**Bayes Theorem (Best Example) :
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Bayes theorem

P(A) prior probability for A
P(B) prior probability for B

P(B|A) conditional probability for B given A
P(A|B) conditional probability for A given B

P(A, B) joint probability for A and B

$$\mathbf{A} \downarrow \mathbf{B}$$

P(A,B) = P(B|A) * P(A)

P(A|B) = P (B|A) * P(A) / P(B)   Theorem

Intuitive meaning: Infering a conditional probability for a cause given a symptom.

<u>Best Example :</u>

Example [edit]

Suppose there is a school having 60% boys and 40% girls as students. The girls wear trousers or skirts in equal numbers; all boys wear trousers. An observer sees a (random) student from a distance; all the observer can see is that this student is wearing trousers. What is the probability this student is a girl? The correct answer can be computed using Bayes' theorem.

The event G is that the student observed is a girl, and the event T is that the student observed is wearing trousers. To compute the posterior probability $P(G|T)$, we first need to know:

- $P(G)$, or the probability that the student is a girl regardless of any other information. Since the observer sees a random student, meaning that all students have the same probability of being observed, and the percentage of girls among the students is 40%, this probability equals 0.4.
- $P(B)$, or the probability that the student is not a girl (i.e. a boy) regardless of any other information (B is the complementary event to G). This is 60%, or 0.6.
- $P(T|G)$, or the probability of the student wearing trousers given that the student is a girl. As they are as likely to wear skirts as trousers, this is 0.5.
- $P(T|B)$, or the probability of the student wearing trousers given that the student is a boy. This is given as 1.
- $P(T)$, or the probability of a (randomly selected) student wearing trousers regardless of any other information. Since $P(T) = P(T|G)P(G) + P(T|B)P(B)$ (via the law of total probability), this is $P(T) = 0.5 \times 0.4 + 1 \times 0.6 = 0.8$.

Given all this information, the **posterior probability** of the observer having spotted a girl given that the observed student is wearing trousers can be computed by substituting these values in the formula:

$$P(G|T) = \frac{P(T|G)P(G)}{P(T)} = \frac{0.5 \times 0.4}{0.8} = 0.25.$$

An intuitive way to solve this is to assume the school has N students. Number of boys = 0.6N and number of girls = 0.4N. If N is sufficiently large, total number of trouser wearers = 0.6N+ 50% of 0.4N. And number of girl trouser wearers = 50% of 0.4N. Therefore, in the population of trousers, girls are (50% of 0.4N)/(0.6N+ 50% of 0.4N) = 25%. In other words, if you separated out the group of trouser wearers, a quarter of that group will be girls. Therefore, if you see trousers, the most you can deduce is that you are looking at a single sample from a subset of students where 25% are girls. And by definition, chance of this random student being a girl is 25%. Every Bayes theorem problem can be solved in this way .