Magnetic susceptibility tensor

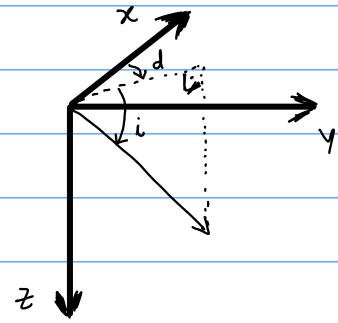
$$\mathbf{m} = \boldsymbol{\chi}\,\mathbf{h}$$

inducing field

magnetization

$$\boldsymbol{\chi} = \begin{bmatrix} \chi_{11} & \chi_{12} & \chi_{13} \\ \chi_{12} & \chi_{22} & \chi_{23} \\ \chi_{13} & \chi_{23} & \chi_{33} \end{bmatrix}$$

susceptibility tensor



$$\mathbf{h} = \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix} = h\,\hat{\mathbf{h}}$$

$$h = \left[h_1^2 + h_2^2 + h_3^2\right]^{1/2}$$

$$\hat{\mathbf{h}} = \begin{bmatrix} \cos i \cos d \\ \cos i \sin d \\ \sin i \end{bmatrix} = \begin{bmatrix} \hat{h}_1 \\ \hat{h}_2 \\ \hat{h}_3 \end{bmatrix}$$

unit vector defining the inducing field direction

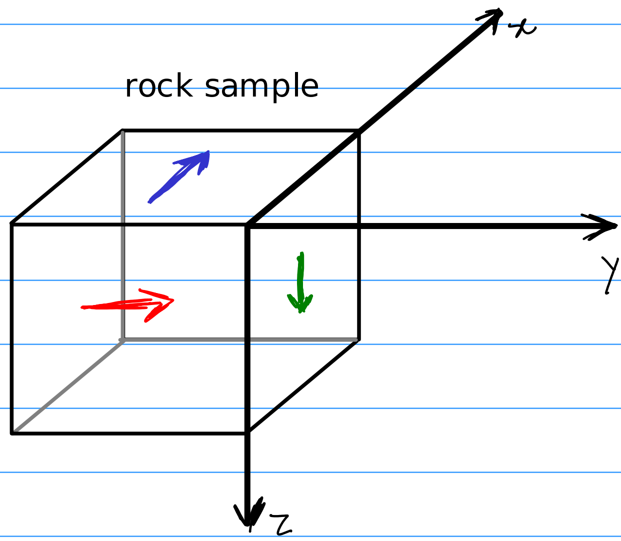
$$\mathbf{h}^T \boldsymbol{\chi}\, h\hat{\mathbf{h}} = \hat{\mathbf{h}}^T \mathbf{m}$$

$$\hat{\mathbf{h}}^T \boldsymbol{\chi}\, \hat{\mathbf{h}} = \sigma$$

$$\sigma = \frac{\hat{\mathbf{h}}^T \mathbf{m}}{h}$$

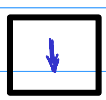
$$\hat{\mathbf{h}}^T \boldsymbol{\chi}\, \hat{\mathbf{h}} = \mathbf{a}^T \mathbf{p}$$

rock sample

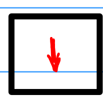


$$\mathbf{a} = \begin{bmatrix} \hat{h}_1^2 \\ \hat{h}_2^2 \\ \hat{h}_3^2 \\ 2\hat{h}_1\hat{h}_2 \\ 2\hat{h}_2\hat{h}_3 \\ 2\hat{h}_1\hat{h}_3 \end{bmatrix}_{6\times 1} \qquad \mathbf{p} = \begin{bmatrix} \chi_{11} \\ \chi_{22} \\ \chi_{33} \\ \chi_{12} \\ \chi_{23} \\ \chi_{13} \end{bmatrix}_{6\times 1}$$

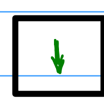
Consider the following inducing field directions:



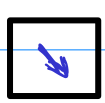
$i = 0°$, $d = 0°$ $\quad \hat{\mathbf{h}} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$



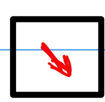
$i = 0°$, $d = 90°$ $\quad \hat{\mathbf{h}} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$



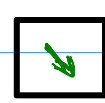
$i = 90°$, $d = 0°$ $\quad \hat{\mathbf{h}} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$



$i = 0°$, $d = 45°$ $\quad \hat{\mathbf{h}} = \begin{bmatrix} \sqrt{2}/2 \\ \sqrt{2}/2 \\ 0 \end{bmatrix}$



$i = 45°$, $d = 90°$ $\quad \hat{\mathbf{h}} = \begin{bmatrix} 0 \\ \sqrt{2}/2 \\ \sqrt{2}/2 \end{bmatrix}$



$i = 45°$, $d = 0°$ $\quad \hat{\mathbf{h}} = \begin{bmatrix} \sqrt{2}/2 \\ 0 \\ \sqrt{2}/2 \end{bmatrix}$

Then, we have

this matrix is called design matrix

$$A p = \sigma \qquad A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1/2 & 1/2 & 0 & 1 & 0 & 0 \\ 0 & 1/2 & 1/2 & 0 & 1 & 0 \\ 1/2 & 0 & 1/2 & 0 & 0 & 1 \end{bmatrix} \qquad \sigma = \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{bmatrix}$$

With the design matrix A and the data vector σ , we may solve the linear system for p and obtain an estimate for the susteptibility tensor χ .

Given a susceptibility tensor χ , we may compute its eigenvectors and eigenvalues:

$$\chi = \underbrace{V}_{\text{main susceptibility directions}} \underbrace{\Lambda}_{\text{main susceptibilities}} V^T , \qquad \Lambda = \begin{bmatrix} \tilde{\chi}_1 & 0 & 0 \\ 0 & \tilde{\chi}_2 & 0 \\ 0 & 0 & \tilde{\chi}_3 \end{bmatrix} , \qquad V^T V = V V^T = I$$

main susceptibilities

main susceptibility directions

* Hext, George R. “The Estimation of Second-Order Tensors, with Related Tests and Designs.” Biometrika 50, no. 3–4 (1963): 353–73. https://doi.org/10.1093/biomet/50.3-4.353.
* Tauxe, Lisa. Paleomagnetic Principles and Practice. Kluwer Academic Publishers, 2003.