ECEN 434: Computer Project

Fall 2021

Assign date: Monday, November 15
Due date: Monday, December 6

Project assignment: Consider the unconstrained problem:

$$\text{minimize} \quad f(\boldsymbol{x}) = -\sum_{i=1}^{m} \ln(1 - \boldsymbol{a}_i^t \boldsymbol{x}) - \sum_{i=1}^{n} \ln(1 - x_i^2)$$

with variable $\boldsymbol{x} \in \mathbb{R}^n$ and $\text{dom}(f) = \{\boldsymbol{x} : \ \boldsymbol{a}_i^t \boldsymbol{x} < 1, \ i = 1, 2, \ldots, m, \ |x_i| < 1, \ i = 1, 2, \ldots, n\}$. Note that we can choose $\boldsymbol{x}^{(0)} = \boldsymbol{0}$ as our initial point. You can generate instances of this problem by randomly choosing $\boldsymbol{a}_i$ from $\mathbb{R}^n$.

a) (5 Points) Derive an analytical expression for the gradient of the objective function. Use the gradient method to solve the problem, using reasonable choices for the backtracking parameters and a stopping criterion of the form $\|\nabla f(\boldsymbol{x})\|_2 \le \eta$. Plot in separate figures the objective value (in log scale) and step length versus iteration number. Once you have determined p^* to high accuracy, you can also plot $f - p^*$ (in log scale) versus iteration. Experiment with the backtracking parameters α and β to see their effect on the total number of iterations required. Carry these experiments out for several instances of the problem, of different sizes.

b) (10 Points) Derive an analytical expression for the Hessian of the objective function. Redo the same problem instances from part a) using damped Newton's method, with backtracking line search and a stopping criterion of the form $\frac{1}{2}\lambda(\boldsymbol{x})^2 \le \epsilon$, where $\lambda(\boldsymbol{x})^2 := \nabla f(\boldsymbol{x})^t \nabla^2 f(\boldsymbol{x})^{-1} \nabla f(\boldsymbol{x})$. Plot in separate figures $f - p^*$ (in log scale) and step length versus iteration number. Describe the convergence behavior that you observe.

c) (5 Points) The cost of Newton's methods is dominated by the cost of evaluating the Hessian and finding its inverse. For large problems, it is sometimes useful to replace the Hessian by a positive definite approximation that makes it easier to form and solve for the search direction. For each of the approximate Newton methods described below, test the method on the same instances from part a) and b) and compare the results to those obtained using damped Newton's method and gradient method.

   i) (Re-using the Hessian) We evaluate and invert the Hessian only every N iterations, where $N > 1$, and use the search direction $\Delta\boldsymbol{x} = -\boldsymbol{H}^{-1}\nabla f(\boldsymbol{x})$, where $\boldsymbol{H}$ is the last Hessian evaluated.

ii) (Diagonal approximation) We replace the Hessian by its diagonal, so we only have to evaluate the n second derivatives $\partial^2 f(\boldsymbol{x})/\partial x_i^2$, and computing the search direction is very easy.

Report requirement:

1) Please include both figures and Matlab/Python codes in your report. We will check both to determine your project grade.

2) For each set of figures that you include in your report, please explicitly state the problem instance and algorithm parameters (backtracking parameters α and β, accuracy parameters η and ϵ) of your choice.

3) Please submit your project report to Google Classroom by Monday, December 6.