



# QUALITY CONTROL ANALYSIS (PHASE I )

ISEN 614 PROJECT BY:

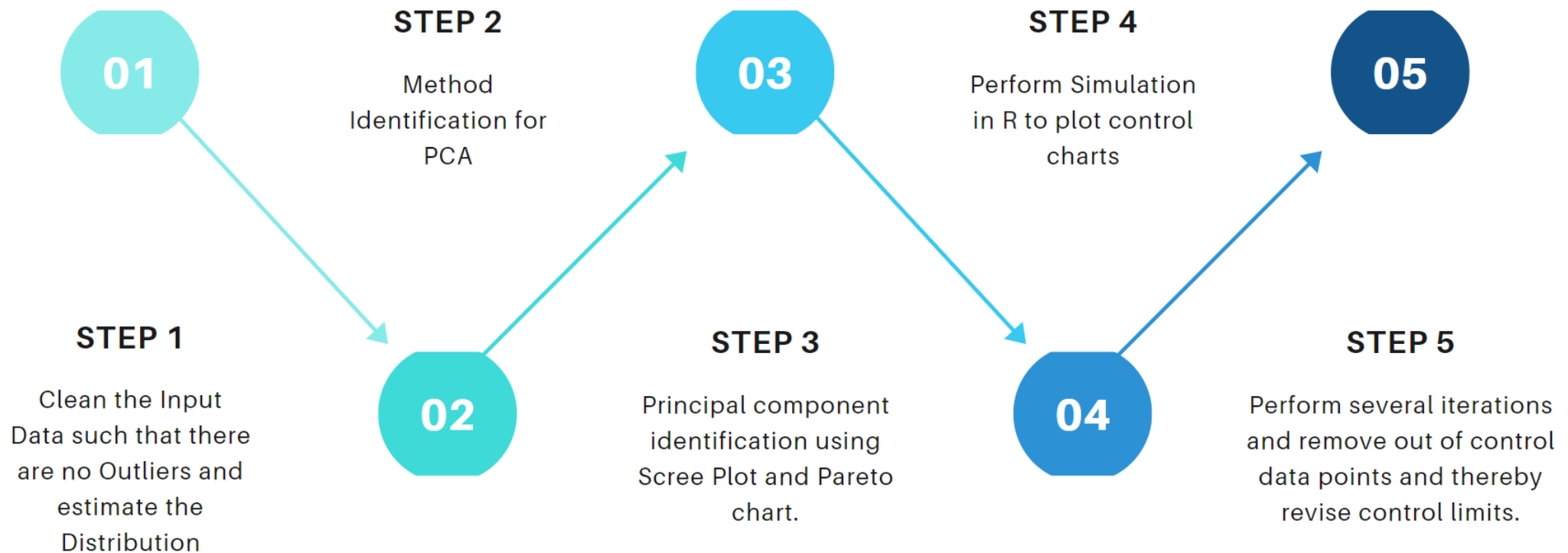
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# PROCEDURE



# EXPLORATORY ANALYSIS

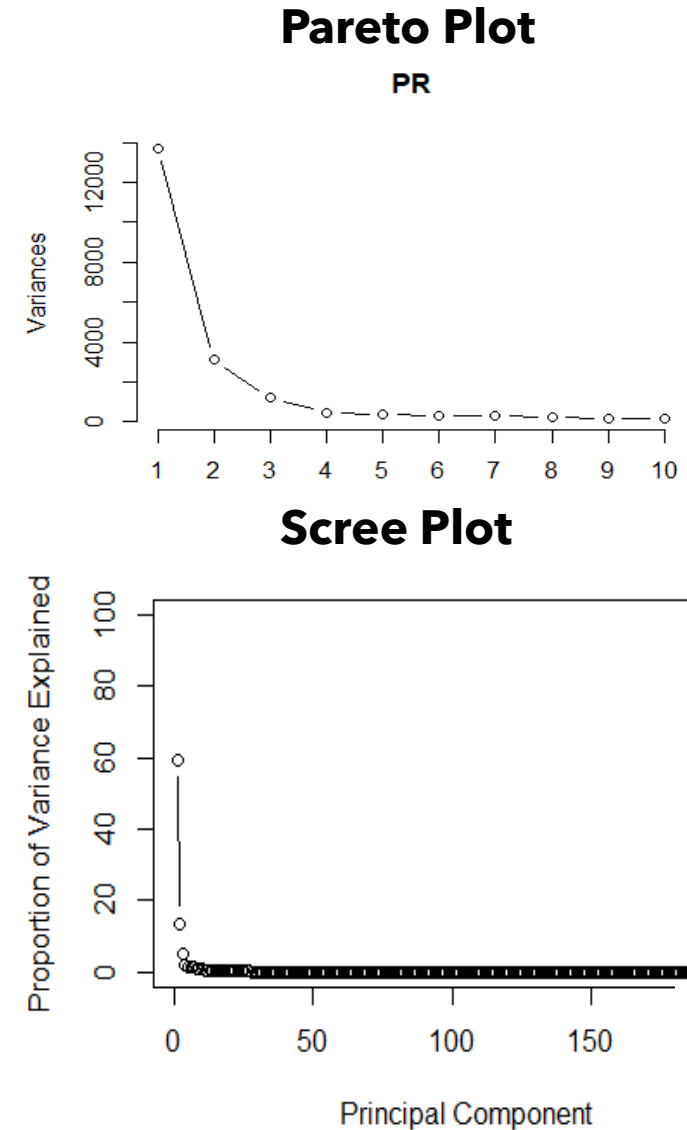
- The sample size of the given data is equal to 1, observations equal to 552 and a predictor size of 209.
- On a subset analysis, it was observed that there is a pattern in how the values varied for all the 209 predictors.
- Principal Components Analysis was done with the sole purpose of finding the variables that explain the maximum variance hence improving the signal to noise ratio.

# CONTROL CHARTS & ITERATIONS

- The data was initially centered and then a covariance matrix of the PCs were computed.
- Popular detection techniques for multivariate data such as Hotelling  $T^2$ , m-CUSUM and m-EWMA were applied.
- Aim was to remove all out of control data points by a series of iterations so that Mean and Covariance for the Phase I analysis could then be estimated from the remaining data.
- In the end we decided to produce results based on Combination of  $T^2$  chart with m-CUSUM and m-EMA control charts.

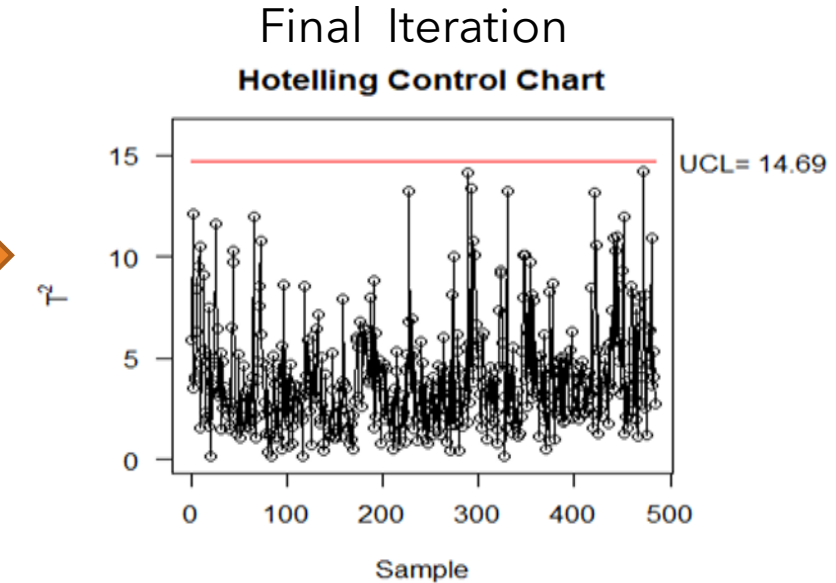
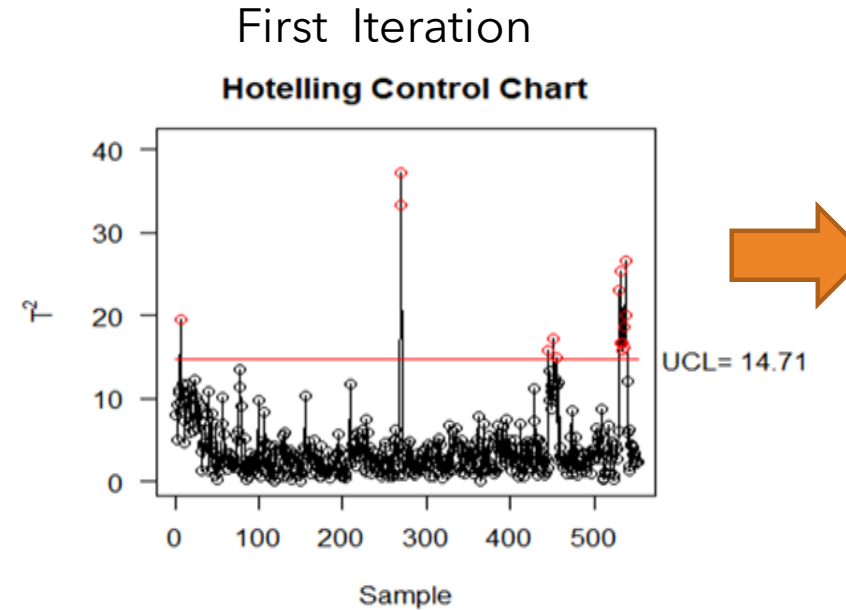
# DIMENSIONALITY REDUCTION

- Scree and Pareto plots were used instead of Minimum Description Length(MDL) to decide the number of Principal Components to be used..
- The elbow bend appears to happen at PC=4 on the Scree plot. Whereas in the Pareto plot at PC=4 the aggregated variance up to this point accounted for around 80% of the total.
- Selecting 4 Principal Components reduced our dimension from 209 originally to effectively being equal to 4. This reduces the detrimental effect termed as "Curse of Dimensionality".



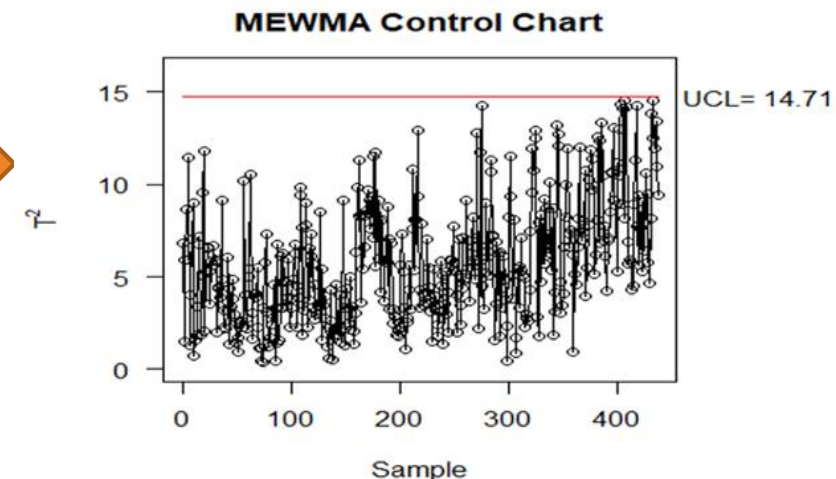
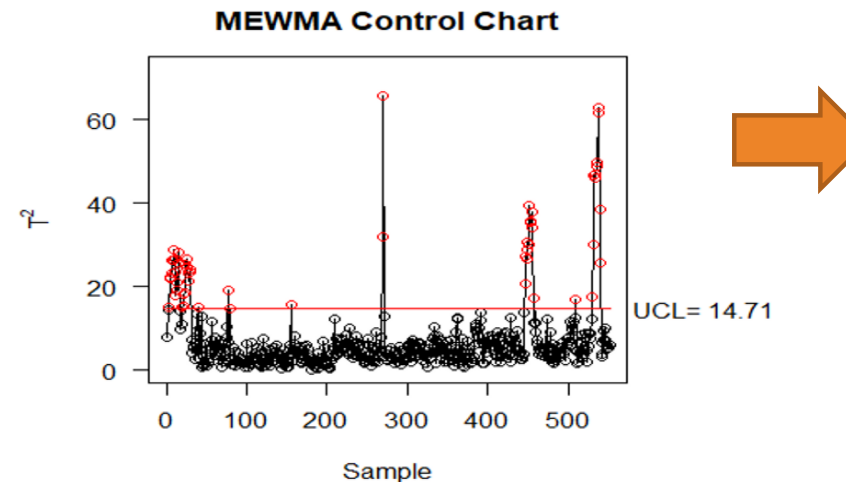
## HOTELLING $T^2$ CHART

- After 13 iterations we find all remaining points to be in-control.
- In total 68 Out of Control data points were removed and 484 data points remain.



## m-EWMA CHART

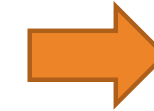
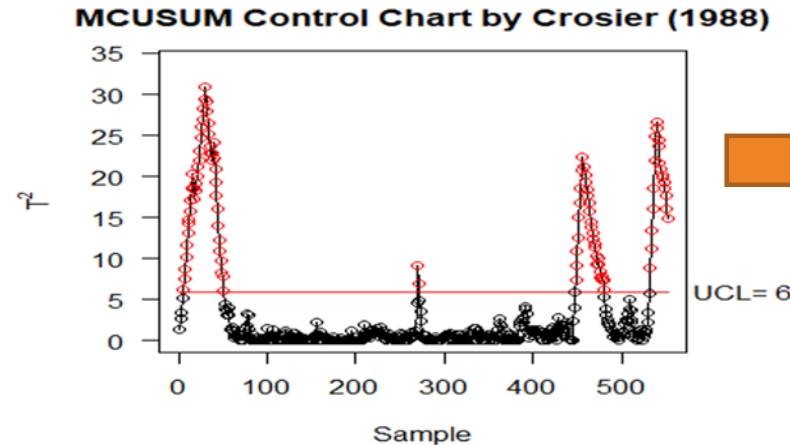
- After 6 iterations we find all remaining points to be in-control.
- In total 115 out of control data points were removed and 437 data points remain



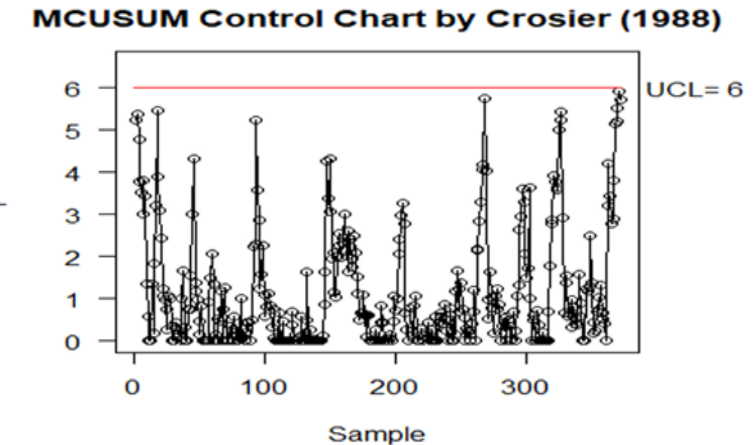
## m-CUSUM CHART

- After 7 iterations we find all remaining points to be in-control.
- In total 180 Out of Control data points were removed and 372 data points remain.

First Iteration

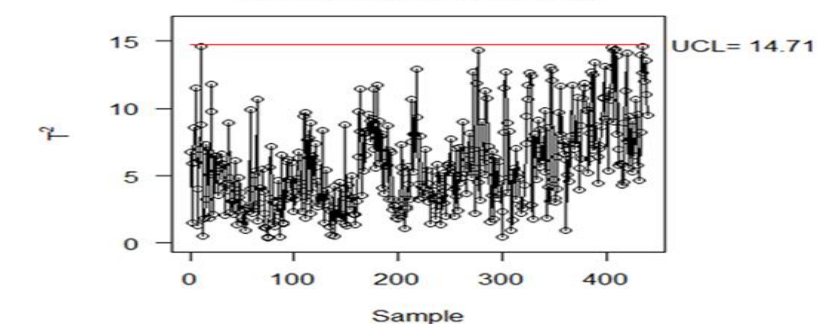
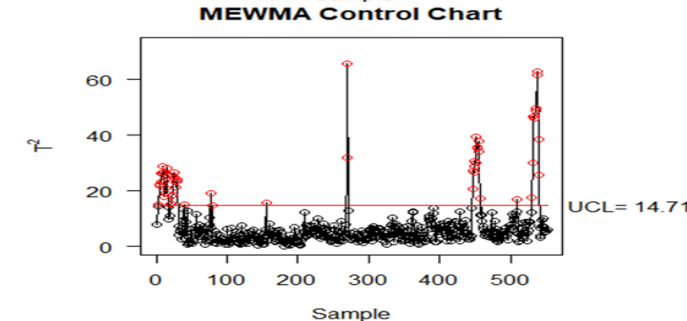
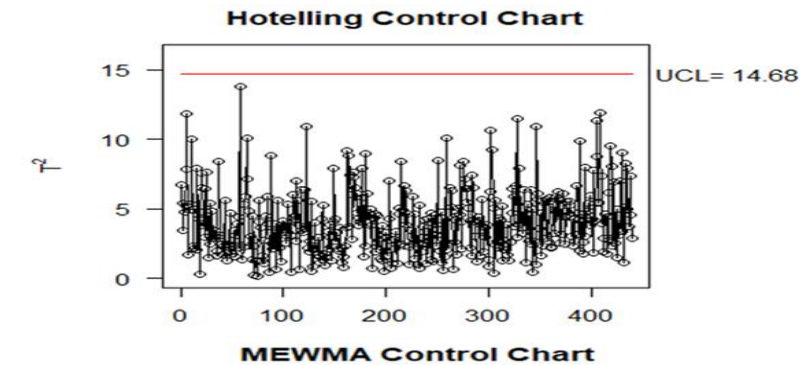
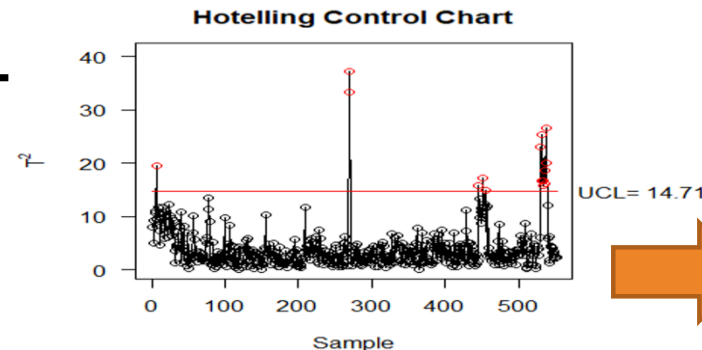


Final Iteration



## Combined $T^2$ & m-EWMA CHART

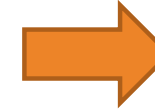
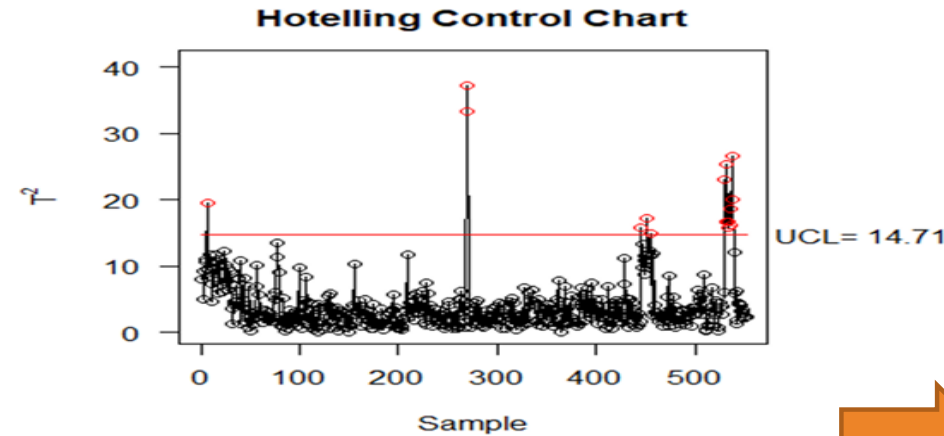
- After 6 iterations we find all remaining points to be in-control.
- In total 113 out of control data points were removed and 439 data points remain



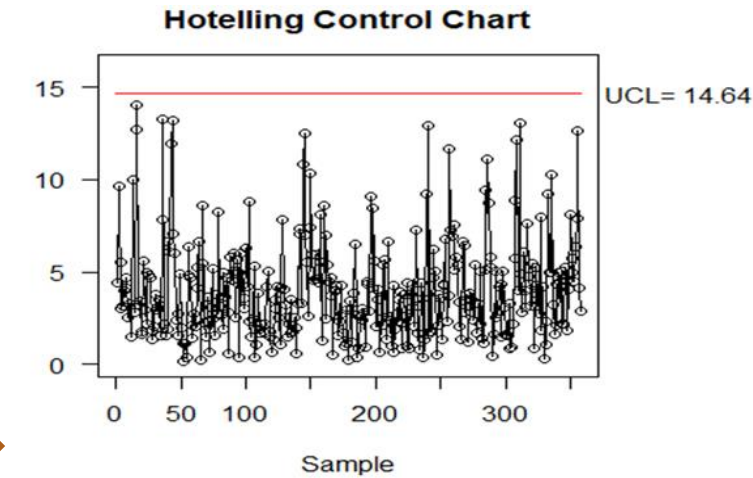
## Combined $T^2$ & m-CUSUM CHART

- After 7 iterations we find all remaining points to be in-control.
- In total 194 out of control data points were removed and 358 data points remain
- $K=1.5$ ,  $h=6$  for m-CUSUM
- Alpha set as 0.005

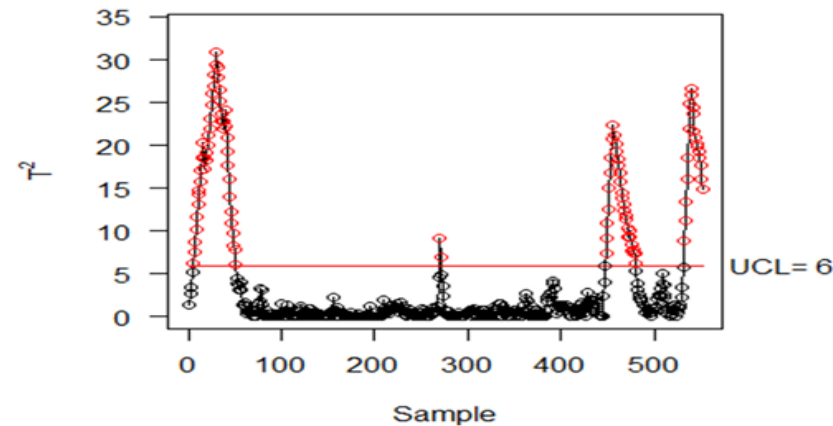
First Iteration



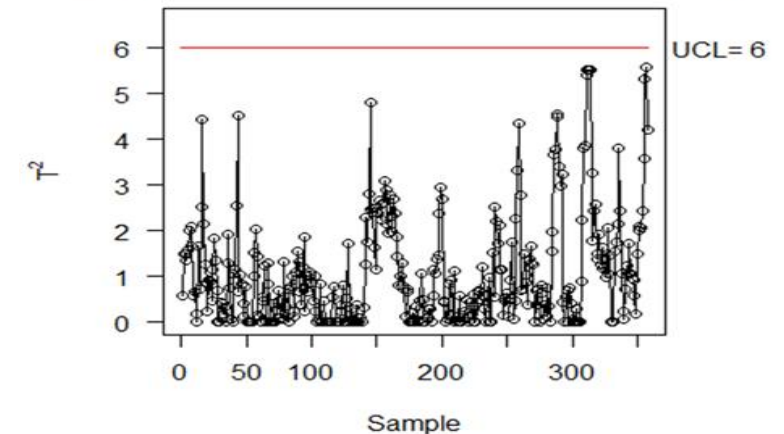
Final Iteration



MCUSUM Control Chart by Crosier (1988)



MCUSUM Control Chart by Crosier (1988)





# CONCLUSION

- Hotelling  $T^2$  control chart method had the most in-control data points after conclusion of Phase 1 analysis (484) whereas the Combined  $T^2$  and CUSUM had the lowest (358).
- $T^2$  took the longest number of iterations (13) to reach its conclusion whereas both m-EWMA and Combined  $T^2$  m-EWMA took the shortest (6 iterations each)
- If our objective is to select the maximum number of in-control points and get an in-control sample without large spikes present, we must choose the Hotelling  $T^2$  chart.
- For detecting small sustained mean shift, we can select M-EWMA chart.
- Provided our goal is to detect both large spikes and small mean shifts, it would be wise to choose a combination of  $T^2$  and MEWMA to select in-control points.

<b><i>Control Chart Type</i></b>	<b><i>ARL0</i></b>	<b><i>No. of Iterations</i></b>	<b><i>In-control data points</i></b>
<b>Hotelling <math>T^2</math></b>	<b>200</b>	<b>13</b>	<b>484</b>
<b>m-CUSUM</b>	<b>200</b>	<b>7</b>	<b>372</b>
<b>m-EWMA</b>	<b>200</b>	<b>6</b>	<b>437</b>
<b>Combined <math>T^2</math> &amp; m-EWMA</b>	<b>200</b>	<b>6</b>	<b>439</b>
<b>Combined <math>T^2</math> &amp; m-CUSUM</b>	<b>200</b>	<b>7</b>	<b>358</b>

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THANK YOU

