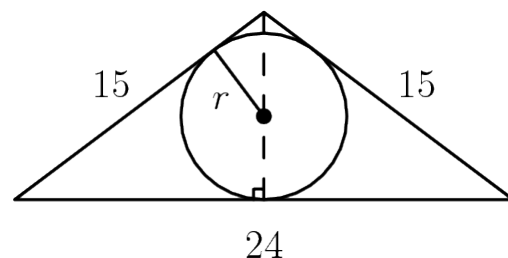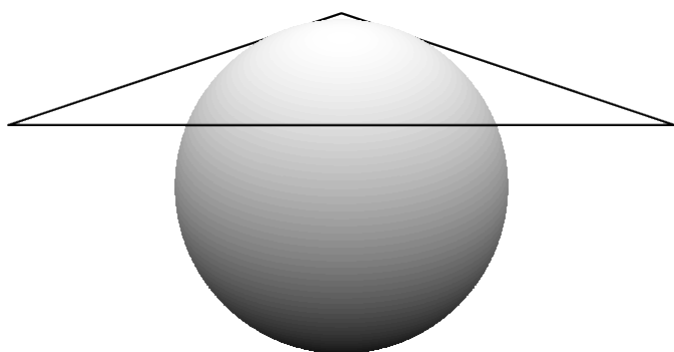# AMC10 2019 A21

A sphere with center O has radius 6. A triangle with sides of length $15, 15$, and 24 is situated in space so that each of its sides is tangent to the sphere. What is the distance between O and the plane determined by the triangle?

A. $2\sqrt{3}$ B. 4   C. $3\sqrt{2}$ D. $2\sqrt{5}$ E. 5

The two triangles in figure 2 are 3-4-5 right triangles scaled up by 3 (9-12-15), so the height is 9. Using triangle similarity, we get $\frac{r}{9-r} = \frac{4}{5}$, so $r = 4$.

Let O be the center of the sphere, let C be the center of the circle that lies on the triangle's plane (the one with radius 4), and let X be the tangency point of the sphere and any side of the triangle. $\triangle OCX$ is a right triangle with hypotenuse length 6 and one side of length 4. Solving for the other side, we get $\sqrt{6^2 - 4^2} = \sqrt{20} = $ D ($2\sqrt{5}$).