3. Big-oh and Little-oh. Use the definitions of *O(f(n))* and limit facts about *o(f(n))* given in class to decide whether each of the following is true or false, and in each case, prove your answer.

A. **$1 + 4n^2$ is $O(n^2)$**

Solution

Let c = 5 and $n_0 = 1$. Then, whenever n ≥ n0,

$1 + 4n^2 \leq n^2 + 4n^2 = 5n^2 = cn^2$.

Hence, $1 + 4n^2$ is $O(n^2)$ → **True**

B. **$n^2 – 2n$ is not O(n)**

Solution

Let proof by contradiction that $n^2 - 2n\ is\ O(n), so\ that\ \lim_{n\to\infty} \frac{n^2-n}{n}$ is finite

$$f(n) = n^2 - 2n$$

$$\lim_{n\to\infty} \frac{n^2 - 2n}{n} = \lim_{n\to\infty} \frac{n-2}{1} = \infty \to not\ finited, so\ "f(n)\ is\ not\ O(n)"\ is\ \boldsymbol{true}.$$

C. **log(n) is o(n)**

Solution

$$f(n) = \log n$$

$$\lim_{n\to\infty} \frac{\log n}{n} = \frac{\infty}{\infty}\ is\ undefined\ , so\ lets\ use\ L'Hopital's\ rule$$

$$\lim_{n\to\infty} \frac{(\log n)\prime}{(n)\prime} = \frac{1/(nln2)}{1} = \lim_{n\to\infty}(1/n)loge = 0$$ → *log(n) is o(n) is* ***true***.

D. **n is not o(n)**

Solution

$$\lim_{n\to\infty} \frac{n}{n} = \lim_{n\to\infty} \frac{1}{1} = 1 \neq 0 \to "n\ is\ not\ o(n)"\ is\ \boldsymbol{true}.$$