6 \| 2 = 1.5$

But $I_1' = \dfrac{10}{7+1.5} = 1.176$

$\therefore \quad I_1 = \dfrac{6 \times 1.176}{8} = 0.88A$

Case (2):



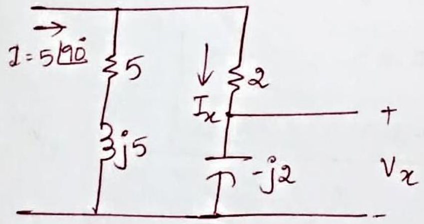

$$I_2 = \frac{6 \times I_2'}{7+6} \qquad 6 \| 7 = 3.23$$

$$I_2' = \frac{10}{2+3.23} = 1.911$$

$$I_2 = \frac{6 \times 1.911}{13} = 0.882\,A.$$

$$\therefore \boxed{I_1 = I_2 = 0.882A} = I$$

+ Find '$V_x$'

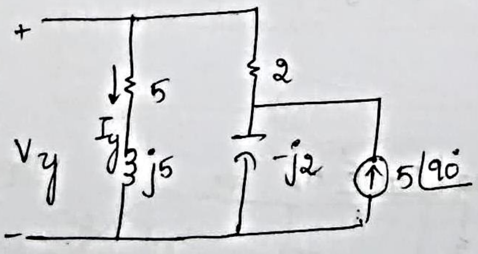



⇒ Soln:- Case (1):- $V_x = (-j2)\, I_x$

$$I_x = \frac{(5+j5)\,(5\angle 90°)}{7+3j} = 4.6424\angle 111.801\, A$$

$$V_x = (-j2)\,(4.6424\angle 111.801) = 9.2848\angle 21.8°\ V$$

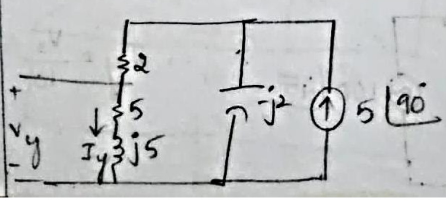
Case (2):-




$$V_y = (5+j5)\, I_y.$$

$$I_y = \frac{0-j2}{7+j3} \times 5\angle 90°$$

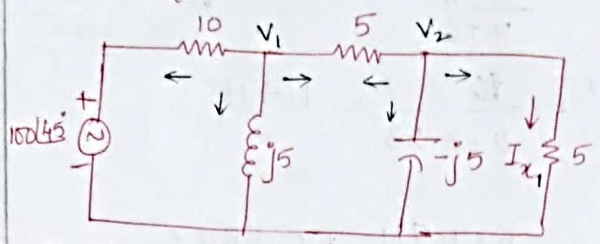
$$I_y = 1.313\angle -23.198\ A$$




$$\therefore V_y = 9.2848\angle 21.8°\ V$$

$$\therefore \boxed{V_x = V_y}$$

5. Find $I_x$




$$I_{x_1} = \frac{V_2}{5}$$

Case(1):

Node(1):

$$\frac{V_1 - 100\angle 45^\circ}{10} + \frac{V_1}{j5} + \frac{V_1 - V_2}{5} = 0$$

$$(0.3 - j0.2)V_1 - 0.2V_2 = 10\angle 45^\circ \longrightarrow (1)$$

Node(2):

$$\frac{V_2 - V_1}{5} + \frac{V_2}{-j5} + \frac{V_2}{5} = 0$$

$$-0.2V_1 + (0.4 + j0.2)V_2 = 0 \longrightarrow (2)$$

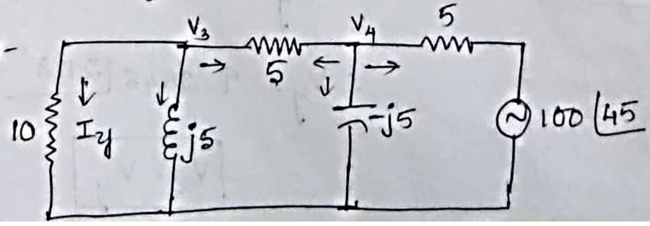
$$\begin{bmatrix} (0.3-j0.2) & -0.2 \\ -0.2 & (0.4+j0.2) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 10\angle 45^\circ \\ 0 \end{bmatrix}$$

$$V_2 = \frac{\begin{vmatrix} 0.3-j0.2 & 10\angle 45^\circ \\ -0.2 & 0 \end{vmatrix}}{\Delta} = \frac{2\angle 45^\circ}{(0.3-j0.2)(0.4+j0.2) - (0.2)^2}$$

$$V_2 = 16.66\angle 54.46^\circ \text{ V}$$

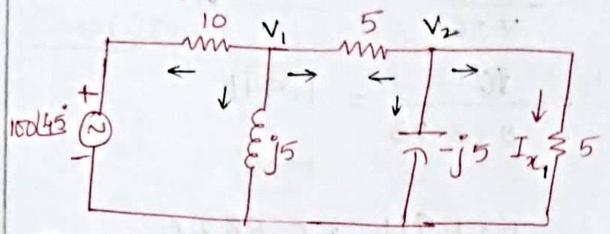
$$\therefore I_{x_1} = \frac{16.66\angle 54.46}{5} = 3.332\angle 54.46 \text{ A}$$

Case(2):-




$$\therefore I_y = \frac{V_3}{10}$$

5. Find $I_x$




$$I_{x_1} = \frac{V_2}{5}$$

→ Case(1):

Node(1):

$$\frac{V_1 - 100\angle 45^\circ}{10} + \frac{V_1}{j5} + \frac{V_1 - V_2}{5} = 0$$

$$(0.3 - j0.2)V_1 - 0.2V_2 = 10\angle 45^\circ \longrightarrow (1)$$

Node(2):

$$\frac{V_2 - V_1}{5} + \frac{V_2}{-j5} + \frac{V_2}{5} = 0$$

$$-0.2V_1 + (0.4 + j0.2)V_2 = 0 \longrightarrow (2)$$

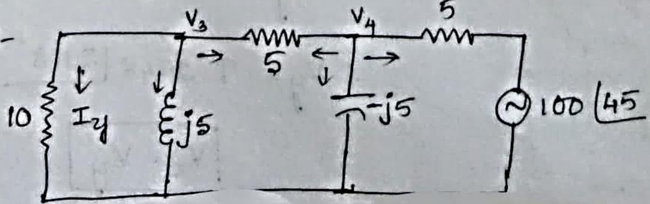
$$\begin{bmatrix} (0.3-j0.2) & -0.2 \\ -0.2 & (0.4+j0.2) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 10\angle 45^\circ \\ 0 \end{bmatrix}$$

$$V_2 = \frac{\begin{vmatrix} 0.3-j0.2 & 10\angle 45^\circ \\ -0.2 & 0 \end{vmatrix}}{\Delta} = \frac{2\angle 45^\circ}{(0.3-j0.2)(0.4+j0.2) - (0.2)^2}$$

$$V_2 = 16.66\angle 54.46^\circ \text{ V}$$

$$\therefore I_{x_1} = \frac{16.66\angle 54.46}{5} = 3.332\angle 54.46 \text{ A}$$

Case(2):-




$$\therefore I_y = \frac{V_3}{10}$$

Node 3:
$$\frac{V_3}{10} + \frac{V_3}{j5} + \frac{V_3 - V_4}{5} = 0$$

$$(0.3 - j0.2)V_3 - 0.2V_4 = 0 \longrightarrow (3)$$

Node 4:
$$\frac{V_4 - V_3}{5} + \frac{V_4}{-j5} + \frac{V_4 - 100\angle 45^\circ}{5} = 0$$

$$-0.2V_3 + (0.4 + j0.2)V_4 = 20\angle 45^\circ \longrightarrow (4)$$

$$\begin{bmatrix} (0.3-j0.2) & -0.2 \\ -0.2 & (0.4+j0.2) \end{bmatrix} \begin{bmatrix} V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 20\angle 45^\circ \end{bmatrix}$$

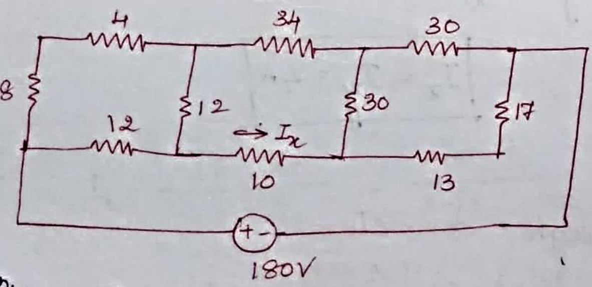
$$V_3 = \frac{\begin{vmatrix} 0 & -0.2 \\ 20\angle 45^\circ & 0.4+j0.2 \end{vmatrix}}{\Delta} = \frac{0.2 \times 20\angle 45^\circ}{(0.3-j0.2)(0.4+j0.2) - (0.2)^2}$$

$$V_3 = 33.33\angle 54.46^\circ \text{ V}$$

$$\therefore I_y = \frac{V_3}{10} = 3.333\angle 54.46^\circ \text{ V}$$

$$\therefore \boxed{I_x = I_y}$$

6. Determine current through '$10\Omega$'

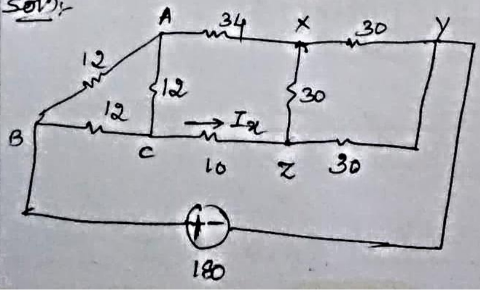



$\Delta$ to Y

$R_A = R_B = R_C$

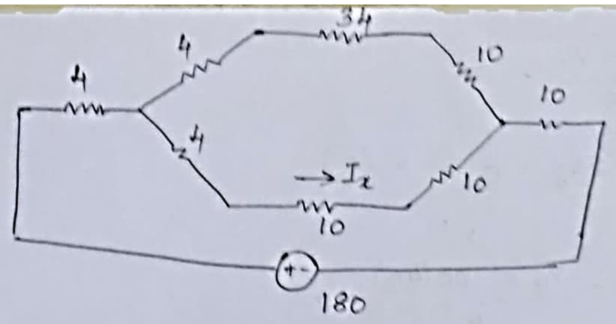
$$R_A = \frac{R_{AB}R_{AC}}{\Sigma R_{AB}} = \frac{12 \times 12}{36} = 4\Omega$$

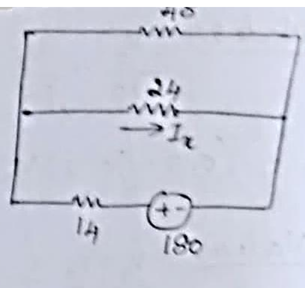
→ Soln:-

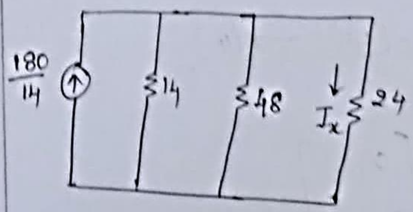


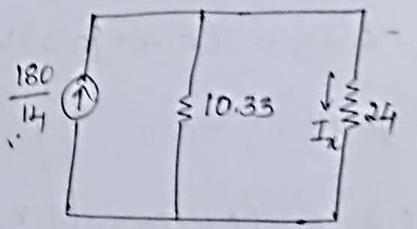

$R_x = R_y = R_z$

$$R_x = \frac{R_{xy}R_{xz}}{\Sigma R_{xz}} = \frac{30 \times 30}{90} = 10\Omega$$



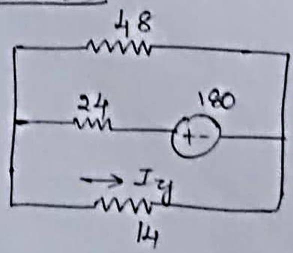



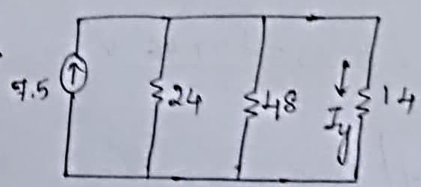




$14 \| 48 = 10.33$

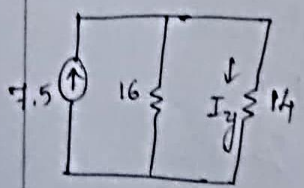
$$I_x = \frac{10.33 \times (180/14)}{34.33} \quad ; \quad \boxed{I_x = 4A}$$

Case(2):-



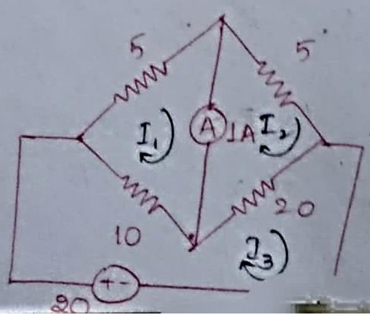

$24 \| 48 = 16\Omega$




$$I_y = \frac{16 \times 7.5}{30} = 4A$$

$$\therefore \boxed{I_x = I_y = 4A}$$

7. Find current through ammeter.



Soln:- Case1:

loop1:- $16I_1 - I_2 - 10I_3 = 0 \longrightarrow (1)$

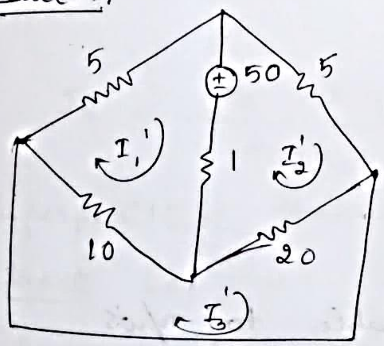
loop2:- $-I_1 + 26I_2 - 20I_3 = 0 \longrightarrow (2)$

loop3:- $-10I_1 - 20I_2 + 30I_3 = 50 \longrightarrow (3)$

$I_1 = 4.59A$ ; $I_2 = 5.41A$

$\therefore I_A = I_2 - I_1 = 0.82A$

Case(2):




loop1: $16I_1' - I_2' - 10I_3' = -50 \rightarrow (1)$

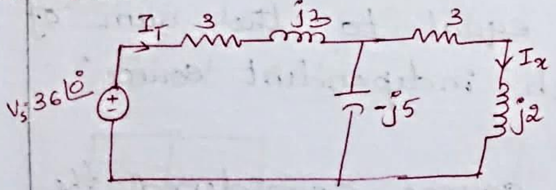
loop2: $-I_1' + 26I_2' - 20I_3' = 50 \rightarrow (2)$

loop3: $-10I_1' - 20I_2' + 30I_3' = 0 \rightarrow (3)$

$I_3' = 0.82A$.

$\therefore \boxed{I_A = I_3' = 0.82A}$

8. Find current '$I_x$' in '$j2\Omega$' impedance.

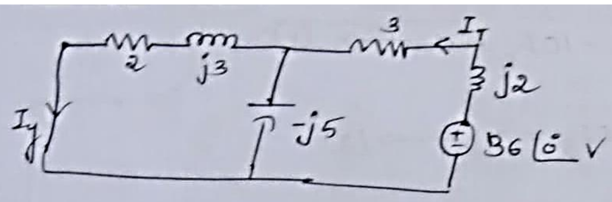



→ Soln:- $I_x = \dfrac{-j5 \times I_T}{-j5 + 3 + j2} = \dfrac{-j5 \cdot I_T}{3 - j3}$

$Z_T = (2 + j3) + [(-j5) \,||\, (3 + j2)] = 6.537 \angle 19.36°$

$\therefore I_T = \dfrac{36\angle 0°}{6.537 \angle 19.36°} = 5.507 \angle -19.36° \ A$

$\therefore I_x = 6.49 \angle -64.36° \ A$




$$I_y = \frac{-j5}{2-j2} \cdot I_T'$$

$$\therefore I_T' = \frac{36\angle 0°}{Z_T'} = 3.672\angle -19.36° \text{ A}$$

$$\therefore Z_T' = \left[(2+j3) \| (-j5)\right] + (3+j2)$$

$$\hookrightarrow = 9.804\angle 19.36° \ \Omega$$

$$\therefore I_y = 6.49\angle -64.36° \text{ A}$$

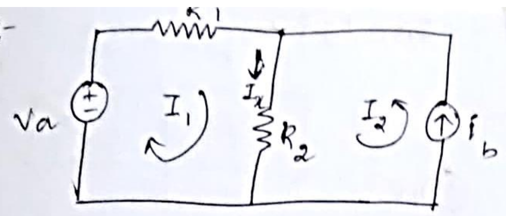
## 2 SUPERPOSITION THEOREM:

This theorem is applicable to n/w's containing multiple srcs.

<u>Statement</u>:- Superposition theorem states that, "in a linear bilateral n/w. containing several independent srcs, the overall response at any time in the n/w. is equal to the sum of responses due to each independent source."

Other than the source considered the remaining srcs are set to zero. i.e. independent Voltage srcs. are replaced by short ckt. & independent current srcs. are replaced by open ckt. Dependent srcs. are left as they are

Proof :-




$\therefore I_x = I_1 + I_2$

$I_2 = i_b$.

$$\therefore V_a = R_1 I_1 + R_2(I_1 + I_2) = (R_1 + R_2) I_1 + R_2 i_b$$

$$\therefore I_1 = \frac{V_a - R_2 i_b}{R_1 + R_2}$$

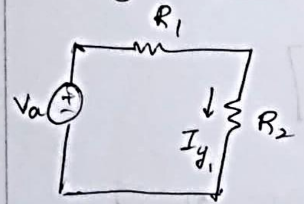
W.K.T.

$$I_x = I_1 + I_2 = \frac{V_a - R_2 i_b}{R_1 + R_2} + i_b.$$

$$I_x = \frac{V_a + R_1 i_b}{R_1 + R_2} \longrightarrow (1)$$

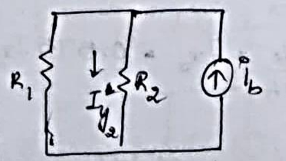
Superposition theorem method :

Case(1): Let V.S. '$V_a$' be present & C.S. '$i_b$' be set to zero [open ckt.]




$$\therefore I_{y_1} = \frac{V_a}{R_1 + R_2}$$

Case(2) :- Let C.S. be present & V.S. be set to zero [short ckt].




$$\therefore I_{y_2} = \frac{R_1 i_b}{R_1 + R_2}$$

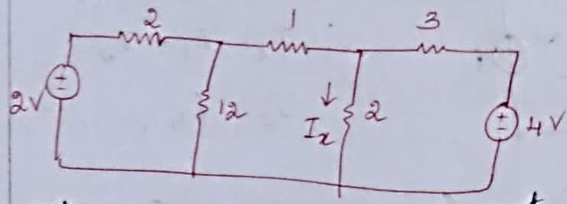
$\therefore$ By SP. theorem, $I_y = I_{y_1} + I_{y_2}$

$$\therefore I_y = \frac{V_a}{R_1 + R_2} + \frac{R_1 i_b}{R_1 + R_2}$$

$$I_y = \frac{V_a + R_1 i_b}{R_1 + R_2} \rightarrow (2)$$

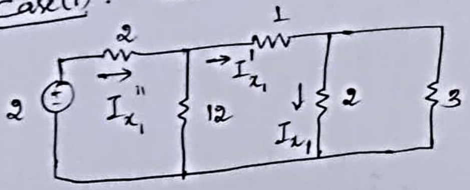
$\therefore (1) = (2)$ ; $\boxed{I_x = I_y}$ Hence superposition theorem verified

1 Find the current in $R = 2\Omega$




⇒ Soln :-

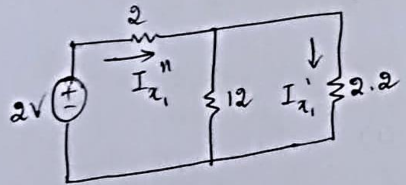
Case(i) :- Let 2V be present & 4V = 0




$$I_{x_1} = \frac{3}{5} \times I_{x_1}' = 0.6\, I_{x_1}'$$

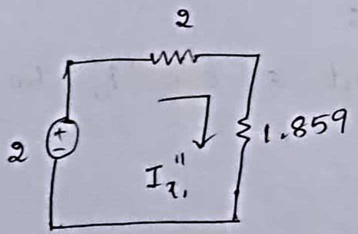
$$[2||3] + 1 = 2.2\,\Omega$$

$$\therefore I_{x_1}' = \frac{12}{14.2} \times I_{x_1}'' = 0.845\, I_{x_1}''$$




$$12||2.2 = \frac{12 \times 2.2}{14.2}$$

$$\hookrightarrow = 1.859\,\Omega$$

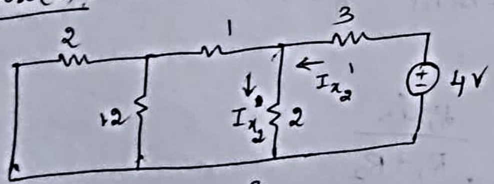



$$\therefore I_{x_1}'' = \frac{2}{3.859} = 0.5182\,A$$

$$\therefore I_{x_1}' = 0.845 \times 0.5182 =$$

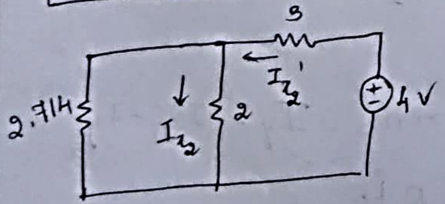
$$\boxed{I_{x_1} = 0.2627\,A}$$

Case(2) :- Let 4V be present.



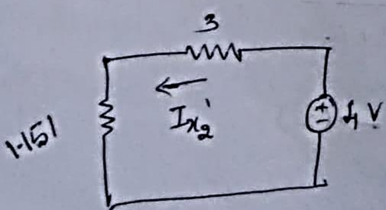

$$[2||12] + 1 = \frac{24}{14} + 1$$

$$\hookrightarrow = 2.714$$




$$\therefore I_{x_2} = \frac{2.714}{4.714} \times I_{x_2}' = 0.575\, I_{x_2}'$$

$$[2||2.714] = 1.151\,\Omega$$



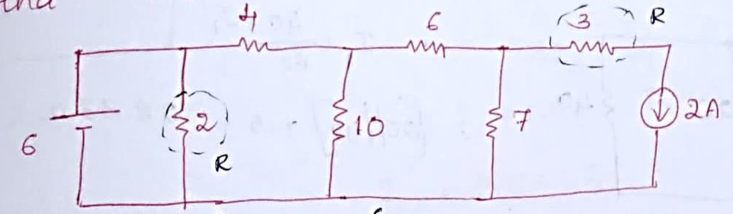

$$I_{x_2}' = \frac{4}{4.151} = 0.9635\,A$$

$$\therefore I_{x_2} = 0.5540\,A$$

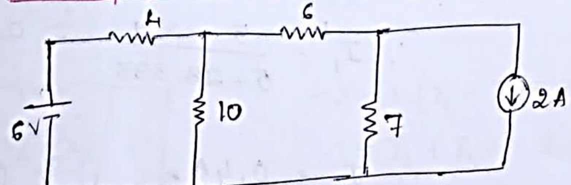
$I_x = I_{x_1} + I_{x_2} = 0.2627 + 0.5540$

$\boxed{I_x = 0.8167A}$

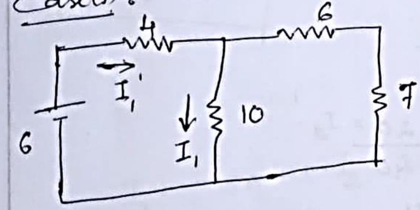
2. Find the current through $10\Omega$



$\Rightarrow$ Sol$^n$:-



Case(1):- Let 6V be present



$6+7=13$.

$I_1 = \frac{13 \times I_1'}{23} = 0.5652 I_1'$

$13 \| 10 = 5.652$.

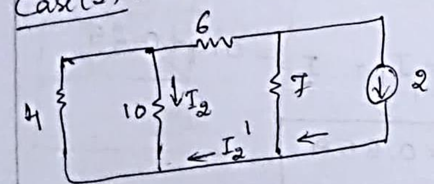
$\therefore I_1' = \frac{6}{4+5.652} = 0.6216$

$I_1 = 0.3513A$

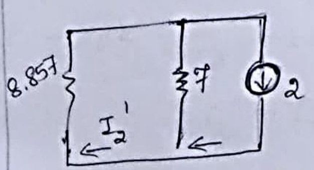
Case(2):- Let 2A be present

$\therefore I_2 = \frac{4}{14} \times (-I_2)'$

$[4\|10] + 6 = 8.857$.



$\therefore I_2' = \frac{7 \times 2}{15.857} = 0.8828$

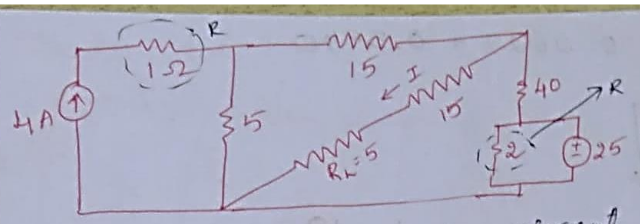


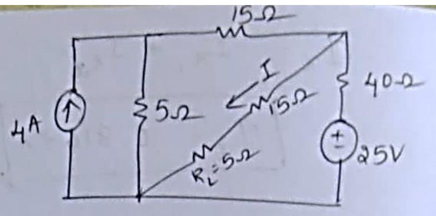
$\therefore I_2 = -0.2522A$.

$\therefore I = I_1 + I_2 = 0.3513 - 0.2522$

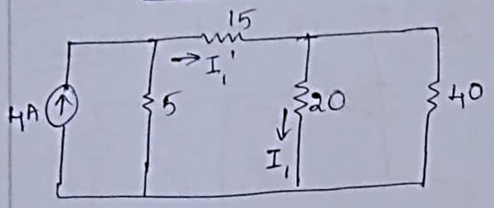
$\boxed{I = 0.099A}$

3. Find current through '$R_L$'



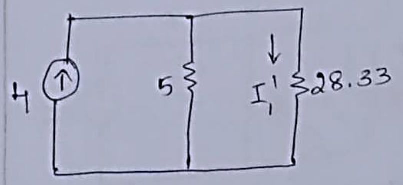




⇒ Sol$^n$:- Case(1): Let 4A be present




$\therefore\ I_1 = \frac{40 \times I_1'}{60}$

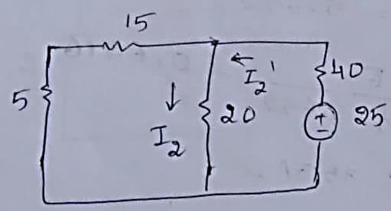
$\therefore\ [20 \| 40] + 15 = 28.33\Omega$.




$\therefore\ I_1' = \frac{5 \times 4}{5 + 28.333} = 0.6$

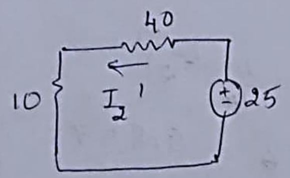
$\therefore\ I_1 = 0.4A$

Case(2): Let 25V be present




$I_2 = \frac{20 \times I_2'}{40}$

$20 \| 20 = 10$



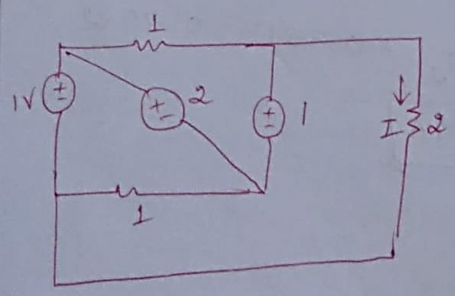

$I_2' = \frac{25}{50} = 0.5$

$\therefore\ I_2 = 0.25$

$\therefore\ I = I_1 + I_2 = 0.4 + 0.25$

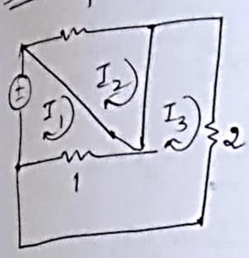
$\boxed{I = 0.65A}$

4- Find the current through 2Ω



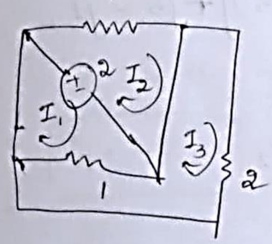
Case (1) : 1V

Soln:



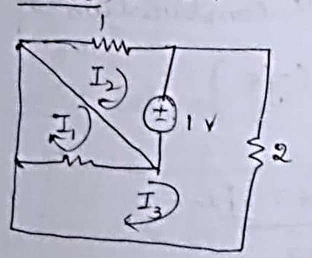
$$1 - (I_1 - I_3) = 0$$
$$-I_1 + I_3 = -1 \rightarrow (1)$$
$$I_2 = 0$$
$$-2I_3 - 1(I_3 - I_1) = 0$$
$$-3I_3 + I_1 = 0 \rightarrow (2)$$
$$\therefore I_1 = 1.5A;\ I_2 = 0;\ I_3 = 0.5A$$
$$I'_{2\Omega} = 0.5A = I_3$$

Case (2) :- 2V.



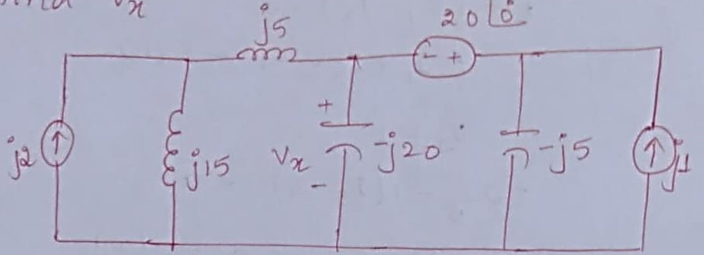
$$-2 - 1(I_1 - I_3) = 0$$
$$-I_1 + I_3 = 2 \rightarrow (1)$$
$$2 - I_2 = 0;\ I_2 = 2$$
$$-2I_3 - 1(I_3 - I_1) = 0$$
$$I_1 - 3I_3 = 0 \rightarrow (2)$$
$$I_1 = -3A;\ I_2 = 2A;\ I_3 = -1A$$
$$I''_{2\Omega} = I_3 = -1$$

Case (3) :- 1V

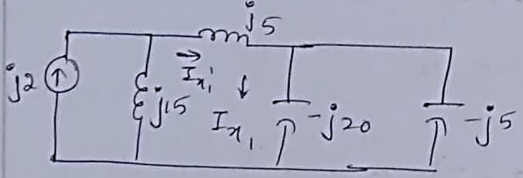


$$-I_1 + I_3 = 0 \rightarrow (1)$$
$$-I_2 - 1 = 0$$
$$I_2 = -1$$
$$1 - 2I_3 - (I_3 - I_1) = 0$$
$$I_1 - 3I_3 = -1 \rightarrow (2)$$
$$I_1 = 0.5;\ I_2 = -1A;\ I_3 = 0.5$$
$$\therefore I'''_{2\Omega} = I_3 = 0.5$$
$$\therefore I_{2\Omega} = I'_{2\Omega} + I''_{2\Omega} + I'''_{2\Omega} = 0.5 - 1 + 0.5$$
$$\boxed{I_{2\Omega} = 0A}$$

5. Find '$V_x$'

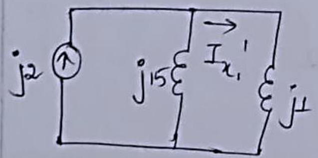



Soln:- Case (1): j2A = present.




$$\therefore I_{x_1} = \frac{-j5}{-j25} \times I_{x_1}' = 0.2 I_{x_1}'$$

$$[(-j20)||(-j5)] + j5 = j1$$

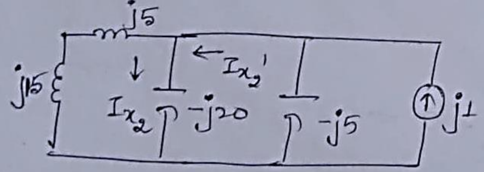



$$\therefore I_{x_1}' = \frac{j15 \times j2}{j16}$$

$$I_{x_1}' = j1.875A$$

$$\therefore I_{x_1} = j0.375A$$

Case(2):- Let j1A be present

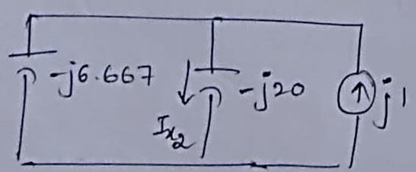



$$I_{x_2} = \frac{j20}{j20 - j20} \times I_{x_2}' = \infty$$

So take ||le combination of $(j20) || (-j5)$

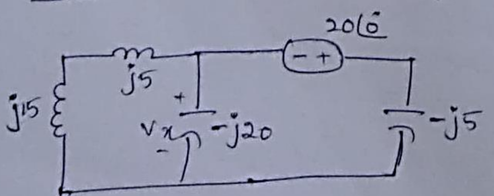
$$\therefore j20 || (-j5) = -j6.667$$

$$I_{x_2} = \frac{-j6.667 \times j1}{-j26.67}$$

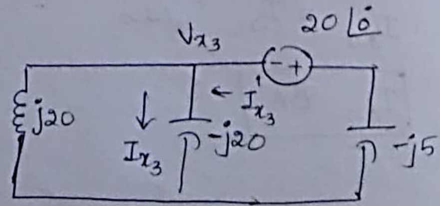



$$I_{x_2} = 0.247j$$

Case (3):- Let 20∠0° be present




→



Node analysis,

$$\frac{V_{x_3}-0}{j20} + \frac{V_{x_3}}{-j20} + \frac{V_{x_3}+20\angle 0^\circ}{-j5} = 0$$

$$V_{x_3} = -20\angle 0^\circ$$

$$\therefore I_{x_3} = \frac{-20\angle 0^\circ}{-j20} = -j1$$

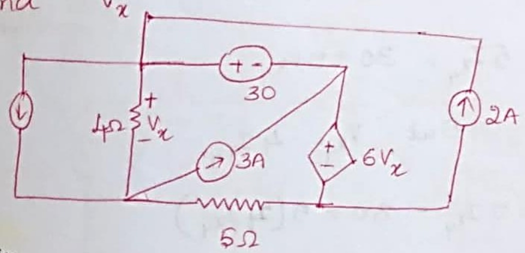
By superposition thm,

$$I_x = I_{x_1} + I_{x_2} + I_{x_3} = -j0.375A$$

$$\therefore V_x = I_x R = -j0.375 \times (-j20)$$

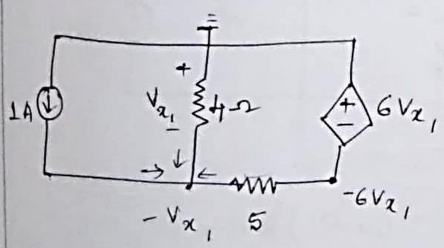
$$\boxed{V_x = -7.5V}$$

6. Find '$V_x$'




⇒ Soln: While applying SPT, dependent srcs. are not set to zero.

Case(1): Let 1A c.s. be present



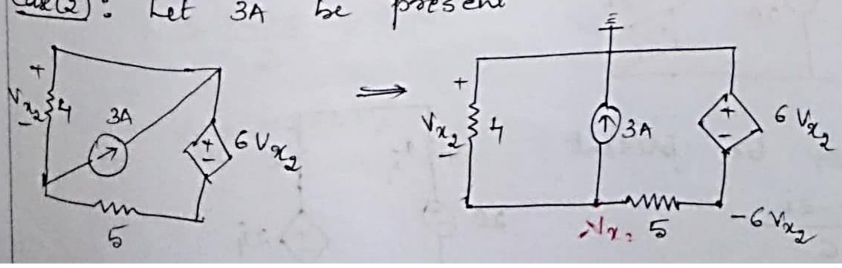

$$1 + \frac{0-(-V_{x_1})}{4} + \frac{(-6V_{x_1})+V_{x_1}}{5} = 0$$

$$1 + 0.25V_{x_1} - V_{x_1} = 0$$

$$1 - 0.75V_{x_1} = 0$$

$$V_{x_1} = 1.33V$$

Case(2): Let 3A be present

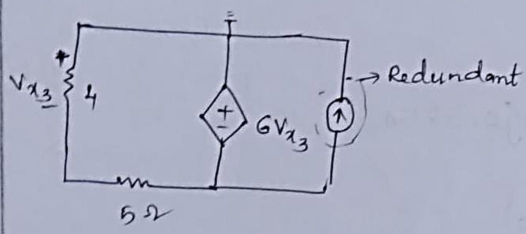


$$3 + \frac{-V_{x_2}}{4} + \frac{[-V_{x_2} - (-6V_{x_2})]}{5} = 0$$

$$3 - 0.25V_{x_2} + V_{x_2} = 0$$

$$3 + 0.75V_{x_2} = 0 \; ; \quad V_{x_2} = -4V$$

Case(3) :- Let 2A C.S. be present.

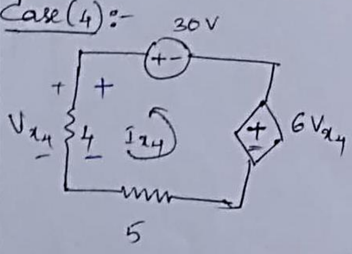



There is no independent src.

$\therefore V_{x_3} = 0$

[ ∵ Anything in $||^{le}$ with V.S. is redundant]

Case(4) :-




$$I_{x_4} = \frac{30 - V_{x_4} + 6V_{x_4}}{5} = \frac{30 + 5V_{x_4}}{5}$$

$$5I_{x_4} = 30 + 5V_{x_4}$$

But $V_{x_4} = \cdot 4I_{x_4}$

$$\therefore 5I_{x_4} = 30 + 5(4I_{x_4})$$

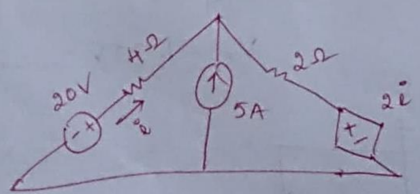
$$-15I_{x_4} = 30; \quad I_{x_4} = -2A$$

$$\therefore V_{x_4} = 4I_{x_4} = -8V$$

∴ By S.P.T.

$$V_x = V_{x_1} + V_{x_2} + V_{x_3} + V_{x_4} = -10.667V$$

7. Find current 'i'

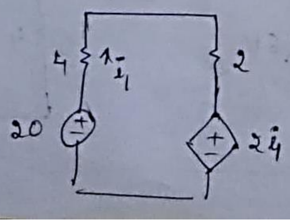



Soln:-

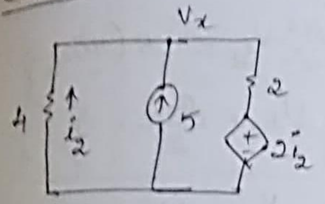
Case (1) : Let 20V be present

$$i_1 = \frac{20 - 2i_1}{6}$$

$\therefore i_1 = 2.5A$



Case (2): Let 5A C.S. be present




$i_2 = \frac{0 - V_x}{4}$

Node analysis.

$5 + i_2 + \frac{2i_2 - V_x}{2} = 0$

$10 + 2i_2 + 2i_2 - V_x = 0$

$10 + 4i_2 - V_x = 0.$

$10 + 4i_2 = V_x = -4i_2$

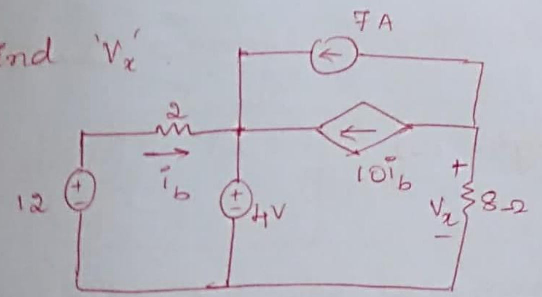
$\therefore 8i_2 = -10;\quad i_2 = -1.25A$

By SP.T,

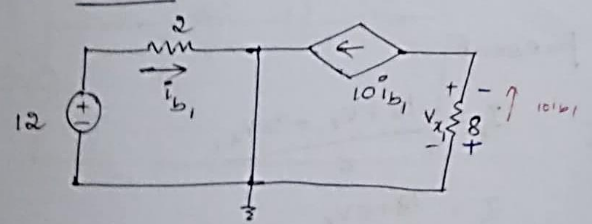
$i = i_1 + i_2 = 2.5 - 1.25$

$i = 1.25A$

8. Find '$V_x$'




Sol$^n$:- Case (1): Let 12V be present.



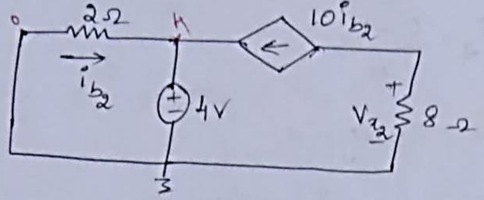

* NOTE:- 8Ω connected across $10i_b'$ is redundant interms of current but not in terms of voltage

$\therefore V_{x_1} = -8(10i_{b_1}) = -80i_{b_1}$

$i_{b_1} = \frac{12-0}{2} = 6A$

$\therefore V_{x_1} = -480V$

Case(2):- Let 4V V.S. be present

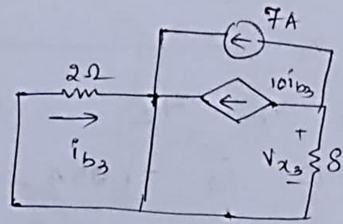



$\therefore V_{x_2} = -8(10 i_{b_2})$

$i_{b_2} = \frac{0-4}{2} = -2$

$V_{x_2} = +160V.$

Case(3):- Let 7A C.S. be present




$\therefore V_{x_3} = -8[10 i_{b_3} + 7)$

$i_{b_3} = 0$

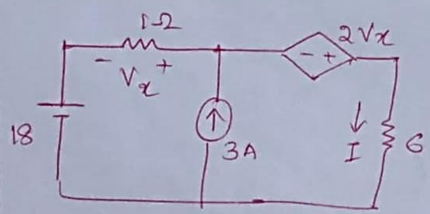
$V_{x_3} = -56V$

By SPT.

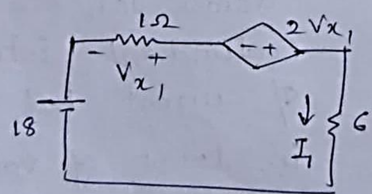
$V_x = V_{x_1} + V_{x_2} + V_{x_3}$

$\boxed{V_x = -376V}$

9. Find current through 6Ω




Soln:- Case(1):- 18V be presem




$I_1 = \frac{18 + V_{x_1} + 2V}{}$

$I_1$

$6I_1 = 18 + 3V_{x_1}.$

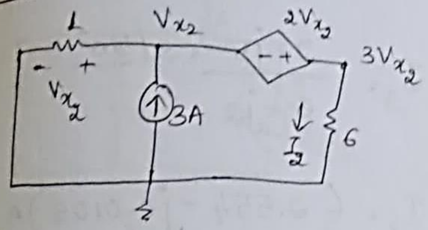
But $V_{x_1} = -I_1$

$\therefore 6I_1 = 18 + 3(-I_1) = 18 - 3I_1$

$9I_1 = 18$

$I_1 = 2A$

Case(2): Let 3A be present




Hint:- If c.s. is present apply node analysis

$$3 + \frac{0 - V_{x_2}}{1} + \frac{0 - 3V_{x_2}}{6} = 0$$

$$3 - V_{x_2} - 0.5V_{x_2} = 0$$

$$3 - 1.5V_{x_2} = 0$$

$$V_{x_2} = 2V$$

$$\therefore I_2 = \frac{3V_{x_2} - 0}{6}$$

$$\hookrightarrow = \frac{6-0}{6}$$

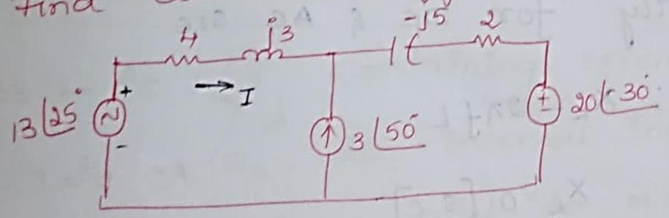
$$I_2 = 1A$$

By SPT,

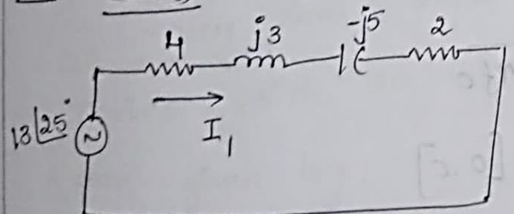
$$I = I_1 + I_2$$

$$\hookrightarrow = 3A$$

10. Find current through $(4+j3)\Omega$



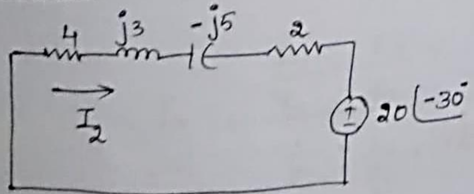

→ Soln: Case(1): Let $13\angle 25^\circ$ be present




$$\therefore I_1 = \frac{13\angle 25^\circ}{6 + j3 - j5}$$

$$I_1 = (1.4925 + j1.4132)A$$

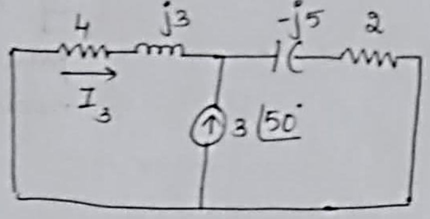
Case(2): Let $20\angle -30^\circ$ be present




$$I_2 = \frac{-20\angle -30^\circ}{6 - j2}$$

$$I_2 = (-3.098 + j0.6339)A$$

Case (3): Let $3\angle 50^\circ$ A be present




$$I_3 = \frac{2-j5}{6-j2} \cdot 3\angle 50^\circ$$

$$I_3 = (2.554 - j0.0105)\,A$$

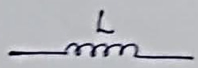
$\therefore$ By SPT,

$$I = I_1 + I_2 + I_3$$

$$I = (-4.1595 + j2.0366)\,A$$

* Practical application of superposition thm.:-

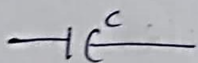
* NOTE: When a n/w contain both AC & DC sources to find the response, SPT must be used because ckt. element behave differently for DC & AC src.




; $X_L = 2\pi f L$

L — DC $\rightarrow$ SC; AC $\rightarrow$ O.C

For $f_{dc} = 0$ ; $X_L = 0$ [S.C]

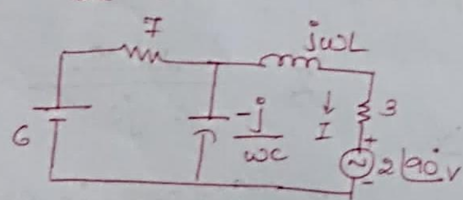



; $X_C = \frac{1}{2\pi f C}$

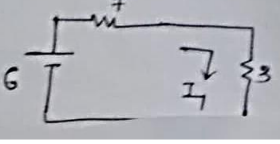
C — DC $\rightarrow$ OC; AC $\rightarrow$ SC

For $f_{dc} = 0$ ; $X_C = \infty$ [O.C]

11. Find the current 'I' through $3\Omega$ [Assume $\omega L = 1$ & $\omega C = 1$]

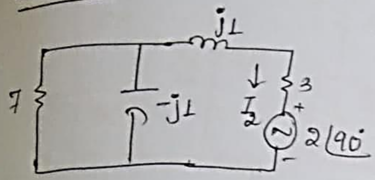



→ Soln:- Case (1): Let 6V be present

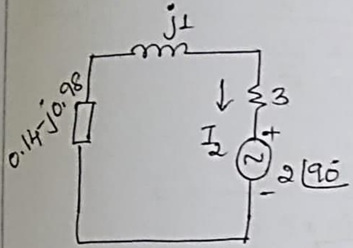



$$I_1 = \frac{6}{10} = 0.6A$$

Case(2) :- Let $2\angle 90^\circ$ be present




$7 \parallel (-j1) = 0.14 - j0.98$




$$I_2 = \frac{-2\angle 90^\circ}{0.14 - j0.98 + j1 + 3}$$

$$I_2 = -0.00405 - j0.6369$$

∴ By SPT, $I = I_1 + I_2 = 0.6 - 0.00405 - j0.6369$

$$\boxed{I = 0.59595 - j0.636\ A}$$

3 ## MILLMAN'S THEOREM :-

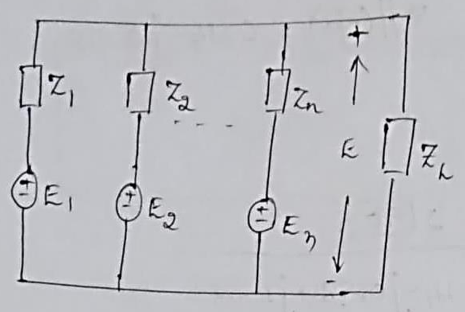
A n/w. containing several independent v.s $E_1, E_2 \ldots E_n$ with internal impedance $Z_1, Z_2 \cdots Z_n$ connected in parallel to a load impedance '$Z_L$' may be replaced by a single v.s. '$E$' in series with an impedance '$Z_L$'. where '$E$' & '$Z$' are given by.

$$E = IZ = \frac{I}{Y} = \frac{I_1 + I_2 + \cdots + I_n}{Y_1 + Y_2 + \cdots + Y_n}$$

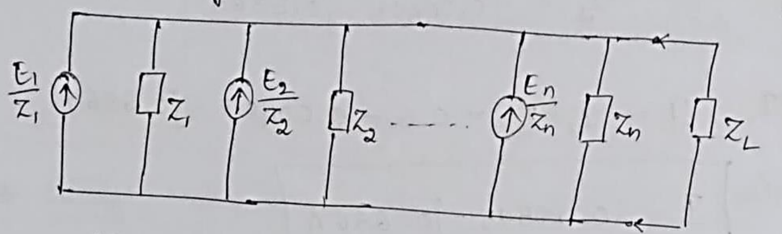
$$\hookrightarrow = \frac{\frac{E_1}{Z_1} + \frac{E_2}{Z_2} + \cdots + \frac{E_n}{Z_n}}{\frac{1}{Z_1} + \frac{1}{Z_2} + \cdots + \frac{1}{Z_n}}$$

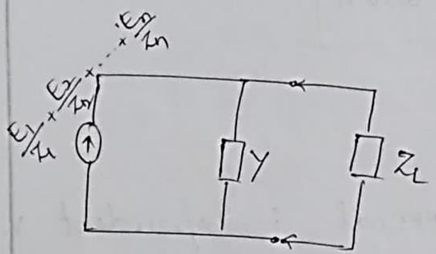
$$\therefore Z = \frac{1}{Y} = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2} + \cdots + \frac{1}{Z_n}}$$

Proof: consider the n/w. given below



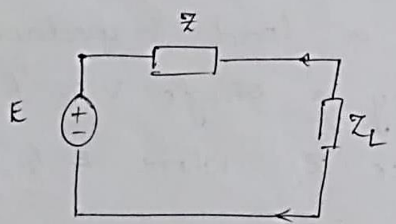

Converting v.s to c.s.




$$Y = \frac{1}{Z_1} + \frac{1}{Z_2} + \dots + \frac{1}{Z_n}$$

$$\therefore Z = \frac{1}{Y} = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2} + \dots + \frac{1}{Z_n}}$$



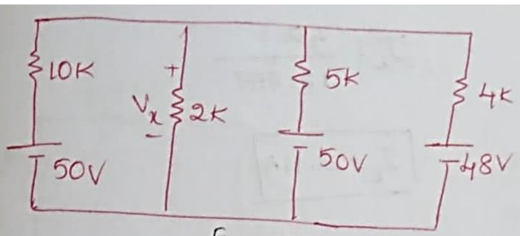

$$E = \frac{\frac{E_1}{Z_1} + \frac{E_2}{Z_2} + \dots + \frac{E_n}{Z_n}}{\frac{1}{Z_1} + \frac{1}{Z_2} + \dots + \frac{1}{Z_n}}$$

* **Note:**

With 3 srcs.

$$E = \frac{\frac{E_1}{Z_1} + \frac{E_2}{Z_2} + \frac{E_3}{Z_3}}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}}$$

1. Find vtg '$V_x$' across 2K resistor by using Millman's theorem.



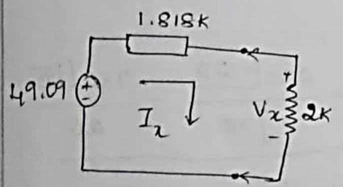

Sol$^n$:

$$E = \frac{\frac{E_1}{Z_1} + \frac{E_2}{Z_2} + \frac{E_3}{Z_3}}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}} = \frac{\frac{50}{10{,}000} + \frac{50}{5000} + \frac{48}{4000}}{\frac{1}{10{,}000} + \frac{1}{5000} + \frac{1}{4000}}$$

$$E = \frac{0.027}{5.5 \times 10^{-4}}$$

$$E = 49.09V$$

$$Z = \frac{1}{Y} = \frac{1}{5.5 \times 10^{-4}} = 1.818k\Omega$$

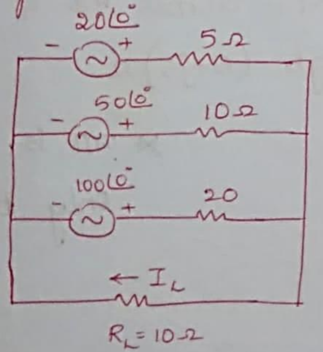



$$I_x = \frac{49.09}{3.818 \times 10^3} = 0.0129A$$

$$\therefore V_x = 2000\, I_x = 2000 \times 0.0154$$

$$\boxed{V_x = 25.8V}$$

2. Using Millman's theorem, find current through $R_L = 10\Omega$




Sol$^n$:-

$$E = \frac{\frac{E_1}{Z_1} + \frac{E_2}{Z_2} + \frac{E_3}{Z_3}}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}} = \frac{\frac{20\angle 0°}{5} + \frac{50\angle 0°}{10} + \frac{100\angle 0°}{20}}{\frac{1}{5} + \frac{1}{10} + \frac{1}{20}} = \frac{14}{0.35}$$

$$E = 40V$$

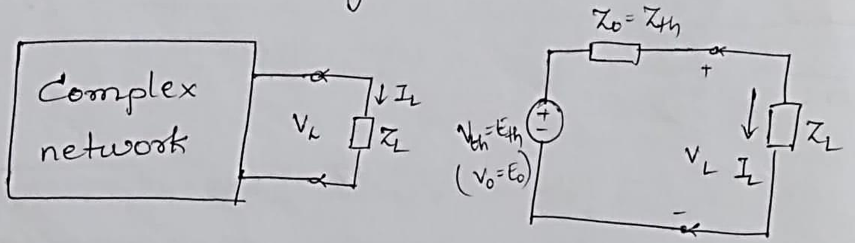
$$Z = \frac{1}{Y} = \frac{1}{0.35} = 2.857\Omega$$

4 ## THEVENIN'S THEOREM :-

A linear bilateral n/w. however complicated the n/w. may be connected to a load impedance '$Z_L$' may be replaced by a single equivalent ckt. containing a voltage source in series with an impedance.

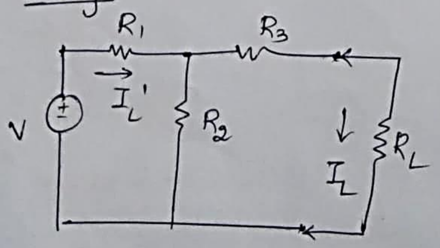
The voltage of voltage source is equal to open circuited voltage across load terminals.

The value of impedance is equal to equivalent impedance of the n/w. as viewed from the load terminals into the n/w replacing all independent voltage source by short ckt & current source by open ckt.




$$\therefore \boxed{I_L = \frac{V_0}{Z_0 + Z_L}} \quad ; \quad \boxed{V_L = \frac{V_0 \times Z_L}{Z_0 + Z_L}}$$

**Proof:-** Consider a linear n/w. given below,




$$I_L = \frac{R_2 \times I_L'}{R_2 + R_3 + R_L}$$

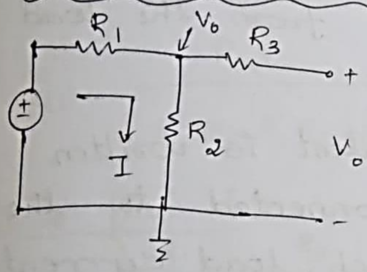
$$\therefore I_L' = \frac{V}{R_1 + \left[R_2 \| (R_3 + R_L)\right]}$$

$$I_L = \frac{R_2}{R_2 + R_3 + R_L}\left[\frac{V}{R_1 + \dfrac{R_2(R_3 + R_L)}{R_2 + R_3 + R_L}}\right]$$

$$I_L = \frac{R_2 V}{R_1R_2 + R_1R_3 + R_2R_3 + R_1R_L + R_2R_L} \longrightarrow (1)$$

To find thevenin's equivalent:-

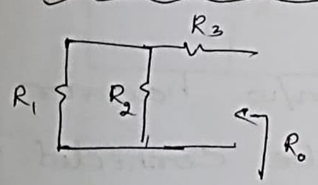
i) To find thevenin's vtg $[V_0]$:



$$V_0 = I R_2$$

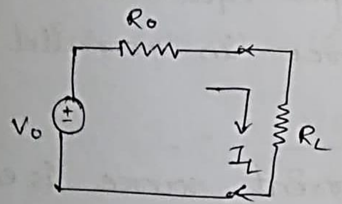
$$\hookrightarrow = \frac{VR_2}{R_1+R_2}.$$

ii) To find thevenin's resistance '$R_0$':



$$R_0 = [R_1 || R_2] + R_3 = \frac{R_1R_2}{R_1+R_2} + R_3$$

$$R_0 = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_1+R_2}$$



$$I_L = \frac{V_0}{R_0 + R_L}$$

$$I_L = \frac{\frac{VR_2}{R_1+R_2}}{\frac{R_1R_2 + R_1R_3 + R_2R_3}{R_1+R_2} + R_L}$$

$$I_L = \frac{VR_2}{R_1R_2 + R_2R_3 + R_1R_3 + R_1R_L + R_2R_L} \longrightarrow (2)$$

(1) = (2)

Hence proved.

* Procedure for solving a n/w using thevenin's theorem

Step1:- The load impedance '$Z_L$' through which voltage (or) current is to be found is removed & an open ckt. is created across load terminal.

step2: The open ckted. vtg. '$V_0$' across load terminal is measured.

step3: All the independent v.s. are replaced by short ckt. & independent c.s. by open ckt.

~~step~~: The equivalent impedance '$Z_0$' looking into the n/w. as viewed from the load terminal is found.

step4: The thevenin's equivalent is written.

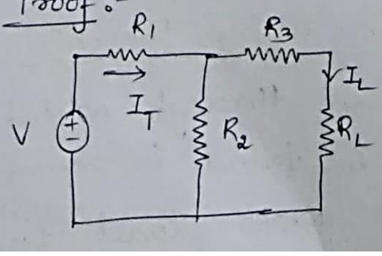
step5: The load '$Z_L$' is connected to the thevenin's equivalent from which load current/ load vtg. can be found.

5. NORTON'S THEOREM:-

In a linear bilateral n/w. however complicated the n/w. may be connected to '$Z_L$' it maybe replaced by simple equivalent ckt containing a current source in parallel with an impedance.

The current of the current source is equal to short ckt current flowing through load terminal & the value of '$Z$' is equal to the equivalent impedance of the n/w. as viewed from the load terminal into the n/w. replacing all independent v.s. by short ckt. & current src by open ckt.

Proof:-




By current division,

$$I_L = \frac{R_2 \cdot I_T}{R_2 + R_3 + R_L} \longrightarrow (1)$$

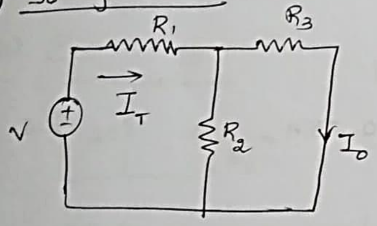
$$I_T = \frac{V}{R_T} \longrightarrow (2)$$

$$R_T = \frac{R_1R_2 + R_2R_3 + R_3R_1 + R_1R_L + R_2R_L}{R_2 + R_3 + R_L} \rightarrow (3)$$

Subs. (3) in (2) & (2) in (1)

$$I_L = \frac{R_2 V}{R_1R_2 + R_2R_3 + R_3R_1 + R_1R_L + R_2R_L} \longrightarrow (4)$$

i) To find $I_o$:-



$$I_o = \frac{R_2 \times I_T}{R_2 + R_3} \longrightarrow (5)$$

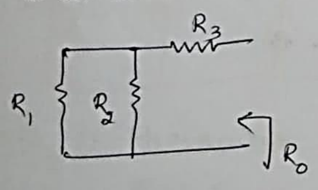
$$I_T = \frac{V}{R_T} \longrightarrow (6)$$

$$R_T = R_1 + \frac{R_2R_3}{R_2 + R_3} \longrightarrow (7)$$

Sub eqn. (7) in (6) & (6) in (5)

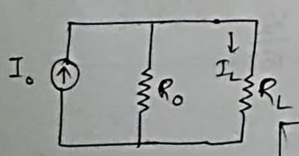
$$\therefore I_o = \frac{VR_2}{R_1R_2 + R_2R_3 + R_1R_3} \longrightarrow (8)$$

ii) To find $R_o$:-



$$R_o = \frac{R_1R_3 + R_2R_3 + R_1R_2}{R_1 + R_2} \rightarrow (9)$$

$\therefore$ Norton's eqn. ckt is,



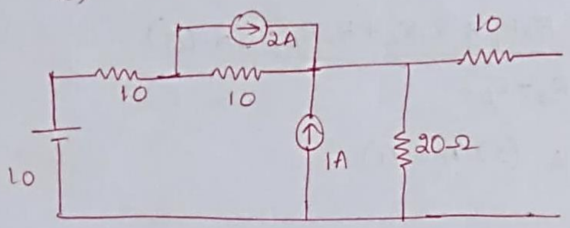
$$\therefore I_L = \frac{R_o \cdot I_o}{R_o + R_L} \longrightarrow (10)$$

subs. (8) & (9) in (10),

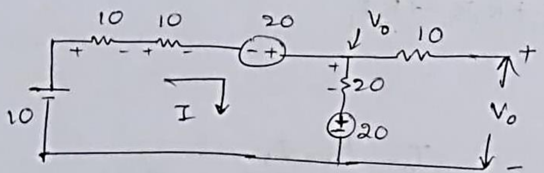
$$\boxed{I_L = \frac{VR_2}{R_1R_2 + R_2R_3 + R_3R_1 + R_1R_L + R_2R_L}} \rightarrow (11)$$

$\therefore$ Norton's thm. is proved.

1) For the n/w shown find.
(i) Thevenin's Equivalent (ii) Norton's equivalent




⇒ Soln:- i) Thevenin's equivalent :- (a) thevenin's vtg:



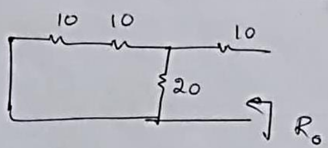

$\therefore V_o = 20 + 20I$

$$\therefore I = \frac{10+20-20}{10+10+20} = 0.25A$$

$\therefore V_o = 20 + 20(0.25)$

$\boxed{V_o = 25V}$

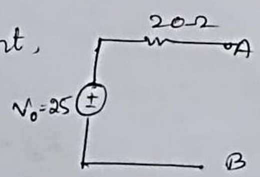
(b) Thevenin's resistance:-




$R_o = [20 \| 20] + 10 = 10 + 10$

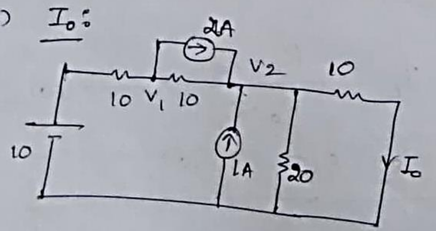
$R_o = 20\Omega$

$\therefore$ Thevenin's equivalent,




ii) Norton's equivalent :-

(a) $I_o$:




$$I_o = \frac{V_2}{10}$$

Node 1 :- $\frac{10-V_1}{10} + \frac{V_2-V_1}{10} - 2 = 0$

$0.2V_1 - 0.1V_2 = -1 \longrightarrow (1)$

Node(2):- $1 + 2 + \frac{V_1 - V_2}{10} + \frac{0 - V_2}{20} + \frac{0 - V_2}{10} = 0$

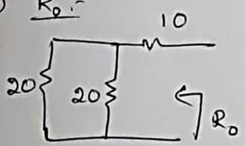
$-0.1V_1 + 0.25V_2 = 3 \longrightarrow (2)$

$$\begin{bmatrix} 0.2 & -0.1 \\ -0.1 & 0.25 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$$

$\therefore V_1 = 1.25V$ ; $V_2 = 12.5V$

$\therefore I_0 = \frac{V_2}{10} = \frac{12.5}{10}$ ; $\boxed{I_0 = 1.25A}$

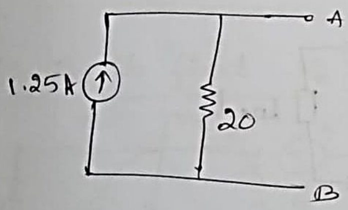
(b) $R_0$:-



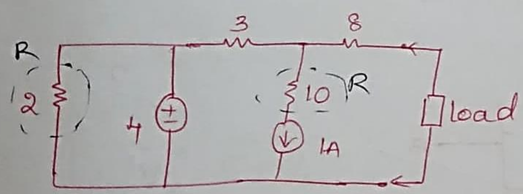

$R_0 = [20 \| 20] + 10$

$\hookrightarrow = 20\Omega$

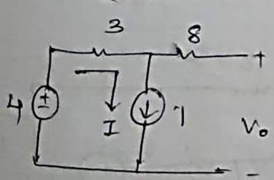
Norton's equivalent,




3. Find thevenin's & norton's equivalent ckt




→ soln:- i) thevenin's equivalent:-




$V_0 = 4 - 3 \cdot I$

But $I = 1A$

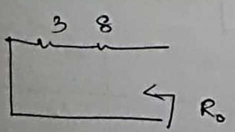
$\therefore V_0 = 4 - 3$ ; $V_0 = 1V$

(or)

$\frac{V_0 - 4}{3} + 1 + 0 = 0$

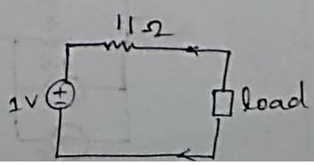
$3 + V_0 - 4 = 0$

$\boxed{V_0 = 1V}$

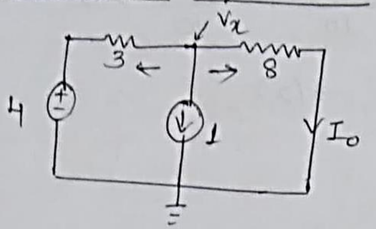



$R_0 = 3 + 8$

$\hookrightarrow = 11\Omega$



ii) Norton's equivalent :-



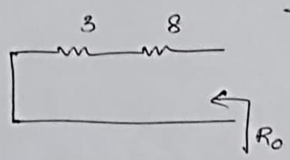

$$I_o = \frac{V_x - 0}{8}$$

Node analysis, $\frac{V_x - 4}{3} + 1 + \frac{V_x}{8} = 0$

$$V_x = 8/11$$

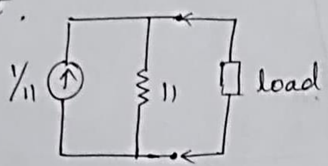
$$\therefore I_o = \frac{8/11}{8}$$

$$\boxed{I_o = \frac{1}{11} A}$$

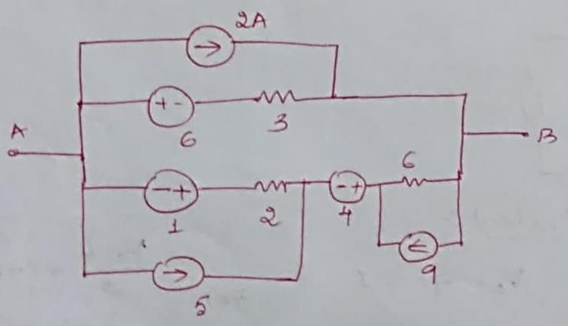



$R_o = 11\Omega$

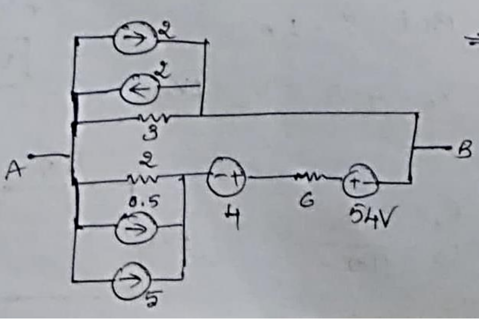
$\therefore$ Norton's equivalent,



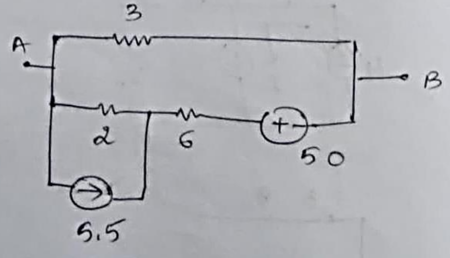

**3. Obtain T.E & N.E**

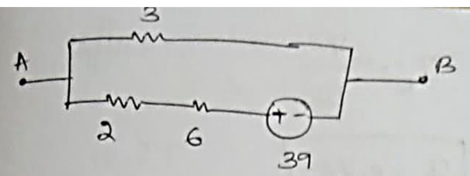



sol$^n$:-

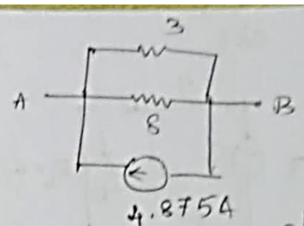



$\Rightarrow$

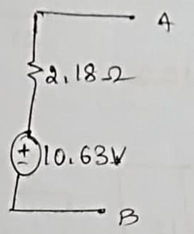




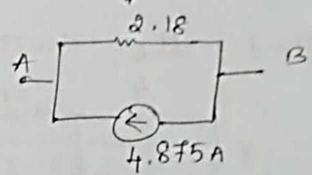

$\rightarrow$




$3 \| 8 = 2.18\,\Omega$

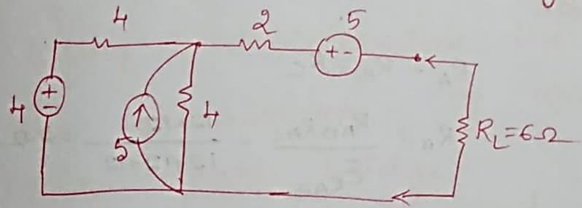



TE

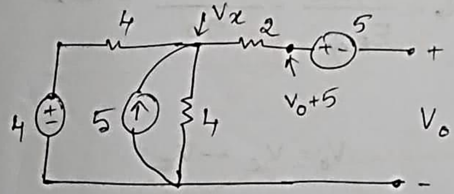



N.E

4. Find the current through $R_L = 6\Omega$




$\Rightarrow$ Soln :- i) To find '$V_o$' :-




$\therefore V_x = 5 + V_o$

$$5 + \frac{4 - V_x}{4} + \frac{0 - V_x}{4} + 0 = 0$$

$$24 - 2V_x = 0$$

$$2V_x = 24$$

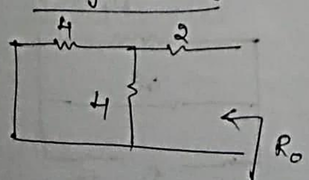
$$\boxed{V_x = 12}$$

$\therefore V_x = 5 + V_o$

$$V_o = V_x - 5 = 12 - 5$$

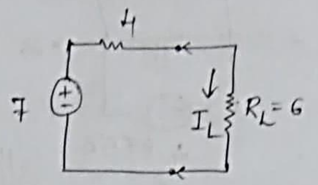
$$\boxed{V_o = 7V}$$

ii) To find '$R_o$' :-




$$R_o = [4 \| 4] + 2 = 4\Omega$$

T.E. ckt,

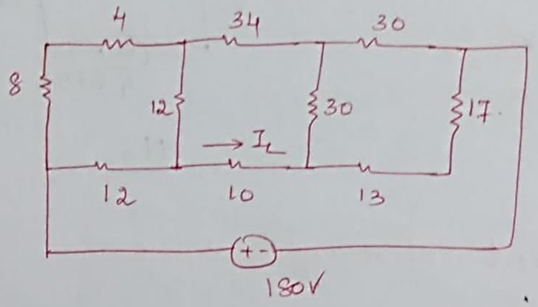


$$I_L = \frac{7}{10}$$

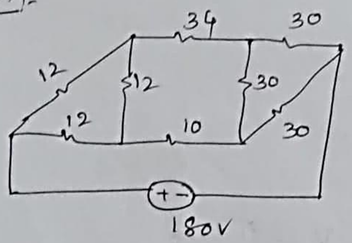
$$\boxed{I_L = 0.7A}$$

M-J-10

5) Find the current in 10Ω



→ soln:-

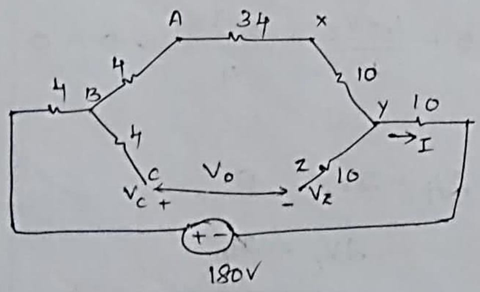


$R_A = R_B = R_C$

$$R_A = \frac{R_{AB}R_{AC}}{\Sigma R_{AB}} = \frac{12\times 12}{12+12+12} = 4\Omega$$

$R_x = R_y = R_z$

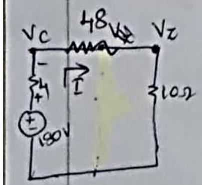
$$R_x = \frac{R_{xy}R_{xz}}{\Sigma R_{xy}} = \frac{30\times 30}{90} = 10\Omega$$



$\therefore V_0 = V_C - V_Z$

$$I = \frac{180}{4+4+34+10+10}$$

$I = 2.903A$.



$V_C = 180 - 4I$

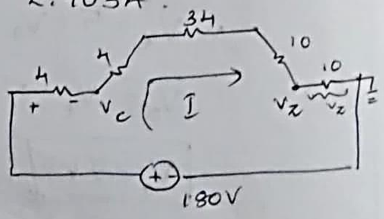
$V_Z = 10I$

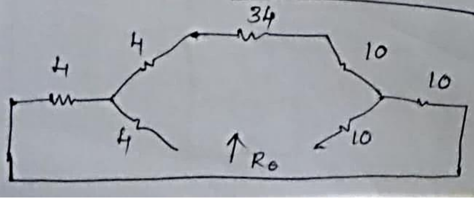
$V_C = 180 - 4I = 168.39V$

$V_Z = 10I = 29.03V$

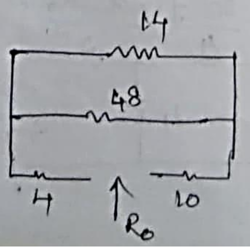
$\therefore V_0 = 168.39 - 29.03$

$$\boxed{V_0 = 139.36V}$$

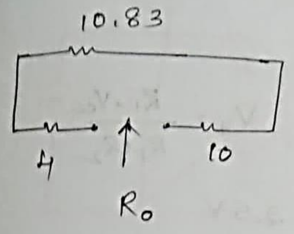




⟹

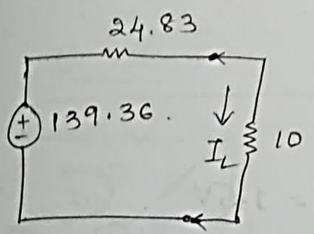


$$14 \| 48 = \frac{14 \times 48}{14 + 48} = 10.83\Omega$$




$$R_o = 10 + 4 + 10.83$$

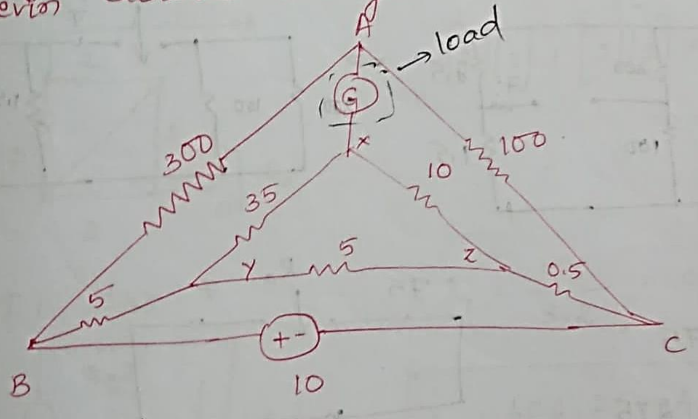
$$R_o = 24.83\Omega$$




$$I_L = \frac{139.36}{34.83}$$

$$\boxed{I_L = 4A}$$

6 Using T.T. determine the current through the galvanometer of 16Ω resistance in Kevin double bridge.

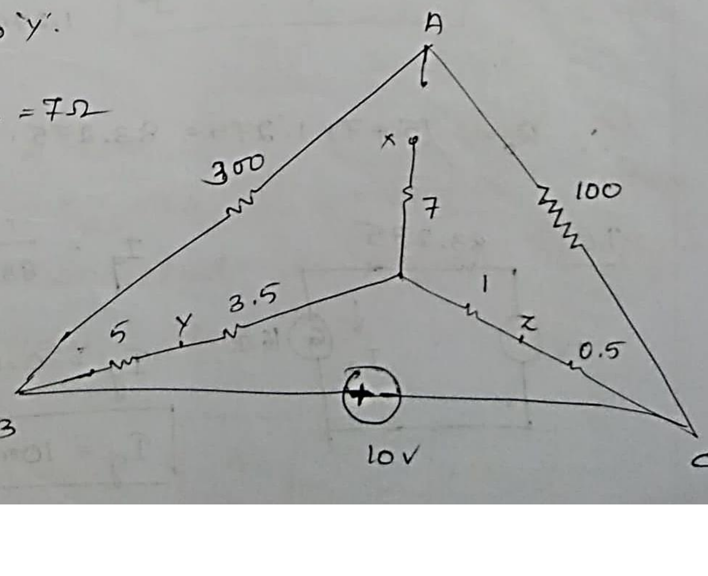


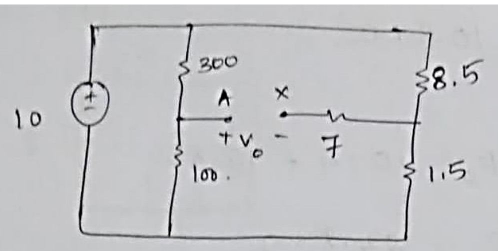

⇒ soln: i) Vo: Convert 'Δ' to 'Y'.

$$R_x = \frac{R_{xy} R_{xz}}{\Sigma R_{xz}} = \frac{35 \times 10}{50} = 7\Omega$$

$$R_y = \frac{5 \times 35}{10} = 3.5\Omega$$

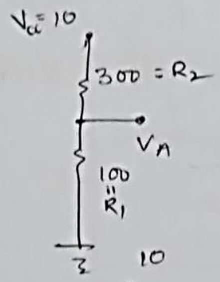
$$R_z = \frac{10 \times 5}{50} = 1\Omega$$



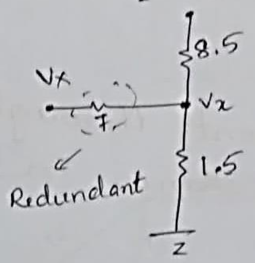



$$V_o = V_A - V_x$$

$$V_A = \frac{R_1 \times V_{cc}}{R_1 + R_2}$$



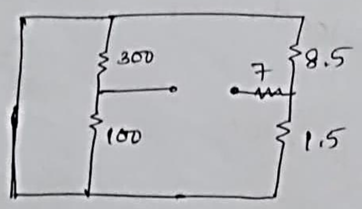

$$V_A = \frac{100 \times 10}{100 + 300} = 2.5\text{V}$$



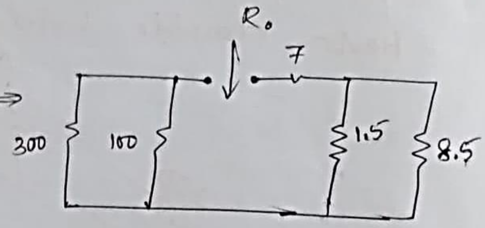

$$V_x = \frac{1.5 \times 10}{1.5 + 8.5} = 1.5\text{V}$$

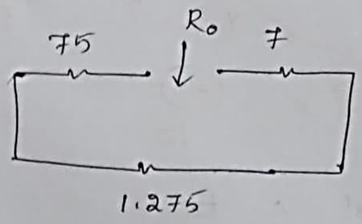
$$\therefore V_o = 2.5 - 1.5 = 1\text{V}$$

ii) $R_o$:




$\Rightarrow$



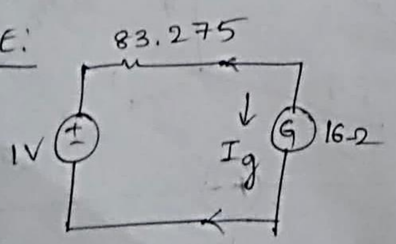




$300 \| 100 = 75$

$1.5 \| 8.5 = 1.275$

$$\therefore R_o = 75 + 7 + 1.275 = 83.275$$

T.E:

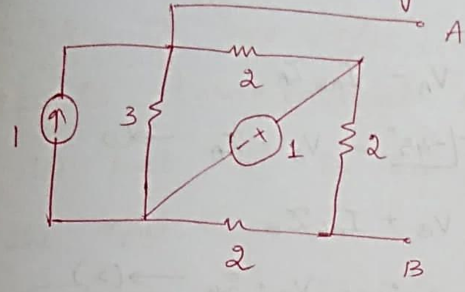



$$I_g = \frac{1}{83.275 + 16}$$

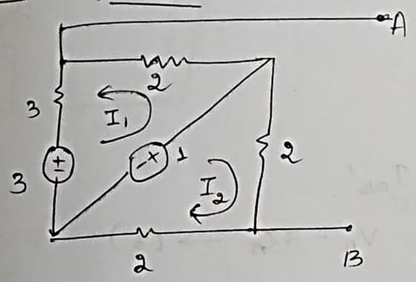
$$I_g = 0.0100$$

$$\boxed{I_g = 10\text{mA}}$$

Determine current through 1Ω resistor connected b/w A & B using T.T.



⇒ Soln:- i) $V_0$:



$V_0 = 2I_2 - 2I_1$

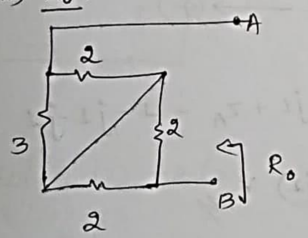
$I_1 = \frac{-3+1}{5} = -\frac{2}{5} = -0.4A$

$I_2 = \frac{1}{4} = 0.25A$

$V_0 = (2 \times 0.25) - (2 \times (-0.4))$

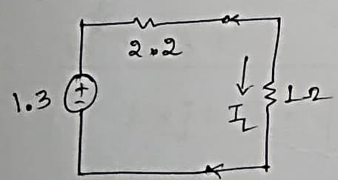
∴ $V_0 = 1.3V$

ii) $R_0$:



$[2||3] + [2||2] = 1.2 + 1 = 2.2\Omega$

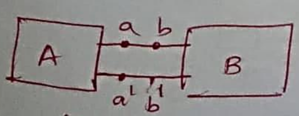
T.E ckt.



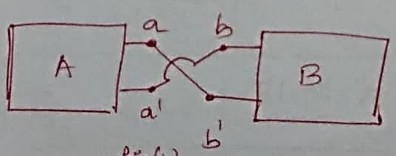
$I_L = \frac{1.3}{3.2} = 0.40625$

$\boxed{I_L = 406.25mA}$

8 When 2 n/w's A & B are connected as shown in fig(a). $I_{ab} = 1A$, $V_{aa'} = \sqrt{2}\angle{-45^\circ}$ V. When the same n/w is connected as shown in fig(b) $I_{a'b} = 3A$, $V_{aa'} = \sqrt{2}\angle{45^\circ}$ V. Find T.E.

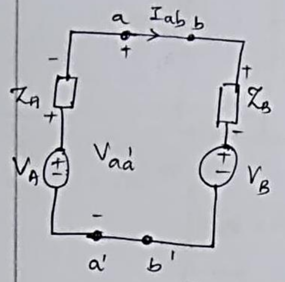


fig(a)



fig(b)

→ Sol⁷:- Let the T.E of n/w A & B be '$V_A$' in series with $Z_A$ & '$V_B$' in series with '$Z_B$'



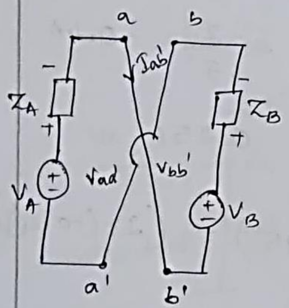

$V_{aa'} = V_A - I_{ab} Z_A$

$1 - j1 = \sqrt{2}\angle{-45^\circ} = V_A - Z_A \rightarrow (1)$

$V_{bb'} = V_B + I_{ab} Z_B$

$\sqrt{2}\angle{45^\circ} = 1 + j1 = V_B + Z_B \rightarrow (2)$

$V_{aa'} = V_{bb'}$ [Same vtg.]




$V_{aa'} = -V_{bb'}$

$\therefore V_{aa'} = V_A - Z_A I_{ab}'$

$\sqrt{2}\angle{45^\circ} = 1 + j1 = V_A - 3Z_A \rightarrow (3)$

$V_{bb'} = V_B - 2Z_B = -\sqrt{2}\angle{45^\circ}$

$-(1+1j) = V_B - 2Z_B \longrightarrow (4)$

(1) − (3)

$V_A - Z_A = 1 - j1$

$V_A - 3Z_A = 1 + j1$

(−) (+) (−) (−)

$2Z_A = -j2$

$Z_A = -j1\,\Omega$.

$\therefore V_A = 1 - j1 + Z_A = 1 - j1 - j1$

$V_A = (1 - j2)\ V$

(2) − (4)

$V_B + Z_B = 1 - j1$

$V_B - 3Z_B = -1 - j1$

(−) (+) (+) (+)

$4Z_B = 2$

$Z_B = 0.5\,\Omega$

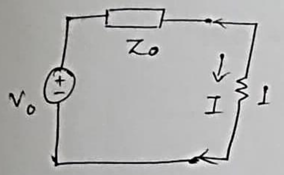
$\therefore V_B = 1 - j1 - Z_B$

$= 1 - j1 - 0.5$

$V_B = (0.5 - j1)\ V$

9. A LTI n/w. shown when terminated with (i) $R = 1\Omega$, $I = 5\angle{-45^\circ}$ A ; (ii) $X_C = 1\Omega$, $I_C = 10\angle{-45^\circ}$ A. Find the T.E. of the n/w. what will be the current if it is terminated with $X_L = 1\Omega$

Soln:- (i) Let $V_0$ is in series with $Z_0$, the TE of the n/w,

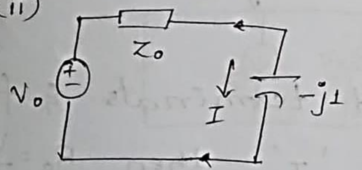


$$I = \frac{V_0}{Z_0 + 1} = 5\angle{-45^\circ}$$

$$Z_0 + 1 = \frac{V_0}{5\angle{-45^\circ}}$$

$$Z_0 + 1 = 0.2\angle{45^\circ}\ V_0 \longrightarrow (1)$$

(ii)



$$I = \frac{V_0}{Z_0 - j1} = 10\angle{-45^\circ}$$

$$Z_0 - j1 = 0.1\angle{-45^\circ}\ V_0 \longrightarrow (2)$$

Solve (1) & (2),

$$\cancel{Z_0} + 1 = 0.2\angle{45^\circ}\ V_0$$

$$\underset{(-)}{\cancel{Z_0}} \underset{(+)}{-j1} = \underset{(-)}{0.1\angle{-45^\circ}}\ V_0$$

$$1 + j1 = \left[0.2\angle{45^\circ} - 0.1\angle{-45^\circ}\right] V_0.$$

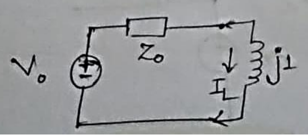
$$V_0 = \frac{1+j1}{0.2\angle{45^\circ} - 0.1\angle{-45^\circ}} = 14.14V$$

From eqn. (1),

$$Z_0 = 0.2\angle{45^\circ}\ V_0 - 1 = \left[0.2\angle{45^\circ} \times 14.14 - 1\right]$$

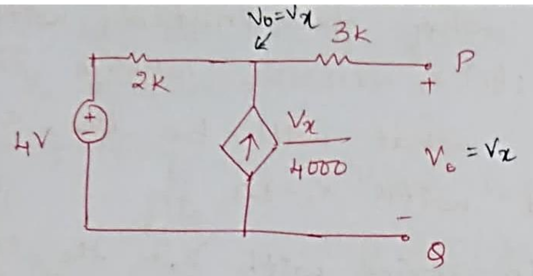
$$Z_0 = (1 + j2)\ \Omega$$

$$I_L = \frac{14.14}{1 + j2 + j1}$$



$$\boxed{I_L = 4.46\angle{-71.56^\circ}\ A}$$

10




$\Rightarrow$ Sol$^n$:- i) $V_o$:

$$\frac{V_x}{4000} + \frac{4 - V_x}{2000} + \cancel{\frac{V_x - V_x}{3k}} = 0$$

$$\boxed{V_x = 8V = V_o}$$

ii) $R_o$:- When a n/w. contains dependent src. the following techniques can to find $R_o[Z_o]$ be used,

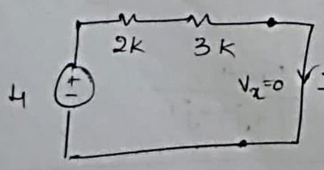
1) Short ckt. the load terminals & find short ckted current '$I_o$', then $R_o = \frac{V_o}{I_o}$.

2) Replace all independent V.S. by short ckt. & c.s. by open ckt. Connect a V.S. across the load terminals of value 'V' volts 1V, 10V... etc, find the current 'I' drawn by the n/w. then $R_o = \frac{V}{I}, \frac{1}{I}, \frac{10}{I}$

$I_o$: When O/P terminals are short ckted. to find $I_o$, $V_x$ becomes '0'. This makes dependent c.s.

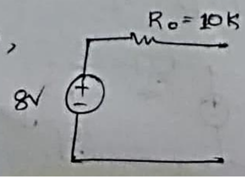
$\frac{V_x}{4000} = 0$ [O.C]

$\therefore I_o = \frac{4}{5k} = 0.8mA$

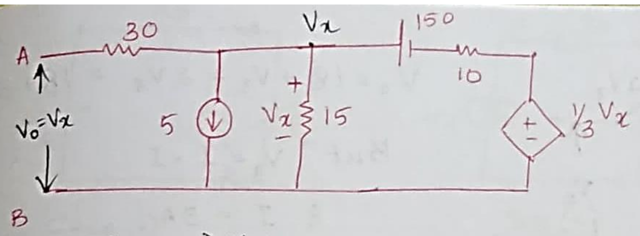



$R_o = \frac{8}{0.8} = 10k\Omega$

$\therefore$ TE is,



11.




Sol$^n$ :- i) $V_0$ :

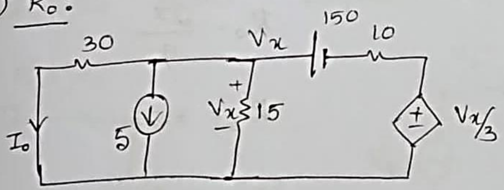
$$0 + 5 + \frac{V_x - 0}{15} + \frac{V_x - 150 - \frac{1}{3}V_x}{10} = 0$$

$$150 + 2V_x + 3\left[V_x - 150 - \frac{V_x}{3}\right] = 0$$

$$4V_x = 300$$

$$\boxed{V_x = 75V = V_0}$$

ii) $R_0$ :



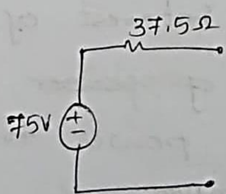

$$\frac{V_x - 0}{30} + 5 + \frac{V_x - 0}{15} + \frac{(V_x - 150) - V_x/3}{10} = 0$$

$$5V_x = 300$$

$$\boxed{V_x = 60 V} \quad ; \quad I_o = \frac{V_x}{30} = \frac{60}{30} = 2A.$$

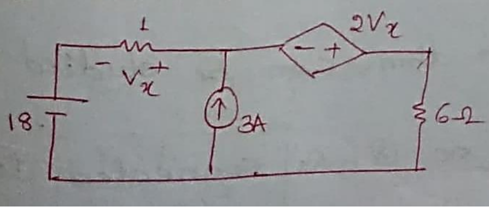
$$R_0 = \frac{V_0}{I_0} = \frac{75}{2}$$

$$\boxed{R_0 = 37.5 \Omega}$$

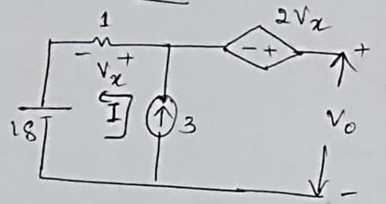



m-J07

12. Find thevenin's vtg., short ckt current & determine the actual current flowing through '6Ω'.



$\Rightarrow$ soln :- i) $V_o$ :




$V_o = 18 + V_x + 2V_x = 18 + 3V_x$

But $V_x = 1 \times I$

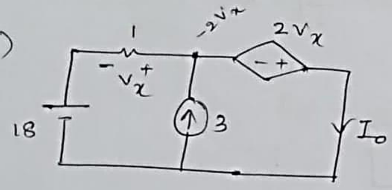
& $I = 3A$

$\therefore V_x = 3V$

$\therefore V_o = 18 + 9$

$V_o = 27V$

ii)




$3 = I_o + \frac{-2V_x - 18}{1}$

But, $V_x = 1(3 - I_o)$

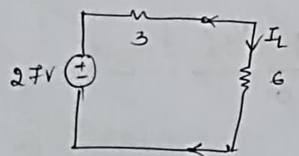
$3I_o = 27$

$I_o = 9A$

$R_o = \frac{27}{9}$

$R_o = 3\Omega$

$\therefore$ T.E is,




$I_L = \frac{27}{9}$

$\boxed{I_L = 3A}$

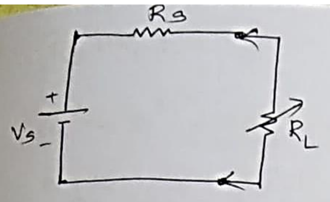
## * Maximum power transfer theorem:-

General interest of any electrical ckt. is the transfer of power from src. to load. At all conditions the power transferred to the load will not be maximum.

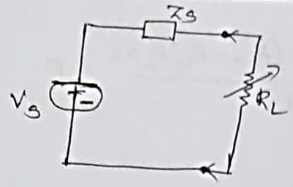
The condition for maximum power depends upon the type of src. i.e. DC/AC & the type of load. i.e R/Z.

Max. power transfer thm. can be studied under following cases.

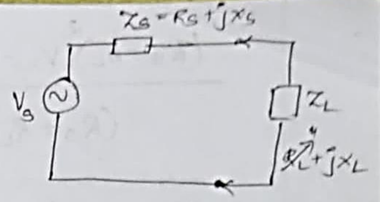
Case (1): DC src. with src. R 'Rs' connected to load resistance 'RL' [RL is variable]




Case (1)




Case (2)

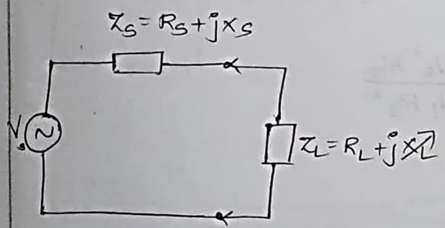



Case (3)

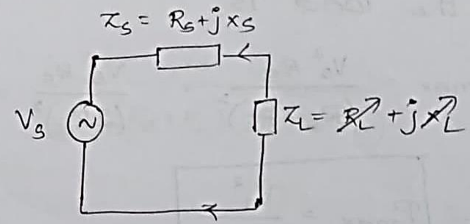
Case(2): AC src with src impedance '$Z_s$' is equal to $R_s + jX_s$ connected to a load resistance '$R_L$' [$R_L$ = Variable]

Case(3): AC src with src impedance $Z_s = R_s + jX_s$ connected to a load impedance, $Z_L = R_L + jX_L$ [$R_L$ = Variable, $X_L$ = constant]

Case(4): AC src with src impedance $Z_s = R_s + jX_s$ connected to load impedance. $Z_L = R_L + jX_L$ $\begin{cases} R_L = \text{Constant} \\ X_L = \text{variable} \end{cases}$




Case (4)

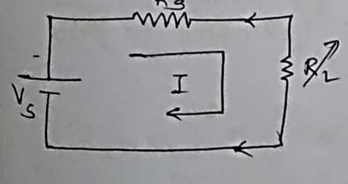



Case (5)

Case(5): AC src. with src impedance $Z_s = R_s + jX_s$ connected to load impedance $Z_L = R_L + jX_L$ [$R_L$ = Variable, $X_L$ = Variable]

Proof:-

Case(1): DC src. with src. resistance '$R_s$' connected to load resistance '$R_L$'




$$I = \frac{V_s}{R_s + R_L}$$

$$P = I^2 R_L = \frac{V_s^2 \cdot R_L}{(R_s + R_L)^2}$$

For max. power transfer, $\frac{dP}{dR_L} = 0$.

$$\frac{d}{dR_L}\left[\frac{V_s^2 \cdot R_L}{(R_s + R_L)^2}\right] = 0$$

$$\frac{(R_S+R_L)^2 V_S - 2(R_S+R_L)R_L V_S}{(R_S+R_L)^4} = 0$$

$$(R_S+R_L)^2 - 2R_L(R_S+R_L) = 0$$

$$R_S^2 + R_L^2 + \cancel{2R_L R_S} - \cancel{2R_L R_S} - 2R_L^2 = 0$$

$$R_S^2 - R_L^2 = 0$$

$$R_S^2 = R_L^2$$

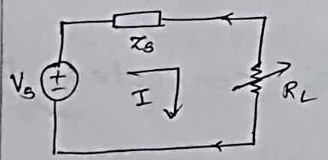
$\boxed{R_S = R_L}$ This is the condition for max. power transfer.

∴ Under this condition the max. power transferred to the load is,

$$P_{max} = \frac{V_S^2 R_S}{(R_S+R_S)^2} = \frac{V_S^2 R_S}{(2R_S)^2} = \frac{V_S^2 R_S}{4R_S^2}$$

$$\therefore \boxed{P_{max} = \frac{V_S^2}{4R_S}}$$

Case(2) AC src with $Z_S = R_S + jX_S$ connected to load resistance '$R_L$' [$R_L$ = variable]




$$I = \frac{V_S}{R_S + R_L + jX_S}$$

$$|I| = \frac{V_S}{\sqrt{(R_S+R_L)^2 + X_S^2}}$$

$$P = |I|^2 R_L = \frac{V_S^2 \cdot R_L}{(R_S+R_L)^2 + X_S^2} \longrightarrow (*)$$

For max. power transfer, $\frac{dP}{dR_L} = 0$;

$$\frac{d}{dR_L}\left[\frac{V_S^2 R_L}{(R_S+R_L)^2 + X_S^2}\right] = 0 \qquad \frac{[(R_S+R_L)^2 + X_S^2](1) - 2R_L(R_S+R_L)}{[(R_S+R_L)^2 + X_S^2]^2} = 0$$

$$(R_S+R_L)^2+X_S^2 - R_L[2(R_S+R_L)] = 0$$

$$R_S^2+R_L^2+\cancel{2R_SR_L}+X_S^2-\cancel{2R_LR_S}-2R_L^2=0$$

$$R_S^2+X_S^2-R_L^2=0$$

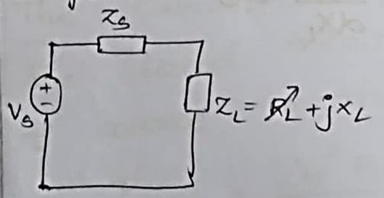
$$R_L^2 = R_S^2+X_S^2 \; ; \quad \boxed{R_L = \sqrt{R_S^2+X_S^2} = |Z_S|}$$

This is the condition for max power transfer.

Replace $R_L \longrightarrow |Z_S|$ in (*)

$$\therefore \; P_{max} = \frac{V_S^2\,|Z_S|}{(R_S+|Z_S|)^2+X_S^2}$$

Case (3):- AC src with $Z_S = R_S + jX_S$ connected to load impedance $Z_L = R_L + jX_L$




$$I = \frac{V_S}{R_S+jX_S+R_L+jX_L}$$

$$I = \frac{V_S}{(R_S+R_L)+j(X_S+X_L)}$$

$$|I| = \frac{V_S}{\sqrt{(R_S+R_L)^2+(X_S+X_L)^2}}$$

$$\therefore \; P = |I|^2 R_L = \frac{V_S^2\,R_L}{(R_S+R_L)^2+(X_S+X_L)^2}$$

For max. power transfer, $\frac{dP}{dR_L} = 0$

$$\frac{d}{dR_L}\left[\frac{V_S^2\,R_L}{(R_S+R_L)^2+(X_S+X_L)^2}\right] = 0$$

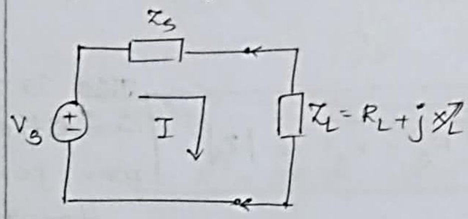
$$(R_S+R_L)^2+(X_S+X_L)^2-2R_L(R_S+R_L)=0$$

$$R_S^2+R_L^2+\cancel{2R_SR_L}+X_S^2+X_L^2+2X_SX_L-\cancel{2R_SR_L}-2R_L^2=0$$

$$R_S^2-R_L^2+X_S^2+X_L^2+2X_SX_L=0$$

$$\therefore \; \boxed{R_L = \sqrt{R_S^2+(X_S+X_L)^2}} \quad \text{(or)} \quad \boxed{R_L = |Z_S+jX_L|}$$

Case(4):- AC src. with $Z_S = R_S + jX_S$ connected to $Z_L = R_L + jX_L$




$$I = \frac{V}{Z_S + Z_L} = \frac{V_S}{(R_S+R_L) + j(X_S+X_L)}$$

$$|I| = \frac{V_S}{\sqrt{(R_S+R_L)^2 + (X_S+X_L)^2}}$$

$$\therefore \quad P = |I|^2 R_L = \frac{V_S^2 R_L}{\left[\sqrt{(R_S+R_L)^2 + (X_S+X_L)^2}\right]^2} = \frac{V_S^2 R_L}{(R_S+R_L)^2 + (X_S+X_L)^2}$$

For max. power ~~transfer~~, $\frac{dP}{dX_L} = 0$

$$0 - R_L\left[2(X_S+X_L)\right] = 0$$

$$X_S + X_L = 0$$

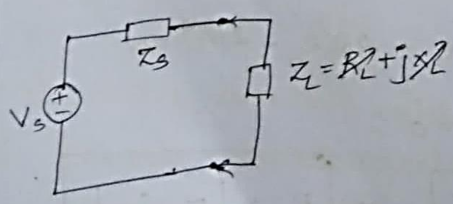
$$X_L = -X_S$$

The condition for max. power transfer is load reactance should be conjugate of source reactance. Under this condition,

$$P_{max} = \frac{V_S^2 R_L}{(R_S+R_L)^2 + (\cancel{-X_L} + \cancel{X_L})^2}$$

$$P_{max} = \frac{V_S^2 R_L}{(R_S+R_L)^2}.$$

** Case(5):- AC source with $Z_S = R_S + jX_S$ connected to a $Z_L = \cancel{R_L} + j\cancel{X_L}$




$$I = \frac{V_S}{(R_S+R_L) + j(X_S+X_L)}$$

$$|I| = \frac{V_S}{\sqrt{(R_S+R_L)^2 + (X_S+X_L)^2}}$$

$$P = |I|^2 R_L = \frac{V_s^2 R_L}{(R_s+R_L)^2 + (X_s+X_L)^2}$$

For max. power transfer, $\frac{dP}{dX_L} = 0$

$$\frac{d}{dX_L}\left[\frac{V_s^2 R_L}{(R_s+R_L)^2 + (X_s+X_L)^2}\right] = 0.$$

$$0 - R_L\left[0 + 2(X_s+X_L)\right] = 0.$$

$$X_s + X_L = 0$$

$$X_L = -X_s.$$

The condition for max. power transfer is the load reactance should be conjugate of source reactance. Under this condition,

$$P_{max} = \frac{V_s^2 R_L}{(R_s+R_L)^2}$$

$\therefore \frac{dP}{dR_L} = 0,$

$$\frac{d}{dR_L}\left[\frac{V_s^2 R_L}{(R_s+R_L)^2}\right] = 0$$

$$(R_s+R_L)^2(1) - R_L\left[2(R_s+R_L)\right] = 0.$$

$$R_s^2 + R_L^2 + \cancel{2R_sR_L} - \cancel{2R_sR_L} - 2R_L^2 = 0$$

$$R_s^2 - R_L^2 = 0$$

$$R_s^2 = R_L^2$$

WKT,

$$Z_L = R_L + jX_L$$

put, $R_L = R_s$ & $X_L = -X_s$

$$\therefore Z_L = R_s - jX_s$$

$$\boxed{Z_L = Z_s^*}$$

$\therefore$ The condition for max. power transfer is the load impedance should be complex conjugate of source impedance.

* Summary :-

i) $R_L = R_S = R_{th}$

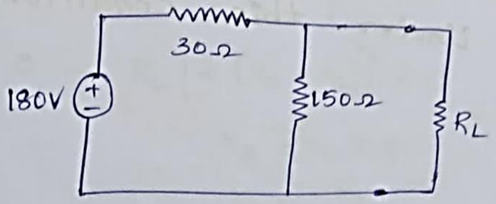
ii) $R_L = |Z_S|$

iii) $R_L = |Z_S + jX_L|$

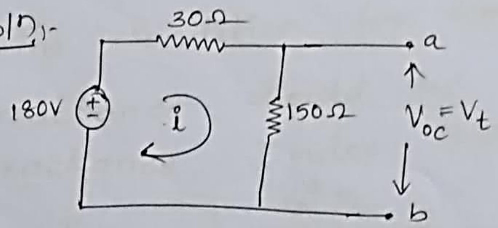
iv) $X_L = -X_S$

v) $Z_L = Z_S^* = Z_{th}^*$

1. Find the load '$R_L$' that will result in maximum power delivered to the load for the ckt. of fig. Also determine the max power $P_{max}$.

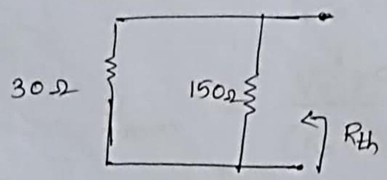



⟹ Soln:-




$$i = \frac{180}{30+150} = 1A$$

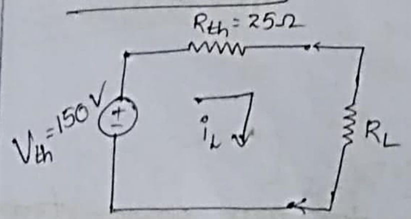
$$V_{oc} = V_{th} = 150 \cdot i = 150V$$




$$R_{th} = 30 \| 150 = \frac{30 \times 150}{180}$$

$$R_{th} = 25\Omega$$

Thevenin's ckt,




∴ For max. power transfer,

$$R_L = R_{th} = 25\Omega$$

$$\therefore P_{max} = \frac{V_{th}^2}{4R_L} = \frac{150^2}{4 \times 25}$$

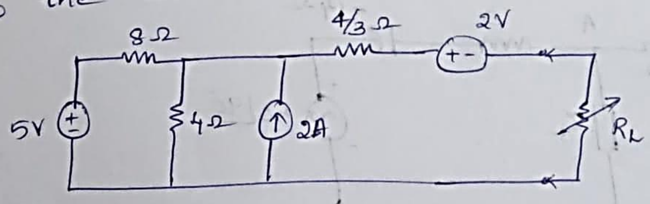
$$\boxed{P_{max} = 225W}$$

$\therefore$ Total power, $P_t = 150 \times i_L$

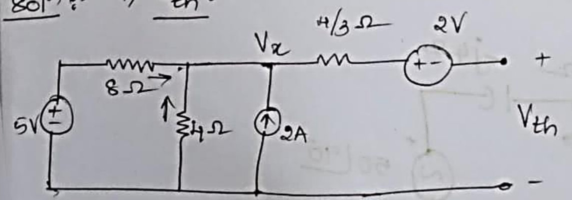
$$P_t = 150 \times \left[\frac{150}{25+25}\right]$$

$$\boxed{P_t = 450 \text{ Watts}}$$

2. Find the value of 'RL' for max. power to be transferred to the load. Also find max. power transferred.




→ Soln :- i) $V_{th}$ :




$V_{th} = V_x - 2$

$$2 + \frac{0 - V_x}{4} + \frac{5 - V_x}{8} = 0$$

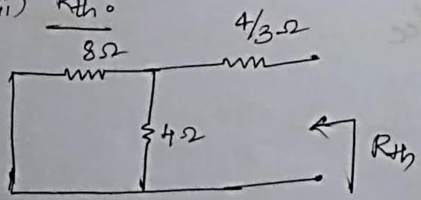
$$16 - 2V_x + 5 - V_x = 0$$

$$\cancel{-}3V_x = \cancel{-}21$$

$$V_x = 7\text{ V}$$

$$\therefore V_{th} = 7 - 2 = 5\text{V}$$

ii) $R_{th}$ :

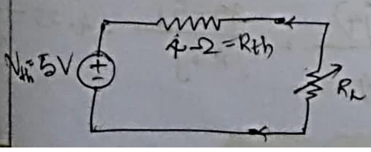



$$R_{th} = \frac{4}{3} + [8 \| 4]$$

$$= \frac{4}{3} + \frac{4 \times 8}{12}$$

$$= 4\Omega$$

Thevenin's ckt,




$\therefore$ For max. power transfer, $R_L = R_{th}$

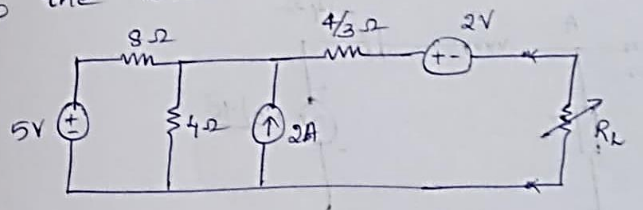
$R_L = 4\Omega$

$\therefore$ Total power, $P_t = 150 \cdot i_L$

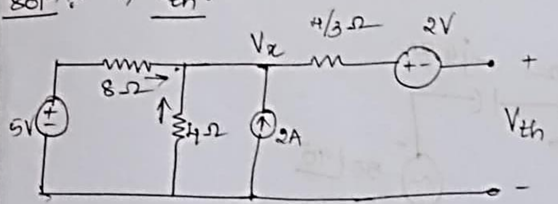
$$P_t = 150 \cdot \left[\frac{150}{25+25}\right]$$

$$\boxed{P_t = 450 \text{ Watts}}$$

2. Find the value of 'R_L' for max. power to be transferred to the load. Also find max. power transferred.




→ Sol$^n$:- i) $V_{th}$:




$$V_{th} = V_x - 2$$

$$2 + \frac{0 - V_x}{4} + \frac{5 - V_x}{8} = 0$$

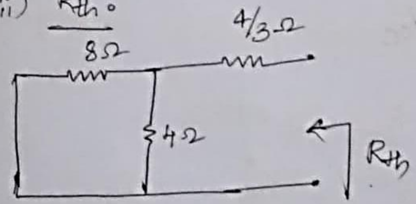
$$16 - 2V_x + 5 - V_x = 0$$

$$-3V_x = -21$$

$$V_x = 7\ V$$

$$\therefore V_{th} = 7 - 2 = 5V$$

ii) $R_{th}$:

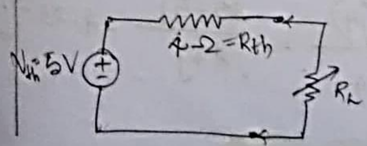



$$R_{th} = \frac{4}{3} + [8 \| 4]$$

$$= \frac{4}{3} + \frac{4 \times 8}{12}$$

$$= 4\Omega$$

Thevenin's ckt,




$\therefore$ For max. power transfer, $R_L = R_{th}$

$R_L = 4\Omega$

Maximum power transfer, $P_{max} = I^2 R_L$

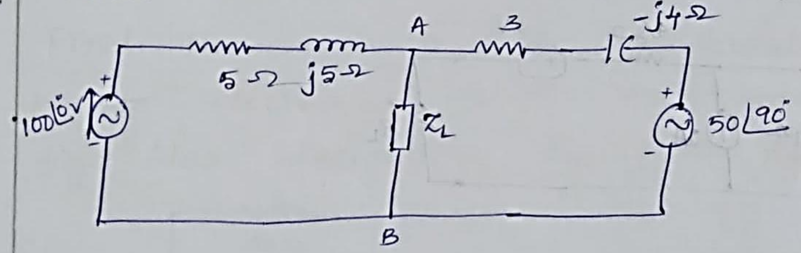
$$= 4 \times \left[\frac{5}{4+4}\right]$$

$$= \frac{20}{8}$$

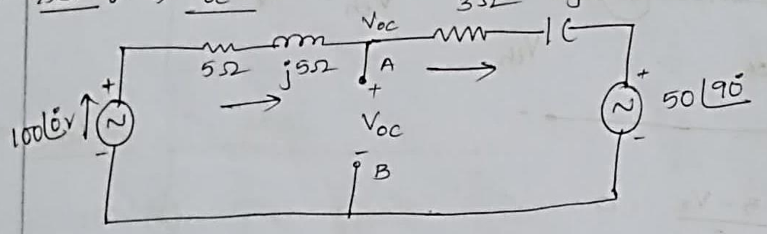
$$\boxed{P_{max} = 1.5625 \text{ Watts}}$$

May-June 10

3. Find the max. P.T. to load '$Z_L$' of the n/w:




→ Soln :- i) $V_{oc}$ :-




$$\frac{100\angle 0° - V_{oc}}{5+j5} = \frac{V_{oc} - 50\angle 90°}{3-j4}$$

$$(3-j4)(100) - (3-j4)V_{oc} = (5+j5)V_{oc} - 50j(5+j5)$$

$$(300-j400) - (3-j4)V_{oc} - (5+j5)V_{oc} + 250j - 250 = 0$$

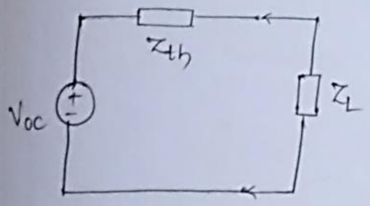
$$(50-j150) = [3+5-j4+j5]V_{oc}$$

$$V_{oc} = \frac{50-j150}{8+j}$$

$$V_{oc} = (3.84 - 19.2j) V$$

ii) $Z_{th}$: $(5+j5) \| (3-j4) = \frac{(5+j5)(3-j4)}{5+j5+3-j4} = \frac{35-5j}{8+j} = 4.23 - 1.15j$

Thevenin's ckt,




From fig., it belongs to case(5),

$$Z_L = Z_{th}^* = 4.23 + 1.15j$$

$$\therefore \quad P_{max} = R_L \cdot I^2$$

$$I = \frac{3.84 - 19.2j}{4.23 - \cancel{1.15j} + 4.23 + \cancel{1.15j}} = 0.45 - 2.26j$$

$\therefore$