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Final Project

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Optical Tweezers

Introduction

Optical tweezers are used to trap, hold and move particles using a light beam and the technique is sometimes referred to as optical levitation, since the particles appear to be levitating (especially when the media is air or vacuum), without any external support. The technique has found its application in various domains (biology, medicine, nanochemistry, quantum optics, etc) and is frequently used to trap particles such as viruses, bacteria, DNA strands, individual atoms, etc [1]. Dr. Arthur Ashkin, who invented optical tweezers in 1986 [2], was awarded the Nobel Prize in Physics in 2018.

Optical tweezers use a single laser beam, focused by a high numerical aperture lens, to trap dielectric particles near the lens' focus. A combination of forces caused by refraction and reflection of the light on the particle create a harmonic oscillator-like situation for the particle at the focus of the beam. Therefore, the particle experiences a restoring force proportional to its displacement from the equilibrium position. It also executes Brownian motion while in the trap, and hence the system is also referred as a Brownian harmonic oscillator [3]. Fig. 1 illustrates the harmonic trap for a particle near the focus of the beam.

In this paper, we'll analyse the forces experienced by a particle in optical tweezers, followed by simulation of an HIV-I virus held using optical tweezers, and analysis of its properties.

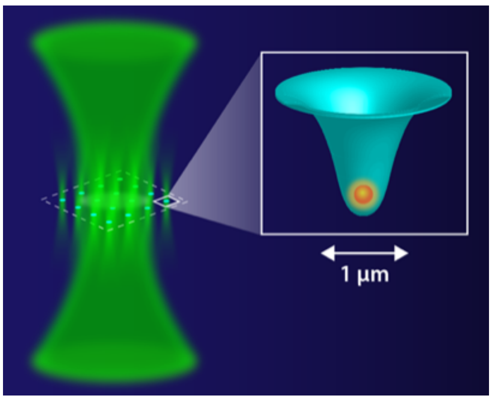

Figure 1: Illustration of harmonic trap for a particle at the focus of a Gaussian beam. The typical beam waist is in μm scale, and is ≈ 1 μm for alkali atom tweezers. Source: APS (`https://physics.aps.org/articles/v11/135`)

Preliminary Analysis

The electromagnetic theory illustrates that light can transfer momentum to particles. For a completely reflective particle, a photon with momentum $\hbar k$, when reflected from its surface imparts a momentum $\Delta p = 2\hbar k$ to the particle. Therefore, the net force due to a light beam, with power P_{beam} can be written as:

$$F = 2\frac{P_{beam}}{v}$$

where $v = c/n$ is the velocity of light in the medium with refractive index n.
This force, caused due to the reflection of light from the particle's surface is called "Radiation Pressure". It is not capable of creating an optical trap, since it can only exert a force in the direction of the incident beam. To successfully trap a particle, we require gradient forces which arise when a beam is focused at a point. Hence, Gaussian beams are usually used for creating optical tweezers.

The refraction of the beam passing through the particles gives rise to gradient forces. The wave vector $\vec{k}$ of a beam passing through the particle changes, which also leads to a change in its momentum. The particle experiences a momentum change in the opposite direction, since the momentum of the entire system is conserved. Fig. 2 presents the particle as a spherical bead. A gaussuan beam is focused using an objective lens, such that the particle lies near its focus. We look at two distinct rays, with wave vectors $\vec{k_1}$ and $\vec{k_2}$. Ray 1 emanates from the central region of the beam profile and creates a force $\vec{F_1}$ when it refracts through the particle. Ray 2 emerges from the peripheral region and creates a force $\vec{F_2}$ when it refracts.

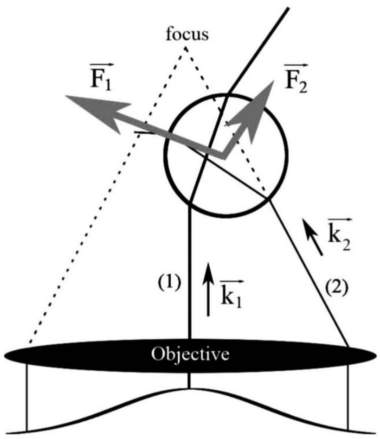

Figure 2: A particle can be modelled as a spherical bead. Various forces acting on it due to refraction of two light ways through it are illustrated. Source: [3]

Since the intensity profile of the beam has a Gaussian profile, $|\vec{F_1}| > |\vec{F_2}|$. Hence, the resultant force is pointing towards the beam focus. These two rays, therefore help us understand the net effect all the rays in the beam - to create a net restoring force towards the direction of the beam focus. Therefore, a balance between the radiation pressure force, and the gradient force is required to create a trap.

The above treatment provides us with a superficial understanding about the functioning of an optical trap. To obtain a more comprehensive understanding we look at two different regimes in which a particle can be treated. The distinction is based on the size of the particle, and the following equation is used to as a criteria:

$$2ka(\mu - 1) << 1 \tag{1}$$

where k is the wave vector in the medium surrounding the beam, a is the radius of the particle and $\mu = n_s/n_m$ is the relative refractive index with n_s being the refractive index of the particle and n_m being the refractive index of the medium. When the particle follows this criteria, it can be considered to be an induced dipole in uniform electric field. This is known as the Rayleigh regime. When the particle does not follow the criteria, it can be treated as a spherical bead (like we did previously) and further results can be derived using ray optics. Hence, this is known as the Geometrical Optics regime.

Since our simulation aims to create optical tweezers for a virus, which follows the criteria given in Eqn 1, we'll focus on understanding the properties of a particle under the Rayleigh regime.

Rayleigh Regime

In this regime, we can approximate the particle to be a dielectric sphere inside a uniform electric field. Therefore, the induced dipole moment of the sphere [3] [4] can be given as:

$$\vec{p} = \frac{K-1}{K+2} a^3 \vec{E} \tag{2}$$

where $\vec{E}$ denotes the electric field outside the sphere and $K = \epsilon/\epsilon_m$, is the ratio of the electric permittivities of the sphere and medium, respectively.
Since the potential energy is given by $U = -\vec{p}.\vec{E}$, and the force on the sphere is given by $\vec{F} = -\vec{\nabla} U$, we obtain:

$$\vec{F} = \vec{\nabla}(\vec{p}.\vec{E}) \tag{3}$$

Using Eqn. 2 and 3, we obtain:

$$\vec{F} = \frac{K-1}{K+2} a^3 \vec{\nabla} E^2 \tag{4}$$

Since this force is proportional to the gradient of the field intensity, it points in the direction of the maximum field intensity, which occurs at the focus. This provides another verification that the particle stays trapped in the focal region of the beam, in Rayleigh regime. Now, we can also define a quantity, "trap stiffness", κ, which is a measure of the strength of the trap, and is given by the gradient of the force at the equilibrium position. Therefore, the x-component of the trap stiffness is denoted as:

$$\kappa_x = -\left(\frac{\partial F_x}{\partial x}\right)_{r_{eq}} = -\frac{K-1}{K+2} a^3 \left(\frac{\partial^2 E^2}{\partial x^2}\right)_{r_{eq}} \tag{5}$$

For small displacements, we can approximate the force acting on the particle to be:

$$F_x = -\kappa_x \Delta x \tag{6}$$

where $\Delta x = x - x_o$ is the displacement from the equilibrium position x_o.

Modelling the HIV-1 virus

Using the data available in reference [5], we can construct a model of a single virus suspend in an appropriate medium. Using this model, we aim to determine the trap stiffness experienced the virus, at various axial positions in the trap.

We can model the HIV-1 virus as a sphere, with a radius $a = 74 \pm 12$ nm. Hence, we accept the median radius of 74 nm as the radius for our model. While using a laser at $\lambda = 830$ nm, the refractive index of the virus was determined to be $n = 1.42$.

The virus is suspended in a medium, called as "complete media" which consists of DMEM supplemented with 10% fetal bovine serum (Hyclone), to a concentration of 4×10^7 virions/ml. The refractive index of this medium was determined to be $n_m = 1.3307$ at $\lambda = 830$ nm.

Setting up the simulation

We set up a Lumerical FDTD simulation to calculate the force experienced by the virus suspended in the complete media. By calculating the slope of the force at the equilibrium position, we can determine the trap stiffness in any axis.

We start off with an example simulation provided on the Lumerical Knowledge Base website [6]. We tweak the model such that the refractive index of the medium is defined as $n_m = 1.3307$, refractive index of the 2D sphere is $n = 1.42$ and its radius is $a = 74$ nm. The light source is defined as a Gaussian source with $\lambda = 830$ nm and NA = 1.2. The beam source is placed 0.6 μm away from the origin. The simulation region, mesh and monitors are modified accordingly. Fig. 3 illustrates the simulation setup.

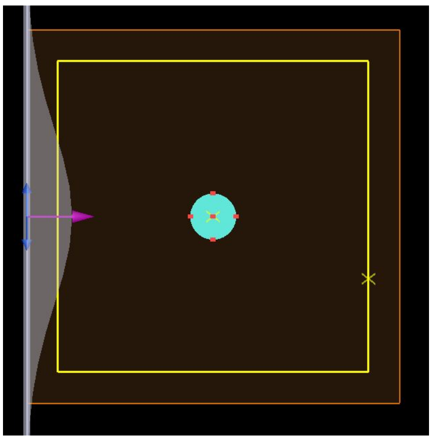

Figure 3: Lumerical model of the HIV-1 virus suspended in complete media and trapped using a Gaussian beam source.

This is a 2D simulation, wherein the beam propagates along the x-axis, hereby also known as the transverse axis. The axial axis is defined as the y-axis. The simulation model uses Maxwell's Stress Tensors to determine the forces acting on the particle. The transverse force F_x is determined as a function of the particle's position along the x-axis. This also allows us to determine the equilibrium position, i.e. the position where $F_x = 0$.

Results

We run the first simulation for a virus placed at the origin. The beam source is hence placed 600 nm away from the particle at $x_{source} = -0.6\mu m$. The force near the equilibrium position is illustrated in fig. 4.

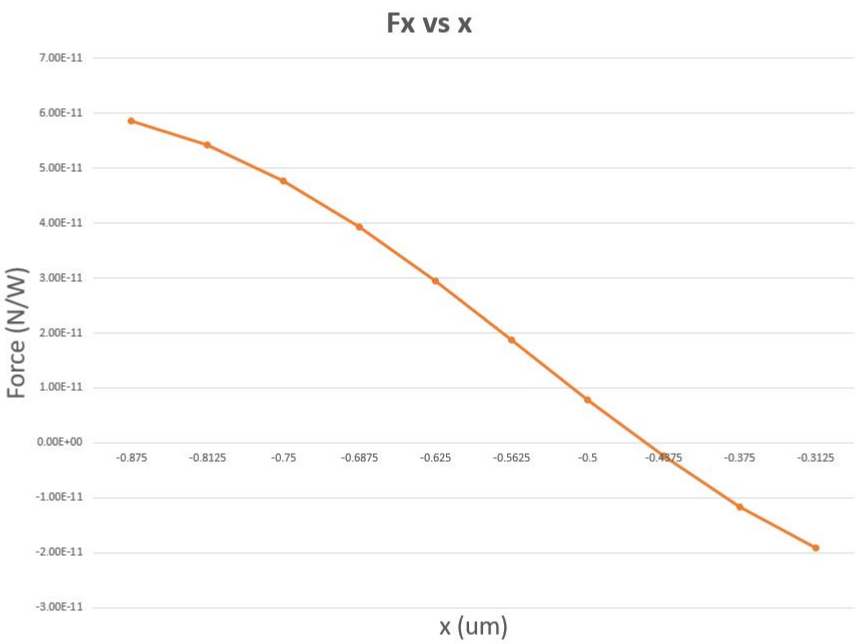

Figure 4: Lumerical model of the HIV-1 virus suspended in complete media and trapped using a Gaussian beam source.

Using this plot, we can determine the equilibrium position as $x_{eq} = -0.45\,\mu m$. Hence, the equilibrium position is $x_{eq} - x_{souce} = 0.6 - 0.45 = 0.15\,\mu m$ away from the source.

The transverse trap stiffness can be determined using the slope of this curve as $\kappa_x = 127.715\,pN/W\,\mu m$

References

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