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University Physics, 14th Edition, Global Edition
– Hugh D. Young and Roger A. Freedman

Chapter 28

Sources of Magnetic Field

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(2018 Fall).
Contents modified by GS Yun,
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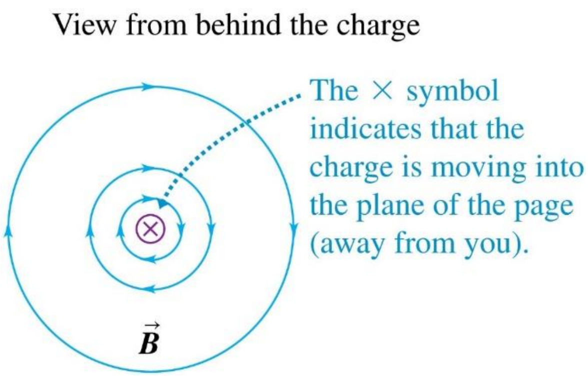
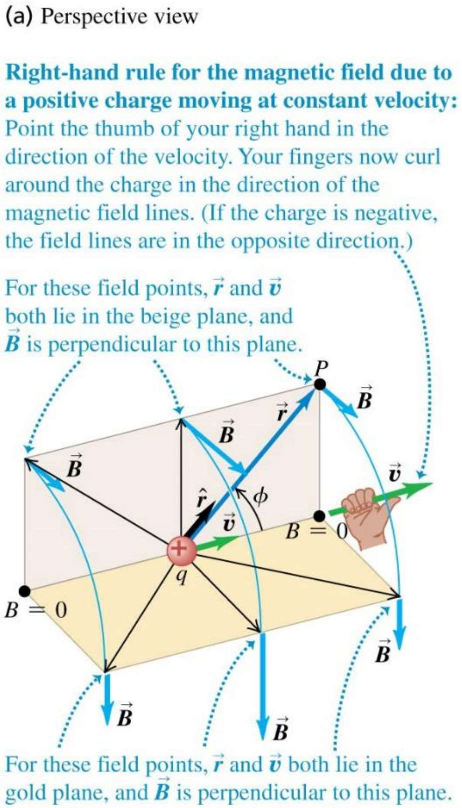
Goals for Chapter 28

- Magnetic field of a moving charged particle
- Magnetic field of a current element – **the law of Biot and Savart**
- Magnetic field of a long straight current-carrying wire
- Force between current-carrying wires
- Magnetic field of a current loop
- **Ampere's law and its application**

28.1 The magnetic field of a moving charge

- A **moving charge** generates a magnetic field that depends on the velocity of the charge, and the distance from the charge.





Magnetic field due to a point charge with constant velocity

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}$$

Magnetic constant (μ_0); Charge (q); Velocity ($\vec{v}$); Unit vector from point charge toward where field is measured ($\hat{r}$); Distance from point charge to where field is measured (r)

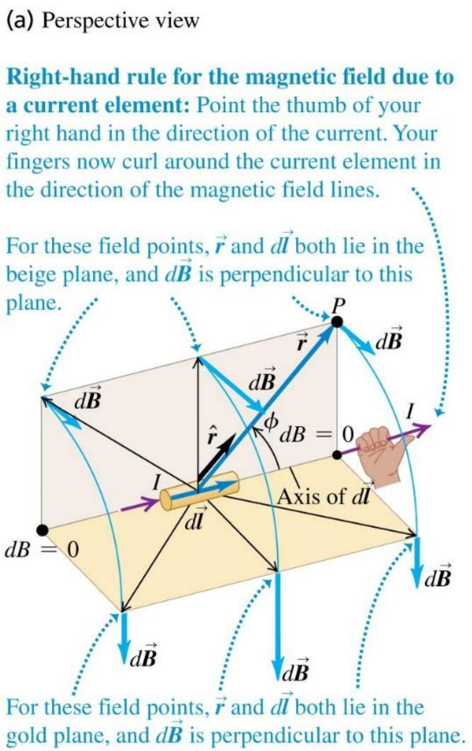
28.2 Magnetic field of a current element

- Superposition principle

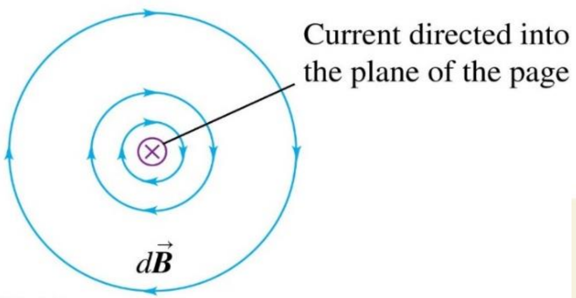
Number of (moving) particles in the line segment $dl = nAdl$

Total magnetic field ($d\vec{B}$) generated by the moving charges in the line segment:

$$
\begin{aligned}
d\vec{B} &= \frac{\mu_0}{4\pi}\frac{(nAdl)q\vec{v}\times\hat{r}}{r^2} \\
&= \frac{\mu_0}{4\pi}\frac{(nAqv)d\vec{l}\times\hat{r}}{r^2} = \frac{\mu_0}{4\pi}\frac{Id\vec{l}\times\hat{r}}{r^2}
\end{aligned}
$$



View along the axis of the current element



Magnetic field due to an infinitesimal current element

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l}\times\hat{r}}{r^2}$$

Magnetic constant

Current

Vector length of element (points in current direction)

Unit vector from element toward where field is measured

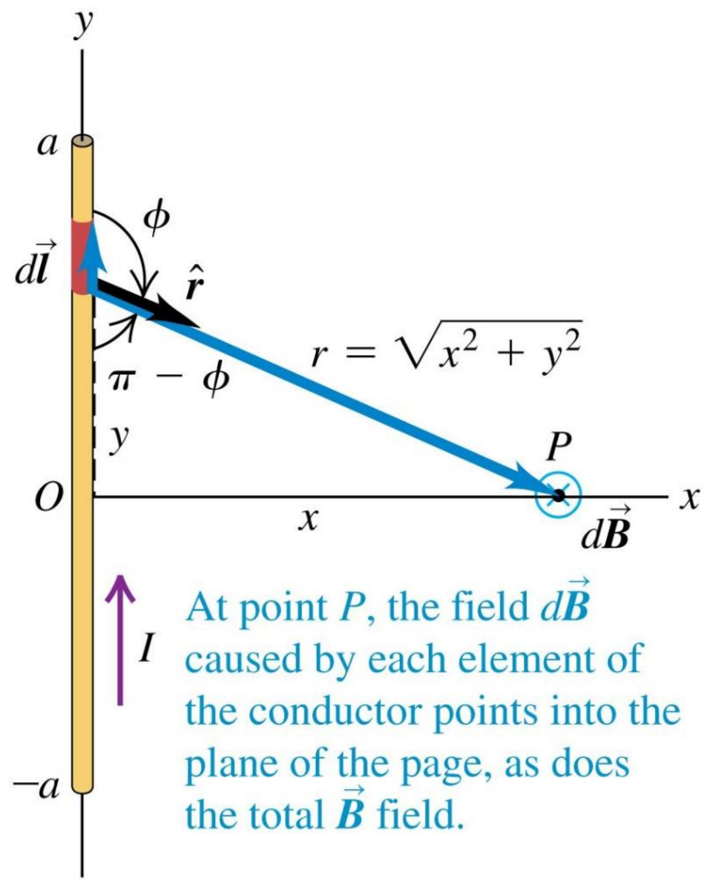
Distance from element to where field is measured

28.3 Magnetic field of a straight current-carrying conductor

- **Law of Biot and Savart:**

$$\vec{B} = \int d\vec{B} = \int \frac{\mu_0}{4\pi} \frac{I d\vec{l} \times \hat{r}}{r^2}$$

- Magnetic field $\vec{B}$ of a conductor with length 2*a* carrying a current *I* at a point a distance *x* from the conductor on its perpendicular bisector.



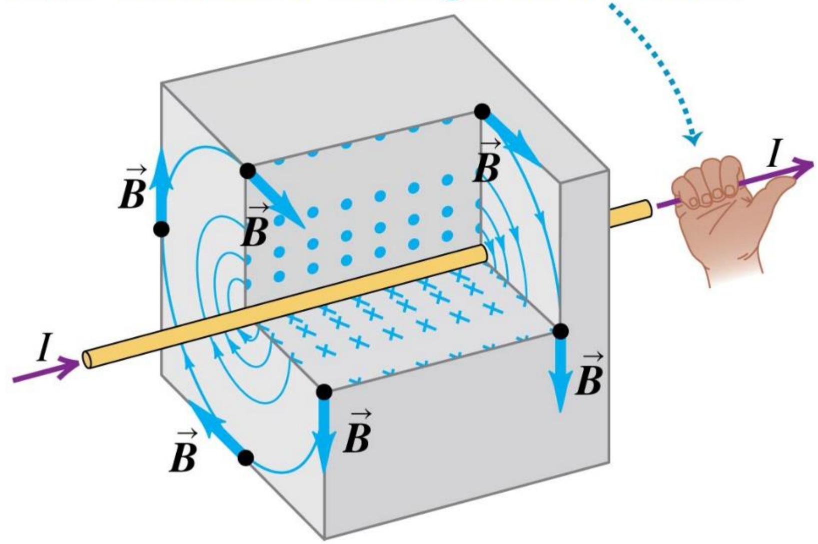
$$\begin{aligned}\vec{B} &= \int_{-a}^{a} \frac{\mu_0 I}{4\pi}\frac{dy}{r^2}\hat{y}\times\hat{r} = \int_{-a}^{a} \frac{\mu_0 I}{4\pi}\frac{dy}{r^2}\hat{y}\times\left(\hat{x}\frac{x}{r} - \hat{y}\frac{y}{r}\right) \\ &= (-\hat{z})\frac{\mu_0 I}{4\pi}\int_{-a}^{a} dy \frac{x}{(x^2+y^2)^{3/2}} = (-\hat{z})\frac{\mu_0 I}{4\pi}\left.\frac{y}{x(x^2+y^2)^{1/2}}\right|_{-a}^{a} = (-\hat{z})\frac{\mu_0 I}{4\pi}\frac{2a}{x\sqrt{x^2+a^2}}\end{aligned}$$

Magnetic field of a long straight current-carrying conductor

- For infinitely long wire, $a \gg x$,

$$B = \frac{\mu_0 I}{4\pi} \frac{2a}{x\sqrt{x^2 + a^2}} \approx \frac{\mu_0 I}{2\pi x}$$

Right-hand rule for the magnetic field around a current-carrying wire: Point the thumb of your right hand in the direction of the current. Your fingers now curl around the wire in the direction of the magnetic field lines.



Magnetic field near a long, straight, current-carrying conductor

$$B = \frac{\mu_0 I}{2\pi r}$$

Magnetic constant

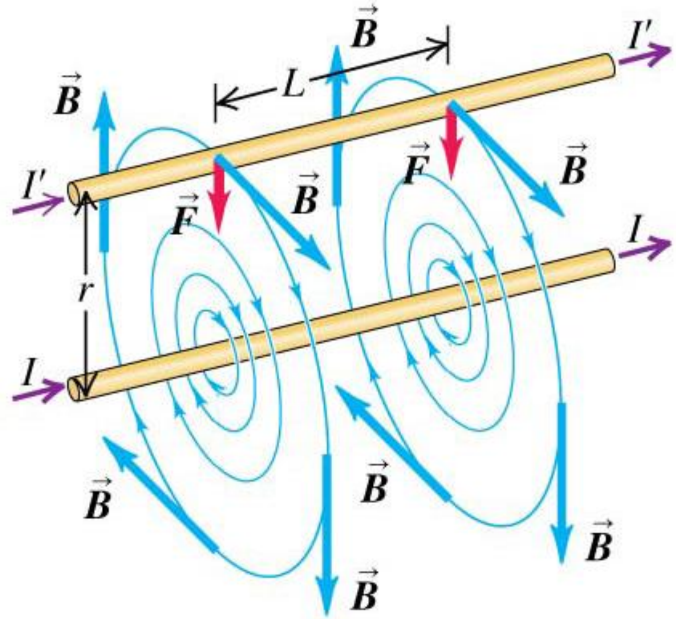
Current

Distance from conductor

28.4 Force between parallel conductors

- Two long, straight, parallel conductors separated by a distance *r* and carrying currents *I* and *I′* in the same direction.

- Each conductor lies in the magnetic field set up by the other, so each experiences a force.



$$\text{Force on } I \text{ by } I' \text{ for length } L$$

$$= LIB' = LI\frac{\mu_0 I'}{2\pi r}$$

Magnetic constant

Current in first conductor

Magnetic force per unit length between two long, parallel, current-carrying conductors

$$\frac{F}{L} = \frac{\mu_0 I I'}{2\pi r}$$

Current in second conductor

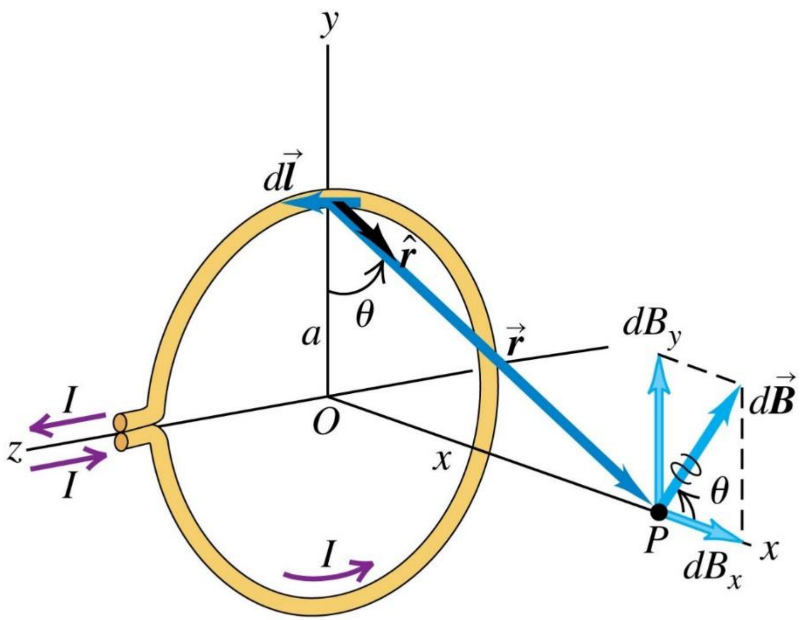
Distance between conductors

Definition of the ampere

- The definition of the SI base unit **ampere, A**:
  - One ampere is that unvarying current that, if present in each of two parallel conductors of infinite length and one meter apart in empty space, causes each conductor to experience a force of exactly 2×10^{-7} newtons per meter of length.
    - $\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$
    - **1 C = 1 A·s**

28.5 Magnetic field of a circular current loop*

- A circular conductor with radius *a* carrying a counterclockwise current *I*.



- From the law of Biot and Savart

$$\vec{B} = \int d\vec{B} = \int \frac{\mu_0 I}{4\pi} \frac{d\vec{l} \times \hat{r}}{r^2}$$

$$B_x = \frac{\mu_0 I}{4\pi} \int dl \frac{\cos\theta}{r^2} = \frac{\mu_0 I}{2} a \frac{a}{r^3}$$

Magnetic constant

Current

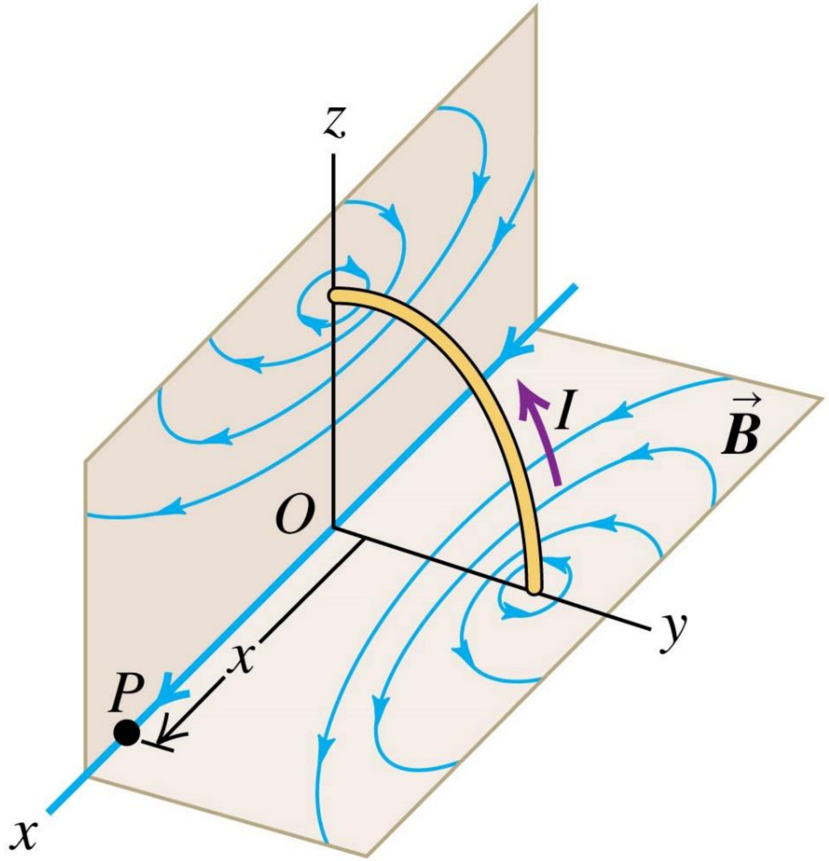
Magnetic field on axis of a circular current-carrying loop

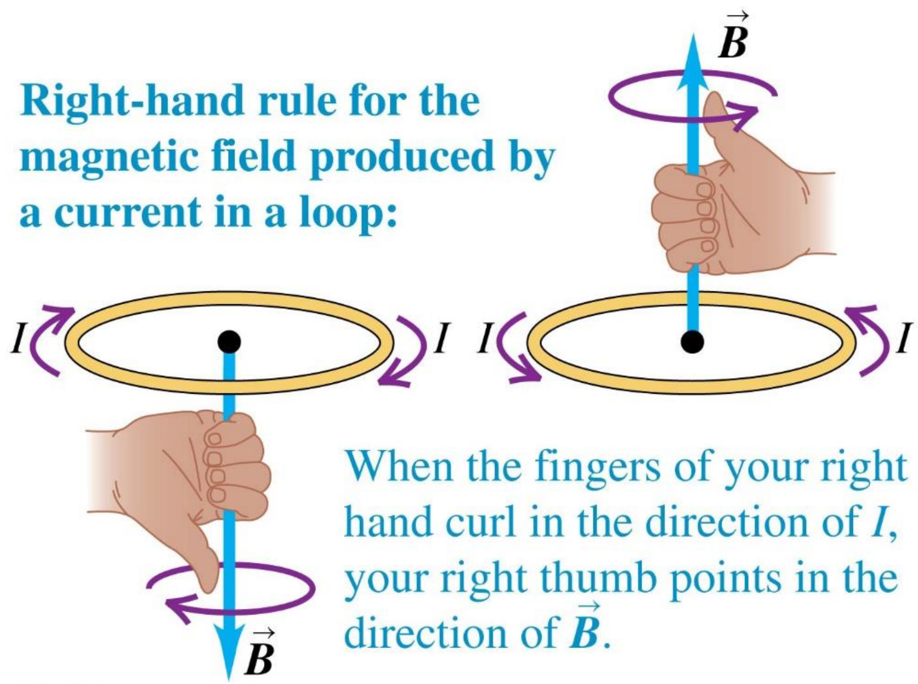
$$B_x = \frac{\mu_0 I a^2}{2(x^2 + a^2)^{3/2}}$$

Radius of loop

Distance along axis from center of loop to field point

Magnetic field lines of a circular current loop





28.6 Ampere's law

- Gauss's law: charge distribution → electric field

Gauss's law: $$\Phi_E = \underbrace{\oint \vec{E} \cdot d\vec{A}}_{} = \frac{Q_{encl}}{\epsilon_0}$$

Electric flux through a closed surface of area A = surface integral of $\vec{E}$

Total charge enclosed by surface

Electric constant

- **Ampere's law: current distribution → magnetic field**

Line integral around a closed path

Ampere's law: $$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

Magnetic constant

Net current enclosed by path

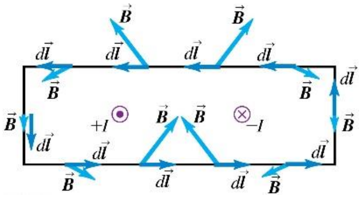
Scalar product of magnetic field and vector segment of path

Symmetry!!!

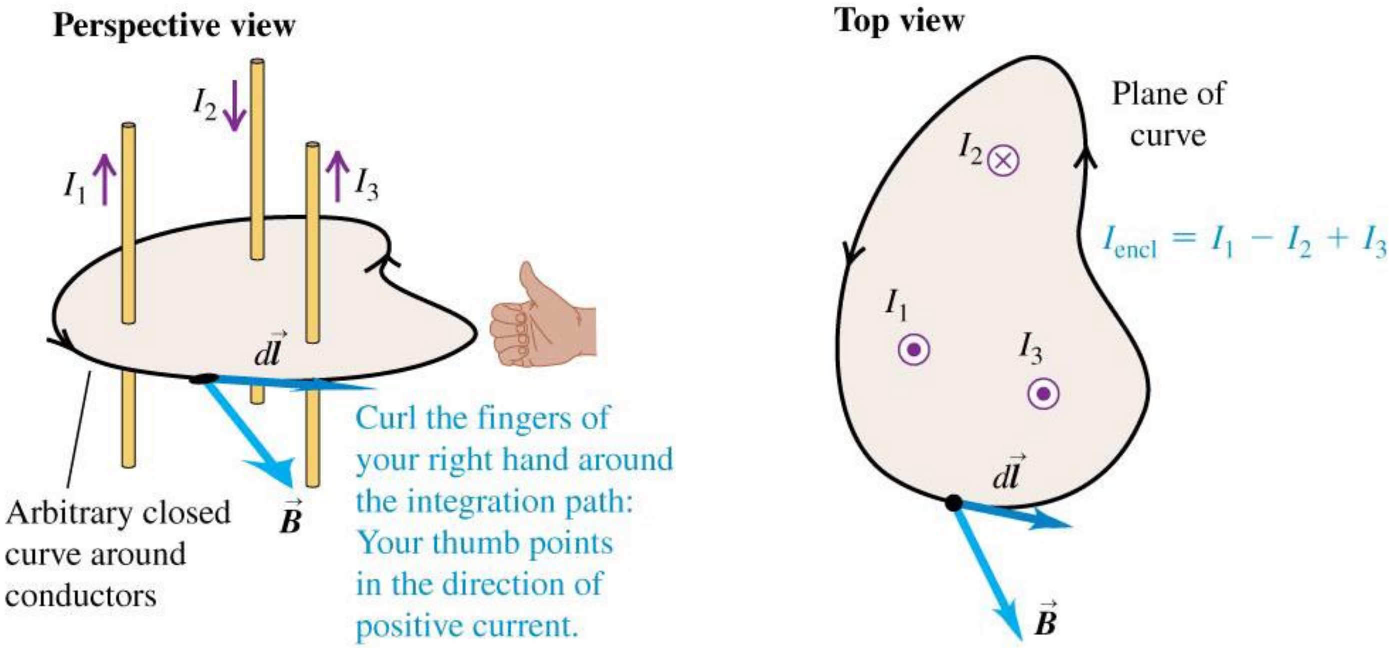
Ampere's law

- The line integral of the total magnetic field is proportional to the algebraic sum of the currents, I_{encl}, passing through the area bounded by the curve.

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{encl}} \Rightarrow$$



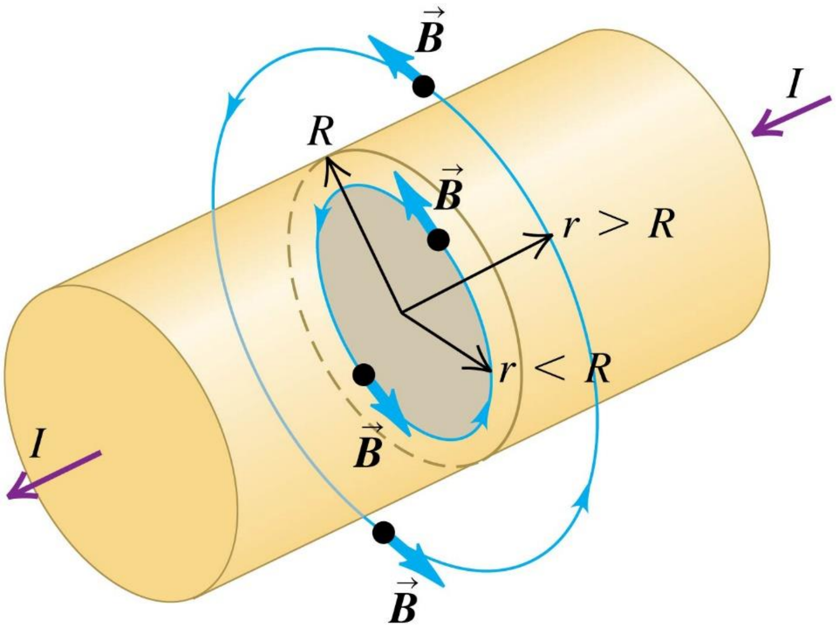
$$I_{\text{encl}} = \int \vec{J}\cdot d\vec{A}$$



28.7 Applications of Ampere's law

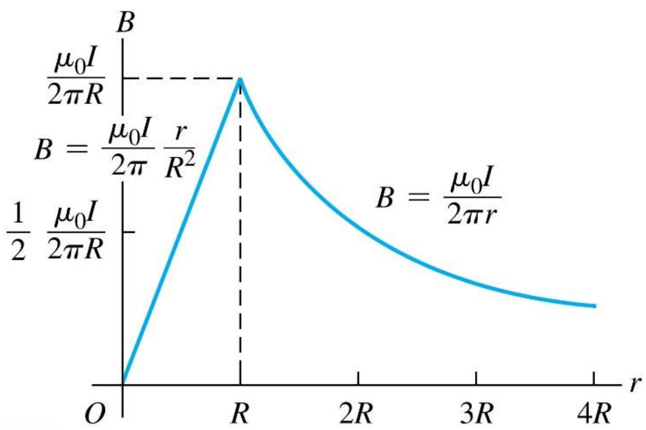
Field around a long cylindrical conductor

- A cylindrical conductor with radius R carries a current I.
- The current is uniformly distributed over the cross-sectional area of the conductor.
- From Ampere's law:



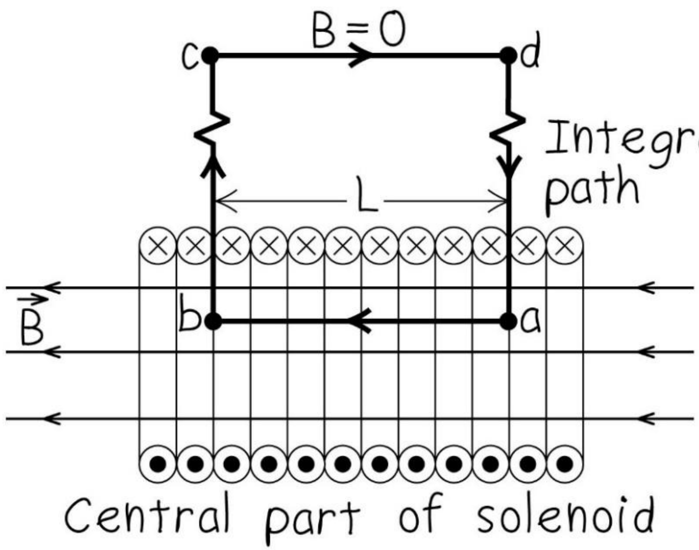
$$B = \frac{\mu_0 I}{2\pi} \frac{r}{R^2} \qquad \text{(inside the conductor, } r < R\text{)}$$

$$B = \frac{\mu_0 I}{2\pi r} \qquad \text{(outside the conductor, } r > R\text{)}$$

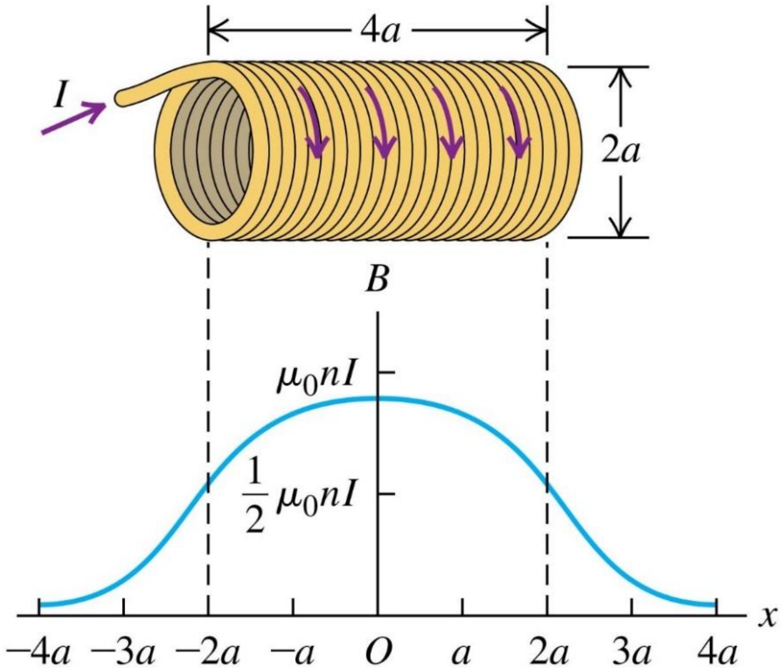


Field of a solenoid

- A solenoid consists of a helical winding of wire on a cylinder.

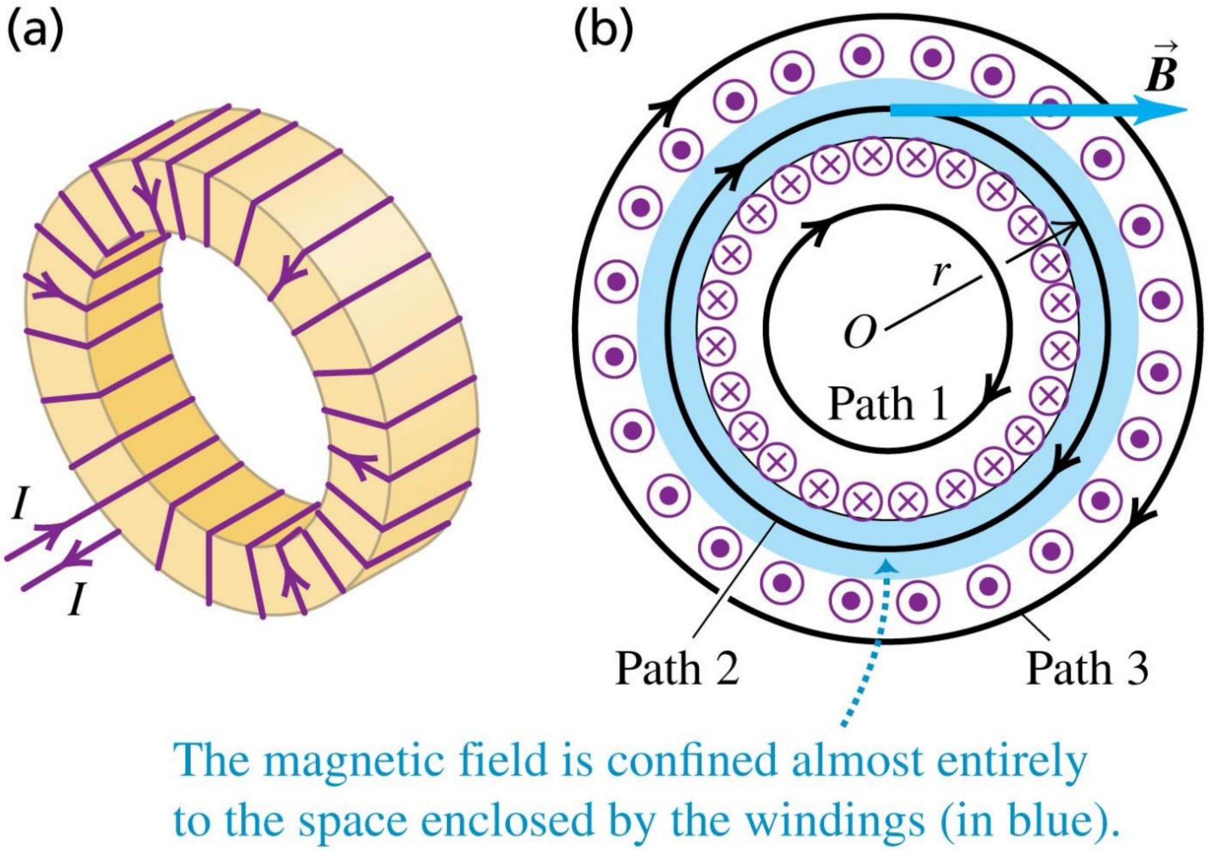


$$BL \approx \mu_0 NI$$
$$B \approx \mu_0 nI$$



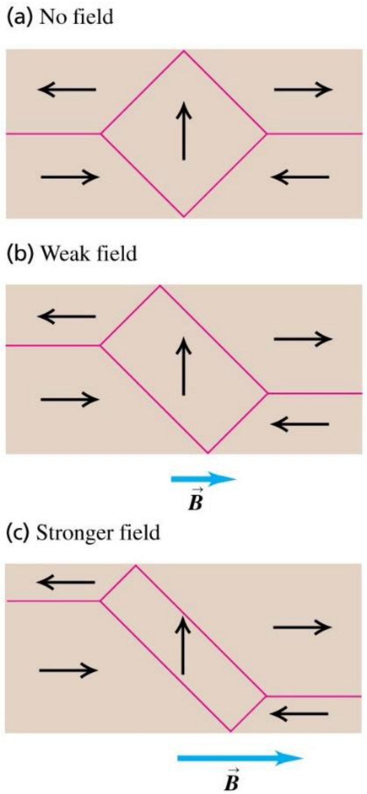
Field of a toroidal solenoid

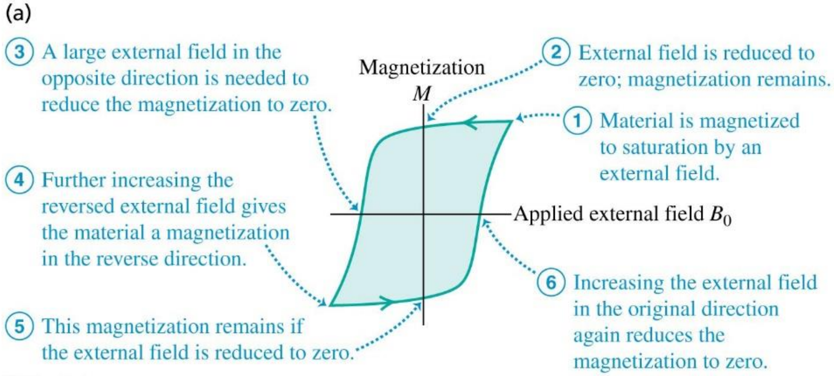
- A doughnut-shaped toroidal solenoid tightly wound with N turns of wire carrying a current I.



28.8 Magnetic materials* - Ferromagnetism

- In **ferromagnetic materials** (such as iron), atomic magnetic moments tend to line up parallel to each other in regions called magnetic domains.
- When there is no externally applied field, the domain magnetizations are randomly oriented.
- When an external magnetic field is present, the domain boundaries shift; the domains that are magnetized in the field direction grow, and those that are magnetized in other directions shrink.





28 Summary

- **Magnetic field of a moving charge**
- **Magnetic field of a current-carrying conductor**
- **Magnetic force between current-carrying conductors**
- **Ampere's law**
- **Magnetic field due to current distributions**