cs109a_hw4_submit

October 17, 2018

1 CS109A Introduction to Data Science:

1.1 Homework 4 - Regularization

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1.1.1 INSTRUCTIONS

- **This homework must be completed individually.**
- To submit your assignment follow the instructions given in Canvas.
- Restart the kernel and run the whole notebook again before you submit.
- As much as possible, try and stick to the hints and functions we import at the top of the homework, as those are the ideas and tools the class supports and is aiming to teach. And if a problem specifies a particular library you're required to use that library, and possibly others from the import list.

Names of people you have worked with goes here: Michel Atoudem Kana

In [1]:
```
#RUN THIS CELL
import requests
from IPython.core.display import HTML
styles = requests.get("https://raw.githubusercontent.com/Harvard-IACS/2018-CS109A/maste
HTML(styles)
```

Out[1]:
```
<IPython.core.display.HTML object>
```

import these libraries

In [2]:
```
import warnings
#warnings.filterwarnings('ignore')
import numpy as np
import pandas as pd
import matplotlib
import matplotlib.pyplot as plt
from sklearn.metrics import r2_score
from sklearn.preprocessing import PolynomialFeatures
from sklearn.linear_model import Ridge
```

```
from sklearn.linear_model import Lasso
from sklearn.linear_model import RidgeCV
from sklearn.linear_model import LassoCV
from sklearn.linear_model import LinearRegression
from sklearn.preprocessing import StandardScaler
from sklearn.model_selection import train_test_split

from sklearn.model_selection import cross_val_score
from sklearn.model_selection import LeaveOneOut
from sklearn.model_selection import KFold

import statsmodels.api as sm
from statsmodels.regression.linear_model import OLS

from pandas.core import datetools

import seaborn as sns

%matplotlib inline
```

```
C:\ProgramData\Anaconda3\lib\site-packages\ipykernel_launcher.py:24: FutureWarning: The pandas
```

2 Continuing Bike Sharing Usage Data

In this homework, we will focus on regularization and cross validation. We will continue to build regression models for the Capital Bikeshare program in Washington D.C. See homework 3 for more information about the Capital Bikeshare data that we'll be using extensively.

Question 1 [20pts] Data pre-processing

1.1 Read in the provided `bikes_student.csv` to a data frame named `bikes_main`. Split it into a training set `bikes_train` and a validation set `bikes_val`. Use `random_state=90`, a test set size of .2, and stratify on month. Remember to specify the data's index column as you read it in.

1.2 As with last homework, the response will be the `counts` column and we'll drop `counts`, `registered` and `casual` for being trivial predictors, drop `workingday` and `month` for being multicollinear with other columns, and `dteday` for being inappropriate for regression. Write code to do this.

Encapsulate this process as a function with appropriate inputs and outputs, and **test** your code by producing `practice_y_train` and `practice_X_train`.

1.3 Write a function to standardize a provided subset of columns in your training/validation/test sets. Remember that while you will be scaling all of your data, you must learn the scaling parameters (mean and SD) from only the training set.

Test your code by building a list of all non-binary columns in your `practice_X_train` and scaling only those columns. Call the result `practice_X_train_scaled`. Display the `.describe()` and verify that you have correctly scaled all columns, including the polynomial columns.

Hint: employ the provided list of binary columns and use `pd.columns.difference()`

`binary_columns = [ 'holiday', 'workingday','Feb', 'Mar', 'Apr', 'May', 'Jun', 'Jul', 'Aug', 'Sept', 'Oct', 'Nov', 'Dec', 'spring', 'summer',` 'fall', 'Mon', 'Tue', 'Wed', 'Thu', 'Fri', 'Sat',   'Cloudy', 'Snow', 'Storm']

1.4 Write a code to augment your a dataset with higher-order features for `temp`, `atemp`, `hum`,`windspeed`, and `hour`. You should include ONLY the pure powers of these columns. So with degree=2 you should produce `atemp^2` and `hum^2` but not `atemp*hum` or any other two-feature interactions.

Encapsulate this process as a function with appropriate inputs and outputs, and test your code by producing `practice_X_train_poly`, a training dataset with quadratic and cubic features built from `practice_X_train_scaled`, and printing `practice_X_train_poly`'s column names and `.head()`.

1.5 Write code to add interaction terms to the model. Specifically, we want interactions between the continuous predictors (`temp`,`atemp`, `hum`,`windspeed`) and the month and weekday dummies (`Feb`, `Mar`...`Dec`, `Mon`, `Tue`, ... `Sat`). That means you SHOULD build `atemp*Feb` and `hum*Mon` and so on, but NOT `Feb*Mar` and NOT `Feb*Tue`. The interaction terms should always be a continuous feature times a month dummy or a continuous feature times a weekday dummy.

Encapsulate this process as a function with appropriate inputs and outputs, and test your code by adding interaction terms to `practice_X_train_poly` and show its column names and `.head()`**

1.6 Combine all your code so far into a function that takes in `bikes_train`, `bikes_val`, the names of columns for polynomial, the target column, the columns to be dropped and produces computation-ready design matrices `X_train` and `X_val` and responses `y_train` and `y_val`. Your final function should build correct, scaled design matrices with the stated interaction terms and any polynomial degree.

2.0.1 Solutions

1.1 Read in the provided `bikes_student.csv` to a data frame named `bikes_main`. Split it into a training set `bikes_train` and a validation set `bikes_val`. Use `random_state=90`, a test set size of .2, and stratify on month. Remember to specify the data's index column as you read it in.

Read the dataset *Import the CSV data file into a dataframe*

```
In [3]: bikes_main = pd.read_csv('data/bikes_student.csv', index_col='Unnamed: 0')
```

Explore the data

```
In [4]: bikes_main.columns.values

Out[4]: array(['dteday', 'hour', 'year', 'holiday', 'workingday', 'temp', 'atemp',
               'hum', 'windspeed', 'casual', 'registered', 'counts', 'Feb', 'Mar',
               'Apr', 'May', 'Jun', 'Jul', 'Aug', 'Sept', 'Oct', 'Nov', 'Dec',
               'spring', 'summer', 'fall', 'Mon', 'Tue', 'Wed', 'Thu', 'Fri',
               'Sat', 'Cloudy', 'Snow', 'Storm', 'month'], dtype=object)

In [5]: bikes_main.shape

Out[5]: (1250, 36)

In [6]: bikes_main.head()
```

```
Out[6]:          dteday  hour  year  holiday  workingday  temp   atemp   hum  \
5887   2011-09-07    19     0        0           1  0.64  0.5758  0.89
10558  2012-03-21     1     1        0           1  0.52  0.5000  0.83
14130  2012-08-16    23     1        0           1  0.70  0.6515  0.54
2727   2011-04-28    13     0        0           1  0.62  0.5758  0.83
8716   2012-01-04     0     1        0           1  0.08  0.0606  0.42

       windspeed  casual  ...   Mon  Tue  Wed  Thu  Fri  Sat  Cloudy  Snow  \
5887      0.0000      14  ...     0    0    1    0    0    0       1     0
10558     0.0896       4  ...     0    0    1    0    0    0       0     0
14130     0.1045      58  ...     0    0    0    1    0    0       0     0
2727      0.2985      18  ...     0    0    0    1    0    0       1     0
8716      0.3284       0  ...     0    0    1    0    0    0       0     0

       Storm  month
5887       0      9
10558      0      3
14130      0      8
2727       0      4
8716       0      1

[5 rows x 36 columns]
```

```
In [7]: bikes_main.describe()
```

```
Out[7]:             hour         year      holiday   workingday         temp  \
count  1250.000000  1250.000000  1250.000000  1250.000000  1250.000000
mean     11.410400     0.514400     0.030400     0.675200     0.494160
std       6.885456     0.499993     0.171754     0.468488     0.192529
min       0.000000     0.000000     0.000000     0.000000     0.040000
25%       5.000000     0.000000     0.000000     0.000000     0.340000
50%      11.000000     1.000000     0.000000     1.000000     0.500000
75%      17.000000     1.000000     0.000000     1.000000     0.660000
max      23.000000     1.000000     1.000000     1.000000     0.940000

             atemp         hum    windspeed       casual   registered  \
count  1250.000000  1250.00000  1250.000000  1250.000000  1250.000000
mean      0.473600     0.63844     0.197306    36.050400   159.288000
std       0.171707     0.18818     0.123924    50.958026   161.031847
min       0.060600     0.00000     0.000000     0.000000     0.000000
25%       0.333300     0.50000     0.104500     3.000000    32.250000
50%       0.484800     0.65000     0.194000    15.000000   113.500000
75%       0.621200     0.80000     0.283600    50.000000   231.750000
max       0.909100     1.00000     0.850700   362.000000   810.000000

          ...          Mon          Tue         Wed          Thu  \
count     ...  1250.000000  1250.000000  1250.00000  1250.000000
mean      ...     0.140800     0.141600     0.15520     0.138400
```

```
std      ...        0.347954     0.348779     0.36224     0.345458
min      ...        0.000000     0.000000     0.00000     0.000000
25%      ...        0.000000     0.000000     0.00000     0.000000
50%      ...        0.000000     0.000000     0.00000     0.000000
75%      ...        0.000000     0.000000     0.00000     0.000000
max      ...        1.000000     1.000000     1.00000     1.000000

               Fri          Sat       Cloudy         Snow   Storm        month
count  1250.000000  1250.000000  1250.000000  1250.000000  1250.0  1250.000000
mean      0.129600     0.148000     0.276800     0.086400     0.0     6.533600
std       0.335997     0.355242     0.447596     0.281066     0.0     3.441503
min       0.000000     0.000000     0.000000     0.000000     0.0     1.000000
25%       0.000000     0.000000     0.000000     0.000000     0.0     4.000000
50%       0.000000     0.000000     0.000000     0.000000     0.0     7.000000
75%       0.000000     0.000000     1.000000     0.000000     0.0    10.000000
max       1.000000     1.000000     1.000000     1.000000     0.0    12.000000

[8 rows x 35 columns]
```

Describe the dataset We will work with a dataset contained in data/bikes_student.csv. Each row in this file represents the number of rides by registered users and casual users in a given hour of a specific date in the years 2011 and 2012. There are 36 attributes in total:

- `Unnamed: 0` (unique row identifier)
- `dteday` (date in the format YYYY-MM-DD, e.g. 2011-01-01)
- `hour` (0 for 12 midnight, 1 for 1:00am, 23 for 11:00pm)
- `year` (0 for 2011, 1 for 2012)
- `holiday` (1 = the day is a holiday, 0 = otherwise)
- `workingday` (1 = the day is a working day, 0 = otherwise)
- `temp` (temperature in Celsius)
- `atemp` (apparent temperature, or relative outdoor temperature, in Celsius)
- `hum` (relative humidity)
- `windspeed` (wind speed)
- `casual` (number of rides that day made by casual riders, not registered in the system)
- `registered` (number of rides that day made by registered riders)
- `count` (total number of rides that day made)
- `Feb` (1 = the day is a Feb day, 0 = otherwise)
- `Mar` (1 = the day is a Mar day, 0 = otherwise)
- `Apr` (1 = the day is a Apr day, 0 = otherwise)
- `May` (1 = the day is a May day, 0 = otherwise)
- `Jun` (1 = the day is a Jun day, 0 = otherwise)
- `Jul` (1 = the day is a Jul day, 0 = otherwise)
- `Aug` (1 = the day is a Aug day, 0 = otherwise)
- `Sep` (1 = the day is a Sep day, 0 = otherwise)
- `Oct` (1 = the day is a Oct day, 0 = otherwise)
- `Nov` (1 = the day is a Nov day, 0 = otherwise)
- `Dec` (1 = the day is a Dec day, 0 = otherwise)
- `spring` (1 = the day is a spring day, 0 = otherwise)

- `summer` (1 = the day is a summer day, 0 = otherwise)
- `fall` (1 = the day is a fall day, 0 = otherwise)
- `Mon` (1 = the day is Monday, 0 = otherwise)
- `Tue` (1 = the day is Tuesday, 0 = otherwise)
- `Wed` (1 = the day is Wednesday, 0 = otherwise)
- `Thu` (1 = the day is Thursday, 0 = otherwise)
- `Fri` (1 = the day is Friday, 0 = otherwise)
- `Sat` (1 = the day is Saturday, 0 = otherwise)
- `Cloudy` (1 = the weather is Mist + Cloudy, Mist + Broken clouds, Mist + Few clouds, Mist, 0 = otherwise)
- `Snow` (1 = the weather is Light Snow, Light Rain + Thunderstorm, 0 = otherwise)
- `Storm` (1 = the weather is Heavy Rain + Thunderstorm + Mist, Snow + Fog, 0 = otherwise)
- `month` (1 = Jan, 2= Feb, 3 = Mar, 4 = Apr, 5 = May, 6 = Jun, 7 = Jul, 8 = Aug, 9 = Sep, 10 = Oct, 11 = Nov, 12 = Dec)

If all binary predictors that represent months have the value 0 for a given row, then that row represents an observation done in the month of January. If all binary predictors that represent week days have the value 0 for a given row, then that row represents an observation made on Sunday. If all binary predictors that represent season have the value 0 for a given row, then that row represents an observation made on winter. If all binary predictors that represent weather have the value 0 for a given row, then that row represents an observation made during clear weather.

** Split the dataset **

Split the data (1250 rows) into a training (80%) and a validation set (20%).

```
In [8]: try:
            bikes_train, bikes_val = train_test_split(bikes_main, test_size = 0.2, random_state
        except:
            # Drop missing lines
            print("Lines with missing values dropped")
            bikes_main = bikes_main.dropna(subset=['month'])
            bikes_train, bikes_val = train_test_split(bikes_main, test_size = 0.2, random_state
```

Verify that each month is represented equally within each set: ca 83 rows per month in the training set and ca. 21 rows per month in the validation set.

```
In [9]: bikes_train.shape

Out[9]: (1000, 36)

In [10]: bikes_train.groupby(['month']).count().dteday

Out[10]: month
         1      82
         2      78
         3      85
         4      82
         5      86
         6      83
         7      86
```

```
8     85
9     82
10    83
11    82
12    86
Name: dteday, dtype: int64
```

```
In [11]: bikes_val.groupby(['month']).count().dteday
```

```
Out[11]: month
1     21
2     19
3     21
4     21
5     21
6     21
7     21
8     21
9     21
10    21
11    21
12    21
Name: dteday, dtype: int64
```

1.2 As with last homework, the response will be the `counts` column and we'll drop `counts`, `registered` and `casual` for being trivial predictors, drop `workingday` and `month` for being multicolinear with other columns, and `dteday` for being inappropriate for regression. Write code to do this.

Encapsulate this process as a function with appropriate inputs and outputs, and test your code by producing `practice_y_train` and `practice_X_train`

** Drop columns **

Function that takes a data frame and a list of columns as parameters, and return a new dataframe where the columns have been dropped.

```
In [12]: def drop_columns(df, cols):
             for col in cols:
                 if col in df:
                     df = df.drop(columns = [col])
             return df
```

Test the function on the train data set and create response and predictor data frames

```
In [13]: cols_to_drop = ['counts', 'registered', 'casual', 'workingday', 'month', 'dteday']
         practice_y_train = drop_columns(bikes_train, bikes_train.columns.difference(['counts'
         practice_X_train = drop_columns(bikes_train, cols_to_drop)
```

Explore the response and predictor data frames ad verify that the specified columns have been correctly dropped

```
In [14]: practice_y_train.head()
```

```
Out[14]:        counts
         15762     111
         4213      170
         14301      16
         15900      24
         14320     306
```

```
In [15]: practice_X_train.head()
```

```
Out[15]:        hour  year  holiday  temp   atemp   hum  windspeed  Feb  Mar  Apr  \
         15762    23     1        0  0.54  0.5152  0.73     0.1045    0    0    0
         4213     11     0        0  0.76  0.6667  0.35     0.2239    0    0    0
         14301     2     1        0  0.66  0.6212  0.69     0.0000    0    0    0
         15900     5     1        0  0.30  0.3030  0.81     0.1343    0    0    0
         14320    21     1        0  0.70  0.6515  0.61     0.1642    0    0    0

                ...   fall  Mon  Tue  Wed  Thu  Fri  Sat  Cloudy  Snow  Storm
         15762  ...      1    0    1    0    0    0    0       0     0      0
         4213   ...      0    0    0    1    0    0    0       0     0      0
         14301  ...      0    0    0    0    0    1    0       0     0      0
         15900  ...      1    0    0    1    0    0    0       1     0      0
         14320  ...      0    0    0    0    0    1    0       1     0      0

         [5 rows x 30 columns]
```

```
In [16]: practice_X_train.columns.values
```

```
Out[16]: array(['hour', 'year', 'holiday', 'temp', 'atemp', 'hum', 'windspeed',
                'Feb', 'Mar', 'Apr', 'May', 'Jun', 'Jul', 'Aug', 'Sept', 'Oct',
                'Nov', 'Dec', 'spring', 'summer', 'fall', 'Mon', 'Tue', 'Wed',
                'Thu', 'Fri', 'Sat', 'Cloudy', 'Snow', 'Storm'], dtype=object)
```

1.3 Write a function to standardize a provided subset of columns in your training/validation/test sets. Remember that while you will be scaling all of your data, you must learn the scaling parameters (mean and SD) from only the training set.

Test your code by building a list of all non-binary columns in your `practice_X_train` and scaling only those columns. Call the result `practice_X_train_scaled`. Display the `.describe()` and verify that you have correctly scaled all columns, including the polynomial columns.

Hint: employ the provided list of binary columns and use `pd.columns.difference()`

`binary_columns = [ 'holiday', 'workingday','Feb', 'Mar', 'Apr', 'May', 'Jun', 'Jul', 'Aug', 'Sept', 'Oct', 'Nov', 'Dec', 'spring', 'summer', 'fall', 'Mon', 'Tue', 'Wed', 'Thu', 'Fri', 'Sat', 'Cloudy', 'Snow', 'Storm']`

** Standardize function **

Function that takes a data frame and a set of columns as parameters, and scales data using the mean and standard deviation of training data (also a parameter of the function). There are other standardization methods, for example dividing the data by their standard deviation only (without removing the mean). This later method is decribed in the class book. In this homework, we substract the mean as it was done in the lab.

The function will be used to standardize predictors prior to regularization.

```python
In [17]: def standardize_columns(df, cols, scaler):
             df_copy = df.copy()
             df_copy[cols] = scaler.transform(df_copy[cols])
             return df_copy
```

** Standardize continuous predictors **

After unecessary columns have been dropped, the remaining predictors can be split into two groups.

- binary predictors: 'holiday', 'Feb', 'Mar', 'Apr', 'May', 'Jun', 'Jul', 'Aug', 'Sept', 'Oct', 'Nov', 'Dec', 'spring', 'summer', 'fall', 'Mon', 'Tue', 'Wed', 'Thu', 'Fri', 'Sat', 'Cloudy', 'Snow', 'Storm'
- continuous predictors: 'hour', 'temp', 'atemp', 'hum', 'windspeed', 'year'

```python
In [18]: binary_columns = ['holiday','Feb', 'Mar', 'Apr',
                'May', 'Jun', 'Jul', 'Aug', 'Sept', 'Oct', 'Nov', 'Dec', 'spring',
                'summer', 'fall', 'Mon', 'Tue', 'Wed', 'Thu', 'Fri', 'Sat',
                'Cloudy', 'Snow', 'Storm']
         continuous_columns = practice_X_train.columns.difference(binary_columns)
         scaler = StandardScaler().fit(practice_X_train[continuous_columns])

         practice_X_train_scaled = standardize_columns(practice_X_train, continuous_columns, sc
```

Verify if the continuous predictors have mean 0 (almost) and standard deviation 1

```python
In [19]: practice_X_train_scaled[continuous_columns].describe()
```

Out[19]:

	atemp	hour	hum	temp	windspeed
count	1.000000e+03	1.000000e+03	1.000000e+03	1.000000e+03	1.000000e+03
mean	-1.256772e-16	-1.994516e-16	5.995204e-17	3.019807e-17	1.301181e-16
std	1.000500e+00	1.000500e+00	1.000500e+00	1.000500e+00	1.000500e+00
min	-2.402605e+00	-1.646163e+00	-3.397602e+00	-2.347976e+00	-1.554205e+00
25%	-8.121270e-01	-9.189949e-01	-7.421467e-01	-7.922693e-01	-7.231056e-01
50%	7.147176e-02	-4.639332e-02	5.448995e-02	3.744066e-02	-1.130295e-02
75%	8.670022e-01	8.262083e-01	8.511266e-01	8.671507e-01	4.634972e-01
max	2.546131e+00	1.698810e+00	1.913309e+00	2.319143e+00	5.211499e+00

	year
count	1.000000e+03
mean	2.686740e-17
std	1.000500e+00
min	-1.018165e+00
25%	-1.018165e+00
50%	9.821591e-01
75%	9.821591e-01
max	9.821591e-01

Verify scaling on validation set

In [20]:
```
standardize_columns(bikes_val, continuous_columns, scaler).describe()
```

Out[20]:
```
             hour        year     holiday  workingday        temp       atemp  \
count  250.000000  250.000000  250.000000  250.000000  250.000000  250.000000
mean     0.066463    0.054009    0.044000    0.652000    0.035781    0.030762
std      1.005123    0.999567    0.205507    0.477292    0.991393    1.006891
min     -1.646163   -1.018165    0.000000    0.000000   -2.140548   -2.402605
25%     -0.773561   -1.018165    0.000000    0.000000   -0.792269   -0.812127
50%      0.099040    0.982159    0.000000    1.000000    0.141154    0.160123
75%      0.971642    0.982159    0.000000    1.000000    0.867151    0.867002
max      1.698810    0.982159    1.000000    1.000000    2.111716    2.192692

              hum   windspeed      casual  registered    ...           Mon  \
count  250.000000  250.000000  250.000000  250.000000    ...     250.00000
mean    -0.034521    0.074947   37.744000  161.832000    ...       0.13200
std      0.996533    0.922962   52.363942  166.616349    ...       0.33917
min     -3.397602   -1.554205    0.000000    1.000000    ...       0.00000
25%     -0.795256   -0.723106    4.000000   39.250000    ...       0.00000
50%     -0.051728   -0.011303   19.000000  118.000000    ...       0.00000
75%      0.784740    0.701295   51.500000  221.000000    ...       0.00000
max      1.913309    3.075296  307.000000  803.000000    ...       1.00000

              Tue        Wed         Thu         Fri         Sat      Cloudy  \
count  250.000000  250.00000  250.000000  250.000000  250.000000  250.000000
mean     0.116000    0.12800    0.180000    0.140000    0.140000    0.264000
std      0.320867    0.33476    0.384958    0.347683    0.347683    0.441684
min      0.000000    0.00000    0.000000    0.000000    0.000000    0.000000
25%      0.000000    0.00000    0.000000    0.000000    0.000000    0.000000
50%      0.000000    0.00000    0.000000    0.000000    0.000000    0.000000
75%      0.000000    0.00000    0.000000    0.000000    0.000000    1.000000
max      1.000000    1.00000    1.000000    1.000000    1.000000    1.000000

             Snow  Storm     month
count  250.000000  250.0  250.0000
mean     0.104000    0.0    6.5360
std      0.305873    0.0    3.4491
min      0.000000    0.0    1.0000
25%      0.000000    0.0    4.0000
50%      0.000000    0.0    7.0000
75%      0.000000    0.0    9.7500
max      1.000000    0.0   12.0000

[8 rows x 35 columns]
```

1.4 Write a code to augment your a dataset with higher-order features for `temp`, `atemp`, `hum`,`windspeed`, and `hour`. You should include ONLY pure powers of these columns. So with degree=2 you should produce `atemp^2` and `hum^2` but not `atemp*hum` or any other two-feature interactions.

Encapsulate this process as a function with apropriate inputs and outputs, and test your code by producing `practice_X_train_poly`, a training dataset with qudratic and cubic features built from `practice_X_train_scaled`, and printing `practice_X_train_poly`'s column names and `.head()`.

** Higher-oder function **

Function that takes a dataframe, a list of columns and a polynomial degree, and creates higher order features for the given columns.

For example if the degree is 3 and the columns list is ['x'], then the function will return a new dataframe with new columns x^2 and x^3 additional to the existing x.

```
In [21]: def create_higher_order(df, cols, degree):
             df_copy = df.copy()
             for col in cols:
                 for order in range(2, degree + 1):
                     col_name = col + '^' + str(order)
                     if not col_name in df:
                         df_copy[col_name] = df[col] ** order
             return df_copy
```

** Create higher order features of continuous variables **

The measured data contains the temperature (temp), relative temperature (atemp), humidity (hum), windspeed and hour.

The data is now augmented with temp^2, temp^3, atemp^2, atemp^3, hum^2, hum^3, windspeed^2, windspeed^3, hour^2, hour^3.

```
In [22]: practice_X_train_poly = create_higher_order(practice_X_train_scaled, ['temp', 'atemp'
```

Verify the new polynomial features

```
In [23]: new_cols = practice_X_train_poly.columns.difference(list(practice_X_train.columns))
         new_cols

Out[23]: Index(['atemp^2', 'atemp^3', 'hour^2', 'hour^3', 'hum^2', 'hum^3', 'temp^2',
                'temp^3', 'windspeed^2', 'windspeed^3'],
               dtype='object')

In [24]: practice_X_train_poly[new_cols].head()

Out[24]:         atemp^2   atemp^3    hour^2    hour^3     hum^2     hum^3    temp^2  \
         15762  0.061889  0.015396  2.885955  4.902689  0.229789  0.110152  0.059960
         4213   1.282270  1.452008  0.002152 -0.000100  2.367854 -3.643615  1.920218
         14301  0.751693  0.651719  1.836827 -2.489443  0.071250  0.019018  0.751950
         15900  0.977818 -0.966912  0.844552 -0.776139  0.817642  0.739341  0.999394
         14320  1.089356  1.136984  1.982303  2.790969  0.024947 -0.003940  1.154718

                  temp^3  windspeed^2  windspeed^3
         15762  0.014682     0.522882    -0.378099
         4213   2.660884     0.051300     0.011619
         14301  0.652054     2.415552    -3.754263
         15900 -0.999091     0.236296    -0.114864
         14320  1.240835     0.061656    -0.015309
```

1.5 Write code to add interaction terms to the model. Specifically, we want interactions between the continuous predictors (`temp`,`atemp`, `hum`,`windspeed`) and the month and weekday dummies (`Feb`, `Mar`...`Dec`, `Mon`, `Tue`, ... `Sat`). That means you SHOULD build `atemp*Feb` and `hum*Mon` and so on, but NOT `Feb*Mar` and NOT `Feb*Tue`. The interaction terms should always be a continuous feature times a month dummy or a continuous feature times a weekday dummy.

Encapsulate this process as a function with appropriate inputs and outputs, and test your code by adding interaction terms to `practice_X_train_poly` and show its column names and `.head()`**

** Interaction terms function **

Function takes a dataframe, a list of continuous predictors, a list of time-related predictors; and returns a new dataframe augmented with interaction terms between each of the continuous predictors and each of the time-related predictors.

```
In [25]: def create_interaction_terms(df, cols_continuous = ['temp', 'atemp', 'hum', 'windspee
                                      cols_time = ['Feb', 'Mar', 'Apr', 'May', 'Jun', 'Jul', '
                                                   'Mon', 'Tue', 'Wed', 'Thu', 'Fri', 'Sat']):
             df_copy = df.copy()
             for colc in cols_continuous:
                 for colb in cols_time:
                     col_name = colc + '*' + colb
                     if not col_name in df_copy:
                         df_copy[col_name] = df_copy[colc] * df_copy[colb]
             return df_copy
```

** Create interaction terms between countinuous predictors and months/weekday **

```
In [26]: practice_X_train_poly_inter = create_interaction_terms(practice_X_train_poly)
```

Verify the new interaction terms

```
In [27]: new_cols = practice_X_train_poly_inter.columns.difference(list(practice_X_train_poly.
         new_cols
```

```
Out[27]: Index(['atemp*Apr', 'atemp*Aug', 'atemp*Dec', 'atemp*Feb', 'atemp*Fri',
                'atemp*Jul', 'atemp*Jun', 'atemp*Mar', 'atemp*May', 'atemp*Mon',
                'atemp*Nov', 'atemp*Oct', 'atemp*Sat', 'atemp*Sept', 'atemp*Thu',
                'atemp*Tue', 'atemp*Wed', 'hum*Apr', 'hum*Aug', 'hum*Dec', 'hum*Feb',
                'hum*Fri', 'hum*Jul', 'hum*Jun', 'hum*Mar', 'hum*May', 'hum*Mon',
                'hum*Nov', 'hum*Oct', 'hum*Sat', 'hum*Sept', 'hum*Thu', 'hum*Tue',
                'hum*Wed', 'temp*Apr', 'temp*Aug', 'temp*Dec', 'temp*Feb', 'temp*Fri',
                'temp*Jul', 'temp*Jun', 'temp*Mar', 'temp*May', 'temp*Mon', 'temp*Nov',
                'temp*Oct', 'temp*Sat', 'temp*Sept', 'temp*Thu', 'temp*Tue', 'temp*Wed',
                'windspeed*Apr', 'windspeed*Aug', 'windspeed*Dec', 'windspeed*Feb',
                'windspeed*Fri', 'windspeed*Jul', 'windspeed*Jun', 'windspeed*Mar',
                'windspeed*May', 'windspeed*Mon', 'windspeed*Nov', 'windspeed*Oct',
                'windspeed*Sat', 'windspeed*Sept', 'windspeed*Thu', 'windspeed*Tue',
                'windspeed*Wed'],
               dtype='object')
```

In [28]: practice_X_train_poly_inter[new_cols].head()

```
Out[28]:        atemp*Apr  atemp*Aug  atemp*Dec  atemp*Feb  atemp*Fri  atemp*Jul  \
         15762        0.0   0.000000        0.0        0.0   0.000000        0.0
         4213         0.0   0.000000        0.0        0.0   0.000000        0.0
         14301        0.0   0.867002        0.0        0.0   0.867002        0.0
         15900       -0.0  -0.000000       -0.0       -0.0  -0.000000       -0.0
         14320        0.0   1.043722        0.0        0.0   1.043722        0.0

                atemp*Jun  atemp*Mar  atemp*May  atemp*Mon      ...         \
         15762   0.000000        0.0        0.0        0.0      ...
         4213    1.132373        0.0        0.0        0.0      ...
         14301   0.000000        0.0        0.0        0.0      ...
         15900  -0.000000       -0.0       -0.0       -0.0      ...
         14320   0.000000        0.0        0.0        0.0      ...

                windspeed*Mar  windspeed*May  windspeed*Mon  windspeed*Nov  \
         15762           -0.0           -0.0           -0.0           -0.0
         4213             0.0            0.0            0.0            0.0
         14301           -0.0           -0.0           -0.0           -0.0
         15900           -0.0           -0.0           -0.0           -0.0
         14320           -0.0           -0.0           -0.0           -0.0

                windspeed*Oct  windspeed*Sat  windspeed*Sept  windspeed*Thu  \
         15762      -0.723106           -0.0            -0.0           -0.0
         4213        0.000000            0.0             0.0            0.0
         14301      -0.000000           -0.0            -0.0           -0.0
         15900      -0.486103           -0.0            -0.0           -0.0
         14320      -0.000000           -0.0            -0.0           -0.0

                windspeed*Tue  windspeed*Wed
         15762      -0.723106      -0.000000
         4213        0.000000       0.226495
         14301      -0.000000      -0.000000
         15900      -0.000000      -0.486103
         14320      -0.000000      -0.000000

         [5 rows x 68 columns]
```

1.6 Combine all your code so far into a function that takes in `bikes_train`, `bikes_val`, the names of columns for polynomial, the target column, the columns to be dropped and produces computation-ready design matrices `X_train` and `X_val` and responses `y_train` and `y_val`. Your final function should build correct, scaled design matrices with the stated interaction terms and any polynomial degree.

Function for computation-ready design matrices

Function that accepts the following parameters:

- a train dataframe

- a validation dataframe
- a polynomial degree
- a list of columns for polynomial features. This list is also the list of continuous predictors.
- the response variable
- a list of columns to remove.

The function performs the following actions: * retrieves the response variable from the input dataframe * drops bad columns * create polynomial terms for continuous predictors up to the given degree * create interaction terms between continuous predictors and month/weekday binary variables * standardize all continuous predictors

The function standardizes only the continuous predictors, their corresponding polynomial features and interaction terms are not standardized. Standardizing the predictors would help reduce multicollinearity.

```
In [29]: def get_design_mats(train_df, val_df,  degree,
                             columns_forpoly=['temp', 'atemp', 'hum','windspeed', 'hour'],
                             target_col='counts',
                             bad_columns=['counts', 'registered', 'casual', 'workingday', 'mon
             # retrieve response variables
             y_train = train_df[[target_col]]
             y_val = val_df[[target_col]]
             # drop bad variables
             x_train = drop_columns(train_df, bad_columns)
             x_val = drop_columns(val_df, bad_columns)
             # get binary and continuous columns
             binary_columns = ['holiday','Feb', 'Mar', 'Apr',
                'May', 'Jun', 'Jul', 'Aug', 'Sept', 'Oct', 'Nov', 'Dec', 'spring',
                'summer', 'fall', 'Mon', 'Tue', 'Wed', 'Thu', 'Fri', 'Sat',
                'Cloudy', 'Snow', 'Storm']
             continuous_columns = x_train.columns.difference(binary_columns)
             # standardize continuous predictors
             scaler = StandardScaler().fit(x_train[continuous_columns])
             x_train = standardize_columns(x_train, continuous_columns, scaler)
             x_val = standardize_columns(x_val, continuous_columns, scaler)
             # create polynomial terms
             x_train = create_higher_order(x_train, columns_forpoly, degree)
             x_val = create_higher_order(x_val, columns_forpoly, degree)
             # create interaction terms
             x_train = create_interaction_terms(x_train)
             x_val = create_interaction_terms(x_val)
             return x_train, y_train, x_val, y_val
```

Test the design matrices for train and validation

```
In [30]: X_train, y_train, X_val, y_val = get_design_mats(bikes_train, bikes_val, 3)
```

```
In [31]: X_val.describe()
```

```
Out[31]:              hour        year     holiday        temp       atemp         hum  \
         count  250.000000  250.000000  250.000000  250.000000  250.000000  250.000000
```

```
mean      0.066463    0.054009    0.044000    0.035781    0.030762   -0.034521
std       1.005123    0.999567    0.205507    0.991393    1.006891    0.996533
min      -1.646163   -1.018165    0.000000   -2.140548   -2.402605   -3.397602
25%      -0.773561   -1.018165    0.000000   -0.792269   -0.812127   -0.795256
50%       0.099040    0.982159    0.000000    0.141154    0.160123   -0.051728
75%       0.971642    0.982159    0.000000    0.867151    0.867002    0.784740
max       1.698810    0.982159    1.000000    2.111716    2.192692    1.913309

        windspeed         Feb         Mar         Apr       ...        \
count  250.000000   250.00000  250.000000  250.000000       ...
mean     0.074947     0.07600    0.084000    0.084000       ...
std      0.922962     0.26553    0.277944    0.277944       ...
min     -1.554205     0.00000    0.000000    0.000000       ...
25%     -0.723106     0.00000    0.000000    0.000000       ...
50%     -0.011303     0.00000    0.000000    0.000000       ...
75%      0.701295     0.00000    0.000000    0.000000       ...
max      3.075296     1.00000    1.000000    1.000000       ...

       windspeed*Sept  windspeed*Oct  windspeed*Nov  windspeed*Dec  \
count      250.000000     250.000000     250.000000     250.000000
mean         0.003352      -0.023718      -0.014692       0.020918
std          0.226063       0.264793       0.292966       0.304081
min         -1.554205      -1.554205      -1.554205      -1.554205
25%         -0.000000      -0.000000       0.000000      -0.000000
50%          0.000000      -0.000000       0.000000       0.000000
75%          0.000000       0.000000       0.000000       0.000000
max          1.294596       1.532394       1.532394       2.600496

       windspeed*Mon  windspeed*Tue  windspeed*Wed  windspeed*Thu  \
count     250.000000     250.000000     250.000000     250.000000
mean        0.018963       0.032439      -0.004730      -0.002445
std         0.363190       0.357898       0.313457       0.327324
min        -1.554205      -1.554205      -1.554205      -1.554205
25%         0.000000      -0.000000       0.000000       0.000000
50%         0.000000       0.000000      -0.000000      -0.000000
75%        -0.000000       0.000000      -0.000000       0.000000
max         2.600496       3.075296       1.532394       2.007194

       windspeed*Fri  windspeed*Sat
count     250.000000     250.000000
mean        0.010335       0.025519
std         0.337981       0.388775
min        -1.554205      -1.554205
25%        -0.000000       0.000000
50%        -0.000000       0.000000
75%        -0.000000      -0.000000
max         1.769397       3.075296
```

```
[8 rows x 108 columns]
```

Check the number of rows in the train and validation sets

```
In [32]: X_train.shape, y_train.shape, X_val.shape, y_val.shape
```

```
Out[32]: ((1000, 108), (1000, 1), (250, 108), (250, 1))
```

Check the data in the training set

```
In [33]: print(y_train.columns.values)
         print(X_train.columns.values)
```

```
['counts']
['hour' 'year' 'holiday' 'temp' 'atemp' 'hum' 'windspeed' 'Feb' 'Mar'
 'Apr' 'May' 'Jun' 'Jul' 'Aug' 'Sept' 'Oct' 'Nov' 'Dec' 'spring' 'summer'
 'fall' 'Mon' 'Tue' 'Wed' 'Thu' 'Fri' 'Sat' 'Cloudy' 'Snow' 'Storm'
 'temp^2' 'temp^3' 'atemp^2' 'atemp^3' 'hum^2' 'hum^3' 'windspeed^2'
 'windspeed^3' 'hour^2' 'hour^3' 'temp*Feb' 'temp*Mar' 'temp*Apr'
 'temp*May' 'temp*Jun' 'temp*Jul' 'temp*Aug' 'temp*Sept' 'temp*Oct'
 'temp*Nov' 'temp*Dec' 'temp*Mon' 'temp*Tue' 'temp*Wed' 'temp*Thu'
 'temp*Fri' 'temp*Sat' 'atemp*Feb' 'atemp*Mar' 'atemp*Apr' 'atemp*May'
 'atemp*Jun' 'atemp*Jul' 'atemp*Aug' 'atemp*Sept' 'atemp*Oct' 'atemp*Nov'
 'atemp*Dec' 'atemp*Mon' 'atemp*Tue' 'atemp*Wed' 'atemp*Thu' 'atemp*Fri'
 'atemp*Sat' 'hum*Feb' 'hum*Mar' 'hum*Apr' 'hum*May' 'hum*Jun' 'hum*Jul'
 'hum*Aug' 'hum*Sept' 'hum*Oct' 'hum*Nov' 'hum*Dec' 'hum*Mon' 'hum*Tue'
 'hum*Wed' 'hum*Thu' 'hum*Fri' 'hum*Sat' 'windspeed*Feb' 'windspeed*Mar'
 'windspeed*Apr' 'windspeed*May' 'windspeed*Jun' 'windspeed*Jul'
 'windspeed*Aug' 'windspeed*Sept' 'windspeed*Oct' 'windspeed*Nov'
 'windspeed*Dec' 'windspeed*Mon' 'windspeed*Tue' 'windspeed*Wed'
 'windspeed*Thu' 'windspeed*Fri' 'windspeed*Sat']
```

```
In [34]: y_val.head()
```

Out[34]:

	counts
8512	70
14196	343
8716	9
7913	224
7403	5

```
In [35]: X_train.describe()
```

Out[35]:

	hour	year	holiday	temp	atemp \
count	1.000000e+03	1.000000e+03	1000.000000	1.000000e+03	1.000000e+03
mean	-1.994516e-16	2.686740e-17	0.027000	3.019807e-17	-1.256772e-16
std	1.000500e+00	1.000500e+00	0.162164	1.000500e+00	1.000500e+00
min	-1.646163e+00	-1.018165e+00	0.000000	-2.347976e+00	-2.402605e+00

```
25%   -9.189949e-01 -1.018165e+00     0.000000 -7.922693e-01 -8.121270e-01
50%   -4.639332e-02  9.821591e-01     0.000000  3.744066e-02  7.147176e-02
75%    8.262083e-01  9.821591e-01     0.000000  8.671507e-01  8.670022e-01
max    1.698810e+00  9.821591e-01     1.000000  2.319143e+00  2.546131e+00

                hum     windspeed          Feb          Mar          Apr  \
count  1.000000e+03  1.000000e+03  1000.000000  1000.000000  1000.000000
mean   5.995204e-17  1.301181e-16     0.078000     0.085000     0.082000
std    1.000500e+00  1.000500e+00     0.268306     0.279021     0.274502
min   -3.397602e+00 -1.554205e+00     0.000000     0.000000     0.000000
25%   -7.421467e-01 -7.231056e-01     0.000000     0.000000     0.000000
50%    5.448995e-02 -1.130295e-02     0.000000     0.000000     0.000000
75%    8.511266e-01  4.634972e-01     0.000000     0.000000     0.000000
max    1.913309e+00  5.211499e+00     1.000000     1.000000     1.000000

           ...        windspeed*Sept  windspeed*Oct  windspeed*Nov  \
count      ...           1000.000000    1000.000000    1000.000000
mean       ...             -0.014081      -0.014803       0.001709
std        ...              0.242928       0.281089       0.288773
min        ...             -1.554205      -1.554205      -1.554205
25%        ...              0.000000       0.000000       0.000000
50%        ...             -0.000000       0.000000      -0.000000
75%        ...              0.000000       0.000000       0.000000
max        ...              2.600496       2.600496       2.837498

       windspeed*Dec  windspeed*Mon  windspeed*Tue  windspeed*Wed  \
count    1000.000000    1000.000000    1000.000000    1000.000000
mean       -0.017209      -0.013921      -0.013266       0.004386
std         0.310498       0.370430       0.346023       0.403203
min        -1.554205      -1.554205      -1.554205      -1.554205
25%         0.000000       0.000000       0.000000       0.000000
50%        -0.000000       0.000000      -0.000000       0.000000
75%         0.000000       0.000000       0.000000       0.000000
max         3.312298       4.143398       2.600496       2.600496

       windspeed*Thu  windspeed*Fri  windspeed*Sat
count    1000.000000    1000.000000    1000.000000
mean       -0.011268       0.001801       0.017336
std         0.312060       0.376432       0.428265
min        -1.554205      -1.554205      -1.554205
25%         0.000000       0.000000      -0.000000
50%         0.000000      -0.000000       0.000000
75%         0.000000       0.000000       0.000000
max         3.668597       3.312298       3.668597

[8 rows x 108 columns]
```

In [36]:
```
X_train.head()
```

```
Out[36]:            hour      year  holiday      temp     atemp       hum  windspeed  \
        15762  1.698810  0.982159        0  0.244868  0.248775  0.479363  -0.723106
        4213  -0.046393 -1.018165        0  1.385719  1.132373 -1.538783   0.226495
        14301 -1.355296  0.982159        0  0.867151  0.867002  0.266926  -1.554205
        15900 -0.918995  0.982159        0 -0.999697 -0.988847  0.904236  -0.486103
        14320  1.407943  0.982159        0  1.074578  1.043722 -0.157946  -0.248305

               Feb  Mar  Apr      ...        windspeed*Sept  windspeed*Oct  \
        15762    0    0    0      ...                  -0.0      -0.723106
        4213     0    0    0      ...                   0.0       0.000000
        14301    0    0    0      ...                  -0.0      -0.000000
        15900    0    0    0      ...                  -0.0      -0.486103
        14320    0    0    0      ...                  -0.0      -0.000000

               windspeed*Nov  windspeed*Dec  windspeed*Mon  windspeed*Tue  \
        15762           -0.0           -0.0           -0.0      -0.723106
        4213             0.0            0.0            0.0       0.000000
        14301           -0.0           -0.0           -0.0      -0.000000
        15900           -0.0           -0.0           -0.0      -0.000000
        14320           -0.0           -0.0           -0.0      -0.000000

               windspeed*Wed  windspeed*Thu  windspeed*Fri  windspeed*Sat
        15762      -0.000000           -0.0      -0.000000           -0.0
        4213        0.226495            0.0       0.000000            0.0
        14301      -0.000000           -0.0      -1.554205           -0.0
        15900      -0.486103           -0.0      -0.000000           -0.0
        14320      -0.000000           -0.0      -0.248305           -0.0

        [5 rows x 108 columns]
```

Check the data in the validation set

```
In [37]: print(y_val.columns.values)
         print(X_val.columns.values)
```

```
['counts']
['hour' 'year' 'holiday' 'temp' 'atemp' 'hum' 'windspeed' 'Feb' 'Mar'
 'Apr' 'May' 'Jun' 'Jul' 'Aug' 'Sept' 'Oct' 'Nov' 'Dec' 'spring' 'summer'
 'fall' 'Mon' 'Tue' 'Wed' 'Thu' 'Fri' 'Sat' 'Cloudy' 'Snow' 'Storm'
 'temp^2' 'temp^3' 'atemp^2' 'atemp^3' 'hum^2' 'hum^3' 'windspeed^2'
 'windspeed^3' 'hour^2' 'hour^3' 'temp*Feb' 'temp*Mar' 'temp*Apr'
 'temp*May' 'temp*Jun' 'temp*Jul' 'temp*Aug' 'temp*Sept' 'temp*Oct'
 'temp*Nov' 'temp*Dec' 'temp*Mon' 'temp*Tue' 'temp*Wed' 'temp*Thu'
 'temp*Fri' 'temp*Sat' 'atemp*Feb' 'atemp*Mar' 'atemp*Apr' 'atemp*May'
 'atemp*Jun' 'atemp*Jul' 'atemp*Aug' 'atemp*Sept' 'atemp*Oct' 'atemp*Nov'
 'atemp*Dec' 'atemp*Mon' 'atemp*Tue' 'atemp*Wed' 'atemp*Thu' 'atemp*Fri'
 'atemp*Sat' 'hum*Feb' 'hum*Mar' 'hum*Apr' 'hum*May' 'hum*Jun' 'hum*Jul'
 'hum*Aug' 'hum*Sept' 'hum*Oct' 'hum*Nov' 'hum*Dec' 'hum*Mon' 'hum*Tue'
```

```
 'hum*Wed' 'hum*Thu' 'hum*Fri' 'hum*Sat' 'windspeed*Feb' 'windspeed*Mar'
 'windspeed*Apr' 'windspeed*May' 'windspeed*Jun' 'windspeed*Jul'
 'windspeed*Aug' 'windspeed*Sept' 'windspeed*Oct' 'windspeed*Nov'
 'windspeed*Dec' 'windspeed*Mon' 'windspeed*Tue' 'windspeed*Wed'
 'windspeed*Thu' 'windspeed*Fri' 'windspeed*Sat']
```

In [38]:
```
y_val.head()
```

Out[38]:

```
       counts
8512       70
14196     343
8716        9
7913      224
7403        5
```

In [39]:
```
X_val.describe()
```

Out[39]:

```
             hour        year     holiday        temp       atemp         hum  \
count  250.000000  250.000000  250.000000  250.000000  250.000000  250.000000
mean     0.066463    0.054009    0.044000    0.035781    0.030762   -0.034521
std      1.005123    0.999567    0.205507    0.991393    1.006891    0.996533
min     -1.646163   -1.018165    0.000000   -2.140548   -2.402605   -3.397602
25%     -0.773561   -1.018165    0.000000   -0.792269   -0.812127   -0.795256
50%      0.099040    0.982159    0.000000    0.141154    0.160123   -0.051728
75%      0.971642    0.982159    0.000000    0.867151    0.867002    0.784740
max      1.698810    0.982159    1.000000    2.111716    2.192692    1.913309

        windspeed        Feb         Mar         Apr       ...         \
count  250.000000  250.00000  250.000000  250.000000       ...
mean     0.074947    0.07600    0.084000    0.084000       ...
std      0.922962    0.26553    0.277944    0.277944       ...
min     -1.554205    0.00000    0.000000    0.000000       ...
25%     -0.723106    0.00000    0.000000    0.000000       ...
50%     -0.011303    0.00000    0.000000    0.000000       ...
75%      0.701295    0.00000    0.000000    0.000000       ...
max      3.075296    1.00000    1.000000    1.000000       ...

       windspeed*Sept  windspeed*Oct  windspeed*Nov  windspeed*Dec  \
count      250.000000     250.000000     250.000000     250.000000
mean         0.003352      -0.023718      -0.014692       0.020918
std          0.226063       0.264793       0.292966       0.304081
min         -1.554205      -1.554205      -1.554205      -1.554205
25%         -0.000000      -0.000000       0.000000      -0.000000
50%          0.000000      -0.000000       0.000000       0.000000
75%          0.000000       0.000000       0.000000       0.000000
max          1.294596       1.532394       1.532394       2.600496

       windspeed*Mon  windspeed*Tue  windspeed*Wed  windspeed*Thu  \
```

```
count     250.000000     250.000000     250.000000     250.000000
mean        0.018963       0.032439      -0.004730      -0.002445
std         0.363190       0.357898       0.313457       0.327324
min        -1.554205      -1.554205      -1.554205      -1.554205
25%         0.000000      -0.000000       0.000000       0.000000
50%         0.000000       0.000000      -0.000000      -0.000000
75%        -0.000000       0.000000      -0.000000       0.000000
max         2.600496       3.075296       1.532394       2.007194

       windspeed*Fri  windspeed*Sat
count     250.000000     250.000000
mean        0.010335       0.025519
std         0.337981       0.388775
min        -1.554205      -1.554205
25%        -0.000000       0.000000
50%        -0.000000       0.000000
75%        -0.000000      -0.000000
max         1.769397       3.075296

[8 rows x 108 columns]
```

In [40]: X_val.head()

Out[40]:

```
           hour      year  holiday      temp     atemp       hum  windspeed  \
8512  -0.191827 -1.018165        1 -0.792269 -1.076915 -1.167020   2.600496
14196  0.826208  0.982159        0  0.763437  0.690282  0.744908  -0.723106
8716  -1.646163  0.982159        0 -2.140548 -2.402605 -1.167020   1.057594
7913  -0.337261 -1.018165        0 -0.999697 -1.165566 -0.635928   1.057594
7403  -1.355296 -1.018165        0 -0.584842 -0.458687  1.594654  -1.554205

       Feb  Mar  Apr       ...        windspeed*Sept  windspeed*Oct  \
8512     0    0    0       ...                   0.0            0.0
14196    0    0    0       ...                  -0.0           -0.0
8716     0    0    0       ...                   0.0            0.0
7913     0    0    0       ...                   0.0            0.0
7403     0    0    0       ...                  -0.0           -0.0

       windspeed*Nov  windspeed*Dec  windspeed*Mon  windspeed*Tue  \
8512        0.000000       2.600496       2.600496            0.0
14196      -0.000000      -0.000000      -0.000000           -0.0
8716        0.000000       0.000000       0.000000            0.0
7913        0.000000       1.057594       0.000000            0.0
7403       -1.554205      -0.000000      -0.000000           -0.0

       windspeed*Wed  windspeed*Thu  windspeed*Fri  windspeed*Sat
8512        0.000000       0.000000            0.0            0.0
14196      -0.000000      -0.000000           -0.0           -0.0
8716        1.057594       0.000000            0.0            0.0
```

```
7913        0.000000     1.057594          0.0          0.0
7403       -0.000000    -1.554205         -0.0         -0.0

[5 rows x 108 columns]
```

Question 2 [20pts]: Regularization via Ridge
2.1 For each degree in 1 through 8:

1. Build the training design matrix and validation design matrix using the function `get_design_mats` with polynomial terms up through the specified degree.
2. Fit a regression model to the training data.
3. Report the model's score on the validation data.

2.2 Discuss patterns you see in the results from 2.1. Which model would you select, and why?

2.3 Let's try regularizing our models via ridge regression. Build a table showing the validation set R^2 of polynomial models with degree from 1-8, regularized at the levels $\lambda = (.01, .05, .1, .5, 1, 5, 10, 50, 100)$. Do not perform cross validation at this point, simply report performance on the single validation set.

2.4 Find the best-scoring degree and regularization combination.

2.5 It's time to see how well our selected model will do on future data. Read in the provided test dataset, do any required formatting, and report the best model's R^2 score. How does it compare to the validation set score that made us choose this model?

2.6 Why do you think our model's test score was quite a bit worse than its validation score? Does the test set simply contain harder examples, or is something else going on?

2.0.2 Solutions

2.1 For each degree in 1 through 8:

1. Build the training design matrix and validation design matrix using the function `get_design_mats` with polynomial terms up through the specified degree.
2. Fit a regression model to the training data.
3. Report the model's score on the validation data.

** Polynomial models fitting and validation **

For each polynomial degree, we compute the design matrices and fit a polynomial linear regression on the training dataset. We use the model to predict the counts on the validation dataset. The Rˆ2 score is returned not only for the validation dataset but also for the training dataset. We return the coefficients variance and the model with best validation R2 score as well.

The following function performs the above steps for a given max_degree.

```
In [41]: def poly_fit(max_degree = 8):
             r2_scores_val = []
             r2_scores_train = []
             coefs_var = []
             residuals = []
```

```
    poly_degrees = range(1,max_degree + 1)
    best_model = None
    best_score_dict = {"degree": 1, "score": 0}
    for degree in poly_degrees:
        # build the design matrices
        X_train, y_train, X_val, y_val = get_design_mats(bikes_train, bikes_val, degre
        # fit a regression model on training data
        model = LinearRegression().fit(X_train.values, y_train.values)
        # predict on training data
        y_train_predict_ols = model.predict(X_train.values)
        # predict on validation data
        y_val_predict_ols = model.predict(X_val.values)
        # calculate R2 score
        r2_train = r2_score(y_train.values, y_train_predict_ols)
        r2_scores_train.append(r2_train)
        r2_val = r2_score(y_val.values, y_val_predict_ols)
        r2_scores_val.append(r2_val)
        # calculate coefficients variance
        coefs_var.append(np.var(model.coef_))
        # calculate residuals on validation data
        residuals.append(y_val.values - y_val_predict_ols)
        if r2_val > best_score_dict['score']:
            best_score_dict = dict({"degree": degree, "score": r2_val, "predictors": X
            best_model = model
    df_scores = pd.DataFrame(dict({"poly_degrees": poly_degrees,
                                "r2_scores_val": r2_scores_val,
                                "r2_scores_train": r2_scores_train,
                                "coefs_variance": coefs_var,
                                "residuals": residuals}))
    df_scores.poly_degrees = df_scores.poly_degrees.astype('int')
    return df_scores, best_score_dict, best_model
```

Reporting scores
We run the above function for up to 8 degree.

```
In [42]: df_poly_scores, best_poly_score_info, best_poly_model = poly_fit(8)
```

The model's scores on the validation data is obtained as follows (together with coefficients variance):

```
In [43]: df_poly_scores[['poly_degrees', 'r2_scores_val', 'coefs_variance']]
```

Out[43]:

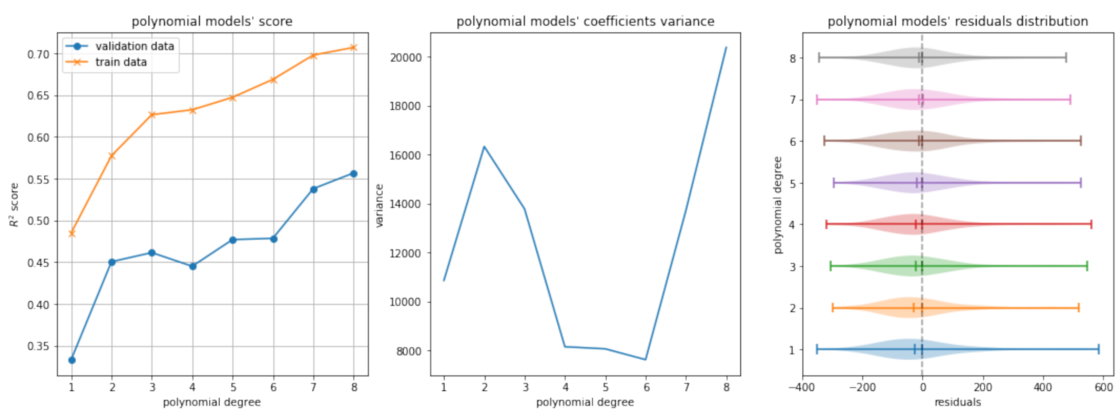
	poly_degrees	r2_scores_val	coefs_variance
0	1	0.333359	10858.122684
1	2	0.450571	16334.719550
2	3	0.461470	13781.201503
3	4	0.445117	8147.802440
4	5	0.477027	8063.536955
5	6	0.478536	7620.035254

6	7	0.537901	13722.642613
7	8	0.556701	20384.061970

We now plot how the R2 score and variance change with increasing model complexity (polynomial order).

```
In [44]: def poly_scores_plot(df):
             fig, ax = plt.subplots(1,3, figsize=(18,6))
             ax[0].plot(df.poly_degrees, df.r2_scores_val, ls='-', marker='o', label = 'valida
             ax[0].plot(df.poly_degrees, df.r2_scores_train, ls='-', marker='x', label = 'trai
             ax[0].set_ylabel('$R^2$ score')
             ax[0].set_xlabel('polynomial degree');
             ax[0].set_title('polynomial models\' score')
             ax[0].legend()
             ax[0].grid()
             ax[1].plot(df.poly_degrees, df.coefs_variance, label = 'normalized variance')
             ax[1].set_ylabel('variance')
             ax[1].set_xlabel('polynomial degree');
             ax[1].set_title('polynomial models\' coefficients variance')
             for degree in range(1, len(df.residuals)+1):
                 ax[2].violinplot(df.residuals[degree-1], positions=[degree],
                           showmeans=True, showmedians=True, showextrema=True, vert=False)
             ax[2].set_ylabel('polynomial degree')
             ax[2].set_xlabel('residuals');
             ax[2].set_title('polynomial models\' residuals distribution')
             ax[2].axvline(x=0, color='black', ls='--', alpha=0.4)
```

```
In [45]: poly_scores_plot(df_poly_scores)
```



2.2 Discuss patterns you see in the results from 2.1. Which model would you select, and why?**

The R2 score on training data increases as new polynomial features are added to the model (up to 8th power). This trend is expected.

The R2 score on validation data follows a similar trend as on training data; except of a decrease of R2 score when the 4th polynomial order is added. We might need to investigate some multicollinearity effect introduced by those polynomials. By adding a 5th order the increasing R2 trend is resumed.

Most residuals for all models are found below zero for all polynomial orders.

Normally we would choose the model that provides the best R2 score 0.556 on validation data. Th R2 score indicates the proportion of variance in the counts of bikes riding that can be explained using the predictors. High values of R2 are signs for good model fitness. Therefore the model with 8 degree of polynomials is the one we would select.

However higher model complexity introduces higher variance in the predicted response as shown in the middle plot above. This means that changing the validation data a little bit would come with wider changes in the predicted response. This reduces the significance of regression coefficients and has negative impact on prediction quality. The polynomial degree 6 has the lowest variance ratio with R2 score 0.478 that is still acceptable. We recall having a R2 score 0.445 in homework 3 using forward features selection on a quadratic polynomial model.

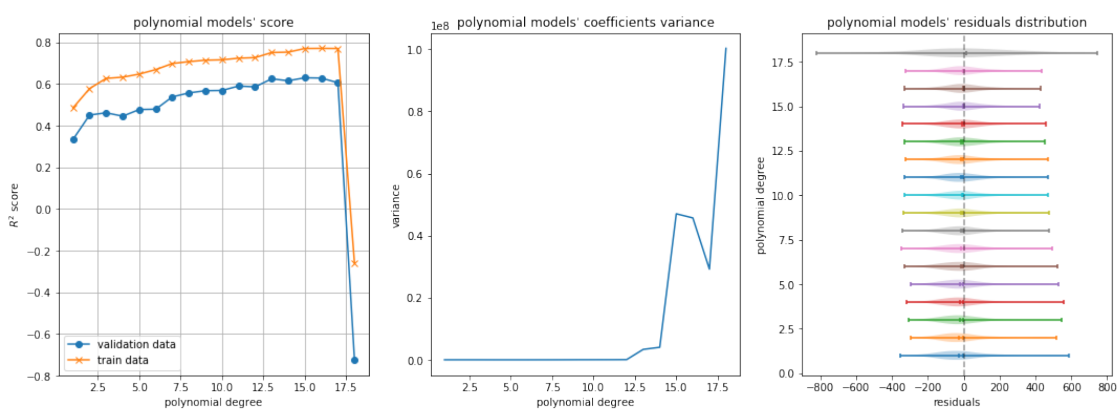
If we investigate further by increasing the degree of polynomials, we obtain the plots below. The residual sum of squares becomes higher than the total variance in the original data with polynomial order 17.

At this moment, we would select the model with polynomial terms up to degree 6 because of it seems to offer the most acceptable prediction error, variance and bias.

```
In [46]: df_poly_scores_18, best_poly_score_info_18, best_poly_model_18 = poly_fit(18)
```

```
In [47]: print("Best R2 score " + str(best_poly_score_info_18['score']) + " obtained with poly
         poly_scores_plot(df_poly_scores_18)
```

```
Best R2 score 0.6294230386344067 obtained with polynomial degree 15 but at the cost of a large
```



2.3 Let's try regularizing our models via ridge regression. Build a table showing the validation set R^2 of polynomial models with degree from 1-8, regularized at the levels $\lambda = (.01, .05, .1, .5, 1, 5, 10, 50, 100)$. Do not perform cross validation at this point, simply report performance on the single validation set.

Model selection with Ridge regularization

Previously we created different linear regression models with polynomial features. We found out that they provide a good R2 score on validation data. However we were not sure about the importance of predictors and the quality of our estimates in general. Variable selection can be performed by looking for the best subset of predictors, this means searching among 2ˆ193 models.

Stepwise forward selection requires less computing effort (18722 possible models). Stepwise backward selection and mixed of both would have a similar computational impact. The best subset of predictors might still miss key predictors, which are significant for the response.

Here we try regularization using a new loss function that augments the MSE with a penalty on large coefficients (except of the intercept). We start with Ridge shrinkage that imposes a constraint on the sum of squares of estimates by shrinking them towards zero. A shrinkage parameter lambda is used to tune the penalty effect. This reduces the variance of those coefficients, what could provide more quality in our predictions and better interpretation of the importance and role of predictors on the response.

The below function gets a polynomial degree and a list of lambdas as parameters. It creates a ridge regression for each of the lambdas, using predictors which are augmented with their polynomial features up to the specified degree. The function returns the R2 scores obtained on the validation set.

```
In [48]: def ridge_fit(max_degree = 8, lambdas = [0, .01, .05, .1, .5, 1, 5, 10, 50, 100]):
             r2_scores_dict = dict()
             coefs_variance_dict = dict()
             residuals_dict = dict()
             poly_degrees = range(1,max_degree + 1)
             r2_scores_dict['poly_degrees'] = poly_degrees
             coefs_variance_dict['poly_degrees'] = poly_degrees
             residuals_dict['poly_degrees'] = poly_degrees
             best_score_dict = {'degree': 0, 'lambda': 0, 'score': 0}
             best_model = None
             for lamb in lambdas:
                 r2_scores_val = []
                 coefs_variance = []
                 residuals = []
                 for degree in poly_degrees:
                     # build the design matrices
                     X_train, y_train, X_val, y_val = get_design_mats(bikes_train, bikes_val, 
                     # fit a regularized regression model on training data
                     ridge = Ridge(alpha = lamb)
                     model = ridge.fit(X_train.values, y_train.values)
                     # predict on validation data
                     y_val_predict_ols = model.predict(X_val.values)
                     # calculate R2 score
                     r2 = r2_score(y_val.values, y_val_predict_ols)
                     r2_scores_val.append(r2)
                     # retrieve coefficients
                     coefs_variance.append(np.var(model.coef_))
                     # calculate residuals
                     residuals.append(y_val.values - y_val_predict_ols)
                     if (r2 > best_score_dict['score']) & (lamb != 0):
                         best_score_dict = {'degree': degree, 'lambda': lamb, 'score': r2, 'pr
                         best_model = model
                 r2_scores_dict[r'$\lambda = ' + str(lamb) + '$'] = r2_scores_val
                 coefs_variance_dict[r'$\lambda = ' + str(lamb) + '$'] = coefs_variance
```

```python
            residuals_dict[r'$\lambda = ' + str(lamb) + '$'] = residuals
        return pd.DataFrame(r2_scores_dict), pd.DataFrame(coefs_variance_dict), pd.DataFra
```

```python
In [49]: df_ridge_scores, df_ridge_coefs_variance, df_ridge_residuals, best_ridge_score_info, l
```

Report Ridge R2 scores

The following table shows the R2 scores obtained when predicting bikes riding with polynomial Ridge regression model on the validation dataset. The models were trained using the training dataset and candidate λs.

The R2 scores are displayed for each value of lambda, the Ridge tuning parameter. We added a lambda=0 for comparison purpose which gives the same results as those obtained with ordinary linear regression (without Ridge).

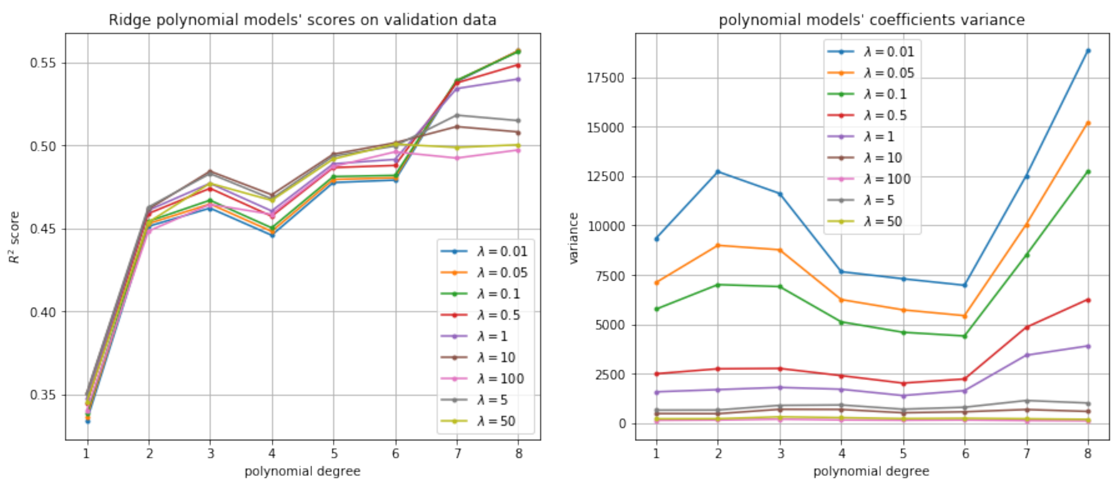
```python
In [50]: df_ridge_scores
```

```
Out[50]:    poly_degrees  $\lambda = 0$  $\lambda = 0.01$  $\lambda = 0.05$  \
         0             1       0.327582          0.334079          0.336303
         1             2       0.446213          0.451156          0.452777
         2             3       0.461159          0.462133          0.464616
         3             4       0.445153          0.445810          0.448139
         4             5       0.477031          0.477541          0.479419
         5             6       0.478533          0.479001          0.480442
         6             7       0.537902          0.538235          0.538780
         7             8       0.556620          0.556880          0.556861

            $\lambda = 0.1$  $\lambda = 0.5$  $\lambda = 1$  $\lambda = 5$  \
         0         0.338252         0.344601        0.347245        0.350852
         1         0.454184         0.458863        0.460860        0.462742
         2         0.466878         0.474055        0.477064        0.482930
         3         0.450257         0.457269        0.460419        0.467707
         4         0.481121         0.486543        0.488813        0.493436
         5         0.481841         0.487837        0.491471        0.499539
         6         0.539033         0.537512        0.534090        0.518093
         7         0.556306         0.548498        0.539885        0.514831

            $\lambda = 10$  $\lambda = 50$  $\lambda = 100$
         0        0.350720        0.345421         0.340400
         1        0.461501        0.453643         0.448234
         2        0.484346        0.477066         0.464374
         3        0.470200        0.466792         0.458405
         4        0.494610        0.491686         0.487166
         5        0.501450        0.500695         0.496005
         6        0.511204        0.498667         0.492277
         7        0.508011        0.500281         0.497137
```

The following function plots the R^2 score on validation data, obtained using a Ridge model for each polynomial degree and lambda combination, as well as the corresponding variance of models coefficients after Ridge schrinkage.

```
In [51]: def ridge_scores_plot(df_scores, df_coefs_variance):
             lambdas = df_scores.columns.difference(['poly_degrees', '$\lambda = 0$']).values
             fig, ax = plt.subplots(1,2, figsize=(15,6))
             for lamb in lambdas:
                 ax[0].plot(df_scores.poly_degrees, df_scores[lamb], ls='-', marker='o', marke
             ax[0].set_ylabel('$R^2$ score')
             ax[0].set_xlabel('polynomial degree');
             ax[0].set_title('Ridge polynomial models\' scores on validation data')
             ax[0].legend()
             ax[0].grid()
             for lamb in lambdas:
                 ax[1].plot(df_coefs_variance.poly_degrees, df_coefs_variance[lamb], ls='-', ma
             ax[1].set_ylabel('variance')
             ax[1].set_xlabel('polynomial degree');
             ax[1].set_title('polynomial models\' coefficients variance')
             ax[1].legend()
             ax[1].grid()
```

```
In [52]: ridge_scores_plot(df_ridge_scores, df_ridge_coefs_variance)
```



When higher values of λ are used, this penalizes high model complexity, causing models with higher polynomial terms to produce a weak R2 score.

2.4 Find the best-scoring degree and regularization combination.

```
In [53]: print("The best-scoring degree appears to be " + str(best_ridge_score_info['degree'])
```

```
The best-scoring degree appears to be 8 with Ridge tuning parameter 0.01. The corresponding re
```

The variance of coefficients obtained with each degree and lambda combination is given in the table below.

```
In [54]: df_ridge_coefs_variance
```

Out[54]:

	poly_degrees	$\lambda = 0$	$\lambda = 0.01$	$\lambda = 0.05$
0	1	5.222566e+30	9363.134729	7122.321011
1	2	3.652825e+30	12718.887049	8994.156287
2	3	7.280216e+28	11618.054508	8769.794417
3	4	9.350728e+25	7661.355726	6252.269933
4	5	4.481185e+25	7304.584351	5733.383863
5	6	7.707326e+23	6975.605831	5440.594851
6	7	2.296147e+22	12506.720923	10042.072200
7	8	7.360081e+21	18831.150135	15203.833136

	$\lambda = 0.1$	$\lambda = 0.5$	$\lambda = 1$	$\lambda = 5$
0	5772.116055	2503.624150	1589.076581	664.420445
1	7008.703487	2752.361189	1699.541380	669.285740
2	6908.175510	2771.296449	1811.546327	903.990796
3	5125.374055	2407.848583	1719.682692	924.209603
4	4599.811538	2029.424269	1402.063655	709.994021
5	4412.093598	2237.038878	1649.032736	807.935927
6	8493.599176	4847.689072	3437.079191	1149.306835
7	12737.220785	6257.953598	3908.009121	1024.328113

	$\lambda = 10$	$\lambda = 50$	$\lambda = 100$
0	490.106225	223.470299	149.419614
1	482.849758	227.107165	163.457885
2	705.026394	325.598688	206.166343
3	697.429542	282.549506	167.378675
4	530.450043	233.330485	153.310133
5	579.042948	250.201631	163.548431
6	693.392328	225.271901	137.307618
7	596.845952	195.441777	122.861194

2.5 It's time to see how well our selected model will do on future data. Read in the provided test dataset `data/bikes_test.csv`, do any required formatting, and report the best model's R^2 score. How does it compare to the validation set score that made us choose this model?

Read test dataset

```
In [55]: bikes_test = pd.read_csv('data/bikes_test.csv', index_col='Unnamed: 0')

         bikes_test.head()
```

Out[55]:

	dteday	hour	year	holiday	workingday	temp	atemp	hum
7955	2011-12-03	3	0	0	0	0.24	0.2424	0.70
113	2011-01-05	22	0	0	1	0.18	0.1970	0.55
701	2011-02-01	14	0	0	1	0.22	0.2576	0.80
12221	2012-05-29	10	1	0	1	0.74	0.6970	0.70
7255	2011-11-03	22	0	0	1	0.40	0.4091	0.82

```
       windspeed  casual  ...   Mon  Tue  Wed  Thu  Fri  Sat  Cloudy  Snow  \
```

```
7955       0.1343       4  ...       0     0     0     0     0     1        0     0
113        0.1343       1  ...       0     0     1     0     0     0        0     0
701        0.0896       5  ...       0     1     0     0     0     0        1     0
12221      0.2985      67  ...       0     1     0     0     0     0        0     0
7255       0.0000      21  ...       0     0     0     1     0     0        0     0

       Storm  month
7955       0     12
113        0      1
701        0      2
12221      0      5
7255       0     11

[5 rows x 36 columns]
```

Create design matrices for test data

We will perform similar data cleaning and preparation on the test dataset as we did for the training and validation dataset. We use the same function get_design_mats for this purposes.

In [56]:
```
X_train_best, y_train_best, X_test_best, y_test_best = get_design_mats(bikes_train, bi
```

Explore the test dataset

In [57]:
```
X_train_best.columns.values
```

Out[57]:
```
array(['hour', 'year', 'holiday', 'temp', 'atemp', 'hum', 'windspeed',
        'Feb', 'Mar', 'Apr', 'May', 'Jun', 'Jul', 'Aug', 'Sept', 'Oct',
        'Nov', 'Dec', 'spring', 'summer', 'fall', 'Mon', 'Tue', 'Wed',
        'Thu', 'Fri', 'Sat', 'Cloudy', 'Snow', 'Storm', 'temp^2', 'temp^3',
        'temp^4', 'temp^5', 'temp^6', 'temp^7', 'temp^8', 'atemp^2',
        'atemp^3', 'atemp^4', 'atemp^5', 'atemp^6', 'atemp^7', 'atemp^8',
        'hum^2', 'hum^3', 'hum^4', 'hum^5', 'hum^6', 'hum^7', 'hum^8',
        'windspeed^2', 'windspeed^3', 'windspeed^4', 'windspeed^5',
        'windspeed^6', 'windspeed^7', 'windspeed^8', 'hour^2', 'hour^3',
        'hour^4', 'hour^5', 'hour^6', 'hour^7', 'hour^8', 'temp*Feb',
        'temp*Mar', 'temp*Apr', 'temp*May', 'temp*Jun', 'temp*Jul',
        'temp*Aug', 'temp*Sept', 'temp*Oct', 'temp*Nov', 'temp*Dec',
        'temp*Mon', 'temp*Tue', 'temp*Wed', 'temp*Thu', 'temp*Fri',
        'temp*Sat', 'atemp*Feb', 'atemp*Mar', 'atemp*Apr', 'atemp*May',
        'atemp*Jun', 'atemp*Jul', 'atemp*Aug', 'atemp*Sept', 'atemp*Oct',
        'atemp*Nov', 'atemp*Dec', 'atemp*Mon', 'atemp*Tue', 'atemp*Wed',
        'atemp*Thu', 'atemp*Fri', 'atemp*Sat', 'hum*Feb', 'hum*Mar',
        'hum*Apr', 'hum*May', 'hum*Jun', 'hum*Jul', 'hum*Aug', 'hum*Sept',
        'hum*Oct', 'hum*Nov', 'hum*Dec', 'hum*Mon', 'hum*Tue', 'hum*Wed',
        'hum*Thu', 'hum*Fri', 'hum*Sat', 'windspeed*Feb', 'windspeed*Mar',
        'windspeed*Apr', 'windspeed*May', 'windspeed*Jun', 'windspeed*Jul',
        'windspeed*Aug', 'windspeed*Sept', 'windspeed*Oct',
        'windspeed*Nov', 'windspeed*Dec', 'windspeed*Mon', 'windspeed*Tue',
        'windspeed*Wed', 'windspeed*Thu', 'windspeed*Fri', 'windspeed*Sat'],
       dtype=object)
```

Predict and get R2 score on test data

The previous investigations identified the best tuning parameter for the Ridge regularization, combined with the best polynomial order. This model is now fit and used on test data below.

```
In [58]: ridge = Ridge(alpha = best_ridge_score_info['lambda'])
         model = ridge.fit(X_train_best.values, y_train_best.values)
```

```
In [59]: y_test_predict_ridge = model.predict(X_test_best.values)
         r2_score_ridge_test = r2_score(y_test_best.values, y_test_predict_ridge)
         print("Test score " + str(r2_score_ridge_test))
         print("Validation score " + str(best_ridge_score_info['score']))
```

```
Test score 0.5668964072043828
Validation score 0.5568797750209155
```

2.6 Why do you think our model's test score was quite a bit worse than its validation score? Does the test set simply contain harder examples, or is something else going on?

It is surpising that the Ridge regression with high variance is performing so well on our test data. The model's test score is a bit higher than its validation score. A reason could be that the distribution of test samples is quite similar to those of the validation set. We observed that after scaling the test data using the mean and std learned from the training data, the std of the test data is close to 1 and their mean is almost zero.

It might be interesting to verify why we are overfitting on the test set. A reason could be that the validation set inculded outliers.

We will perform cross-validation for reducing variability biais, as well as a Lasso regression in order to force some variables to zero and better understand the effect of each variable on the response.

Question 3 [20pts]: Comparing Ridge, Lasso, and OLS

3.1 Build a dataset with polynomial degree 1 and fit an OLS model, a Ridge model, and a Lasso model. Use `RidgeCV` and `LassoCV` to select the best regularization level from among `(.1,.5,1,5,10,50,100)`.

Note: On the lasso model, you will need to increase `max_iter` to 100,000 for the optimization to converge.

3.2 Plot histograms of the coefficients found by each of OLS, ridge, and lasso. What trends do you see in the magnitude of the coefficients?

3.3 The plots above show the overall distribution of coefficient values in each model, but do not show how each model treats individual coefficients. Build a plot which cleanly presents, for each feature in the data, 1) The coefficient assigned by OLS, 2) the coefficient assigned by ridge, and 3) the coefficient assigned by lasso.

Hint: Bar plots are a possible choice, but you are not required to use them

Hint: use `xticks` to label coefficients with their feature names

3.4 What trends do you see in the plot above? How do the three approaches handle the correlated pair `temp` and `atemp`?

2.0.3 Solutions

3.1 Build a dataset with polynomial degree 1 and fit an OLS model, a Ridge model, and a Lasso model. Use `RidgeCV` and `LassoCV` to select the best regularization level from among `(.1,.5,1,5,10,50,100)`.

Note: On the lasso model, you will need to increase `max_iter` to 100,000 for the optimization to converge.

Build a training and validation dataset with polynomial degree 1

We use the get_design_mats function to create a design matrix.

```
In [60]: X_train_1, y_train_1, X_val_1, y_val_1 = get_design_mats(bikes_train, bikes_val, degre
         X_train_1, y_train_1, X_test_1, y_test_1 = get_design_mats(bikes_train, bikes_test, de
```

Fit an OLS model

We fit a linear regression model on the training data without polynomial features of higher order.

```
In [61]: ols_model = LinearRegression().fit(X_train_1.values, y_train_1.values)
         y_val_predicted_ols = ols_model.predict(X_val_1.values)
         r2_score_val_ols = r2_score(y_val_1.values, y_val_predicted_ols)
         y_test_predicted_ols = ols_model.predict(X_test_1.values)
         r2_score_test_ols = r2_score(y_test_1.values, y_test_predicted_ols)
```

Fit a Ridge model

In the previous question we performed regularization on a polynomial multiple linear regression model using Ridge cost function by choosing the tuning parameter and polynomial degree that provided the best R^2 score on validation dataset.

It could however happen by chance that the validation data set was clustered around the regression line of our chosen model. In this case although we got a good validation score, we could experience a much worse score on production.

By using cross-validation we can select the best tuning parameter with the lowest cross-validation error. Instead of just using one single validation set, we split our data in chunks (folds) on approximately equal size. For each value of the tuning parameter, we use one fold as validation set, and the remaining folds as training set. We average the errors and at the end we chose the λ value that provides the lowest mean error.

For this purpose let's use the built-in, automated cross validation feature of sklearn for Ridge regression. Since our data has a time predictor, we shuffle the data explicitely before searching for the optimal λ via cross-validation.

We use a fold size of 20, as recommended by law of thumb in the lecture.

```
In [62]: from sklearn.model_selection import KFold
         alphas_list = (.1,.5,1,5,10,50,100)
         splitter = KFold(n_splits = int(X_train_1.shape[0]/20), random_state=42, shuffle=True)

         ridge_cv_model = RidgeCV(alphas=alphas_list, cv=splitter, normalize=False)
         ridge_cv_model.fit(X_train_1.values, y_train_1.values)

Out[62]: RidgeCV(alphas=(0.1, 0.5, 1, 5, 10, 50, 100),
             cv=KFold(n_splits=50, random_state=42, shuffle=True),
             fit_intercept=True, gcv_mode=None, normalize=False, scoring=None,
             store_cv_values=False)

In [63]: print("The Ridge tuning parameter for obtained via cross-validation is " + str(ridge_
```

```
The Ridge tuning parameter for obtained via cross-validation is 50
```

Now let's fit a multilinear regression model with Ridge regularization on the whole training training set.

```
In [64]: ridge_model = Ridge(alpha = ridge_cv_model.alpha_)
         ridge_model.fit(X_train_1.values, y_train_1.values)
         y_val_predicted_ridge = ridge_model.predict(X_val_1.values)
         r2_score_val_ridge = r2_score(y_val_1.values, y_val_predicted_ridge)
         y_test_predicted_ridge = ridge_model.predict(X_test_1.values)
         r2_score_test_ridge = r2_score(y_test_1.values, y_test_predicted_ridge)
```

Fit a Lasso model

Whereas a Ridge regularization adds a penalty term as sum of sqares of coefficients magnitude, Lasso uses the sum of their absolute values, nulling those with no effect on the response.

We use cross-validation in order to find the best tuning parameter.

```
In [65]: lasso_cv_model = LassoCV(alphas=alphas_list, cv=splitter, max_iter = 100000)
         lasso_cv_model.fit(X_train_1.values, y_train_1.values)
```

```
C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\coordinate_descent.py:1094: Da
  y = column_or_1d(y, warn=True)
```

```
Out[65]: LassoCV(alphas=(0.1, 0.5, 1, 5, 10, 50, 100), copy_X=True,
             cv=KFold(n_splits=50, random_state=42, shuffle=True), eps=0.001,
             fit_intercept=True, max_iter=100000, n_alphas=100, n_jobs=1,
             normalize=False, positive=False, precompute='auto', random_state=None,
             selection='cyclic', tol=0.0001, verbose=False)
```

```
In [66]: print("The Lasso tuning parameter for obtained via cross-validation is " + str(lasso_
```

```
The Lasso tuning parameter for obtained via cross-validation is 0.5
```

Now let's fit a multilinear regression model with Lasso regularization on the whole training training set.

```
In [67]: lasso_model = Lasso(alpha = lasso_cv_model.alpha_)
         lasso_model.fit(X_train_1.values, y_train_1.values)
         y_val_predicted_lasso = lasso_model.predict(X_val_1.values)
         r2_score_val_lasso = r2_score(y_val_1.values, y_val_predicted_lasso)
         y_test_predicted_lasso = lasso_model.predict(X_test_1.values)
         r2_score_test_lasso = r2_score(y_test_1.values, y_test_predicted_lasso)
```

The R2 scores of all three approaches are given below.

```
In [68]: pd.DataFrame({'model': ['osl', 'ridge', 'lasso'],
                       'R2 score on val': [r2_score_val_ols, r2_score_val_ridge, r2_score_val_l
                       'R2 score on test': [r2_score_test_ols, r2_score_test_ridge, r2_score_te
```

Out[68]:

	model	R2 score on val	R2 score on test
0	osl	0.333359	0.349485
1	ridge	0.345421	0.388293
2	lasso	0.343814	0.374765

3.2 Plot histograms of the coefficients found by each of OLS, ridge, and lasso. What trends do you see in the magnitude of the coefficients?

Retrieve model coefficients

We retrieve predictor coefficients from our OLS model

```
In [69]: coefs = pd.DataFrame({'predictors': X_train_1.columns.values, 'ols': ols_model.coef_.r
         coefs.ols = round(coefs.ols.astype('float'), 2)
```

We retrieve predictor coefficients from our Ridge model

```
In [70]: coefs['ridge'] = ridge_model.coef_.ravel()
         coefs.ridge = round(coefs.ridge.astype('float'), 2)
```

We retrieve predictor coefficients from our Lasso model

```
In [71]: coefs['lasso'] = lasso_model.coef_.ravel()
         coefs.lasso = round(coefs.lasso.astype('float'), 2)
```

Let's have a look at the coefficients

```
In [72]: coefs.head()
```

Out[72]:

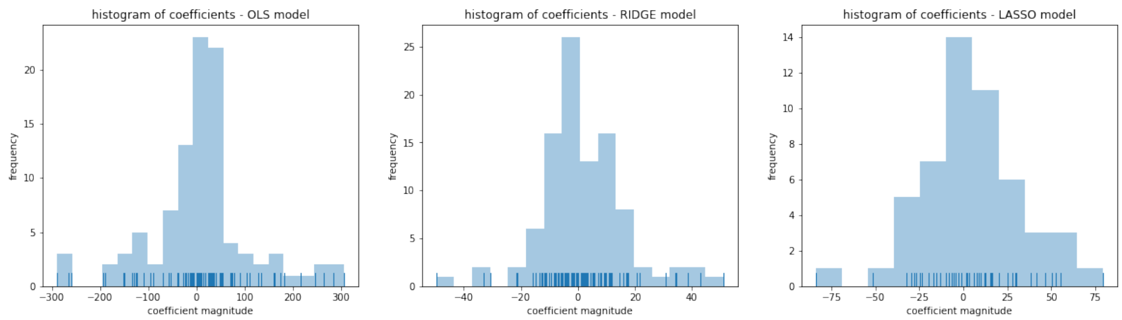
	predictors	ols	ridge	lasso
0	hour	52.90	51.09	52.84
1	year	40.15	38.61	38.49
2	holiday	-12.03	-7.13	-0.00
3	temp	162.09	42.81	79.63
4	atemp	-95.86	34.37	1.63

Coefficients histograms

Below is a distribution plot of all coefficients for each model.

```
In [73]: fig, ax = plt.subplots(1,3,figsize=(20,5))
         sns.distplot(coefs.ols, rug=True, kde=False, norm_hist=False, ax=ax[0])
         ax[0].set_title('histogram of coefficients - OLS model')
         ax[0].set_xlabel('coefficient magnitude')
         ax[0].set_ylabel('frequency')
         sns.distplot(coefs.ridge, rug=True, kde=False, norm_hist=False, ax=ax[1])
         ax[1].set_title('histogram of coefficients - RIDGE model')
         ax[1].set_xlabel('coefficient magnitude')
         ax[1].set_ylabel('frequency')
         sns.distplot(coefs.lasso[coefs.lasso!=0], rug=True, kde=False, norm_hist=False, ax=ax
         ax[2].set_title('histogram of coefficients - LASSO model')
         ax[2].set_xlabel('coefficient magnitude')
         ax[2].set_ylabel('frequency')
         plt.show()
```

```
C:\ProgramData\Anaconda3\lib\site-packages\matplotlib\axes\_axes.py:6462: UserWarning: The 'no
  warnings.warn("The 'normed' kwarg is deprecated, and has been "
C:\ProgramData\Anaconda3\lib\site-packages\matplotlib\axes\_axes.py:6462: UserWarning: The 'no
  warnings.warn("The 'normed' kwarg is deprecated, and has been "
C:\ProgramData\Anaconda3\lib\site-packages\matplotlib\axes\_axes.py:6462: UserWarning: The 'no
  warnings.warn("The 'normed' kwarg is deprecated, and has been "
```



The histograms show smaller magnitude for regularized coefficients. With the OLS model, coefficients are distributed in a wider values range. Ridge and Lasso models have shrinked coefficients with less variance. Lasso caused a lot of coefficients to be zero (we did not show them in the Lasso histogram in order to keep the remaining distribution visible, the list of zero coefficients is given below).

Outliers coefficient are particularly large in the OLS model. Their magnitude is significantly reduced by Ridge and set to zero by Lasso. Most coefficients are found around zero.

The overall variance of the coefficients for each model is given below.

```
In [74]: np.var(coefs.ols), np.var(coefs.ridge), np.var(coefs.lasso)

Out[74]: (10858.067095835073, 223.46875107246984, 422.9162923469388)
```

The following coefficients were removed by Lasso regression. The corresponding predictors or interaction terms have very little or no effect on the number of bike rides.

```
In [75]: coefs[coefs.lasso==0].predictors.values

Out[75]: array(['holiday', 'windspeed', 'Feb', 'May', 'Jul', 'Aug', 'Nov', 'Dec',
               'summer', 'Mon', 'Fri', 'Sat', 'Storm', 'temp*Feb', 'temp*Mar',
               'temp*Apr', 'temp*May', 'temp*Jun', 'temp*Jul', 'temp*Aug',
               'temp*Oct', 'temp*Nov', 'temp*Fri', 'temp*Sat', 'atemp*Apr',
               'atemp*Jun', 'atemp*Aug', 'atemp*Sept', 'atemp*Oct', 'atemp*Nov',
               'atemp*Dec', 'atemp*Mon', 'atemp*Tue', 'atemp*Wed', 'atemp*Thu',
               'hum*Mar', 'hum*Jun', 'hum*Jul', 'hum*Dec', 'hum*Fri',
               'windspeed*Feb', 'windspeed*Apr', 'windspeed*Dec', 'windspeed*Tue',
               'windspeed*Wed', 'windspeed*Fri'], dtype=object)
```

3.3 The plots above show the overall distribution of coefficient values in each model, but do not show how each model treats individual coefficients. Build a plot which cleanly presents, for each feature in the data, 1) The coefficient assigned by OLS, 2) the coefficient assigned by ridge, and 3) the coefficient assigned by lasso.

Hint: Bar plots are a possible choice, but you are not required to use them

Hint: use `xticks` to label coefficients with their feature names

We first sort the coefficients by overall magnitude.

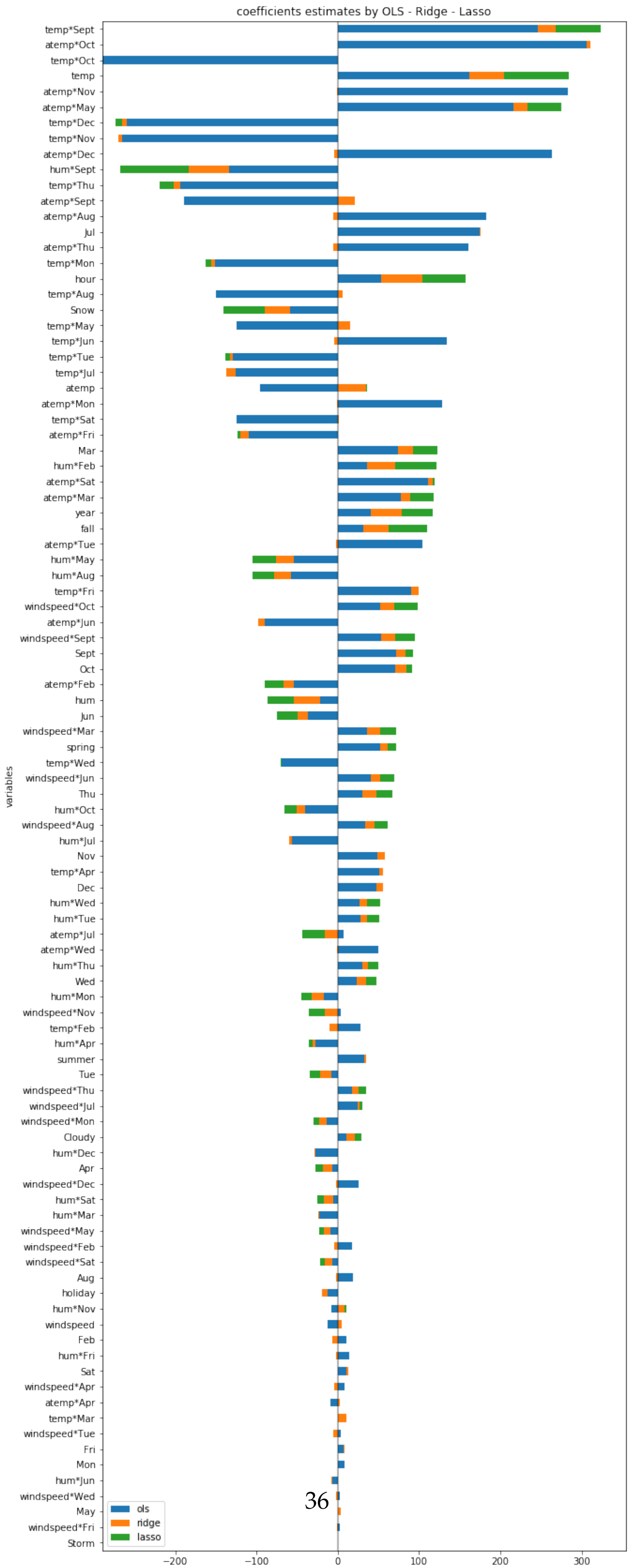
In [76]:
```
coefs['all'] = abs(coefs.ols) + abs(coefs.ridge) + abs(coefs.lasso)
coefs = coefs.sort_values(by = ['all'])
```

The stacked barplot below shows the magnitude of the coefficients associated with the predictors. The predictors with the cummulative largest coefficient magnitude are displayed at the beginning of the plot. The bar shows the magnitude obtained with each of the three models (OLS - blue, Ridge - orange, Lasso - green).

Bars oriented to the left identify those predictors which have a negative effect on the predicted count of ridership. The remaining ones have a positive effect. A step increase of the later would imply a step increase in the count by the magnitude of the coefficient.

Lasso provides the best interpretability. For example humidity has a considerable impact on ridership during certain months (Sept). High temperature brings definetely more ridership. It is strange that Storm has no effect on the ridership. This is probably because of the lack of data during stormy weather.

In [77]:
```
coefs.plot.barh(x='predictors', y=['ols', 'ridge', 'lasso'], stacked=True, legend=Tru
plt.axvline(x=0, color='black', alpha=0.4)
plt.xlabel('magnitude')
plt.ylabel('variables')
plt.title('coefficients estimates by OLS - Ridge - Lasso')
plt.show()
```


coefficients estimates by OLS - Ridge - Lasso
variables
temp*Sept
atemp*Oct
temp*Oct
temp
atemp*Nov
atemp*May
temp*Dec
temp*Nov
atemp*Dec
hum*Sept
temp*Thu
atemp*Sept
atemp*Aug
Jul
atemp*Thu
temp*Mon
hour
temp*Aug
Snow
temp*May
temp*Jun
temp*Tue
temp*Jul
atemp
atemp*Mon
temp*Sat
atemp*Fri
Mar
hum*Feb
atemp*Sat
atemp*Mar
year
fall
atemp*Tue
hum*May
hum*Aug
temp*Fri
windspeed*Oct
atemp*Jun
windspeed*Sept
Sept
Oct
atemp*Feb
hum
Jun
windspeed*Mar
spring
temp*Wed
windspeed*Jun
Thu
hum*Oct
windspeed*Aug
hum*Jul
Nov
temp*Apr
Dec
hum*Wed
hum*Tue
atemp*Jul
atemp*Wed
hum*Thu
Wed
hum*Mon
windspeed*Nov
temp*Feb
hum*Apr
summer
Tue
windspeed*Thu
windspeed*Jul
windspeed*Mon
Cloudy
hum*Dec
Apr
windspeed*Dec
hum*Sat
hum*Mar
windspeed*May
windspeed*Feb
windspeed*Sat
Aug
holiday
hum*Nov
windspeed
Feb
hum*Fri
Sat
windspeed*Apr
atemp*Apr
temp*Mar
windspeed*Tue
Fri
Mon
hum*Jun
windspeed*Wed
May
windspeed*Fri
Storm
ols
ridge
lasso
−200
−100
0
100
200
300
magnitude

3.4 What trends do you see in the plot above? How do the three approaches handle the correlated pair `temp` and `atemp`?

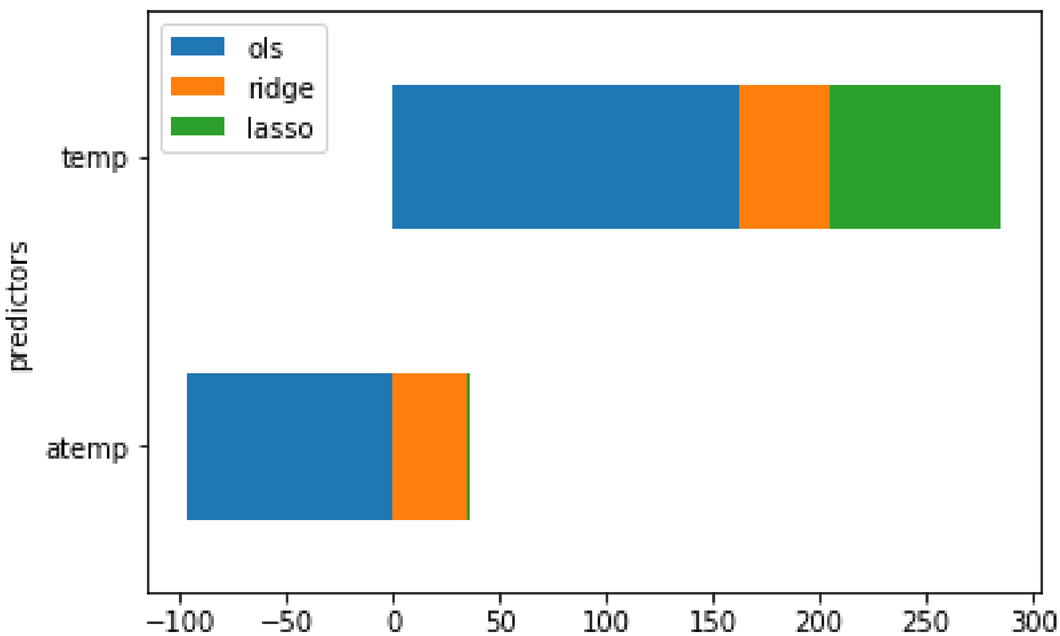
When the coefficient is large, the sign of coefficients appears to be the same regardless the approach used. The coefficients which were set to zero by Lasso tend to be handled differently by Ridge and OLS when it comes to their sign.

OLS handles temp as a positive predictor of ridership, and atemp as a negative predictore of ridership. This is strange because both predictors are positively correlated. We would expect temp and atemp to behave in the same direction. This is not the case for Lasso and Ridge approaches, where both temp and atemp predict an increase in ridership.

Ridge assumes that both temp and atemp would have approximately the same impact on the response. Lasso however seems to have detected the collinearity between both and tried to shrink atemp towards zero.

```
In [78]: coefs[(coefs.predictors=='temp')|(coefs.predictors=='atemp')].plot.barh(x='predictors
```

```
Out[78]: <matplotlib.axes._subplots.AxesSubplot at 0x1e7f49ff278>
```



Question 4 [20 pts]: Reflection

These problems are open-ended, and you are not expected to write more than 2-3 sentences. We are interested in seeing that you have thought about these issues; you will be graded on how well you justify your conclusions here, not on what you conclude.

4.1 Reflect back on the `get_design_mats` function you built. Writing this function useful in your analysis? What issues might you have encountered if you copy/pasted the model-building code instead of tying it together in a function? Does a `get_design_mat` function seem wise in general, or are there better options?

The get_design_mats function was useful for better code visibility, debugging, it promoted less lines of code, allowing to reuse the same code logic through the questions. If the data structure is the same throughout the project, then using such a function is wise for exploring various polynomial and regularized models. An improvement of get_design_mats could be adding a flag for enabling/disabling the creation of interaction terms, and also providing the list of predictors that could interact as a parameter to the function. get_design_mats could also handle exceptions.

4.2 What are the costs and benefits of applying ridge/lasso regularization to an overfit OLS model, versus setting a specific degree of polynomial or forward selecting features for the model?

Adding polynomials in OLS improves the validation score while increasing prediction variance and risk of overfitting; Ridge/Lasso reduces the variance. Polynomial terms bring multicollinearity issues with them, what negatively affects interpretability; by shrinking the coefficients of less important predictors, Ridge/Lasso helps avoiding that issue. Forward selection identifies the subset of predictors which give the less error, but this approach can miss important predictors. Lasso can help in the special case of high dimensionality where there are more predictors than observations. The down side is, Ridge/Lasso comes at the cost of computation power especcially when the data set is very big.

** 4.3** This pset posed a purely predictive goal: forecast ridership as accurately as possible. How important is interpretability in this context? Considering, e.g., your lasso and ridge models from Question 3, how would you react if the models predicted well, but the coefficient values didn't make sense once interpreted?

Although prediction was more important than interpretability in this context, having for example two strange coefficients for temp and atemp, or storm should also be considered closely, as they could affect prediction on another test dataset. Inference should be ideally investigated even if the main goal is prediction.

4.4 Reflect back on our original goal of helping BikeShare predict what demand will be like in the week ahead, and thus how many bikes they can bring in for maintenance. In your view, did we accomplish this goal? If yes, which model would you put into production and why? If not, which model came closest, what other analyses might you conduct, and how likely do you think they are to work

For any given month, day of week, weather forecast and hour of the day, our models are able to predict the number of rides that will be made by both registered and casual users with an acceptable R2 score (see summary table below). However without a proper analysis of coefficients variance, bias, residuals and coefficients p-values/CI, we think that the prediction goal was not fully achieved.

We suggest a model obtained with interaction terms, polynomial model with degree 6 regularized with Lasso using a tuning parameter obtained via cross-validation. This model would predict with R2 score around 0.478, would bring low coefficients variance, and would consider only significant predictors.

```
In [79]: pd.DataFrame({"Model": ['original', 'polynomial degree 2', 'polynomial degree 2 with 
```

Out[79]:

	Model	Test R2
0	original	0.406
1	polynomial degree 2	0.496
2	polynomial degree 2 with forward selection	0.445
3	polynomial degree 6 with int. terms	0.478
4	multilinear polynomia degree 8 with inter. terms	0.556
5	polynomial degree 8 with Ridge lamba 0.1	0.556

```
6             polynomial degree 1 with inter. terms    0.349
7   polynomial degree 1 with inter. terms and Ridge    0.388
8   polynomial degree 1 with inter. terms and Lasso    0.383
```

your answer here