Problem 2

a. $\mathbf{4^n}$ **is** $\boldsymbol{O(2^n)}$**?** → false

$$\lim_{n\to\infty}\frac{4^n}{2^n}=\lim_{n\to\infty}\frac{(2^n)^2}{2^n}=\lim_{n\to\infty}2^n=\infty\ \ ,which\ is\ not\ finite, so\ \ it\ is\ \boldsymbol{false}.$$

b. $\boldsymbol{\log n}$ **is** $\boldsymbol{\Theta(\log_3 n)}$**?** → True

1. $\lim\limits_{n\to\infty}\frac{\log n}{\log_3 n}=\lim\limits_{n\to\infty}\frac{\frac{\log_3 n}{\log_3 2}}{\log_3 n}=\lim\limits_{n\to\infty}\frac{1}{\log_3 2}=c_1\to(1)$
2. $\lim\limits_{n\to\infty}\frac{\log_3 n}{\log n}=\lim\limits_{n\to\infty}\frac{\frac{\log n}{\log 3}}{\log n}=\lim\limits_{n\to\infty}\frac{1}{\log 3}=c_2\to(2)$

From (1) and (2) above: $\log n\ \ is\ \Theta(\log_3 n)$.

c. $\boldsymbol{\frac{n}{2}\log\frac{n}{2}}$ **is** $\boldsymbol{\Theta(n\log n)}$**?** → True

1. $\lim\limits_{n\to\infty}\frac{\frac{n}{2}\log\frac{n}{2}}{n\log n}=\lim\limits_{n\to\infty}\frac{\log\frac{n}{2}}{2\log n}=\lim\limits_{n\to\infty}\frac{\log n-\log 2}{2\log n}=\lim\limits_{n\to\infty}\frac{1-\frac{\log 2}{\log n}}{2}=\frac{1}{2}\ \to c_1\to(1)$
2. $\lim\limits_{\mathrm{n}\to\infty}\frac{\mathrm{n}\log\mathrm{n}}{\frac{\mathrm{n}}{2}\log\frac{\mathrm{n}}{2}}=\lim\limits_{\mathrm{n}\to\infty}\frac{2\log\mathrm{n}}{\log\frac{\mathrm{n}}{2}}=\lim\limits_{n\to\infty}\frac{2\log n}{\log n-\log 2}=\lim\limits_{n\to\infty}\frac{2}{1-\frac{\log 2}{\log n}}=2\to c_2\to(2)$

From (1) and (2): $\frac{n}{2}\log\frac{n}{2}\ \ is\ \Theta(n\log n)$.