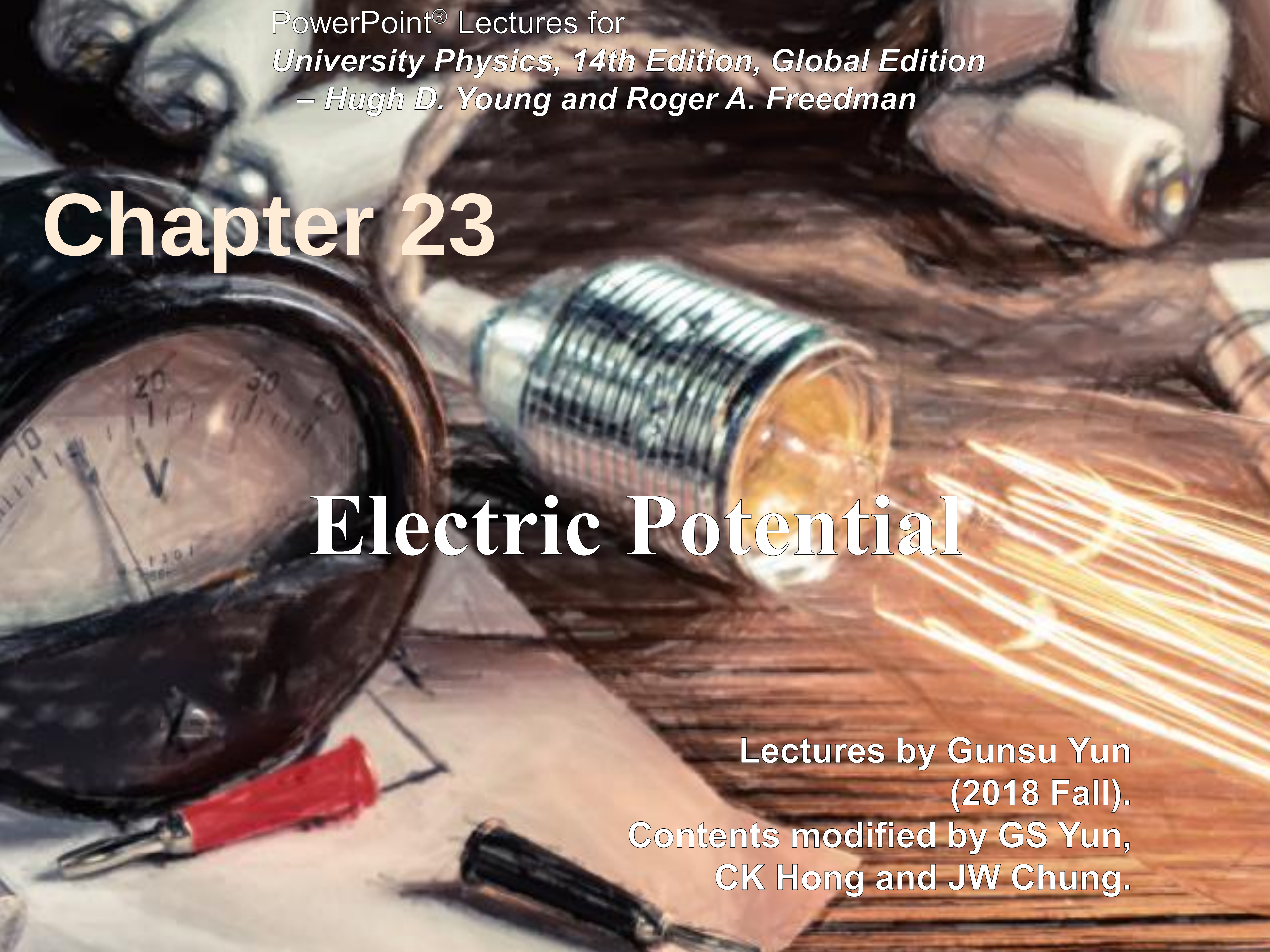

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– Hugh D. Young and Roger A. Freedman

Chapter 23

Electric Potential

Lectures by Gunsu Yun
(2018 Fall).
Contents modified by GS Yun,
CK Hong and JW Chung.

Goals for Chapter 23

- **Electric potential energy**
- **Electric potential**
- Equipotential surfaces
- Electric potential → electric field

(chap.21) **Substance** (Charge) → Force

→ **Field**, Field lines; Diploe

(chap.22) → **Flux, Gauss's Law**

(chap.23) **Force (-Work)** → Potential energy → Potential;

Equipotential/Gradient → **Field**

23.1 Electric potential energy

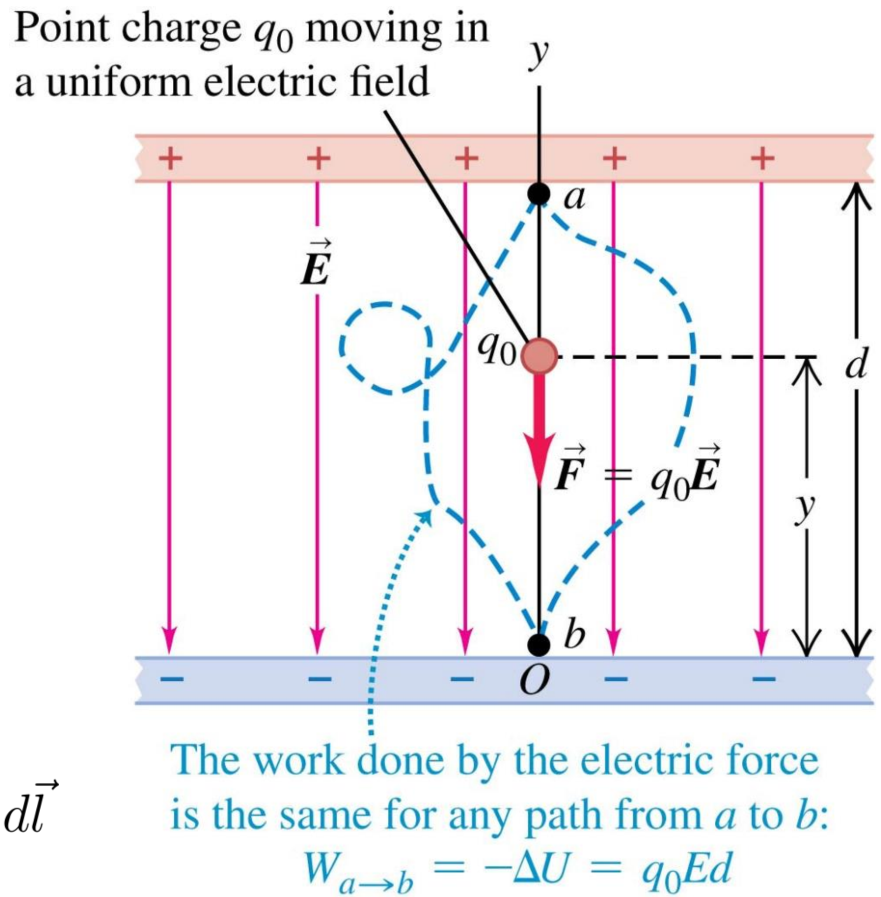
Electric potential energy in a uniform field

- In the figure, a uniform field exerts a downward force on a positive test charge.

- As the charge moves downward from point *a* to point *b*, **the work done by the field is independent of the path** the particle takes.

  → conservative force

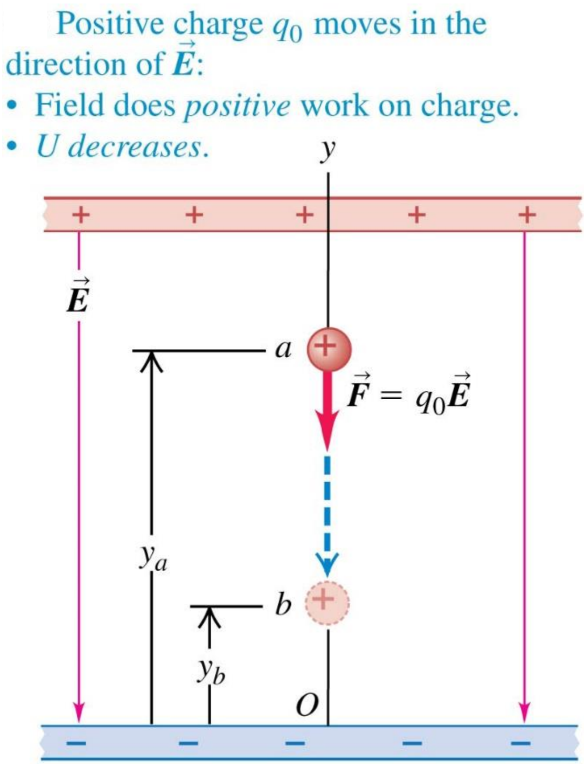
  → potential energy

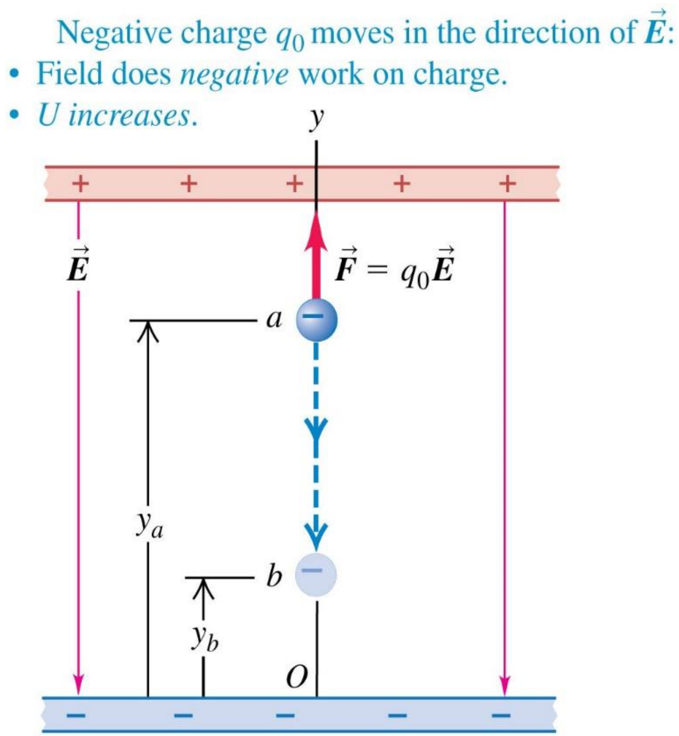


$$U = -\int_C \vec{F} \cdot d\vec{l} = -\int_C q_0\vec{E} \cdot d\vec{l}$$

A positive(negative) charge moving in a uniform field

- If the positive(negative) charge moves in the direction of the field, the field does positive(negative) work on the charge.
- The potential energy decreases(increases).



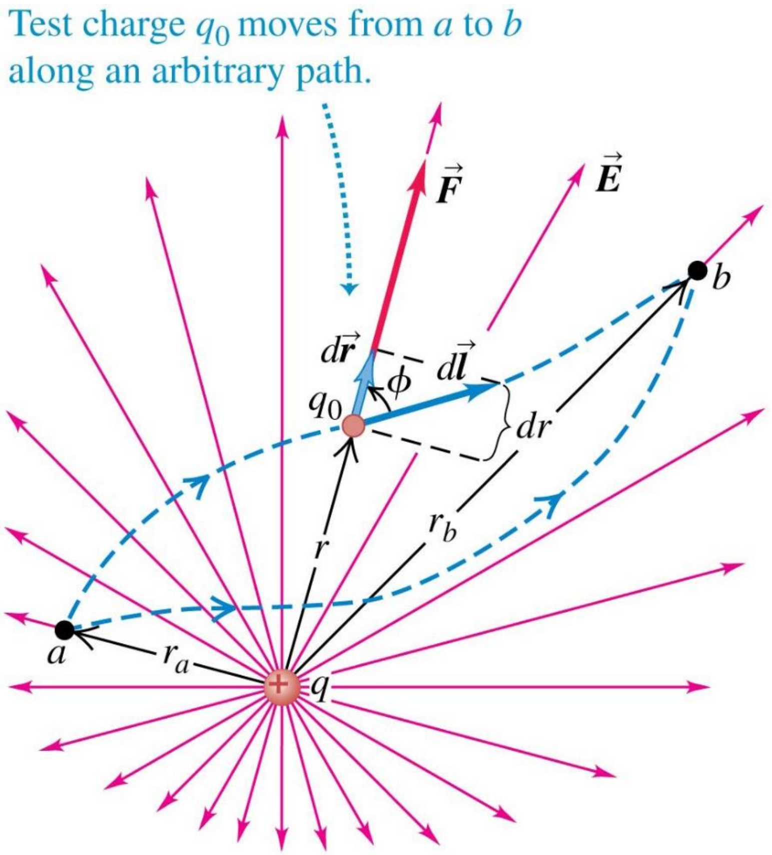


Electric potential energy of two point charges

- The work done by the electric field of one point charge on another does not depend on the path taken.

- Therefore, the electric potential energy only depends on the distance between the charges.

$$
\begin{aligned}
\Delta U &= U_b - U_a = -W_{a\to b} \\
&= -\int_a^b \vec{F}\cdot d\vec{l} \\
&= -\int_{r_a}^{r_b} \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r^2}dr \\
&= \frac{qq_0}{4\pi\epsilon_0}\left(\frac{1}{r_b}-\frac{1}{r_a}\right)
\end{aligned}
$$



Electric potential energy of two point charges

Electric potential energy of two point charges

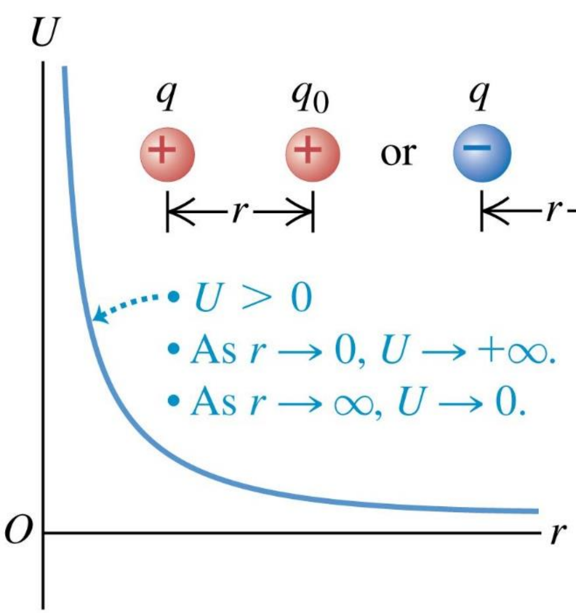
$$U = \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r}$$

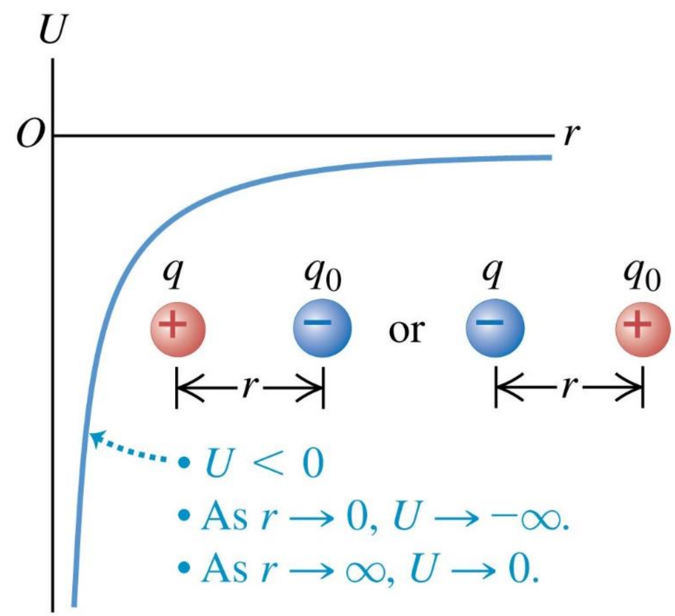
Values of two charges

Distance between two charges

Electric constant

- Potential energy is defined to be **zero** when the charges are infinitely far apart.

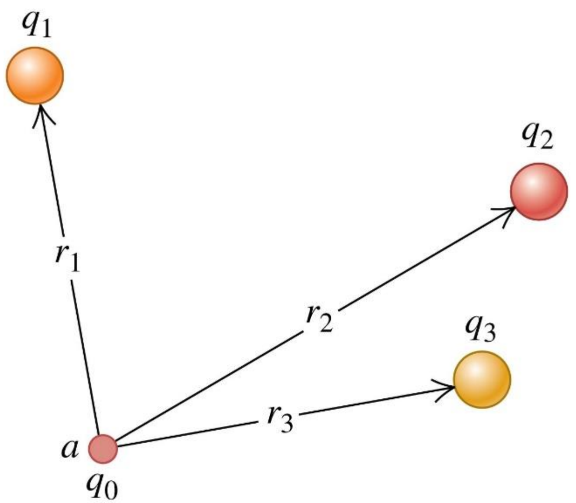




Electrical potential energy with several point charges

- The potential energy associated with q_0 depends on the other charges and their distances from q_0.

- The electric potential energy is the **algebraic sum**:



Electric potential energy of point charge q_0 and collection of charges $q_1, q_2, q_3, \ldots$

$$U = \frac{q_0}{4\pi\epsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \cdots\right) = \frac{q_0}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i}$$

Electric constant

Distances from q_0 to $q_1, q_2, q_3, \ldots$

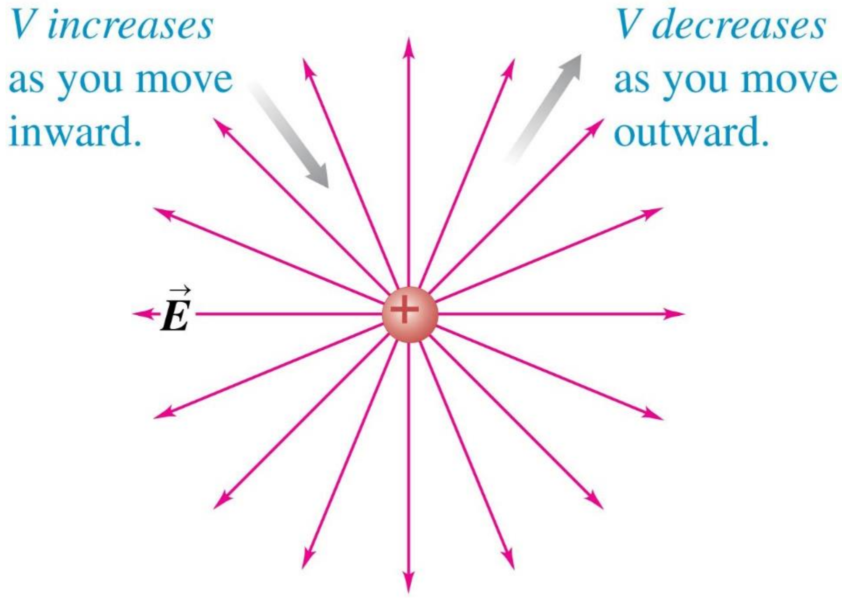
cf. Total potential energy U of several of point charges initially separated by ~~in~~finite distances:

$$U_{\text{tot}} = \frac{1}{4\pi\epsilon_0}\sum_{i<j}\frac{q_i q_j}{r_{ij}}$$

23.2 Electric potential

- **Electric Potential(V)** is electric potential energy per unit charge.
- The electric potential of b with respect to a ($V_{ba} = V_b - V_a$) equals the minus of the work done by the electric force when a unit charge moves from b to a.
- SI unit of electric potential: **volt**, **V** = J/C

$$\begin{aligned} V_b - V_a &\equiv (U_b - U_a)/q_0 \\ &= -\int_a^b q_0^{-1}\vec{F}\cdot d\vec{l} \\ &= -\int_a^b \vec{E}\cdot d\vec{l} \end{aligned}$$



Electric potential of point charges

- The potential due to a single point charge is:

$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$

- Like electric field, potential is independent of the test charge q_0 that we use to define it.

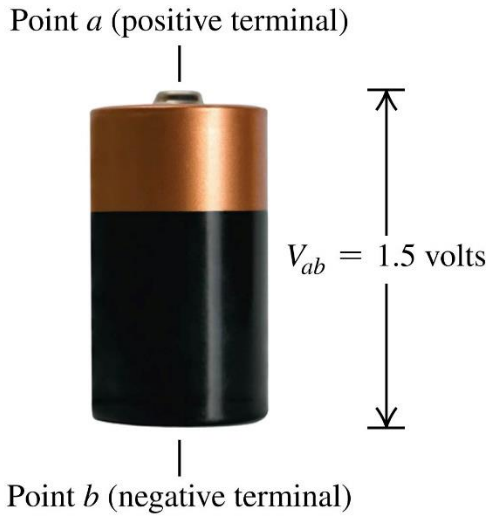
- For a collection of point charges:

$$V = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i}$$

Typical electric potential differences

TABLE 17–1 Some Typical Potential Differences (Voltages)

Source	Voltage (approx.)
Thundercloud to ground	10^8 V
High-voltage power line	10^5–10^6 V
Power supply for TV tube	10^4 V
Automobile ignition	10^4 V
Household outlet	10^2 V
Automobile battery	12 V
Flashlight battery	1.5 V
Resting potential across nerve membrane	10^{-1} V
Potential changes on skin (EKG and EEG)	10^{-4} V



The electron volt (a unit of energy)

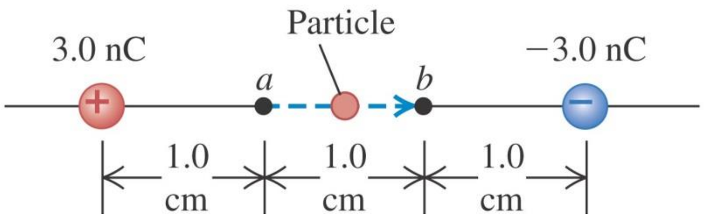
- $\Delta U = q\,\Delta V$
- If a particle with the charge of e (=1.602× 10^{-19} C) moves across a potential difference of 1 V, the change in kinetic (or potential) energy is defined as one **electron volt (eV)**:

$$1\ \text{eV} = 1.602 \times 10^{-19}\ \text{J}$$

- Pohang Accelerator Laboratory: ~1–10 GeV

Moving through a potential difference

In Fig. 23.15 a dust particle with mass $m = 5.0 \times 10^{-9}$ kg $= 5.0\ \mu$g and charge $q_0 = 2.0$ nC starts from rest and moves in a straight line from point a to point b. What is its speed v at point b?



The mechanical energy K+U is conserved since only conservative electric force acts on the particle.

$$0 + q_0 V_a = \tfrac{1}{2} m v^2 + q_0 V_b \quad \Rightarrow \quad v = \sqrt{\frac{2q_0(V_a - V_b)}{m}}$$

$$V_a = k\left(\frac{q}{r_{1a}} + \frac{-q}{r_{2a}}\right) = kq\left(\frac{1}{r_{1a}} - \frac{1}{r_{2a}}\right) = 1350 \text{ V}$$

$$V_b = -1350 \text{ V}$$

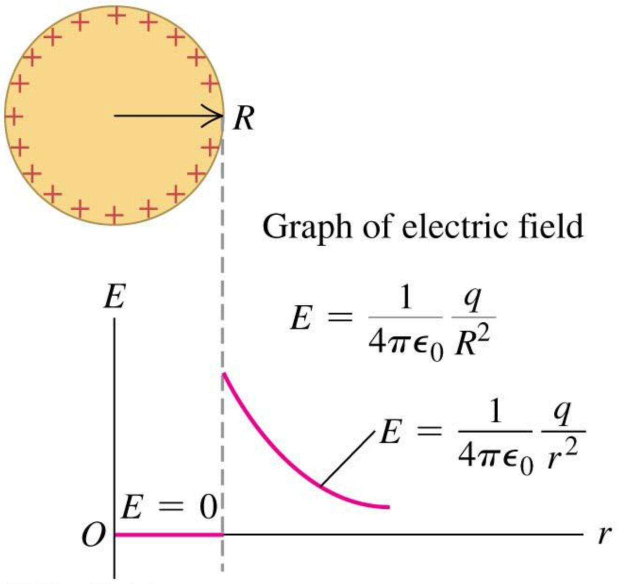
$$\Rightarrow \quad v = \sqrt{\frac{2(2.0 \times 10^{-9}\ \text{C})(2700\ \text{V})}{5.0 \times 10^{-9}\ \text{kg}}} = 46 \text{ m/s}$$

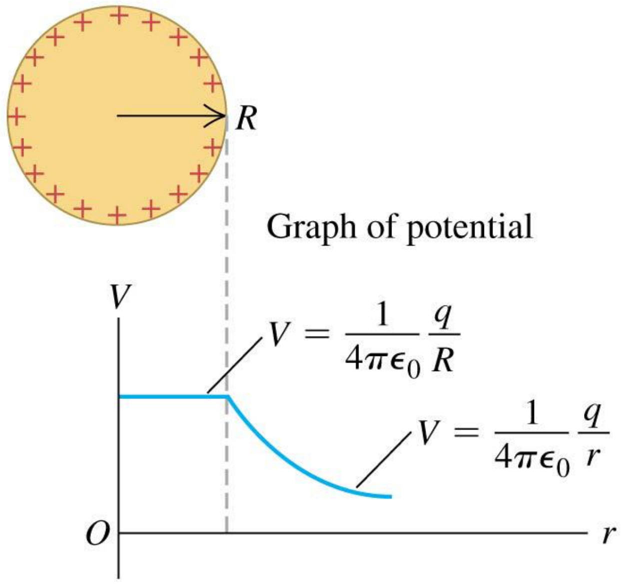
23.3 Calculating electric potential

Electric potential and field of a charged conductor

- A solid conducting sphere of radius *R* has a total charge *q*.
- The electric field inside the sphere is zero.
- Outside, $\vec{E} = \hat{r}\frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$

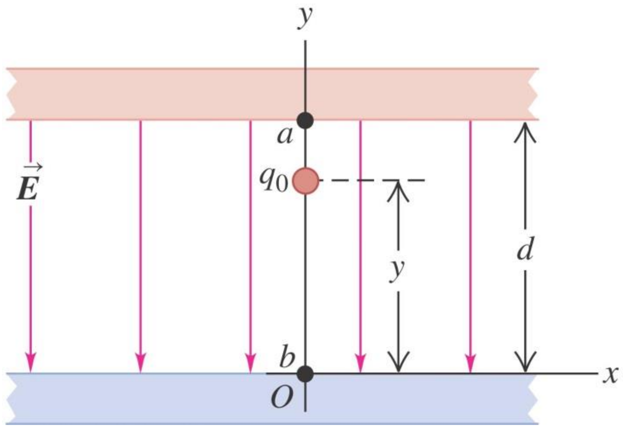
$$V = -\int_{\infty}^{r} \vec{E} \cdot d\vec{l}$$





Oppositely charged parallel plates

Find the potential at any height y between the two oppositely charged parallel plates discussed in Section 23.1 (Fig. 23.18).



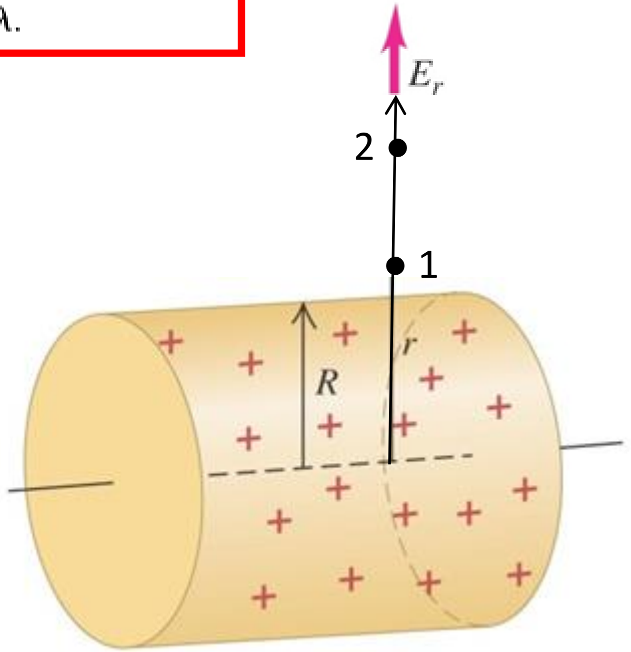
$$V(y) = \frac{U(y)}{q_0} = \frac{yq_0E}{q_0} = yE$$

$$V_a - V_b = Ed \text{ and } E = \frac{V_a - V_b}{d} = \frac{V_{ab}}{d}$$

(Volt/meter)

Line charge

Find the potential at a distance r from a very long line of charge with linear charge density (charge per unit length) λ.



$$\begin{aligned} V_2 - V_1 &= -\int_1^2 \vec{E}\cdot d\vec{l} = -\int_1^2 E_r dr = \frac{-\lambda}{2\pi\epsilon_0}\int_{r_1}^{r_2}\frac{dr}{r} \\ &= \frac{-\lambda}{2\pi\epsilon_0}\ln r\Big|_{r_1}^{r_2} = \frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{r_1}{r_2}\right) \end{aligned}$$

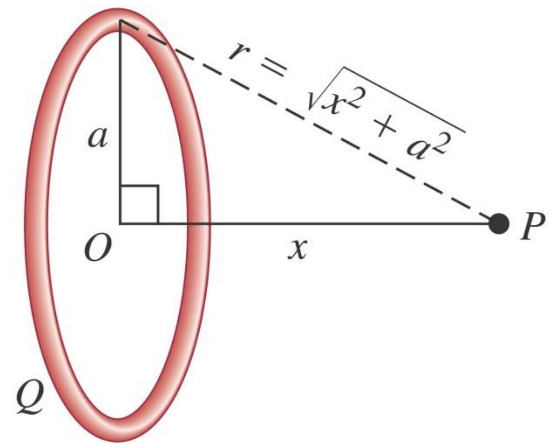
Taking $V = 0$ at the surface of the conducting wire of radius R,

$$V(r) = \frac{\lambda}{2\pi\epsilon_0}\ln\frac{R}{r} \text{ (for } r >= R)$$

How about $V(r)$ for $r < R$?

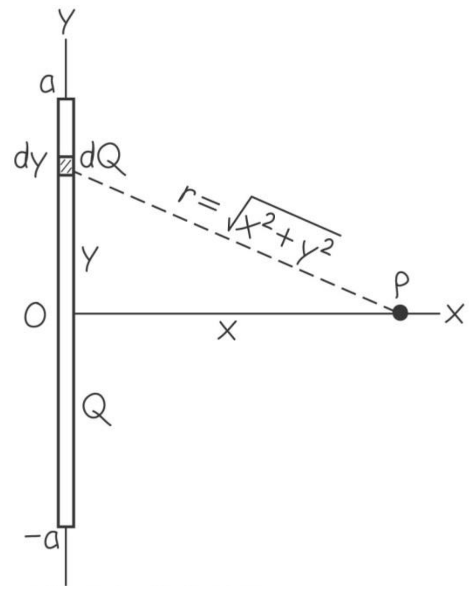
A ring of charge

Electric charge Q is distributed uniformly around a thin ring of radius a (Fig. 23.20). Find the potential at a point P on the ring axis at a distance x from the center of the ring.



$$V(x) = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r} = \frac{1}{4\pi\epsilon_0}\frac{1}{\sqrt{x^2+a^2}}\int dq = \frac{1}{4\pi\epsilon_0}\frac{Q}{\sqrt{x^2+a^2}}$$

Positive electric charge Q is distributed uniformly along a line of length $2a$ lying along the y-axis between $y = -a$ and $y = +a$ (Fig. 23.21). Find the electric potential at a point P on the x-axis at a distance x from the origin.

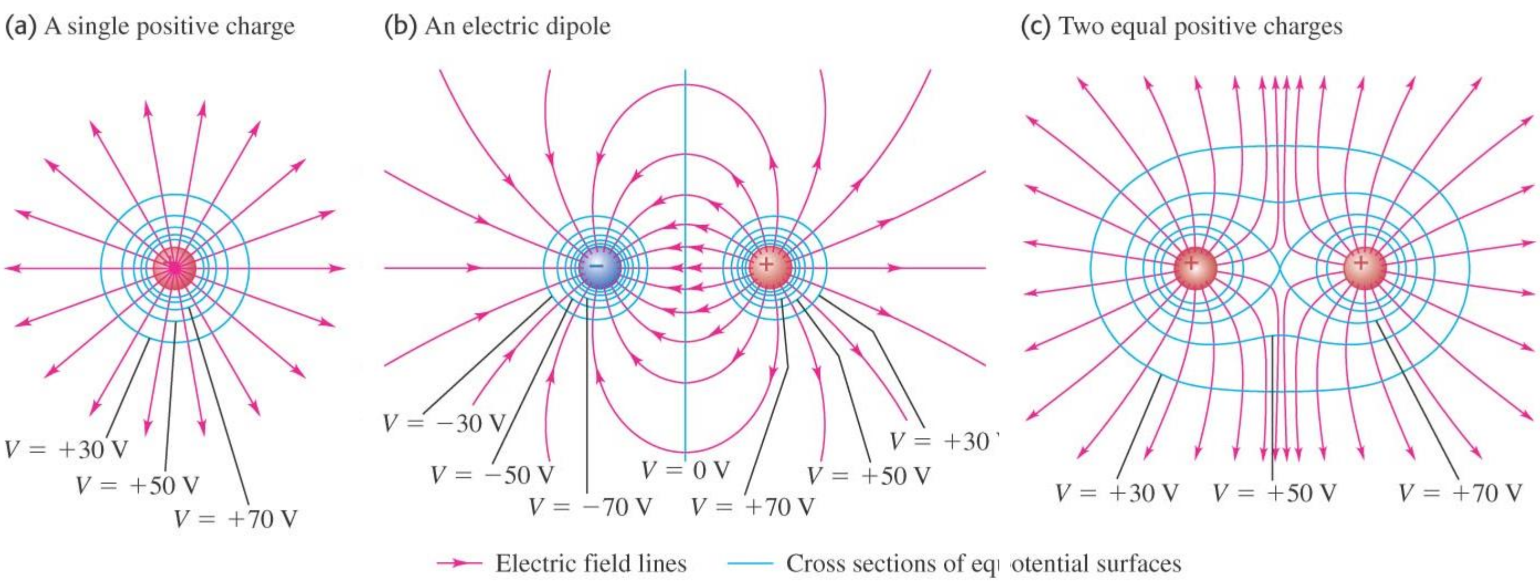


$$\begin{aligned} V(x) &= \frac{1}{4\pi\epsilon_0}\int_{y=-a}^{y=a}\frac{dQ}{r} = \frac{1}{4\pi\epsilon_0}\left(\frac{Q}{2a}\right)\int_{-a}^{a}\frac{dy}{\sqrt{y^2+x^2}} \\ &= \frac{1}{4\pi\epsilon_0}\left(\frac{Q}{2a}\right)\ln\left(\sqrt{y^2+x^2}+y\right)\Big|_{-a}^{a} \\ &= \frac{1}{4\pi\epsilon_0}\left(\frac{Q}{2a}\right)\ln\left(\frac{\sqrt{a^2+x^2}+a}{\sqrt{a^2+x^2}-a}\right) \end{aligned}$$

For $x \gg a$, $V(x) = \frac{1}{4\pi\epsilon_0}\frac{Q}{x}$.

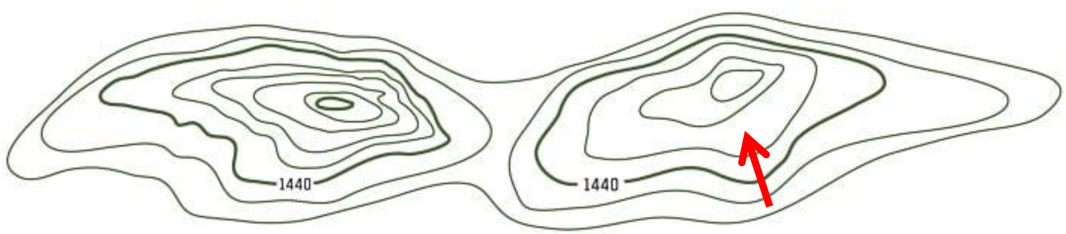
23.4 Equipotential surfaces and field lines

- An **equipotential surface** is a surface on which the electric potential is the same.
- Field lines and equipotential surfaces are always **mutually perpendicular**.
- Examples of equipotential surfaces (blue lines) and electric field lines (red lines):

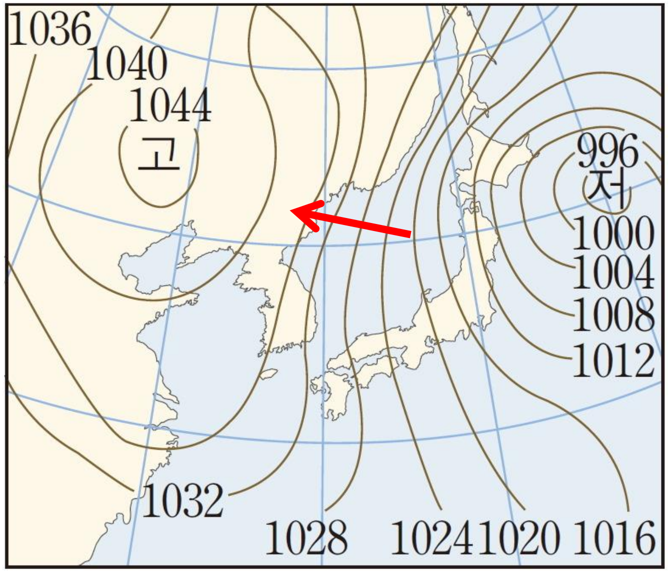


Equipotential and potential gradient

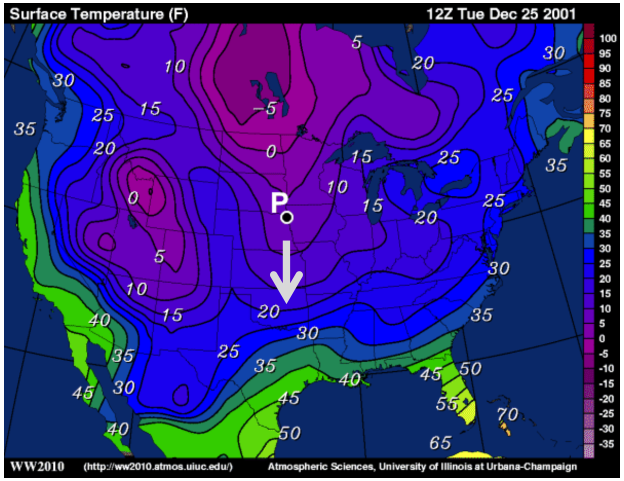
Topographic map:
contour lines (등고선)



Pressure map (기압도)

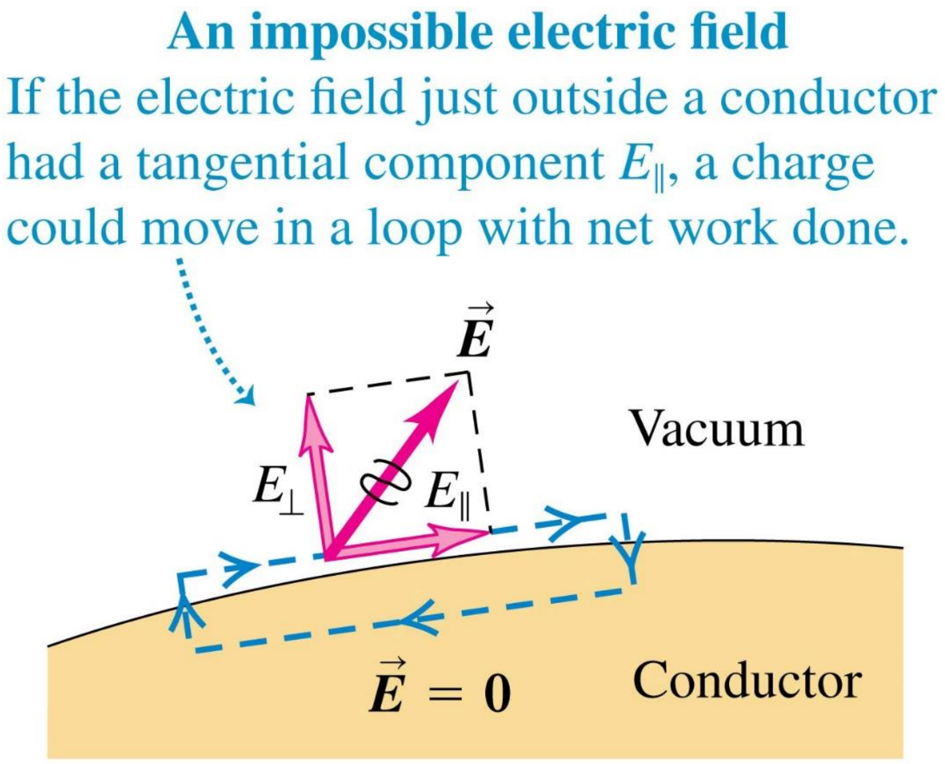


Temperature map



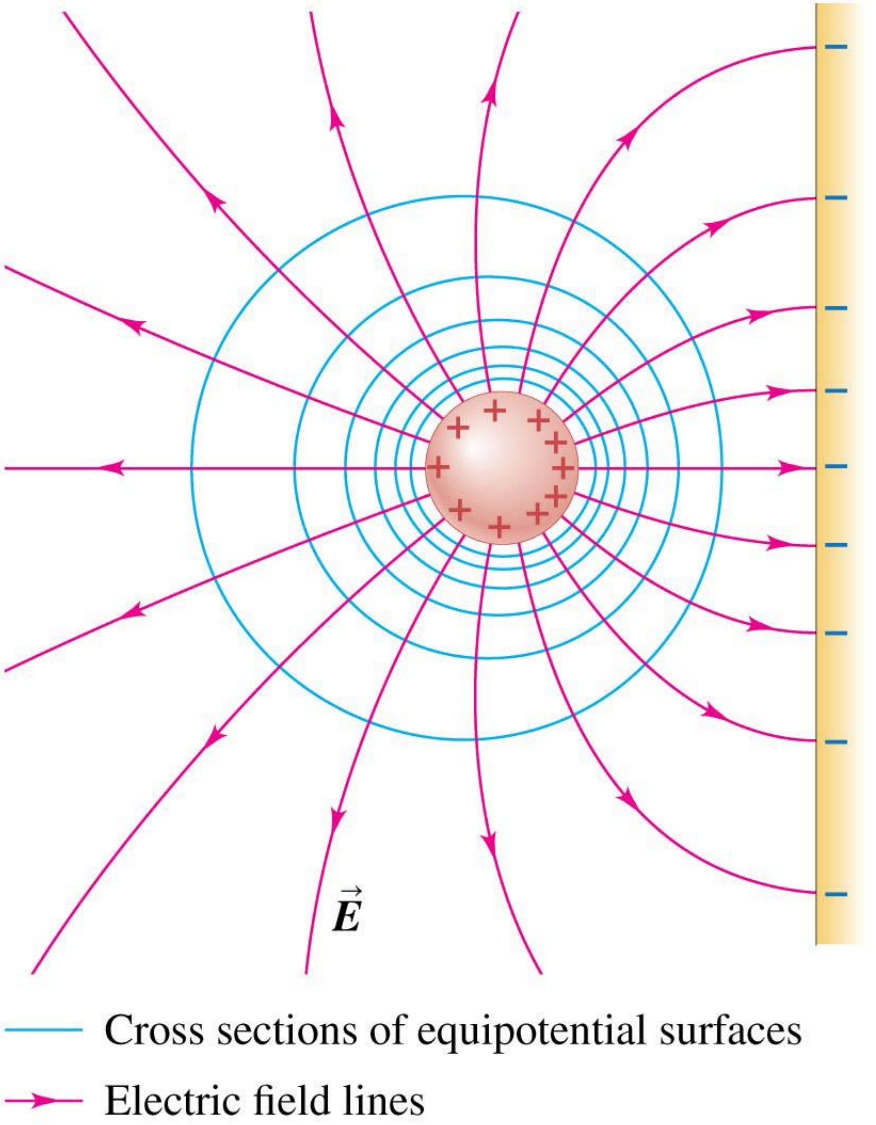
Equipotentials and conductors

- If the electric field had a **tangential component** at the surface of a conductor, a net amount of work would be done on a test charge by moving it around a loop as shown here—which is impossible because the electric force is conservative.



Equipotentials and conductors

- When all charges are **at rest** (*electro-statics*):
  - ✓ the surface of a conductor is always an **equipotential surface**.
  - ✓ the electric field just outside a conductor is always **perpendicular to the surface**.



23.5 Potential gradient

- The components of the electric field can be found by taking **partial derivatives** of the electric potential:

$$V = -\int \vec{E}\cdot d\vec{l} = -\left(\int E_x dx + \int E_y dy + \int E_z dz\right) \Rightarrow$$

Electric field components found from potential:

Each electric field component ...

$$E_x = -\frac{\partial V}{\partial x} \qquad E_y = -\frac{\partial V}{\partial y} \qquad E_z = -\frac{\partial V}{\partial z}$$

... equals the negative of the corresponding partial derivative of electric potential function V.

- Equivalently, the electric field is the negative **gradient** of the potential:

$$\vec{E} = -\nabla V = -\left(\hat{x}\frac{\partial V}{\partial x} + \hat{y}\frac{\partial V}{\partial y} + \hat{z}\frac{\partial V}{\partial z}\right)$$

Potential gradient

From Eq. (23.14) the potential at a radial distance r from a point charge q is $V = q/4\pi\epsilon_0 r$. Find the vector electric field from this expression for V.

$$\vec{E} = \hat{r}E_r \quad \text{by symmetry, where}$$

$$E_r = -\frac{\partial V}{\partial r} = -\frac{\partial}{\partial r}\left(\frac{q}{4\pi\epsilon_0 r}\right) = \frac{q}{4\pi\epsilon_0 r^2}$$

Component-wise:

Noting that $r = \left(x^2 + y^2 + z^2\right)^{1/2}$ and $\frac{\partial r}{\partial x} = \frac{x}{r}$,

$$E_x = -\frac{\partial V}{\partial x} = -\frac{\partial V}{\partial r}\frac{\partial r}{\partial x} = \frac{q}{4\pi\epsilon_0 r^2}\left(\frac{x}{r}\right)$$

Similiarly,

$$E_y = -\frac{\partial V}{\partial y} = \frac{q}{4\pi\epsilon_0 r^2}\left(\frac{y}{r}\right)$$

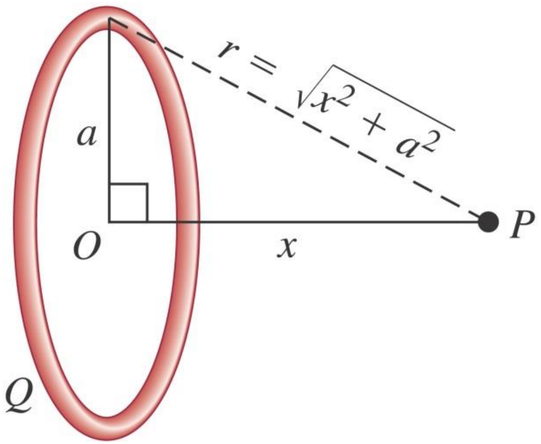
$$E_z = -\frac{\partial V}{\partial z} = \frac{q}{4\pi\epsilon_0 r^2}\left(\frac{z}{r}\right)$$

$$\Rightarrow \vec{E} = \frac{q}{4\pi\epsilon_0 r^2}\left(\frac{\hat{x}x + \hat{y}y + \hat{z}z}{r}\right) = \frac{q}{4\pi\epsilon_0 r^2}\hat{r}$$

In Example 23.11 (Section 23.3) we found that for a ring of charge with radius a and total charge Q, the potential at a point P on the ring's symmetry axis a distance x from the center is

$$V = \frac{Q}{4\pi\epsilon_0}\frac{1}{\left(x^2 + a^2\right)^{1/2}}$$

$$E_x = -\frac{\partial V}{\partial x} = \frac{Q}{4\pi\epsilon_0}\frac{x}{\left(x^2 + a^2\right)^{3/2}}$$



23 Summary

- Electric potential energy

- Electric potential

- Equipotential surfaces

- Finding electric field from electric potential