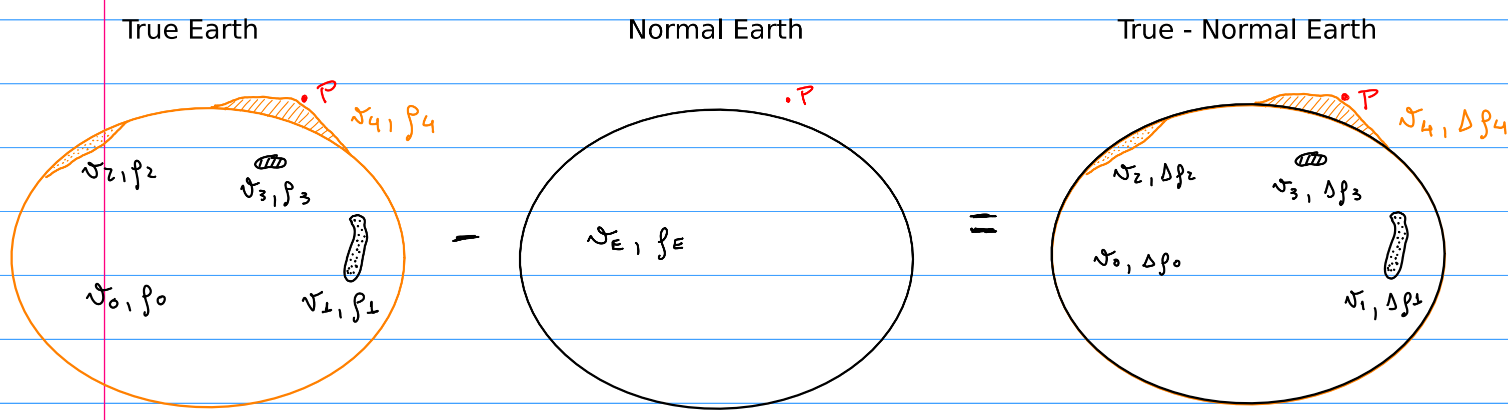
Gravity modeling



$\vartheta_E = \vartheta_0 \cup \vartheta_1 \cup \vartheta_2 + \vartheta_3$

elipsoid volume

$P = (x, y, z)$

$\Delta\rho_0 = \rho_0 - \rho_E$

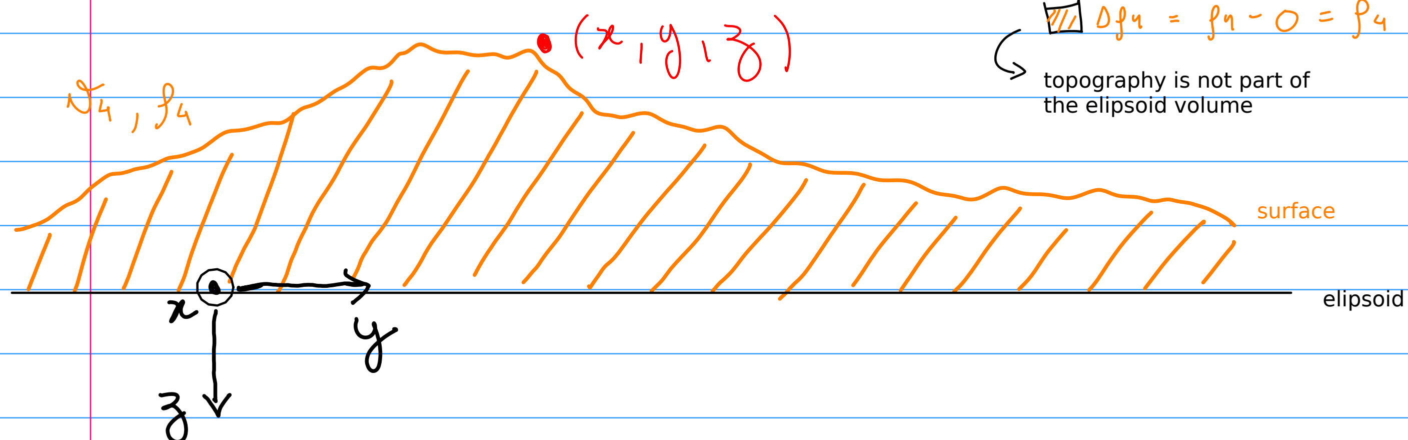
$\Delta\rho_1 = \rho_1 - \rho_E$

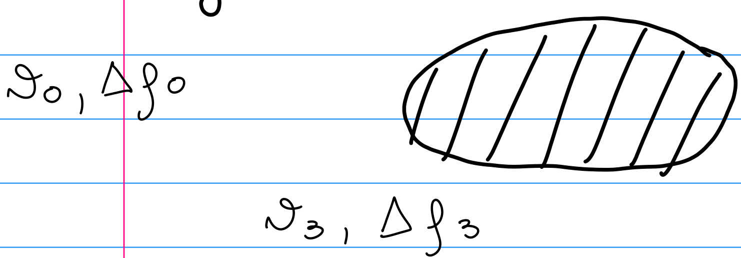
$\Delta\rho_2 = \rho_2 - \rho_E$

$\Delta\rho_3 = \rho_3 - \rho_E$

$\Delta\rho_4 = \rho_4 - 0 = \rho_4$

topography is not part of the elipsoid volume





Consider that this geological body is our target

gravitational aceleration produced by the geological body

$$\mathbf{a}_P = \mathbf{a}_P^{(0)} + \mathbf{a}_P^{(1)} + \mathbf{a}_P^{(2)} + \mathbf{a}_P^{(3)} + \mathbf{a}_P^{(4)}$$

centrifugal component

gravitational component

$$\mathbf{g}_P = \mathbf{c}_P + \mathbf{a}_P$$

gravity vector

$$\mathbf{a}_P^{(k)} = \mathbf{a}^{(k)}(x, y, z) = G \iiint_{\vartheta_k} \left(\nabla \frac{1}{r}\right) \rho_k(x', y', z')\, dv$$

gravity disturbance vector

gravity vector

normal gravity vector

$$\delta \mathbf{g}_P = \mathbf{g}_P - \boldsymbol{\gamma}_P$$

centrifugal component

gravitational component

$$\boldsymbol{\gamma}_P = \mathbf{c}_P + \mathbf{a}_P^{(E)}$$

$$\mathbf{a}_P^{(E)} = G \iiint_{v_E} \left(\nabla \frac{1}{r} \right) \rho_E(x', y', z') \, dv$$

purely gravitational effect

$$\delta \mathbf{g}_P = \cancel{\mathbf{c}_P} + \mathbf{a}_P - \cancel{\mathbf{c}_P} - \mathbf{a}_P^{(E)} = \mathbf{a}_P - \mathbf{a}_P^{(E)}$$

$$\delta \mathbf{g}_P = \left(\sum_{k=0}^{4} G \iiint_{v_k} \left(\nabla \frac{1}{r} \right) \rho_k(x', y', z') \, dv \right) - G \iiint_{v_E} \left(\nabla \frac{1}{r} \right) \rho_E(x', y', z') \, dv$$

$$= \left(\sum_{k=0}^{3} G \iiint_{v_k} \left(\nabla \frac{1}{r} \right) \Delta \rho_k(x', y', z') \, dv \right) + G \iiint_{v_4} \left(\nabla \frac{1}{r} \right) \rho_4(x', y', z') \, dv$$

$$\mathbf{g}_P = \boldsymbol{\gamma}_P + \delta \mathbf{g}_P \,, \quad \| \boldsymbol{\gamma}_P \| \gg \| \delta \mathbf{g}_P \|$$

normal gravity

gravity

$\| \mathbf{g}_P \|$ $\quad$ $\| \boldsymbol{\gamma}_P \|$

gravity disturbance

$$\delta g_P = g_P - \gamma_P$$

$$\approx \left(\cancel{\gamma_P} + \hat{\mathbf{z}}^T \delta \mathbf{g}_P \right) - \cancel{\gamma_P}$$

Consider a coordinate system satisfying:

$$\hat{\boldsymbol{\gamma}}_P = \hat{\mathbf{z}} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},$$

for any point P in the study area.

$$\delta g_P \approx \left(\sum_{k=0}^{3} G \iiint_{v_k} \partial_z \frac{1}{r} \, \Delta \rho_k(x', y', z') \, dv \right) + G \iiint_{v_4} \partial_z \frac{1}{r} \, \rho_4(x', y', z') \, dv$$

By neglecting the gravitational effects of ,

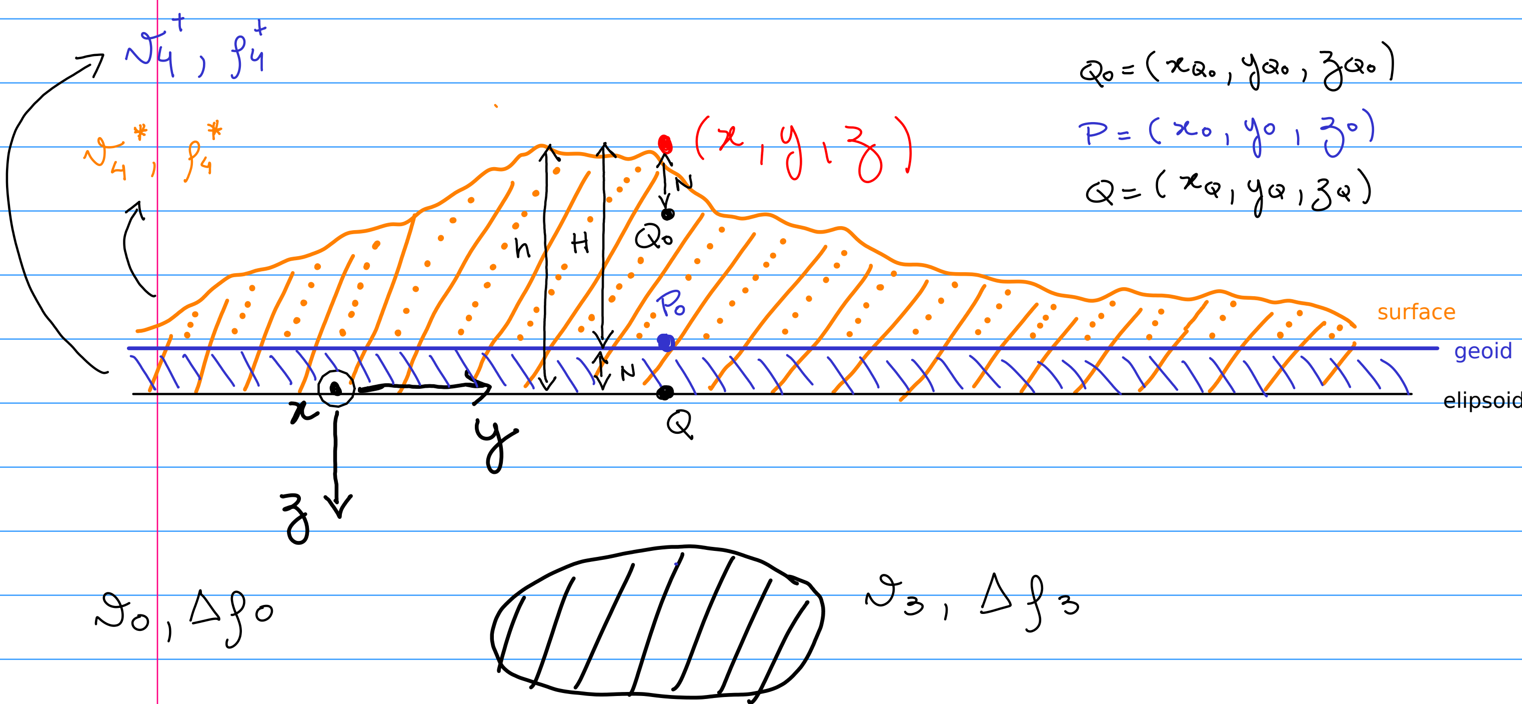
$$\delta g_P \approx G \iiint_{v_3} \partial_z \frac{1}{r} \, \Delta\rho_3(x',y',z')\, dv + G \iiint_{v_4} \partial_z \frac{1}{r} \, \rho_4(x',y',z')\, dv$$

vertical component of the gravitational aceleration produced by the geological body with volume v_3

vertical component of the gravitational aceleration produced by the topography

By removing the gravitational effect of the topography, we obtain:

$$\delta g_P^B \approx G \iiint_{v_3} \partial_z \frac{1}{r} \, \Delta\rho_3(x',y',z')\, dv$$



$$a_P + a_P = G \iiint_{v_4} \partial_z \frac{1}{r} \, \rho_4(x',y',z')\, dv$$

vertical component of the gravitational aceleration produced by the mass located between the elipsoid and the surface (the topography)

$$a_P = G \iiint_{v_4^*} \partial_z \frac{1}{r} \, \rho_4^*(x',y',z')\, dv$$

vertical component of the gravitational aceleration produced by the mass located between the geoid and the surface

$$a_P = G \iiint_{v_4^+} \partial_z \frac{1}{r} \, \rho_4^+(x',y',z')\, dv$$

vertical component of the gravitational aceleration produced by the mass located between the elipsoid and the geoid

$$\Delta g_P^B = g_P - \Big(\gamma_Q + \overbrace{\frac{\partial \gamma}{\partial h} H}^{\approx \frac{\partial \gamma}{\partial h} h - \frac{\partial \gamma}{\partial h} N}\Big) - a_P \approx \left\{ g_P - \gamma_P \right\} + \frac{\partial \gamma}{\partial h} N - a_P$$

$$\approx \left\{ G \iiint_{v_3} \partial_z \frac{1}{r} \, \Delta\rho_3(x',y',z') \, dv + G \iiint_{v_4} \partial_z \frac{1}{r} \, \rho_4(x',y',z') \, dv \right\} + \frac{\partial \gamma}{\partial h} N - a_P$$

vertical component of the gravitational aceleration produced by the geological body with volume v_3

vertical component of the gravitational aceleration produced by the mass located between the elipsoid and the surface (the topography)

$$\approx G \iiint_{v_3} \partial_z \frac{1}{r} \, \Delta\rho_3(x',y',z') \, dv + \left(\frac{\partial \gamma}{\partial h} N + a_P \right)$$

geophysical indirect effect
(Hinze et al, 2005)

$$\Delta g_P^B \approx \delta g_P^B + \left(\frac{\partial \gamma}{\partial h} N + a_P \right)$$

$\approx 0{,}3086$ mGal/m

$$\boxed{a_P \approx 2\pi G \tilde{\rho}_4^{+} N}$$