ECE 901 – Nanophotonics
Problem Set 3
Due: November 16, 2020

<u>**Notes:**</u>

(a) You are encouraged to discuss the homework with other students in the class, and you can work together. However, all of your code must be your own, all of the figures must be your own, and all of the writeup must be your own.

(b) Your writeup should be typed up, including all necessary equations using a proper math font, and all figures appropriately labeled and captioned. I recommend using either MS Word (with its built-in equation editor) or Latex to prepare your writeups, though you are free to use whatever you prefer.

  a. No "one problem exception" like last time: please type up the entire solution set, since this one is shorter

(c) Your homework writeup must be in pdf format, with your code in the text (at the end, or throughout the writeup).

  - Your neat and properly commented working code should also be uploaded as well (e.g., working .m files if written in Matlab)

(d) All plots should be clearly labeled (axis labels, units, etc.), and should be computer generated. Figures should be drawn using some sort of figure-making software (PowerPoint, Illustrator, etc.), though neat touch-screen-drawn sketches may be acceptable.

(e) If you used data from a reference (something from a publication, internet database, etc.), the reference should be given in your writeup.

<u>Problem 1</u>:

(4 pts) Using the Fresnel equations and Snell's law, derive the expression for the Brewster angle θ_B, which is the angle for which there is no p-polarized reflection.

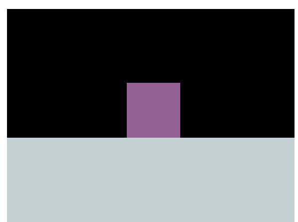
<u>Problem 2:</u>

(5 pts) Using the basic fabrication processes outlined in Lecture, draw and label the steps to make the coupled waveguide shown above, consisting of two ribs with two different heights. Start with an SOI wafer, and label every step (metal deposition, resist spinning, exposure through a mask, developing of resist, metal etching, oxygen plasma, reactive ion etching of silicon).

Since we did not discuss the details of the anti-reflective coating (ARC) in class, you do not have to include it in the process. It is often not used or necessary anyway, in part because many traditional lithography tools are not particularly narrowband.

Problem 3:

(a) (2 pts) Using the built-in mode solver in Lumerical, calculate the effective index $n_{eff} = \beta/k_0$ of the bound modes of a waveguide made up of gallium arsenide (GaAs) with a square cross section, 300 nm x 300 nm, floating in free space, at a free space wavelength of 1.55 µm. Note that there are two degenerate modes because of the symmetry of the geometry.
(b) (2 pt) Make a color plot of the intensity of the total electric field in this mode as a function of position. Make sure your resolution is good enough so the plot is not too grainy.
(c) (2 pts) Plot the dispersion relation (ω vs β) for this fundamental mode, covering the range of free space wavelengths λ_0 from $1\ \mu\text{m}$ to $1.7\ \mu\text{m}$. Use at least 8-10 data points to make the plot.
(d) (2 pts) Now plot the group velocity as a function of ω.
(e) (2 pts) Now put that waveguide directly on top of a large substrate made up of silicon dioxide (SiO_2) and calculate the modes at $\lambda_0 = 1.55\ \mu\text{m}$. For clarity, the geometry is shown in the figure below. Find n_{eff} of the two fundamental modes. Did n_{eff} increase or decrease? Explain why this makes sense.



Problem 4

Assume a parallel plate waveguide is made of two PEC plates, with spacing d, and refractive index n = 2 + 0.001i in between the plates. Use the Helmholtz equation we found in class to find the even modes (*i.e.*, the modes where the instantaneous electric field is symmetric about the mid-point of the waveguide) only. The problem is readily solved by guessing a reasonable solution, and working out the Helmholtz equation.

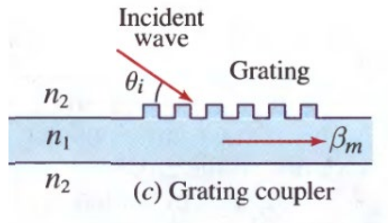
Find:

(a) (3 pt) The propagation constant β in terms of ω, d, and any fundamental constants of nature you need

(b) (2 pt) The field profile $E(y, \omega, t)$, where y is the direction that cuts across the waveguide

(c) (2 pt) Assume $d = 2\ \mu\text{m}$ and the frequency corresponds to a free-space wavelength of $1\ \mu\text{m}$. For the lowest-order mode, calculate what % of the initial intensity is left after propagation for 1 cm.

Problem 5:



In lecture, we found that a grating with wavelength Λ can be thought of as having a wave-vector with magnitude k_g, which facilitates coupling. This coupling can be between an incident plane wave and a waveguide mode, like in the lecture, or between two waveguide modes, or between two plane waves.

(a) (3 pts) Given a wave inside of a waveguide propagating to the right with some effective index n_{eff}, what value of Λ would couple that wave to a waveguide mode with the same n_{eff}, but propagating to the left? Write your answer in terms of the free space wavelength λ_0 and n_{eff}.

(b) (2 pts) A slab waveguide is infinite in the third dimension, and consists of a 200 nm thick slab of silicon sitting on top of a semi-infinite silicon oxide substrate (see picture below). Using either analytical techniques or the Lumerical mode solver, calculate the effective index and mode profile of the fundamental mode for $\lambda_0 = 1.55\ \mu m$.

(c) (3 pts) Set up a 2D simulation in Lumerical where you inject the fundamental mode into the waveguide and allow the light to propagate for 6 microns. Terminate the simulation with absorbing boundary conditions. Set up profile monitors before and after the source so that you can measure the efficiency with which the light is transmitted from one end of the waveguide to another (transmission monitor), and the efficiency with which light is reflected (reflection monitor). Make a plot of the transmission and reflection as a function of free space wavelength, where the source λ_0 ranges from $1.3\ \mu m$ to $1.7\mu m$. You can use the same mode profile you calculated for $\lambda_0 = 1.55\ \mu m$ because the mode profile does not change too much across that wavelength range. You should expect the transmission to be very high and the reflection very low because you are just sending a waveguide mode down the waveguide with no obstacles.

(d) (3 pts) Now introduce a grating into the waveguide, made of 10 silicon ribs that stick out of the waveguide. Let the size of each rib be 60 nm x 60 nm. Set the grating period to the value you calculated in (a). Like in (c), make a plot of the transmission and reflection as a function of λ_0. Comment on the result given what you found in part (a).

Note: to aid you in parts (c) and (d) of this problem, here is a screenshot of my setup: