Generator-Aware Bayesian Variational Autoencoders

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Abstract

In some scenarios, it is important to have uncertainty in the generator. However, naively using a BNN as the generator will be limited by (mutual info argument). Here we show how to build a GABVAE.

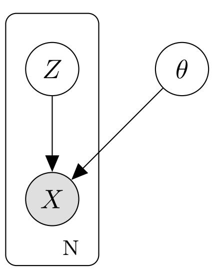
1 Introduction

VAEs [Kingma and Welling, 2014] overfit in both the recognition and generator networks. The recognition overfitting is solvable. The generator overfitting gets worse with higher latent dimensionality Q and input dimensionality D.

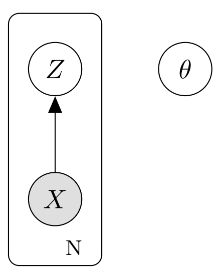
A BVAE is a VAE where we are being Bayesian with the parameters θ of the generator network.

Meaning of z is dependent on θ. Therefore the recognition networks needs to be generator-aware. Here we introduce the Generator-Aware Bayesian Variational Autoencoders (GABVAE).

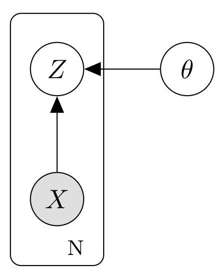
2 Generator-Aware Bayesian VAE



(a) Generative model



(b) VAE Inference model



(c) BVAE Inference model

Figure 1: Probabilistic graphical models

2.1 How to summarize the generator?

- Pass the whole network (hypo-network)
- Pass the latent variable (MNF)
- Other summary statistics

Another option is to use HVI [Salimans et al., 2015].

2.2 Mutual Info

Let $\mathbf{x_i} \in \mathbf{X}$ denote the i-th data point of the training data, $\mathbf{z_i} \in \mathbf{Z}$ its corresponding latent representation and θ the parameter variables of the generator network. Assuming that the true posterior factorises as follows,

$$
\begin{aligned}
p(\mathbf{z_1}, \mathbf{z_2}, \ldots, \mathbf{z_n}, \theta | \mathbf{x_1}, \mathbf{x_2}, \ldots, \mathbf{x_n}) &= p(\theta|\mathbf{X}) p(\mathbf{Z}|\mathbf{X}, \theta) && (1) \\
&= p(\theta|\mathbf{X}) \prod_{i=1}^{N} p(\mathbf{z_i}|\mathbf{x_i}, \theta) && (2)
\end{aligned}
$$

Then, some optimal approximate posterior with the same factorial structure, $q^*(\mathbf{Z}, \theta|\mathbf{X}, \phi_{ga}) = q^*(\theta|\mathbf{X}, \phi_{ga}) \prod_{i=1}^{N} q^*(\mathbf{z_i}|\mathbf{x_i}, \theta, \phi_{ga})$, parameterised by ϕ_{ga} ("ga" stands for generator-aware) from a rich enough variational family would be able to match the true posterior and set the KL-divergence to zero. More formally,

$$D_{KL}\Big(q^*(\theta|\mathbf{X}, \phi_{ga}) \prod_{i=1}^{N} q^*(\mathbf{z_i}|\mathbf{x_i}, \theta, \phi_{ga}) || p(\mathbf{Z}, \theta|\mathbf{X})\Big) = 0 \tag{3}$$

However, if our approximate posterior does not take into account θ when predicting $\mathbf{z}$ given $\mathbf{x}$, it is possible that the KL-divergence remain positive even under the optimal approximation, however rich the variational family may be.

$$D_{KL}\Big(q^*(\theta|\mathbf{X}, \phi_{ga}) \prod_{i=1}^{N} q^*(\mathbf{z_i}|\mathbf{x_i}, \phi_{ga}) || p(\mathbf{Z}, \theta|\mathbf{X})\Big) > 0 \tag{4}$$

The loss of information could be measured by the KL-divergence between these two approximations.

$$
\begin{aligned}
& D_{KL}\Big(q^*(\mathbf{Z}, \theta|\mathbf{X}) || q^*(\mathbf{Z}|\mathbf{X}) q^*(\theta|\mathbf{X})\Big) && (5) \\
=& \int_\theta \int_{\mathbf{Z}} q^*(\mathbf{Z}, \theta|\mathbf{X}) \log \frac{q^*(\mathbf{Z}, \theta|\mathbf{X})}{q^*(\mathbf{Z}|\mathbf{X}) q^*(\theta|\mathbf{X})} && (6) \\
=& \mathcal{I}(\mathbf{Z}, \theta) && (7)
\end{aligned}
$$

where $\mathcal{I}(\mathbf{Z}, \theta)$ is the mutual information between the variables $\mathbf{Z}$ and θ.

3 Related Work

4 Experimental Results

Objective maximized during training:

$$E_{q(\theta,z|x)}\left[log\left(\frac{p(x,z)p(\theta)}{q(z|x,\theta)q(\theta)}\right)\right] \tag{8}$$

Objective evaluated on the test set:

$$E_{q(\theta)}\left[E_{q(z'|x',\theta)}\left[log\left(\frac{p(x',z'|\theta)}{q(z'|x',\theta)}\right)\right]\right] \tag{9}$$

4.1 Choice of Generator-Aware model

4.2 Generator-Aware with non-factorized inference and generator networks

		q(W)	
		BNN	MNF
q(z)	factorized	167, 168	162, 194
	NF	166, 168	161, 185

Table 1: NLL on test set. Left is no GA, right is naive GA.

5 Conclusion

We've introduced GABVAEs.

References

[Kingma and Welling, 2014] Kingma, D. P. and Welling, M. (2014). Auto-Encoding Variational Bayes. *In ICLR*.

[Salimans et al., 2015] Salimans, T., Kingma, D. P., and Welling, M. (2015). Markov chain monte carlo and variational inference: Bridging the gap. *In ICML*.

6 Appendix

Objective maximized during training:

$$
\begin{aligned}
p(x) &= \int_{\theta}\int_{z} p(x, z, \theta) && (10)\\
&= E_{q(\theta,z|x)}\left[\frac{p(x, z, \theta)}{q(\theta, z|x)}\right] && (11)\\
&= E_{q(\theta,z|x)}\left[\frac{p(x, z)p(\theta)}{q(z|x, \theta)q(\theta)}\right] && (12)\\
log(p(x)) &= log\left(E_{q(\theta,z|x)}\left[\frac{p(x, z)p(\theta)}{q(z|x, \theta)q(\theta)}\right]\right) && (13)\\
&\geq E_{q(\theta,z|x)}\left[log\left(\frac{p(x, z)p(\theta)}{q(z|x, \theta)q(\theta)}\right)\right] && (14)
\end{aligned}
$$

Objective evaluated on the test set:

$$
\begin{aligned}
p(x'|x) &= \int_{\theta}\int_{z'} p(x', z', \theta|x) && (15)\\
&= \int_{\theta}\int_{z} p(x', z'|\theta, x)p(\theta|x) && (16)\\
&= \int_{\theta}\int_{z} p(x', z'|\theta)p(\theta|x) && (17)\\
q(x'|x) &= \int_{\theta}\int_{z} p(x', z'|\theta)q(\theta) && (18)\\
&= E_{q(\theta)}\left[\int_{z} p(x', z'|\theta)\right] && (19)\\
&= E_{q(\theta)}\left[E_{q(z'|x')}\left[\frac{p(x', z'|\theta)}{q(z'|x', \theta)}\right]\right] && (20)\\
log(q(x'|x)) &= log\left(E_{q(\theta)}\left[E_{q(z'|x')}\left[\frac{p(x', z'|\theta)}{q(z'|x', \theta)}\right]\right]\right) && (21)\\
&\geq E_{q(\theta)}\left[E_{q(z'|x')}\left[log\left(\frac{p(x', z'|\theta)}{q(z'|x', \theta)}\right)\right]\right] && (22)
\end{aligned}
$$