MTH 101: Calculus I

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Tutorial 1 (combined)
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Negation

- Consider a statement S.
- The negation of S, denoted by not-S, is a statement which is TRUE when S is FALSE, and is FALSE when S is TRUE.
- Often, to prove that a mathematical statement S is true, it is easier to work with not-S. We consider the negation not-S of S and prove that it is false.
  For example, to prove

  S: There are infinitely many prime numbers,

  we consider

  not-S: There are finitely many prime numbers,

  and prove that not-S is false, which is easier than to prove that S is true.

Problem 1

Write the negation of the following statement?

- S: The bakery shop is closed today.
- not-S: The bakery shop is not closed today.
- Note that you could also write

  not-S: The bakery shop is open today,

  since open and closed are opposites of each other. So you can recover S by taking not-not-S.

Problem 2

Write the negation of the following statement?

- S: $\sqrt{9} = 4$.
- not-S: $\sqrt{9} \neq 4$.
- Note that the negation of S is NOT

  $\sqrt{9} = \pm 3$,

  since then you cannot recover S but taking negation of $\sqrt{9} = \pm 3$.

Problem 3

Identify the quantifier, set of context, and the property in the given statement. Write it mathematically using symbols. Also write its negation.

- S: Some of my shoes are torn.
- Quantifier: There exists (existential)
  Set of context X: collection of my shoes
  Property P: torn
- Can be written mathematically as:

  S: $\exists x \in X(x \text{ is torn})$
- Negation is:

  not-S: Given any of my shoe, it is not torn.
- Mathematically,

  not-S: $\forall x \in X(x \text{ is not torn})$

Problem 4

Identify the quantifier, set of context, and the property in the given statement. Write it mathematically using symbols. Also write its negation.

- S: Every book in my bookshelf is a fiction.
- Quantifier: For all (universal)
  Set of context X: Books in my bookshelf
  Property P: fiction
- Can be written mathematically as:

  S: $\forall x \in X(x \text{ is a fiction})$
- Negation is:

  not-S: There exists a book in my bookshelf which is not a fiction.
- Mathematically,

  not-S: $\exists x \in X(x \text{ is not a fiction})$.

Problem 5

Identify the quantifier, set of context, and the property in the given statement. Write it mathematically using symbols. Also write its negation.

- S: All prime numbers are odd.
- Quantifier: For all (universal)
  Set of context X: Set of prime numbers
  Property P: odd
- Can be written mathematically as:

  S: $\forall x \in X(x \text{ is odd})$
- Negation is:

  not-S: There is a prime number which is not odd.
- Mathematically,

  not-S: $\exists x \in X(x \text{ is not odd})$.

Problem 6 – Multiple Quantifiers

Identify the quantifiers, sets of context, and the property in the given statement. Write it mathematically using symbols. Also write its negation.

- S: There is a shelf in the library which has a mathematics book.
- The statement S has multiple quantifiers: 'there is', 'has a' (both existential)
- First consider:
  Quantifier: There is, Set of context X: Shelfs in the library, Property P: Has a mathematics book.
- Mathematically, S: $\exists x \in X$(x has a mathematics book)
- Now "x has a mathematics book" is itself a statement with a quantifier:
  Quantifier: There is, Set of context B_x: Books on the shelf x, Property P: mathematics book.
- Hence, S: $\exists x \in X(\exists y \in B_x$(y is a mathematics book)).
- Negation not-S: Given any shelf in the library, it does not have a mathematics book.
- Mathematically, not-S: $\forall x \in X(\forall y \in B_x$(y is not a mathematics book)).

Problem 7 – Multiple Quantifiers – order matters

- Consider the following statement:
  S: For every integer x, there is an integer y such that $y > x$.
- Observe that y depends on x in S.
- Mathematically,
  S: $\forall x \in X(\exists y \in X(y > x))$,
  where X denote the set of integers.
- Suppose we interchange the order of quantifiers, what happens? That is, we interchange $\forall x \in X$ and $\exists y \in X$.
- $\exists y \in X(\forall x \in X(y > x))$. Is this statement same as S?
- NO! Clearly, there does not exist any integer y such that all integers x satisfy $y > x$.
  In fact, taking x to be y violates $y > x$.
- Note that: the negation of S is
  "There exists an integer x such that $y \leq x$ for every integer y".
  Mathematically, not-S: $\exists x \in X(\forall y \in X(y \leq x))$

Problem 8 – Negation of a compound statement

Identify whether the given compound statement is a conjunction or a disjunction. Negate the statement.

- Let A,B be subset of a set X. Consider the statement:
  S1: $x \notin A \cup B$.
  When is S true? When x lies in the complement of $A \cup B$. That is, x neither belongs to A nor to B.

- We can write:
  S1: $x \in (A \cup B)^c$,
  which can also be written as
  S1: $x \in A^c \cap B^c$,
  which can also be written as
  S1: $x \in A^c$ and $x \in B^c$.

- Hence S1 is a conjunction of two statements.

- Hence
  not-S1: $x \in A$ or $x \in B$.

Problem 9 – Negation of a compound statement

Identify whether the given compound statement is a conjunction or a disjunction. Negate the statement.

- Consider the statement:
  S1: The bag belongs to either Saloni or Devjyoti.
  When is S true? When the bag belongs to at least one of Saloni and Devjyoti.

- We can write:
  S1: The bag belongs to Saloni or the bag belongs to Devjyoti.

- Hence, S1 is a disjunction of two statements.

- Negation of S1 is:
  not-S1: The bag belongs to neither Saloni nor Devjyoti.

- Also,
  not-S1: The bag does not belong to Saloni and the bag does not belong to Devjyoti.

Problem 10 – Implications and their converse

This problem was not discussed in the tutorial.

- An implication is a statement of the form:

  "If S then T".

- Consider the implication:
  S1: If it is raining, then I will carry an umbrella.

- The converse of S1 is:
  If I will carry an umbrella, then it is raining.