GENERAL SOURCES

$$f(x,y,\overset{\text{fixed}}{z_0}) = \int_{z_0}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} s(x',y',z')\,\psi(x-x',y-y',\overset{\text{fixed}}{z_0}-z')\,dx'dy'dz'$$

$s(x',y',z')$ is null outside the integration volume

$$F[f(x,y,z)] = \int_{z_0}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} s(x',y',z')\,F[\psi(x-x',y-y',z_0-z')]\,dx'dy'dz'$$

$$F[\psi(x-x',y-y';z_0-z)] = F[\psi(x,y;z_0-z')]\,e^{-i(k_x x'+k_y y')}$$

$$F[f(x,y,z)] = \int_{z_0}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} s(x',y',z')\,F[\psi(x,y,z_0-z')]\,e^{-i(k_x x'+k_y y')}\,dx'dy'dz'$$

$$= \int_{z_0}^{\infty} F[\psi(x,y,z_0-z')] \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} s(x',y',z')\,e^{-i(k_x x'+k_y y')}\,dx'dy'dz'$$

$$= \int_{z_0}^{\infty} F[\psi(x,y,z_0-z')]\,\underbrace{F[s(z')]}_{\substack{\text{FOURIER transform of the function}\\ \text{s at a fixed z coordinate } z'}}\,dz'$$

GRAVITY SOURCES

$$g_z = \gamma \int_{z_0}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \rho(x',y',z')\,\psi(x-x',y-y',z_0-z')\,dx'dy'dz'$$

$$F[g_z] = \gamma \int_{z_0}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \rho(x',y',z')\,F[\psi(x,y,z_0)]\,e^{-i(K_x x' + K_y y')}\,dx'dy'dz' =$$

$$= \gamma \int_{z_0}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} 2\pi e^{|K|(z_0-z')}\,\underbrace{\rho(x',y',z')\,e^{-i(K_x x' + K_y y')}}\,dx'dy'dz' =$$

$$= 2\pi\gamma e^{|K|z_0} \int_{z_0}^{\infty} F[\rho(z')]\,e^{-|K|z'}\,dz'$$

MAGNETIC SOURCES

$$\Delta T(x,y,z_0) \approx -\hat{f}^{T}\,\overline{\nabla}_p V(x,y,z_0)$$

$$\approx -\hat{f}_x \frac{\partial V(x,y,z_0)}{\partial x} - \hat{f}_y \frac{\partial V(x,y,z_0)}{\partial y} - \hat{f}_z \frac{\partial V(x,y,z_0)}{\partial z}$$

$$V(x,y,z_0) = -C_m \int_{z_0}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} m_x(x',y',z')\frac{\partial}{\partial x}\frac{1}{r} + m_y(x',y',z')\frac{\partial}{\partial y}\frac{1}{r} + m_z(x',y',z')\frac{\partial}{\partial z}\frac{1}{r}\,dx'dy'dz'$$

$$F[V(x,y,z_0)] = -C_m \int_{z_0}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} m_x(x',y',z')\,iK_x \frac{2\pi e^{|K|(z_0-z')}}{|K|} e^{-i(K_x x'+K_y y')} + m_y(x',y',z')\,iK_y \frac{2\pi e^{|K|(z_0-z')}}{|K|} e^{-i(K_x x'+K_y y')} + m_z(x',y',z')\,2\pi e^{|K|(z_0-z')} e^{-i(K_x x'+K_y y')}\,dx'dy'dz'$$

$$\mathcal{F}\left[V(x,y,z_0)\right] =$$

$$= - C_m \int_{z_0}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} 2\pi \overbrace{\left\{ m_z(x',y',z') + i\,\frac{k_x m_x(x',y',z') + k_y m_y(x',y',z')}{|k|} \right\}}^{\Theta_m(x',y',z')} \times$$

$$\times\, e^{-i(k_x x' + k_y y')}\, e^{|k|(z_0 - z')}\, dx'\,dy'\,dz' =$$

$$= -2\pi C_m\, e^{|k| z_0} \int_{z_0}^{\infty} \mathcal{F}[\Theta_m(z')]\, e^{-|k| z'}\, dz'$$

$$\mathcal{F}\left[\Delta T(x,y,z_0)\right] = -\hat{f}_x\, i k_x\, \mathcal{F}\left[V(x,y,z_0)\right] - \hat{f}_y\, i k_y\, \mathcal{F}\left[V(x,y,z_0)\right] +$$

$$- \hat{f}_z \boxed{\frac{\partial}{\partial z}\mathcal{F}\left[V(x,y,z_0)\right]}\ \circledast$$

$\circledast$

$$\frac{\partial}{\partial z}\mathcal{F}\left[V(x,y,z)\right] = -2\pi C_m \int_{z_0}^{\infty} \mathcal{F}[\Theta_m(z')]\, e^{-|k| z'}\, dz'\, \frac{\partial}{\partial z} e^{|k| z} =$$

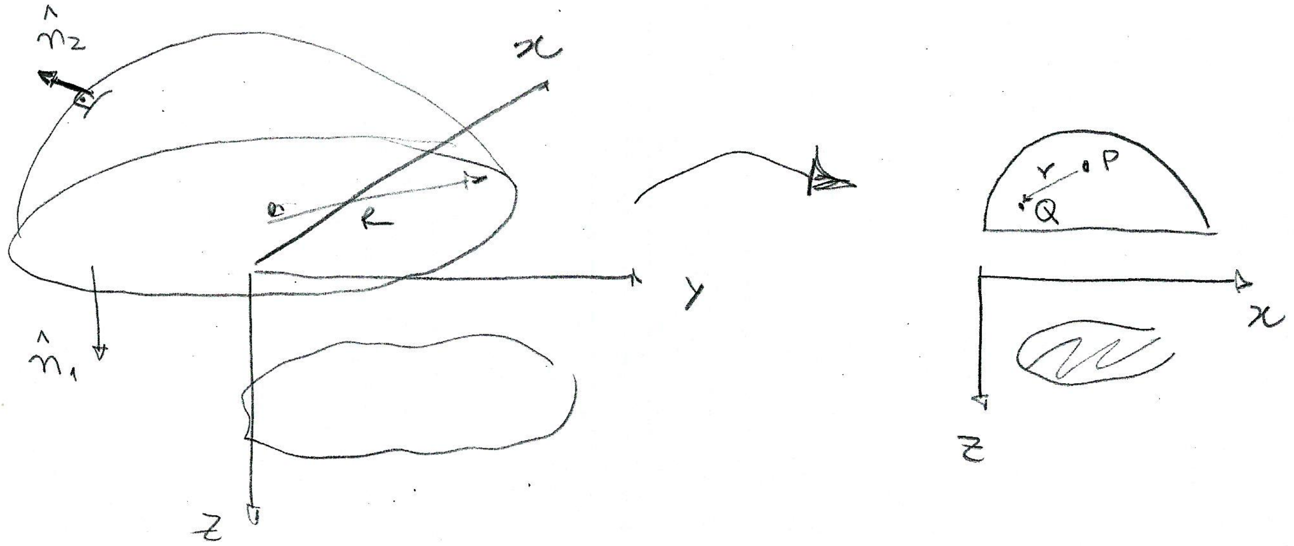
$$= |k| \underbrace{\left\{ -2\pi C_m\, e^{|k| z} \int_{z_0}^{\infty} \mathcal{F}[\Theta_m(z')]\, e^{-|k| z'}\, dz' \right\}}_{\mathcal{F}\left[V(x,y,z)\right]}$$

$$\mathcal{F}\left[\Delta T(x,y,z_0)\right] = 2\pi C_m \, e^{|k|z_0} \int_{z_0}^{\infty} \mathcal{F}\left[\Theta_m(z')\right] e^{-|k|z'} dz' \times$$

$$\times \, |k| \underbrace{\left\{ \hat{f}_z + i \frac{k_x \hat{f}_x + k_y \hat{f}_y}{|k|} \right\}}_{\Theta_f}$$

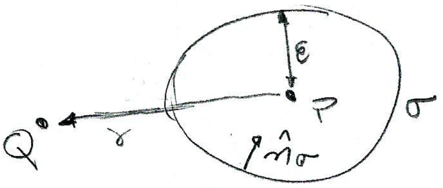
$$\boxed{\mathcal{F}\left[\Delta T(x,y,z_0)\right] = 2\pi C_m |k| \, e^{|k|z_0} \, \Theta_f \int_{z_0}^{\infty} \mathcal{F}\left[\Theta_m(z')\right] e^{-|k|z'} dz'}$$

Upward continuation



$$\iiint_V (u\nabla^2 v - v\nabla^2 u)\,dv = \iint_S \left(u\frac{\partial}{\partial n}v - v\frac{\partial}{\partial n}u\right) ds$$

$v = 1/r$



$$\frac{\partial}{\partial n_\sigma} = -\frac{\partial}{\partial r}$$

$$\iint_\sigma u\left(-\frac{\partial}{\partial r}\frac{1}{r}\right) - \frac{1}{r}\left(-\frac{\partial}{\partial r}u\right) ds =$$

$$= \int_0^{2\pi}\int_0^{\pi}\left(\frac{u}{\varepsilon^2} + \frac{1}{\varepsilon}\frac{\partial u}{\partial r}\right)\varepsilon^2\,\text{sen}\,\theta\,d\theta\,d\phi =$$

$$\boxed{\begin{gathered}\int_0^{\pi}\text{sen}\,\theta\,d\theta = \\ -\left[\cos\theta\right]_0^{\pi} = -\left[-1-(1)\right] = \\ = -(-2) = 2\end{gathered}}$$

$$= \underbrace{\int_0^{2\pi}\int_0^{\pi} u\,\text{sen}\,\theta\,d\theta\,d\phi}_{4\pi\tilde{u}} + \int_0^{2\pi}\int_0^{\pi}\varepsilon\frac{\partial u}{\partial r}\,d\theta\,d\phi$$

$$\iiint_{v} -\frac{1}{r}\nabla^2 u \, dv = 4\pi u(P) + \iint_{S}\left(u\frac{\partial}{\partial n}\frac{1}{r} - \frac{1}{r}\frac{\partial u}{\partial n}\right) ds$$

$$u(x,y,z_0) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{(d-z_0)\, u(x',y',d)}{[(x-x')^2+(y-y')^2+(z_0-d)^2]^{3/2}}\, dx'\, dy'$$

$$z_0 - d = -\Delta z$$

$$F[u(x,y,z_0)] = F[u(x,y,z_0+\Delta z)] \cdot F[x,y,z_0+\Delta z)]$$

$$\psi[u(x,y,z_0)) = \frac{1}{2\pi}\frac{-\Delta z}{(x^2+y^2+\Delta z^2]^{3/2}} = -\frac{1}{2\pi}\frac{\partial}{\partial \Delta z}\frac{1}{r}$$

$$F(\psi(x,y,z_0+\Delta z)] = -\frac{1}{2\pi}\frac{\partial}{\partial z}\left[2\pi\frac{e^{-|K|\Delta z}}{|K|}\right] =$$

$$= e^{-|K|\Delta z}, \quad \Delta z > 0$$

$$F[u(x,y,z_0)] = e^{-|K|\Delta z} F[u(x,y,z_0+\Delta z)]$$

Reduction-to-the-pole

$$\mathcal{F}\left[\Delta T(x,y,z_0)\right] = 2\pi C_m |K| e^{|K| z_0} \Theta_f \Theta_m \int_{z_0}^{\infty} \mathcal{F}[m(z')]\, e^{-|K| z'} dz'$$

direção de magnetização constante

$$\mathcal{F}\left[\Delta T^*(x,y,z_0)\right] = \mathcal{F}\left[\Delta T(x,y,z_0)\right] \mathcal{F}\left[\psi^*\right]$$

$$\mathcal{F}\left[\psi^*\right] = \frac{\Theta_f^* \Theta_m^*}{\Theta_f \Theta_m}$$

$$\Theta_f = \hat{f}_z + i \frac{k_x \hat{f}_x + k_y \hat{f}_y}{|K|}$$

$$\Theta_m = \hat{m}_z + i \frac{k_x \hat{m}_x + k_y \hat{m}_y}{|K|}$$