Criando a Árvore de Decisão?

outlook temperature humidity windy

- Temos 4 atributos “candidatos” a ser o nó raiz.
- Qual devemos escolher?
  - Buscar o que tenha maior ganho de informação!

outlook	temperature	humidity	windy	play
sunny	hot	high	FALSE	no
sunny	hot	high	TRUE	no
overcast	hot	high	FALSE	yes
rainy	mild	high	FALSE	yes
rainy	cool	normal	FALSE	yes
rainy	cool	normal	TRUE	no
overcast	cool	normal	TRUE	yes
sunny	mild	high	FALSE	no
sunny	cool	normal	FALSE	yes
rainy	mild	normal	FALSE	yes
sunny	mild	normal	TRUE	yes
overcast	mild	high	TRUE	yes
overcast	hot	normal	FALSE	yes
rainy	mild	high	TRUE	no

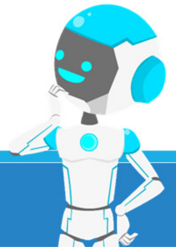
Entropia

Teoria da Informação

$$E(S) = -\sum_{i=1}^{n} p_i \log_2 pi$$

Se todas as instancias de S pertencem a mesma classe E(S) = 0

Se S contem o mesmo número de instancia para cada classe, E(s) = 1



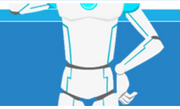
Cálculo da Entropia - Classe

$E(S) = -\sum_{i=1}^{n} p_i \log_2 pi$

$E(S) = \left(-\frac{9}{14}\log_2\left(\frac{9}{14}\right)\right) + \left(-\frac{5}{14}\log_2\left(\frac{5}{14}\right)\right)$

$E(S)$ = 0,94

outlook	temperature	humidity	windy	play
sunny	hot	high	FALSE	no
sunny	hot	high	TRUE	no
overcast	hot	high	FALSE	yes
rainy	mild	high	FALSE	yes
rainy	cool	normal	FALSE	yes
rainy	cool	normal	TRUE	no
overcast	cool	normal	TRUE	yes
sunny	mild	high	FALSE	no
sunny	cool	normal	FALSE	yes
rainy	mild	normal	FALSE	yes
sunny	mild	normal	TRUE	yes
overcast	mild	high	TRUE	yes
overcast	hot	normal	FALSE	yes
rainy	mild	high	TRUE	no



Cálculo da Entropia - Outlook

$E(S) = -\sum_{i=1}^{n} p_i \log_2 pi$

$Outlook\ (sunny - para\ yes\ e\ no)$

$E(S) = \left(-\frac{2}{5}\log_2\left(\frac{2}{5}\right)\right) + \left(-\frac{3}{5}\log_2\left(\frac{3}{5}\right)\right) = 0{,}97$

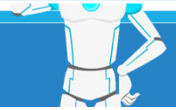
$Outlook\ (overcast - para\ yes\ e\ no)$

$E(S) = \left(-\frac{4}{4}\log_2\left(\frac{4}{4}\right)\right) + \left(-\frac{0}{4}\log_2\left(\frac{0}{4}\right)\right)$ =0

$Outlook\ (rainy - para\ yes\ e\ no)$

$E(S) = \left(-\frac{3}{5}\log_2\left(\frac{3}{5}\right)\right) + \left(-\frac{2}{5}\log_2\left(\frac{2}{5}\right)\right)$ =0,97

outlook	temperature	humidity	windy	play
sunny	hot	high	FALSE	no
sunny	hot	high	TRUE	no
overcast	hot	high	FALSE	yes
rainy	mild	high	FALSE	yes
rainy	cool	normal	FALSE	yes
rainy	cool	normal	TRUE	no
overcast	cool	normal	TRUE	yes
sunny	mild	high	FALSE	no
sunny	cool	normal	FALSE	yes
rainy	mild	normal	FALSE	yes
sunny	mild	normal	TRUE	yes
overcast	mild	high	TRUE	yes
overcast	hot	normal	FALSE	yes
rainy	mild	high	TRUE	no



Cálculo da Entropia - Temperature

$E(S) = -\sum_{i=1}^{n} p_i \log_2 pi$

$Temperature\ (hot - para\ yes\ e\ no)$

$E(S) = \left(-\frac{2}{4}\log_2\left(\frac{2}{4}\right)\right) + \left(-\frac{2}{4}\log_2\left(\frac{2}{4}\right)\right) = 1$

$Temperature\ (mild - para\ yes\ e\ no)$

$E(S) = \left(-\frac{4}{6}\log_2\left(\frac{4}{6}\right)\right) + \left(-\frac{2}{6}\log_2\left(\frac{2}{6}\right)\right)$ =0,91

$Temperature\ (cold - para\ yes\ e\ no)$

$E(S) = \left(-\frac{3}{4}\log_2\left(\frac{3}{4}\right)\right) + \left(-\frac{1}{4}\log_2\left(\frac{1}{4}\right)\right)$ =0,81

outlook	temperature	humidity	windy	play
sunny	hot	high	FALSE	no
sunny	hot	high	TRUE	no
overcast	hot	high	FALSE	yes
rainy	mild	high	FALSE	yes
rainy	cool	normal	FALSE	yes
rainy	cool	normal	TRUE	no
overcast	cool	normal	TRUE	yes
sunny	mild	high	FALSE	no
sunny	cool	normal	FALSE	yes
rainy	mild	normal	FALSE	yes
sunny	mild	normal	TRUE	yes
overcast	mild	high	TRUE	yes
overcast	hot	normal	FALSE	yes
rainy	mild	high	TRUE	no

Cálculo da Entropia - Humidity

$E(S) = -\sum_{i=1}^{n} p_i \log_2 pi$

Humidity (high – para yes e no)

$$E(S) = \left(-\frac{3}{7}\log_2\left(\frac{3}{7}\right)\right) + \left(-\frac{4}{7}\log_2\left(\frac{4}{7}\right)\right) = 0{,}98$$

Humidity (normal – para yes e no)

$$E(S) = \left(-\frac{6}{7}\log_2\left(\frac{6}{7}\right)\right) + \left(-\frac{1}{7}\log_2\left(\frac{1}{7}\right)\right) = 0{,}59$$

outlook	temperature	humidity	windy	play
sunny	hot	high	FALSE	no
sunny	hot	high	TRUE	no
overcast	hot	high	FALSE	yes
rainy	mild	high	FALSE	yes
rainy	cool	normal	FALSE	yes
rainy	cool	normal	TRUE	no
overcast	cool	normal	TRUE	yes
sunny	mild	high	FALSE	no
sunny	cool	normal	FALSE	yes
rainy	mild	normal	FALSE	yes
sunny	mild	normal	TRUE	yes
overcast	mild	high	TRUE	yes
overcast	hot	normal	FALSE	yes
rainy	mild	high	TRUE	no

Cálculo da Entropia - Windy

$$E(S) = -\sum_{i=1}^{n} p_i \log_2 pi$$

Windy (True – para yes e no)

$$E(S) = \left(-\frac{3}{6}\log_2\left(\frac{3}{6}\right)\right) + \left(-\frac{3}{6}\log_2\left(\frac{3}{6}\right)\right) = 1$$

Windy (False – para yes e no)

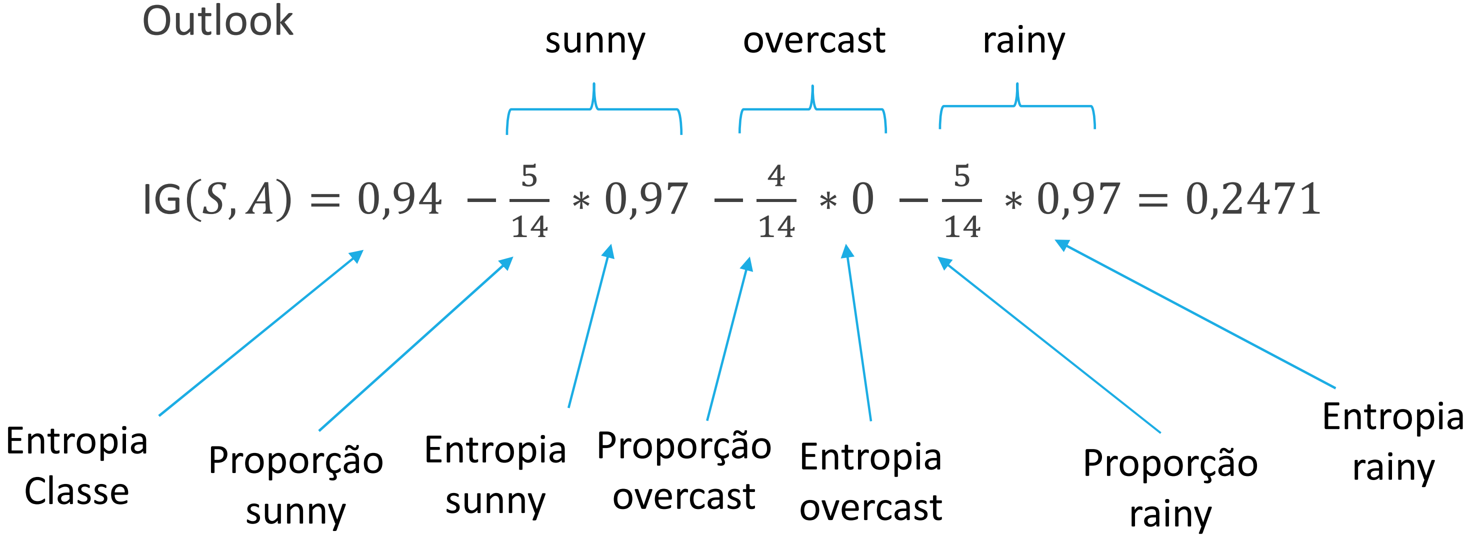
$$E(S) = \left(-\frac{6}{8}\log_2\left(\frac{6}{8}\right)\right) + \left(-\frac{2}{8}\log_2\left(\frac{2}{8}\right)\right) = 0{,}81$$

outlook	temperature	humidity	windy	play
sunny	hot	high	FALSE	no
sunny	hot	high	TRUE	no
overcast	hot	high	FALSE	yes
rainy	mild	high	FALSE	yes
rainy	cool	normal	FALSE	yes
rainy	cool	normal	TRUE	no
overcast	cool	normal	TRUE	yes
sunny	mild	high	FALSE	no
sunny	cool	normal	FALSE	yes
rainy	mild	normal	FALSE	yes
sunny	mild	normal	TRUE	yes
overcast	mild	high	TRUE	yes
overcast	hot	normal	FALSE	yes
rainy	mild	high	TRUE	no

Ganho de Informação (Information Gain)

$$\text{IG}(S,A) = E(S) - \sum_{i=1}^{n} \frac{|S_i|}{|S|} E(S_i)$$

Outlook



$$\text{IG}(S,A) = 0,94 - \overbrace{\frac{5}{14} * 0,97}^{\text{sunny}} - \overbrace{\frac{4}{14} * 0}^{\text{overcast}} - \overbrace{\frac{5}{14} * 0,97}^{\text{rainy}} = 0,2471$$

Entropia Classe

Proporção sunny

Entropia sunny

Proporção overcast

Entropia overcast

Proporção rainy

Entropia rainy

$$\text{IG}(S,A) = E(S) - \sum_{i=1}^{n} \frac{|S_i|}{|S|} E(S_i)$$

Outlook

$$\text{IG}(S,A) = 0,94 - \frac{5}{14} * 0,97 - \frac{4}{14} * 0 - \frac{5}{14} * 0,97 = 0,2471$$

Ganho de Informação (Information Gain)

$$\mathrm{IG}(S, A) = Entropia(S) - \sum_{i=1}^{n} \frac{|S_i|}{|S|} Entropia(S_i)$$

Outlook

$$\mathrm{IG}(S, A) = 0{,}94 - \frac{5}{14} * 097 - \frac{4}{14} * 0 - \frac{5}{14} * 0{,}97 = 0{,}2471$$

Temperatures

$$\mathrm{IG}(S, A) = 0{,}94 - \frac{4}{14} * 1 - \frac{6}{14} * 0{,}91 - \frac{4}{14} * 0{,}81 = \quad 0{,}0328$$

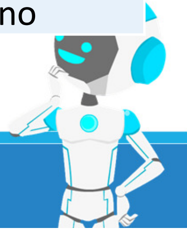
Humidy

$$\mathrm{IG}(S, A) = 0{,}94 - \frac{7}{14} * 0{,}97 - \frac{7}{14} * 0{,}59 = 0{,}16$$

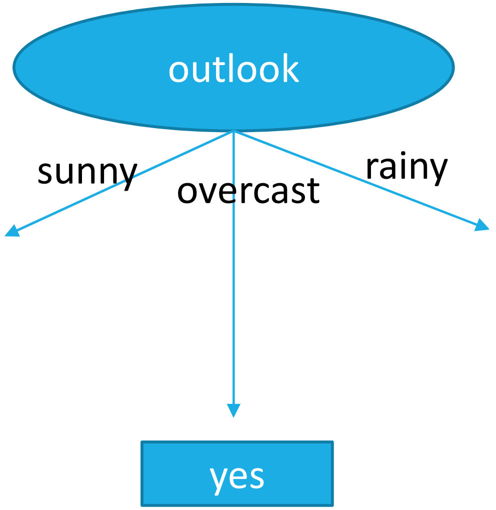
Windy

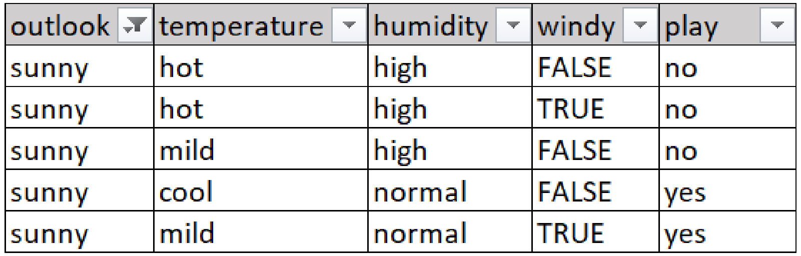
$$\mathrm{IG}(S, A) = 0{,}94 - \frac{6}{14} * 1 - \frac{8}{14} * 0{,}81 = 0{,}048$$

outlook	temperature	humidity	windy	play
sunny	hot	high	FALSE	no
sunny	hot	high	TRUE	no
overcast	hot	high	FALSE	yes
rainy	mild	high	FALSE	yes
rainy	cool	normal	FALSE	yes
rainy	cool	normal	TRUE	no
overcast	cool	normal	TRUE	yes
sunny	mild	high	FALSE	no
sunny	cool	normal	FALSE	yes
rainy	mild	normal	FALSE	yes
sunny	mild	normal	TRUE	yes
overcast	mild	high	TRUE	yes
overcast	hot	normal	FALSE	yes
rainy	mild	high	TRUE	no

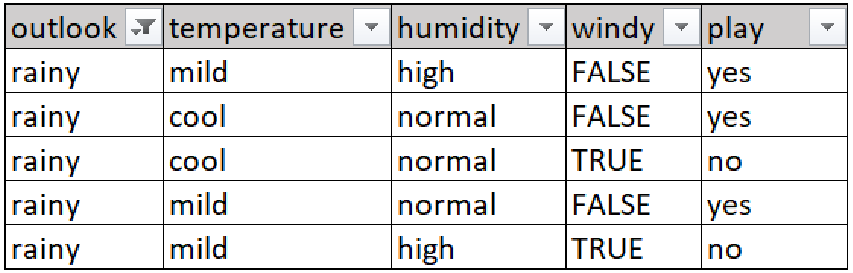


Primeiro nodo

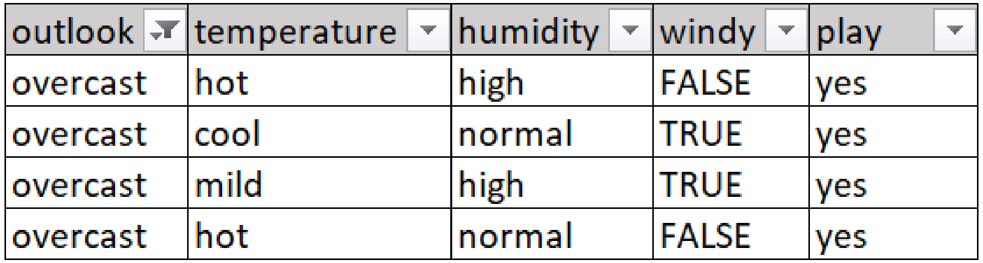




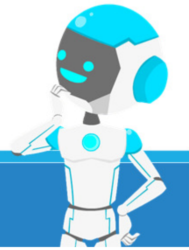
outlook	temperature	humidity	windy	play
sunny	hot	high	FALSE	no
sunny	hot	high	TRUE	no
sunny	mild	high	FALSE	no
sunny	cool	normal	FALSE	yes
sunny	mild	normal	TRUE	yes



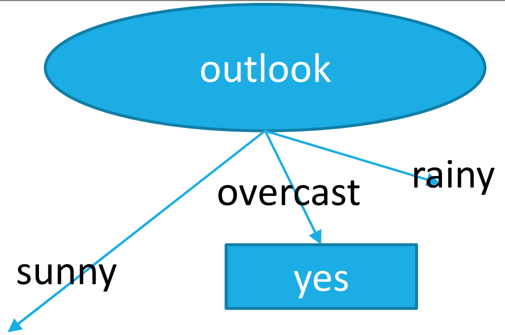
outlook	temperature	humidity	windy	play
rainy	mild	high	FALSE	yes
rainy	cool	normal	FALSE	yes
rainy	cool	normal	TRUE	no
rainy	mild	normal	FALSE	yes
rainy	mild	high	TRUE	no

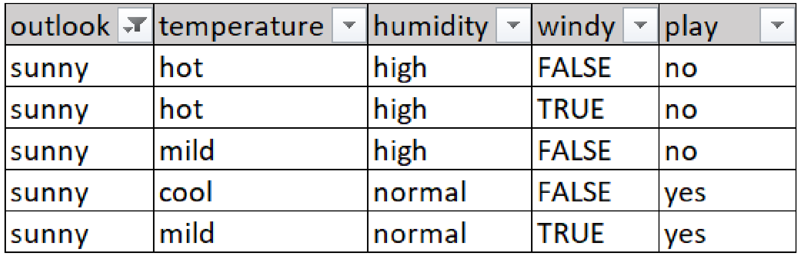


outlook	temperature	humidity	windy	play
overcast	hot	high	FALSE	yes
overcast	cool	normal	TRUE	yes
overcast	mild	high	TRUE	yes
overcast	hot	normal	FALSE	yes



Particionando sunny

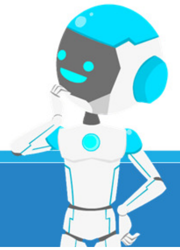




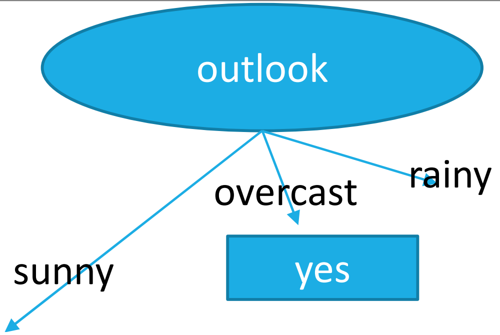
outlook	temperature	humidity	windy	play
sunny	hot	high	FALSE	no
sunny	hot	high	TRUE	no
sunny	mild	high	FALSE	no
sunny	cool	normal	FALSE	yes
sunny	mild	normal	TRUE	yes

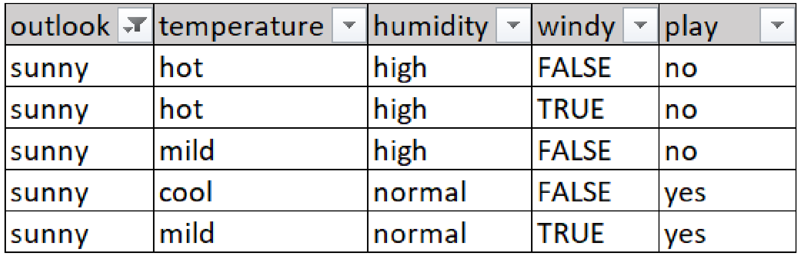
Entropia Classe

$$E(sunny) = \left(-\frac{3}{5}\log_2\left(\frac{3}{5}\right)\right) + \left(-\frac{2}{5}\log_2\left(\frac{2}{5}\right)\right) = 0{,}97$$



Cálculo da Entropia - Temperature





outlook	temperature	humidity	windy	play
sunny	hot	high	FALSE	no
sunny	hot	high	TRUE	no
sunny	mild	high	FALSE	no
sunny	cool	normal	FALSE	yes
sunny	mild	normal	TRUE	yes

Temperatura (hot para yes e no)

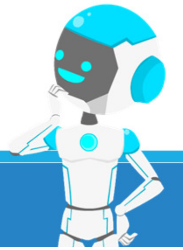
$$E(S) = \left(-\frac{0}{2}\log_2\left(\frac{0}{2}\right)\right) + \left(-\frac{2}{2}\log_2\left(\frac{2}{2}\right)\right) = 0$$

Temperatura (mild para yes e no)

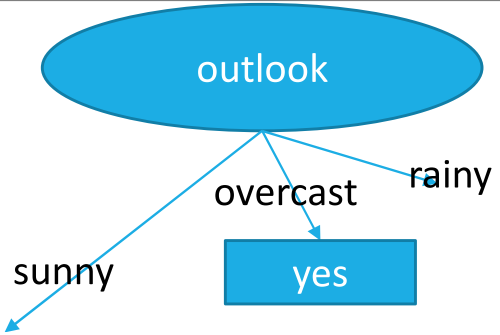
$$E(S) = \left(-\frac{1}{2}\log_2\left(\frac{1}{2}\right)\right) + \left(-\frac{1}{2}\log_2\left(\frac{1}{2}\right)\right) = 1$$

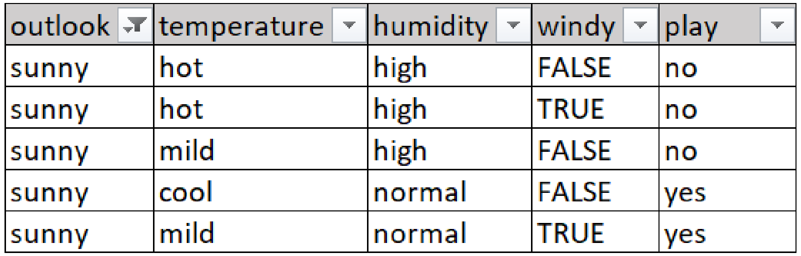
Temperatura (cool para yes e no)

$$E(S) = \left(-\frac{1}{1}\log_2\left(\frac{1}{1}\right)\right) + \left(-\frac{0}{1}\log_2\left(\frac{0}{1}\right)\right) = 0$$



Cálculo da Entropia - Humidity





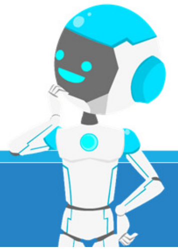
outlook	temperature	humidity	windy	play
sunny	hot	high	FALSE	no
sunny	hot	high	TRUE	no
sunny	mild	high	FALSE	no
sunny	cool	normal	FALSE	yes
sunny	mild	normal	TRUE	yes

Humidity (high para yes e no)

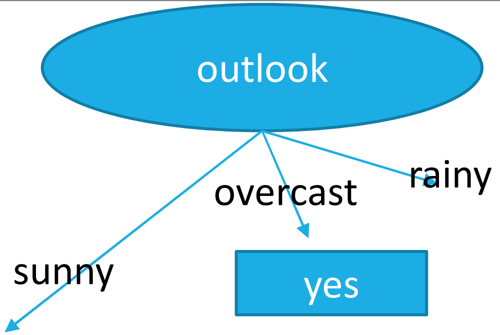
$$E(S) = \left(-\frac{0}{3}\log_2\left(\frac{0}{3}\right)\right) + \left(-\frac{3}{3}\log_2\left(\frac{3}{3}\right)\right) = 0$$

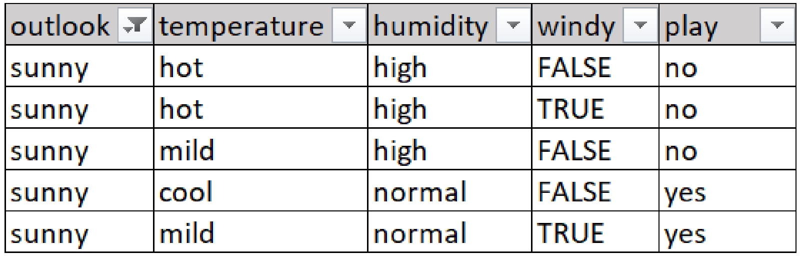
Humidity (normal para yes e no)

$$E(S) = \left(-\frac{2}{2}\log_2\left(\frac{2}{2}\right)\right) + \left(-\frac{0}{2}\log_2\left(\frac{0}{2}\right)\right) = 0$$



Cálculo da Entropia - Windy





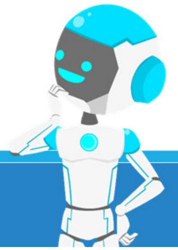
outlook	temperature	humidity	windy	play
sunny	hot	high	FALSE	no
sunny	hot	high	TRUE	no
sunny	mild	high	FALSE	no
sunny	cool	normal	FALSE	yes
sunny	mild	normal	TRUE	yes

windy (False para yes e no)

$$E(S) = \left(-\frac{1}{3}\log_2\left(\frac{1}{3}\right)\right) + \left(-\frac{2}{3}\log_2\left(\frac{2}{3}\right)\right) = 0{,}91$$

Humidity (True para yes e no)

$$E(S) = \left(-\frac{1}{2}\log_2\left(\frac{1}{2}\right)\right) + \left(-\frac{1}{2}\log_2\left(\frac{1}{2}\right)\right) = 1$$



Ganho de Informação (Information Gain)

$$\mathrm{IG}(S,A) = Entropia(S) - \sum_{i=1}^{n} \frac{|S_i|}{|S|} Entropia(S_i)$$

Temperatures

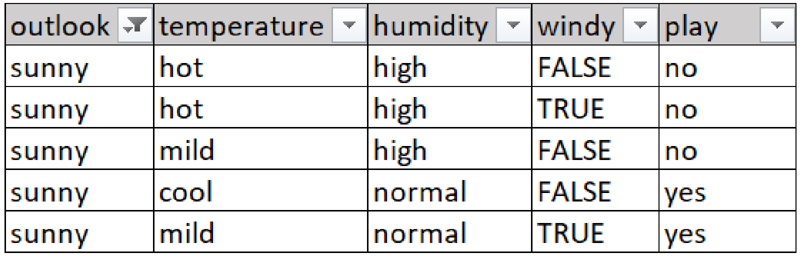
$$\mathrm{IG}(S,A) = 0{,}97 - \frac{2}{5} * 0 - \frac{2}{5} * 1 - \frac{1}{5} * 0 = 0{,}57$$

Humidity

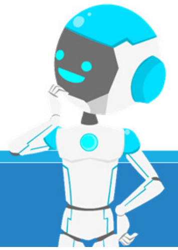
$$\mathrm{IG}(S,A) = 0{,}97 - \frac{3}{5} * 0 - \frac{2}{5} * 0 = 0{,}97$$

Windy

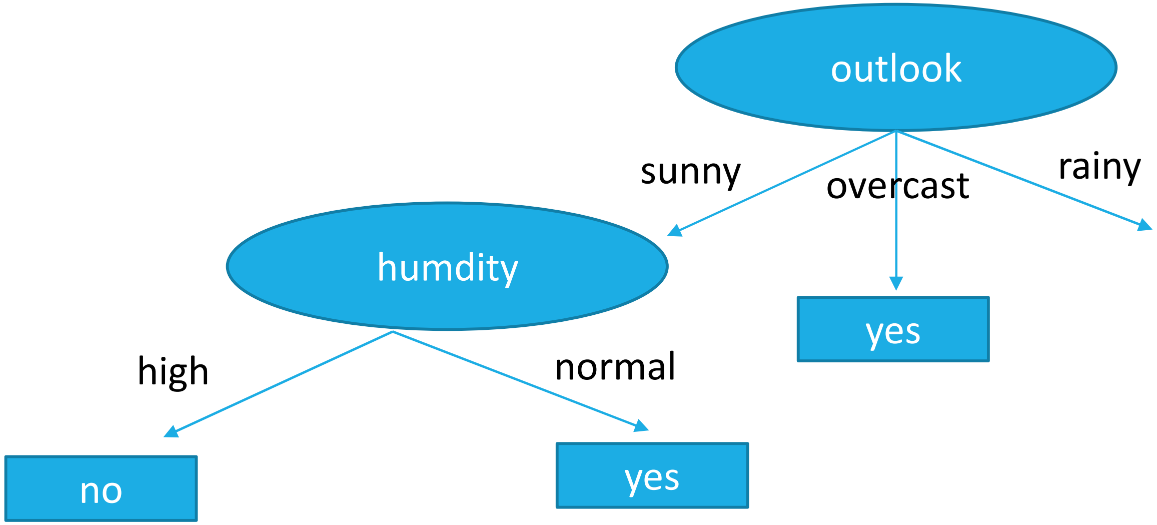
$$\mathrm{IG}(S,A) = 0{,}97 - \frac{3}{5} * 0{,}91 - \frac{2}{5} * 1 = 0{,}024$$

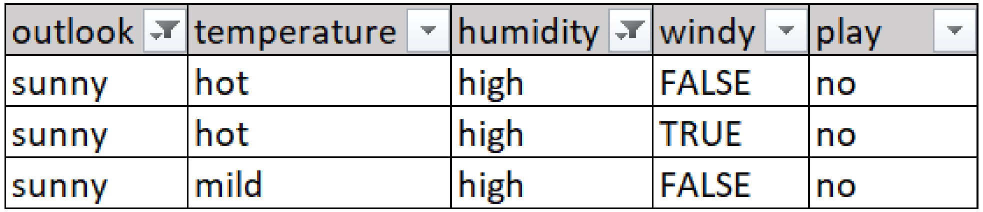


outlook	temperature	humidity	windy	play
sunny	hot	high	FALSE	no
sunny	hot	high	TRUE	no
sunny	mild	high	FALSE	no
sunny	cool	normal	FALSE	yes
sunny	mild	normal	TRUE	yes

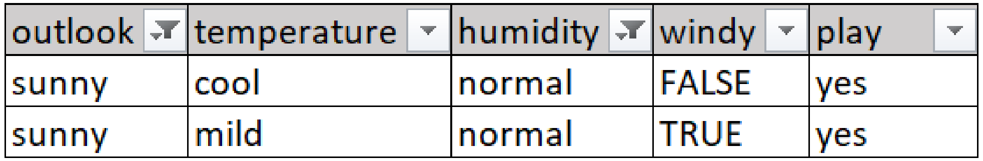


Próximo nodo

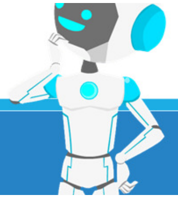




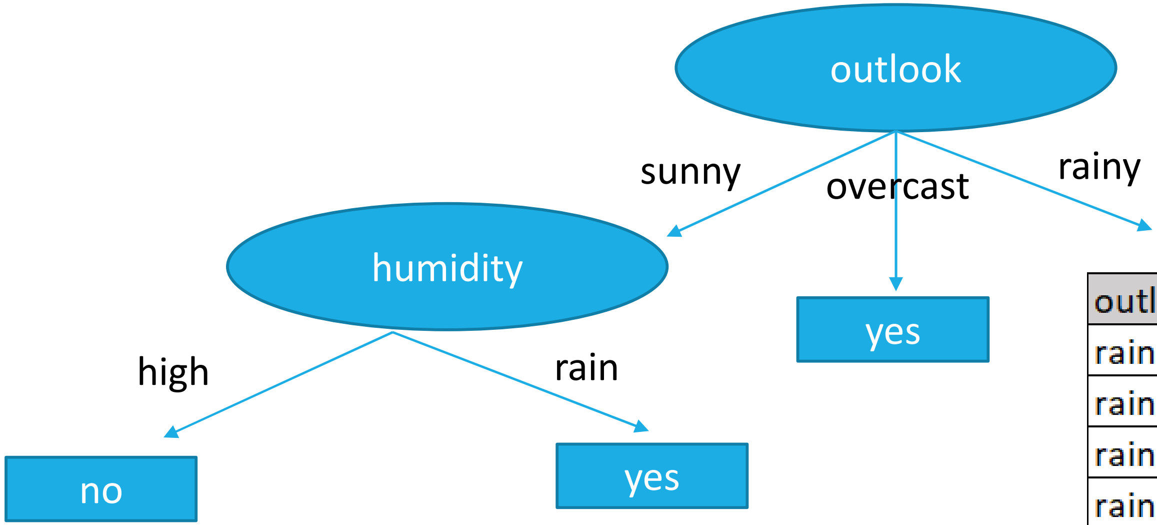
outlook	temperature	humidity	windy	play
sunny	hot	high	FALSE	no
sunny	hot	high	TRUE	no
sunny	mild	high	FALSE	no

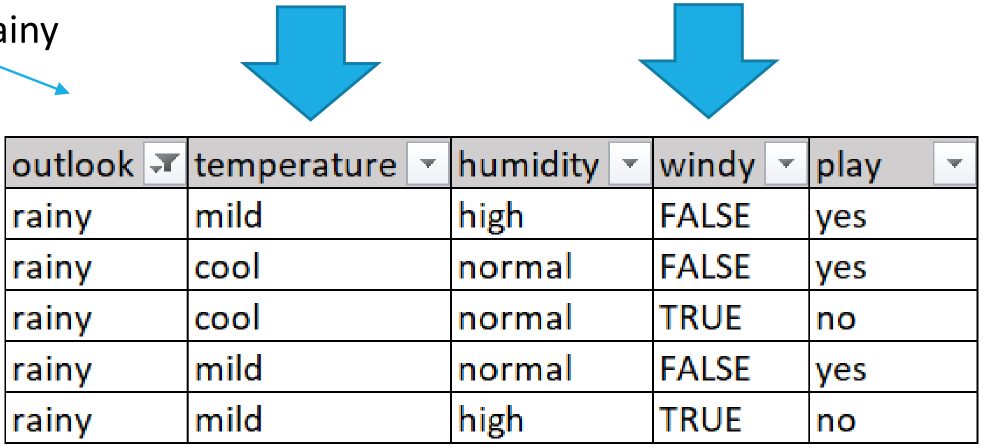


outlook	temperature	humidity	windy	play
sunny	cool	normal	FALSE	yes
sunny	mild	normal	TRUE	yes



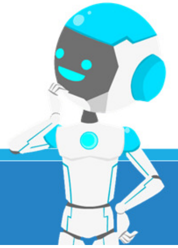
Continuando



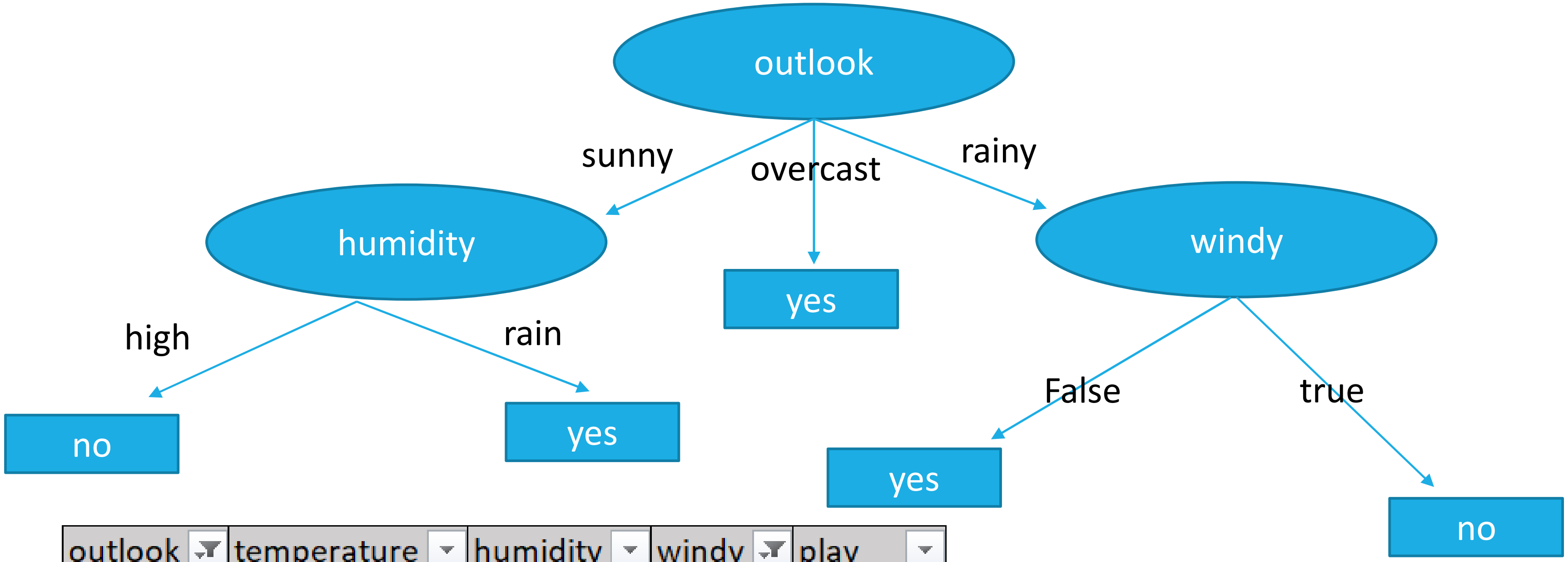


outlook	temperature	humidity	windy	play
rainy	mild	high	FALSE	yes
rainy	cool	normal	FALSE	yes
rainy	cool	normal	TRUE	no
rainy	mild	normal	FALSE	yes
rainy	mild	high	TRUE	no

Windy = 0,97



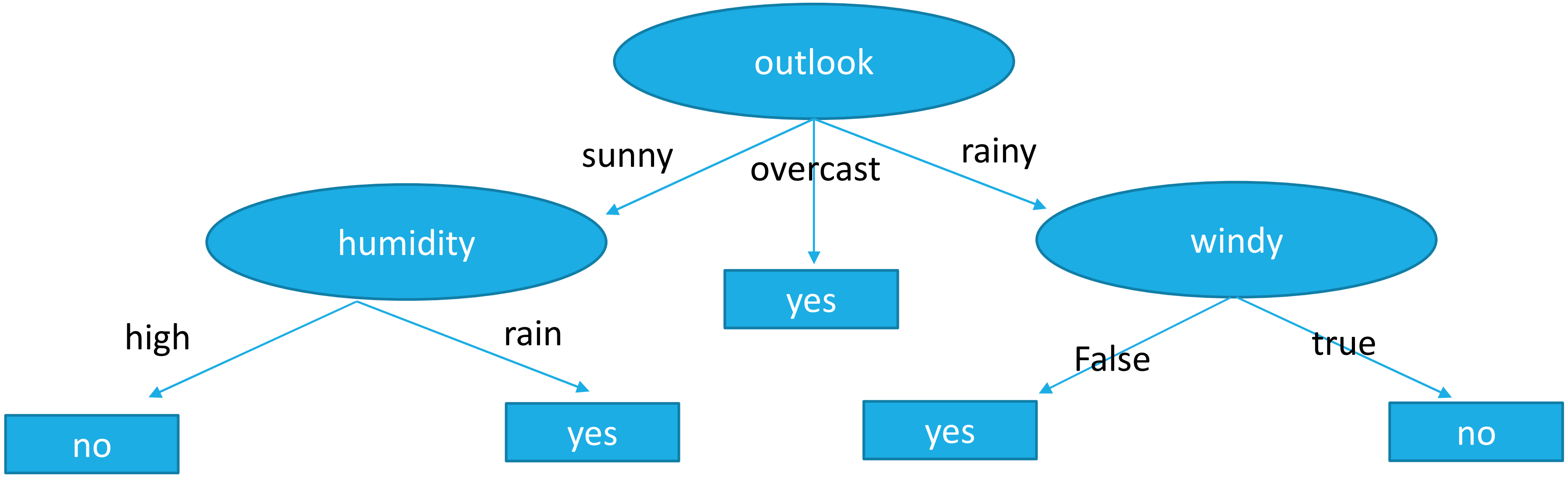
Continuando



outlook	temperature	humidity	windy	play
rainy	mild	high	FALSE	yes
rainy	cool	normal	FALSE	yes
rainy	mild	normal	FALSE	yes

outlook	temperature	humidity	windy	play
rainy	cool	normal	TRUE	no
rainy	mild	high	TRUE	no

Finalizando


outlook
sunny
overcast
rainy
humidity
yes
windy
high
rain
False
true
no
yes
yes
no