AMC10 2019 A20

The numbers $1, 2, ..., 9$ are randomly placed into the 9 squares of a $3x3$ grid. Each square gets one number, and each of the numbers is used once. What is the probability that the sum of the numbers in each row and each column is odd?

A. $\frac{1}{21}$ B. $\frac{1}{14}$ C. $\frac{5}{63}$ D. $\frac{2}{21}$ E. $\frac{1}{7}$

Odd sums can only be formed in two patterns: (e, e, o) or (o, o, o). Looking at possible placements of the es, we can find 9 patterns that work:

- 4 arranged in a square (4 arrangements, with one square in each corner)

e	e	
e	e	

- 2 across from 2, with a space in between (4 arrangements)

e	e	
e	e	

- 4 in each corner (1 arrangement)

e		e
e		e

Therefore, there are $9 \times 4! \times 5!$ arrangements that satisfy the conditions. The probability is $\frac{9 \times 4! \times 5!}{9!}$, which is $\frac{1}{14}$. B $\left(\frac{1}{14}\right)$.