

1. (a) Experimental Units: Leaves. Observational Units: Leaves.
 (b) Plants.
 (c) RCBD.
 (d) The missing d.f. values are given in the table below. Note that we have $I = 2$ treatment groups, and $B = 5$ plants. So d.f. (PLANT) = $B - 1 = 4$, d.f. (VIRUS) = $I - 1 = 1$ and d.f. (Error) = $(I - 1)(B - 1) = 4$. Total d.f. = 9.

Source	DF	Sum of Squares	Mean Square	F value	p-value
PLANT	4		6.32		0.076
VIRUS	1		8.97		0.047
Error	4		1.12		
C. TOTAL	9				

- (d) $F = MS_{VIRUS}/MS_{Error} = \frac{8.97}{1.12} = 8.0$.
 (e) The d.f. for the F-distribution is (1,4).
 (f) If blocks are ignored, rerunning the analysis gives $MSE_{CRD} = (6.32*4 + 1.12*4)/(4+4) = 3.72$ (noting that SSE_{CRD} and d.f. (Error_{CRD}) combine blocks and error). Hence, the efficiency of the RCBD is $MSE_{CRD}/MSE_{RCBD} = 3.72/1.12 = 3.32$.
 (g) Using blocks is a good idea because the efficiency of RCBD versus CRD is bigger than 1. So I would recommend design 1 for future studies.
2. (a) Experimental Units: Pens. Observational Units: Pigs.
 (b) The missing d.f. values are given in the table below. Note that we have $I = 4$ diets, $J = 3$ pens for each diet, and $K = 3$ measurements for each pen. So d.f. (diet) = $I - 1 = 3$, d.f. (pens) = $I(J - 1) = 4 \times 2 = 8$ and d.f. (Error) = $IJ(K - 1) = 24$.

Sounce	DF	Sum of Squares	Mean Square	Expected Mean Square
Diet	3		190.4	$\sigma^2_E + 3 \sigma^2_P + \text{stuff}$
Pen	8		65.05	$\sigma^2_E + 3 \sigma^2_P$
Error	24		1.30	σ^2_E

- (c) The ANOVA estimates are obtained by equating observed and expected MS; that is,

$$\hat{\sigma}_E^2 = MS(\text{Error}) = 1.30$$

and

$$\hat{\sigma}_E^2 + 3\hat{\sigma}_P^2 = MS(\text{pens}) = 65.05 \Rightarrow \hat{\sigma}_P^2 = \frac{65.05 - 1.30}{3} = 21.25$$

(d) The intra-class correlation between two pigs in the same pen is

$$\frac{\sigma_P^2}{\sigma_E^2 + \sigma_P^2} = \frac{21.25}{1.30 + 21.25} = 0.942$$

3. (a) Full factorial treatment design.

(b) (i) simple effect; (ii) main effect; (iii) interaction effect.

(c) Contrast (ii) $(\mu_{FN} + \mu_{FP})/2 - (\mu_{NN} + \mu_{NP})/2$.

Contrast (iii) $(\mu_{FP} - \mu_{NP}) - (\mu_{FN} - \mu_{NN})$.

Since this is a balanced design, we just need to check $\sum c_{ij}k_{ij} = 0$ or not.

$$\begin{aligned}\sum c_{ij}k_{ij} &= c_{NN}k_{NN} + c_{NP}k_{NP} + c_{FN}k_{FN} + c_{FP}k_{FP} \\ &= (-1/2) \times 1 + (-1/2) \times (-1) + (1/2) \times (-1) + (1/2) \times (1) = 0\end{aligned}$$

Hence, the two contrasts are orthogonal.