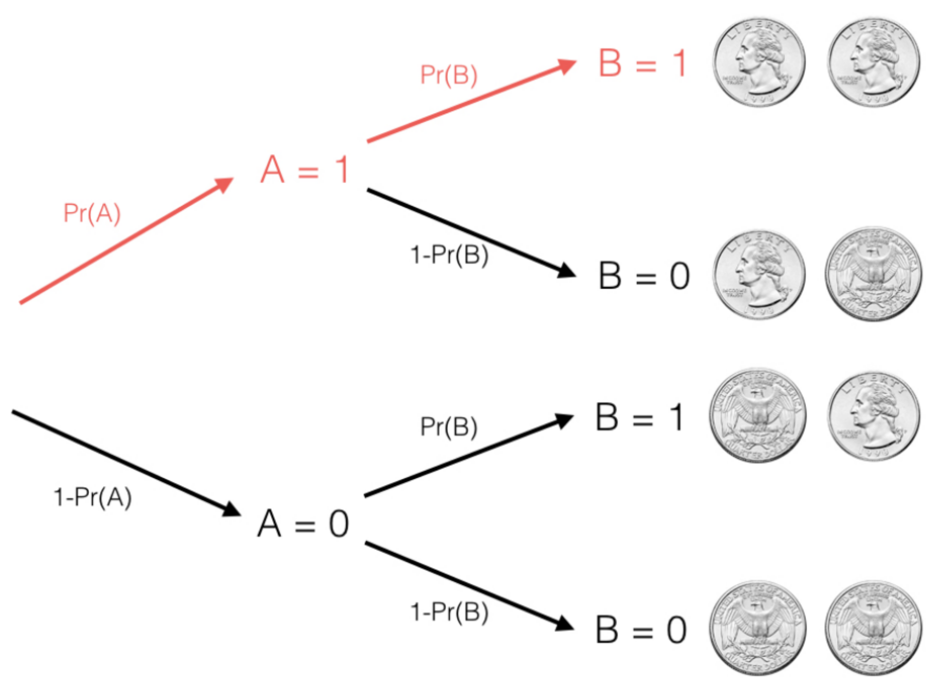
dsmath-practice

1 Review DataCamp ch 2 (laws of probability)

1. Consider two events A and B each with outcomes 0 or 1:

   What is `1 - (Pr(A=0) + Pr(A=1) = Pr(B=0) + Pr(B=1))`?

2. What does this figure illustrate?

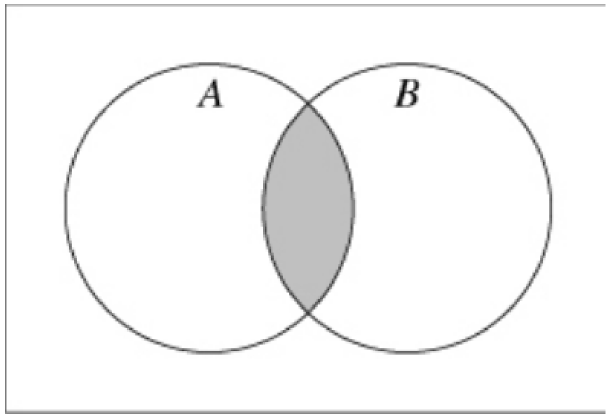


   1. A decision tree (to organize decisions)
   2. An option tree (to organize options)
   3. The "simultaneous" flip of two fair coins A and B
   4. Flipping one fair coin twice (with replacement)
   5. Two independent events A and B
   6. Probabilities of an event A and an event B
   7. Probabilities of two events that are stochastically "coupled"
   8. `Pr(A=0) + Pr(A=1) = Pr(B=0) + Pr(B=1) = 1`
   9. `Pr(A \cap B)`: the probability of getting A and B
   10. `Pr(A \cup B)`: the probability of getting A or B

3. What is `Pr(A \cap B)` (AND) for independent events `A` and `B`?

   `Pr(A \cap B) = Pr(A) \times Pr(B)`

4. What is `Pr(A \cap B)` as a Venn diagram?



5. What happens to the Venn diagram if the probabilities are not equal?

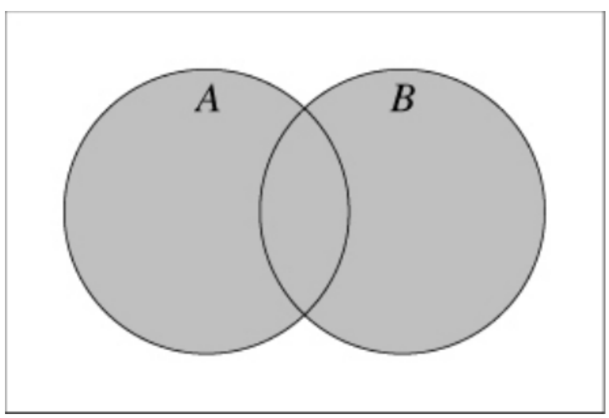
   The area of the circle represents the state space and hence it is a measure for the probability: non-equal probabilities means that the circles have a different radius.

6. How do you simulate `Pr(A \cap B)` if `Pr(A)=0.4` and `Pr(B)=.2`?

```
n <- 100000
prob <- c(0.4,0.2) # probability to get heads in A, B
A <- rbinom(n, 1, prob[1])
B <- rbinom(n, 1, prob[2])
mean(A & B) # probability to get heads for A and B
```

```
[1] 0.08144
```

7. What is `Pr(A \cup B)` (OR) as a Venn diagram?



8. What is `Pr(A \cup B)` for mutually exclusive events `A` and `B`?

   `Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B)`

9. How do you simulate `Pr(A \cup B)` if `Pr(A) = 0.2` and `Pr(B) = 0.6`?

```
n <- 100000
prob <- c(0.2,0.6) # probability to get heads in A, B
A <- rbinom(n, 1, prob[1])
B <- rbinom(n, 1, prob[2])
mean(A | B) # probability to get heads for A or B (or both)
```

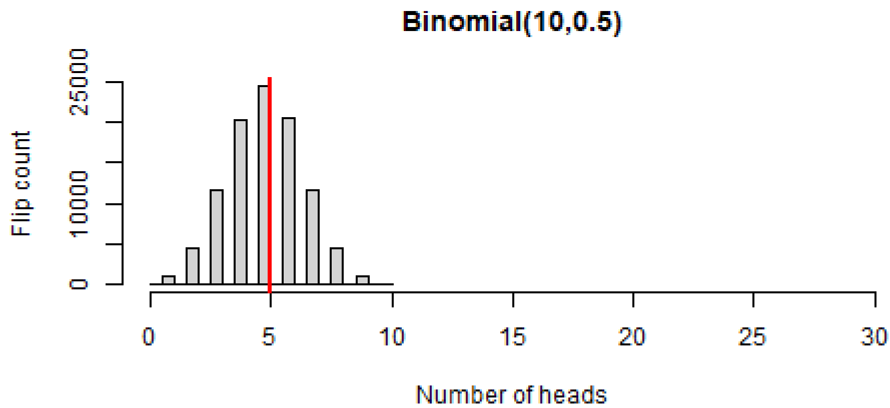
```
[1] 0.67866
```

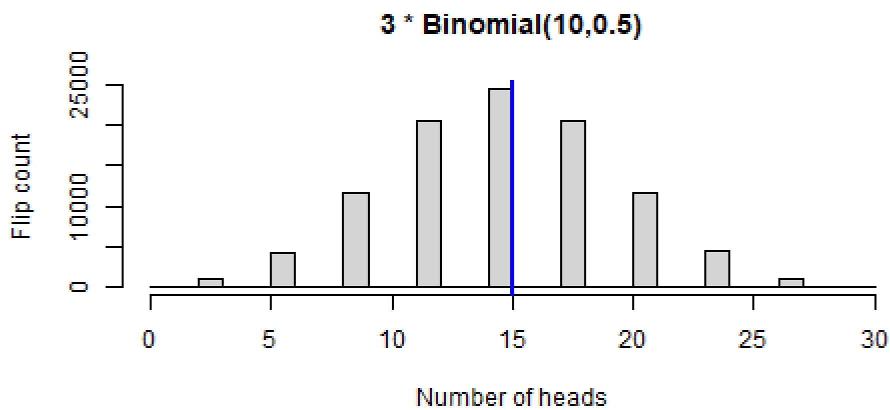
10. Multiplying a random variable by a factor will also multiply the expected value by that factor.

    Create a histogram of `X \tilde Binomial(10,0.5)` and a histogram `Y\tilde3*X`, put them into one plot array, and show the expected value in each histogram.

    Remember that the x-axis scales for each plot are chosen based on the plot data - you have to align them to make the plots comparable.

```
par(mfrow=c(2,1))
flips_X <- rbinom(100000,10,.5)
hist(flips_X,
     main="Binomial(10,0.5)",
     breaks=30,
     xlab="Number of heads",
     ylab="Flip count",
     xlim=c(0,30))
abline(v=mean(flips_X),col="red", lwd=2)
flips_Y <- 3 * rbinom(100000,10,.5)
hist(flips_Y,
     main="3 * Binomial(10,0.5)",
     breaks = 30,
     xlab="Number of heads",
     ylab="Flip count",
     xlim=c(0,30))
abline(v=mean(flips_Y),col="blue", lwd=2)
```





11. The multiplication rule for random variables also applies to the spread. Show how the spread changes from X to Y using a function.

```
var(flips_Y)-var(flips_X)
var(flips_X)
var(flips_Y)
var(flips_X) * 3^2  # var(k * X) = var(x) * k^2
```

```
[1] 20.17118
[1] 2.506824
[1] 22.678
[1] 22.56141
```

12. What is the expected value of the binomial distributions X + Y? Print the expected value of X, Y and X+Y

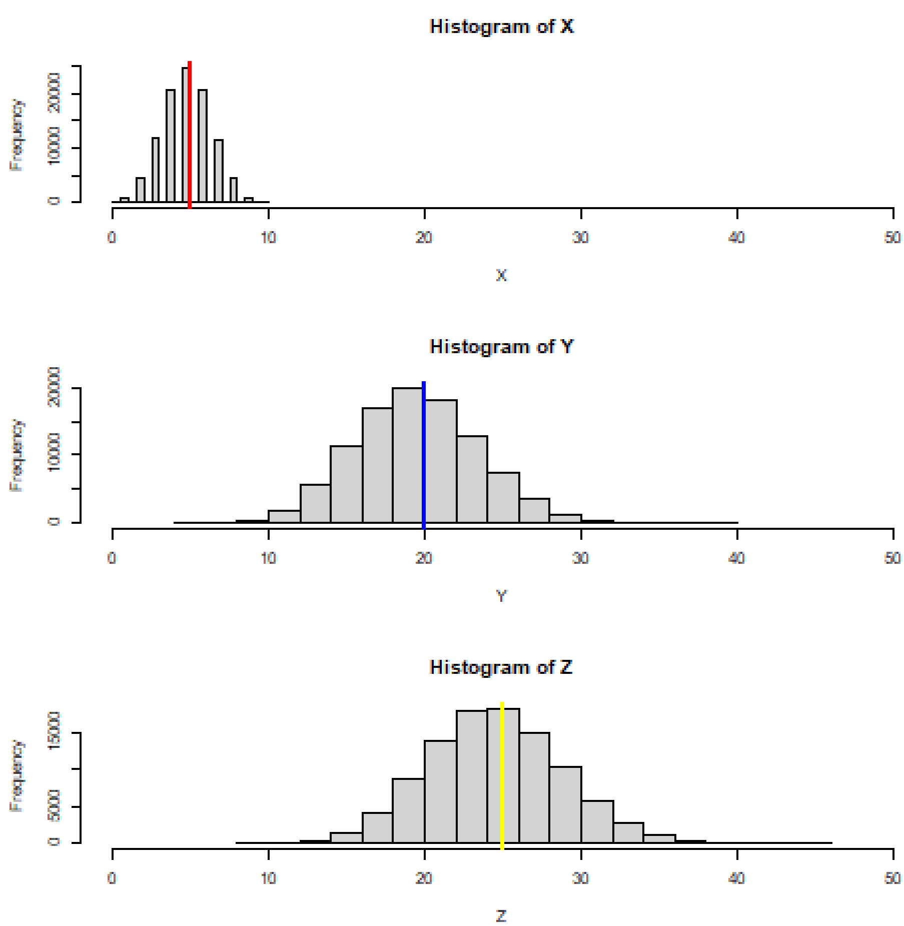
```
X <- rbinom(100000,10,.5)
mean(X)
Y <- rbinom(100000,100,.2)
mean(Y)
Z <- X + Y
mean(Z)
```

```
[1] 4.99982
[1] 19.99409
[1] 24.99391
```

13. Can you put the three distributions X, Y, and X + Y into one plot array to make them comparable, and indicate the expected values with thick red, blue and yellow vertical lines?

    Remember that the x-axis scales for each plot are chosen based on the plot data - you have to align them to make the plots comparable.

```
par(mfrow=c(3,1))
X <- rbinom(100000,10,.5)
hist(X,xlim=c(0,50))
abline(v=mean(X), col="red",lwd=2)
Y <- rbinom(100000,100,.2)
hist(Y,xlim=c(0,50))
abline(v=mean(Y), col="blue",lwd=2)
Z <- X + Y
hist(Z,xlim=c(0,50))
abline(v=mean(Z), col="yellow",lwd=2)
```



14. Do expected value and variance follow simple additive laws if the underlying distributions are not independent? Why or why not?

    Only the expected value is still additive if the distributions are not independent, while the variance is no longer additive.

    The reason is that the variance contains an explicit reference to the mean while the expected value does not.

    The mean is affected because without independence, we cannot simply compare the centrality of the two distributions. It's as if some of the points in a plot were from another function. Before comparing, you have to map the distributions on a third to make them measureable together.

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