1. Problem 1

Prove that $F_n > \left(\frac{4}{3}\right)^n$

i. Base case: $n = 5$

Given : F(0) = 1 , F(1) = 1

$F(5) = F(4) + F(3)$

$$= \big(F(3) + F(2)\big) + \big(F(2) + F(1)\big)$$
$$= \big((F(2) + F(1)\big) + ((F(1) + F(0))) + ((F(1) + F(0)) + 1)$$

$= 1 + 1 + 2 = 5$

$since \left(\frac{4}{3}\right)^5 = 4.21$

$\therefore F(5) > \left(\frac{4}{3}\right)^5$

ii. Induction step:

Assume $F(n) > \left(\frac{4}{3}\right)^n$ $and\ \ we\ need\ to\ \ prove\ that\ F(n+1) > \left(\frac{4}{3}\right)^{n+1}$

$L.H.S = F(n+1) = F(n) + F(n-1), using\ the\ assumption\ hypothesis:$

$L.H.S > \left(\frac{4}{3}\right)^n + \left(\frac{4}{3}\right)^{n-1}$

$> \left(\frac{4}{3}\right)^n + \left(\frac{4}{3}\right)^n \div \frac{4}{3} > \left(\frac{4}{3}\right)^n \left(1 + \frac{1}{\frac{4}{3}}\right)$

$> 1\frac{3}{4}\left(\frac{4}{3}\right)^n > 1.75\left(\frac{4}{3}\right)^n$

$R.H.S = \left(\frac{4}{3}\right)^{n+1} = \left(\frac{4}{3}\right)^n . \frac{4}{3} = 1.33\left(\frac{4}{3}\right)^n$

$\boxed{\boldsymbol{Since\ 1.75\left(\frac{4}{3}\right)^n > 1.33\left(\frac{4}{3}\right)^n}}$

$\therefore\ F(n) > \left(\frac{4}{3}\right)^n$