AMC10 2019 A19

What is the least possible value of

$$(x + 1)(x + 2)(x + 3)(x + 4) + 2019$$

where x is a real number?

A. 2017   B. 2018   C. 2019   D. 2020   E. 2021

Since $(x + 1)(x + 2)(x + 3)(x + 4)$ can be equal to 0, we can rule out D and E.

$(x+1)(x+4)(x+2)(x+3) = (x^2+5x+4)(x^2+5x+6)$. If we set $y = (x^2+5x+5)$, the equation can be rewritten as $(y - 1)(y + 1) + 2019 = y^2 + 2018$. Since y^2 is positive, $y = 0$ gives us the minimum value. We can see that $x^2 + 5x + 5 = 0$ has real roots, therefore, y can be equal to 0 and our answer is B (2018).