Geocentric Cartesian Coordinates/System (GCC/S)
" Geodetic " (GGC/S)
" Spherical " (GSC/S)
Topocentric Cartesian " (TCC/S)

GGC → GCC

$$X = (N+h)\cos\varphi\cos\lambda$$
$$Y = (N+h)\cos\varphi\sin\lambda$$
$$Z = [N(1-e^2)+h]\sin\varphi$$

prime vertical radius of curvature

$$N = a/\sqrt{1-e^2\sin^2\varphi}$$

$$e^2 = (a^2-b^2)/b^2$$

a : major semiaxis

b : minor semiaxis

GCC → GGC

Hirvonen-Moritz algorithm

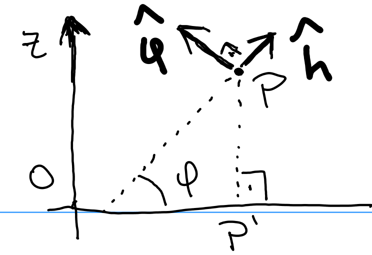
input : $X, Y, Z, a, b,$ itmax

k = 0

- $p = \sqrt{X^2+Y^2}$
- $\varphi_0 = \tan^{-1}\left(\dfrac{Z}{p(1-e^2)}\right)$
- $N_0 = a/\sqrt{1-e^2\sin^2\varphi_0}$
- $h_0 = \dfrac{p}{\cos\varphi_0} - N_0$

for k = 1: itmax

- $\varphi = \tan^{-1}\left[\dfrac{Z}{p}\left(1-e^2\dfrac{N_0}{N_0+h_0}\right)^{-1}\right]$
- $N = a/\sqrt{1-e^2\sin^2\varphi}$
- $h = \dfrac{p}{\cos\varphi} - N$
- $\varphi_0 \leftarrow \varphi$
- $N_0 \leftarrow N$
- $h_0 \leftarrow h$



Unit vectors relating GCC and GGC

$$\hat{h}(h,\varphi,\lambda) = \begin{bmatrix} \cos\varphi\cos\lambda \\ \cos\varphi\sin\lambda \\ \sin\lambda \end{bmatrix}$$

- direction of increasing h
- outward normal of ellipsoid

$$\hat{\varphi}(h,\varphi,\lambda) = \begin{bmatrix} -\sin\varphi\cos\lambda \\ -\sin\varphi\sin\lambda \\ \cos\varphi \end{bmatrix}$$

- direction of increasing φ
- tangent to meridian plane
- " " ellipsoid

$$\hat{\lambda}(h,\varphi,\lambda) = \begin{bmatrix} -\sin\lambda \\ \cos\lambda \\ 0 \end{bmatrix}$$

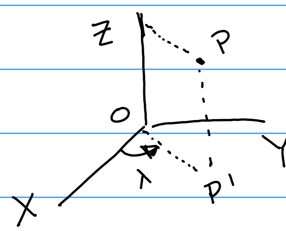
- direction of increasing λ
- tangent to Ellipsoid

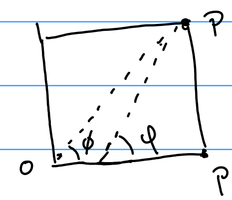
GSC → GCC

$$X = r\cos\phi\cos\lambda$$
$$Y = r\cos\phi\sin\lambda$$
$$Z = r\sin\phi$$

GCC → GSC

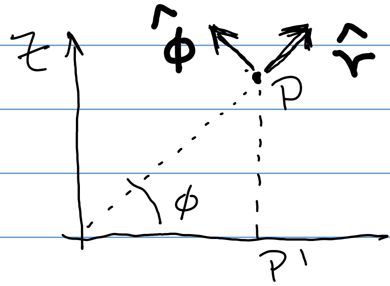
$$r = \sqrt{X^2 + Y^2 + Z^2}$$
$$\phi = \sin^{-1}\left(\frac{Z}{r}\right)$$
$$\lambda = \tan^{-1}\left(Y/X\right)$$





$$\tan\varphi = \frac{a^2}{b^2}\tan\phi$$

Unit vectors relating GSC and GCC



$$\hat{r}(r,\phi,\lambda) = \begin{bmatrix} \cos\phi\cos\lambda \\ \cos\phi\sin\lambda \\ \sin\phi \end{bmatrix} \quad \hat{\phi}(r,\phi,\lambda) = \begin{bmatrix} -\sin\phi\cos\lambda \\ -\sin\phi\sin\lambda \\ \cos\phi \end{bmatrix} \quad \hat{\lambda}(r,\phi,\lambda) = \begin{bmatrix} -\sin\lambda \\ \cos\lambda \\ 0 \end{bmatrix}$$

TCC

- Origin at a point Q
- Q may be defined with GCC, GGC or GSC

Consider two points defined with GCC

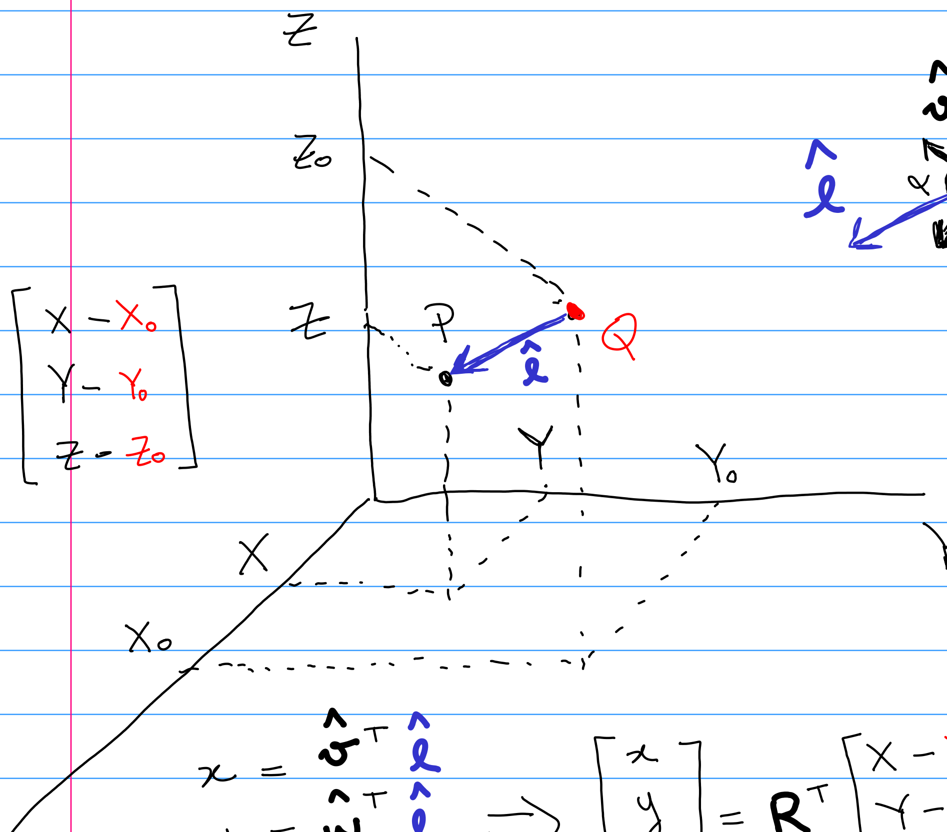
$Q = (X_0, Y_0, Z_0) \quad P = (X, Y, Z)$

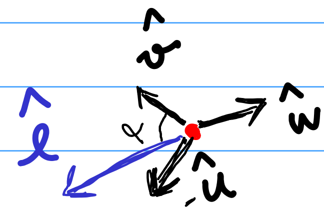
$\hat{v} = \hat{\varphi}$ or $\hat{\Phi}$

$\hat{w} = \hat{\lambda}$

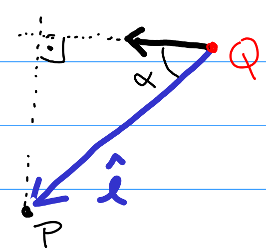
$\hat{u} = \hat{h}$ or $\hat{r}$

$$\hat{l} = \begin{bmatrix} X - X_0 \\ Y - Y_0 \\ Z - Z_0 \end{bmatrix}$$





$\|\hat{v}^T \hat{l}\|$



$$x = \hat{v}^T \hat{l}$$
$$y = \hat{w}^T \hat{l}$$
$$z = \hat{u}^T \hat{l}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \mathbf{R}^T \begin{bmatrix} X - X_0 \\ Y - Y_0 \\ Z - Z_0 \end{bmatrix}$$

$\mathbf{R}^T = \mathbf{R}^{-1}$ ok?

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix} + \mathbf{R} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$