

MTH 101: Calculus I

Nikita Agarwal

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Problem Set for Week 4 (Part 1)

Problems

For the sets $A_1 - A_7$, find a lower bound, an upper bound, supremum, infimum (if they exist). Also argue whether they are bounded or not.

1. $A_1 = \mathbb{Z}$.
2. $A_2 = \{\pi + 1/n : n \in \mathbb{N}\}$.
3. $A_3 = \{\pi - n : n \geq 1\}$.
4. $A_4 = \{x \in \mathbb{R} : |x^2 - 3| > 1\}$.
5. $A_5 = \{1 + 1/n^2 : n \in \mathbb{N}\}$.
6. $A_6 = \left\{ \frac{1}{3} \pm \frac{n}{3n+1} : n \in \mathbb{N} \right\}$.
7. $A_7 = \left\{ \frac{m+n}{mn} : m, n \in \mathbb{N} \right\}$.
8. Let A be a nonempty subset of \mathbb{R} which is bounded. Suppose $\inf(A) = \sup(A)$. What can you say about the set A ?
(Hint: Can it have more than one element?)
9. Use the Archimedian property to prove that for any real number x , there exists a natural number n such that $n > x$.
10. Use the Archimedian property to prove that for any real number $x > 0$, there exists a natural number n such that $1/n < x$.