MTH 101: Calculus I

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Problem Set for Tutorial 2 on January 11, 2025

Problems (Implications)

1. Write the converse of the following implication:
   "If it is a holiday, then I will not go to the class".

2. Write the following statement as a conjunction of two implications:
   "$|x| < a$ if and only if $x \in (-a, a)$".

3. Write the negation of the following implication:
   "For an integer n, if n^2 is divisible by 10, then n is divisible by 10".

4. Write the contrapositive of the following implication:
   "For an integer x, if $x^2 - 6x + 5$ is even, then x is odd".

5. Write the contrapositive of the following statement:
   S: "For an integer n, if $n^3 - 1$ is even, then n is odd".
   Prove S using its contrapositive.

6. Write the negation of the following statement:
   S: "There is no rational number x such that $x^2 = 2$".
   Prove S by contradiction.

7. Give a direct proof that the following statements are equivalent for two given sets A and B.
   (i) $A \subseteq B$.
   (ii) $A \cap B = A$.
   (iii) $A \cup B = B$.

   Prove $(i) \Rightarrow (ii)$, $(ii) \Rightarrow (iii)$ and $(iii) \Rightarrow (i)$.

8. Give a counterexample to disprove the following statement:
   "For real numbers x and y, $|x| > |y|$ if $x > y$".

Problems (Sets)

Prove the following statements:

9. Prove that $(0, 1)$ and $\mathbb{R}$ have the same cardinality.

10. If X, Y are countably infinite sets, then the set $X \cup Y$ is also a countably infinite set.

11. The set of rational numbers, $\mathbb{Q}$, is countably infinite.

12. The power set of $\mathbb{N}$, denoted as $\mathcal{P}(\mathbb{N})$, is uncountable. Moreover, $|\mathbb{N}| < |\mathcal{P}(\mathbb{N})|$.

13. If X is an uncountable set, then $|\mathbb{N}| < |X|$.

Use the Pigeonhole Principle to prove the following statements.

14. Among any 13 people, at least two share a birth month.

15. Suppose $a_1, \ldots, a_n$ are integers. Then some "consecutive sum" $a_k + a_{k+1} + a_{k+2} + \cdots + a_{k+m}$ is divisible by n.