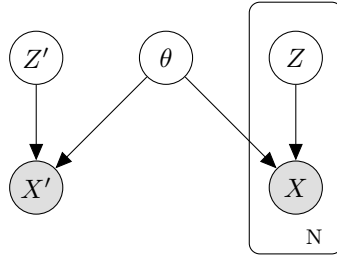# BVAE

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Figure 1: Graphical model of BVAE. X is the training set. X' is the test set.

1 BVAE Objectives

1.1 Objective maximized during training

$$p(x) = \int_\theta \int_z p(x, z, \theta) \tag{1}$$

$$= E_{q(\theta, z|x)} \left[\frac{p(x, z, \theta)}{q(\theta, z|x)} \right] \tag{2}$$

$$= E_{q(\theta, z|x)} \left[\frac{p(x, z)p(\theta)}{q(z|x, \theta)q(\theta)} \right] \tag{3}$$

$$log(p(x)) = log \left(E_{q(\theta, z|x)} \left[\frac{p(x, z)p(\theta)}{q(z|x, \theta)q(\theta)} \right] \right) \tag{4}$$

$$\geq E_{q(\theta, z|x)} \left[log \left(\frac{p(x, z)p(\theta)}{q(z|x, \theta)q(\theta)} \right) \right] \tag{5}$$

1.2 Objective evaluated on the test set

$$p(x'|x) = \int_\theta \int_{z'} p(x', z', \theta|x) \tag{6}$$

$$= \int_\theta \int_z p(x', z'|\theta, x)p(\theta|x) \tag{7}$$

$$= \int_\theta \int_z p(x', z'|\theta)p(\theta|x) \tag{8}$$

$$q(x'|x) = \int_\theta \int_z p(x', z'|\theta)q(\theta) \tag{9}$$

$$= E_{q(\theta)}\left[\int_z p(x', z'|\theta)\right] \tag{10}$$

$$= E_{q(\theta)}\left[E_{q(z'|x')}\left[\frac{p(x', z'|\theta)}{q(z'|x', \theta)}\right]\right] \tag{11}$$

$$log(q(x'|x)) = log\left(E_{q(\theta)}\left[E_{q(z'|x')}\left[\frac{p(x', z'|\theta)}{q(z'|x', \theta)}\right]\right]\right) \tag{12}$$

$$\geq E_{q(\theta)}\left[E_{q(z'|x')}\left[log\left(\frac{p(x', z'|\theta)}{q(z'|x', \theta)}\right)\right]\right] \tag{13}$$

1.3 Other

$$logp(x) = log\int p(x|\theta)q(\theta) = logE_q\left[p(x|\theta)\right] \tag{14}$$

2 2D Plots

Equation used to plot $q_T(z_T)$

$$q_T(z_T) = E_{q(v_T)}\left[\frac{q_0(T_1^{-1}(...(T_T^{-1}(z_T, v_T))}{q(v_T)}\right] \tag{15}$$