**Aggregate Method:**

- Let's say $c_i$ be the cost of $i^{th}$ operation, It is given that, $c_i = i$ if i an exact power of 2 and $c_i = 1$ otherwise. This implies we will have a cost like this.

| Operation : i | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| Cost : $c_i$ | 1 | 2 | 1 | 4 | 1 | 1 | 1 | 8 | 1 |

$$\sum_{i=1}^{n} c_i = \sum_{j=0}^{logn} 2^j + \sum_{j=1}^{n} 1 - \sum_{j=1}^{logn-1} 1$$

$$= \sum_{j=1}^{logn} 2^j + n - (\log n - 1)$$

$$<= n + \sum_{j=1}^{logn} 2^j \text{ for all } n > 0.$$

$$<= n + (2n - 1)$$

$$< 3n => O(3n)$$

Average Cost of Operation = Total Cost / No of Operations = 3n / n = 3.

So, the amortized cost per operation = O (3) = O (1).

**Accounting Analysis Method:**

| Operation : i | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| Cost : $c_i$ | 1 | 2 | 1 | 4 | 1 | 1 | 1 | 8 | 1 |
| Amortized Cost | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |

Let's assume amortized cost be 3 and we need to prove that 3 is actually amortized value.

This implies amortized Cost = $\sum_{i=1}^{n} c_{i\ (amortized)} = 3n$

The summation of cost for each operation of i from time to time cannot be greater than the value of the sum of 3 as the value of i increases.

$$\sum_{i=1}^{n} c_{i\ (amortized)} >= \sum_{i=1}^{n} c_i$$

This implies the amortized cost the operation become O (3) = O (1).