

# BVAE

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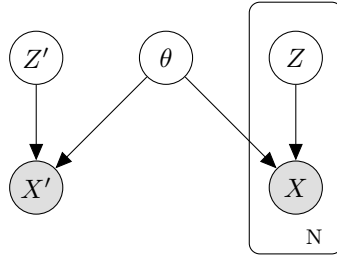


Figure 1: Graphical model of BVAE.  $X$  is the training set.  $X'$  is the test set.

## 1 BVAE Objectives

### 1.1 Objective maximized during training

$$p(x) = \int_{\theta} \int_z p(x, z, \theta) \quad (1)$$

$$= E_{q(\theta, z|x)} \left[ \frac{p(x, z, \theta)}{q(\theta, z|x)} \right] \quad (2)$$

$$= E_{q(\theta, z|x)} \left[ \frac{p(x, z)p(\theta)}{q(z|x)q(\theta)} \right] \quad (3)$$

$$\log(p(x)) = \log \left( E_{q(\theta, z|x)} \left[ \frac{p(x, z)p(\theta)}{q(z|x)q(\theta)} \right] \right) \quad (4)$$

$$\geq E_{q(\theta, z|x)} \left[ \log \left( \frac{p(x, z)p(\theta)}{q(z|x)q(\theta)} \right) \right] \quad (5)$$

## 1.2 Objective evaluated on the test set

$$p(x'|x) = \int_{\theta} \int_{z'} p(x', z', \theta | x) \quad (6)$$

$$= \int_{\theta} \int_z p(x', z' | \theta, x) p(\theta | x) \quad (7)$$

$$= \int_{\theta} \int_z p(x', z' | \theta) p(\theta | x) \quad (8)$$

$$q(x'|x) = \int_{\theta} \int_z p(x', z' | \theta) q(\theta) \quad (9)$$

$$= E_{q(\theta)} \left[ \int_z p(x', z' | \theta) \right] \quad (10)$$

$$= E_{q(\theta)} \left[ E_{q(z'|x')} \left[ \frac{p(x', z' | \theta)}{q(z' | x')} \right] \right] \quad (11)$$

$$\log(q(x'|x)) = \log \left( E_{q(\theta)} \left[ E_{q(z'|x')} \left[ \frac{p(x', z' | \theta)}{q(z' | x')} \right] \right] \right) \quad (12)$$

$$\geq E_{q(\theta)} \left[ E_{q(z'|x')} \left[ \log \left( \frac{p(x', z' | \theta)}{q(z' | x')} \right) \right] \right] \quad (13)$$

## 2 2D Plots

Equation used to plot  $q_T(z_T)$

$$q_T(z_T) = E_{q(v_T)} \left[ \frac{q_0(T_1^{-1}(\dots(T_T^{-1}(z_T, v_T)))}{q(v_T)} \right] \quad (14)$$