

MTH 101: Calculus I

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Problem Set for Week 4 (Part 2)

Problems

1. By induction on n , prove that for all real numbers x_1, x_2, \dots, x_n ,
 $|x_1 + x_2 + \dots + x_n| \leq |x_1| + |x_2| + \dots + |x_n|.$
2. By induction on n , prove that for all real numbers x_1, x_2, \dots, x_n ,
 $|x_1 + x_2 + \dots + x_n| \geq |x_1| - |x_2| - \dots - |x_n|.$
3. Consider the sequence $x_n = 1/n$, for $n \in \mathbb{N}$. Take it as a fact that x_n converges to 0. For various values of ϵ give below, produce a cut-off N as in the definition of convergence of a sequence.
 - a) $\epsilon = 1/10$.
 - b) $\epsilon = 1/100$.
 - c) $\epsilon = 1$.
 - d) $\epsilon = 2/99$.
4. Consider the sequence $x_n = 1/n^2$, for $n \in \mathbb{N}$. Take it as a fact that x_n converges to 0. For various values of ϵ give below, produce a cut-off N as in the definition of convergence of a sequence.
 - a) $\epsilon = 1/25$.
 - b) $\epsilon = 1/100$.
 - c) $\epsilon = 1$.
 - d) $\epsilon = 2/99$.

5. Consider the sequence $x_n = \frac{(-1)^n}{10}$, for $n \in \mathbb{N}$. List the first 10 terms of the sequence.

For various values of x, ϵ give below, if you can, produce a cut-off N such that $x_n \in (x - \epsilon, x + \epsilon)$.

- a) $x = 0, \epsilon = 1$.
- b) $x = 0, \epsilon = 1/5$.
- c) $x = 0, \epsilon = 1/100$.
- d) $x = 1/10, \epsilon = 1$.
- e) $x = 1/10, \epsilon = 1/5$.

Plotting interval $(x - \epsilon, x + \epsilon)$ on the number line and analyzing the terms of the sequence is helpful. Try!