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University Physics, 14th Edition, Global Edition
– Hugh D. Young and Roger A. Freedman

Chapter 21

Electric Charge and Electric Field

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(2018 Fall).
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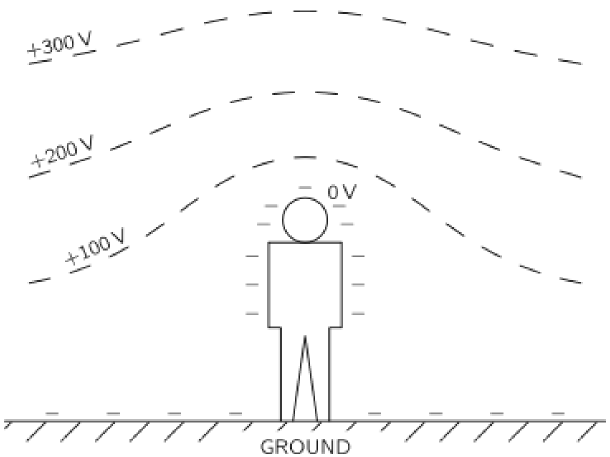
Goals for Chapter 21

- **Electric charge** and **charge conservation**
- Charging objects
- **Electric force** between objects - **Coulomb's law**
- **Electric field**
- Electric field due to many charges – **superposition principle**
- Electric field lines
- Electric dipole

Electromagnetism



(b)

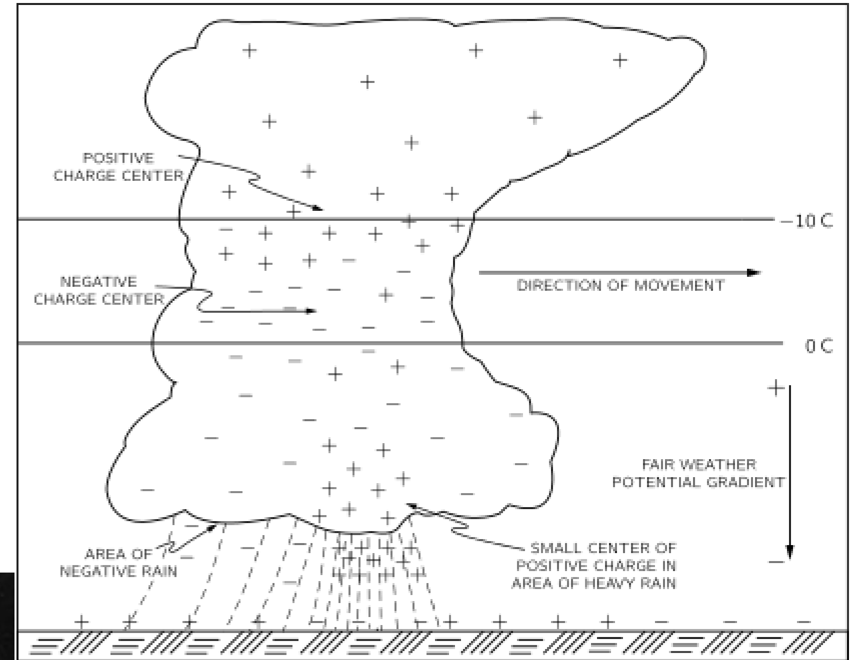
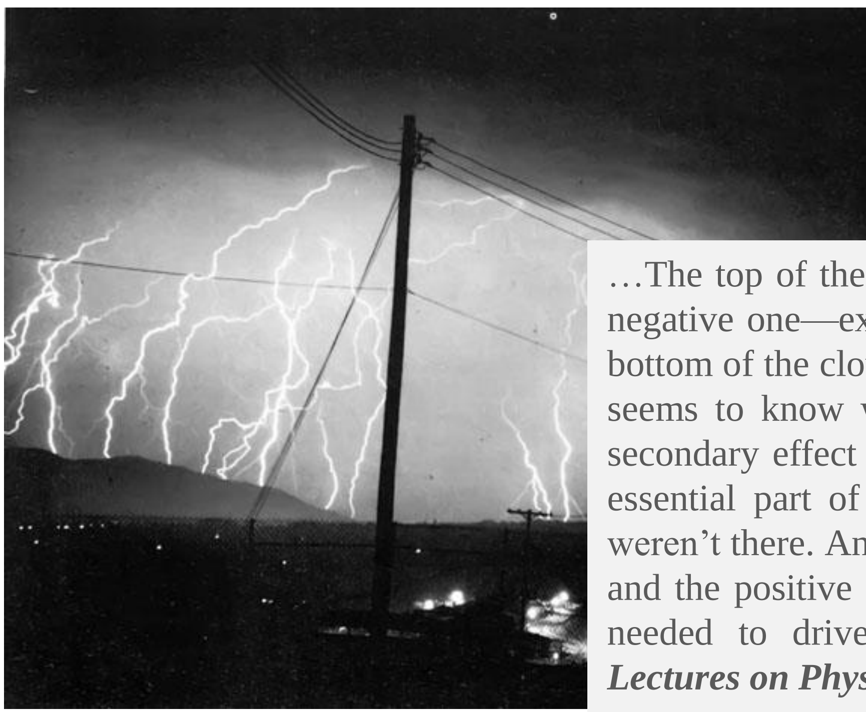


Fig. 9–11. The distribution of electrical charges in a mature thunderstorm cell. [From U.S. Department of Commerce Weather Bureau Report, June 1949.]



…The top of the thunderstorm has a positive charge, and the bottom a negative one—except for a small local region of positive charge in the bottom of the cloud, which has caused everybody a lot of worry. No one seems to know why it is there, how important it is—whether it is a secondary effect of the positive rain coming down, or whether it is an essential part of the machinery. Things would be much simpler if it weren't there. Anyway, the predominantly negative charge at the bottom and the positive charge at the top have the correct sign for the battery needed to drive the earth negative. (quoted from ***The Feynman Lectures on Physics***)

21.1 Electric charge

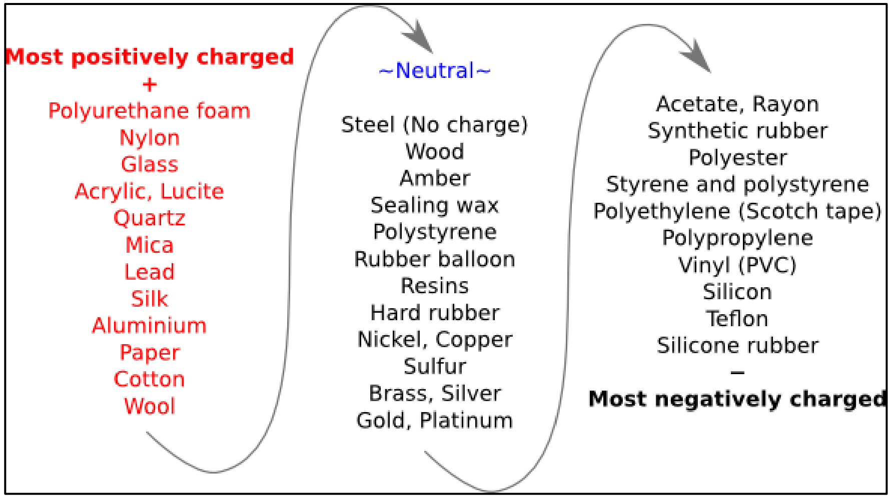
이종의 전하(electric charges, 電荷)의 분리 또는 재결합과 관련된 현상

- Separation of "positive" and "negative" charges: ***Triboelectric effect.***

(e.g.) **Amber** (琥珀, el-ektron ἤλ-εκτρον in Greek) **rubbed with cloth.**

(e.g.) Static discharge in winter

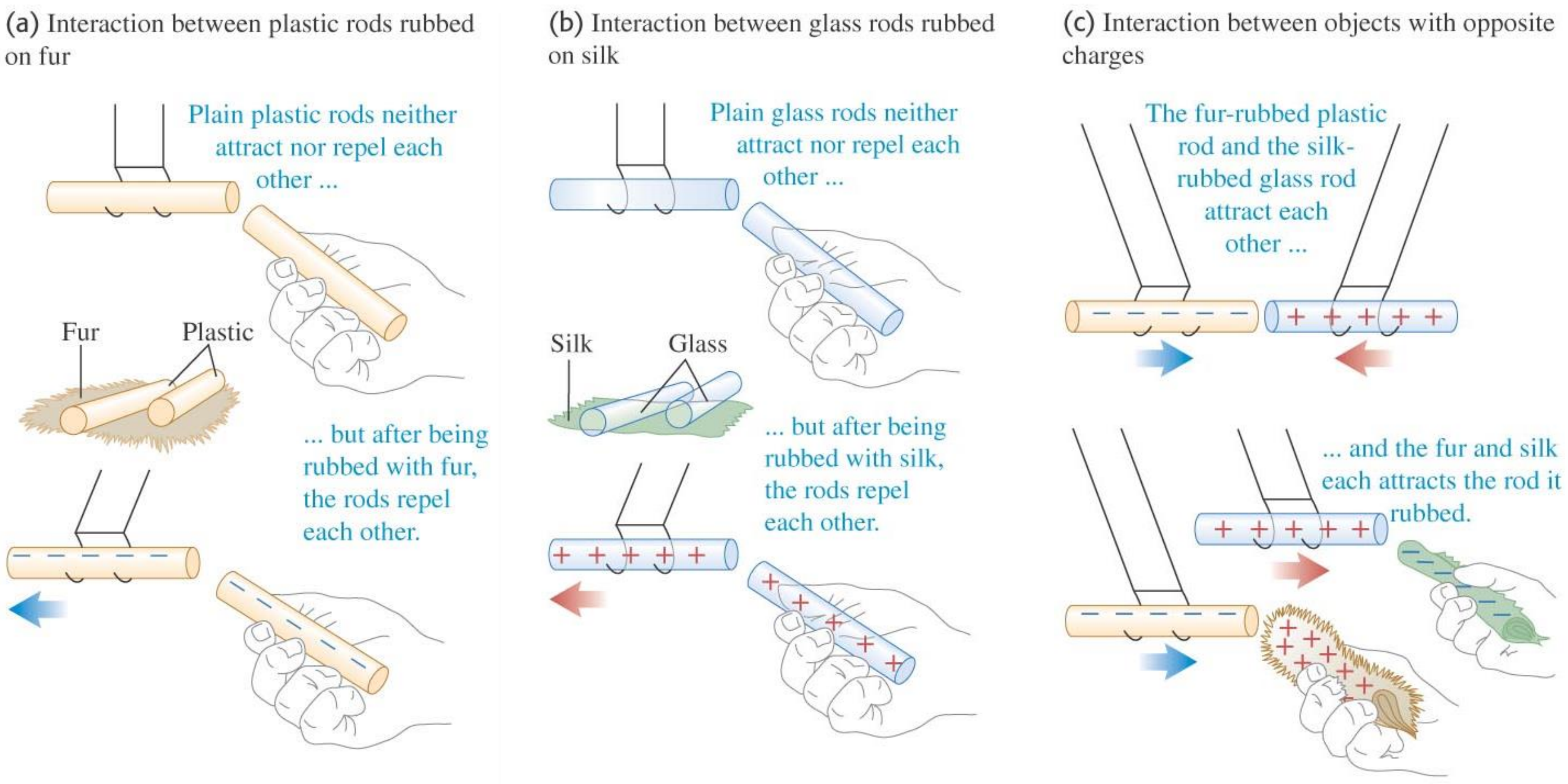
- Recombination of positive and negative charges: (e.g.) Lightning



Triboelectric series (from wikipedia data)

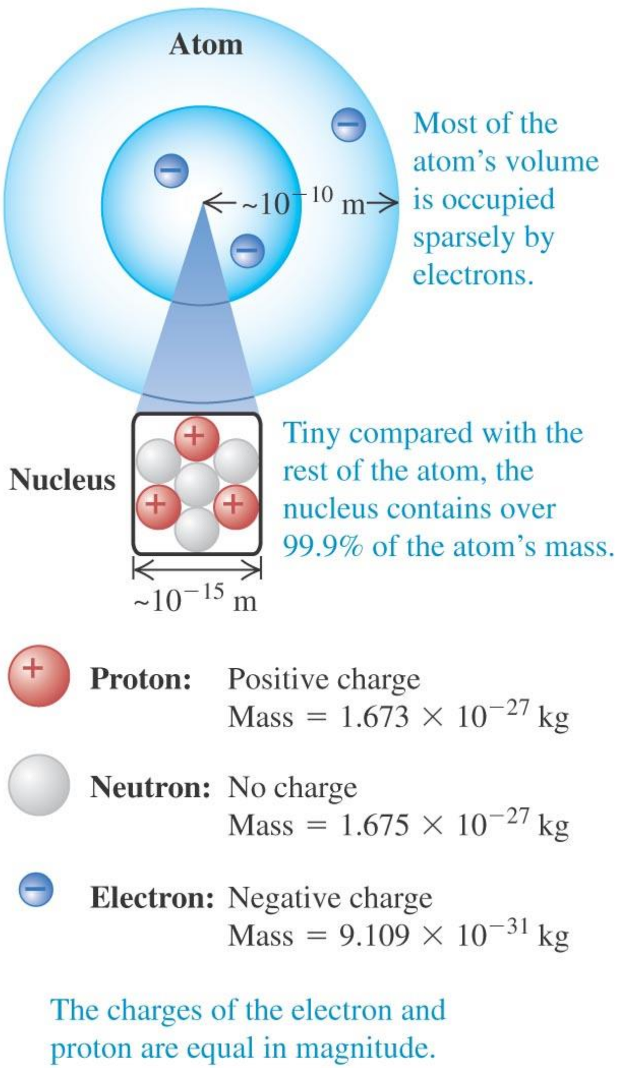
Positive and negative charges

- Two kinds of electric charges - **positive and negative** (cf. neutral)
- Same charges repel each other. Opposite charges attract each other.



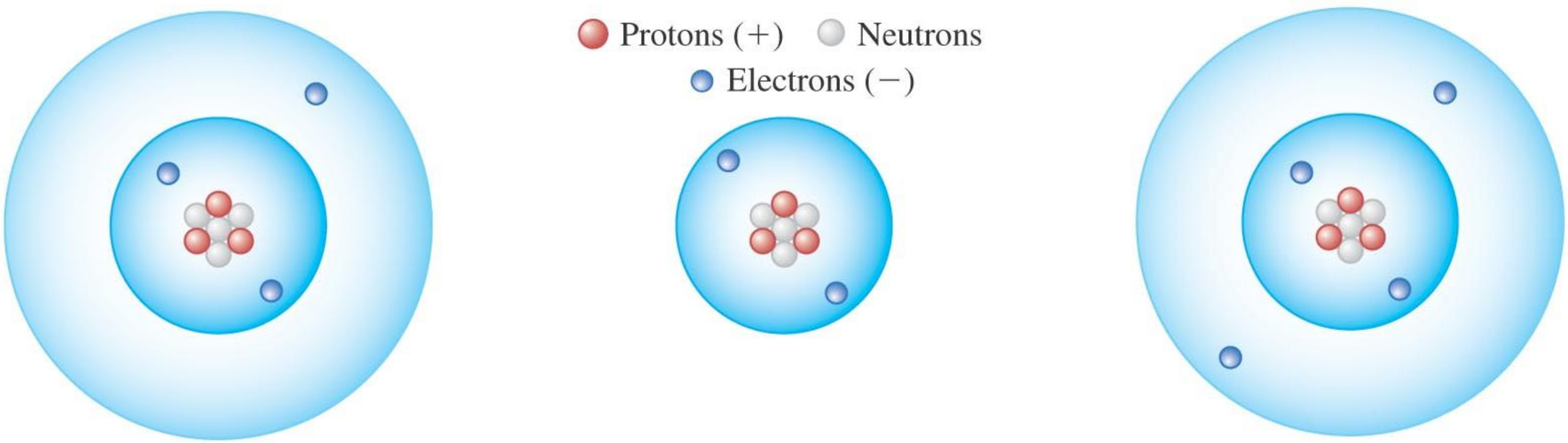
Electric charge and the structure of matter

- **Atom = nucleus(+) + electrons(-)**
- **Nucleus = protons(+) + neutrons(0)**
- The electron "shells" are a schematic representation of the actual electron distribution, a diffuse cloud many times larger than the nucleus (quantum mechanics)



Atoms and ions

- A **neutral atom** has the same number of protons as electrons.
- A **positive ion** is an atom with one or more electrons removed. A **negative ion** has one or more electrons.



(a) Neutral lithium atom (Li):

3 protons (3+)
4 neutrons
3 electrons (3−)

Electrons equal protons:
Zero net charge

(b) Positive lithium ion (Li^+):

3 protons (3+)
4 neutrons
2 electrons (2−)

Fewer electrons than protons:
Positive net charge

(c) Negative lithium ion (Li^-):

3 protons (3+)
4 neutrons
4 electrons (4−)

More electrons than protons:
Negative net charge

Conservation of charge

- The proton and electron have the same magnitude charge.
- The magnitude of charge of the electron or proton is a natural unit of charge, ***e***. All observable charge is **quantized** in this unit.
    - $Q = ne$, $e = 1.6021 \text{ x } 10^{-19}$ C, n = integer,
    - C, coulomb - unit of electric charge
- **Principle of charge conservation:** the algebraic sum of all the electric charges in any closed system is constant.

$$e^{+} + e^{-} \leftrightarrow \gamma + \gamma,$$

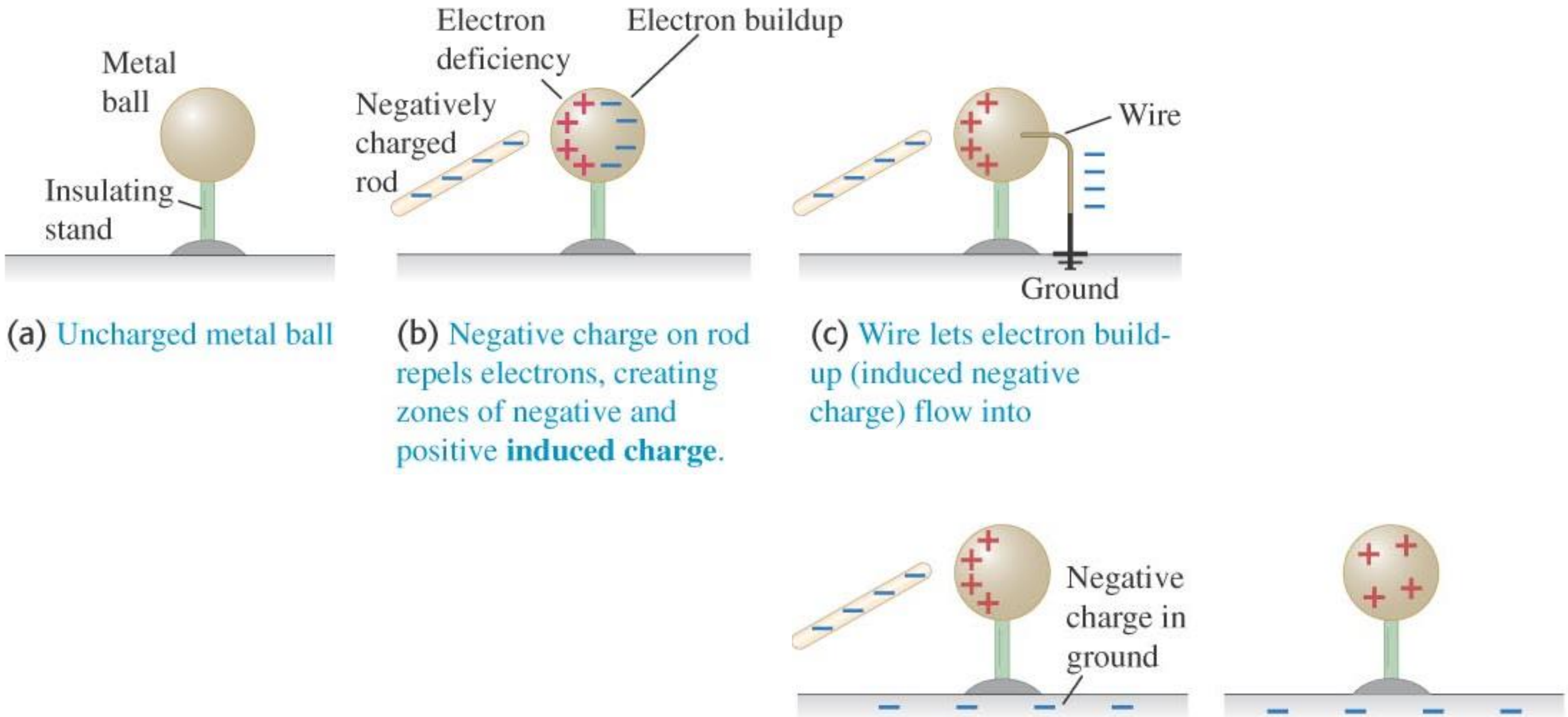
$$\gamma \rightarrow e^{+} + e^{-}$$

$$U_{92}^{238} \rightarrow Th_{90}^{234} + He_{2}^{4}$$

21.2 Conductors, insulators, and induced charge

- **Conductor**: charges can move easily – free electrons
- **Insulator**: no free charges
- **Semiconductor**: free electrons and holes can be generated
- **Superconductor**: electrical resistance ~ 0

Charging by induction



(a) Uncharged metal ball

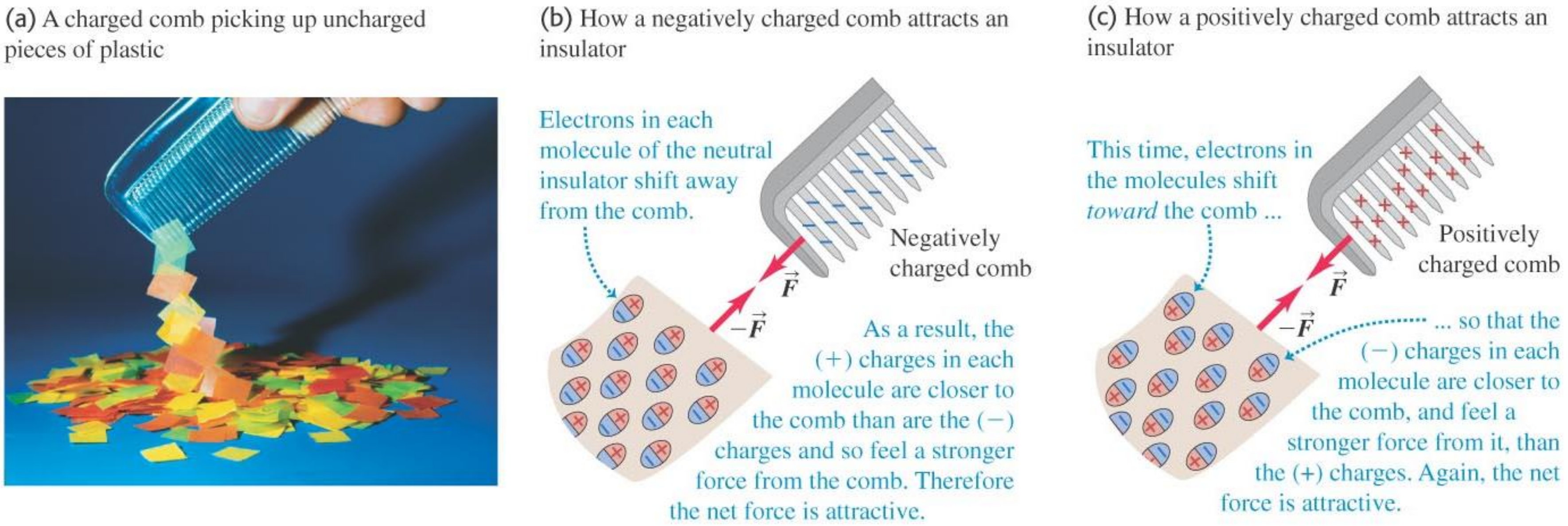
(b) Negative charge on rod repels electrons, creating zones of negative and positive **induced charge**.

(c) Wire lets electron build-up (induced negative charge) flow into

(d) Wire removed; ball now has only an electron-deficient region of positive charge.

(e) Rod removed; electrons rearrange themselves, ball has overall electron deficiency (net positive charge).

Electric forces on uncharged objects



- The negatively charged plastic comb causes a slight shifting of charge within the molecules of the neutral insulator, an effect called **polarization**.

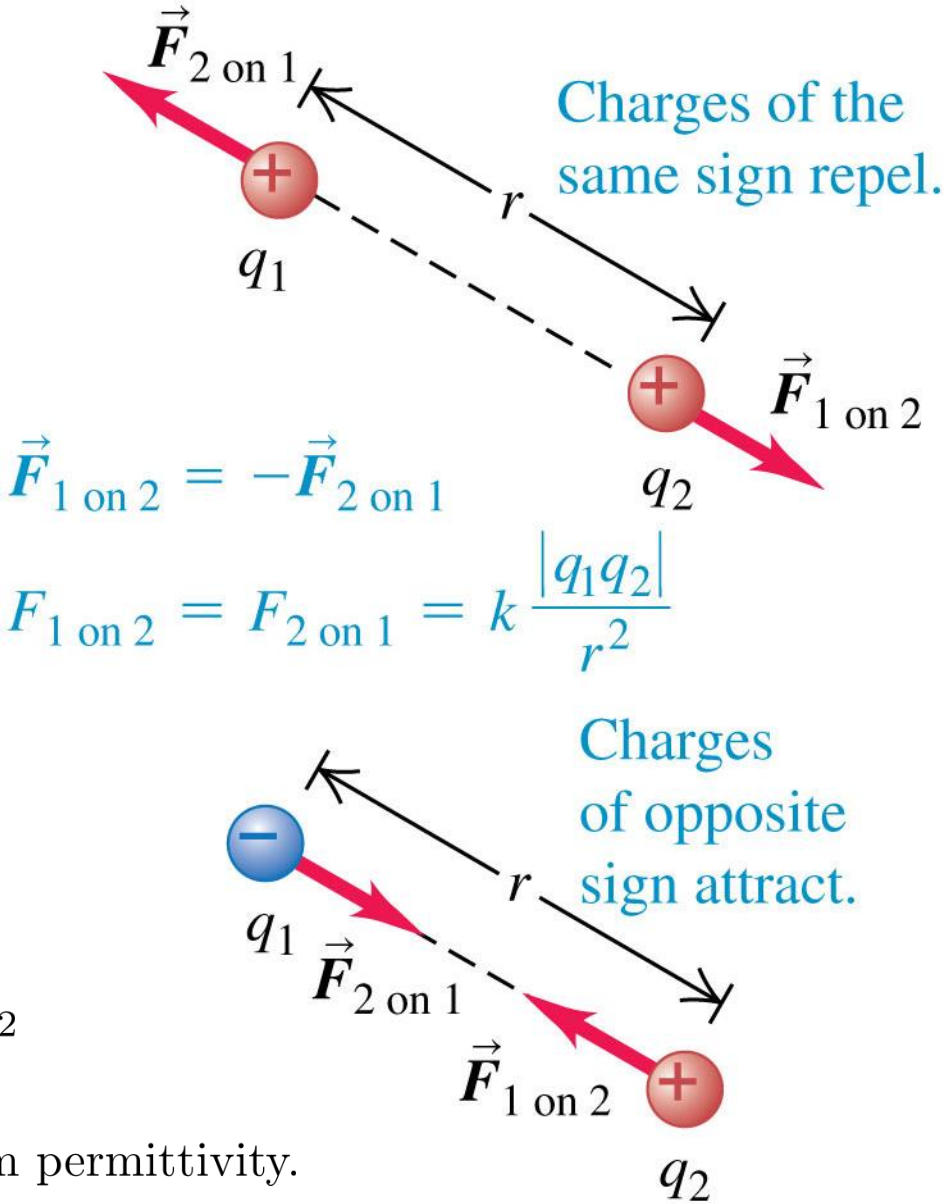
21.3 Coulomb's Law

- **Coulomb's Law**: The magnitude of the electric force between two **point charges** is directly proportional to the product of their charges and inversely proportional to the square of the distance between them.

$$F = k\frac{q_1 q_2}{r^2}$$

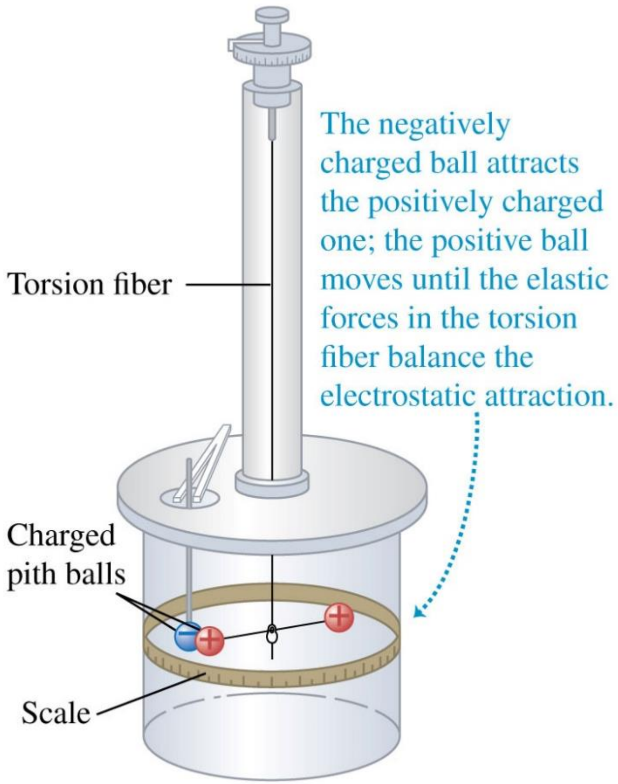
$$k = \frac{1}{4\pi\epsilon_0} \approx 8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$$

ϵ_0: an electric constant called vacuum permittivity.



Measuring the electric force between point charges

- Coulomb studied the interaction forces of charged particles in detail in 1784.
- He used a torsion balance similar to the one used 13 years later by Cavendish to study the much weaker gravitational interaction.



- Coulomb force and gravitational force between electron and proton in a H atom

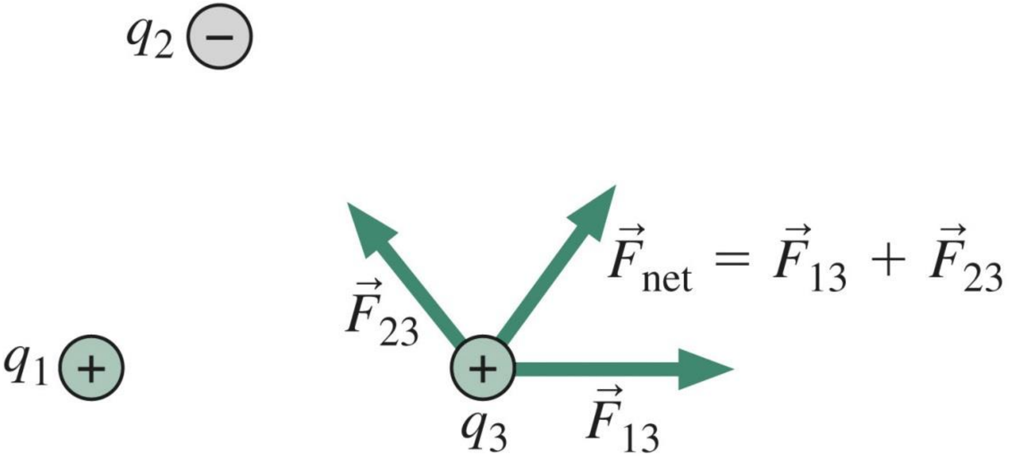
$$F_C = 9\times10^9 \frac{(1.6\times10^{-19})^2}{(0.5\times10^{-10})^2} \approx 10^{-7}\ N \qquad F_G = 6.7\times10^{-11}\frac{10^{-30}\cdot10^{-27}}{(0.5\times10^{-10})^2} \approx 10^{-47}\ N$$

$$F_C/F_G \sim 10^{40}$$

The Superposition Principle

- The net electric force is the sum of the individual forces.

$$\overrightarrow{F_i} = \sum_j \overrightarrow{F_{ji}}$$

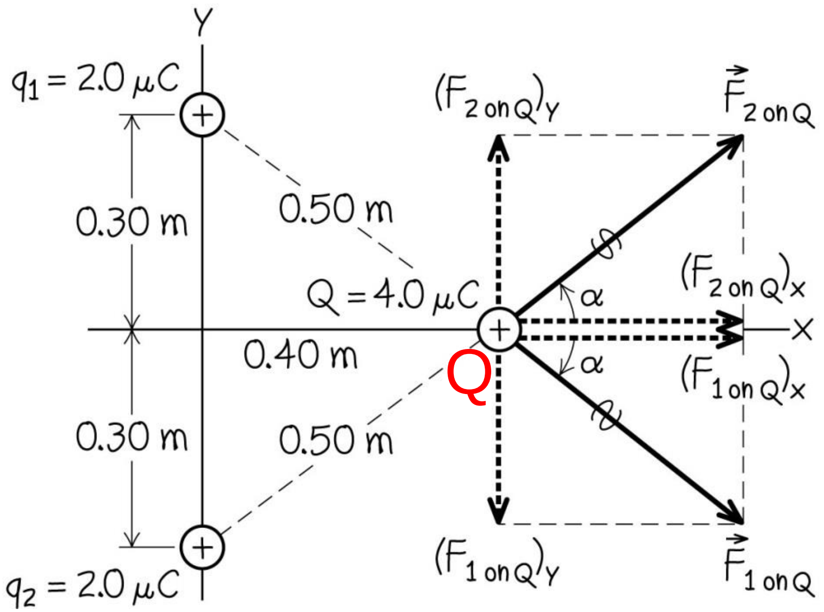


Vector addition of electric forces

Two equal positive charges $q_1 = q_2 = 2.0\ \mu C$ are located at $x = 0$, $y = 0.30$ m and $x = 0$, $y = -0.30$ m, respectively. What are the magnitude and direction of the total electric force that q_1 and q_2 exert on a third charge $Q = 4.0\ \mu C$ at $x = 0.40$ m, $y = 0$?

$$F_{1Q} = F_{2Q} = \frac{kq_1 Q}{\Delta x^2 + \Delta y^2} \approx 0.29\ \mathrm{N}$$

$$F_{1Q,x} = F_{1Q}\cos\alpha \approx 0.23\ \mathrm{N}$$



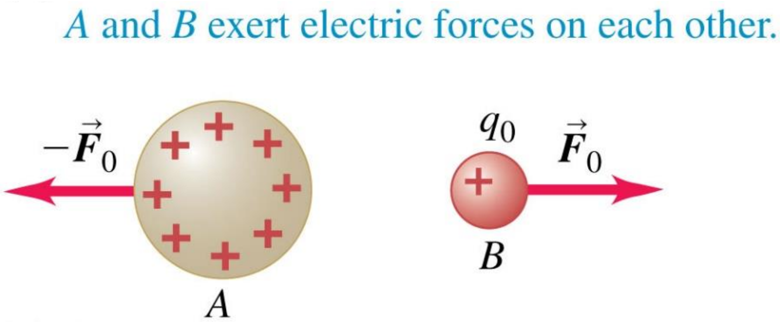
The total force and along the x-axis is

$$F_x = F_{1Q,x} + F_{2Q,x} \approx 0.46\ \mathrm{N}$$

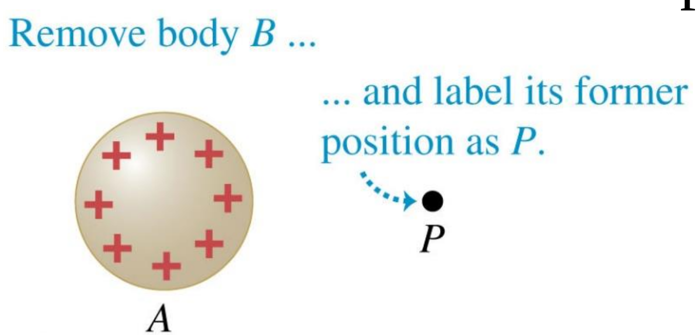
The total force along the y-axis is zero by symmetry.

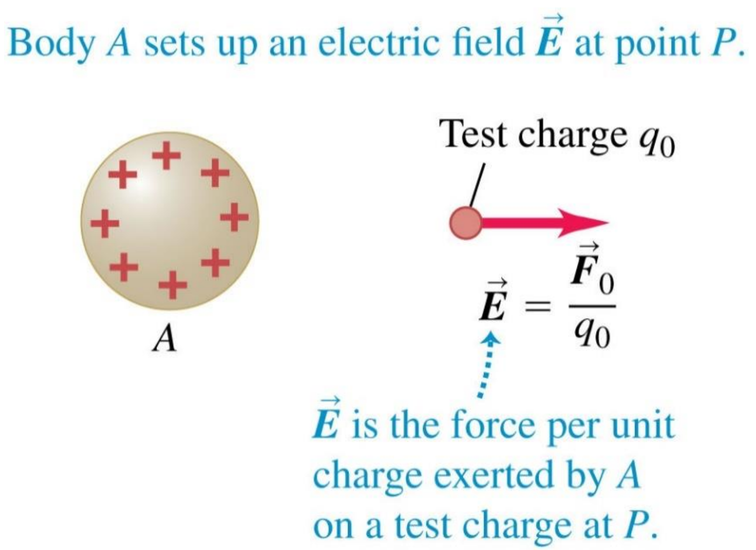
$$F_y = F_{1Q,y} + F_{2Q,y} = 0$$

21.4 Electric field and electric forces



- We can say that charge A somehow modifies the properties of the space at point P.

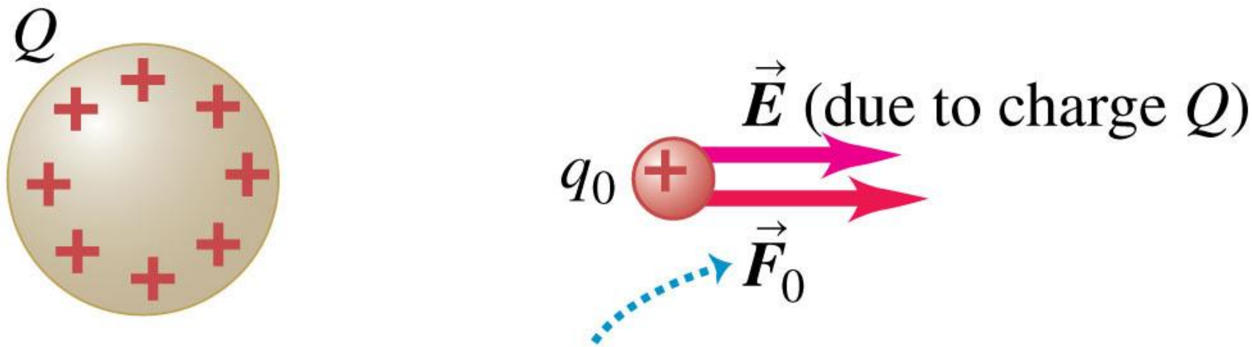




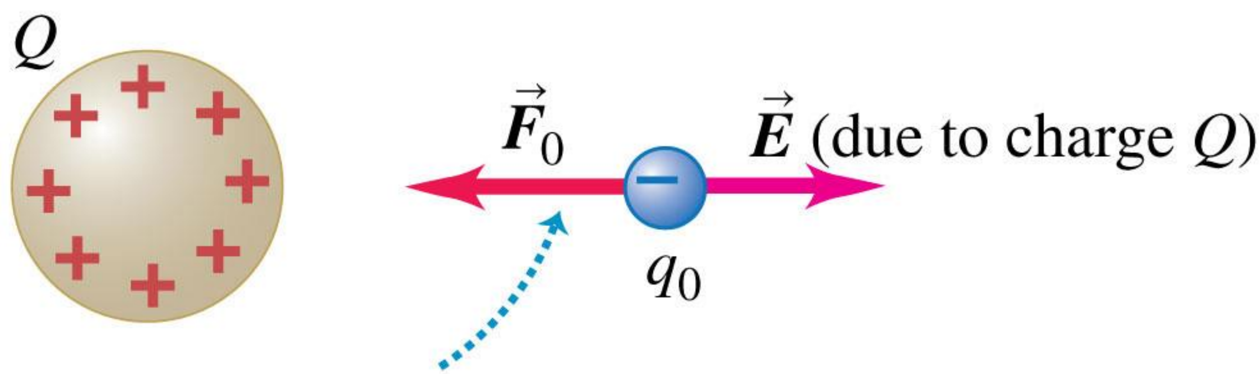
$$\vec{F} = q\vec{E} \leftrightarrow \vec{E} = \frac{\vec{F}}{q}$$

- SI unit for electric field: N/C

Electric force produced by an electric field



The force on a positive test charge q_0 points in the direction of the electric field.



The force on a negative test charge q_0 points opposite to the electric field.

The electric field of a point charge

- Using a unit vector that points away from the origin, we can write a vector equation that gives both the magnitude and the direction of the electric field:

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

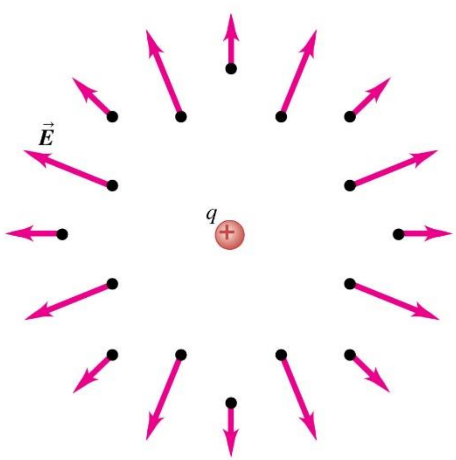
Electric field due to a point charge

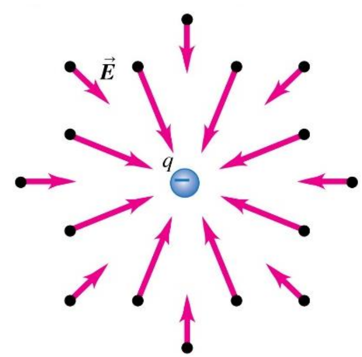
Value of point charge

Unit vector from point charge toward where field is measured

Distance from point charge to where field is measured

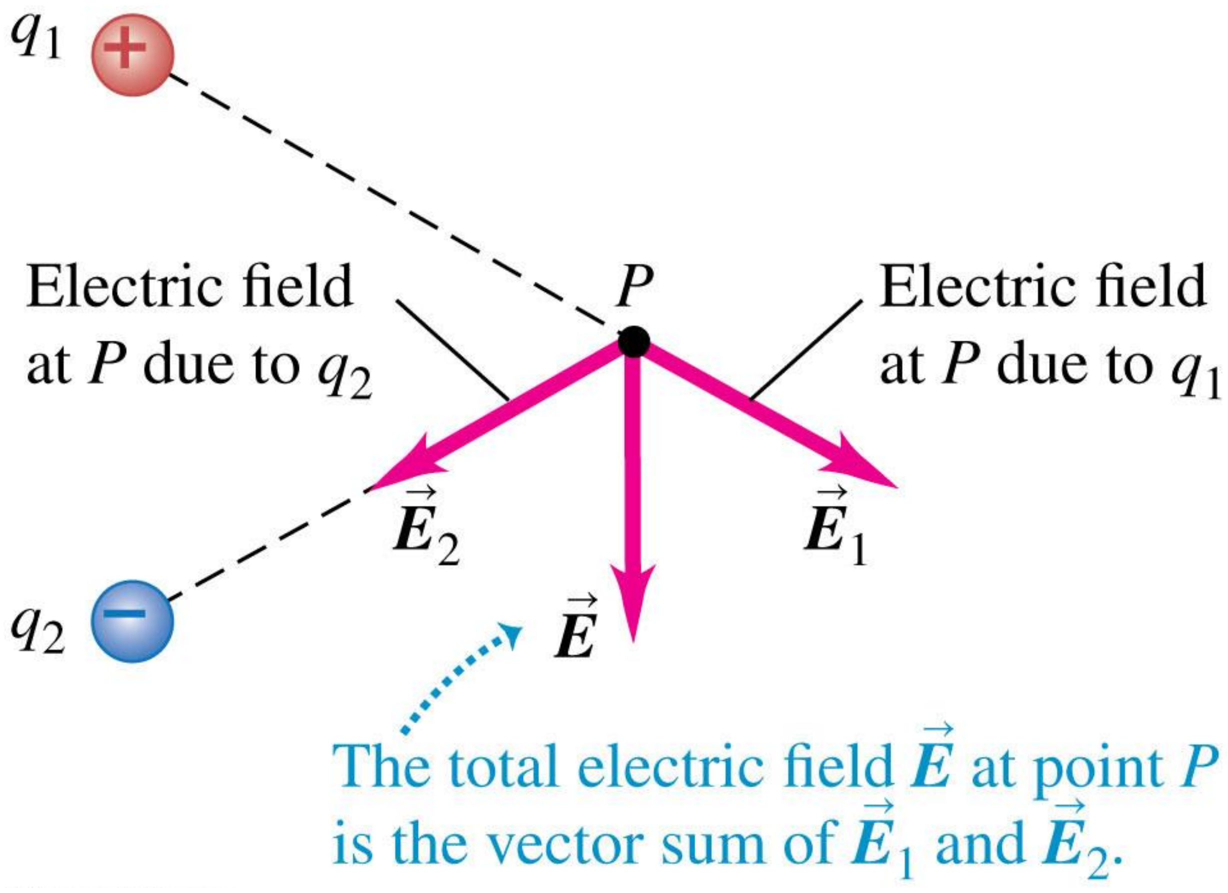
Electric constant





Superposition of electric fields

- The total electric field at a point is the vector sum of the fields due to all the charges present.



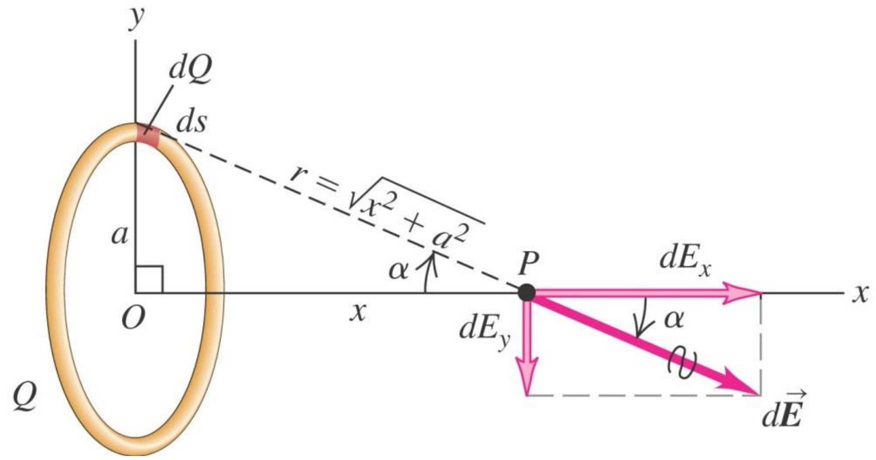
21.5 Electric-field calculation: Field of a ring of charge

Charge Q is uniformly distributed around a conducting ring of radius a (Fig. 21.23). Find the electric field at a point P on the ring axis at a distance x from its center.

$$\vec{E} = \int d\vec{E} = \int \hat{r}\frac{kdq}{r^2}$$

$$dE = \frac{1}{4\pi\epsilon_0}\frac{dQ}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{\lambda ds}{x^2+a^2}$$

λ: line charge density

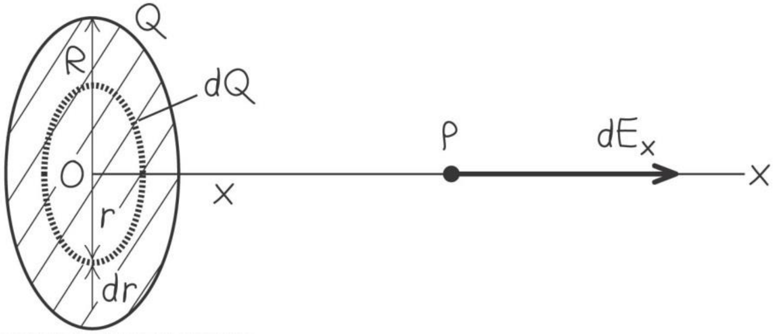


$$\begin{aligned} E_x &= \int dE\cos\alpha \\ &= \frac{x}{4\pi\epsilon_0(x^2+a^2)^{3/2}}\oint \lambda ds \\ &= \frac{xQ}{4\pi\epsilon_0(x^2+a^2)^{3/2}} = \begin{cases} \dfrac{Q}{4\pi\epsilon_0 x^2} & \text{for } x \gg a \\ 0 & \text{for } x = 0 \end{cases} \end{aligned}$$

Field of a uniformly charged disk

A nonconducting disk of radius R has a uniform positive surface charge density σ. Find the electric field at a point along the axis of the disk a distance x from its center. Assume that x is positive.

$dQ = \sigma 2\pi r dr$:
the charge in the ring of radius r and width dr.



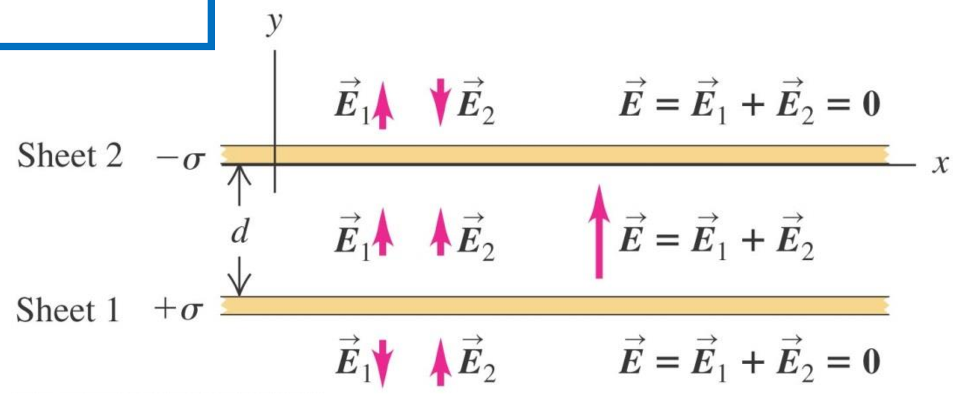
$$dE_x = \frac{1}{4\pi\epsilon_0}\frac{\sigma 2\pi r dr \cdot x}{(x^2+r^2)^{3/2}}$$

$$\begin{aligned} E_x &= \int_0^R \frac{x(2\pi\sigma r dr)}{4\pi\epsilon_0(x^2+r^2)^{3/2}} = \frac{x\sigma}{4\epsilon_0}\int_0^R \frac{2r dr}{(x^2+r^2)^{3/2}} \\ &= \frac{x\sigma}{2\epsilon_0}\left[\frac{1}{x} - \frac{1}{\sqrt{x^2+R^2}}\right] \\ &= \begin{cases} \boxed{\sigma/2\epsilon_0 \quad \text{for } x \ll R} \\ \frac{Q}{4\pi\epsilon_0 x^2} \quad \text{for } x \gg R \end{cases} \end{aligned}$$

→ constant, i.e. independent of the distance from the disk !!

Field of two oppositely charged infinite sheets

Two infinite plane sheets with uniform surface charge densities $+\sigma$ and $-\sigma$ are placed parallel to each other with separation d (Fig. 21.26). Find the electric field between the sheets, above the upper sheet, and below the lower sheet.



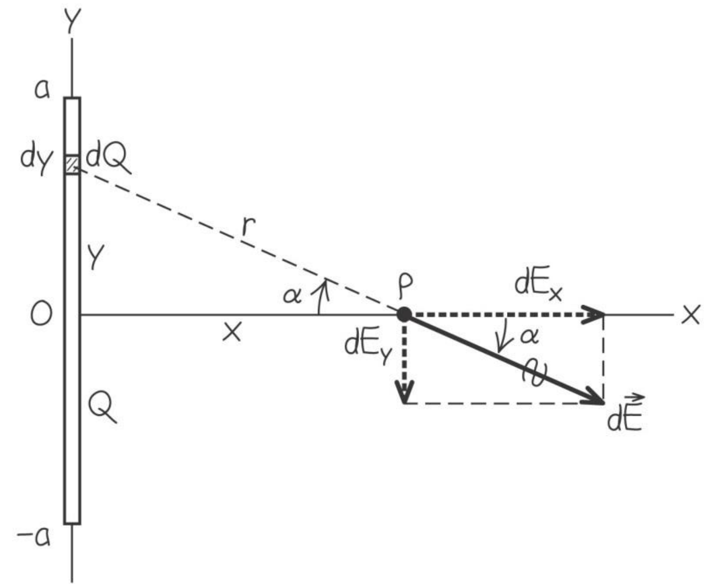
$$E_1 = E_2 = \frac{\sigma}{2\epsilon_0}$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \begin{cases} 0 & \text{above/below the sheets} \\ \boxed{\hat{y}\dfrac{\sigma}{\epsilon_0} \quad \text{between the sheets}} \end{cases}$$

➔ Capacitor!! (storage of electric fields)

Field of a charged line segment

Positive charge Q is distributed uniformly along the y-axis between $y = -a$ and $y = +a$. Find the electric field at point P on the x-axis at a distance x from the origin.



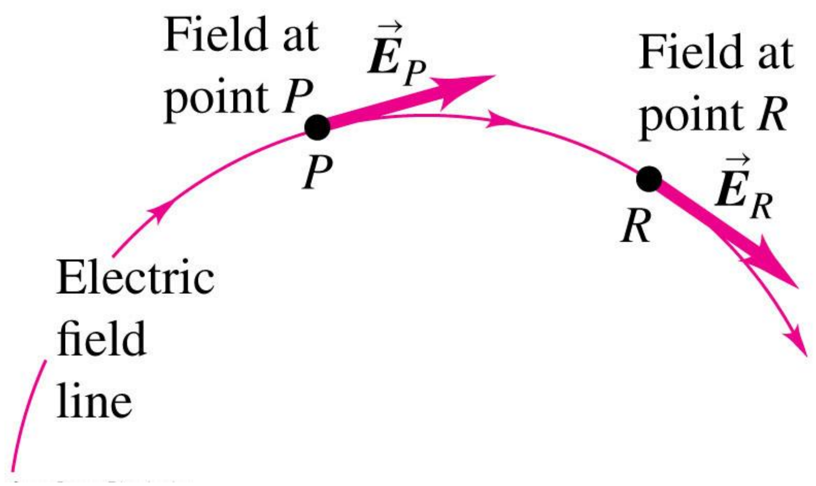
$$dE = \frac{1}{4\pi\epsilon_0}\frac{dQ}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{Q}{2a}\frac{dy}{(x^2+y^2)}$$

$$\vec{E} = \hat{x}E_x \text{ (by symmetry).}$$

$$\begin{aligned} E_x &= \int_{-a}^{+a} dE_x = \frac{Q}{4\pi\epsilon_0}\frac{1}{x(x^2+y^2)^{1/2}} \\ &= \begin{cases} \dfrac{Q}{4\pi\epsilon_0 x^2} & \text{for } x \gg a \\ \left(\dfrac{Q}{2a}\right)\dfrac{1}{2\pi\epsilon_0 x} & \text{for } x \ll a \end{cases} \end{aligned}$$

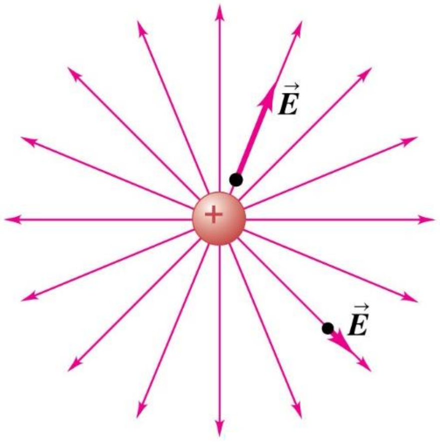
21.6 Electric field lines

- An **electric field line** is an imaginary line or curve whose **tangent** at any point is the **direction** of the electric field vector at that point. The line density is proportional to the **magnitude** of the electric field.

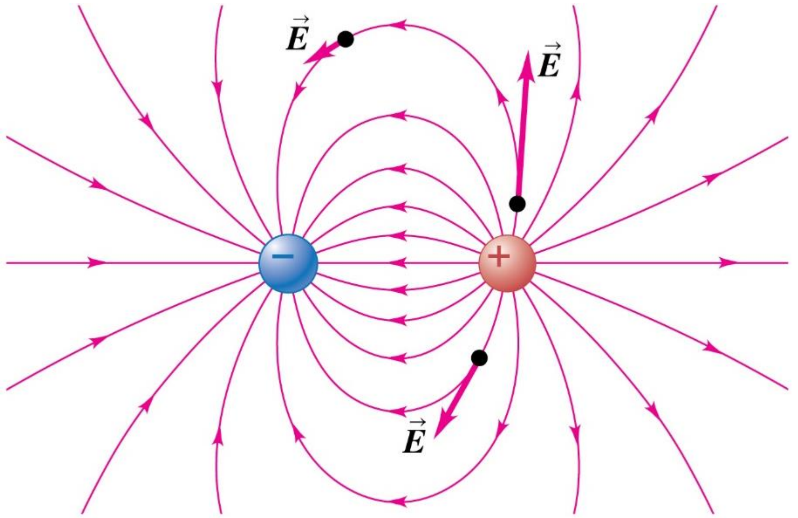


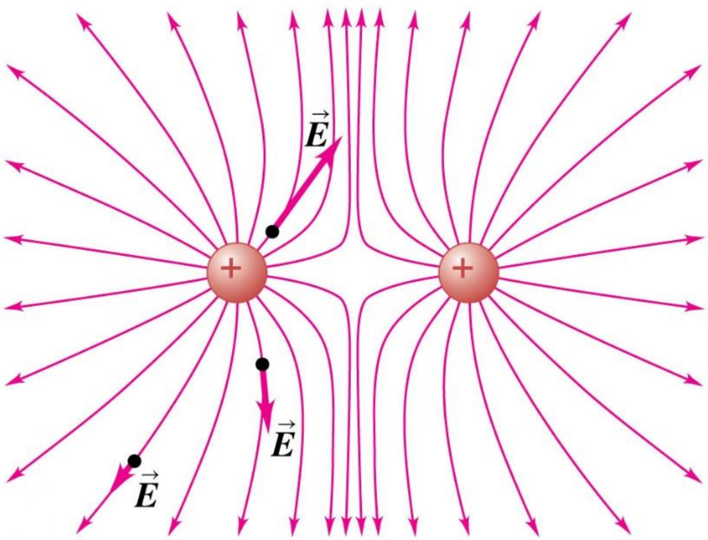
✓ Electric field lines never cross each other.

Examples of electric field lines



- Positive point charge
- Dipole
- Two equal positive charges





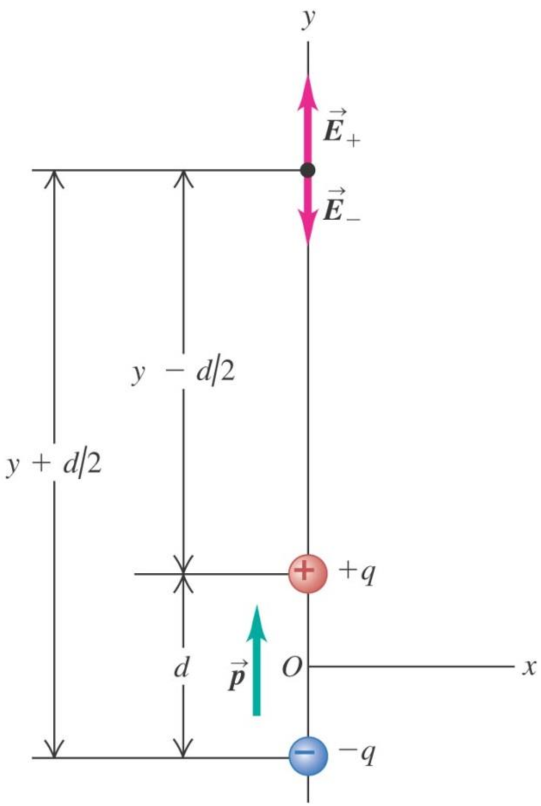
21.7 Electric dipoles

- An **electric dipole** consists of two point charges of equal magnitude but opposite signs, held a short distance apart.
  - The dipole is electrically neutral, but the separation of its charges results in an electric field.
- **Electric dipole moment**: $\vec{p} = q\,\vec{d}$

$\vec{E}_p // \vec{p}$

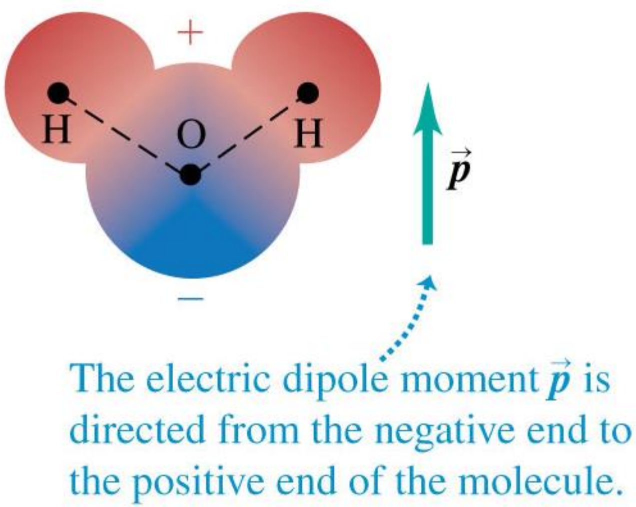
$$\begin{aligned} E_p &= E^+ - E^- = \frac{q}{4\pi\epsilon_0}\left[\left(y-\frac{d}{2}\right)^{-2} - \left(y+\frac{d}{2}\right)^{-2}\right] \\ &\approx \frac{q}{4\pi\epsilon_0 y^2}\frac{2d}{y} \quad (\text{for } y \gg d) \\ &= \frac{1}{2\pi\epsilon_0}\frac{p}{y^3} \end{aligned}$$

E_p decays much faster than E_q

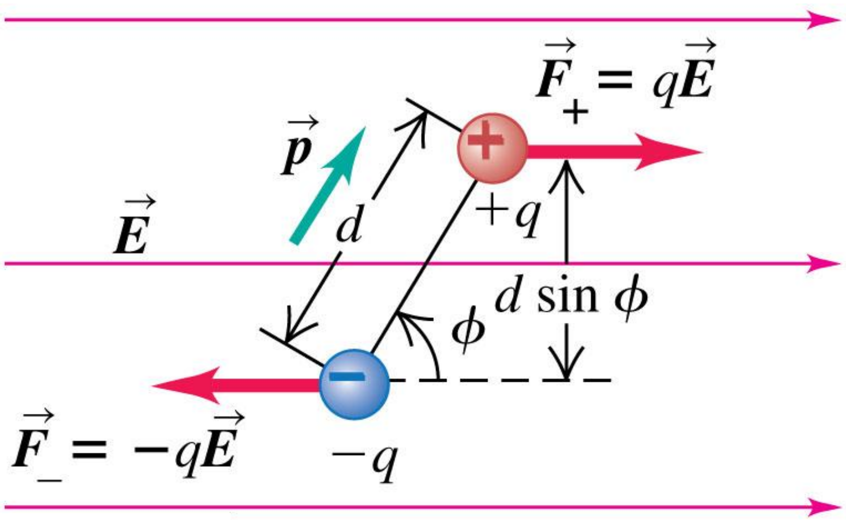


The water molecule is an electric dipole

- The **water molecule** as a whole is electrically neutral, but the chemical bonds within the molecule cause a displacement of charge.
- The result is a net negative charge on the oxygen end of the molecule and a net positive charge on the hydrogen end, forming an electric dipole.



Force, torque, and energy of a dipole



- A dipole in a uniform electric field experiences **no net force** but a **torque** that tends to align the dipole moment with the field:

$$\vec{\tau} = \vec{p} \times \vec{E}.$$

- A dipole in an electric field has a (rotational) **potential energy**:

$$U = -\vec{p} \cdot \vec{E}$$

$$U = -\int \tau \, d\phi = \int pE \sin\phi \, d\phi = -pE\cos\phi$$

21 Summary

- Electric charge
- Electric force and Coulomb's law
- Electric field
- Superposition of electric fields
- Electric field lines
- Electric dipoles