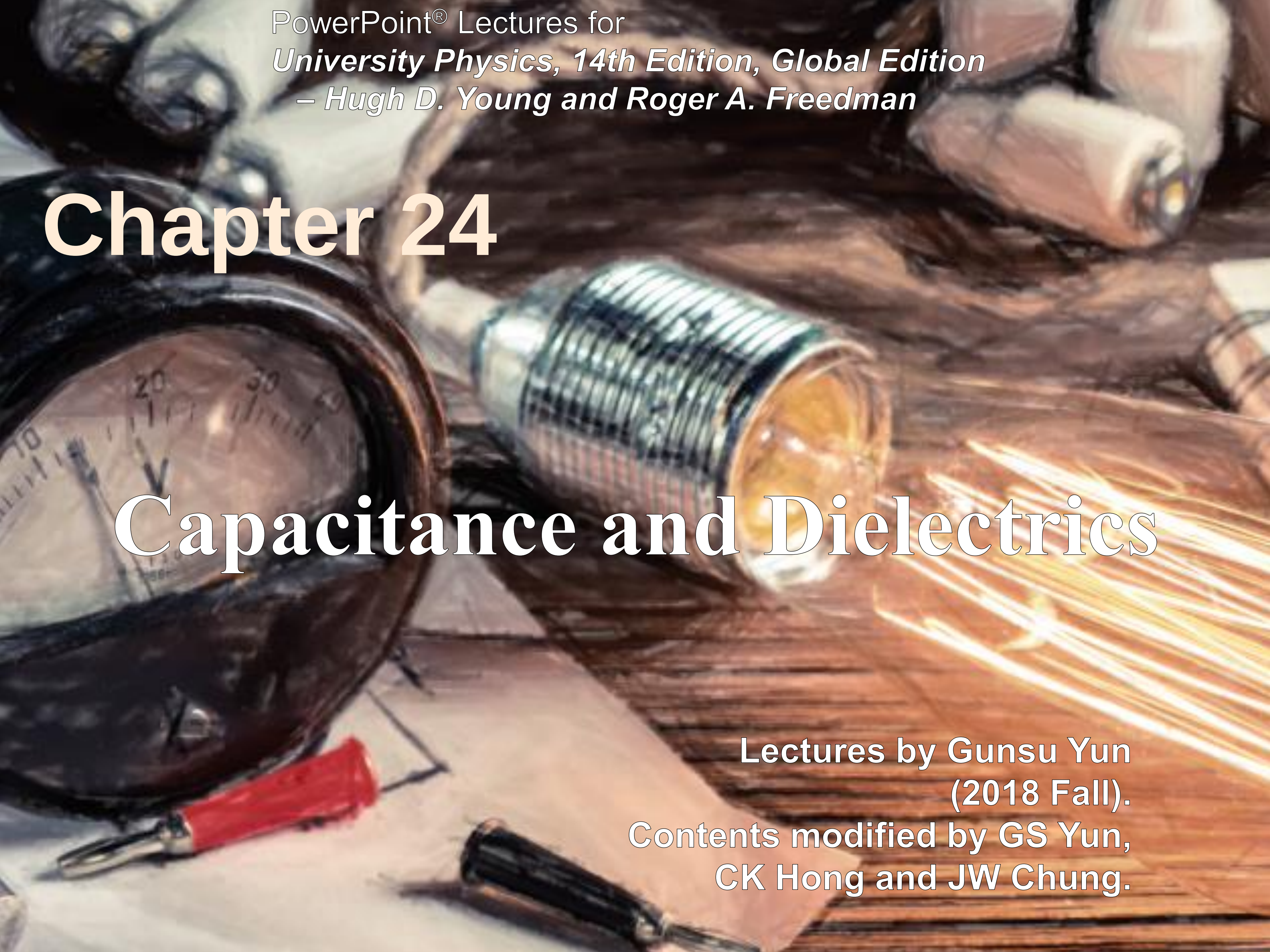

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# Chapter 24

# Capacitance and Dielectrics

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(2018 Fall).
Contents modified by GS Yun,
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# Goals for Chapter 24

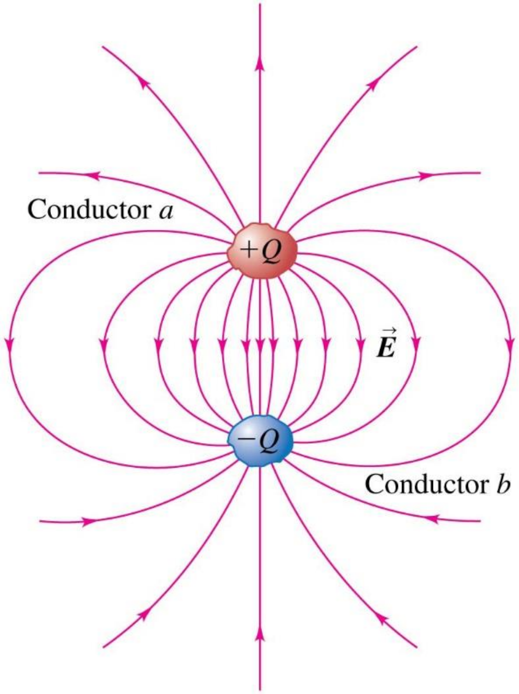
- **Capacitors** – store electric charge
- **Capacitance**
- Capacitors **in series** and **parallel**
- Energy stored in a capacitor
- **Dielectrics**

# 24.1 Capacitors and capacitance

- Two conductors separated by an insulator (or a vacuum) form a **capacitor**.

- When the capacitor is charged, the two conductors have charges with **equal magnitude and opposite sign**, and the net charge on the capacitor as a whole is zero.

- Symbols of capacitor: 




# Capacitance

- One common way to charge a capacitor is to connect the two conductors to opposite terminals of a battery.

- The **potential difference** $V_{ab}$ between the conductors becomes equal to the voltage of the battery.

- The ratio of charge to potential difference is called the **capacitance** $C$ of the capacitor:

**Capacitance** of a capacitor

$$C = \frac{Q}{V_{ab}}$$

Magnitude of charge on each conductor

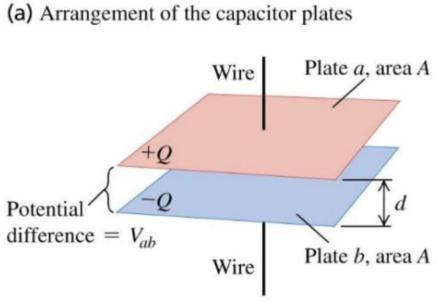
Potential difference between conductors ($a$ has charge $+Q$, $b$ has charge $-Q$)

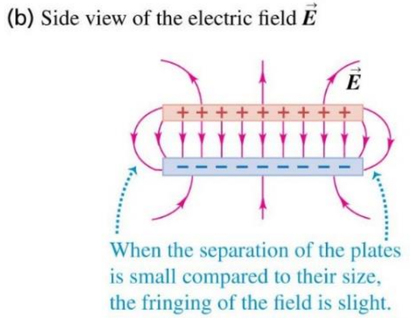
- SI unit of capacitance: **farad, F** (= C/V)

- The capacitance depends on the **geometry** of the capacitor.

# Parallel-plate capacitor

- The field between the plates is essentially uniform, and the charges are uniformly distributed over their opposing surfaces.



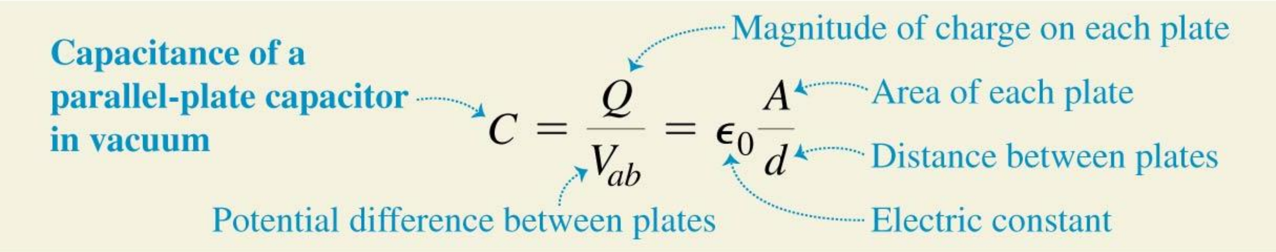




$$E = \frac{\sigma}{\epsilon_0} \quad \text{(from chap.22)}$$

$$C = \frac{Q}{V_{ab}} = \frac{\sigma A}{Ed} = \frac{\epsilon_0 E\, A}{Ed} = \frac{\epsilon_0 A}{d}$$

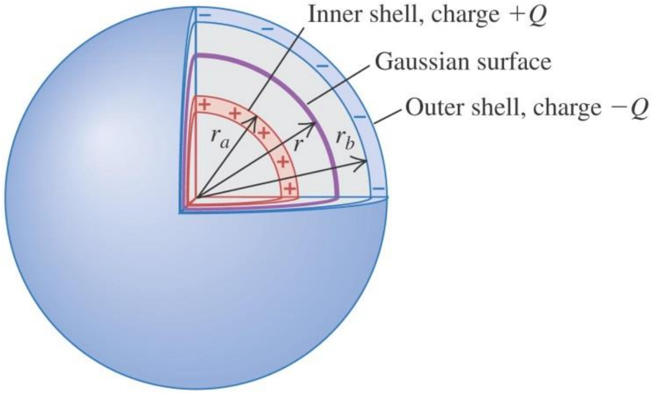
$$= \frac{\epsilon_0 \Phi_E}{V_{ab}}$$

← Storage capacity of charge (or electric flux)



# A spherical capacitor

Two concentric spherical conducting shells are separated by vacuum (Fig. 24.5). The inner shell has total charge $+Q$ and outer radius $r_a$, and the outer shell has charge $-Q$ and inner radius $r_b$. Find the capacitance of this spherical capacitor.



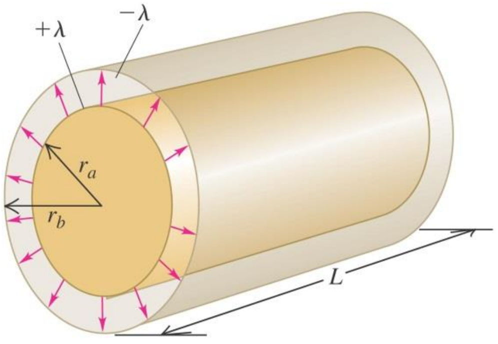

$$V_{ab} = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

$$\Rightarrow C = \frac{Q}{|V_{ab}|} = 4\pi\epsilon_0 \frac{r_b r_a}{r_b - r_a}$$

$C \to 4\pi\epsilon_0 r_a$ if $r_b \to \infty$.

# A cylindrical capacitor

Two long, coaxial cylindrical conductors are separated by vacuum (Fig. 24.6). The inner cylinder has radius $r_a$ and linear charge density $+\lambda$. The outer cylinder has inner radius $r_b$ and linear charge density $-\lambda$. Find the capacitance per unit length for this capacitor.




$$V_{ab} = \frac{\lambda}{2\pi\epsilon_0}\ln\frac{r_b}{r_a}$$

$$\Rightarrow C = \frac{\lambda L}{V_{ab}} = \frac{2\pi\epsilon_0 L}{\ln(r_b/r_a)}$$

The capacitance per unit length, $\dfrac{C}{L} = \dfrac{2\pi\epsilon_0}{\ln(r_b/r_a)}$

# 24.2 Capacitors in series and parallel

## Capacitors in series

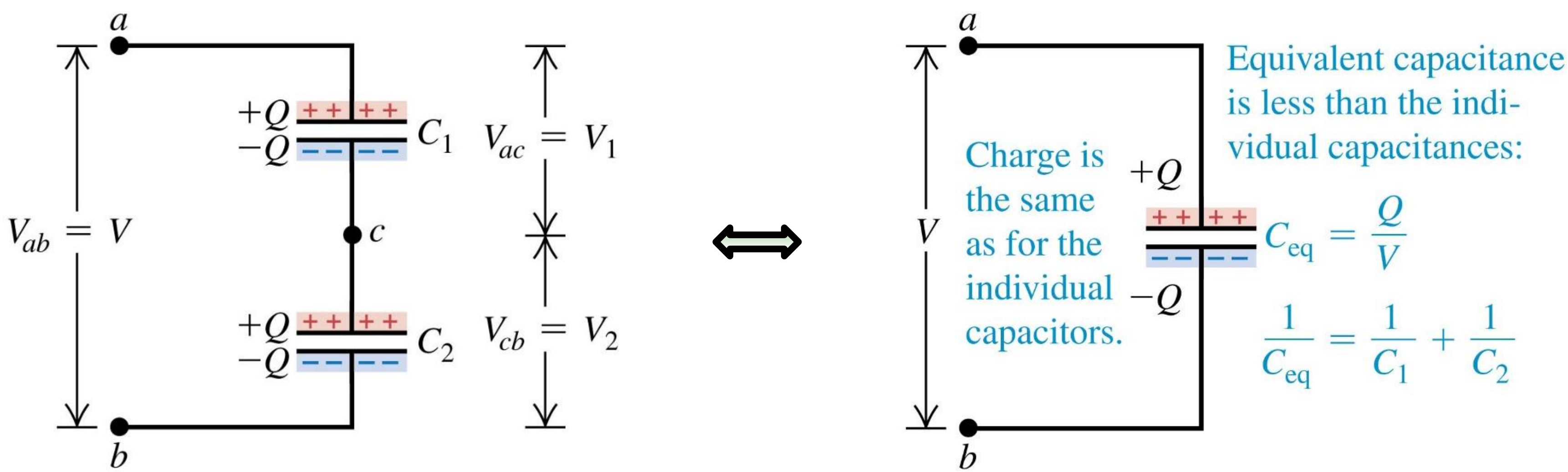
- Capacitors are **in series** if they are connected one after the other. Both capacitors have the **same charge** $Q$.

- **Equivalent capacitor**

$$V_{ab} = V_1 + V_2$$

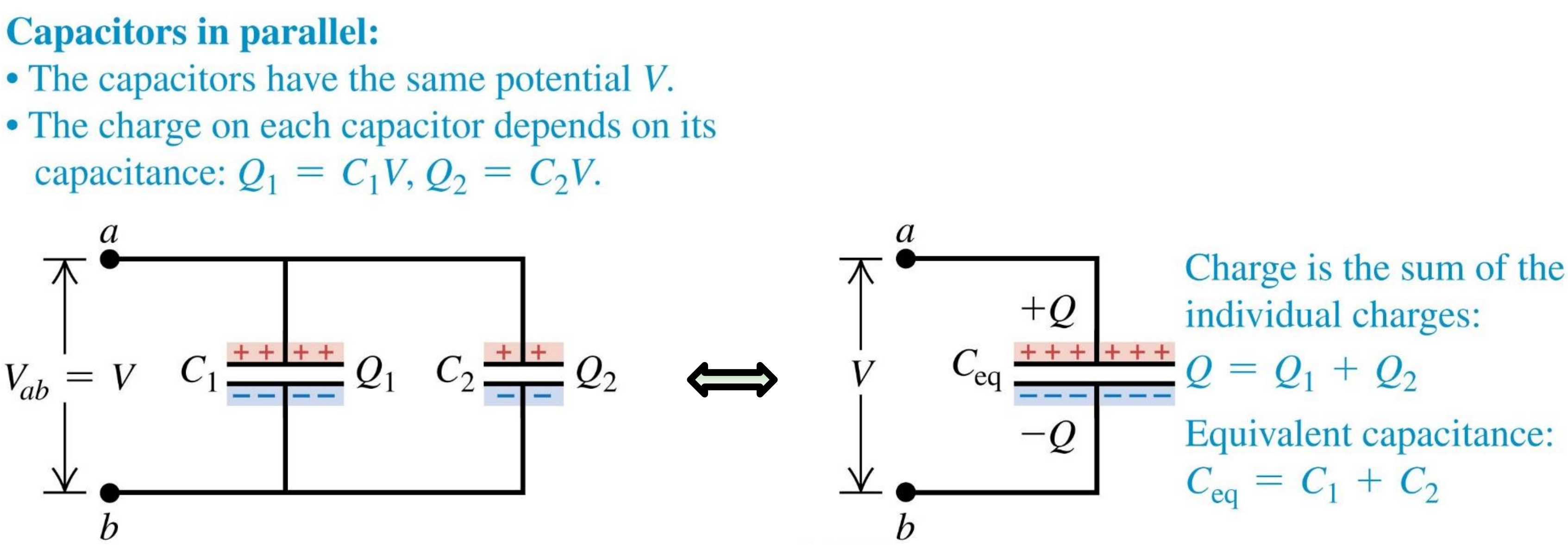
$$\frac{Q}{C_{eq}} \equiv \frac{Q}{C_1} + \frac{Q}{C_2} = \left(\frac{1}{C_1} + \frac{1}{C_2}\right) Q$$

**Capacitors in series:**
- The capacitors have the same charge $Q$.
- Their potential differences add: $V_{ac} + V_{cb} = V_{ab}$.



# Capacitors in parallel

- Capacitors are connected **in parallel** between *a* and *b*. The potential difference $V_{ab}$ is the same for all the capacitors.




$$
\begin{aligned}
Q &= Q_1 + Q_2 \\
C_{\text{eq}} V &= C_1 V + C_2 V \\
\Rightarrow C_{\text{eq}} &= C_1 + C_2
\end{aligned}
$$

# Several capacitors in series and parallel

- In series, the charges on the individual capacitors are same.

**Capacitors in series:**
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$$

Equivalent capacitance of series combination

Capacitances of individual capacitors

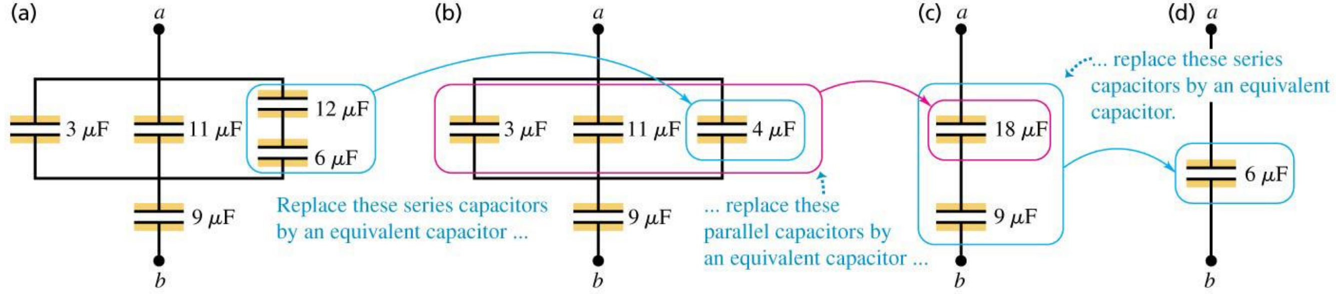
- In parallel, the voltage differences of the individual capacitors are same.

**Capacitors in parallel:**
$$C_{eq} = C_1 + C_2 + C_3 + \cdots$$

Equivalent capacitance of parallel combination

Capacitances of individual capacitors

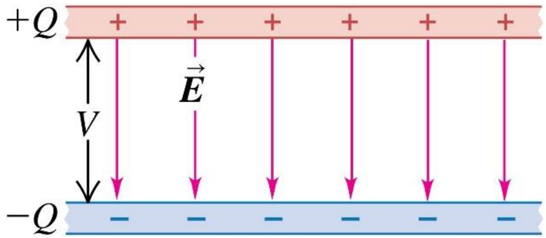
- Capacitor network



# 24.3 Energy storage in capacitors and E-field energy

- The work $dW$ needed to add charge $dQ$ in a capacitor already at the voltage difference $V$:

$$dW = V\,dQ = \frac{Q}{C}dQ$$




- Total work required to charge $Q$:

$$W = \int_0^Q \frac{Q}{C}dQ = \frac{Q^2}{2C}$$

- The potential energy stored in a capacitor is:

Magnitude of charge on each plate

Potential energy stored in a capacitor

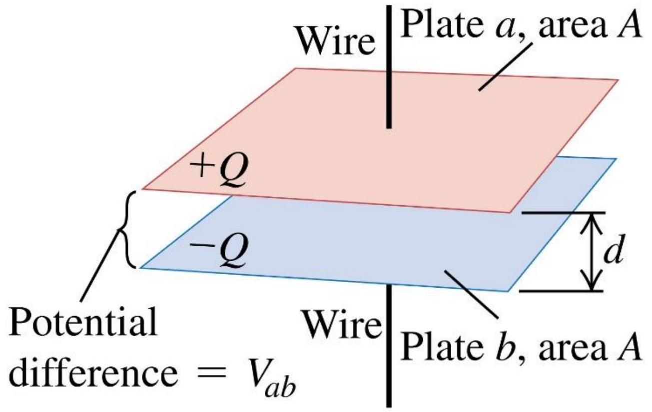
$$U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

Capacitance

Potential difference between plates

# Electric field energy density

- The capacitor energy is considered to be stored in the electric field between the plates.




- The **energy density of the electric field** is

$$u = \frac{U}{Ad} \quad = \quad \frac{1}{2}\frac{CV^2}{Ad} = \frac{1}{2}\epsilon_0 \frac{V^2}{d^2} = \frac{1}{2}\epsilon_0 E^2 \quad \left( = \frac{\sigma^2}{2\epsilon_0} \right)$$

Electric energy density in a vacuum

$$u = \tfrac{1}{2}\epsilon_0 E^2$$

Magnitude of electric field

Electric constant

# 24.4 Dielectrics

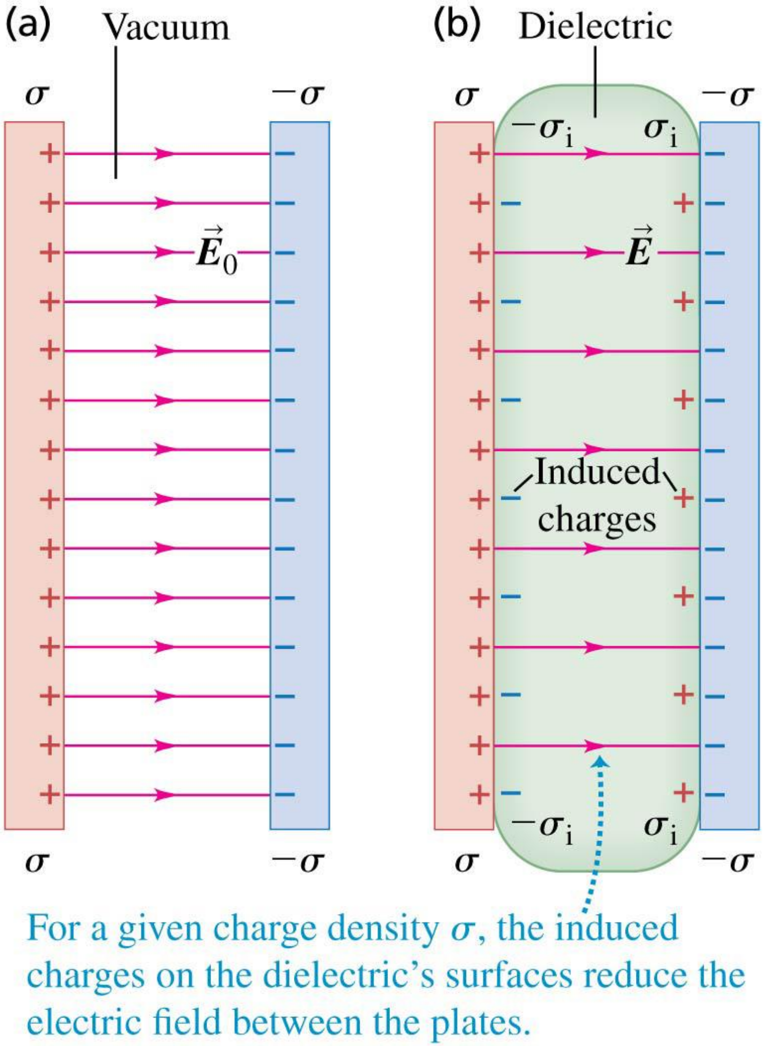
- A **dielectric** is a nonconducting material such as glass, paraffin, or polystyrene.

- When a dielectric is inserted between the plates of a capacitor, **the electric field decreases by a factor *K* (dielectric constant)** due to **induced surface charges via polarization** within the dielectric.

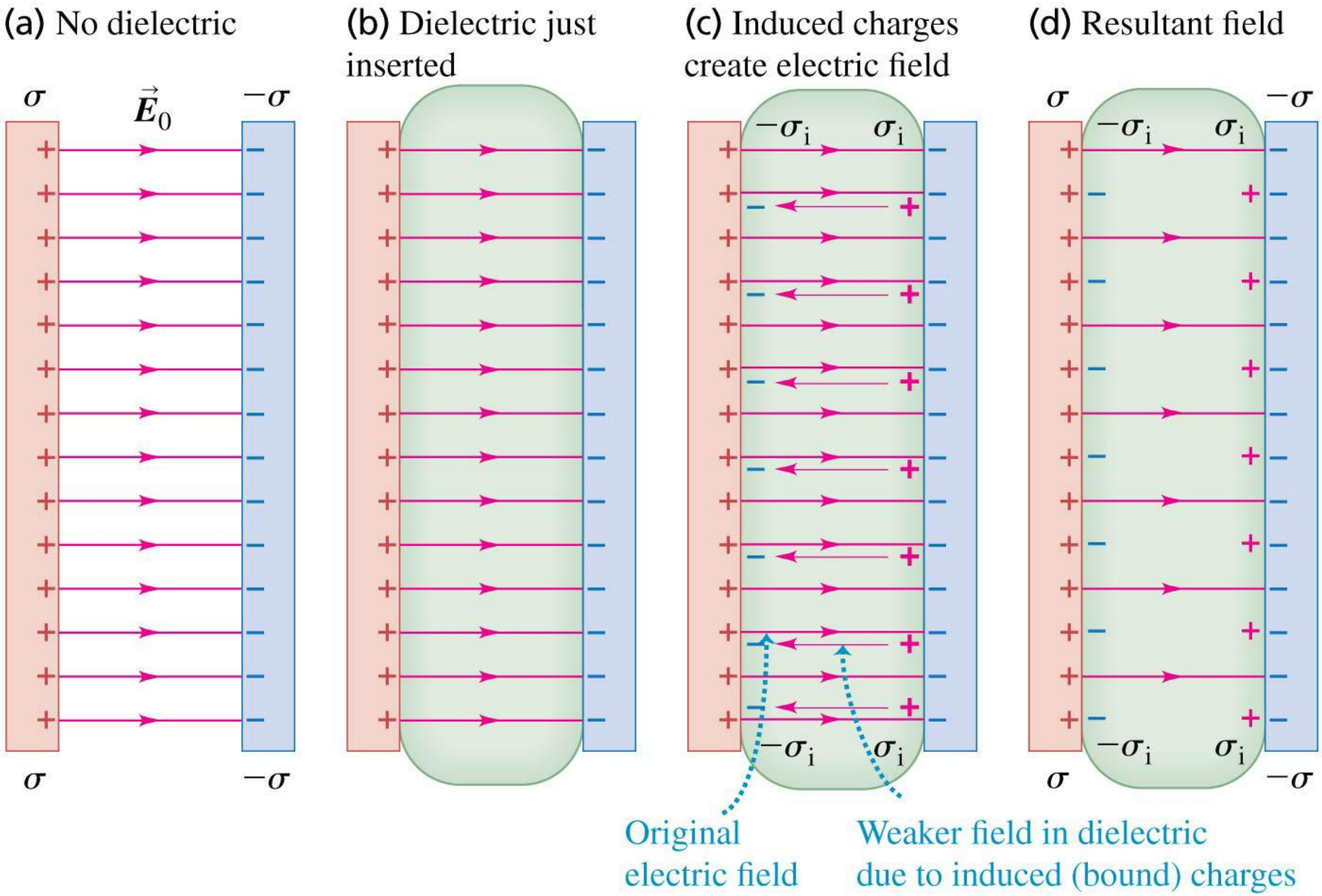
$$E = \frac{1}{K}E_0 = \frac{\sigma}{K\epsilon_0} \equiv \frac{\sigma}{\epsilon}$$

$\epsilon = K\epsilon_0$, permittivity

$E_0$ : electric field produced by "free" charges.



# Behavior of a dielectric in four steps



# Capacitator filled with a dielectric material

$$\begin{aligned} Q &= CV = CEd = CE_0 d/K \\ &= Q_0 = C_0 E_0 d \quad \Rightarrow C = KC_0 \end{aligned}$$

- The new capacitance $C$ is greater than $C_0$ by the factor $K$.

Capacitance of a parallel-plate capacitor, dielectric between plates

Dielectric constant

Area of each plate

Permittivity $= K\epsilon_0$

$$C = KC_0 = K\epsilon_0 \frac{A}{d} = \epsilon \frac{A}{d}$$

Capacitance without dielectric

Electric constant

Distance between plates

- The stored energy increases with $K$ for a given $V$: $U = \frac{1}{2}CV^2 = \frac{1}{2}KC_0V^2$
- The stored energy decreases with $K$ for a given $Q$: $U = \frac{Q^2}{2C} = \frac{Q^2}{2KC_0}$
- The energy density increases with $K$ for a given $E$:

Dielectric constant

Permittivity $= K\epsilon_0$

Electric energy density in a dielectric

$$u = \frac{1}{2}K\epsilon_0 E^2 = \frac{1}{2}\epsilon E^2$$

Electric constant

Magnitude of electric field

# 24.6 Gauss's law in dielectrics

- The Gaussian surface is a rectangular box that lies half in the conductor and half in the dielectric.

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{\text{encl-total}}}{\epsilon_0}$$

$$\oint K\vec{E}\cdot d\vec{A} = \oint \vec{E}_0\cdot d\vec{A} = \frac{\sigma A}{\epsilon_0} = \frac{Q_{\text{encl-free}}}{\epsilon_0}$$

- Gauss's law becomes:

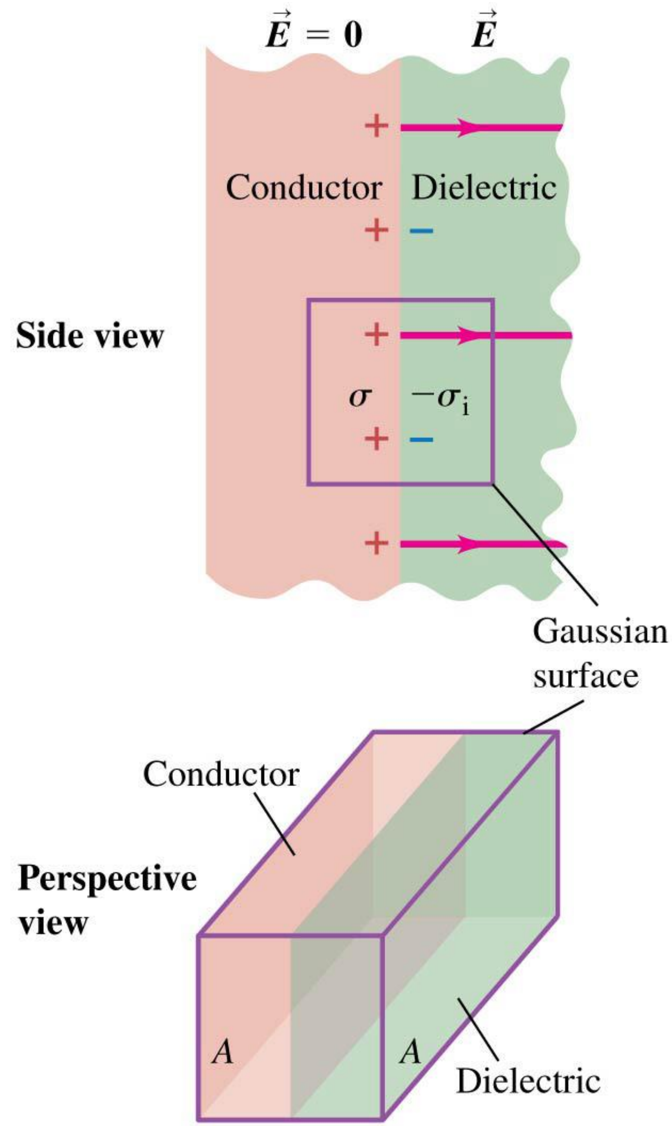
Dielectric constant

$$\underbrace{\oint K\vec{\boldsymbol{E}}\cdot d\vec{\boldsymbol{A}}}_{\text{Surface integral of } K\vec{E} \text{ over a closed surface}} = \frac{Q_{\text{encl-free}}}{\epsilon_0}$$

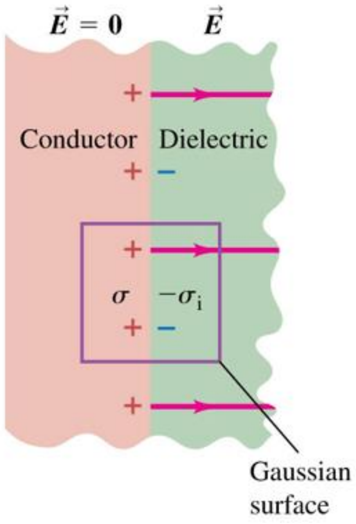
Total free charge enclosed by surface

Electric constant

where $Q_{\text{encl-free}}$ is the total **free charge** $(=\sigma A)$ enclosed by the Gaussian surface.



# Induced charge density




From Gauss's law

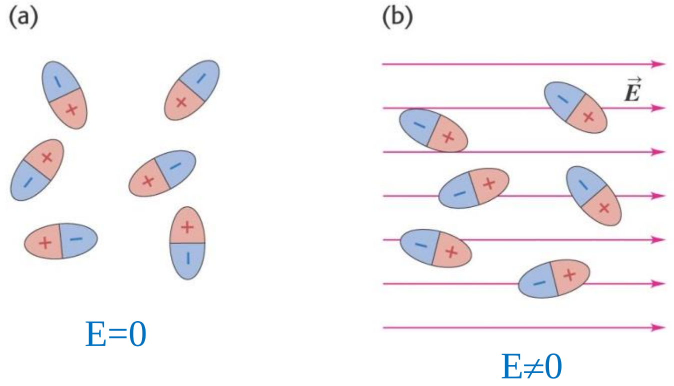
$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl-total}}}{\epsilon_0} \Rightarrow E = \frac{\sigma - \sigma_i}{\epsilon_0} \equiv \frac{E_0}{K} = \frac{\sigma}{\epsilon_0 K}$$

$\Rightarrow$ the induced charge density,

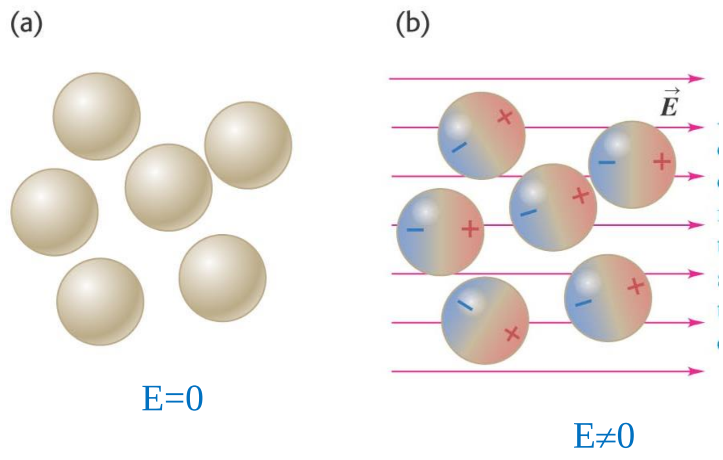
$$\sigma_i = \left(1 - \frac{1}{K}\right)\sigma$$

# 24.5 Molecular model of induced charge*

- **Polar molecules**, such as $H_2O$, have **intrinsic dipoles**. These tend to align along an external electric field.




- **Nonpolar molecules** become polar when an external electric field is applied - **polarization**. These **induced dipoles** align along the external electric field



# Table 24.1—Some dielectric constants

**TABLE 24.1** Values of Dielectric Constant $K$ at 20°C

| Material | $K$ | Material | $K$ |
|---|---|---|---|
| Vacuum | 1 | Polyvinyl chloride | 3.18 |
| Air (1 atm) | 1.00059 | Plexiglas® | 3.40 |
| Air (100 atm) | 1.0548 | Glass | 5–10 |
| Teflon | 2.1 | Neoprene | 6.70 |
| Polyethylene | 2.25 | Germanium | 16 |
| Benzene | 2.28 | Glycerin | 42.5 |
| Mica | 3–6 | $\boxed{\text{Water}}$ | $\boxed{80.4}$ |
| Mylar | 3.1 | Strontium titanate | 310 |

# 24 Summary

- Capacitors and capacitance

- Capacitors in series and parallel

- Energy in a capacitor

- Dielectrics