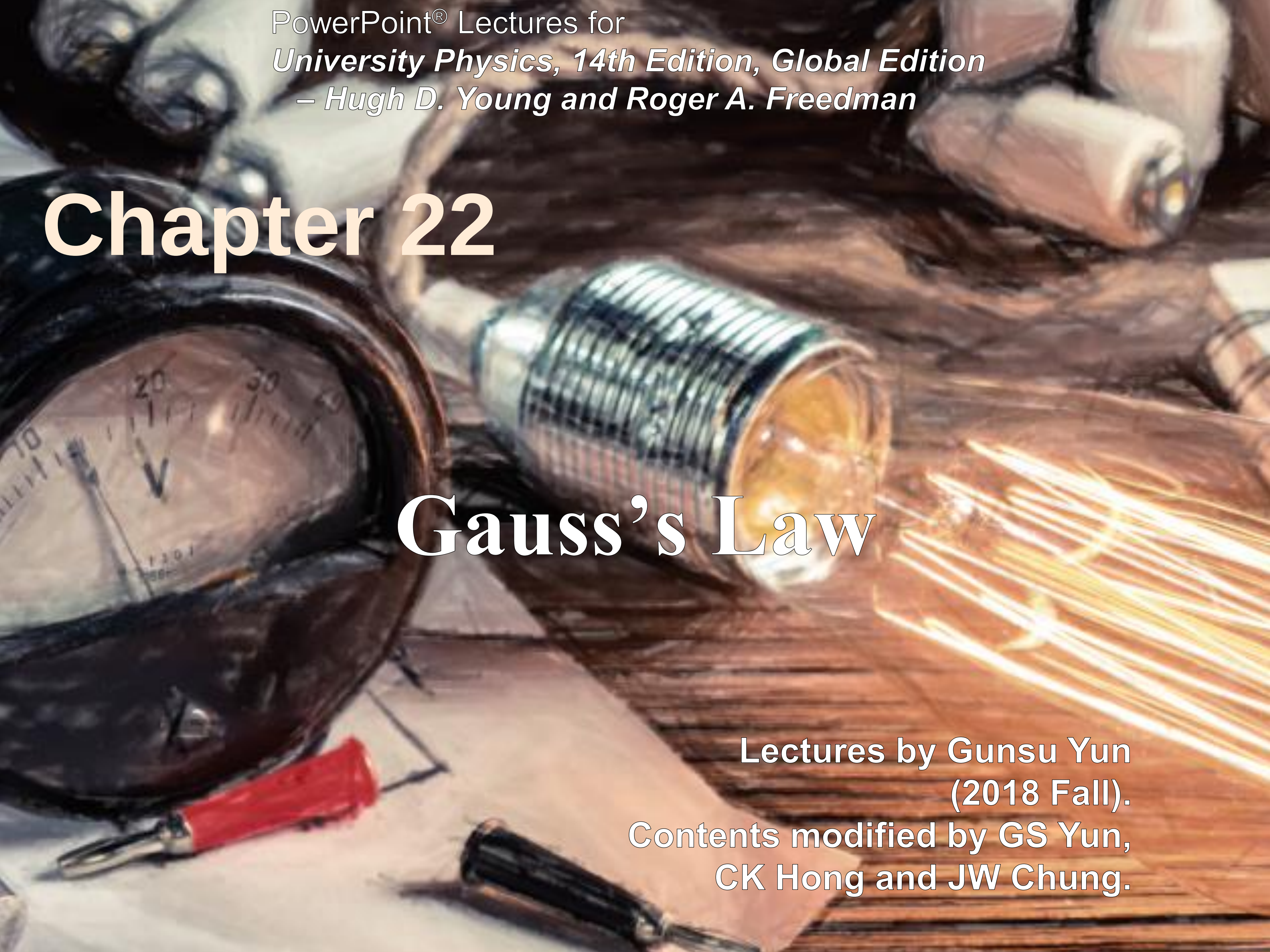

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# Chapter 22

# Gauss's Law

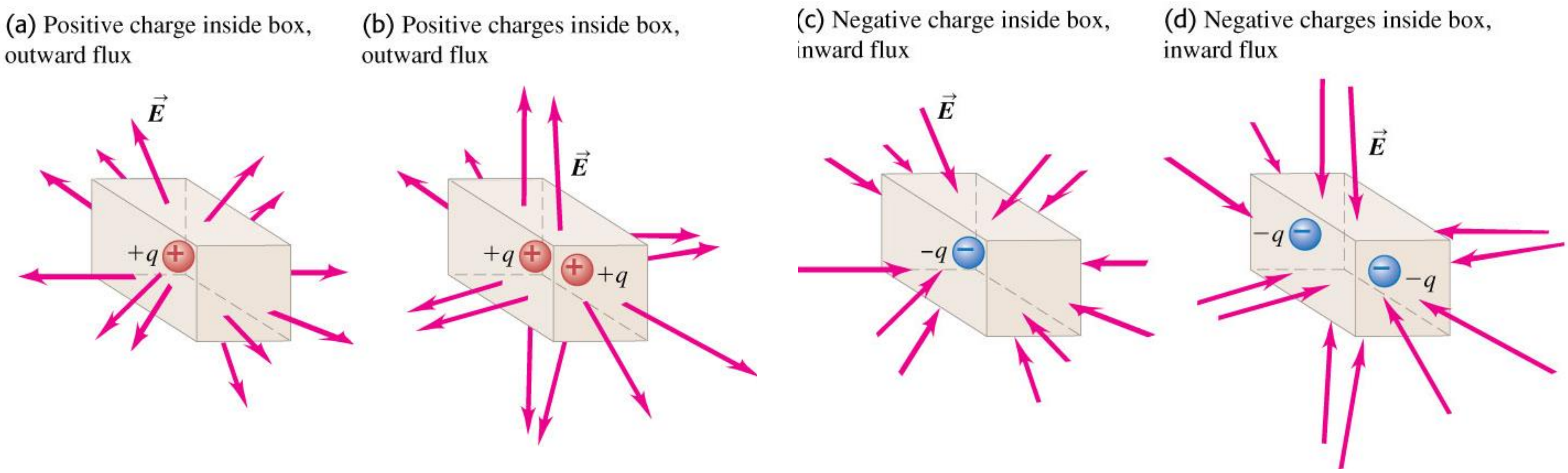
**Lectures by Gunsu Yun**
**(2018 Fall).**
**Contents modified by GS Yun,**
**CK Hong and JW Chung.**

# Goals for Chapter 22

- Electric field at a **surface** → charge within the surface
- **Electric flux**
- **Gauss's law**
- Charge distribution of a conductor

# 22.1 Charge and electric flux

- **Electric flux** ~ the number of electric field lines passing through a surface
- Positive charge within the box produces outward electric flux through the surface of the box, and negative charge produces inward flux.



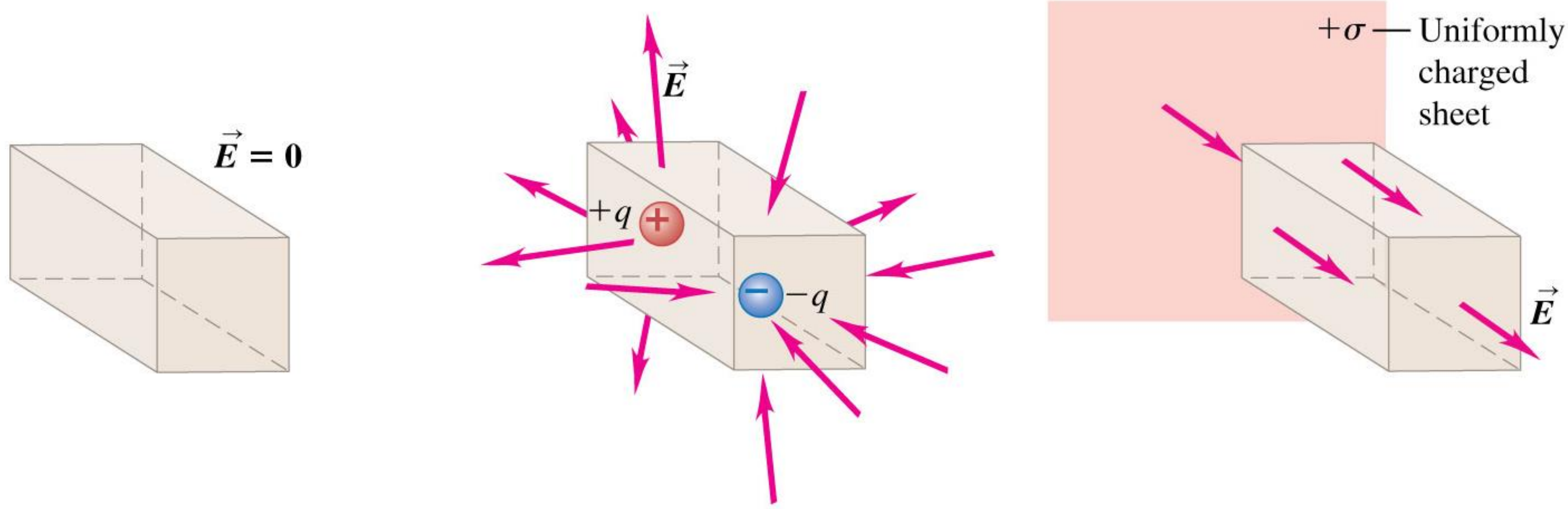
# Zero net charge inside a box

- Three cases in which there is zero *net* charge inside a box and no net electric flux through the surface of the box.

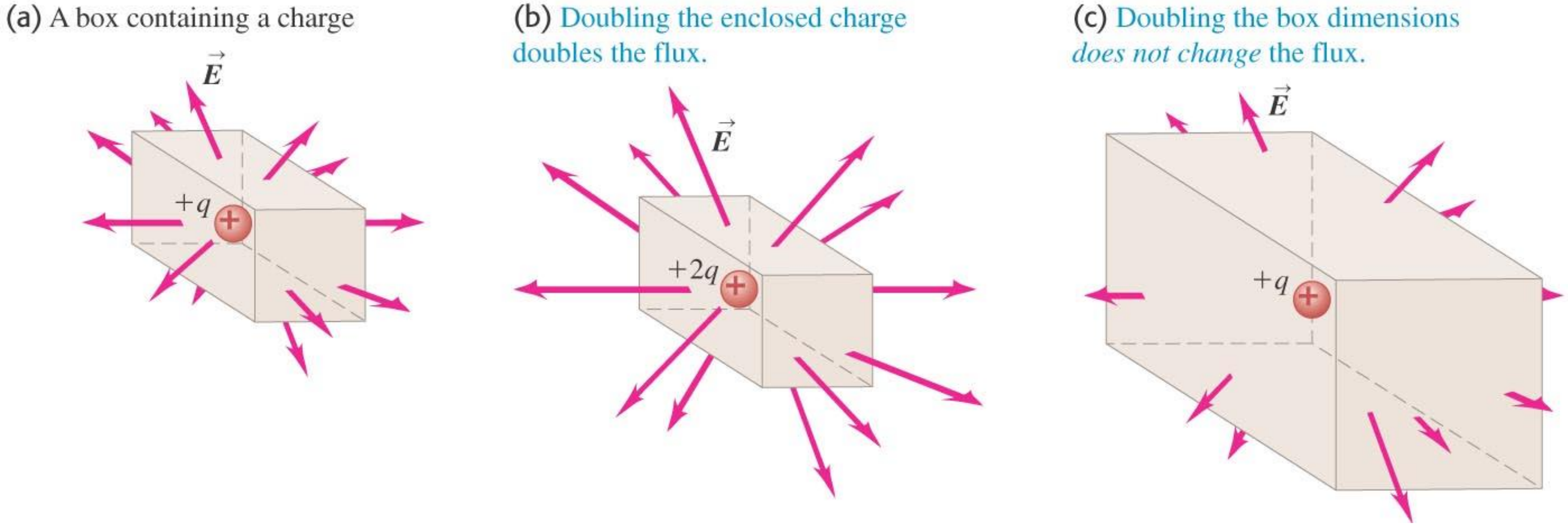
**(a)** No charge inside box, zero flux

**(b)** Zero *net* charge inside box, inward flux cancels outward flux.

**(c)** No charge inside box, inward flux cancels outward flux.

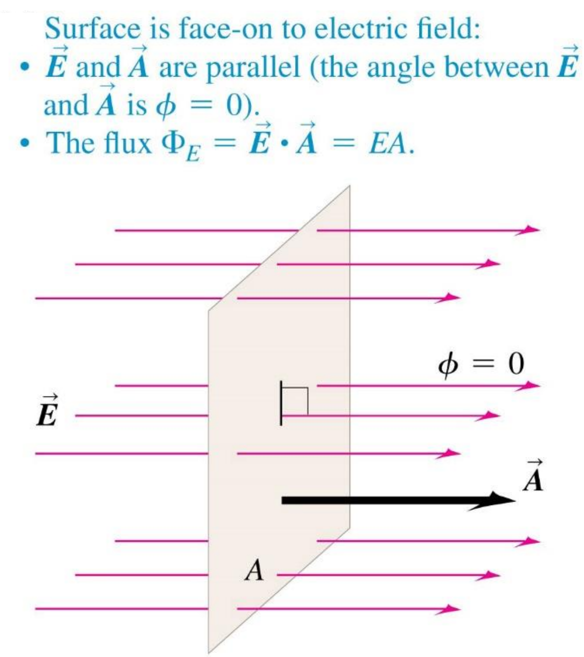


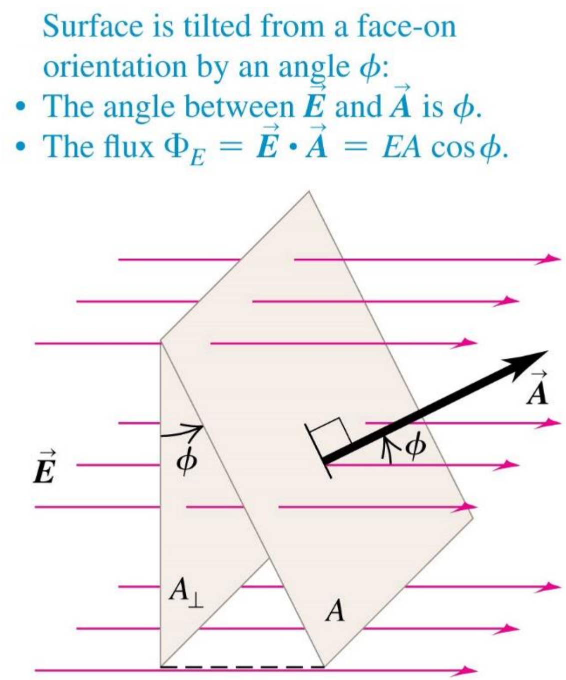
# What affects the flux through a box?




- The net electric flux is **directly proportional** to the net charge within the surface.
- The net electric flux is **independent** of the size of the closed surface.

# 22.2 Calculating electric flux




- Area (surface) Vector:

$$\vec{A} = \hat{n}A$$

$\hat{n}$, unit normal vector.

$$\Phi_E = \vec{E} \cdot \vec{A} = EA\cos\phi$$

(for uniform field and flat surface)

# Flux of a nonuniform electric field

- In general, the flux through a surface must be computed using a **surface integral** over the area:

Magnitude of electric field $\vec{E}$

Component of $\vec{E}$ perpendicular to surface

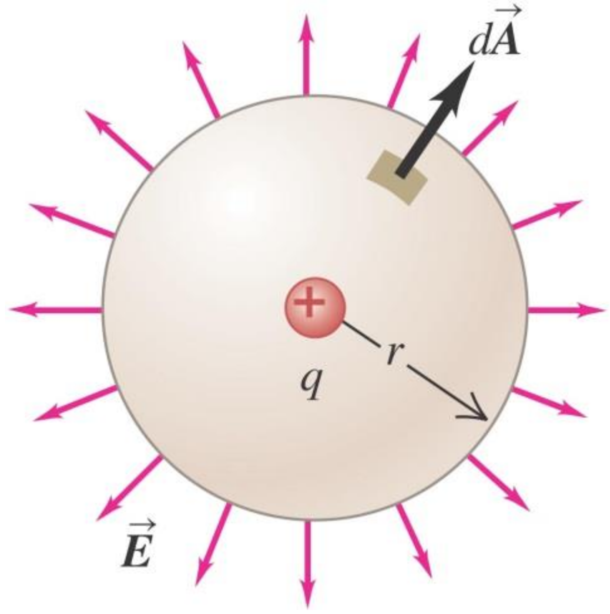
**Electric flux** through a surface

$$\Phi_E = \int E\cos\phi \,\underbrace{dA}_{\text{Element of surface area}} = \int E_\perp \,\underbrace{dA}_{\text{Element of surface area}} = \int \vec{\boldsymbol{E}} \cdot \underbrace{d\vec{\boldsymbol{A}}}_{\text{Vector element of surface area}}$$

Angle between $\vec{E}$ and normal to surface

- SI unit for electric flux: $\mathrm{m}^2 \cdot \mathrm{N/C}$

# Electric flux through a sphere

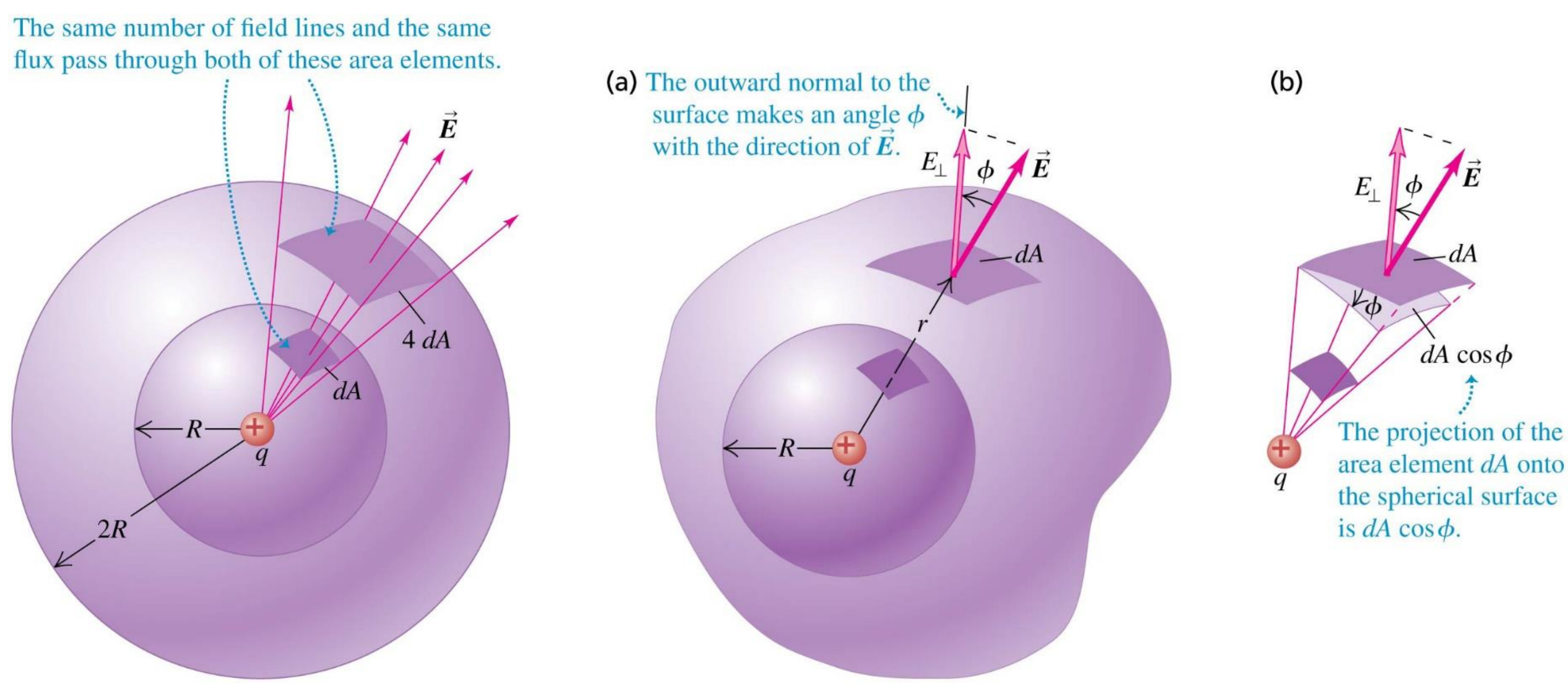



$$\Phi_E = EA = \frac{q}{4\pi\epsilon_0} 4\pi r^2 = \frac{q}{\epsilon_0}$$

independent of $r$ → The flux is a conserved quantity!

# 22.3 Gauss's law

- **Gauss's law** is completely equivalent to Coulomb's law and provides a different way to express the relationship between electric charge and electric field.

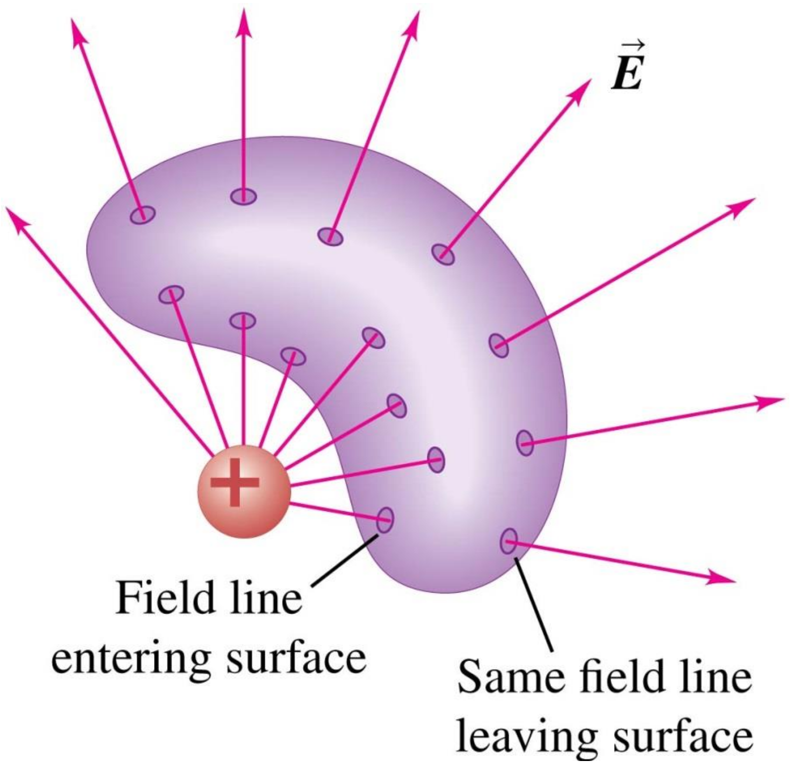


# Gauss's law in a vacuum

- For a closed surface enclosing no charge:

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = 0$$

- If an electric field line from the external charge enters the surface at one point, it must leave at another.



# General form of Gauss's law

- **Gauss's law: the total electric flux through a closed surface is equal to the total (net) electric charge inside the surface, divided by $\varepsilon_0$:**

**Gauss's law:**

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\mathrm{encl}}}{\epsilon_0}$$

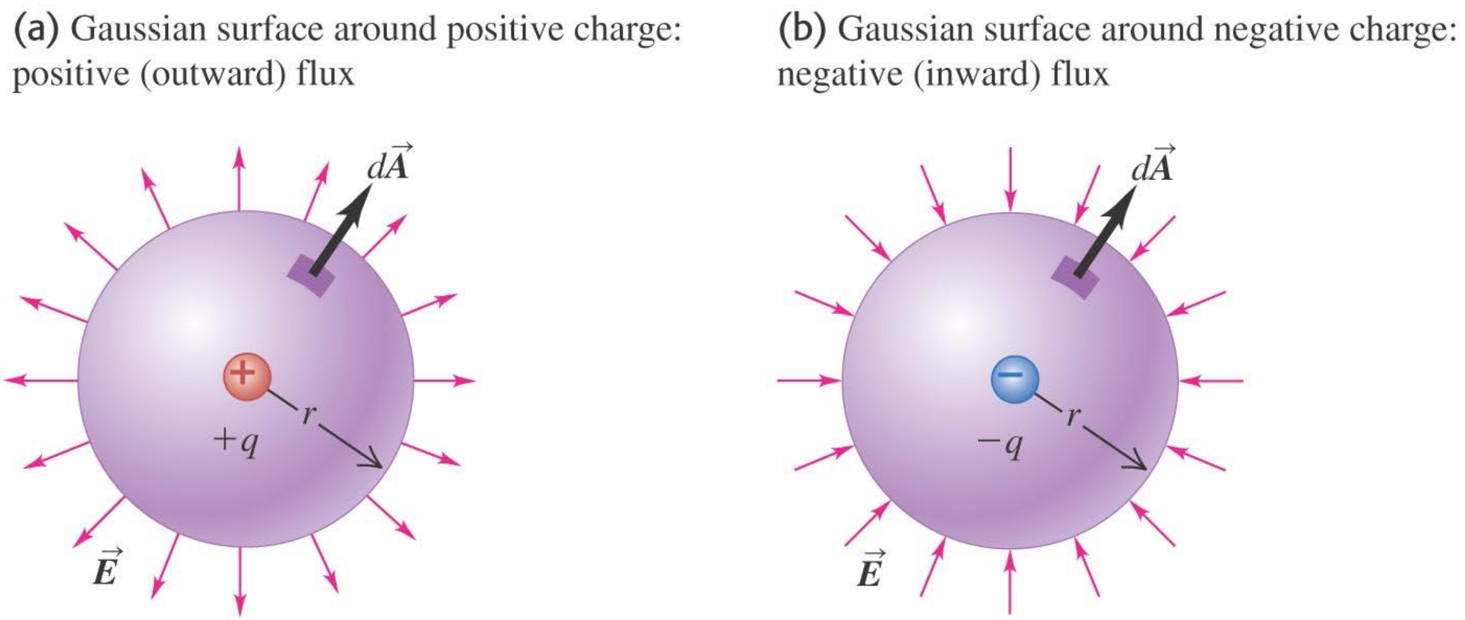
Total charge enclosed by surface

Electric flux through a closed surface of area $A$ = surface integral of $\vec{E}$

Electric constant

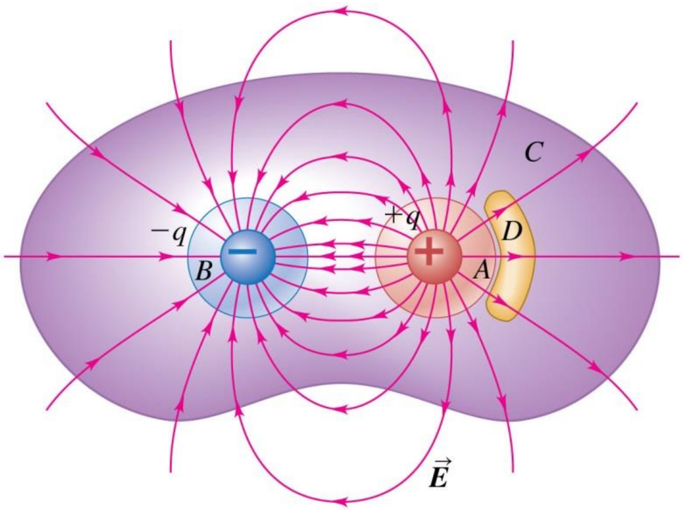
- Gauss's law is very powerful for various symmetrical charge distributions.

# Positive and negative flux



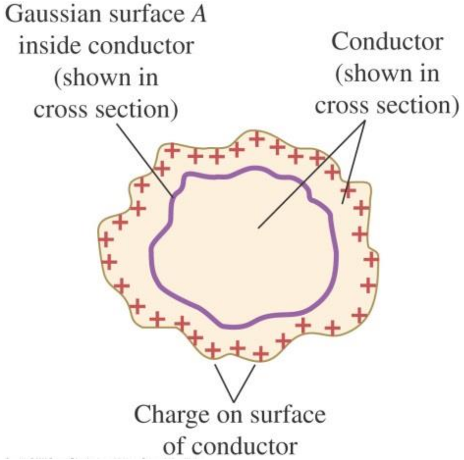

For (a/b)

$$
\begin{aligned}
\Phi_E &= \oint E_\perp dA = \oint \left(\frac{\pm q}{4\pi\epsilon_0 r^2}\right) \\
&= \frac{\pm q}{4\pi\epsilon_0 r^2}\oint dA = \frac{\pm q}{4\pi\epsilon_0 r^2}4\pi r^2 \\
&= \frac{\pm q}{\epsilon_0}
\end{aligned}
$$



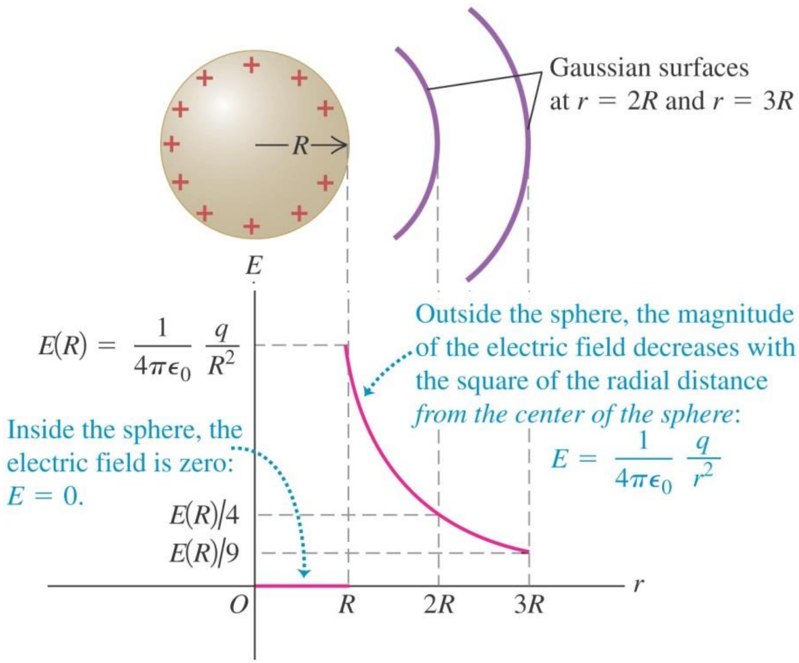
# 22.4 Applications of Gauss's law

- Under electrostatic conditions, the electric field inside a conductor is 0. If not, free charge within the conductor should move.
- $\Phi_E = 0$ for any Gaussian surface within the conductor.
- Excess charge is 0 within the conductor and resides only on the surface .




We place a total positive charge $q$ on a solid conducting sphere with radius $R$ (Fig. 22.18). Find $\vec{E}$ at any point inside or outside the sphere.

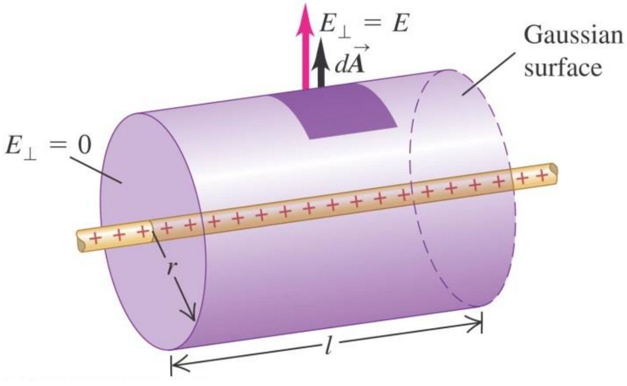
$$E(4\pi r^2) = \frac{q}{\epsilon_0} \Rightarrow E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$



# Field of a uniform line charge

Electric charge is distributed uniformly along an infinitely long, thin wire. The charge per unit length is $\lambda$ (assumed positive). Find the electric field using Gauss's law.

Cylindrical symmetry




For a cylindrical Gaussian surface of radius *r*,

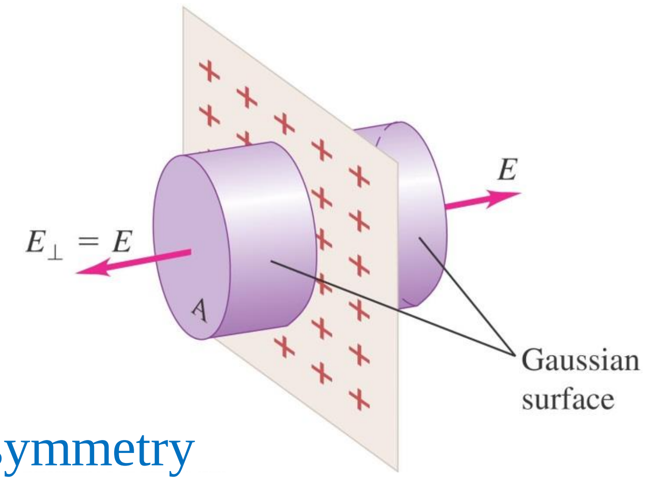
$$\Phi_E = 2\pi r l E = \frac{\lambda l}{\epsilon_0} \quad \Rightarrow \quad \vec{E} = \hat{r}\frac{\lambda}{2\pi\epsilon_0 r}$$

# Field of an infinite plane sheet of charge

Use Gauss's law to find the electric field caused by a thin, flat, infinite sheet with a uniform positive surface charge density $\sigma$.

For a cylindrical Gaussian surface of crosssection *A*,

$$\Phi_E = 2AE = \frac{\sigma A}{\epsilon_0} \quad \Rightarrow \quad E = \frac{\sigma}{2\epsilon_0}$$



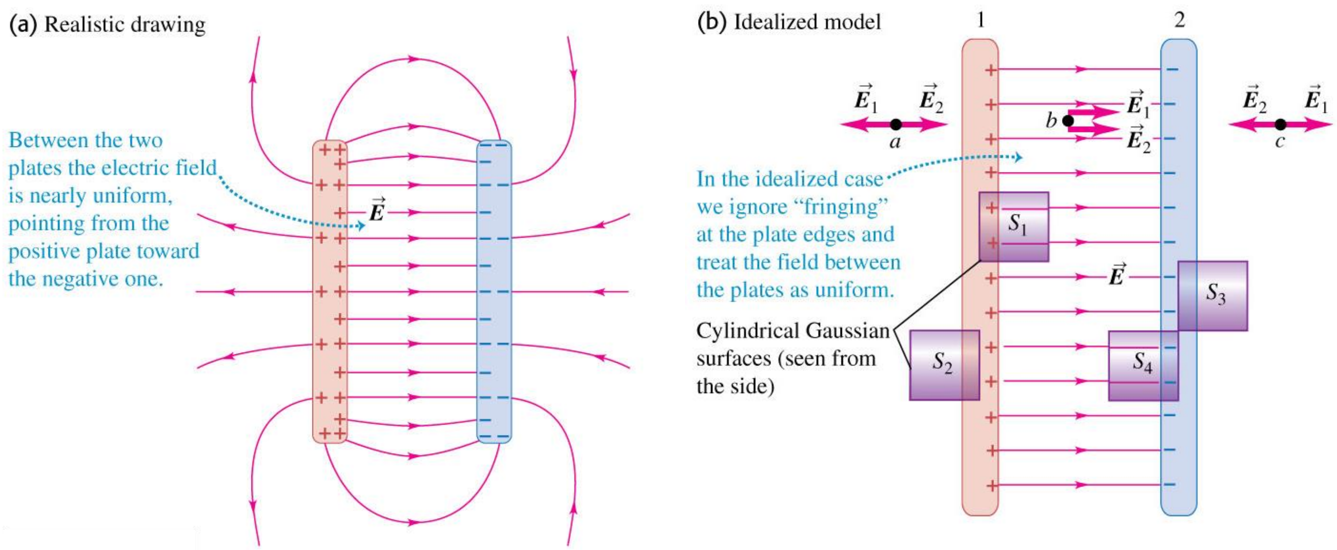

Plane symmetry

# Field between two parallel conducting plates

Two large plane parallel conducting plates are given charges of equal magnitude and opposite sign; the surface charge densities are $+\sigma$ and $-\sigma$. Find the electric field in the region between the plates.

For Gaussian surfaces $S_1$ and $S_4$, $\quad \Phi_E = AE = \dfrac{\sigma A}{\epsilon_0} \quad \Rightarrow E = \dfrac{\sigma}{\epsilon_0}$

For Gaussian surfaces $S_2$ and $S_3$, $\quad E = 0.$

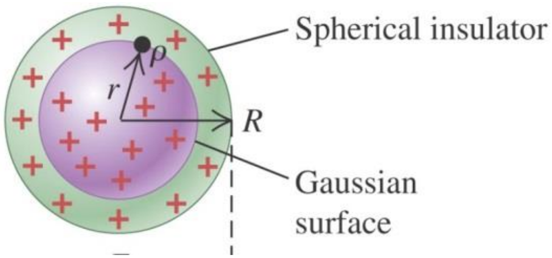



**Superposition principle** gives the same results.

# A uniformly charged sphere

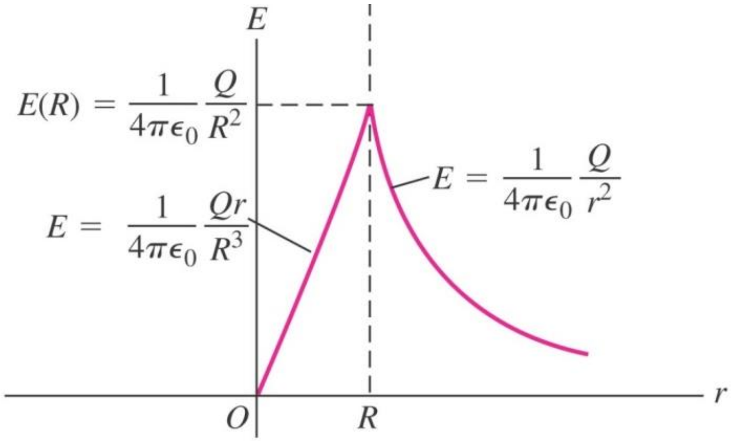
Positive electric charge $Q$ is distributed uniformly *throughout the volume* of an *insulating* sphere with radius $R$. Find the magnitude of the electric field at a point $P$ a distance $r$ from the center of the sphere.

Volume charge density $\rho$: $\quad \rho = Q/\left(\frac{4}{3}\pi R^3\right)$




For a spherical Gaussian surface with radius $r < R$,

$$Q_{\text{encl}}(r) = \rho V_{\text{encl}}(r) = Q\frac{r^3}{R^3}$$

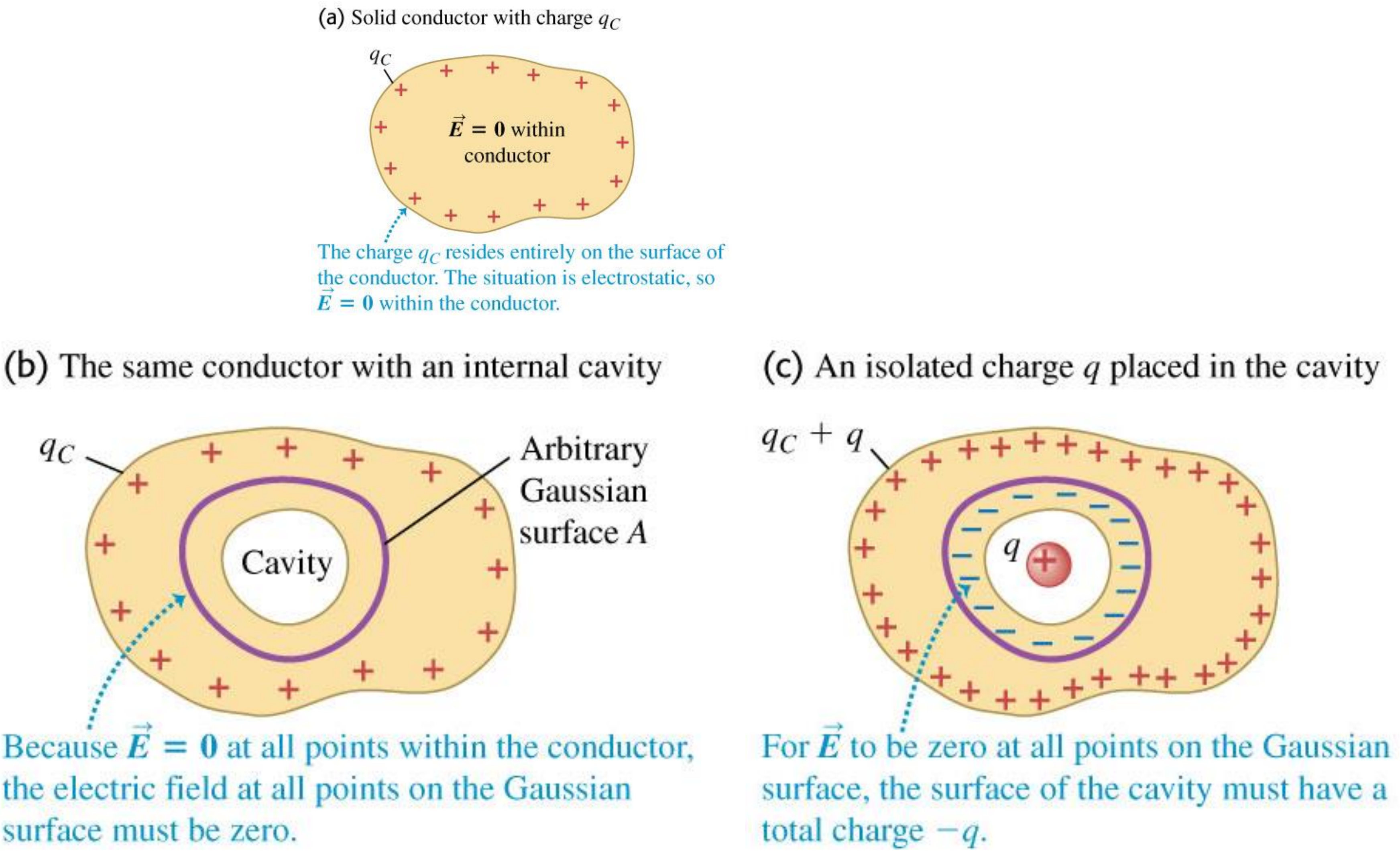



From Gauss's law,

$$\Phi_E(r) = 4\pi r^2 E(r) = \frac{Q_{\text{encl}}(r)}{\epsilon_0} \quad \Rightarrow E(r) = \frac{Q}{4\pi\epsilon_0}\frac{r}{R^3}$$

# 22.5 Charges on conductors

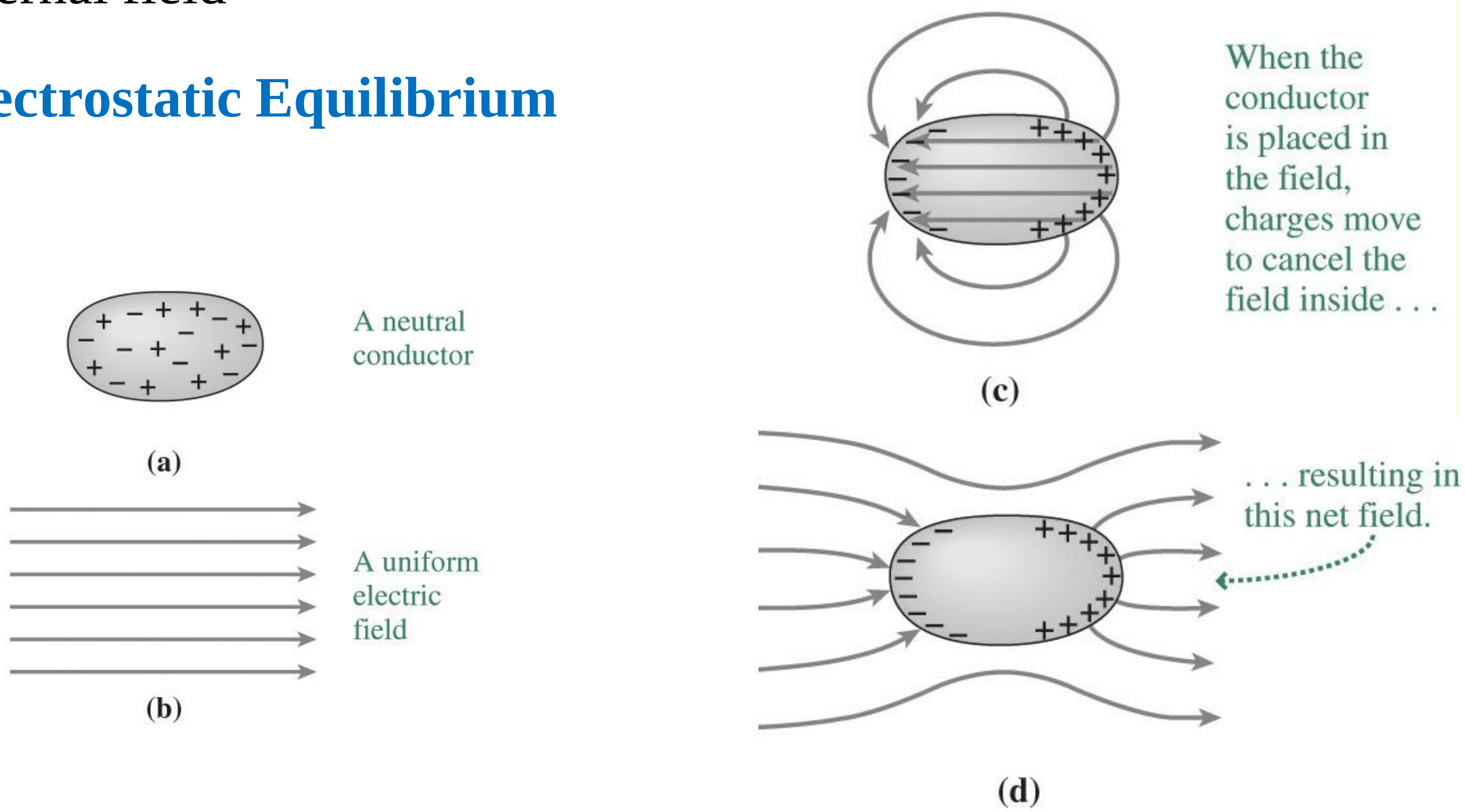
- Charges in a conductor having a cavity within it.



# Charges on conductors in an external electric field

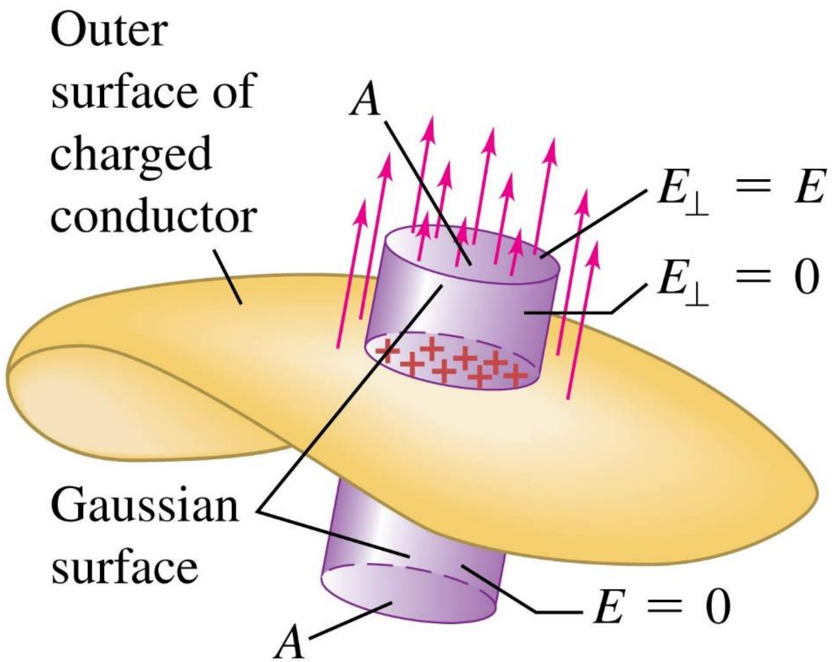
- Charges in conductors are free to move, and respond to an applied electric field until they become stationary with zero internal field

- **Electrostatic Equilibrium**



# Field at the surface of a conductor

- Gauss's law can be used to show that the direction of the electric field at the surface of any conductor is always **perpendicular to the surface**.
- The magnitude of the electric field just outside a charged conductor is proportional to the surface charge density $\sigma$.




**Electric field at surface of a conductor, $\vec{E}$** perpendicular to surface

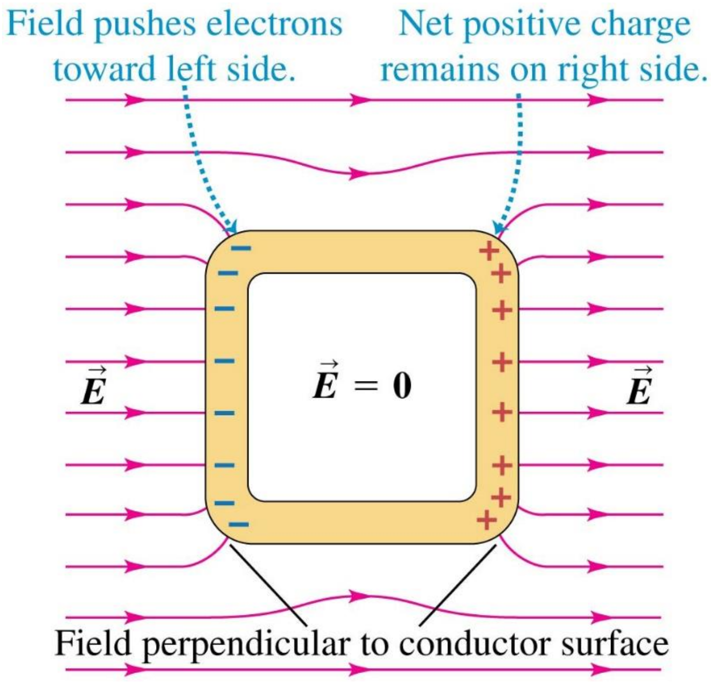
$$E_\perp = \frac{\sigma}{\epsilon_0}$$

Surface charge density ($\sigma$)

Electric constant ($\epsilon_0$)

# Electrostatic shielding

- A conducting box is immersed in an external electric field.
- The field of the induced charges on the box combines with the external field to give **zero total field inside the box**.




# 22 Summary

- Electric flux

- Gauss's law

- Various symmetric charge distributions