Posterior Probability :
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https://www.youtube.com/watch?v=F6cHOAhoJm4

Example 1

Consider the stock price per share of Hooli. The probability of the stock price going up is **90%**. This probability will vary with general market conditions. The probability of a good market and the stock price going up is **67.5%**. Find the probability of a good market given that the stock price went up.

Solution.

$$P(\text{good} \mid \text{up}) = \frac{P(\text{up and good})}{P(\text{up})} = \frac{0.675}{0.9} = 0.75$$

The probability of a good market given that the stock price went up is **75%**.

Example 2

Extending from Example 1, suppose that the market is good **73.5%** of the time. Find the probability of the stock price per share of Hooli going up given a good market.

Solution.

$$P(\text{up} \mid \text{good}) = \frac{P(\text{good and up})}{P(\text{good})} = \frac{0.675}{0.735} = 0.918$$

The probability of the stock price going up given a good market is **91.8%**.

Chain Rule

The chain rule of probability can be used to compute the *joint* probability of both events A and B occurring. It is defined by

$$\mathbf{P}(\text{A and B}) = \mathbf{P}(\text{A}) \times \mathbf{P}(\text{B}|\text{A}).$$

Re-arranging the chain rule gives the conditional probability formula

$$\mathbf{P}(\text{B}|\text{A}) = \frac{\mathbf{P}(\text{A and B})}{\mathbf{P}(\text{A})}.$$

If event B actually occurs, that is

$$\mathbf{P}(\text{B}) > \mathbf{0},$$

then we can compute the probability of A given B by

$$\mathbf{P}(\text{A}|\text{B}) = \frac{\mathbf{P}(\text{B and A})}{\mathbf{P}(\text{B})}$$

Posterior Probabilities

Now, we have a formula to compute posterior probabilities

$$\mathbf{P}(\text{A}|\text{B}) = \frac{\mathbf{P}(\text{A}) \times \mathbf{P}(\text{B}|\text{A})}{\mathbf{P}(\text{B})}.$$

Given the *prior* probabilities

$$\mathbf{P}(\text{A}) \text{ and } \mathbf{P}(\text{B}),$$

and the *conditional* probability

$$\mathbf{P}(\text{B}|\text{A}),$$

we can compute the *posterior* probability

$$\mathbf{P}(\text{A}|\text{B}) = \frac{\mathbf{P}(\text{A}) \times \mathbf{P}(\text{B}|\text{A})}{\mathbf{P}(\text{B})}.$$

Example 3

The stock price of Hooli per share is considered. The probability of it going up is **90%**. Given the stock price went up, the market was good **75%** of the time, fair **20%** of the time, and bad **5%** of the time. When the stock price went down, those numbers were **60%**, **30%**, and **10%**, respectively. Use this information to find the probability of the stock price going up given a fair market.

Solution.

$$\mathbf{P}\left(\begin{matrix}\text{stock}\\\text{price}\end{matrix}\right) \times \mathbf{P}\left(\begin{matrix}\text{market}\\\text{condition}\end{matrix}\,\middle|\,\begin{matrix}\text{stock}\\\text{price}\end{matrix}\right) = \mathbf{P}\left(\begin{matrix}\text{stock}\\\text{price}\end{matrix}\ \textbf{and}\ \begin{matrix}\text{market}\\\text{condition}\end{matrix}\right)$$

	up	down	
good	$0.9 \times 0.75 = 0.675$	$0.1 \times 0.6 = 0.06$	$P(\text{good}) = 0.735$
fair	$0.9 \times 0.2 = 0.18$	$0.1 \times 0.3 = 0.03$	$P(\text{fair}) = 0.21$
bad	$0.9 \times 0.05 = 0.045$	$0.1 \times 0.1 = 0.01$	$P(\text{bad}) = 0.055$
	$P(\text{up}) = 0.9$	$P(\text{down}) = 0.1$	

$$P(\text{up} \mid \text{fair}) = \frac{P(\text{up}) \times P(\text{fair} \mid \text{up})}{P(\text{fair})} = \frac{0.9 \times 0.2}{0.21} = \frac{0.18}{0.21} = \boxed{0.857}$$