# CSED211 : Introduction to Computer SW Architecture

## Lecture 2: Bits, Bytes, Integers

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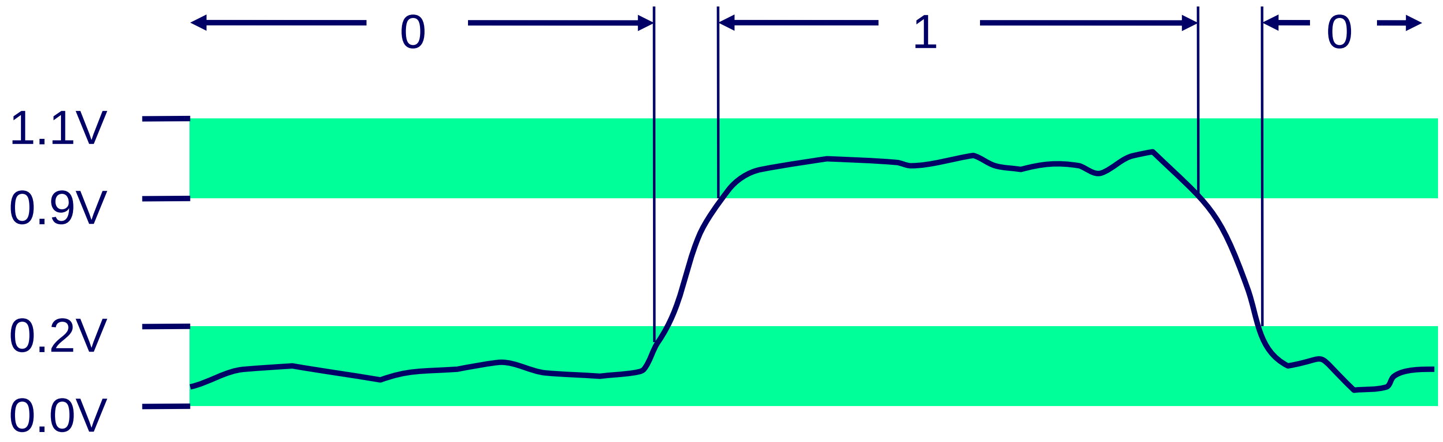
*Disclaimer:
Most slides are taken from author’s lecture slides.

# Today: Bits, Bytes, and Integers

- Representing information as bits
- Bit-level manipulations
- Integers
  - Representation: unsigned and signed
  - Conversion, casting
  - Expanding, truncating
  - Addition, negation, multiplication, shifting
- Representation in memory, pointers, strings

# Everything is bits

- Each bit is 0 or 1
- By encoding/interpreting sets of bits in various ways
  - Computers determine what to do (instructions)
  - … and represent and manipulate numbers, sets, strings, etc…
- Why bits?  Electronic Implementation
  - Easy to store with bi-stable elements
  - Reliably transmitted on noisy and inaccurate wires



# For example, can count in binary

- Base 2 Number Representation
  - Represent $15213_{10}$ as $11101101101101_2$
  - Represent $1.20_{10}$ as $1.0011001100110011[0011]\ldots_2$
  - Represent $1.5213 \text{ X } 10^4$ as $1.1101101101101_2 \text{ X } 2^{13}$

# Encoding Byte Values

- Byte = 8 bits
  - Binary $00000000_2$ to $11111111_2$
  - Decimal: $0_{10}$ to $255_{10}$
  - Hexadecimal $00_{16}$ to $FF_{16}$
    - Base 16 number representation
    - Use characters '0' to '9' and 'A' to 'F'
    - Write $FA1D37B_{16}$ in C as
      - 0xFA1D37B
      - 0xfa1d37b

| Hex | Decimal | Binary |
|---|---|---|
| 0 | 0 | 0000 |
| 1 | 1 | 0001 |
| 2 | 2 | 0010 |
| 3 | 3 | 0011 |
| 4 | 4 | 0100 |
| 5 | 5 | 0101 |
| 6 | 6 | 0110 |
| 7 | 7 | 0111 |
| 8 | 8 | 1000 |
| 9 | 9 | 1001 |
| A | 10 | 1010 |
| B | 11 | 1011 |
| C | 12 | 1100 |
| D | 13 | 1101 |
| E | 14 | 1110 |
| F | 15 | 1111 |

# Example Data Representations

| C Data Type | Typical 32-bit | Typical 64-bit | x86-64 |
|---|---|---|---|
| **`char`** | 1 | 1 | 1 |
| **`short`** | 2 | 2 | 2 |
| **`int`** | 4 | 4 | 4 |
| **`long`** | 4 | 8 | 8 |
| **`float`** | 4 | 4 | 4 |
| **`double`** | 8 | 8 | 8 |
| **`long double`** | − | − | 10/16 |
| pointer | 4 | 8 | 8 |

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# Boolean Algebra

- Developed by George Boole in 19th Century
  - Algebraic representation of logic
    - Encode "True" as 1 and "False" as 0

**And**

- **A&B = 1 when both A=1 and B=1**

| & | 0 | 1 |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 1 |

**Or**

- **A|B = 1 when either A=1 or B=1**

| \| | 0 | 1 |
|---|---|---|
| 0 | 0 | 1 |
| 1 | 1 | 1 |

**Not**

- **~A = 1 when A=0**

| ~ | |
|---|---|
| 0 | 1 |
| 1 | 0 |

**Exclusive-Or (Xor)**

- **A^B = 1 when either A=1 or B=1, but not both**

| ^ | 0 | 1 |
|---|---|---|
| 0 | 0 | 1 |
| 1 | 1 | 0 |

# General Boolean Algebras

- Operate on Bit Vectors
  - Operations applied bitwise

```
  01101001     01101001     01101001
& 01010101   | 01010101   ^ 01010101   ~ 01010101
  01000001     01111101     00111100     10101010
```

- All of the Properties of Boolean Algebra Apply

# Ex: Representing & Manipulating Sets

- Representation
  - Width w bit vector represents subsets of {0, …, w–1}
  - $a_j = 1$ if $j \in A$

    - 01101001 { 0, 3, 5, 6 }
    - *76543210*

    - 01010101 { 0, 2, 4, 6 }
    - *76543210*

- Operations
  - & Intersection 01000001 { 0, 6 }
  - | Union 01111101 { 0, 2, 3, 4, 5, 6 }
  - ^ Symmetric difference 00111100 { 2, 3, 4, 5 }
  - ~ Complement 10101010 { 1, 3, 5, 7 }

# Bit-Level Operations in C

- Operations &, |, ~, ^ Available in C
  - Apply to any “integral” data type
    - long, int, short, char, unsigned
  - View arguments as bit vectors
  - Arguments applied bit-wise
- Examples (Char data type)
  - ~0x41 → 0xBE
    - $\sim 01000001_2 \rightarrow 10111110_2$
  - ~0x00 → 0xFF
    - $\sim 00000000_2 \rightarrow 11111111_2$
  - 0x69 & 0x55 → 0x41
    - $01101001_2 \ \& \ 01010101_2 \rightarrow 01000001_2$
  - 0x69 | 0x55 → 0x7D
    - $01101001_2 \mid 01010101_2 \rightarrow 01111101_2$

# Contrast: Logic Operations in C

- Contrast to Logical Operators
  - &&, ||, !
    - View 0 as "Fa
    - Anythi
    - Always
    - Early t

Watch out for && vs. & (and || vs. |)…
one of the more common oopsies in
C programming

- Examples (char data type)
  - !0x41 → 0x00
  - !0x00 → 0x01
  - !!0x41 → 0x01
  - 0x69 && 0x55 → 0x01
  - 0x69 || 0x55 → 0x01
  - p && *p (avoids null pointer access)

# Shift Operations

- Left Shift: `x << y`
  - Shift bit-vector **x** left **y** positions
    - Throw away extra bits on left
    - Fill with 0's on right
- Right Shift: `x >> y`
  - Shift bit-vector **x** right **y** positions
    - Throw away extra bits on right
  - Logical shift
    - Fill with 0's on left
  - Arithmetic shift
    - Replicate most significant bit on left
- Undefined Behavior
  - Shift amount < 0 or ≥ word size

| | |
|---|---|
| Argument **x** | 01100010 |
| << 3 | 00010*000* |
| Log. >> 2 | *00*011000 |
| Arith. >> 2 | *00*011000 |

| | |
|---|---|
| Argument **x** | 10100010 |
| << 3 | 00010*000* |
| Log. >> 2 | *00*101000 |
| Arith. >> 2 | *11*101000 |

# Today: Bits, Bytes, and Integers

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# Encoding Integers

Unsigned

$$B2U(X) = \sum_{i=0}^{w-1} x_i \cdot 2^i$$

Two's Complement

$$B2T(X) = -x_{w-1} \cdot 2^{w-1} + \sum_{i=0}^{w-2} x_i \cdot 2^i$$

Sign Bit

```
short int x =  15213;
short int y = -15213;
```

- C `short` 2 bytes long

| | Decimal | Hex | Binary |
|---|---|---|---|
| **x** | 15213 | **3B 6D** | **00111011 01101101** |
| **y** | -15213 | **C4 93** | **11000100 10010011** |

- Sign Bit
  – For 2's complement, most significant bit indicates sign
    - 0 for nonnegative
    - 1 for negative

# Encoding Example (Cont.)

```
x =      15213: 00111011 01101101
y =     -15213: 11000100 10010011
```

| Weight | 15213 | | -15213 | |
|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 |
| 2 | 0 | 0 | 1 | 2 |
| 4 | 1 | 4 | 0 | 0 |
| 8 | 1 | 8 | 0 | 0 |
| 16 | 0 | 0 | 1 | 16 |
| 32 | 1 | 32 | 0 | 0 |
| 64 | 1 | 64 | 0 | 0 |
| 128 | 0 | 0 | 1 | 128 |
| 256 | 1 | 256 | 0 | 0 |
| 512 | 1 | 512 | 0 | 0 |
| 1024 | 0 | 0 | 1 | 1024 |
| 2048 | 1 | 2048 | 0 | 0 |
| 4096 | 1 | 4096 | 0 | 0 |
| 8192 | 1 | 8192 | 0 | 0 |
| 16384 | 0 | 0 | 1 | 16384 |
| -32768 | 0 | 0 | 1 | -32768 |
| **Sum** | | **15213** | | **-15213** |

# Numeric Ranges

- Unsigned Values
  - *UMin* = 0

    000…0
  - *UMax* = $2^w - 1$

    111…1
- Two's Complement Values
  - *TMin* = $-2^{w-1}$

    100…0
  - *TMax* = $2^{w-1} - 1$

    011…1
- Other Values
  - Minus 1

    111…1

Values for $W = 16$

| | Decimal | Hex | Binary |
|---|---|---|---|
| **UMax** | **65535** | **FF FF** | **11111111 11111111** |
| **TMax** | **32767** | **7F FF** | **01111111 11111111** |
| **TMin** | **-32768** | **80 00** | **10000000 00000000** |
| **-1** | **-1** | **FF FF** | **11111111 11111111** |
| **0** | **0** | **00 00** | **00000000 00000000** |

# Values for Different Word Sizes

| | W | | | |
|---|---|---|---|---|
| | **8** | **16** | **32** | **64** |
| **UMax** | 255 | 65,535 | 4,294,967,295 | 18,446,744,073,709,551,615 |
| **TMax** | 127 | 32,767 | 2,147,483,647 | 9,223,372,036,854,775,807 |
| **TMin** | -128 | -32,768 | -2,147,483,648 | -9,223,372,036,854,775,808 |

- Observations
  - $|TMin| = TMax + 1$
    - Asymmetric range
  - $UMax = 2 * TMax + 1$

- **C Programming**
  - #include <limits.h>
  - Declares constants, e.g.,
    - ULONG_MAX
    - LONG_MAX
    - LONG_MIN
  - Values platform specific

# Unsigned & Signed Numeric Values

| *X* | B2U(*X*) | B2T(*X*) |
|---|---|---|
| 0000 | 0 | 0 |
| 0001 | 1 | 1 |
| 0010 | 2 | 2 |
| 0011 | 3 | 3 |
| 0100 | 4 | 4 |
| 0101 | 5 | 5 |
| 0110 | 6 | 6 |
| 0111 | 7 | 7 |
| 1000 | 8 | –8 |
| 1001 | 9 | –7 |
| 1010 | 10 | –6 |
| 1011 | 11 | –5 |
| 1100 | 12 | –4 |
| 1101 | 13 | –3 |
| 1110 | 14 | –2 |
| 1111 | 15 | –1 |

- Equivalence
  - Same encodings for nonnegative values
- Uniqueness
  - Every bit pattern represents unique integer value
  - Each representable integer has unique bit encoding
- $\Rightarrow$ Can Invert Mappings
  - $\mathrm{U2B}(x) = \mathrm{B2U}^{-1}(x)$
    - Bit pattern for unsigned integer
  - $\mathrm{T2B}(x) = \mathrm{B2T}^{-1}(x)$
    - Bit pattern for two's comp integer

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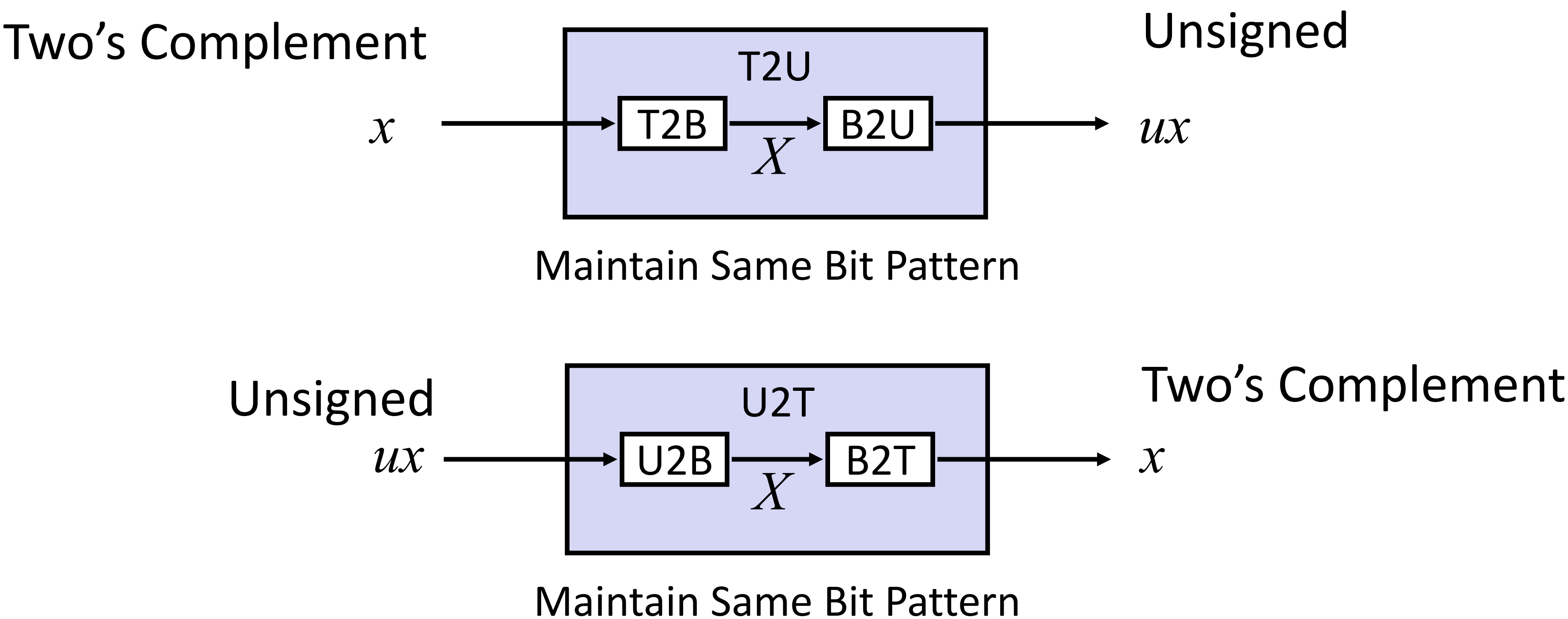
# Mapping Between Signed & Unsigned




- Mappings between unsigned and two's complement numbers:
  keep bit representations and reinterpret

# Mapping Signed ↔ Unsigned

| Bits | Signed | Unsigned |
|---|---|---|
| 0000 | 0 | 0 |
| 0001 | 1 | 1 |
| 0010 | 2 | 2 |
| 0011 | 3 | 3 |
| 0100 | 4 | 4 |
| 0101 | 5 | 5 |
| 0110 | 6 | 6 |
| 0111 | 7 | 7 |
| 1000 | -8 | 8 |
| 1001 | -7 | 9 |
| 1010 | -6 | 10 |
| 1011 | -5 | 11 |
| 1100 | -4 | 12 |
| 1101 | -3 | 13 |
| 1110 | -2 | 14 |
| 1111 | -1 | 15 |

$\rightarrow$ T2U $\rightarrow$

$\leftarrow$ U2T $\leftarrow$

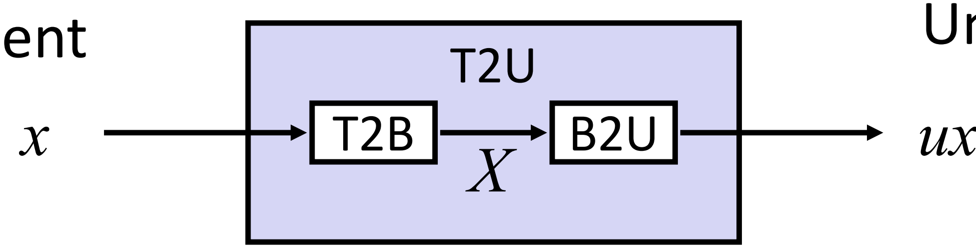
# Mapping Signed ↔ Unsigned

| Bits | Signed | | Unsigned |
|---|---|---|---|
| 0000 | 0 | | 0 |
| 0001 | 1 | | 1 |
| 0010 | 2 | | 2 |
| 0011 | 3 | = | 3 |
| 0100 | 4 | | 4 |
| 0101 | 5 | | 5 |
| 0110 | 6 | | 6 |
| 0111 | 7 | | 7 |
| 1000 | -8 | | 8 |
| 1001 | -7 | | 9 |
| 1010 | -6 | | 10 |
| 1011 | -5 | +/- 16 | 11 |
| 1100 | -4 | | 12 |
| 1101 | -3 | | 13 |
| 1110 | -2 | | 14 |
| 1111 | -1 | | 15 |

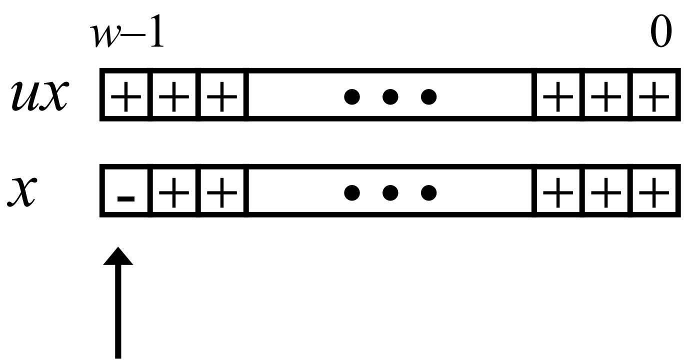
# Relation between Signed & Unsigned

Two's Complement

Unsigned




Maintain Same Bit Pattern




Large negative weight

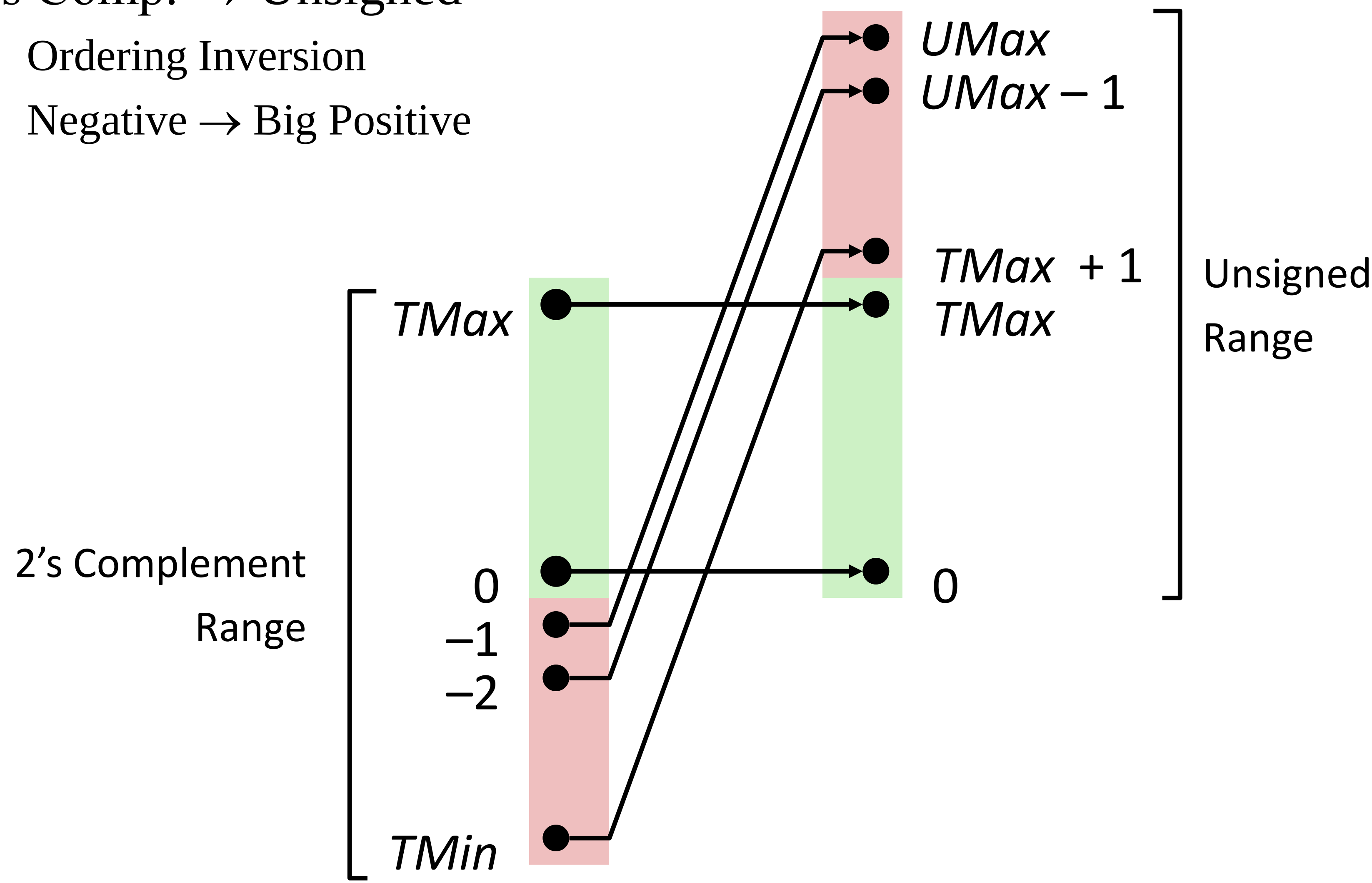
*becomes*

Large positive weight

$$ux = \begin{cases} x & x \geq 0 \\ x + 2^w & x < 0 \end{cases}$$

# Conversion Visualized

- 2's Comp. → Unsigned
  - Ordering Inversion
  - Negative → Big Positive



# Signed vs. Unsigned in C

- Constants
  - By default are considered to be signed integers
  - Unsigned if have "U" as suffix

    **0U, 4294967259U**
- Casting
  - Explicit casting between signed & unsigned same as U2T and T2U

```
int tx, ty;
unsigned ux, uy;
tx = (int) ux;
uy = (unsigned) ty;
```

  - Implicit casting also occurs via assignments and procedure calls

```
tx = ux;
uy = ty;
```

# Casting Surprises

- Expression Evaluation
  - If there is a mix of unsigned and signed in single expression, ***signed values implicitly cast to unsigned***
  - Including comparison operations **<, >, ==, <=, >=**
  - Examples for $W = 32$: **TMIN = -2,147,483,648 ,** **TMAX = 2,147,483,647**

| $\text{Constant}_1$ | $\text{Constant}_2$ | Relation | Evaluation |
|---|---|---|---|
| 0 | 0U | == | unsigned |
| -1 | 0 | < | signed |
| -1 | 0U | > | unsigned |
| 2147483647 | -2147483647-1 | > | signed |
| 2147483647U | -2147483647-1 | < | unsigned |
| -1 | -2 | > | signed |
| (unsigned)-1 | -2 | > | unsigned |
| 2147483647 | 2147483648U | < | unsigned |
| 2147483647 | (int) 2147483648U | > | signed |

# Summary
# Casting Signed ↔ Unsigned: Basic Rules

- Bit pattern is maintained
- But reinterpreted
- Can have unexpected effects: adding or subtracting $2^w$

- Expression containing signed and unsigned int
  - `int` is cast to `unsigned`!!

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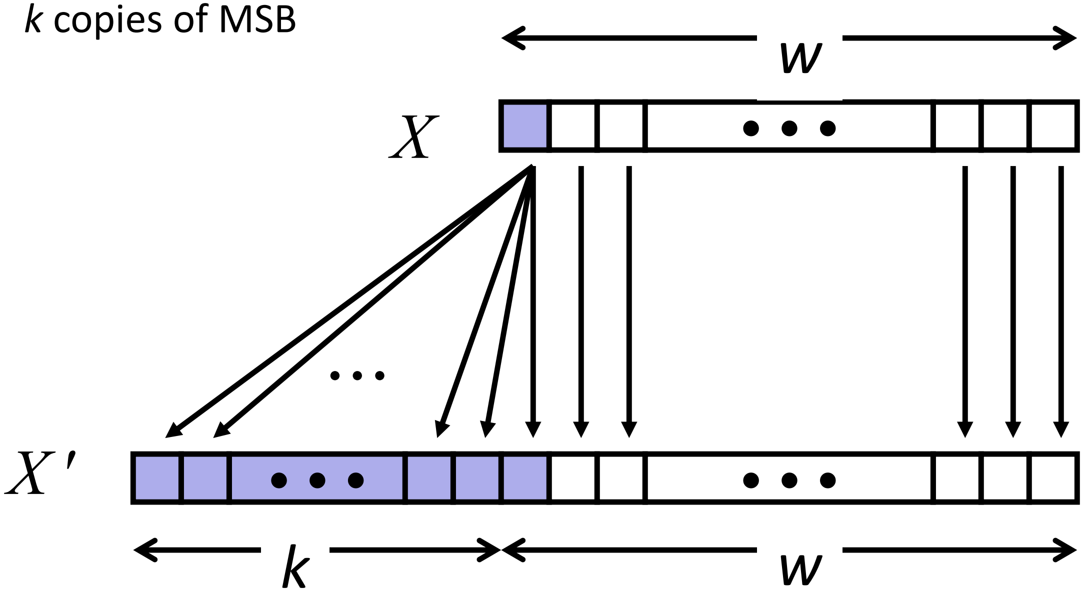
  - **Expanding, truncating**
  - Addition, negation, multiplication, shifting
- Summary

# Sign Extension

- Task:
  - Given $w$-bit signed integer $x$
  - Convert it to $w$+$k$-bit integer with same value
- Rule:
  - Make $k$ copies of sign bit:
  - $X' = \underbrace{x_{w-1}, \ldots, x_{w-1}}_{k \text{ copies of MSB}}, x_{w-1}, x_{w-2}, \ldots, x_0$



# Sign Extension Example

```
short int x =  15213;
int       ix = (int) x;
short int y = -15213;
int       iy = (int) y;
```

| | Decimal | Hex | Binary |
|---|---|---|---|
| x | 15213 | 3B 6D | 00111011 01101101 |
| ix | 15213 | 00 00 3B 6D | 00000000 00000000 00111011 01101101 |
| y | -15213 | C4 93 | 11000100 10010011 |
| iy | -15213 | FF FF C4 93 | 11111111 11111111 11000100 10010011 |

- Converting from smaller to larger integer data type
- C automatically performs sign extension

# Summary: Expanding, Truncating: Basic Rules

- Expanding (e.g., short int to int)
  - Unsigned: zeros added
  - Signed: sign extension
  - Both yield expected result

- Truncating (e.g., unsigned to unsigned short)
  - Unsigned/signed: bits are truncated
  - Result reinterpreted
  - Unsigned: mod operation
  - Signed: similar to mod
  - For small numbers yields expected behaviour

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# Negation: Complement & Increment

- Claim: Following Holds for 2's Complement

  **`~x + 1 == -x`**

- Complement
  - Observation: **`~x + x == 1111…111 == -1`**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | x | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 1 |
| + | ~x | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 |
| | -1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

- Complete Proof?

# Complement & Increment Examples

x = 15213

| | Decimal | Hex | Binary |
|---|---|---|---|
| `x` | 15213 | `3B 6D` | `00111011 01101101` |
| `~x` | -15214 | `C4 92` | `11000100 10010010` |
| `~x+1` | -15213 | `C4 93` | `11000100 10010011` |
| `y` | -15213 | `C4 93` | `11000100 10010011` |

x = 0

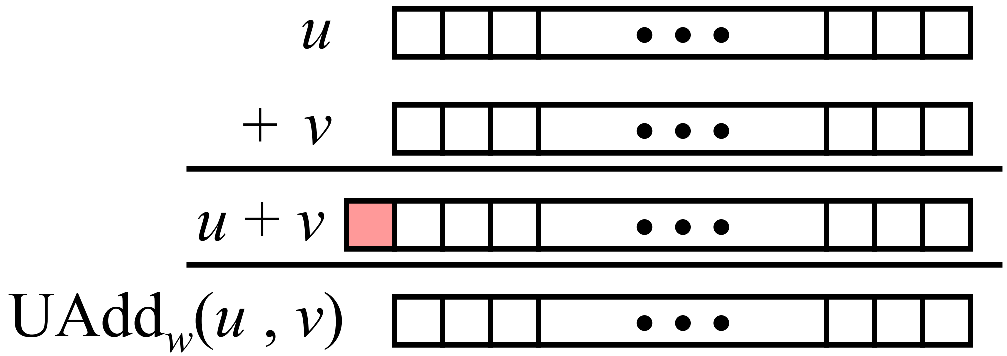
| | Decimal | Hex | Binary |
|---|---|---|---|
| `0` | 0 | `00 00` | `00000000 00000000` |
| `~0` | -1 | `FF FF` | `11111111 11111111` |
| `~0+1` | 0 | `00 00` | `00000000 00000000` |

# Unsigned Addition

Operands: *w* bits

True Sum: *w*+1 bits

Discard Carry: *w* bits




- Standard Addition Function
  - Ignores carry output
- Implements Modular Arithmetic

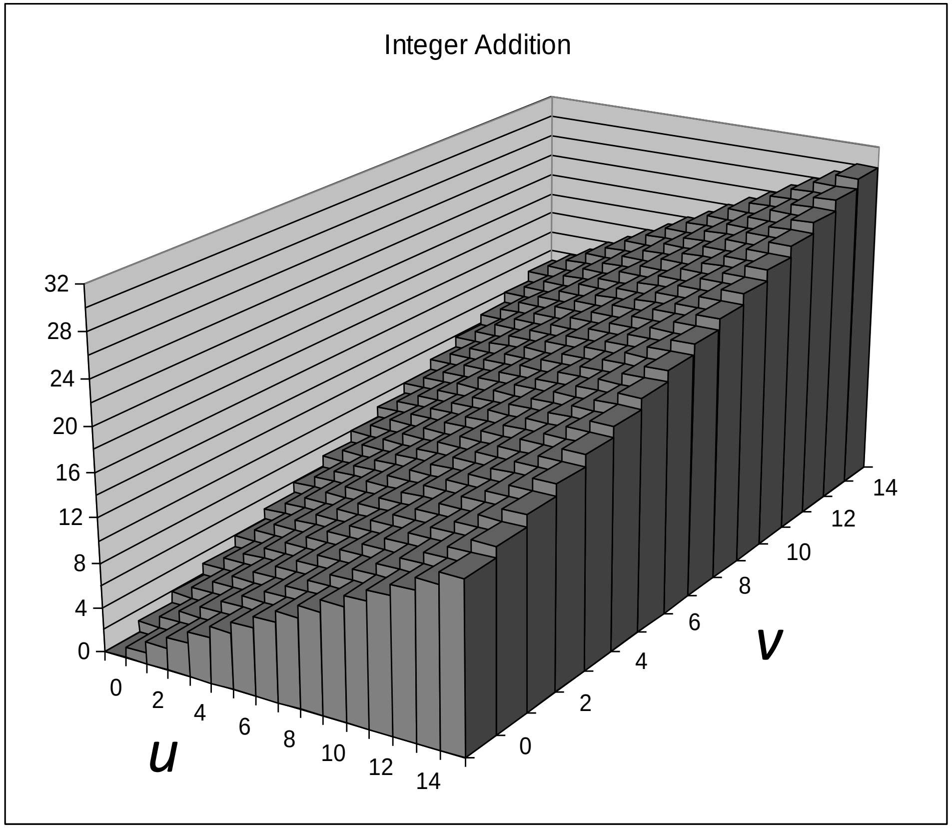
$$s \;=\; \mathrm{UAdd}_w(u\ ,\ v) \;=\; u + v \bmod 2^w$$

$$UAdd_w(u,v) \;=\; \begin{cases} u+v & u+v < 2^w \\ u+v-2^w & u+v \ge 2^w \end{cases}$$

# Visualizing (Mathematical) Integer Addition

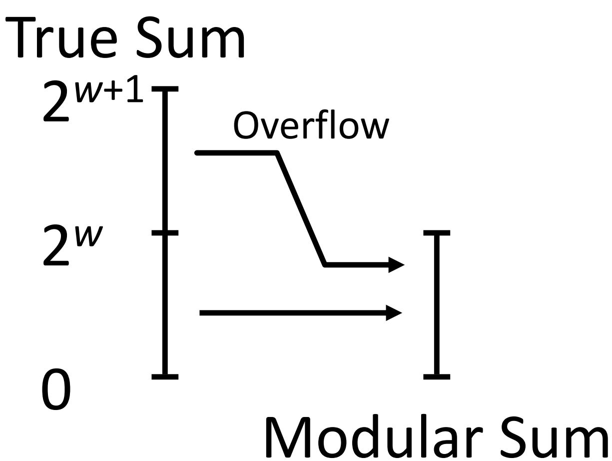
- Integer Addition
  - 4-bit integers *u*, *v*
  - Compute true sum $\text{Add}_4(u\ ,\ v)$
  - Values increase linearly with *u* and *v*
  - Forms planar surface

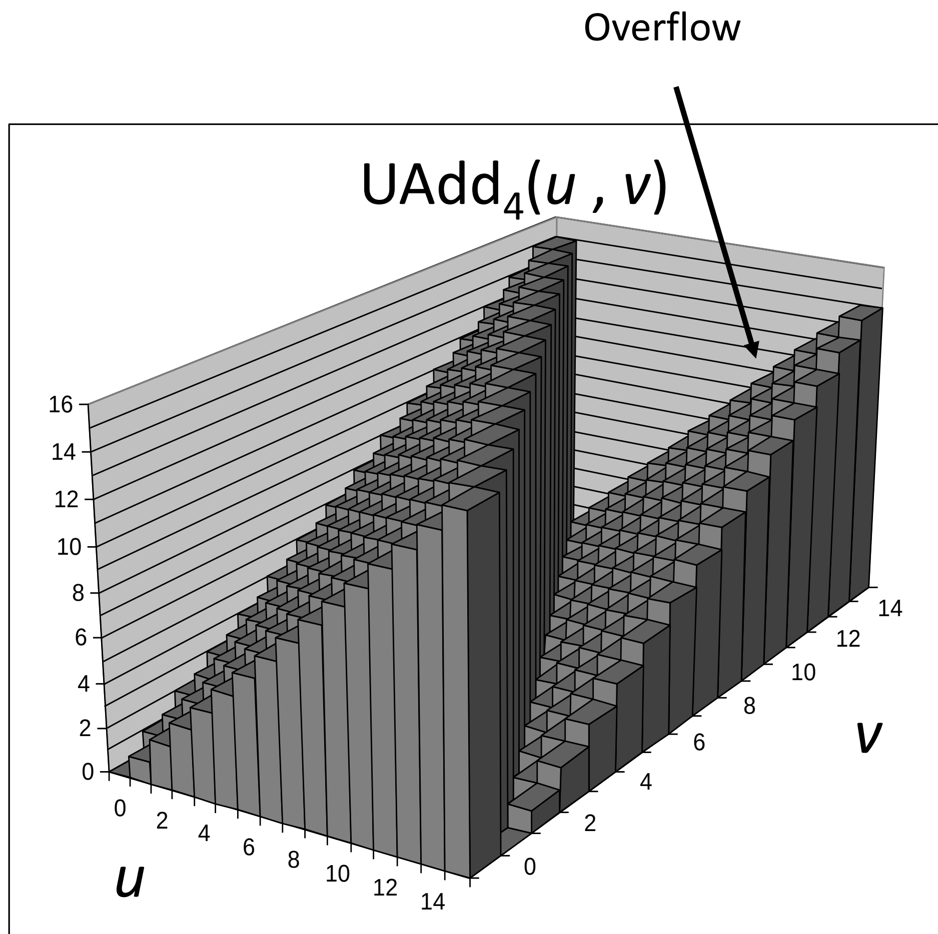
$\text{Add}_4(u\ ,\ v)$



# Visualizing Unsigned Addition

- Wraps Around
  - If true sum $\geq 2^w$
  - At most once



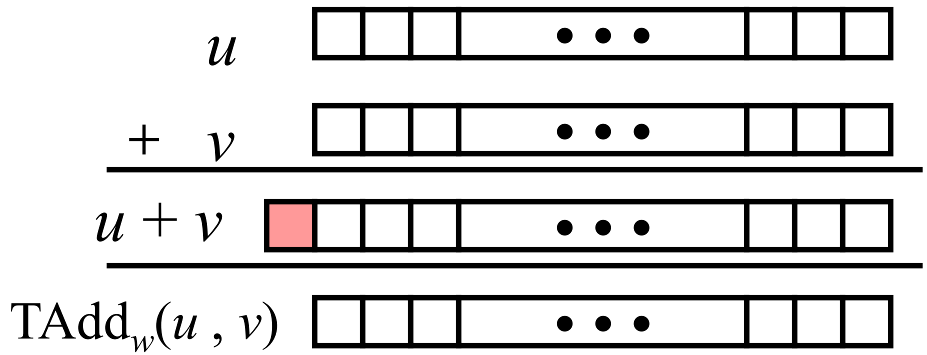



# Two's Complement Addition

Operands: *w* bits

True Sum: *w*+1 bits

Discard Carry: *w* bits



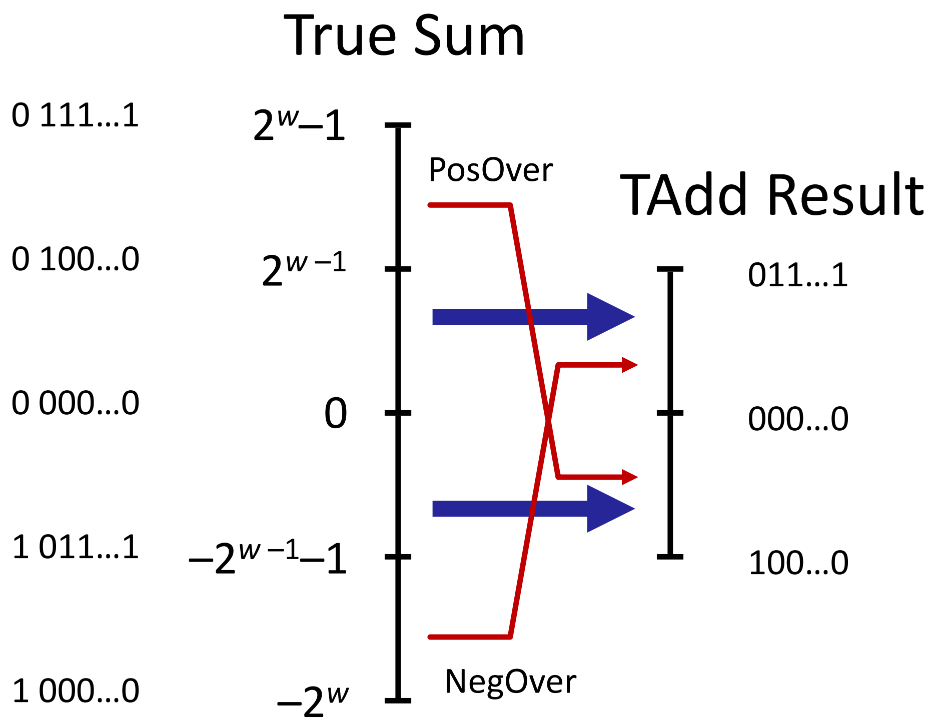

- TAdd and UAdd have Identical Bit-Level Behavior
  - Signed vs. unsigned addition in C:

```
int s, t, u, v;
s = (int) ((unsigned) u + (unsigned) v);
t = u + v
```

  - Will give `s == t`

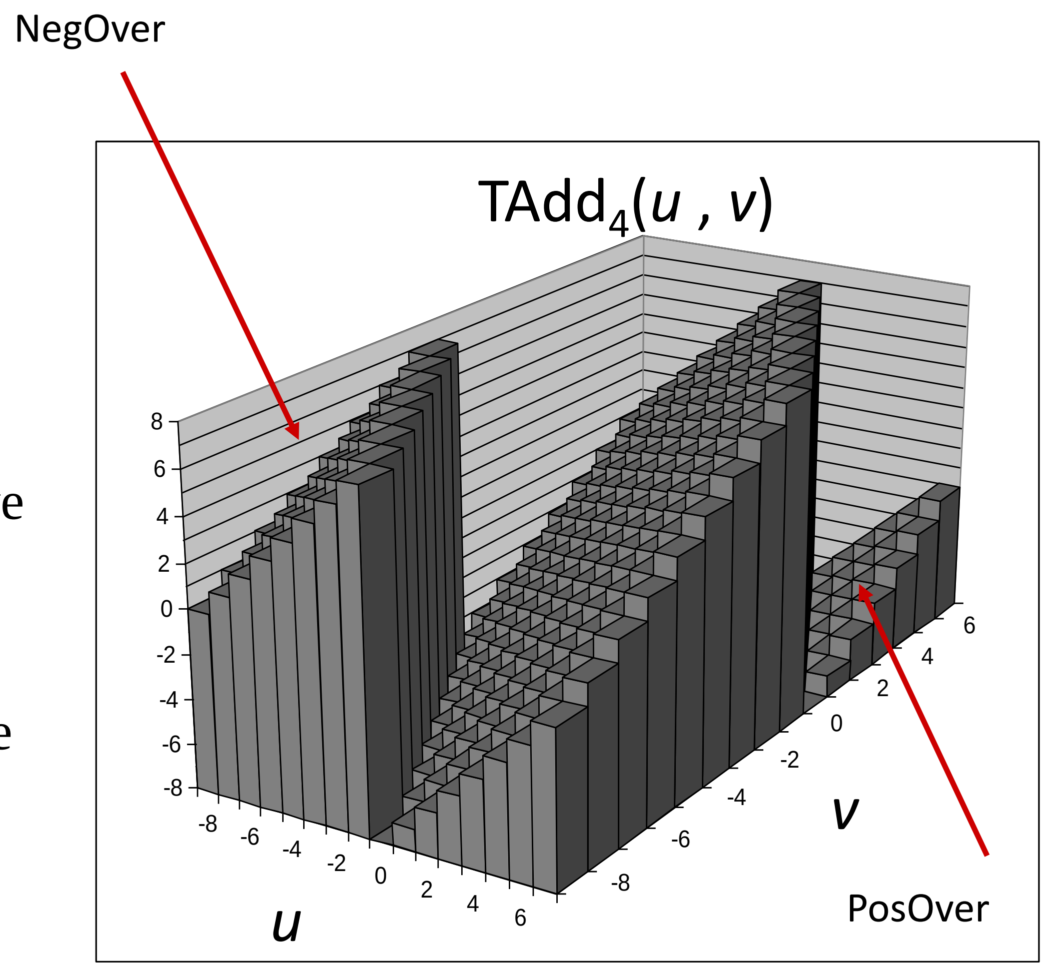
# TAdd Overflow

- Functionality
  - True sum requires $w$+1 bits
  - Drop off MSB
  - Treat remaining bits as 2's comp. integer



# Visualizing 2's Complement Addition

- Values
  - 4-bit two's comp.
  - Range from -8 to +7
- Wraps Around
  - If sum $\geq 2^{w-1}$
    - Becomes negative
    - At most once
  - If sum $< -2^{w-1}$
    - Becomes positive
    - At most once



# Multiplication

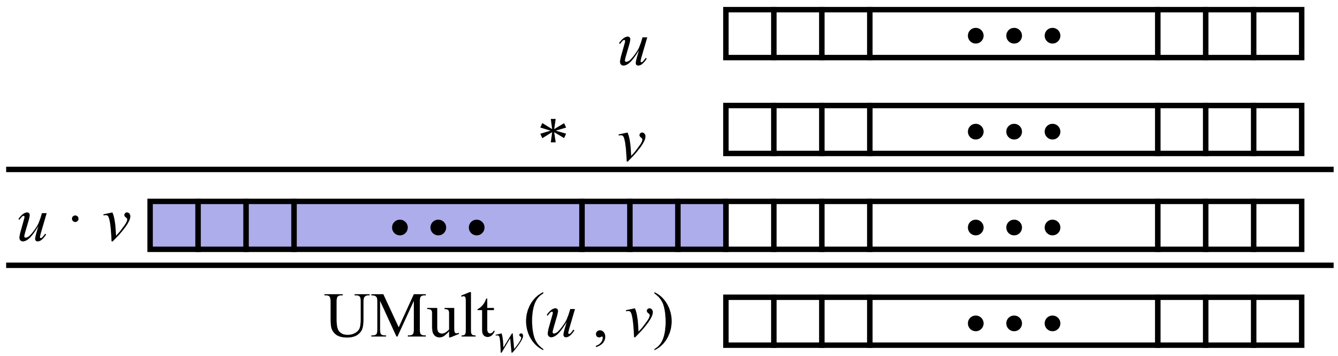
- Computing Exact Product of $w$-bit numbers $x$, $y$
  - Either signed or unsigned
- But, exact results can be bigger than $w$ bits
  - Unsigned: up to $2w$ bits
    - Result range: $0 \le x * y \le (2^w - 1)^2 = 2^{2w} - 2^{w+1} + 1$
  - Two's complement min: Up to $2w$–1 bits
    - Result range: $x * y \ge (-2^{w-1})*(2^{w-1}-1) = -2^{2w-2} + 2^{w-1}$
  - Two's complement max: Up to $2w$ bits, but only for $(TMin_w)^2$
    - Result range: $x * y \le (-2^{w-1})^2 = 2^{2w-2}$
- Maintaining Exact Results
  - Would need to keep expanding word size with each product computed
  - Done in software by "arbitrary precision" arithmetic packages

# Unsigned Multiplication in C

Operands: *w* bits

True Product: 2**w* bits

Discard *w* bits: *w* bits




- Standard Multiplication Function
  - Ignores high order *w* bits
- Implements Modular Arithmetic

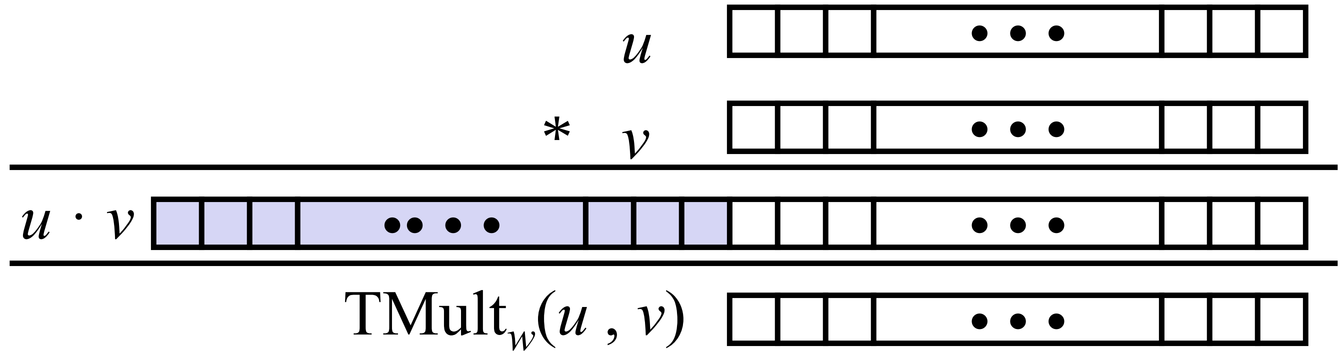
$$\mathrm{UMult}_w(u, v) = u \cdot v \bmod 2^w$$

# Signed Multiplication in C

Operands: *w* bits

True Product: 2**w* bits

Discard *w* bits: *w* bits




- Standard Multiplication Function
  - Ignores high order *w* bits
  - Some of which are different for signed vs. unsigned multiplication
  - Lower bits are the same

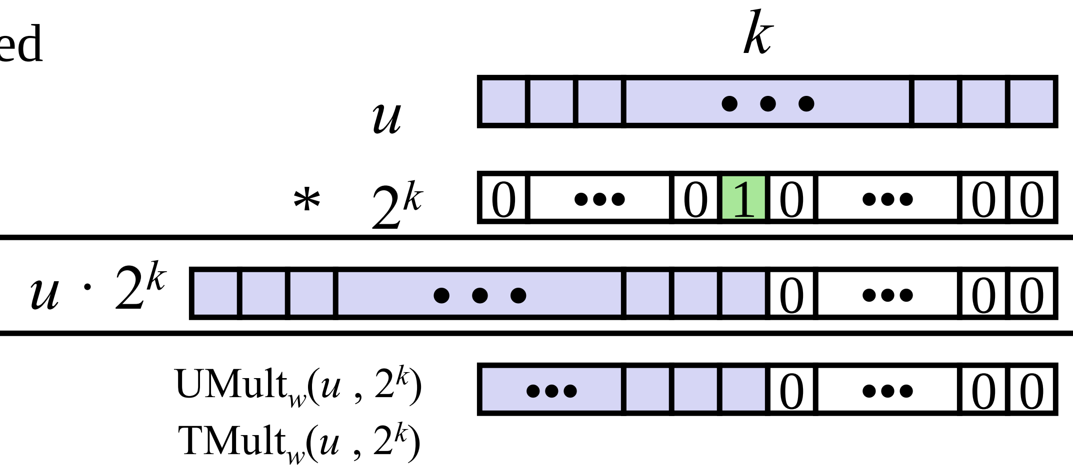
# Power-of-2 Multiply with Shift

- Operation
  - `u << k` gives `u *` $2^k$
  - Both signed and unsigned

Operands: *w* bits

True Product: *w*+*k* bits

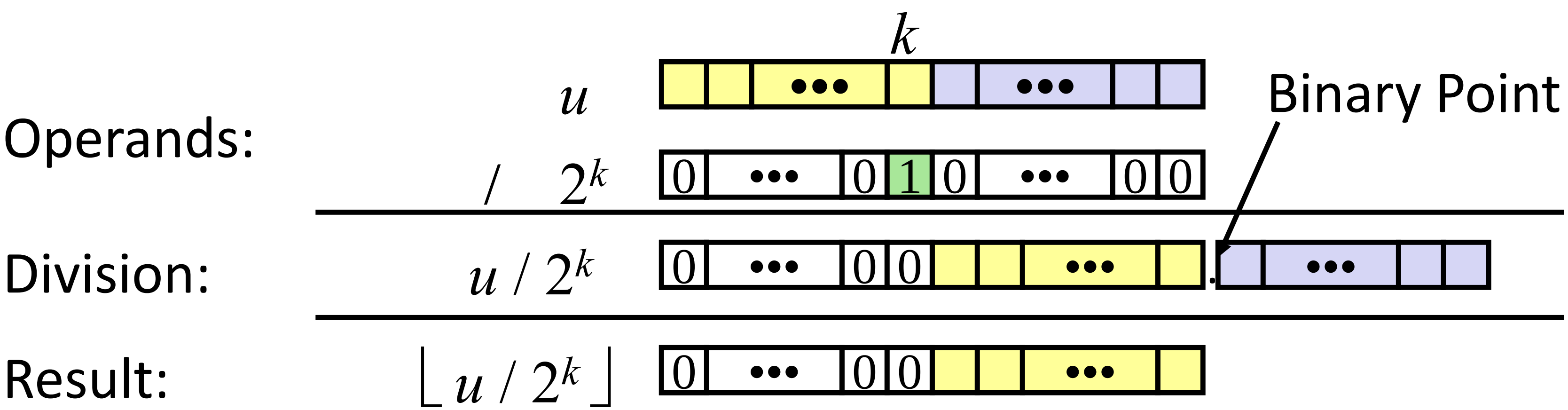
Discard *k* bits: *w* bits




- Examples
  - `u << 3          ==   u * 8`
  - `u << 5 - u << 3    ==    u * 24`
  - Most machines shift and add faster than multiply
    - Compiler generates this code automatically

# Unsigned Power-of-2 Divide with Shift

- Quotient of Unsigned by Power of 2
  - `u >> k` gives $\lfloor$ `u / ` $2^k \rfloor$
  - Uses logical shift




| | Division | Computed | Hex | Binary |
|---|---|---|---|---|
| `x` | 15213 | 15213 | `3B 6D` | `00111011 01101101` |
| `x >> 1` | 7606.5 | 7606 | `1D B6` | `00011101 10110110` |
| `x >> 4` | 950.8125 | 950 | `03 B6` | `00000011 10110110` |
| `x >> 8` | 59.4257813 | 59 | `00 3B` | `00000000 00111011` |

# Arithmetic: Basic Rules

- Addition:
  - Unsigned/signed: Normal addition followed by truncate, same operation on bit level
  - Unsigned: addition mod $2^w$
    - Mathematical addition + possible subtraction of $2^w$
  - Signed: modified addition mod $2^w$ (result in proper range)
    - Mathematical addition + possible addition or subtraction of $2^w$

- Multiplication:
  - Unsigned/signed: Normal multiplication followed by truncate, same operation on bit level
  - Unsigned: multiplication mod $2^w$
  - Signed: modified multiplication mod $2^w$ (result in proper range)

# Why Should I Use Unsigned?

- *Don't* use without understanding implications
  - Easy to make mistakes

```
unsigned i;
for (i = cnt-2; i >= 0; i--)
  a[i] += a[i+1];
```

  - Can be very subtle

```
#define DELTA sizeof(int)
int i;
for (i = CNT; i-DELTA >= 0; i-= DELTA)
  . . .
```

- *Do* Use When Performing Modular Arithmetic
  - Multiprecision arithmetic
- *Do* Use When Using Bits to Represent Sets
  - Logical right shift, no sign extension

# Counting Down with Unsigned

- Proper way to use unsigned as loop index

```
unsigned i;
for (i = cnt-2; i < cnt; i--)
  a[i] += a[i+1];
```

- See Robert Seacord, *Secure Coding in C and C++*
  - C Standard guarantees that unsigned addition will behave like modular arithmetic
    - 0 – 1 → *UMax*
- Even better

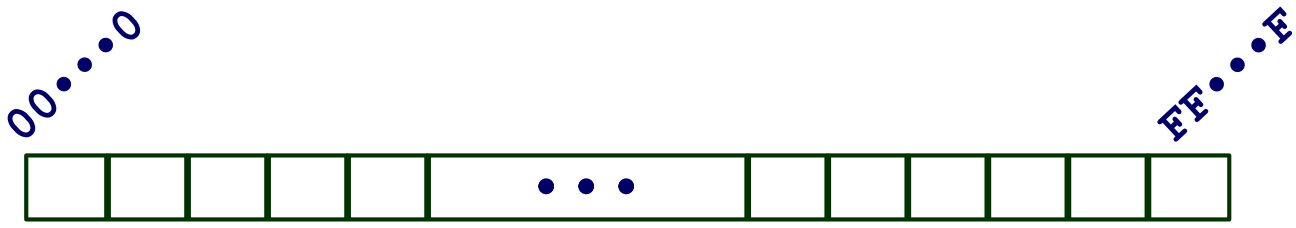
```
size_t i;
for (i = cnt-2; i < cnt; i--)
  a[i] += a[i+1];
```

  - Data type **size_t** defined as unsigned value with length = word size
  - Code will work even if **cnt** = *UMax*
  - What if **cnt** is signed and < 0?

# Today: Integers

- Representation: unsigned and signed
- Conversion, casting
- Expanding, truncating
- Addition, negation, multiplication, shifting
- Representation in memory, pointers, strings

# Byte-Oriented Memory Organization




- Programs Refer to Virtual Addresses
  - Conceptually, envision it as a very large array of bytes
    - In reality, it's not, but can think of it that way
  - An address is like an index into that array
    - and, a pointer variable stores an address

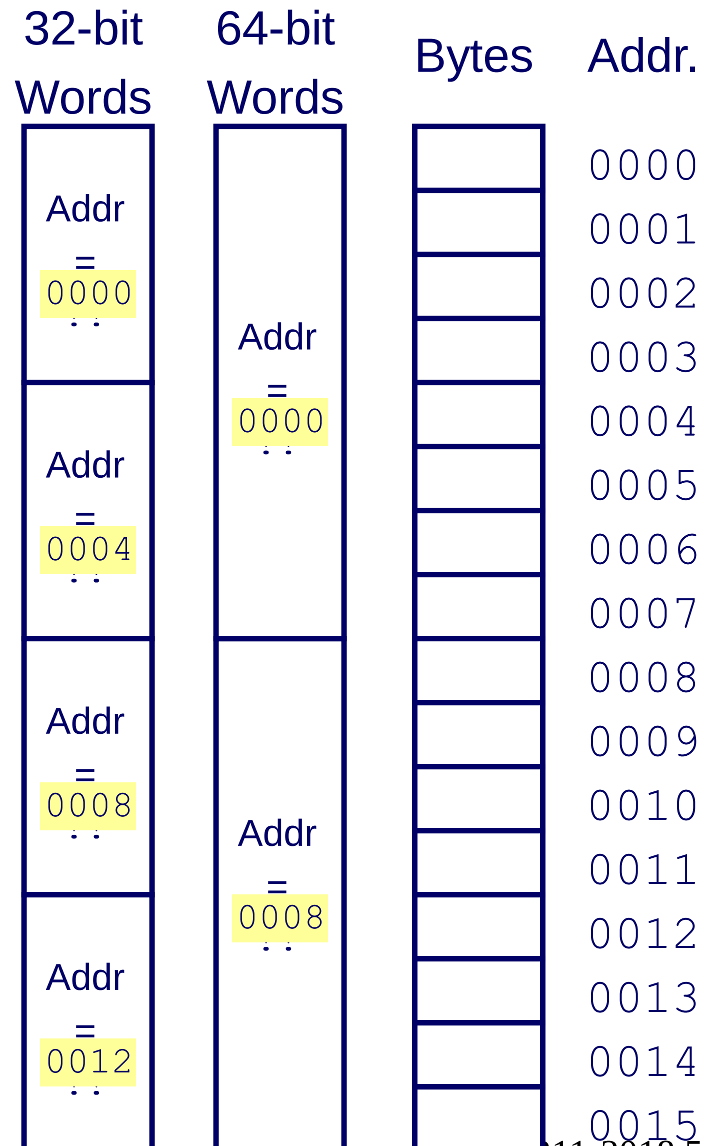
- Note: system provides private address spaces to each "process"
  - Think of a process as a program being executed
  - So, a program can clobber its own data, but not that of others

# Machine Words

- Any given computer has a “Word Size”
  - Nominal size of integer-valued data
    - and of addresses
  - Until recently, most machines used 32 bits (4 bytes) as word size
    - Limits addresses to 4GB ($2^{32}$ bytes)
  - Increasingly, machines have 64-bit word size
    - Potentially, could have 18 EB (exabytes) of addressable memory
    - That’s 18.4 X $10^{18}$
  - Machines still support multiple data formats
    - Fractions or multiples of word size
    - Always integral number of bytes

# Word-Oriented Memory Organization

- Addresses Specify Byte Locations
  - Address of first byte in word
  - Addresses of successive words differ by 4 (32-bit) or 8 (64-bit)



# Example Data Representations

| C Data Type | Typical 32-bit | Typical 64-bit | x86-64 |
|---|---|---|---|
| **`char`** | 1 | 1 | 1 |
| **`short`** | 2 | 2 | 2 |
| **`int`** | 4 | 4 | 4 |
| **`long`** | 4 | 8 | 8 |
| **`float`** | 4 | 4 | 4 |
| **`double`** | 8 | 8 | 8 |
| **`long double`** | − | − | 10/16 |
| pointer | 4 | 8 | 8 |

# Byte Ordering

- How should bytes within a multi-byte word be ordered in memory?
- Conventions
  - Big Endian: Sun, PPC Mac, Internet
    - Least significant byte has highest address
  - Little Endian: x86
    - Least significant byte has lowest address

# Byte Ordering Example

- Big Endian
  - Least significant byte has highest address
- Little Endian
  - Least significant byte has lowest address
- Example
  - Variable x has 4-byte representation 0x01234567
  - Address given by &x is 0x100

Big Endian

| | | 0x100 | 0x101 | 0x102 | 0x103 | | |
|---|---|---|---|---|---|---|---|
| | | 01 | 23 | 45 | 67 | | |

Little Endian

| | | 0x100 | 0x101 | 0x102 | 0x103 | | |
|---|---|---|---|---|---|---|---|
| | | 67 | 45 | 23 | 01 | | |

# Reading Byte-Reversed Listings

- Disassembly
  - Text representation of binary machine code
  - Generated by program that reads the machine code
- Example Fragment

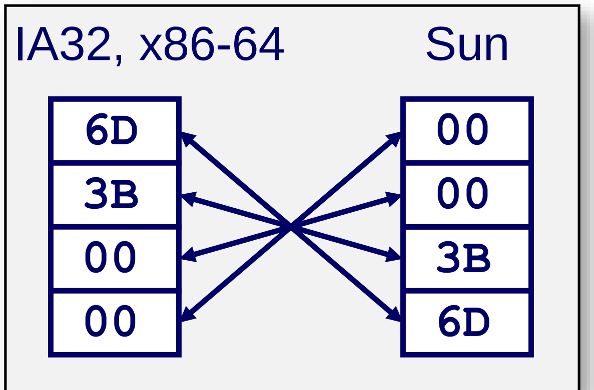
| Address | Instruction Code | Assembly Rendition | |
|---|---|---|---|
| `8048365:` | `5b` | `pop` | `%ebx` |
| `8048366:` | `81 c3 ab 12 00 00` | `add` | `$0x12ab,%ebx` |
| `804836c:` | `83 bb 28 00 00 00 00` | `cmpl` | `$0x0,0x28(%ebx)` |

- Deciphering Numbers
  - Value: 0x12ab
  - Pad to 32 bits: 0x000012ab
  - Split into bytes: 00 00 12 ab
  - Reverse: ab 12 00 00

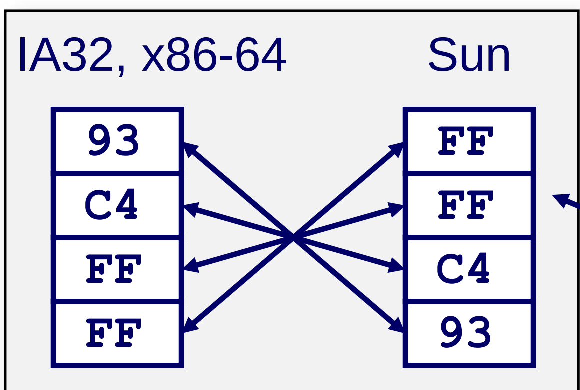
# Representing Integers

| | | | | |
|---|---|---|---|---|
| Decimal: | 15213 | | | |
| Binary: | 0011 | 1011 | 0110 | 1101 |
| Hex: | 3 | B | 6 | D |

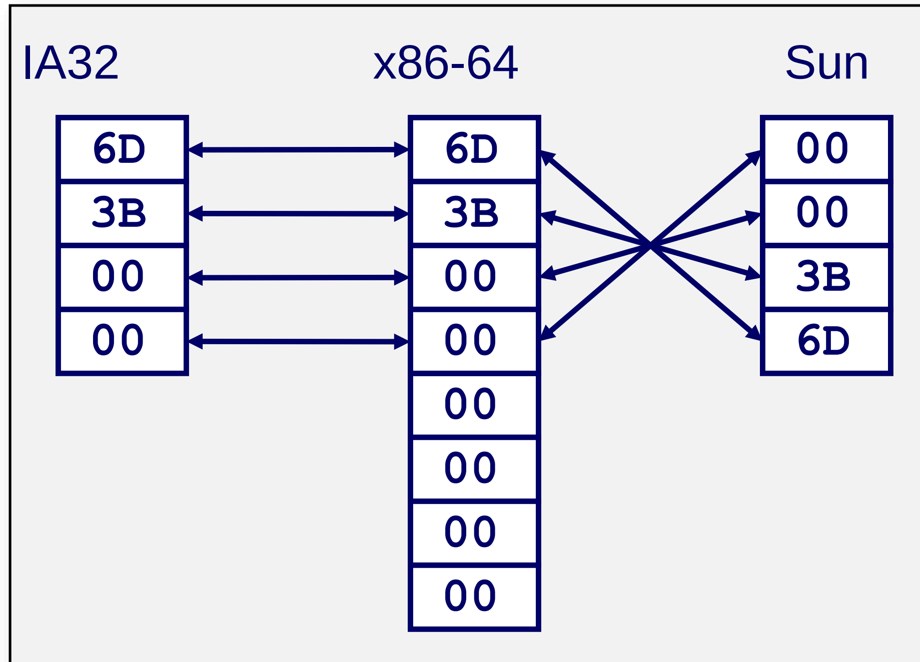
int A = 15213;




int B = −15213;




long int C = 15213;




Two’s complement representation
(Covered later)

# Examining Data Representations

- Code to Print Byte Representation of Data
  - Casting pointer to unsigned char * creates byte array

```
typedef unsigned char *pointer;

void show_bytes(pointer start, int len){
  int i;
  for (i = 0; i < len; i++)
    printf(”%p\t0x%.2x\n",start+i, start[i]);
  printf("\n");
}
```

Printf directives:

%p: Print pointer

%x: Print Hexadecimal

# `show_bytes` Execution Example

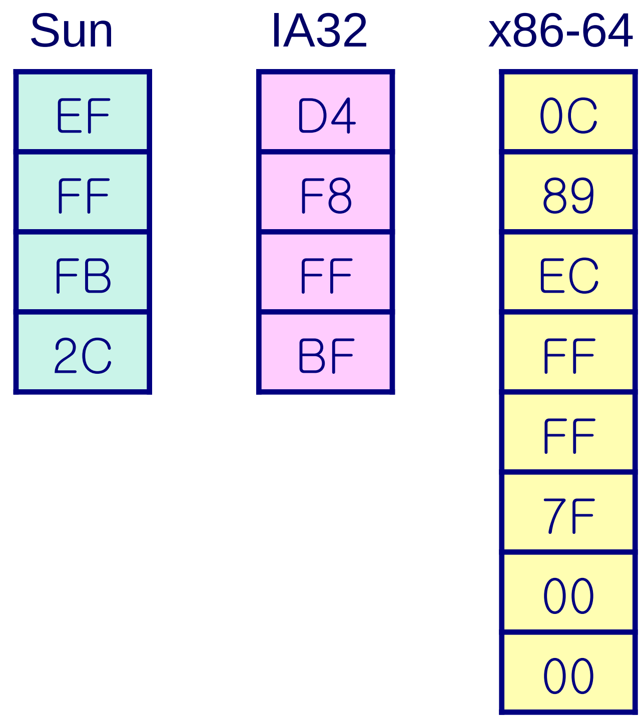
```
int a = 15213;
printf("int a = 15213;\n");
show_bytes((pointer) &a, sizeof(int));
```

Result (Linux):

```
int a = 15213;
0x11ffffcb8 0x6d
0x11ffffcb9 0x3b
0x11ffffcba 0x00
0x11ffffcbb 0x00
```

# Representing Pointers

```
int B = -15213;
int *P = &B;
```



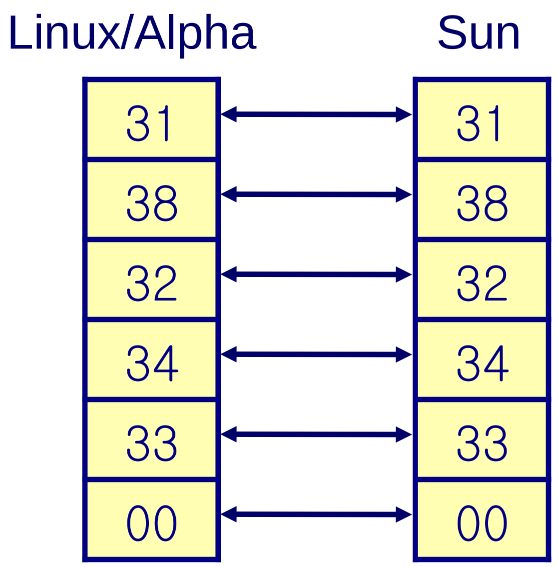

Different compilers & machines assign different locations to objects

Even get different results each time run program

# Representing Strings

```
char S[6] = "18243";
```

- Strings in C
  - Represented by array of characters
  - Each character encoded in ASCII format
    - Standard 7-bit encoding of character set
    - Character "0" has code 0x30
      - Digit *i* has code 0x30+*i*
  - String should be null-terminated
    - Final character = 0
- Compatibility
  - Byte ordering not an issue



# Integer C Puzzles

- Assume 32-bit word size, two's complement integers
- For each of the following C expressions: true or false? Why?

Initialization

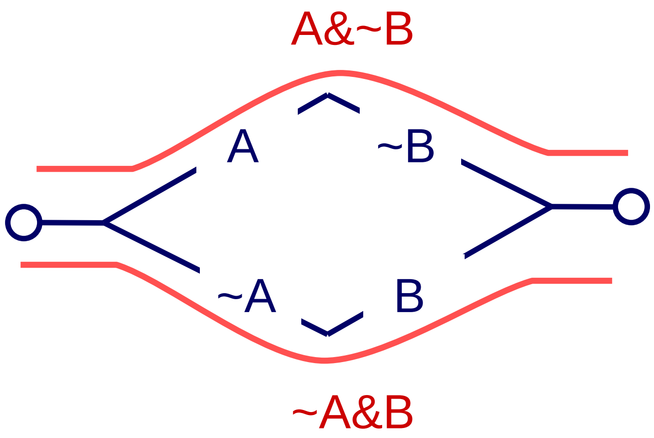
```
int x = foo();

int y = bar();

unsigned ux = x;

unsigned uy = y;
```

- x < 0 ⇒ ((x*2) < 0)
- ux >= 0
- x & 7 == 7 ⇒ (x<<30) < 0
- ux > -1
- x > y ⇒ -x < -y
- x * x >= 0
- x > 0 && y > 0 ⇒ x + y > 0
- x >= 0 ⇒ -x <= 0
- x <= 0 ⇒ -x >= 0
- (x|-x)>>31 == -1
- ux >> 3 == ux/8
- x >> 3 == x/8
- x & (x-1) != 0

# Extras

# Application of Boolean Algebra

- Applied to Digital Systems by Claude Shannon
  - 1937 MIT Master's Thesis
  - Reason about networks of relay switches
    - Encode closed switch as 1, open switch as 0




Connection when

A&~B | ~A&B

= A^B

# Binary Number Property

Claim

$$1 + 1 + 2 + 4 + 8 + \ldots + 2^{w-1} = 2^{w}$$

$$1 + \sum_{i=0}^{w-1} 2^{i} = 2^{w}$$

- w = 0:
  - $1 = 2^{0}$
- Assume true for w-1:
  - $\underbrace{1 + 1 + 2 + 4 + 8 + \ldots + 2^{w-1}}_{=\ 2^{w}} + 2^{w} = 2^{w} + 2^{w} = 2^{w+1}$

# Code Security Example

```
/* Kernel memory region holding user-accessible data */
#define KSIZE 1024
char kbuf[KSIZE];

/* Copy at most maxlen bytes from kernel region to user buffer */
int copy_from_kernel(void *user_dest, int maxlen) {
    /* Byte count len is minimum of buffer size and maxlen */
    int len = KSIZE < maxlen ? KSIZE : maxlen;
    memcpy(user_dest, kbuf, len);
    return len;
}
```

- Similar to code found in FreeBSD's implementation of getpeername
- There are legions of smart people trying to find vulnerabilities in programs

# Typical Usage

```
/* Kernel memory region holding user-accessible data */
#define KSIZE 1024
char kbuf[KSIZE];

/* Copy at most maxlen bytes from kernel region to user buffer */
int copy_from_kernel(void *user_dest, int maxlen) {
    /* Byte count len is minimum of buffer size and maxlen */
    int len = KSIZE < maxlen ? KSIZE : maxlen;
    memcpy(user_dest, kbuf, len);
    return len;
```

```
#define MSIZE 528

void getstuff() {
    char mybuf[MSIZE];
    copy_from_kernel(mybuf, MSIZE);
    printf("%s\n", mybuf);
}
```

# Malicious Usage

```
/* Declaration of library function memcpy */
void *memcpy(void *dest, void *src, size_t n);
```

```
/* Kernel memory region holding user-accessible data */
#define KSIZE 1024
char kbuf[KSIZE];

/* Copy at most maxlen bytes from kernel region to user buffer */
int copy_from_kernel(void *user_dest, int maxlen) {
    /* Byte count len is minimum of buffer size and maxlen */
    int len = KSIZE < maxlen ? KSIZE : maxlen;
    memcpy(user_dest, kbuf, len);
    return len;
}
```

```
#define MSIZE 528

void getstuff() {
    char mybuf[MSIZE];
    copy_from_kernel(mybuf, -MSIZE);
    . . .
}
```

# Mathematical Properties

- Modular Addition Forms an *Abelian Group*
  - **Closed** under addition

    $0 \leq \mathrm{UAdd}_w(u , v) \leq 2^w - 1$
  - **Commutative**

    $\mathrm{UAdd}_w(u , v) = \mathrm{UAdd}_w(v , u)$
  - **Associative**

    $\mathrm{UAdd}_w(t, \mathrm{UAdd}_w(u , v)) = \mathrm{UAdd}_w(\mathrm{UAdd}_w(t, u ), v)$
  - **0** is additive identity

    $\mathrm{UAdd}_w(u , 0) = u$
  - Every element has additive **inverse**
    - Let $\mathrm{UComp}_w (u ) = 2^w - u$

      $\mathrm{UAdd}_w(u , \mathrm{UComp}_w (u )) = 0$

# Mathematical Properties of TAdd

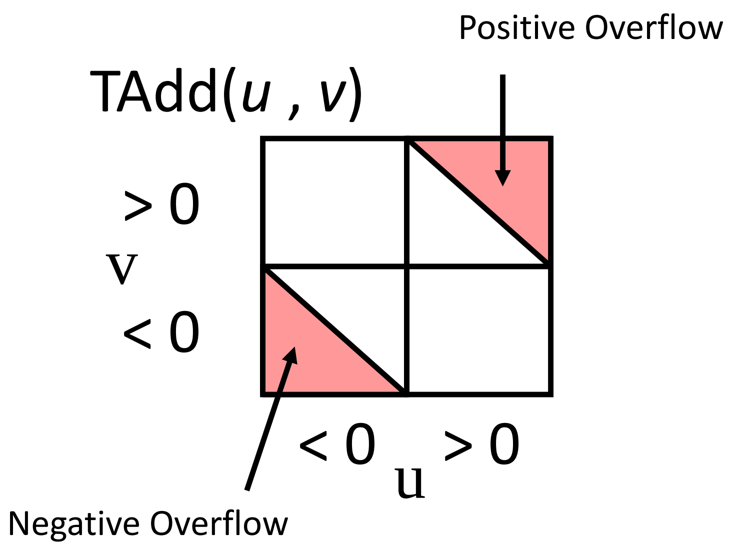
- Isomorphic Group to unsigneds with UAdd
  - $\text{TAdd}_w(u, v) = \text{U2T}(\text{UAdd}_w(\text{T2U}(u), \text{T2U}(v)))$
    - Since both have identical bit patterns

- Two's Complement Under TAdd Forms a Group
  - Closed, Commutative, Associative, 0 is additive identity
  - Every element has additive inverse

$$TComp_w(u) = \begin{cases} -u & u \neq TMin_w \\ TMin_w & u = TMin_w \end{cases}$$

# Characterizing TAdd

- Functionality
  - True sum requires *w*+1 bits
  - Drop off MSB
  - Treat remaining bits as 2's comp. integer




$$TAdd_w(u,v) = \begin{cases} u+v+2^w & u+v < TMin_w \quad \text{(NegOver)} \\ u+v & TMin_w \le u+v \le TMax_w \\ u+v-2^w & TMax_w < u+v \quad \text{(PosOver)} \end{cases}$$

# XDR Code

```
void* copy_elements(void *ele_src[], int ele_cnt, size_t ele_size) {
    /*
     * Allocate buffer for ele_cnt objects, each of ele_size bytes
     * and copy from locations designated by ele_src
     */
    void *result = malloc(ele_cnt * ele_size);
    if (result == NULL)
        /* malloc failed */
        return NULL;
    void *next = result;
    int i;
    for (i = 0; i < ele_cnt; i++) {
        /* Copy object i to destination */
        memcpy(next, ele_src[i], ele_size);
        /* Move pointer to next memory region */
        next += ele_size;
    }
    return result;
}
```

# XDR Vulnerability

malloc(ele_cnt * ele_size)

- What if:
  - **`ele_cnt`** $= 2^{20} + 1$
  - **`ele_size`** $= 4096$ $= 2^{12}$
  - Allocation= ??

- How can I make this function secure?

# Compiled Multiplication Code

C Function

```
int mul12(int x)
{
  return x*12;
}
```

Compiled Arithmetic Operations

```
  leaq (%rax,%eax,2), %rax
  salq $2, %rax
```

Explanation

```
  t <- x+x*2
  return t << 2;
```

- C compiler automatically generates shift/add code when multiplying by constant

# Compiled Unsigned Division Code

C Function

```
unsigned udiv8(unsigned x)
{
  return x/8;
}
```

Compiled Arithmetic Operations

```
  shrq $3, %rax
```

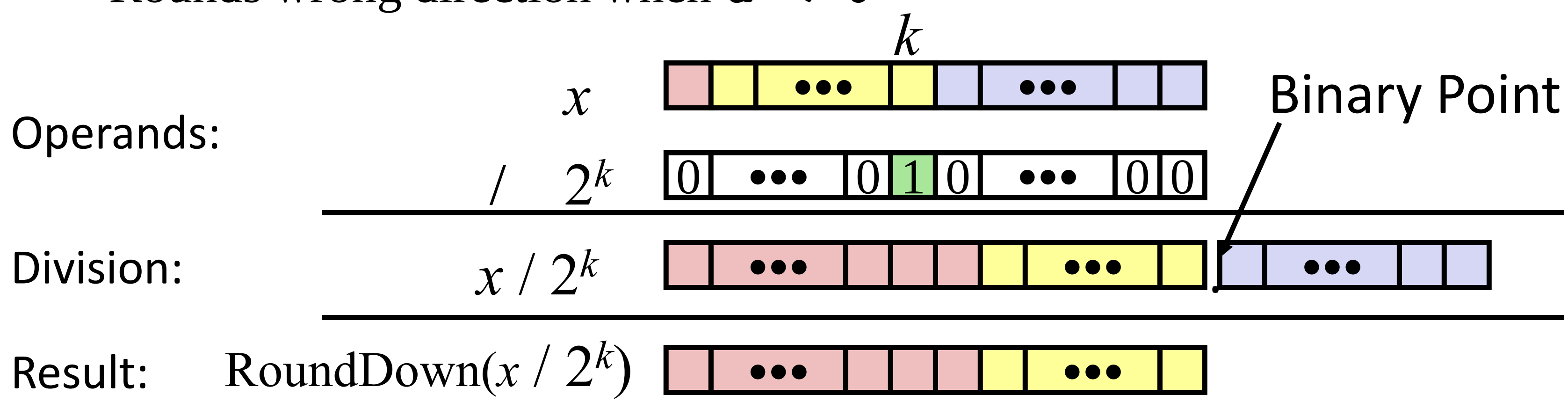
Explanation

```
  # Logical shift
  return x >> 3;
```

- Uses logical shift for unsigned
- For Java Users
  - Logical shift written as >>>

# Signed Power-of-2 Divide with Shift

- Quotient of Signed by Power of 2
  - `x >> k` gives $\lfloor$ `x / ` $2^k \rfloor$
  - Uses arithmetic shift
  - Rounds wrong direction when `u < 0`

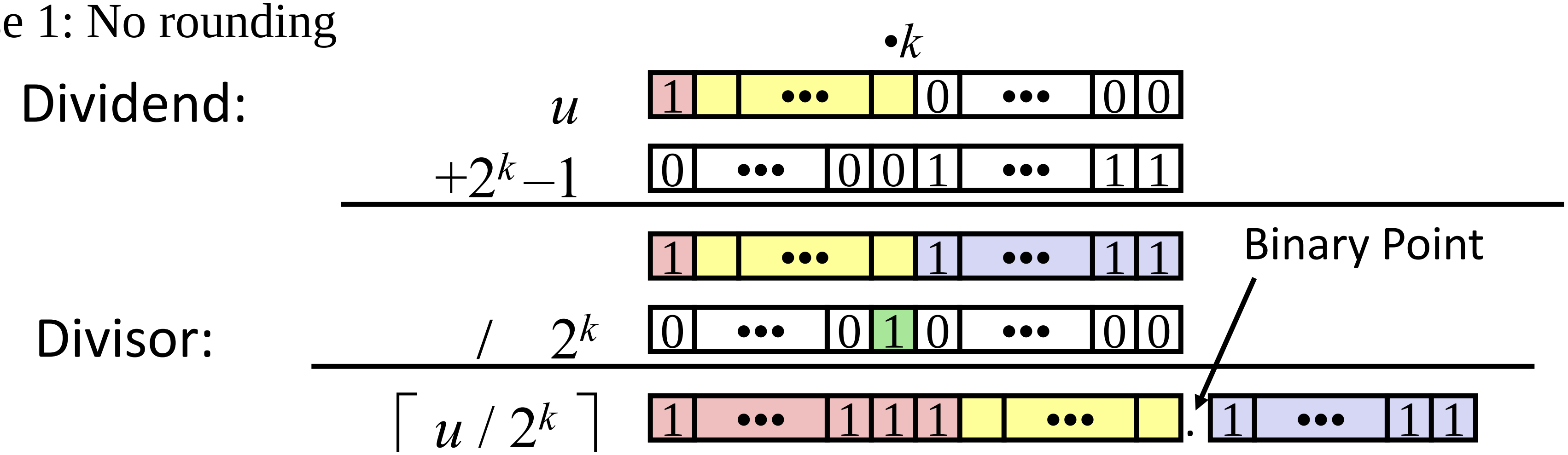



| | Division | Computed | Hex | Binary |
|---|---|---|---|---|
| `y` | -15213 | -15213 | `C4 93` | `11000100 10010011` |
| `y >> 1` | -7606.5 | -7607 | `E2 49` | `11100010 01001001` |
| `y >> 4` | -950.8125 | -951 | `FC 49` | `11111100 01001001` |
| `y >> 8` | -59.4257813 | -60 | `FF C4` | `11111111 11000100` |

# Correct Power-of-2 Divide

- Quotient of Negative Number by Power of 2
  - Want $\lceil$ **`x / 2`**$^k$ $\rceil$ (Round Toward 0)
  - Compute as $\lfloor$ **`(x+2`**$^k$**`-1)/ 2`**$^k$ $\rfloor$
    - In C: **`(x + (1<<k)-1) >> k`**
    - Biases dividend toward 0

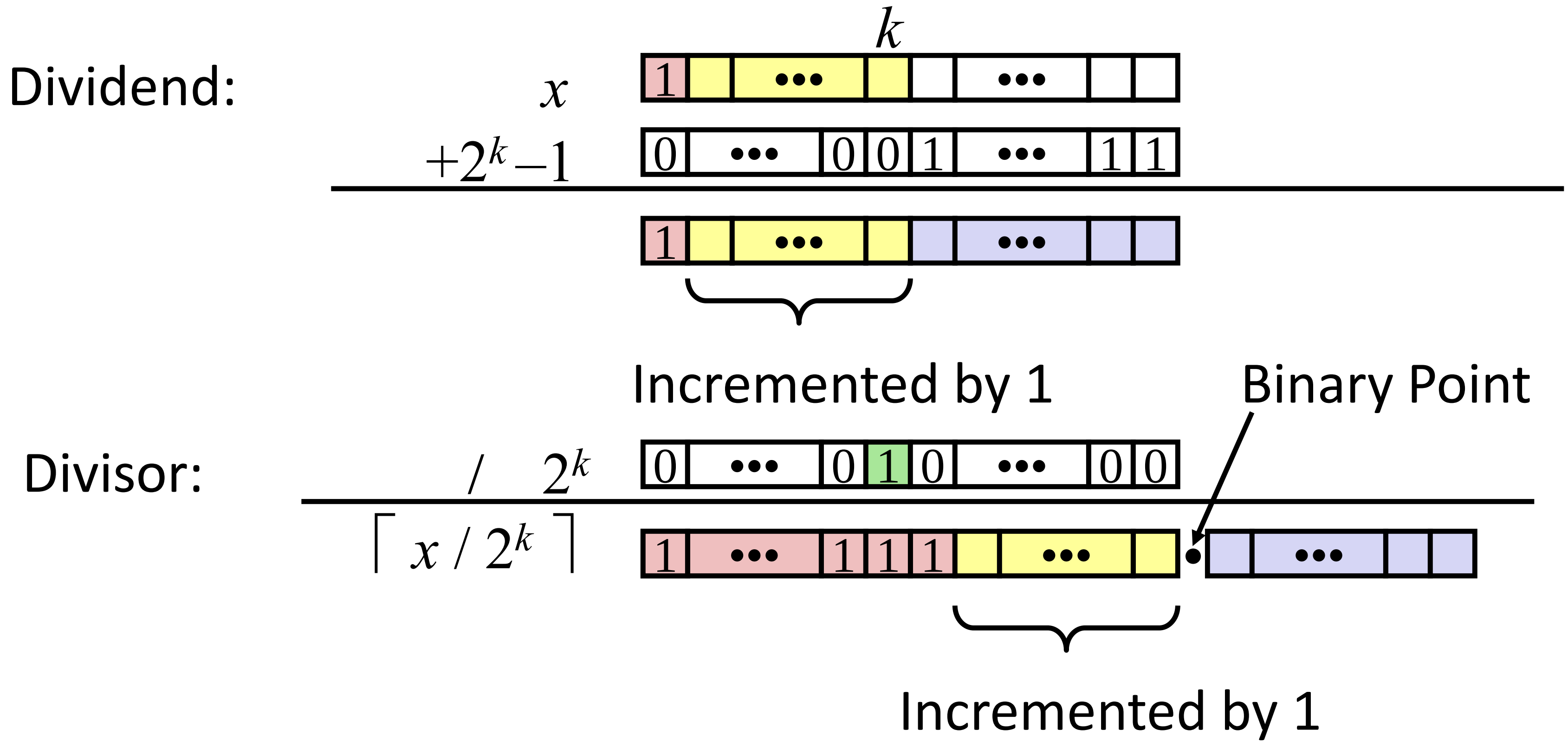
Case 1: No rounding




*Biasing has no effect*

# Correct Power-of-2 Divide (Cont.)

Case 2: Rounding




*Biasing adds 1 to final result*

# Compiled Signed Division Code

C Function

```
int idiv8(int x)
{
  return x/8;
}
```

Compiled Arithmetic Operations

```
  testq %rax, %rax
  js    L4
L3:
  sarq $3, %rax
  ret
L4:
  addq $7, %rax
  jmp  L3
```

Explanation

```
  if x < 0
    x += 7;
  # Arithmetic shift
  return x >> 3;
```

- Uses arithmetic shift for int
- For Java Users
  - Arith. shift written as >>

# Arithmetic: Basic Rules

- Unsigned ints, 2's complement ints are isomorphic rings: isomorphism = casting

- Left shift
  - Unsigned/signed: multiplication by $2^k$
  - Always logical shift

- Right shift
  - Unsigned: logical shift, div (division + round to zero) by $2^k$
  - Signed: arithmetic shift
    - Positive numbers: div (division + round to zero) by $2^k$
    - Negative numbers: div (division + round away from zero) by $2^k$
      Use biasing to fix

# Properties of Unsigned Arithmetic

- Unsigned Multiplication with Addition Forms Commutative Ring
  - Addition is commutative group
  - Closed under multiplication

    $0 \leq \mathrm{UMult}_w(u\,,\,v) \leq 2^w - 1$
  - Multiplication Commutative

    $\mathrm{UMult}_w(u\,,\,v) = \mathrm{UMult}_w(v\,,\,u)$
  - Multiplication is Associative

    $\mathrm{UMult}_w(t, \mathrm{UMult}_w(u\,,\,v)) = \mathrm{UMult}_w(\mathrm{UMult}_w(t, u\,), v)$
  - 1 is multiplicative identity

    $\mathrm{UMult}_w(u\,,\,1) = u$
  - Multiplication distributes over addtion

    $\mathrm{UMult}_w(t, \mathrm{UAdd}_w(u\,,\,v)) = \mathrm{UAdd}_w(\mathrm{UMult}_w(t, u\,), \mathrm{UMult}_w(t, v))$

# Properties of Two's Comp. Arithmetic

- Isomorphic Algebras
  - Unsigned multiplication and addition
    - Truncating to *w* bits
  - Two's complement multiplication and addition
    - Truncating to *w* bits
- Both Form Rings
  - Isomorphic to ring of integers mod $2^w$
- Comparison to (Mathematical) Integer Arithmetic
  - Both are rings
  - Integers obey ordering properties, e.g.,

    $u > 0 \quad\Rightarrow\quad u + v > v$

    $u > 0, v > 0 \quad\Rightarrow\quad u \cdot v > 0$
  - These properties are not obeyed by two's comp. arithmetic

    *TMax* `+ 1` == *TMin*

    `15213 * 30426` == `-10030` (16-bit words)