

Goals for Chapter 27

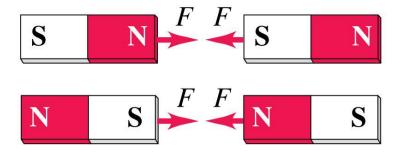
- Magnets
- Magnetic field, field line, and flux
- Magnetic force on moving charges
- Motion of charged particles in a magnetic field
- Magnetic force and torque on a current-carrying conductor



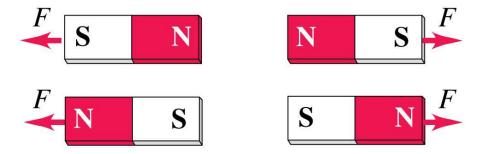
27.1 Magnetism - Magnetic poles

- North pole or N pole of a bar magnet points north.
- The other end is a south pole or S pole.
- Opposite poles attract each other, and like poles repel each other, as shown.

(a) Opposite poles attract.

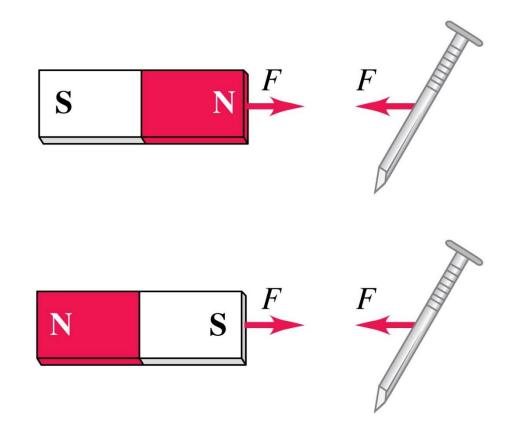


(b) Like poles repel.



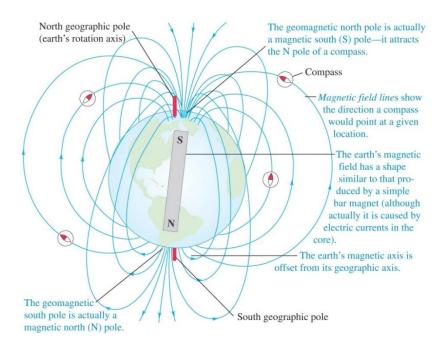
Magnetism and certain metals*

• An object that contains iron but is not itself magnetized is attracted by a permanent magnet.



Magnetic field of the earth*

- The earth itself is a giant magnet. Its geographic north pole is close to a magnetic south pole.
- The earth's magnetic axis is not quite parallel to the axis of rotation → magnetic declination or magnetic variation.
- The magnetic field is not horizontal \rightarrow magnetic inclination.

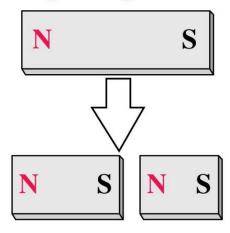


Magnetic monopoles

- Magnetic poles always come in pairs
- There is no experimental evidence for magnetic monopoles.

In contrast to electric charges, magnetic poles always come in pairs and can't be isolated.

Breaking a magnet in two ...



... yields two magnets, not two isolated poles.

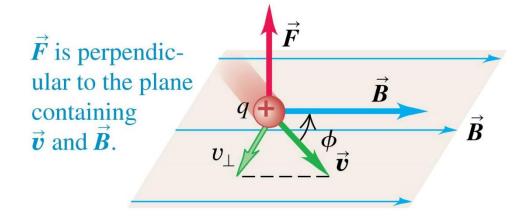
27.2 Magnetic field

- Magnetic field is a vector field: \vec{B}
- The direction of \vec{B} is defined as the direction in which the north pole of a compass needle tends to point.
- Sources of magnetic field
 - Permanent magnets
 - Moving charges
 - Currents

The magnetic force on a moving charge

- The magnitude of the magnetic force on a moving particle is proportional to the product of the charge and the speed of the particle.
- The direction is perpendicular to the velocity of the particle and also to the magnetic field.

A charge moving at an angle ϕ to a magnetic field experiences a magnetic force with magnitude $F = |q|v_{\perp}B = |q|vB \sin \phi$.



Particle's charge

Magnetic force on a moving charged particle

$$\vec{F} = \vec{q}\vec{v} \times \vec{B}$$

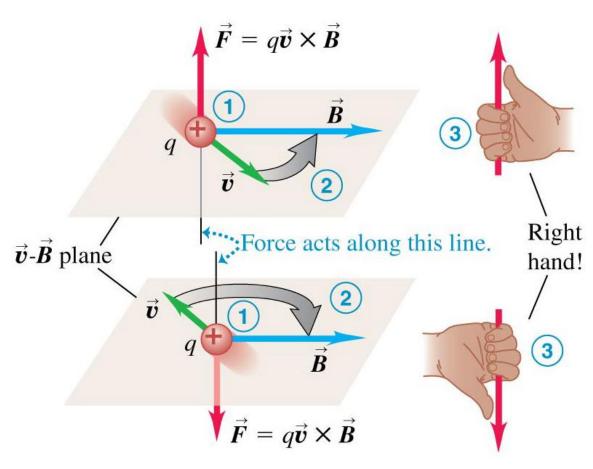
Magnetic field

Particle's velocity

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$
 - Lorentz force

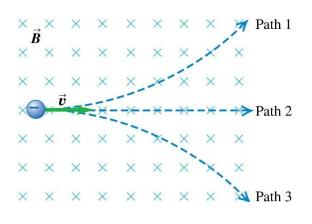
Right-hand rule for magnetic force

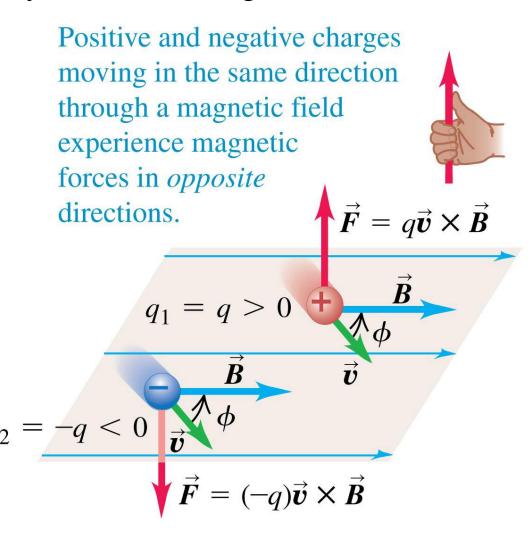
• The **right-hand rule** gives the direction of the force on a positive charge.



Equal velocities but opposite signs

- Imagine two charges of the same magnitude but opposite sign moving with the same velocity in the same magnetic field.
- The magnetic forces on the charges are equal in magnitude but opposite in direction.



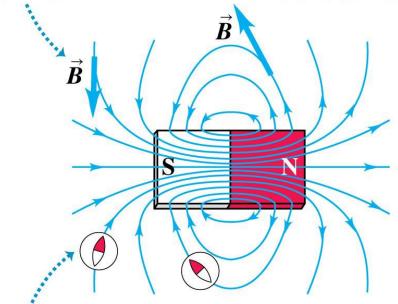


27.3 Magnetic field lines and magnetic flux

- Magnetic field is depicted by magnetic field lines.
- The direction of the magnetic field is tangent to the magnetic field line.
- Field lines never intersect.
- Magnetic field lines are not lines of magnetic force.

At each point, the field line is tangent to the magnetic-field vector \vec{B} .

The more densely the field lines are packed, the stronger the field is at that point.



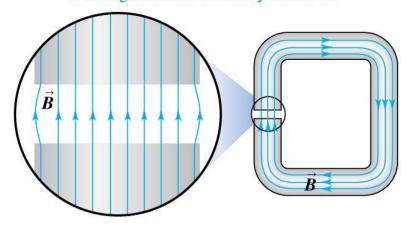
At each point, the field lines point in the same direction a compass would ...

... therefore, magnetic field lines point *away* from N poles and toward S poles.

Examples of magnetic field

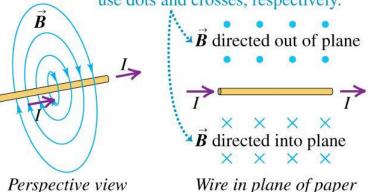
(a) Magnetic field of a C-shaped magnet

Between flat, parallel magnetic poles, the magnetic field is nearly uniform.

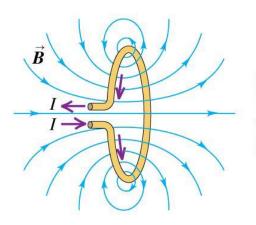


(b) Magnetic field of a straight current-carrying wire

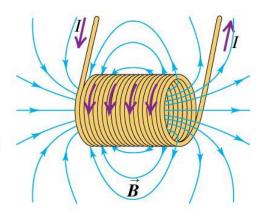
To represent a field coming out of or going into the plane of the paper, we use dots and crosses, respectively.



(c) Magnetic fields of a current-carrying loop and a current-carrying coil (solenoid)



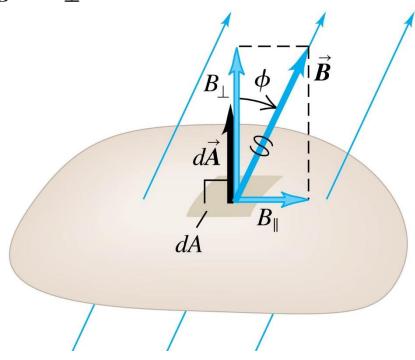
Notice that the field of the loop and, especially, that of the coil look like the field of a bar magnet (see Fig. 27.11).



Magnetic flux

- Any surface can be divided into elements of **infinitesimally small** area dA.
- The magnetic flux through the area element is defined to be

$$d\Phi_B = B_{\perp} dA$$
.



Magnetic flux

• The **total magnetic flux** through the surface is the sum of the contributions from the individual area elements:

Magnitude of magnetic field
$$\vec{B}$$
 perpendicular to surface

Magnetic flux
through a surface $\Phi_B = \int \vec{B} \cos \phi \, dA = \int \vec{B} \cdot dA = \int \vec{B} \cdot dA$

Angle between \vec{B} Element of Vector element and normal to surface surface area

The magnetic flux through any closed surface is zero:

Gauss's law for magnetism:

The total magnetic flux through any closed surface ... $\oint \vec{B} \cdot d\vec{A} = 0 \quad ... \text{ equals zero.}$

Units of magnetic field and magnetic flux

• SI unit of **magnetic field** *B*: **tesla**, **T**

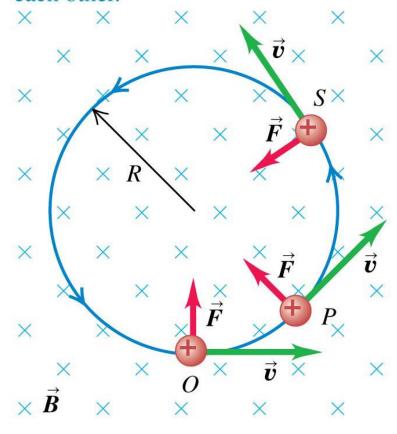
- Another unit of B: gauss, G (1 G = 10^{-4} T)
- The magnetic field of the earth is on the order of 10^{-4} T or 1 G.
- SI unit of magnetic flux Φ_R : weber, Wb
 - $1 \text{ Wb} = 1 \text{ T} \cdot \text{m}^2$

27.4 Motion of charged particles in a magnetic field

- When a charged particle moves in a magnetic field, it is acted on by the magnetic force.
- The force is always perpendicular to the velocity, so it cannot change the speed of the particle (no work done).

$$qvB = m\frac{v^2}{R} \to R = \frac{mv}{qB}$$

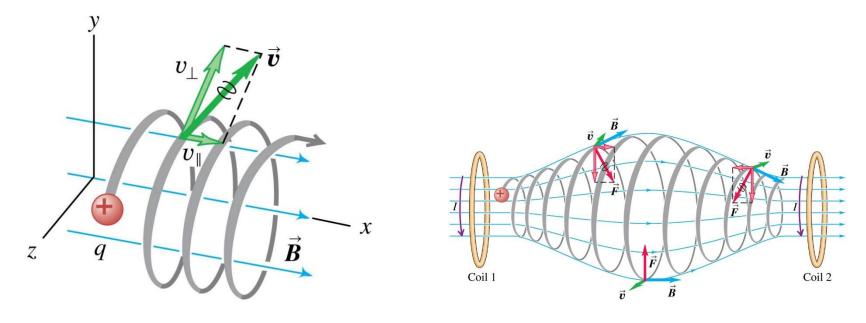
A charge moving at right angles to a uniform \vec{B} field moves in a circle at constant speed because \vec{F} and \vec{v} are always perpendicular to each other.



Helical motion

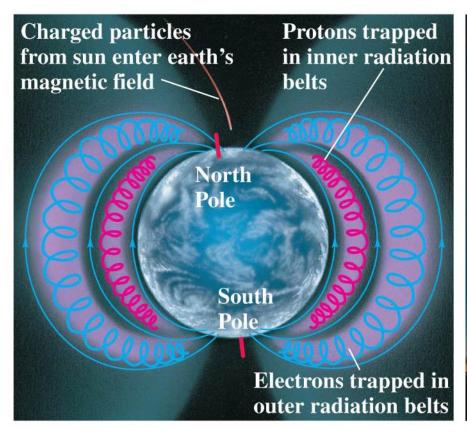
- If the particle has velocity components parallel to and perpendicular to the field, its path is a helix.
- The speed and kinetic energy of the particle remain constant.

This particle's motion has components both parallel (v_{\parallel}) and perpendicular (v_{\perp}) to the magnetic field, so it moves in a helical path.



The Van Allen radiation belts*

• Near the poles, charged particles from these belts can enter the atmosphere, producing the aurora borealis ("northern lights") and aurora australis ("southern lights").





27.5 Applications of motion of charged particles

Velocity selector

Particles of specific speed are selected by balancing the forces due to electric and magnetic fields.

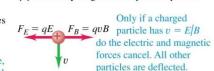
$$0 = \sum F_y = -qE + qvB \Rightarrow v = \frac{E}{B}$$

Thomson's e/m experiment

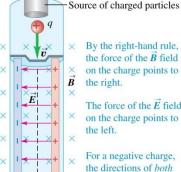
The first measurement of the ratio, e/m_e , of an electron in 1897 by J. J. Thomson (1856-1940), who was credited with the first discovery of the electron.

$$|q_e V| = \frac{1}{2} m_e v^2 = \frac{1}{2} m_e \frac{E^2}{B^2}$$
 $\Rightarrow \frac{|q_e|}{m_e} = \frac{E^2}{2B^2 |V|}$
$$\frac{e}{m_e} = 1.758820150(44) \times 10^{11} \text{ C/kg}$$

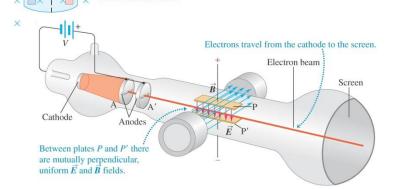
(a) Schematic diagram of velocity selector



(b) Free-body diagram for a positive particle



forces are reversed.



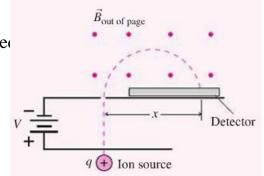
Mass spectrometers

Masses of velocity-selected ions were first measured by Francis Aston in 1919 by guiding the ions into a uniform magnetic field. Isotopes of the elements were discovered by this method.

$$\frac{1}{2}mv^2 = qV \rightarrow mv^2 = 2qV$$

$$m\frac{v^2}{r} = qvB \rightarrow m^2v^2 = (qBr)^2$$

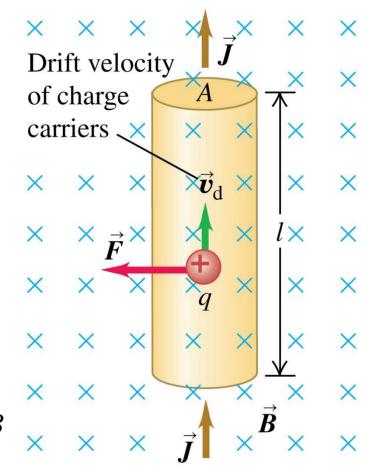
$$\Rightarrow m = \frac{(qBr)^2}{2qV} = \frac{qr^2B^2}{2V}$$



27.6 Magnetic force on a current-carrying conductor

- A straight segment of a conducting wire with length *l* and cross-sectional area *A*.
- The magnitude of the force on a single charge is $F = qv_dB$.
- If the number of charges per unit volume is *n*, then the **total force** on all the charges in this segment is

$$F = (nAl)(qv_dB) = (nqv_dA)(lB) = I \, lB$$

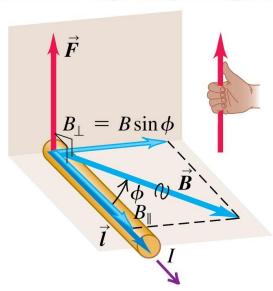


Magnetic force on a current-carrying conductor

• The force is always perpendicular to both the conductor and the field, with the direction determined by the same **right-hand rule** used for a moving positive charge.

Force \vec{F} on a straight wire carrying a positive current and oriented at an angle ϕ to a magnetic field \vec{B} :

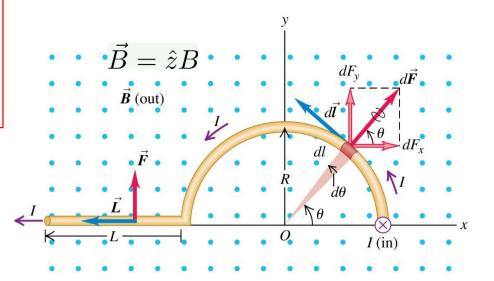
- Magnitude is $F = IlB_{\perp} = IlB \sin \phi$.
- Direction of \vec{F} is given by the right-hand rule.



Magnetic force on a curved conductor

In Fig. 27.30 the magnetic field \vec{B} is uniform and perpendicular to the plane of the figure, pointing out of the page. The conductor, carrying current I to the left, has three segments: (1) a straight segment with length L perpendicular to the plane of the figure, (2) a semicircle with radius R, and (3) another straight segment with length L parallel to the x-axis. Find the total magnetic force on this conductor.

$$\begin{array}{rcl} d\vec{F} & = & Id\vec{l} \times \vec{B} \\ \vec{F} & = & \int Id\vec{l} \times \vec{B} = - \int I\vec{B} \times d\vec{l} \end{array}$$



For segment (1),
$$\vec{F}_1 = I\vec{L} \times \vec{B} = IL\hat{z} \times B\vec{z} = 0$$

For segment (3), $\vec{F}_3 = ILB(-\hat{x}) \times \vec{z} = ILB\hat{y}$ For the curved segment (2), $dl = Rd\theta$ and $d\vec{l} = dl (-\sin\theta \hat{x} + \cos\theta \hat{y})$ at angle θ .

$$\vec{F} = \vec{F_1} + \vec{F_2} + \vec{F_3} = IB(L + 2R)\hat{y}$$

$$\vec{F}_{2} = \int IBd\vec{l} \times \hat{z}$$

$$= \int_{0}^{\pi} IBRd\theta \left(-\hat{x}\sin\theta + \hat{y}\cos\theta\right) \times \hat{z}$$

$$= \int_{0}^{\pi} IBRd\theta \left(\hat{y}\sin\theta + \hat{x}\cos\theta\right)$$

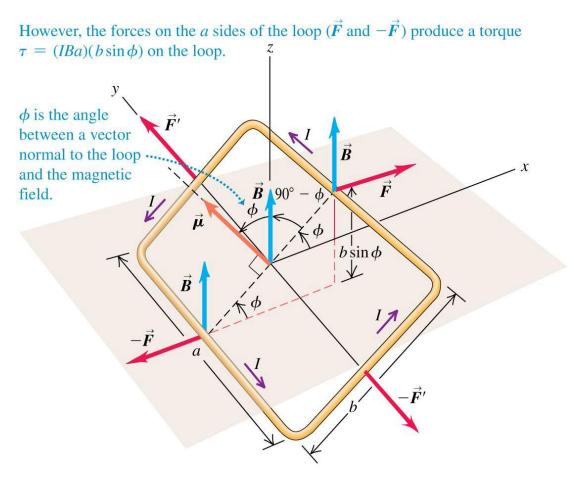
$$= IB(2R)\hat{y}$$

27.7 Force and torque on a current loop

• The net force on a current loop in a uniform magnetic field is zero.

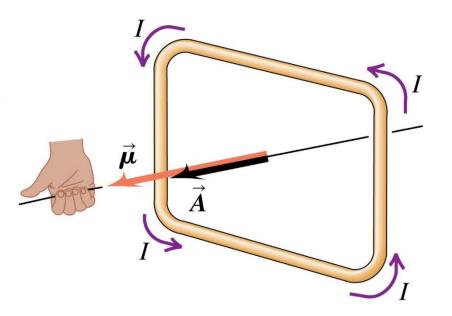
(a)

The two pairs of forces acting on the loop cancel, so no net force acts on the loop.



Magnetic torque on a current loop

- A magnetic moment μ is defined with magnitude *IA* and direction as shown.
- The net torque on the loop is given by the vector product:



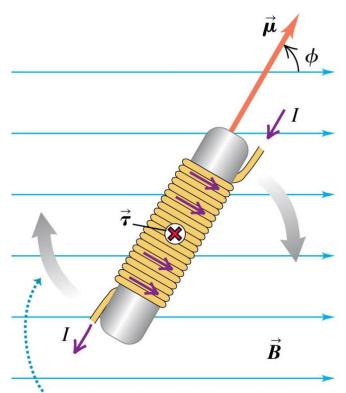
Vector magnetic torque Magnetic dipole moment on a current loop
$$\vec{\tau} = \vec{\mu} \times \vec{B} + \cdots$$
 Magnetic field

Potential energy for a magnetic dipole
$$U = -\vec{\mu} \cdot \vec{B} = -\mu B \cos \phi$$
 Angle between in a magnetic field Magnetic field

Magnetic torque on a coil

• The magnetic moment μ of a coil composed of N turns of loops:

$$\mu = NIA$$



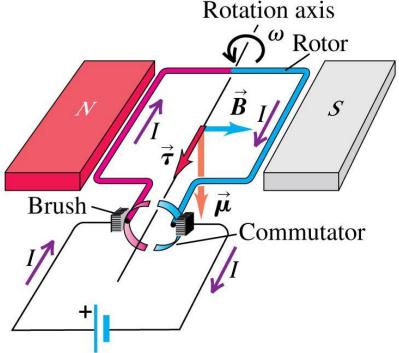
The torque tends to make the solenoid rotate clockwise in the plane of the page, aligning magnetic moment $\vec{\mu}$ with field \vec{B} .

27.8 The direct-current motor

- A simple dc motor
- The **rotor** is a wire loop that is free to rotate about an axis; the rotor ends are attached to the two curved conductors that form the **commutator**.

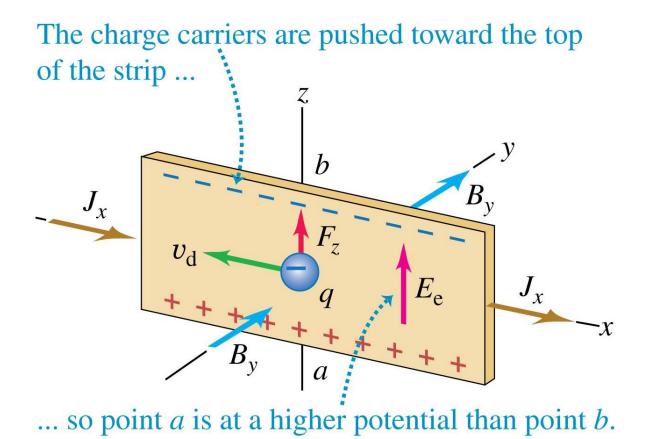
 Current flows into the red side of the rotor and out of the blue side.
 Rotation a

 Therefore the magnetic torque causes the rotor to spin counter-clockwise.



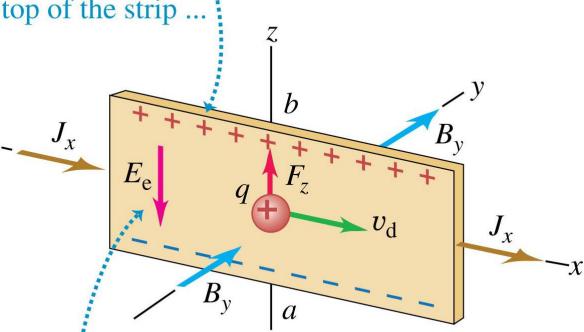
27.9 The Hall effect: negative charge carriers*

• When a current is placed in a magnetic field, the **Hall emf** emerges whether the charge carriers are negative or positive.



The Hall effect: positive charge carriers

The charge carriers are again pushed toward the top of the strip ...:



... so the polarity of the potential difference is opposite to that for negative charge carriers.

27 Summary

- Magnetic forces
- Magnetic field and flux
- Motion in a magnetic field
- Magnetic force and torque on a conductor
- Electric motors
- Hall effect