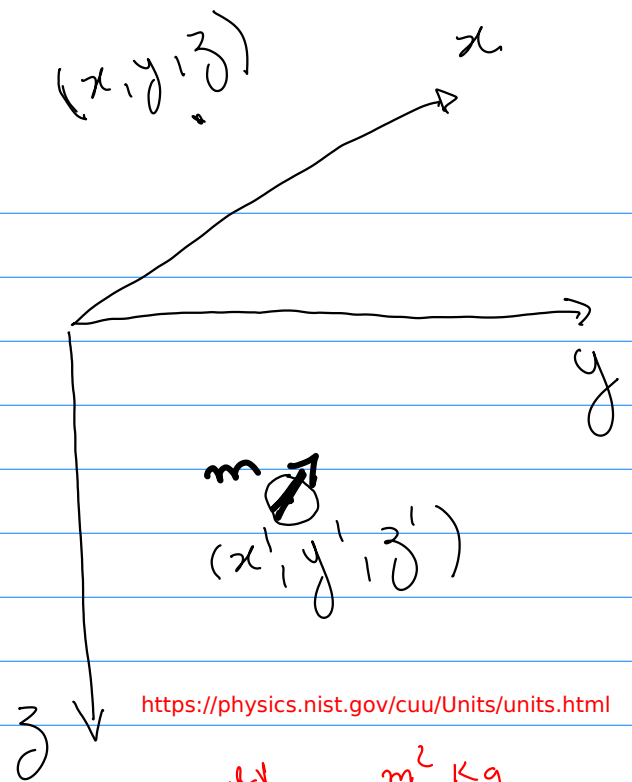


Dipolo

$$\mathcal{B}(x, y, z) ?$$

condições magnetostáticas, isto é
ausência de corrente elétrica
(Jackson, 1975, p 175, p 196)

JACKSON, J. D., 1975, Classical Electrodynamics. Chapman and Hall.
ISBN: 0-471-43132-X.



<https://physics.nist.gov/cuu/Units/units.html>

potencial magnético escalar

$$V(x, y, z) = -C_m \nabla \frac{1}{r} \cdot \mathbf{m} \quad \left[\frac{H A}{m} \right] = \left[\frac{m \cdot \frac{kg}{s^2 A}}{s^2 A} \right] = [Tm]$$

Henry $H = \frac{m^2 \cdot kg}{s^2 A^2}$
Tesla (T)

$$\mathbf{r} = \begin{bmatrix} x-x' \\ y-y' \\ z-z' \end{bmatrix} \quad r = \left[(x-x')^2 + (y-y')^2 + (z-z')^2 \right]^{1/2}$$

$$\nabla \frac{1}{r} = \begin{bmatrix} \frac{\partial}{\partial x} \frac{1}{r} \\ \frac{\partial}{\partial y} \frac{1}{r} \\ \frac{\partial}{\partial z} \frac{1}{r} \end{bmatrix}$$

$$\left[\frac{1}{m^2} \right]$$

$$\mathbf{m} = m \hat{\mathbf{m}} \quad \hat{\mathbf{m}} = \begin{bmatrix} \cos I' \cos D' \\ \cos I' \sin D' \\ \sin I' \end{bmatrix}$$

$$C_m = 10^9 \frac{\mu_0}{4\pi} \frac{H}{m}$$

$$m = \text{volume} \times \text{intensidade de magnetização total}$$

$[m^3]$

$$[A/m]$$

$$\nabla \frac{1}{r} = -\frac{1}{r^3} \mathbf{r}$$

indução magnética

$$\mathbf{B}(x, y, z) = -\nabla V(x, y, z) = - \begin{bmatrix} \partial_x V \\ \partial_y V \\ \partial_z V \end{bmatrix}$$

$$-\partial_x V = -\partial_x \left[-cm \left(\partial_x \frac{1}{r} m_x + \partial_y \frac{1}{r} m_y + \partial_z \frac{1}{r} m_z \right) \right]$$

$$\partial_x \frac{1}{r} = -\frac{1}{r^2} \frac{z(x-x')}{r^3} = -\frac{(x-x')}{r^3}$$

$$\partial_y \frac{1}{r} = -\frac{y-y'}{r^3} \quad \partial_z \frac{1}{r} = -\frac{z-z'}{r^3}$$

$$\nabla \frac{1}{r} = -\frac{1}{r^3} \mathbf{r}$$

$$\begin{aligned} \partial_x x \frac{1}{r} &= \left(-1 \cdot \frac{1}{r^3} \right) + -(x-x') \left(-\frac{3}{r^2} \right) \frac{1}{r^5} z(x-x') \\ &= \frac{3(x-x')^2}{r^5} - \frac{1}{r^3} \end{aligned}$$

$$\begin{aligned} \partial_x y \frac{1}{r} &= -(y-y') \left(-\frac{3}{r^2} \right) \frac{1}{r^5} z(x-x') \\ &= \frac{3(x-x')(y-y')}{r^5} \end{aligned}$$

$$- \partial_x V = \mu_m \left[\partial_{xx} \frac{1}{r} \quad \partial_{xy} \frac{1}{r} \quad \partial_{xz} \frac{1}{r} \right] m$$

$$- \partial_y V = \mu_m \left[\partial_{xy} \frac{1}{r} \quad \partial_{yy} \frac{1}{r} \quad \partial_{yz} \frac{1}{r} \right] m$$

$$- \partial_z V = \mu_m \left[\partial_{xz} \frac{1}{r} \quad \partial_{yz} \frac{1}{r} \quad \partial_{zz} \frac{1}{r} \right] m$$

$$E_x \quad \partial_{xy} \frac{1}{r} = \partial_{yx} \frac{1}{r}$$

$$E_x \quad \partial_{xx} \frac{1}{r} + \partial_{yy} \frac{1}{r} + \partial_{zz} \frac{1}{r}$$

$$E_x \quad \partial_x \frac{1}{r} = - \partial_{x'} \frac{1}{r}$$

$$\mathbf{B}(x, y, z) = \mu_m \begin{bmatrix} \partial_{xx} \frac{1}{r} & \partial_{xy} \frac{1}{r} & \partial_{xz} \frac{1}{r} \\ \partial_{xy} \frac{1}{r} & \partial_{yy} \frac{1}{r} & \partial_{yz} \frac{1}{r} \\ \partial_{xz} \frac{1}{r} & \partial_{yz} \frac{1}{r} & \partial_{zz} \frac{1}{r} \end{bmatrix} m$$

$$\mathbf{B}(x, y, z) = \mu_m \mathbf{H}(x, y, z, x', y', z') m$$

$$\left[T \right] \quad \left[\frac{H}{m} \right] \quad \left[\frac{1}{m^3} \right] \quad \left[A \cdot m^2 \right]$$

$$\frac{kg m^2}{s^2 A^2} \frac{1}{m} \frac{1}{m^3} A m^2 = \frac{kg}{A s^2} = T$$

$$\mathbf{F}_0 = \|\mathbf{F}_0\| \begin{bmatrix} \cos I_0 \cos D_0 \\ \cos I_0 \sin D_0 \\ \sin I_0 \end{bmatrix} \quad \|\mathbf{F}_0\| \gg \|\mathbf{B}\|$$

$$\Delta T(x, y, z) = \|\mathbf{F}_0 + \mathbf{B}\| - \|\mathbf{F}_0\|$$

$$\tilde{\Delta T}(x, y, z) = \hat{\mathbf{F}}_0^T \mathbf{B}(x, y, z)$$

$$= \sum \hat{\mathbf{F}}_0^T \mathbf{H}(x, y, z, x'_j, y'_j, z'_j) m_j$$

Ex: mostre que a anomalia de campo total aproximada é harmônica e a anomalia de campo total sem aproximação não é harmônica

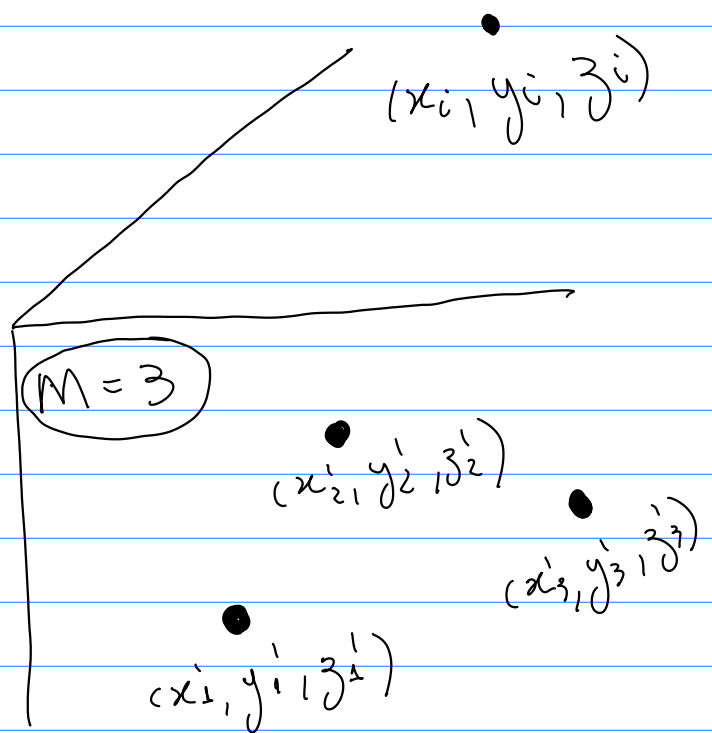
$$\mathbf{B}_{ij} \equiv \mathbf{B}(x_i, y_i, z_i) = \sum \mathbf{H}(x_i, y_i, z_i, x'_j, y'_j, z'_j) m_j$$

$$\mathbf{B}_{ij} = \sum \mathbf{H}_{ij} m_j$$

$$\mathbf{B}_i = \sum_{j=0}^{M-1} \mathbf{B}_{ij}$$

$$V_i = \sum_{j=0}^{M-1} V_{ij}$$

$$\tilde{\Delta T}_i = \sum_{j=0}^{M-1} \tilde{\Delta T}_{ij}$$



Ex: mostre que

$$1) \quad \nabla^2 \tilde{\Delta T}(x, y, z) = 0$$

$$2) \quad \nabla^2 \Delta T(x, y, z) \neq 0$$

$$1) \quad \tilde{\nabla} T(x, y, z) = F_x B_x(x, y, z) + F_y B_y(x, y, z) + F_z B_z(x, y, z)$$

$$B_x(x, y, z) = G_m \left[\partial_{xx} \frac{1}{r} \quad \partial_{xy} \frac{1}{r} \quad \partial_{xz} \frac{1}{r} \right] \cdot m$$

$$\nabla^2 B_x = 0$$

$$\partial_{xx} B_x = G_m \left[\partial_{xx} \left(\partial_{xx} \frac{1}{r} \right) + \partial_{xx} \left(\partial_{xy} \frac{1}{r} \right) + \partial_{xx} \left(\partial_{xz} \frac{1}{r} \right) \right] \cdot m$$

$$\nabla^2 \tilde{\Delta T} = G_m \left[\underset{\substack{\downarrow \\ = 0}}{F_x \nabla^2 B_x} + F_y \nabla^2 B_y + F_z \nabla^2 B_z \right]$$