CS435 Algorithm

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- Lab#1
- Group #

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Math Problem 1. Which of the following functions are increasing? eventually nondecreasing? If you remember techniques from calculus, you can make use of those.

a) $f(x) = -x^2$
b) $f(x) = x^2 + 2x + 1$
c) $f(x) = x^3 + x$

Solution:

To know if a function $f(x)$ is increasing/eventually decreasing

- Get its derivative, $\acute{f}(x)$.
- Get the zero of the derivative $x_0, where\ f'(x) = 0$.
- Test $f'(x)$ before and after x_0.

a) $f(x) = -x^2$

$f'(x) = -2x = 0 \to x_0 = 0 \to (1)$

Test $f'(x)$ before and after x_0:

$let\ x = 1 \to f'(x) = -2(1) = -2 \to negative\ (2)$

$let\ x = -1 \to f'(x) = -2(-1) = 2 \to positive\ (3)$

From (1), (2), and (3):
$f'(x)\ is\ increasing\ for\ the\ range\ [-\infty, 0], and\ decreasing\ for\ the\ range\ [0, \infty].$

b) $f(x) = x^2 + 2x + 1$

$f'(x) = 2x + 2 = 0 \to x_0 = -1 \to (1)$

Test $f'(x)$ before and after x_0:
$let\ x = -2 \to f'(x) = -2 \to negative\ (2)$

$let\ x = 0 \to f'(x) = 2 \to positive(3)$

From (1), (2), and (3):
$f'(x)\ is\ decreasing\ for\ the\ range\ [-\infty, -1], and\ increasing\ for\ the\ range\ [-1, \infty].$

c) $f(x) = x^3 + x = x(x^2 + 1)$

$f'(x) = 3x^2 + 1$

We note that for any value of x, $f'(x)$ will be positive. This means that
$f(x)\ is\ increasing\ for\ [-\infty, \infty].$

Math Problem 2. Consider the following pairs and functions *f, g*. Decide if it is correct to say that, asymptotically, *f* grows no faster than *g, g* grows no faster than *f*, or both.

(1) f(x)=2x^2, g(x) = x^2 + 1

(2) f(x) = x^2, g(x) = x^3

(3) f(x)=4x + 1, g(x) = x^2 – 1

Solution:

To know how a function $f(x)$ grows with respect to another function $g(x)$, we divide them:

- If $\lim_{x\to\infty} \frac{f(x)}{g(x)} = 0$, then $f(x)$ asymptotically grows no faster than $g(x)$.
- If $\lim_{x\to\infty} \frac{f(x)}{g(x)} = \infty$, then $g(x)$ asymptotically grows no faster than $f(x)$.
- If $\lim_{x\to\infty} \frac{f(x)}{g(x)} = c\,, where\ c\ is\ a\ constant$, then $f(x)$ and $g(x)$ asymptotically grow with the same rate.

1) $f(x) = 2x^2, g(x) = x^2 + 1$

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = \lim_{x\to\infty} \frac{2x^2}{x^2+1} = \lim_{x\to\infty} \frac{2}{1+\frac{1}{x^2}} = \frac{2}{1+0} = 2$$

$\therefore f(x)\ and\ g(x)\ asymptotically\ grow\ with\ the\ same\ rate.$

2) $f(x) = x^2, g(x) = x^3$

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = \lim_{x\to\infty} \frac{x^2}{x^3} = \lim_{x\to\infty} \frac{1}{x} = \frac{1}{\infty} = 0$$

$\therefore f(x)\ asymptotically\ grows\ no\ faster\ than\ g(x).$

3) $f(x) = 4x + 1, g(x) = x^2 - 1$

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = \lim_{x\to\infty} \frac{4x+1}{x^2+1} \to Apply\ L'Hopital's\ rule:$$

$$\lim_{x\to\infty} \frac{4}{2x} = \frac{4}{\infty} = 0$$

$\therefore f(x)\ asymptotically\ grows\ no\ faster\ than\ g(x).$