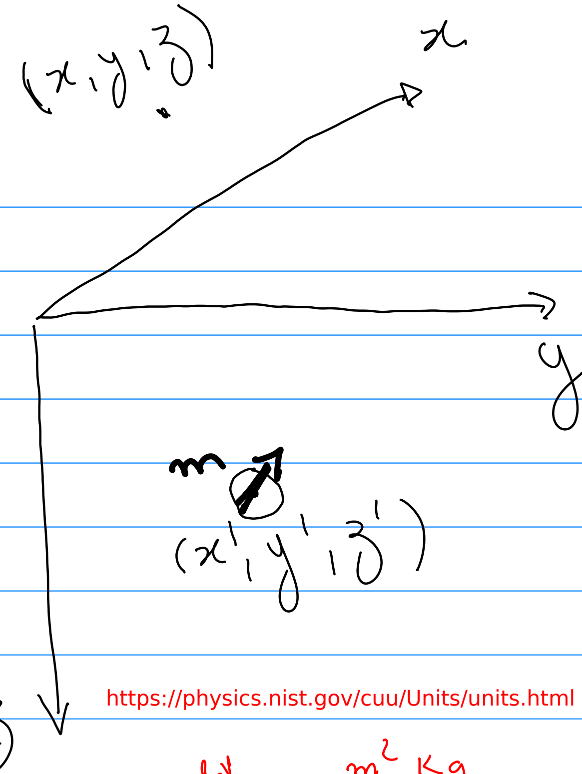
Dipolo



$$\mathbf{B}(x,y,z) \;?$$

condições magnetostáticas, isto é ausência de corrente elétrica (Jackson, 1975, p 175, p 196)

JACKSON, J. D., 1975, Classical Electrodynamics. Chapman and Hall.
ISBN: 0-471-43132-X.

https://physics.nist.gov/cuu/Units/units.html

HENRY $H = \dfrac{m^2\,Kg}{s^2\,A^2}$

TESLA (T)

potencial magnético escalar

$$V(x,y,z) = -C_m \, \nabla \frac{1}{r}^{T} \mathbf{m} \qquad \left[\frac{HA}{m}\right] = \left[\frac{m\,Kg}{s^2\,A}\right] = [Tm]$$

$$\mathbf{r} = \begin{bmatrix} x - x' \\ y - y' \\ z - z' \end{bmatrix} \qquad r = \left[(x-x')^2 + (y-y')^2 + (z-z')^2\right]^{1/2}$$

$$\nabla \frac{1}{r} = \begin{bmatrix} \partial_x \frac{1}{r} \\ \partial_y \frac{1}{r} \\ \partial_z \frac{1}{r} \end{bmatrix} \qquad \left[\frac{1}{m^2}\right]$$

$$\mathbf{m} = m\,\hat{\mathbf{m}}$$

$$\hat{\mathbf{m}} = \begin{bmatrix} \cos I' \cos D' \\ \cos I' \sin D' \\ \sin I' \end{bmatrix}$$

$$C_m = 10^9 \frac{\mu_0}{4\pi} \quad \frac{H}{m}$$

m = volume x intensidade magnetização total

$[m^3]$ $[A/m]$

$$\nabla \frac{1}{r} = -\frac{1}{r^3}\mathbf{r}$$

indução magnética

$$\mathbf{B}(x,y,z) = -\nabla V(x,y,z) = -\begin{bmatrix} \partial_x V \\ \partial_y V \\ \partial_z V \end{bmatrix}$$

$$-\partial_x V = -\partial_x \left[-c_m \left(\partial_x \frac{1}{r} m_x + \partial_y \frac{1}{r} m_y + \partial_z \frac{1}{r} m_z \right) \right]$$

$$\partial_x \frac{1}{r} = -\frac{1}{\cancel{2}} \frac{\cancel{2}(x-x')}{r^3} = -\frac{(x-x')}{r^3}$$

$$\partial_y \frac{1}{r} = -\frac{y-y'}{r^3} \qquad \partial_z \frac{1}{r} = -\frac{z-z'}{r^3}$$

$$\nabla \frac{1}{r} = \frac{-1}{r^3} \mathbf{r}$$

$$\partial_{xx} \frac{1}{r} = \left(-1 \cdot \frac{1}{r^3} \right) + -(x-x')\left(-\frac{3}{2}\right) \frac{1}{r^5} 2(x-x')$$

$$= \frac{3(x-x')^2}{r^5} - \frac{1}{r^3}$$

$$\partial_{xy} \frac{1}{r} = -(y-y')\left(-\frac{3}{2}\right) \frac{1}{r^5} 2(x-x')$$

$$= \frac{3(x-x')(y-y')}{r^5}$$

$$-\partial_x V = C_m \left[\partial_{xx}\frac{1}{r} \quad \partial_{xy}\frac{1}{r} \quad \partial_{xz}\frac{1}{r}\right] \mathbf{m}$$

$$-\partial_y V = C_m \left[\partial_{xy}\frac{1}{r} \quad \partial_{yy}\frac{1}{r} \quad \partial_{yz}\frac{1}{r}\right] \mathbf{m}$$

$$-\partial_z V = C_m \left[\partial_{xz}\frac{1}{r} \quad \partial_{yz}\frac{1}{r} \quad \partial_{zz}\frac{1}{r}\right] \mathbf{m}$$

Ex $\partial_{xy}\frac{1}{r} = \partial_{yx}\frac{1}{r}$

Ex $\partial_{xx}\frac{1}{r} + \partial_{yy}\frac{1}{r} + \partial_{zz}\frac{1}{r}$

Ex $\partial_x \frac{1}{r} = -\partial_{x'}\frac{1}{r}$

$$\mathbf{B}(x,y,z) = C_m \begin{bmatrix} \partial_{xx}\frac{1}{r} & \partial_{xy}\frac{1}{r} & \partial_{xz}\frac{1}{r} \\ \partial_{xy}\frac{1}{r} & \partial_{yy}\frac{1}{r} & \partial_{yz}\frac{1}{r} \\ \partial_{xz}\frac{1}{r} & \partial_{yz}\frac{1}{r} & \partial_{zz}\frac{1}{r} \end{bmatrix} \mathbf{m}$$

$$\mathbf{B}(x,y,z) = C_m \, \mathbf{H}(x,y,z,x',y',z')\, \mathbf{m}$$

$$[T] \qquad \left[\frac{H}{m}\right] \qquad \left[\frac{1}{m^3}\right] \qquad [A \cdot m^2]$$

$$\frac{Kg\, m^2}{s^2 A^2}\ \frac{1}{m}\ \frac{1}{m^3}\ A\ m^2 = \frac{Kg}{A\, s^2} = T$$

$$\mathbf{F}_0 = \|\mathbf{F}_0\| \begin{bmatrix} \cos I_0 \cos D_0 \\ \cos I_0 \sin D_0 \\ \sin I_0 \end{bmatrix} \qquad \|\mathbf{F}_0\| >> \|\mathbf{B}\|$$

$$\Delta T(x, y, z) = \|\mathbf{F}_0 + \mathbf{B}\| - \|\mathbf{F}_0\|$$

$$\tilde{\Delta T}(x, y, z) = \hat{\mathbf{F}}_0^T \mathbf{B}(x, y, z)$$

$$= C_m \hat{\mathbf{F}}_0^T \mathbf{H}(x, y, z, x', y', z') \mathbf{m}$$

Ex: mostre que a anomalia de campo total aproximada é harmônica
e a anomalia de campo total sem aproximação não é harmônica

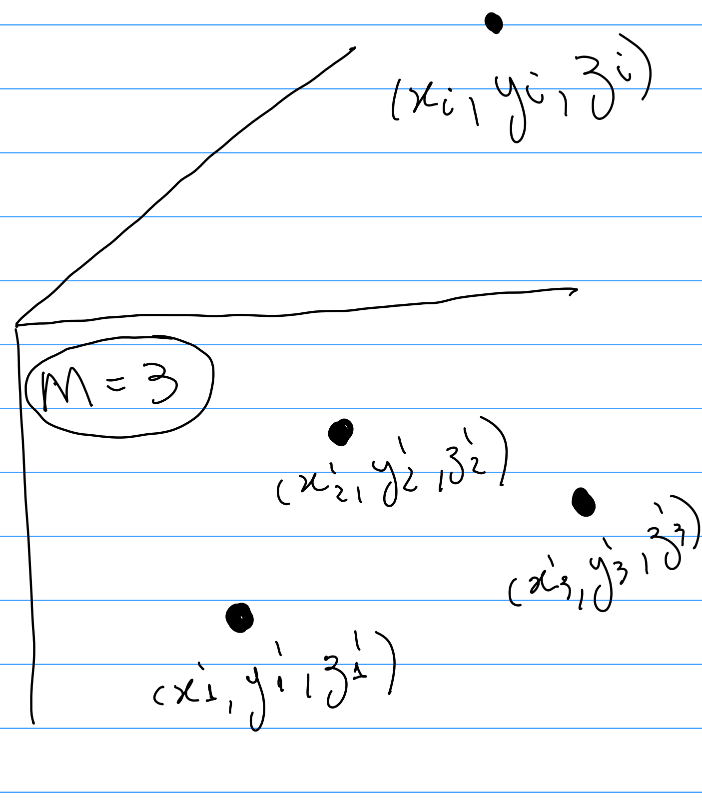
$$\mathbf{B}_{ij} \equiv \mathbf{B}(x_i, y_i, z_i) = C_m \mathbf{H}(x_i, y_i, z_i, x'_j, y'_j, z'_j) \mathbf{m}_j$$

$$\mathbf{B}_{ij} = C_m \mathbf{H}_{ij} \mathbf{m}_j$$

$$\mathbf{B}_i = \sum_{j=0}^{M-1} \mathbf{B}_{ij}$$

$$V_i = \sum_{j=0}^{M-1} V_{ij}$$

$$\tilde{\Delta T}_i = \sum_{j=0}^{M-1} \tilde{\Delta T}_{ij}$$



Ex: Mostre que

1) $\nabla^2 \tilde{\Delta} T(x,y,z) = 0$

2) $\nabla^2 \Delta T(x,y,z) \neq 0$

1) $\Delta \tilde{T}(x,y,z) = F_x B_x(x,y,z) + F_y B_y(x,y,z) + F_z B_z(x,y,z)$

$$B_x(x,y,z) = C_m \left[\partial_{xx}\frac{1}{r} \quad \partial_{xy}\frac{1}{r} \quad \partial_{xz}\frac{1}{r} \right] \cdot \mathbf{m}$$

$$\nabla^2 B_x = 0$$

$$\partial_{xx} B_x = C_m \left[\partial_{xx}\left(\partial_{xx}\frac{1}{r}\right) + \partial_{xx}\left(\partial_{xy}\frac{1}{r}\right) + \right.$$
$$\left. + \partial_{xx}\left(\partial_{xz}\frac{1}{r}\right) \right] \cdot \mathbf{m}$$

$$\nabla^2 \tilde{\Delta} T = C_m \left[\underbrace{F_x \nabla^2 B_x}_{=0} + F_y \nabla^2 B_y + F_z \nabla^2 B_z \right]$$