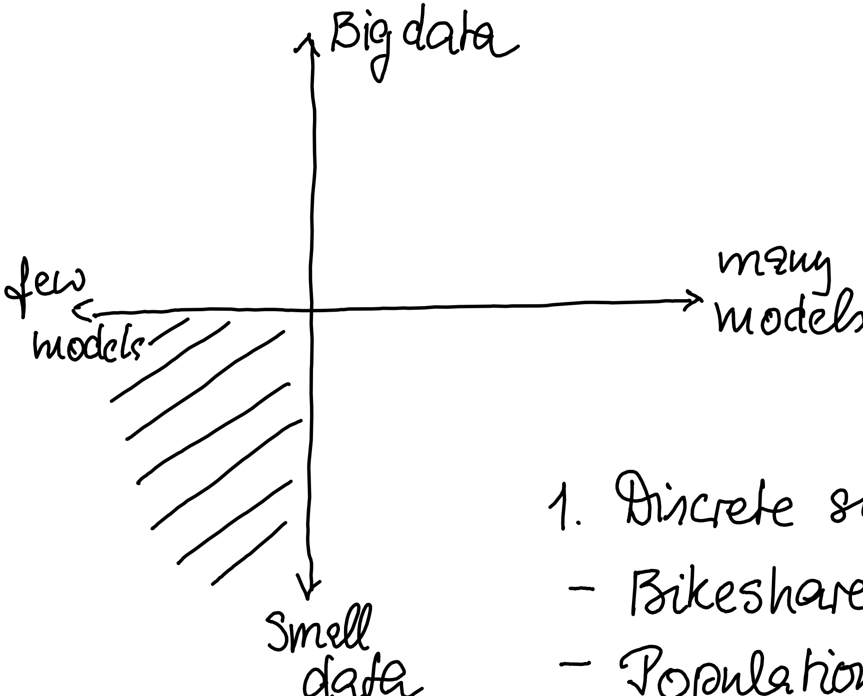
Modeling & simulation (FA23)



1. Discrete systems*:
   - Bikeshare model
   - Population model

* the dependent variable (number of bikes, world population) was discrete, not continuous ⇒ difference, not differential equations

2. Monte Carlo simulation ⇒ physics
3. Hidden Markov Models ⇒ cybersecurity

Why do we bother with discrete systems?

1) Though they're simple, they help us understand more complex systems
2) They compute easily $\Rightarrow$ cost effective
3) To develop performance metrics (like "consumer happiness") useful to optimize systems.
4) Decision support (parameter tuning)

What's next?

1) First order systems. Example: Newton's Law of Cooling

$$\frac{dT}{dt} = -r(T - T_0) \quad \begin{cases} T: \text{temp.} \\ T_0: \text{environment temp.} \end{cases}$$

r is a heat transfer coefficient (const).

2) Second order systems, e.g. Newton's 2nd law of motion: $F = ma$, or

$$\frac{d^2y}{dt^2} = a = F/m \quad \begin{cases} \text{second order} \\ \text{diff. equation} \end{cases}$$

Further study:

- Bender, EA: An introduction to Mathematical modeling (1978) ← Dover classic

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Ad 2. The core idea of MC is to use random sampling to solve deterministic (non-stochastic, non-probabilistic) problems.

Running an MC many times generates a distribution of possible outcomes, which can be used to assess statistical properties of a system.

Biggest constraint: quality of random numbers!

Further reading:

Sobol, A Primer for the Monte Carlo Method (1994)

Ad 3. Hidden Markov Models

why?

B/c the state of the system is not directly observable.

HMMs describe "Markov processes" where the future states of the process depend only on its current state, and not on how it arrived there (the history) — you only see the process results but not the process itself.

Useful whenever the data are sequential (language, time series, genomics etc.)

Further study:

Dover classic

Bharucha-Reid, Elements of the Theory of Markov Processes & their Applications (1960)

Lastly, here is a brand new book on stochastic modeling by a machine learning expert and author whom I like very much – he writes exceptionally well:

R. Kneusel, No Starch 2024: The Art of Randomness.

Includes Markov Chain MC and a bunch of fun applications.

To appear Jan '24 but you can get it now (I have it).

Cheers!