EMA 601/NE 602
Problem set #1
Due 9/29 at 1 pm on Canvas, in pdf format

Guidelines:

- Typed solutions are preferred, but you may scan your handwritten solutions as long as the writing is legible. I find camscanner (available on both Android and IOS mobile platforms) to be a decent application for scanning documents into digital formats, but you may use any software of your preference.
- Please submit any code you write for derivations and plots.
- Please compile your responses into a single pdf document.
- You are welcome to work together on the problems, but you must write your own responses and code.
- This problem set involves usage of a numerical computing tool. Some example codes for using the plotting, data fitting, and symbolic math functions of Matlab will be provided shortly. However, you may use any tool you prefer.

1. (15 points) **Planck's radiation law**
   In class, we derived the Planck's radiation law for the spectral distribution of blackbody radiation, which relates the intensity per wavelength of the emitted electromagnetic radiation by an object to its temperature:
   $$\frac{dI}{d\lambda} = \frac{8\pi hc^2}{\lambda^5}\frac{1}{e^{hc/(\lambda k_B T)} - 1}$$
   where h is the Planck's constant ($h = 6.626 \times 10^{-34}\,Js$), λ is the wavelength of electromagnetic radiation, k_B is the Boltzmann constant ($k_B = 1.38 \times 10^{-23} J/K$), and T is the object's temperature in K.
   (a) (6 points) Derive an expression that relates the wavelength (λ_{max}) producing the maximum value of dI/λ as a function of T. For this calculation, it may be useful to utilize a numerical solver.
   (b) (6 points) Using data provided in this spreadsheet, fit the measured spectral emission from the sun to the Planck's radiation law and show that the temperature of the sun is $5780\ K$. Plot the data and fitted curve on the same figure.
   (c) (2 points) What is the wavelength at the peak of the spectral distribution? Use your result to evaluate the correctness of the expression you derived in (a).
   (d) (1 point) Based on your result in (a), calculate the wavelengths at the peak of the spectral distribution from our human bodies ($T = 335\ K$), building walls ($T = 298\ K$), and space ($T = 2.73\ K$).

2. (10 points) **Doppler shift of reflected photon**
   In space, a photon with energy $E_0 = h\nu_0$ and momentum $\vec{p}_0$ hits the surface of an object initially at rest with mass m at normal incidence.
   (a) (2 points) What is the momentum of the object after the interaction, assuming that the surface of the object is perfectly absorptive?
   (b) (2 points) What is the momentum of the object after the interaction, assuming that the surface of the object perfectly reflective?
   (c) (3 points) In the case that the object's surface is perfectly reflective, what is the frequency shift of the reflected photon? This is known as the Doppler shift.

(d) (3 points) The momentum transfer from solar radiation to a reflective sail (typically called a solar sail) is used to propel spacecrafts. For analyzing the dynamics of a solar sail system, it is useful to define the radiation pressure associated with electromagnetic radiation. For a perfectly absorbing object, radiation pressure is calculated by $P = I/c$, where I is the incident power per unit area and c is the speed of light. Using data from the same spreadsheet used in 1(b), calculate the *net* radiation pressure on a perfectly reflective solar sail located close to the earth's orbit. Assume that the solar radiation is impinging on the sail at normal incidence.

3. (15 points) **Photo-absorption by an electron**
   (a) (6 points) A free electron with velocity v_1 absorbs a photon with energy $h\nu$, *increasing* its velocity to v_2. Are both energy and momentum conserved in this case?
   (b) (6 points) How is Compton scattering different from the scenario described in (a)? How are energy and momentum conserved in Compton scattering?
   (c) (3 points) What is the condition under which the absorption of a photon is allowed?

4. (10 points) **Neutron de Broglie wave**
   (a) (5 points) What is the velocity and kinetic energy of a neutron with de Broglie wavelength $\lambda_{dB} = 1\, nm$?
   (b) (5 points) Free neutrons are unstable and undergo beta decay with a mean lifetime of 15 minutes. A neutron interferometer setup is located 100 m away from the neutron source. What fraction of neutrons is lost between the source and interferometer setup?

5. (10 points) **One-dimensional barrier well**
   A one-dimensional potential well has a barrier of height 1.5 eV on the right-hand side and a barrier higher than this on the left-hand side. This potential well has an energy eigenstate for an electron at 1.3 eV. All energy values are relative to the bottom of the well.
   State the general form of the wavefunction solution (without solving for the expression of the normalizing constant) in the following two cases and give the values for any wavevector magnitude k and/or decay constant κ in these wavefunctions:
   (a) (5 points) Inside the well
   (b) (5 points) In the barrier on the right side