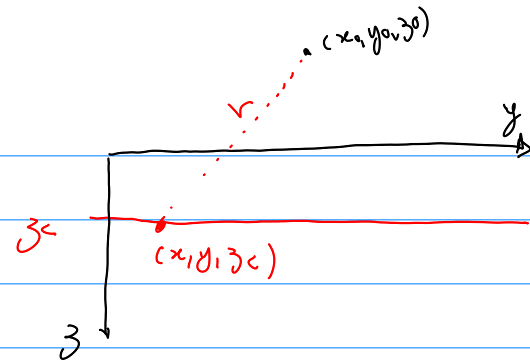
Application of the Dirichlet problem (Upward continuation integral)



$$U(x_0,y_0,z_0) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} U(x,y,z_c)\left(-\partial_z \frac{1}{r}\right) dx\,dy$$

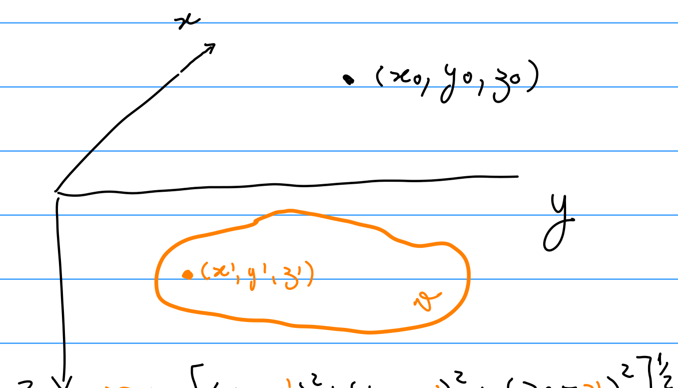
$$\partial_z \frac{1}{r} = \left(-\frac{z_0 - z_c}{r^3}\right)(-1) = -\partial_{z_0}\frac{1}{r}$$

$$U(x_0,y_0,z_0) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p(x,y,z_c)\,\overbrace{\partial_z \frac{1}{r}}^{\text{vertical component of the gravitational attraction produced by a point of mass}}\,dx\,dy \quad , \qquad p(x,y,z_c) = \frac{U(x,y,z_c)}{2\pi}$$

Consider that $U(x_0,y_0,z_0)$ represents the gravity disturbance $\delta g(x_0,y_0,z_0)$

$$\delta g(x_0,y_0,z_0) = G \iiint_v \overset{\text{density}}{\rho(x',y',z')}\; \partial_{z_0} \frac{1}{r}\, dv$$

derivatives with respect to the coordinates of the observation point



gravity disturbance is harmonic outside the sources

$$\nabla^2 \delta g(x_0,y_0,z_0) = G \iiint_v \rho(x',y',z')\, \partial_{z_0}\Big(\underbrace{\cancel{\nabla^2 \frac{1}{r}}}_{=0}\Big)\, dv = 0$$

$$r = \left[(x_0-x')^2 + (y_0-y')^2 + (z_0-z')^2\right]^{1/2}$$

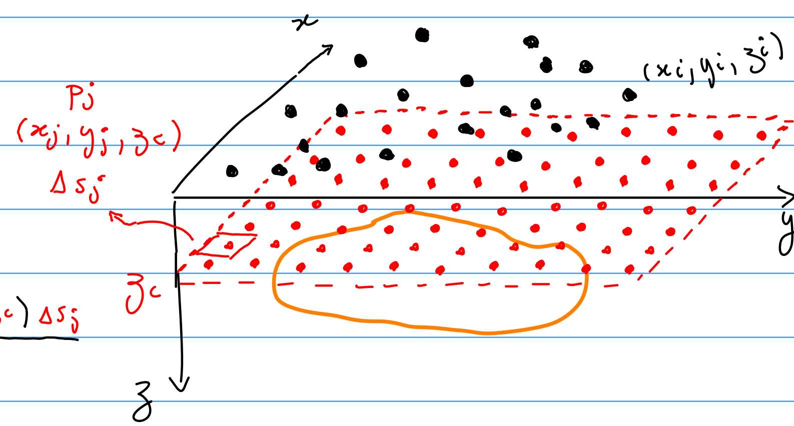
Consider now a set of gravity disturbance values at coordinates (x_i, y_i, z_i) , $z_i < z_c \ \forall z_i$

$$\delta g_i^o \equiv \delta g^o(x_i,y_i,z_i) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p(x,y,z_c)\,\partial_z \frac{1}{r}\, dx\,dy \quad , \quad p(x,y,z_c) = \frac{\delta g(x,y,z_c)}{2\pi}$$

$$\delta g_i^o \approx \sum_{j=1}^{M} p_j\, h_{ij}$$

$$r_{ij} = \left[(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_c)^2\right]^{1/2}$$

$$h_{ij} = \partial_{z_0}\frac{1}{r_{ij}} = \frac{z_c - z_i}{r_{ij}^3} \qquad p_j \approx \frac{\delta g(x_j,y_j,z_c)\,\Delta s_j}{2\pi}$$



$$\delta g_1^o \approx p_1 h_{11} + \cdots + p_m h_{1m}$$

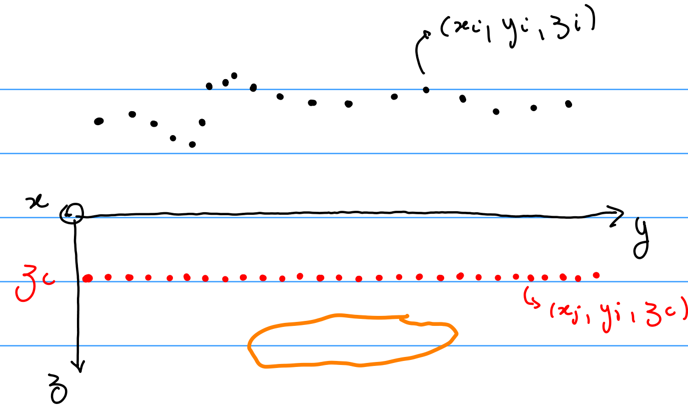
$$\vdots \qquad \vdots \qquad \vdots$$

$$\delta g_N^o \approx p_1 h_{N1} + \cdots + p_m h_{Nm}$$

$$\underbrace{\delta \mathbf{g}^o}_{N \times 1} \approx \underbrace{\mathbf{H}}_{N \times m} \; \underbrace{\mathbf{p}}_{m \times 1} \rightarrow p_j$$

equivalent layer

equivalent source



solve this linear system for estimating $\mathbf{p}$

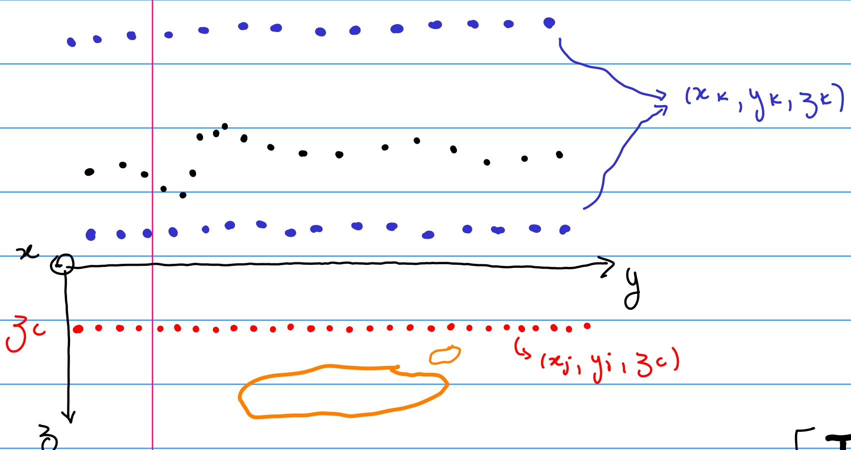
estimated gravity disturbance on the layer

$$\delta \mathbf{g}_{m \times 1} = 2\pi \, \mathbf{S}^{-1}_{m \times m} \, \mathbf{p}^*_{m \times 1}$$

estimated parameter vector

By estimating the parameter vector $\mathbf{p}^*$ we are actually computing a downward continuation of the harmonic function (in this case, the gravity disturbance)

$$\mathbf{S} = \begin{bmatrix} \Delta s_1 & & \\ & \ddots & \\ & & \Delta s_m \end{bmatrix}_{m \times m}$$



transformed data

transformation matrix

$$\underset{L \times 1}{\mathbf{t}} \approx \underset{L \times m}{\mathbf{T}} \; \underset{m \times 1}{\mathbf{p}^*}$$

upward/downward continuation

$$[\mathbf{T}]_{kj} = \frac{z_c - z_k}{r_{kj}^3}$$

z_k above/below z_i

vertical derivative

$$t_k = \partial_z \delta g_k \qquad [\mathbf{T}]_{kj} = \frac{3(z_c - z_k)^2}{r_{kj}^5} - \frac{1}{r_{kj}^3}$$

This application of the upward continuation integral with the gravity disturbance can be made with any harmonic function (i.e., gravitational potential and its derivatives, components of magnetic induction, approximated total-field anomaly, etc.)

References:

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