BVAE

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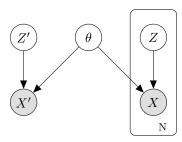


Figure 1: Graphical model of BVAE. X is the training set. X' is the test set.

1 **BVAE** Objectives

Objective maximized during training 1.1

$$p(x) = \int_{\theta} \int_{z} p(x, z, \theta)$$
 (1)

$$= E_{q(\theta,z|x)} \left[\frac{p(x,z,\theta)}{q(\theta,z|x)} \right] \tag{2}$$

$$= E_{q(\theta,z|x)} \left[\frac{p(x,z)p(\theta)}{q(z|x,\theta)q(\theta)} \right]$$
 (3)

$$J_{\theta} J_{z}$$

$$= E_{q(\theta,z|x)} \left[\frac{p(x,z,\theta)}{q(\theta,z|x)} \right]$$

$$= E_{q(\theta,z|x)} \left[\frac{p(x,z)p(\theta)}{q(z|x,\theta)q(\theta)} \right]$$

$$log(p(x)) = log \left(E_{q(\theta,z|x)} \left[\frac{p(x,z)p(\theta)}{q(z|x,\theta)q(\theta)} \right] \right)$$

$$\geq E_{q(\theta,z|x)} \left[log \left(\frac{p(x,z)p(\theta)}{q(z|x,\theta)q(\theta)} \right) \right]$$

$$(5)$$

$$\geq E_{q(\theta,z|x)} \left[log \left(\frac{p(x,z)p(\theta)}{q(z|x,\theta)q(\theta)} \right) \right]$$
 (5)

1.2 Objective evaluated on the test set

$$p(x'|x) = \int_{\theta} \int_{z'} p(x', z', \theta|x) \tag{6}$$

$$= \int_{\theta} \int_{z} p(x', z'|\theta, x) p(\theta|x) \tag{7}$$

$$= \int_{\theta} \int_{z} p(x', z'|\theta) p(\theta|x) \tag{8}$$

$$q(x'|x) = \int_{\theta} \int_{z} p(x', z'|\theta) q(\theta)$$
(9)

$$= E_{q(\theta)} \left[\int_{z} p(x', z'|\theta) \right] \tag{10}$$

$$= E_{q(\theta)} \left[E_{q(z'|x')} \left[\frac{p(x', z'|\theta)}{q(z'|x', \theta)} \right] \right]$$
 (11)

$$log(q(x'|x)) = log\left(E_{q(\theta)}\left[E_{q(z'|x')}\left[\frac{p(x',z'|\theta)}{q(z'|x',\theta)}\right]\right]\right) \tag{12}$$

$$\geq E_{q(\theta)} \left[E_{q(z'|x')} \left[log \left(\frac{p(x', z'|\theta)}{q(z'|x', \theta)} \right) \right] \right] \tag{13}$$

2 2D Plots

Equation used to plot $q_T(z_T)$

$$q_T(z_T) = E_{q(v_T)} \left[\frac{q_0(T_1^{-1}(...(T_T^{-1}(z_T, v_T)))}{q(v_T)} \right]$$
(14)