ECE747:Nanophotonics

University of Wisconsin - Madison
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Problem Set 1

Sanket Deshpande

All calculations and figures (except Prob. 3(b)) have been obtained using MATLAB. The scripts for all relevant questions is appended at the end of the document.

Problem 1:

a. To prove: $\nabla \times \mathbf{E} = \iota \mathbf{k} \times \mathbf{E}$
The plane wave equation is given as:

$$\mathbf{E} = \mathbf{E_o} e^{\iota(\mathbf{k}.\mathbf{r} - \omega t)}$$

Let's define the Cartesian coordinate system such that the wave propagates in the z-direction, such that the wave vector can be represented as $\mathbf{k} = k\ \hat{z}$. Hence, we can rewrite the Electric Field vector as:

$$\mathbf{E} = E_{o,x} e^{\iota(kz-\omega t)}\ \hat{x} + E_{o,y} e^{\iota(kz-\omega t)}\ \hat{y}$$

We can determine the curl of $\mathbf{E}$ as:

$$\nabla \times \mathbf{E} = \frac{\partial}{\partial z} E_{o,x} e^{\iota(kz-\omega t)}\ \hat{y}\ -\ \frac{\partial}{\partial z} E_{o,y} e^{\iota(kz-\omega t)}\ \hat{x}$$
$$\nabla \times \mathbf{E} = \iota k E_{o,x} e^{\iota(kz-\omega t)}\ \hat{y}\ -\ \iota k E_{o,y} e^{\iota(kz-\omega t)}\ \hat{x}$$

$$\nabla \times \mathbf{E} = \iota k [E_{o,x} e^{\iota(kz-\omega t)}\ \hat{y}\ -\ E_{o,y} e^{\iota(kz-\omega t)}\ \hat{x}] \tag{1}$$

The cross product of $\mathbf{k}$ and $\mathbf{E}$ can be determined as:

$$\mathbf{k} \times \mathbf{E} = k E_{o,x} e^{\iota(kz-\omega t)}\ \hat{y}\ -\ k E_{o,y} e^{\iota(kz-\omega t)}\ \hat{x}$$

$$\iota \mathbf{k} \times \mathbf{E} = \iota k [E_{o,x} e^{\iota(kz-\omega t)}\ \hat{y}\ -\ E_{o,y} e^{\iota(kz-\omega t)}\ \hat{x}] \tag{2}$$

From (1) and (2), proved that $\nabla \times \mathbf{E} = \iota \mathbf{k} \times \mathbf{E}$

b. The divergence of $\mathbf{E}$ can be written as:

$$\nabla . \mathbf{E} = \frac{\partial}{\partial x} E_{o,x} e^{\iota(kz-\omega t)}\ +\ \frac{\partial}{\partial y} E_{o,y} e^{\iota(kz-\omega t)}\ +\ \frac{\partial}{\partial z} E_{o,z} e^{\iota(kz-\omega t)}$$
$$\nabla . \mathbf{E} = 0\ +\ 0\ +\ \iota k E_{o,z} e^{\iota(kz-\omega t)}$$
$$\nabla . \mathbf{E} = \iota \mathbf{k} . \mathbf{E}$$

Note that here we are assuming that the amplitude terms ($E_{o,x}$, $E_{o,y}$ & $E_{o,z}$) are constants.

Problem 2:

a. To calculate the force (per unit length) between the wires, we need to calculate the Electric Field due to one wire at the other wire - by applying Gauss Law.
Consider a Gaussian cylinder with radius equal to the separation d between the wires, length equal to L, and such that its axis is placed along the length of the upper wire. The following figure illustrates further and establishes the coordinate system.

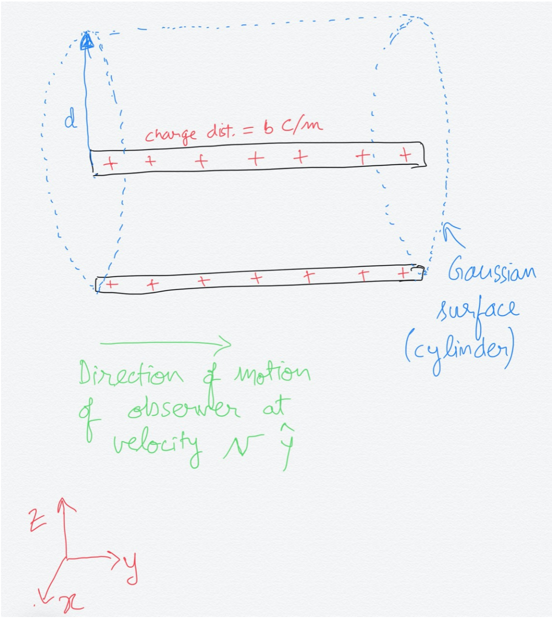

Figure 1: Forming the Gaussian surface, and describing the coordinate system

Applying Gauss Law for the flux coming out of the side of this cylinder:

$$E(2\pi dL) = \frac{Q_{enc}}{\epsilon_o}$$
$$E(2\pi dL) = \frac{bL}{\epsilon_o}$$
$$E = \frac{b}{2\pi\epsilon_o d}$$

On the bottom wire, the electric field can be denoted as:

$$\mathbf{E} = -\frac{b}{2\pi\epsilon_o d}\,\hat{z}$$

Hence, the force per unit length can be denoted as:

$$\mathbf{F}_{bottom} = -\frac{b^2}{2\pi\epsilon_o d}\,\hat{z}$$
$$And,\ similarly\ \mathbf{F}_{top} = \frac{b^2}{2\pi\epsilon_o d}\,\hat{z}$$

b. The observer starts moving with a velocity v in the $+y$ direction. Hence, they perceive current, I flowing through the wires:

$$\mathbf{I} = \frac{charge}{unit\ time} = \frac{\frac{charge}{unit\ length}}{\frac{unit\ time}{unit\ length}} = charge\ per\ unit\ length * velocity\ (-\hat{y})$$

$$\mathbf{I} = -bv\,\hat{y}$$

The magnetic field due to the upper wire on the bottom wire can be determined by using Ampere's law. A loop can be made around the upper wire, with radius equal to d.

$$B(2\pi d) = \mu_o bv$$

$$B = \frac{\mu_o bv}{2\pi d}$$

The direction of the magnetic field is given by the "Right Hand Thumb Rule". Hence,

$$\mathbf{B} = \frac{\mu_o bv}{2\pi d}\,\hat{x}$$

For the observer, the bottom wire has the velocity:

$$\mathbf{v} = -v\,\hat{y}$$

The magnetic force (per unit length) on the bottom wire can be calculated using Lorentz Force Law:

$$\mathbf{F}_{magnetic,bottom} = b(\mathbf{v} \times \mathbf{B})$$

$$\mathbf{F}_{magnetic,bottom} = \frac{\mu_o b^2 v^2}{2\pi d}\,\hat{z}$$

$$And,\ similarly\ \mathbf{F}_{magnetic,top} = -\frac{\mu_o b^2 v^2}{2\pi d}\,\hat{z}$$

c. If the observer is moving at relativistic speeds, they will observer a different charge per unit length, b', because the "unit length" contracts. But, the net force between both the wires (or, acting on each wire) should remain the same for an observer at rest and an observer moving as described in part (b). In this part, all forces calculated are for the bottom wire.
For the stationary observer, the net force on the wire is:

$$\mathbf{F}_{stationary} = -\frac{b^2}{2\pi\epsilon_o d}\,\hat{z}$$

For the observer moving as described in part (b), the net force on the wire is:

$$\mathbf{F}_{motion} = -\frac{b'^2}{2\pi\epsilon_o d}\,\hat{z} + \frac{\mu_o b'^2 v^2}{2\pi d}\,\hat{z}$$

Since the net force on the wire should be the same for both the observers,

$$\mathbf{F}_{stationary} = \mathbf{F}_{motion}$$
$$-\frac{b^2}{2\pi\epsilon_o d} = -\frac{b'^2}{2\pi\epsilon_o d} + \frac{\mu_o b'^2 v^2}{2\pi d}$$
$$\frac{b}{b'} = \sqrt{1 - \epsilon_o\mu_o v^2}$$
$$Note\ that:\ c = \sqrt{\frac{1}{\mu_o\epsilon_o}}$$

Since the charge remains the same, unit lengths for stationary and moving observers can be denoted as l and l' respectively. Therefore, we have:

$$\frac{l'}{l} = \sqrt{1 - \frac{v^2}{c^2}}$$

Hence, the length contracts by a factor of $\sqrt{1 - \frac{v^2}{c^2}}$.

Problem 3:

a. Light with frequency, $f = 300\,THz$ propagates through air and silica glass (SiO_2)

The angular frequencies can be denoted as:

$$\omega_{air} = 2\pi f$$
$$\omega_{SiO_2} = 2\pi f$$

The wavelengths can be denoted as:

$$\lambda_{air} = \frac{c}{f}$$

$$\lambda_{SiO_2} = \frac{\lambda_{air}}{n_{SiO_2(@300THz)}}$$

The wave numbers can be denoted as:

$$k_{air} = \frac{2\pi}{\lambda_{air}}$$

$$k_{SiO_2} = \frac{2\pi}{\lambda_{SiO_2}}$$

The photon energies can be denoted as:

$$E_{ph,air} = hf$$

$$E_{ph,SiO_2} = hf$$

Material	$\boldsymbol{\omega}$ $[rads^{-1}]$	$\boldsymbol{\lambda}$ [μm]	**k** $[m^{-1}]$	$\boldsymbol{E_{ph}}$ [eV]
Air	1.885e+15	1	6.2832e+06	1.2406
Silica Glass	1.885e+15	0.689	9.1131e+06	1.2406

b. In the dispersion relation, the angular frequency, ω is the same for both the mediums, but the wave number, k varies for the mediums.
The relation is derived using the refractive index dataset available in the book,"Malitson, 1965" and obtained from this website.
The following dispersion relations (for both media) are obtained for frequencies ranging from 300 THz to 600 THz:

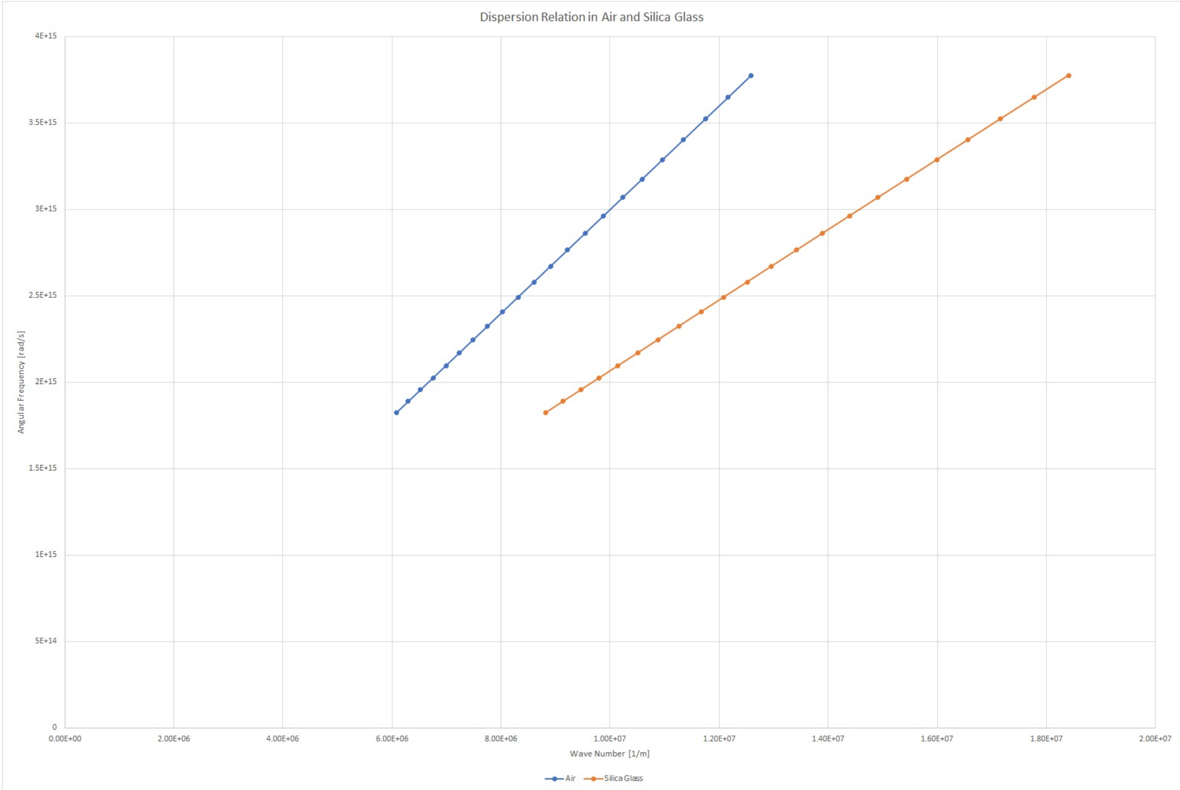

Figure 2: Dispersion relations for frequencies ranging from 300 THz to 600 THz:

c. The plane wave equation is given as:

$$\widetilde{\mathbf{E}} = \widetilde{\mathbf{A}} e^{\iota(\mathbf{kz} - \omega t)}\,\hat{x}$$

And,

$$\widetilde{\mathbf{A}} = 1 + \iota$$

Hence, we can solve this equation as:

$$\widetilde{\mathbf{E}} = (1+\iota)[\cos(kz-\omega t) + \iota\sin(kz-\omega t)]\,\hat{x}$$
$$\widetilde{\mathbf{E}} = [\cos(kz-\omega t) - \sin(kz-\omega t)] + \iota[\cos(kz-\omega t) + \sin(kz-\omega t)]\,\hat{x}$$
$$Re[\widetilde{\mathbf{E}}] = \cos(kz-\omega t) - \sin(kz-\omega t)\,\hat{x}$$

The wavenumber, k, at f=300 THz can be calculated (in air) as:

$$k = \frac{2\pi f}{c} = 6.2832e+06\,m^{-1}$$

At t=0, the wave equation to plot can be denoted as:

$$Re[\widetilde{\mathbf{E}}] = \cos(6.2832e+06\,z) - \sin(6.2832e+06\,z)\,\hat{x}$$

The following plot is obtained for this E-field over a range of, $z = -2\mu m$ to $z = 2\mu m$

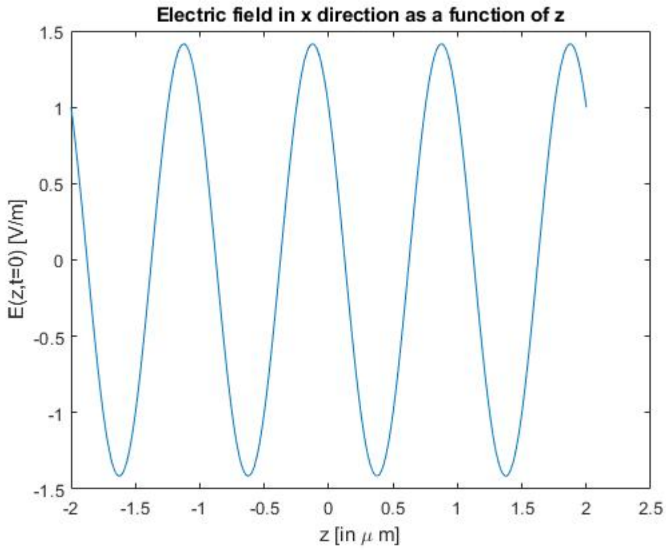

Figure 3: Electric Field of the given plane wave at t=0.

d. The waves can be shown to be moving in opposite direction, by extracting their real part (like in Q3 (c)) and plotting them at various instances.

i. The E-field, $\widetilde{\mathbf{E_1}} = \widetilde{\mathbf{A}} e^{\iota(\mathbf{kz}-\omega t)}\,\hat{x}$, is a right moving wave. This can be seen in the following plot where the wave can be observed to be moving in the right direction, as time progresses.

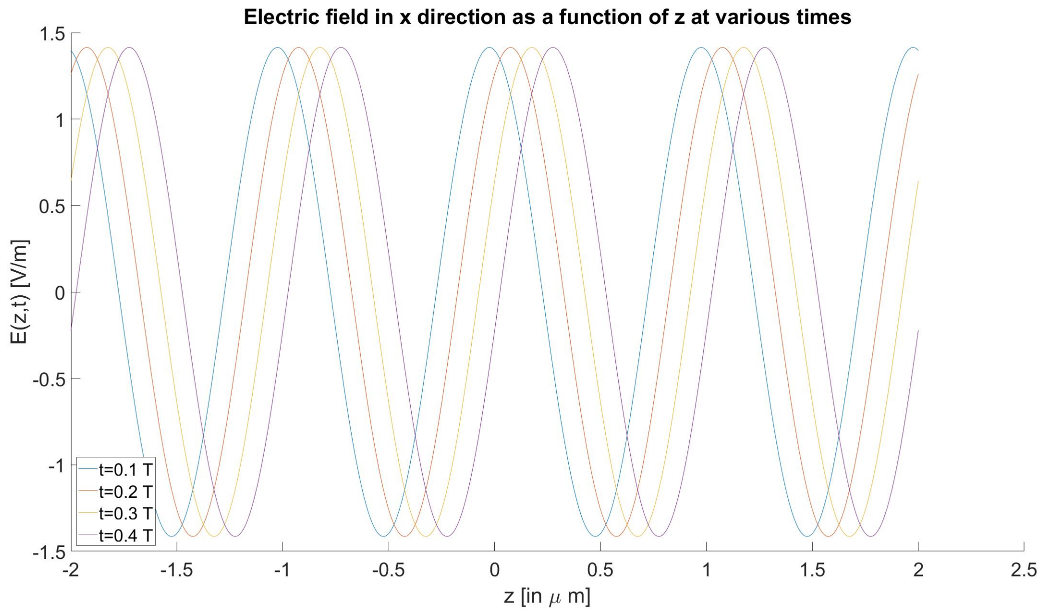

Figure 4: E_1(z,t) can be observed to be a right moving wave

ii. The E-field, $\widetilde{\mathbf{E_2}} = \widetilde{\mathbf{A}} e^{\iota(-\mathbf{kz}-\omega t)}\,\hat{x}$, is a left moving wave. This can be seen in the following plot where the wave can be observed to be moving in the left direction, as time progresses.

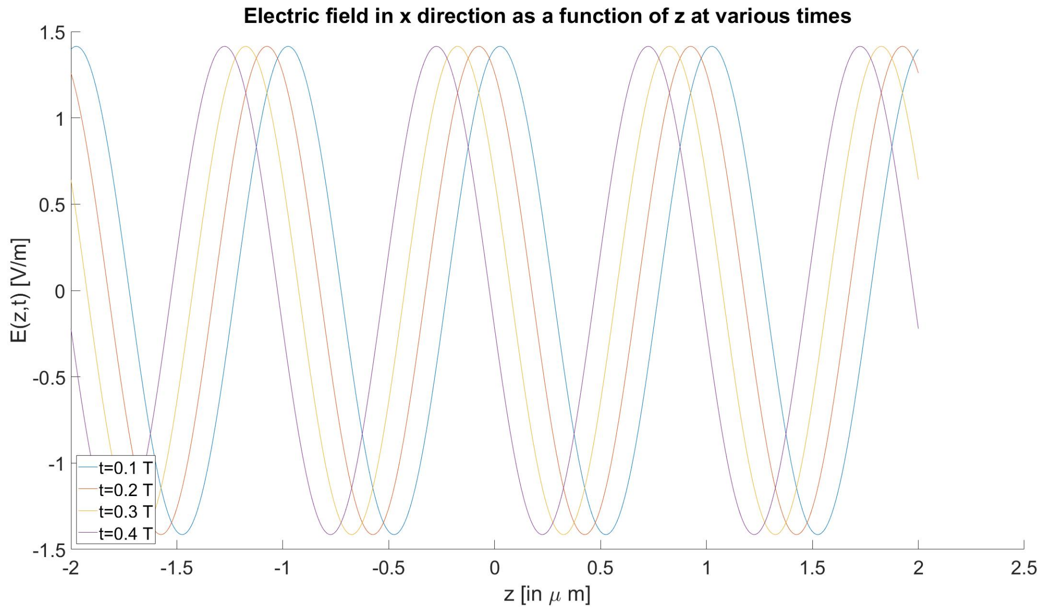

Figure 5: E_2(z,t) can be observed to be a left moving wave

Hence, both the given waves are moving in the opposite direction.

Problem 4:

a. The results are calculated at $t = 0$

$$E = E_z = e^{i\mathbf{k}.\mathbf{r}}$$
$$and, k = \frac{2\pi}{\lambda_o} = 1.2566e + 07\, m^{-1}$$

$$Re[E] = \cos(\mathbf{k}.\mathbf{r})$$
$$Re[E] = \cos(k_x x + k_y y)$$

i. At $\theta = 0^o$

$$k_x = k\cos(0^o) = k$$
$$k_y = k\sin(0^o) = 0$$
$$Therefore, Re[E] = cos(kx)$$

The following plots illustrate this E-field propagating in the X-Y plane.

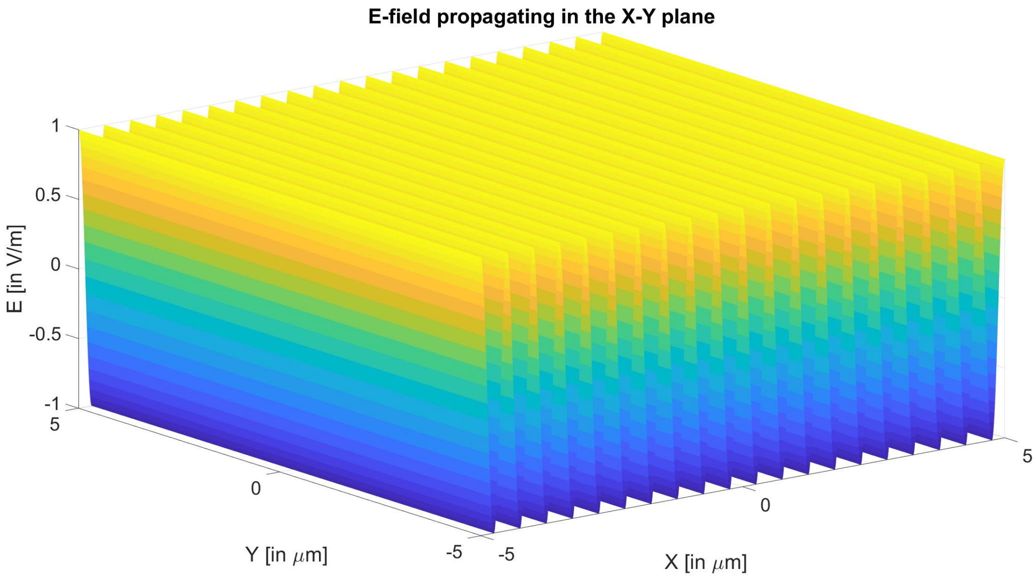

Figure 6: 3D view of the E-field Plane wave

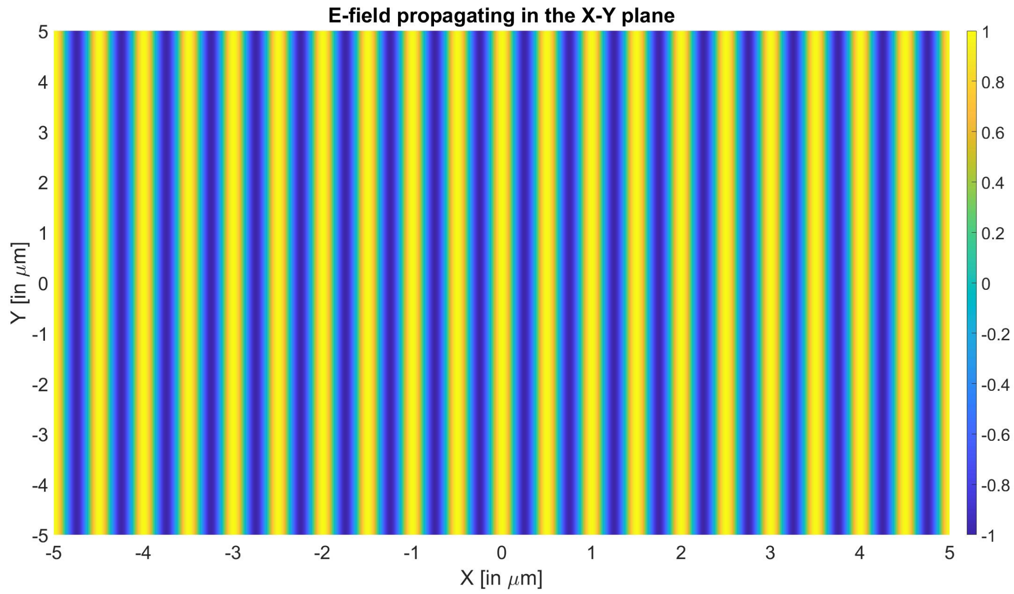

Figure 7: E-field wave in the x-y plane. The color bar indicates the E-Field amplitude.

ii. At $\theta = 10^o$

$$k_x = k\cos(10^o)$$
$$k_y = k\sin(10^o)$$
$$Therefore, Re[E] = \cos[k\cos(10^o)x + k\sin(10^o)y]$$

The following plots illustrate this E-field propagating in the X-Y plane.

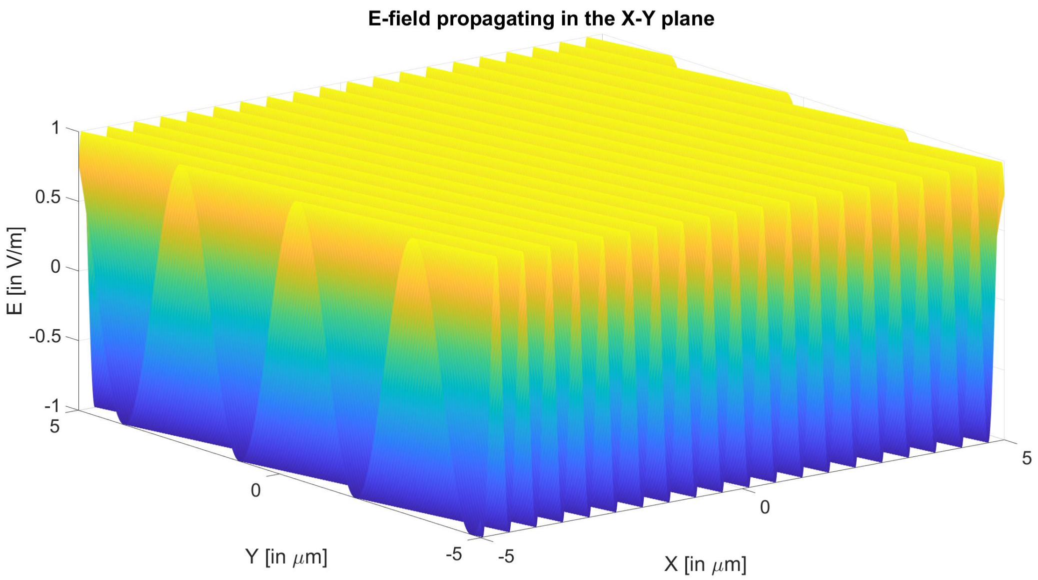

Figure 8: 3D view of the E-field Plane wave

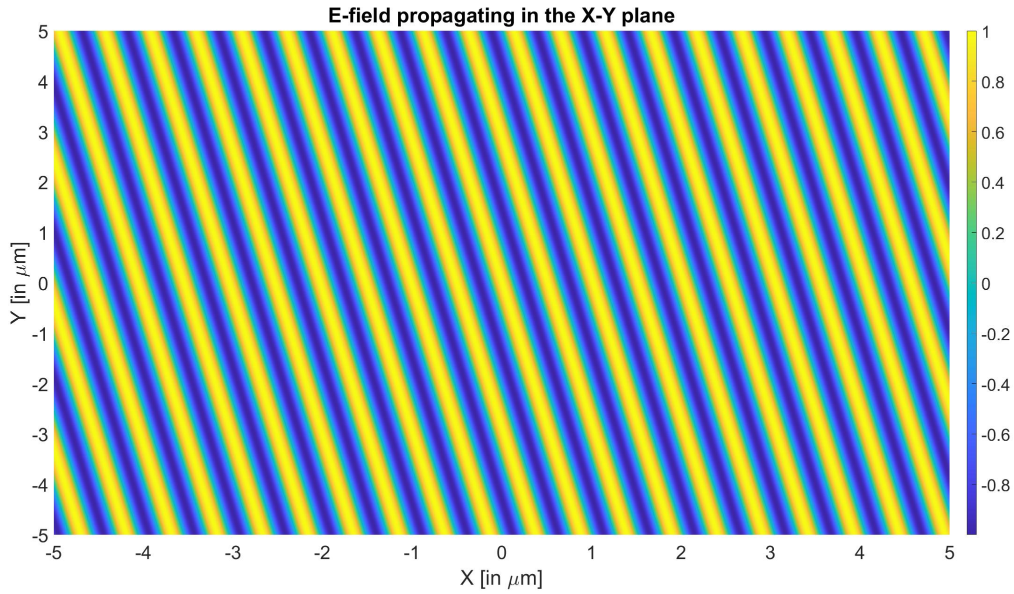

Figure 9: E-field wave in the x-y plane. The color bar indicates the E-Field amplitude.

b. E1, E2, E3 are plane waves polarized in the z-direction. Their amplitude is assumed to be equal, and is set to 1 V/m for convenience. The plots are produced at $t = 0$. E1, E2, E3 have $\theta_1 = -10^o, \theta_2 = -0^o$ & $\theta_3 = 10^o$ respectively. Hence, real part of these plane waves can be denoted as:

$$Re[E_1] = \cos[k\cos(-10^o)x + k\sin(-10^o)y]$$
$$Re[E_2] = \cos[k\cos(0^o)x + k\sin(0^o)y]$$
$$Re[E_3] = \cos[k\cos(10^o)x + k\sin(10^o)y]$$

The real part of the net electric field can hence be represented by:

$$Re[E_{net}] = Re[E_1] + Re[E_2] + Re[E_3];$$

The following plots illustrate this E-field propagating in the X-Y plane.

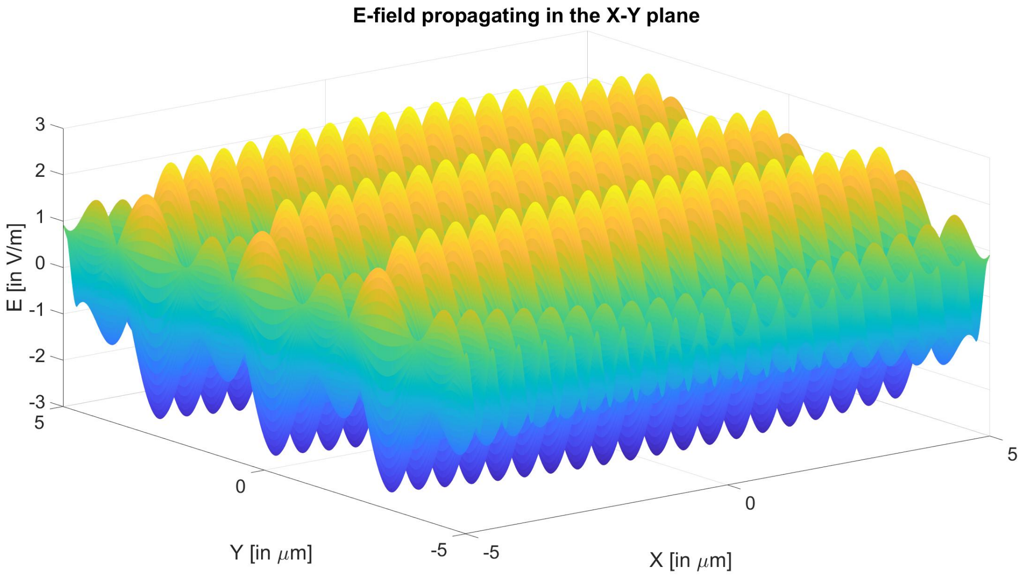

Figure 10: 3D view of the E-field Plane wave

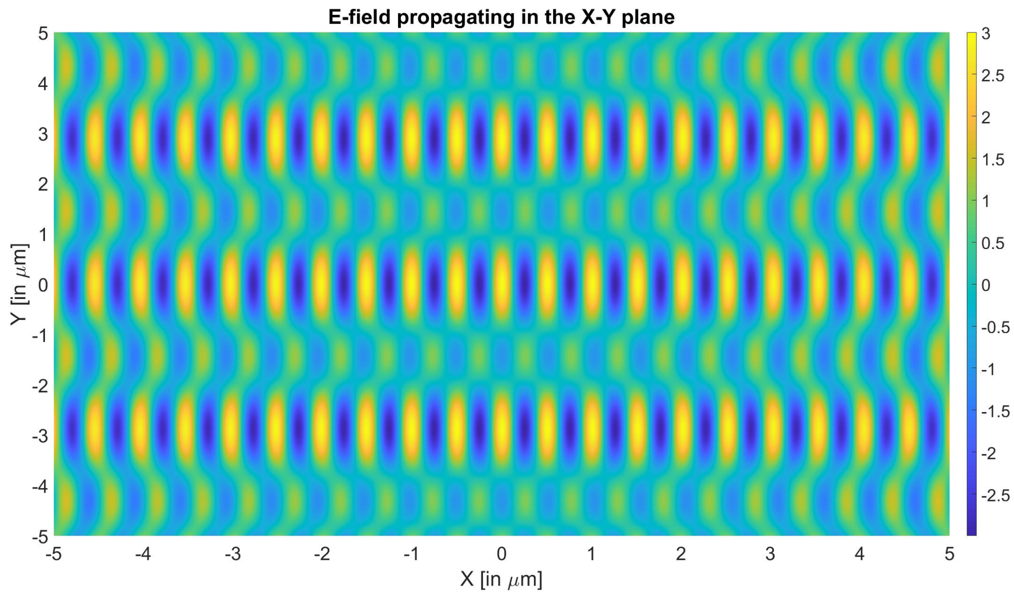

Figure 11: E-field wave in the x-y plane. The color bar indicates the E-Field amplitude.

c. The y component of the wave vector for each wave n, $k_{y,n}$ is said to be linearly distributed over the 30 waves, covering a range from $k\sin(-10^o)$ to $k\sin(10^o)$.

The term σ can be resolved as:

$$\sigma = \frac{k\sin(10^o)}{2} = 1.091063678535367e + 06\, m^{-1}$$

Based on the value for $k_{y,n}$, for each wave, the x-component of the wave vector can be denoted as:

$$k_{x,n} = \sqrt{k^2 - k_{y,n}^2}$$

And this definition can be further used to evaluate the amplitude of each of the waves:

$$A_n = e^{-\frac{k_{y,n}^2}{2\sigma^2}}$$

The real part of any wave, E_n at $t = 0$ can be written as:

$$Re[E_n] = A_n \cos[k_{x,n}x + k_{y,n}y]$$

And the real part of the net electric field, E_{net} can be denoted as:

$$Re[E_{net}] = \sum_{n=1}^{30} A_n \cos[k_{x,n}x + k_{y,n}y]$$

The following plots illustrate this E-field propagating in the X-Y plane.

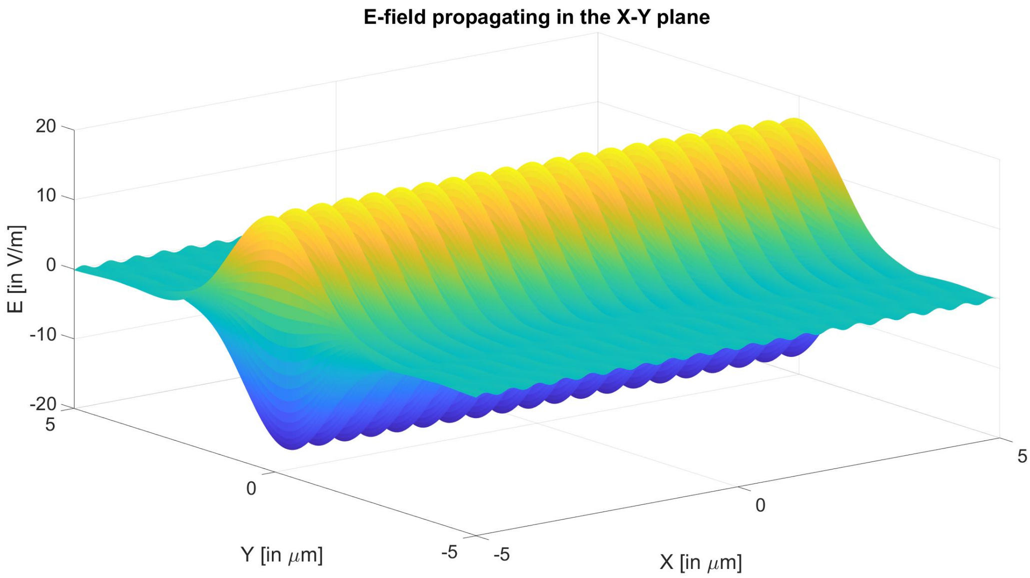

Figure 12: 3D view of the E-field Plane wave

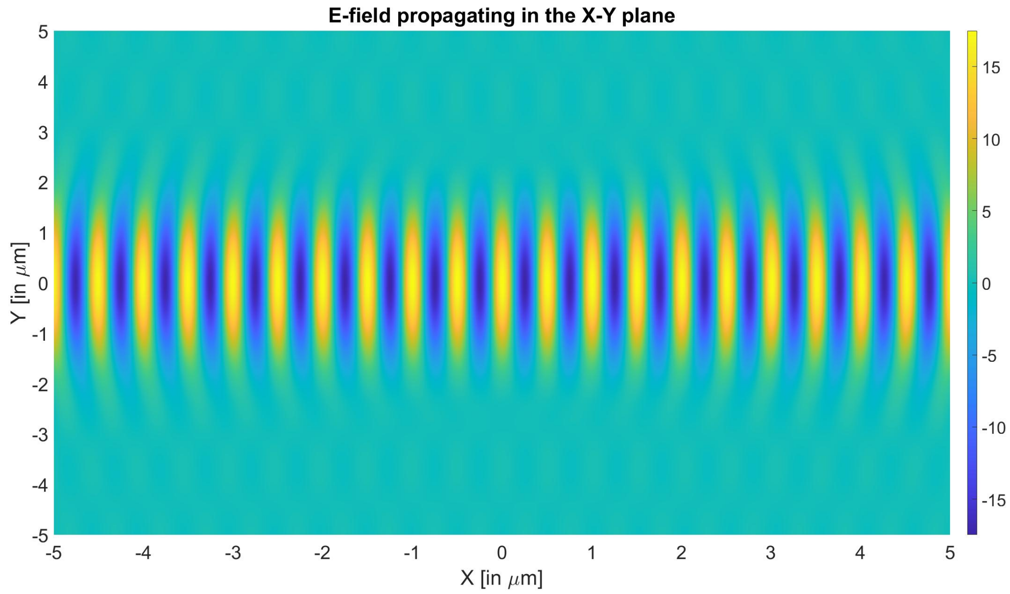

Figure 13: E-field wave in the x-y plane. The color bar indicates the E-Field amplitude.

d. The irradiance of a plane wave is proportional to the square of the amplitude, and can be denoted as:

$$I = \frac{1}{2}\, c\, \epsilon_o\, |E_{net}|^2$$

The following plots illustrate the irradiance of this beam propagating in the X-Y plane.

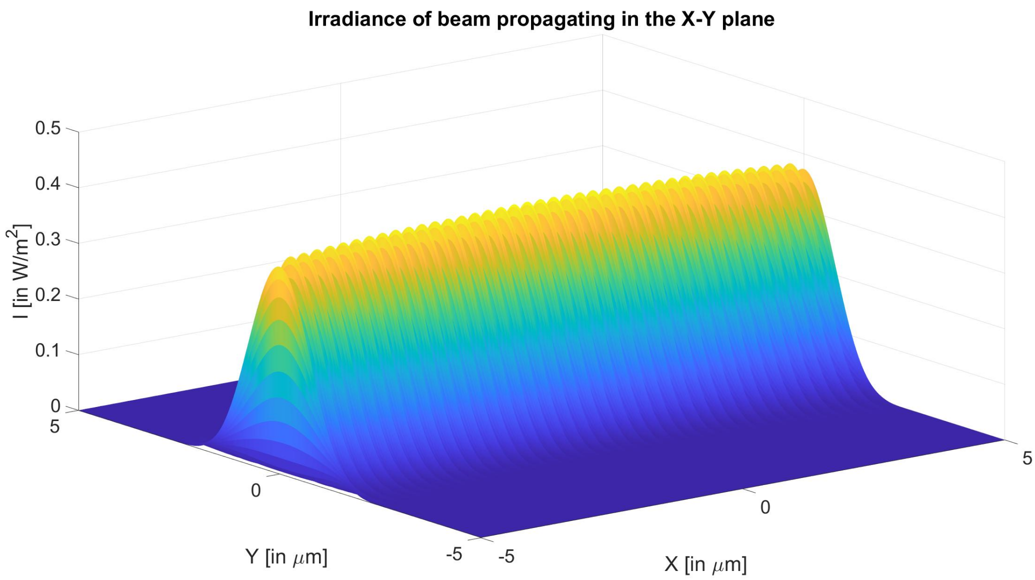

Figure 14: 3D view of the Irradiance of the plane wave

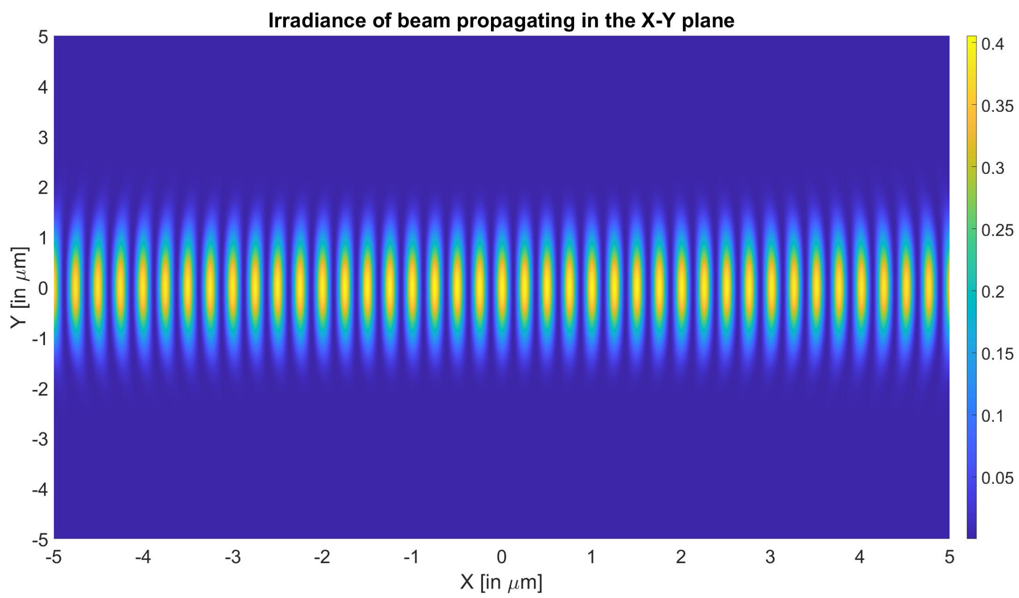

Figure 15: Irradiance of the wave propagating in x-y plane. The color bar indicates the Irradiance amplitude.

Problem 5:

Assume that the medium has no free current density $\mathbf{J} = 0$, and no magnetic response, $\mathbf{M} = 0$, such that $\mathbf{B} = \mu_o \mathbf{H}$.
For this derivation, we start with Ampere's Law with Maxwell's corrections:

$$\nabla \times \mathbf{B} = \mu_o \mathbf{J} + \mu_o \epsilon_o \frac{\partial \mathbf{E}}{\partial t}$$

But, we have assumed the free current density to be zero. Also, taking the curl on both sides, we get,

$$\nabla \times (\nabla \times \mathbf{B}) = \nabla \times \left(\mu_o \epsilon_o \frac{\partial \mathbf{E}}{\partial t} \right)$$
$$\nabla(\nabla . \mathbf{B}) - \nabla^2 . \mathbf{B} = \mu_o \epsilon_o \frac{\partial (\nabla \times \mathbf{E})}{\partial t}$$

From No Name Law, we know that $\nabla . \mathbf{B} = 0$, and from Ampere's Law we know that $\nabla \times \mathbf{E}$ is $-\frac{\partial \mathbf{B}}{\partial t}$. Using these, we get,

$$-\nabla^2 . \mathbf{B} = \mu_o \epsilon_o \frac{\partial}{\partial t} \left(-\frac{\partial \mathbf{B}}{\partial t} \right)$$

We know that $\mathbf{B} = \mu_o \mathbf{H}$. Dividing both sides of the equation by μ_o, we get,

$$\nabla^2 . \left(\frac{\mathbf{B}}{\mu_o} \right) = \mu_o \epsilon_o \frac{\partial}{\partial t} \left(\frac{\partial}{\partial t} \left(\frac{\mathbf{B}}{\mu_o} \right) \right)$$
$$\nabla^2 . \mathbf{H} = \mu_o \epsilon_o \frac{\partial^2 \mathbf{H}}{\partial t^2}$$

Therefore the wave equation for $\mathbf{H}$, is given as:

$$\nabla^2 . \mathbf{H} - \mu_o \epsilon_o \frac{\partial^2 \mathbf{H}}{\partial t^2} = 0 \tag{3}$$

Problem 6:

a. Refractive Index vs Free-Space Wavelength plot

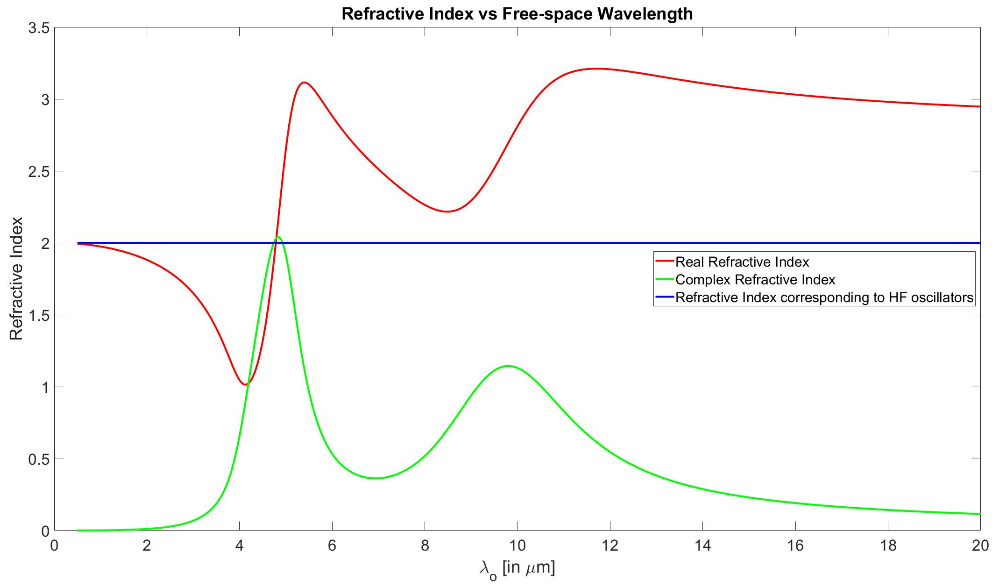

Figure 16: Illustrates refractive indices (real and imaginary) caused by oscillators with relevant resonant frequencies; and refractive index due to oscillators with infinitely higher resonant frequencies.

b. The group velocity inside the media is given as:

$$v_g = \frac{\partial \omega}{\partial k_{media}}$$

Therefore, we must determine the wave numbers inside the given media, using the refractive index obtained in part (a).

$$k_{media} = k_{free\ space} * n_{media}$$

The group velocity can be determined as:

$$v_g = \frac{grad(\omega)}{grad(k_{media})}$$

(credits to Jon King for simplifying this algebra)

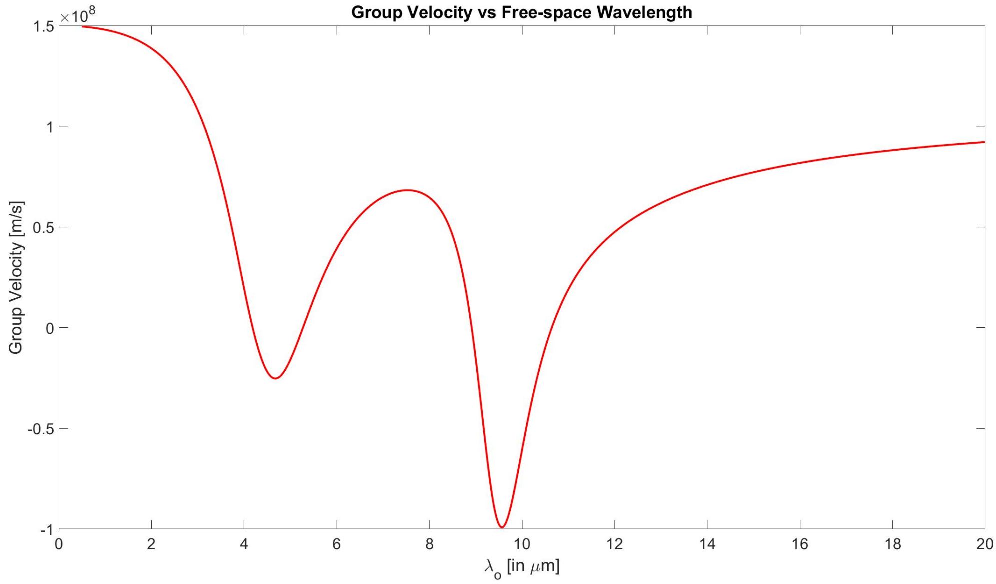

Figure 17: Illustrates the group velocity in the given medium as a function of the free-space wavelength.

MATLAB Scripts

Problem 3:

```
%MATLAB code for Prob Set 1, Q3 (c)

f=300e+12; %freq = 300 THz
k=(2*pi*f)/(3*10^8); %wavenumber, k=2*pi*f/c
z=-2000*10^(-9):1*10^(-9):2000*10^(-9); %z ranges from -2000 nm to 2000 nm

E=cos(k*z)-sin(k*z);% real electric field

plot(z*10^(6), E)
title('Electric field in x direction as a function of z')
xlabel('z [in \mu m]')
ylabel('E(z,t=0) [V/m]')
```

```
%MATLAB code for Prob Set 1, Q3 (d)

f=300e+12; %freq = 300 THz
T=1/f; %time period
k=(2*pi*f)/(3*10^8); %wavenumber, k=2*pi*f/c
w=2*pi*f; %angular velocity
z=-2000*10^(-9):1*10^(-9):2000*10^(-9); %z ranges from -2000 nm to 2000 nm

%E_1 wave is a right moving waves
%E_2 wave is a left moving waves
%Each wave is plotted at 4 different times

%t = 0.1*T
E_1_1=cos(k*z - w*0.1*T)-sin(k*z - w*0.1*T);
E_2_1=cos(-k*z - w*0.1*T)-sin(-k*z - w*0.1*T);

%t = 0.2*T
E_1_2=cos(k*z - w*0.2*T)-sin(k*z - w*0.2*T);
E_2_2=cos(-k*z - w*0.2*T)-sin(-k*z - w*0.2*T);

%t = 0.3*T
E_1_3=cos(k*z - w*0.3*T)-sin(k*z - w*0.3*T);
E_2_3=cos(-k*z - w*0.3*T)-sin(-k*z - w*0.3*T);

%t = 0.4*T
E_1_4=cos(k*z - w*0.4*T)-sin(k*z - w*0.4*T);
E_2_4=cos(-k*z - w*0.4*T)-sin(-k*z - w*0.4*T);


%plotting right moving wave, E_1
hold on
```

```
plot(z*10^(6), E_1_1)
plot(z*10^(6), E_1_2)
plot(z*10^(6), E_1_3)
plot(z*10^(6), E_1_4)
legend({'t=0.1 T','t=0.2 T','t=0.3 T','t=0.4 T'},'Location','southwest')
title('Electric field in x direction as a function of z at various times')
xlabel('z [in \mu m]')
ylabel('E(z,t) [V/m]')
ax = gca;
ax.FontSize = 30;

%plotting left moving wave, E_2
%hold on
%plot(z*10^(6), E_2_1)
%plot(z*10^(6), E_2_2)
%plot(z*10^(6), E_2_3)
%plot(z*10^(6), E_2_4)
%legend({'t=0.1 T','t=0.2 T','t=0.3 T','t=0.4 T'},'Location','southwest')
%title('Electric field in x direction as a function of z at various times')
%xlabel('z [in \mu m]')
%ylabel('E(z,t) [V/m]')
%ax = gca;
%ax.FontSize = 30;
```

Problem 4:

```
%MATLAB code to plot Electric Fields for ECE747 Q4,(a) case i&ii

%defining all the constants
lambda = 500*10^(-9); %given wavelength of the plane wave
k=2*pi/lambda; %modulus of wave vector
theta_1 = 0;  %theta in case #i
theta_2 = 10; %theta in case #ii

%defining components of wave vectors for each wave
k_x_1=k*cos(theta_1*pi/180);
k_y_1=k*sin(theta_1*pi/180);
k_x_2=k*cos(theta_2*pi/180);
k_y_2=k*sin(theta_2*pi/180);

[X,Y] = meshgrid(-5:0.01:5,-5:0.01:5)
%X and Y defined from -5 to 5 micro-meters

E_1=cos(k_x_1*X*10^(-6)+k_y_1*Y*10^(-6)); %E-field for theta=0
E_2=cos(k_x_2*X*10^(-6)+k_y_2*Y*10^(-6)); %E-field for theta=10
surf(X,Y,E_1, 'edgecolor', 'none')
%surf(X,Y,E_2, 'edgecolor', 'none')

title('E-field propagating in the X-Y plane')
xlabel('X [in \mum]')
ylabel('Y [in \mum]')
zlabel('E [in V/m]')
ax = gca;
ax.FontSize = 30;
```

```
%MATLAB code to plot Electric Fields for ECE747 Q4,(b)

lambda = 500*10^(-9); %given wavelength of the plane wave
k=2*pi/lambda; %modulus of wave vector

theta=[-10 0 10];%array representing thetas for E1,2,3 respectively

k_components = zeros(3,2);
%array representing values of x and y components
%of k for all three E-field vectors
%Columns indicate the 3 vectors (sequentially)
%and rows indicate x and y components (sequentially)

for i = 1:1:3
    k_components(i,1)=k*cos(theta(i)*pi/180); %X-components
    k_components(i,2)=k*sin(theta(i)*pi/180); %Y-components
```

```
end

[X,Y] = meshgrid(-5:0.01:5,-5:0.01:5);
%X and Y defined from -5 to 5 micro-meters

E_net=zeros(size(X));%net electric field
for i = 1:1:3
    E_net=E_net+cos(k_components(i,1)*X*10^(-6)+k_components(i,2)*Y*10^(-6));
end
surf(X,Y,E_net, 'edgecolor', 'none')
title('E-field propagating in the X-Y plane')
xlabel('X [in \mum]')
ylabel('Y [in \mum]')
zlabel('E [in V/m]')
ax = gca;
ax.FontSize = 30;
```

```
%MATLAB code to plot Electric Fields for ECE747 Q4,(c) and (d)

%defining all the constants
lambda = 500*10^(-9); %given wavelength of the plane wave
k=2*pi/lambda; %modulus of wave vector
sigma=k*sin(10*pi/180)/2;
ky_min=k*sin(-10*pi/180);
ky_max=k*sin(10*pi/180);

%calculating the wave vectors and amplitudes
ky_array=linspace(ky_min,ky_max,30);
%ky is linearly distributed over 30 waves
kx_array=sqrt(k^2-ky_array.^2);%calculating corresponding kx
A=exp((-ky_array.^2)/(2*sigma^2));%wave amplitudes

[X,Y] = meshgrid(-5:0.01:5,-5:0.01:5);
%X and Y defined from -5 to 5 micro-meters

E_net=zeros(size(X));%net electric field
for i = 1:1:30
    E_net=E_net+A(i)*cos(kx_array(i)*X*10^(-6)+ky_array(i)*Y*10^(-6));
end
%surf(X,Y,E_net, 'edgecolor', 'none')
%title('E-field propagating in the X-Y plane')
%xlabel('X [in \mum]')
%ylabel('Y [in \mum]')
%zlabel('E [in V/m]')
%ax = gca;
%ax.FontSize = 30;
```

```
I=zeros(size(E_net));
I=0.5*3*(10^8)*(8.8542*(10^(-12)))*abs(E_net).^2;
surf(X,Y,I, 'edgecolor', 'none')
title('Irradiance of beam propagating in the X-Y plane')
xlabel('X [in \mum]')
ylabel('Y [in \mum]')
zlabel('I [in W/m^2]')
ax = gca;
ax.FontSize = 30;
```

Problem 6:

```
%MATLAB code for ECE747 Q6 (a)

%defining all parameters
c=3*10^8;%speed of light
lambda_1=10*10^(-6);%resonant wavelength of oscillator #1
lambda_2=5*10^(-6);%resonant wavelength of oscillator #2
f_1=c/lambda_1;%resonant freq of oscillator #1
f_2=c/lambda_2;%rresonant freq of oscillator #2
w_1=2*pi*f_1;%resonant angular freq of oscillator #1
w_2=2*pi*f_2;%resonant angular freq of oscillator #2

ep_inf=4; %permittivity for HF oscillators
A_1=2*(w_1)^2;%lorentz parameter #1
A_2=2*(w_2)^2;%lorentz parameter #2
v_1=w_1/3;%damping coeff #1
v_2=w_2/5;%damping coeff #2

wave_range=linspace(0.5*10^(-6),20*10^(-6),1000);%wavelength range to plot
w_range=2*pi*c./wave_range;%corresponding angular freq range

ep=ep_inf+A_1./(w_1^2-w_range.^2-1i*v_1.*w_range)+
A_2./(w_2^2-w_range.^2-1i*v_2.*w_range);
n=sqrt(ep);%refractive index

n_real=real(n);%real part of the refractive index
n_imag=imag(n);%complex part of refractive index
n_inf=sqrt(ep_inf)*ones(size(wave_range));
%refractive index corresponding to ep_inf

%making the plot
plot(wave_range*10^(6),n_real,'Color','r','Linewidth',3)
hold on
plot(wave_range*10^(6),n_imag,'Color','g','Linewidth',3)
plot(wave_range*10^(6),n_inf,'Color','b','Linewidth',3)
hold off
legend({'Real Refractive Index','Complex Refractive Index',
'Refractive Index corresponding to HF oscillators'},'Location','east')
title('Refractive Index vs Free-space Wavelength')
xlabel('\lambda_o [in \mum]')
ylabel('Refractive Index')
ax = gca;
ax.FontSize = 25;
```

```
%MATLAB code for ECE747 Q6 (b)

%defining all parameters
c=3*10^8;%speed of light
lambda_1=10*10^(-6);%resonant wavelength of oscillator #1
lambda_2=5*10^(-6);%resonant wavelength of oscillator #2
f_1=c/lambda_1;%resonant freq of oscillator #1
f_2=c/lambda_2;%rresonant freq of oscillator #2
w_1=2*pi*f_1;%resonant angular freq of oscillator #1
w_2=2*pi*f_2;%resonant angular freq of oscillator #2

ep_inf=4; %permittivity for HF oscillators
A_1=2*(w_1)^2;%lorentz parameter #1
A_2=2*(w_2)^2;%lorentz parameter #2
v_1=w_1/3;%damping coeff #1
v_2=w_2/5;%damping coeff #2

wave_range=linspace(0.5*10^(-6),20*10^(-6),1000);%wavelength range to plot
w_range=2*pi*c./wave_range;%corresponding angular freq range
k_range=2*pi./wave_range;%corresponding wave number range in free space

ep=ep_inf+A_1./(w_1^2-w_range.^2-1i*v_1.*w_range)+
A_2./(w_2^2-w_range.^2-1i*v_2.*w_range);
n=sqrt(ep);%refractive index

k_range_media=k_range.*n;%wave numbers inside the media
v_g=real(gradient(w_range)./gradient(k_range_media));
%group velocity

%making the plot
plot(wave_range*10^(6),v_g,'Color','r','Linewidth',3)
title('Group Velocity vs Free-space Wavelength')
xlabel('\lambda_o [in \mum]')
ylabel('Group Velocity [m/s]')
ax = gca;
ax.FontSize = 25;
```