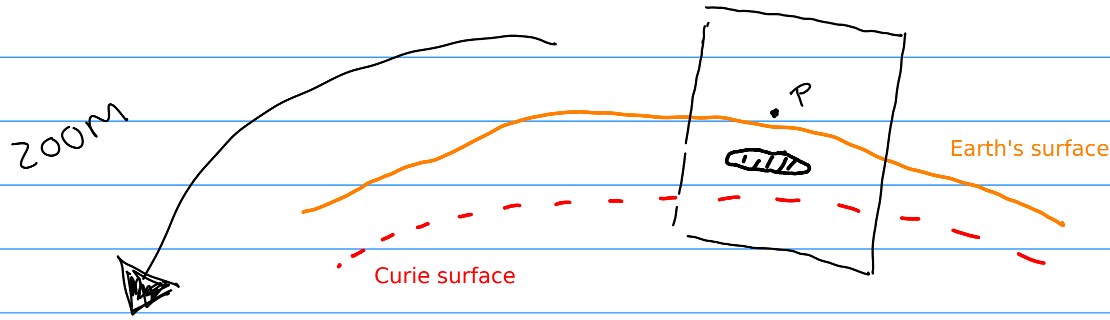
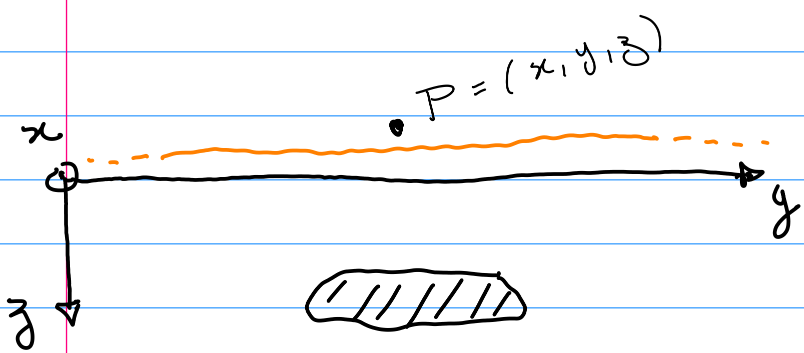
Magnetic modeling





magnetized body with volume v

General case (total magnetization)

$$V(x, y, z) = -C_m \iiint_v \left(\nabla \frac{1}{r} \right)^T \mathbf{h}(x', y', z') \, dv$$

$$\mathbf{B}(x, y, z) = -\nabla V(x, y, z)$$

$$= C_m \iiint_v \underbrace{\left(\nabla^2 \frac{1}{r} \right)}_{\text{matrix containing second derivatives of } 1/r} \mathbf{h}(x', y', z') \, dv$$

$$\mathbf{T}(x, y, z) = \mathbf{F}(x, y, z) + \mathbf{B}(x, y, z)$$

$$\Delta T(x, y, z) = \| \mathbf{T}(x, y, z) \| - \| \mathbf{F}(x, y, z) \|$$

$$\widetilde{\Delta T}(x, y, z) = \hat{\mathbf{F}}^T \mathbf{B}(x, y, z) \qquad \boxed{\widetilde{\Delta T} \approx \Delta T}$$

Constant total-magnetization direction

$$\mathbf{h}(x', y', z') = h(x', y', z')\,\hat{\mathbf{h}} \;, \quad \hat{\mathbf{h}} = \begin{bmatrix} \cos I \cos D \\ \cos I \sin D \\ \sin I \end{bmatrix} = \begin{bmatrix} \hat{h}_x \\ \hat{h}_y \\ \hat{h}_z \end{bmatrix}$$

$$V(x, y, z) = -C_m \iiint_v h(x', y', z') \left(\nabla \frac{1}{r} \right)^T \hat{\mathbf{h}} \, dv$$

$$= -C_m \left\{ \hat{h}_x \iiint_v h(x', y', z')\, \partial_x \frac{1}{r} \, dv \; + \right.$$

$$+ \hat{h}_y \iiint_v h(x', y', z')\, \partial_y \frac{1}{r} \, dv \; +$$

$$\left. + \hat{h}_z \iiint_v h(x', y', z')\, \partial_z \frac{1}{r} \, dv \right\}$$

$$= -\nabla \Theta(x, y, z)^T \hat{\mathbf{h}}$$

$$\Theta(x, y, z) = C_m \iiint_v h(x', y', z') \frac{1}{r} \, dv$$

$$\mathbf{B}(x, y, z) = \nabla^2 \Theta(x, y, z)\, \hat{\mathbf{h}}$$

$$\nabla^2 \Theta(x, y, z) = \begin{bmatrix} \partial_{xx}\Theta & \partial_{xy}\Theta & \partial_{xz}\Theta \\ \partial_{xy}\Theta & \partial_{yy}\Theta & \partial_{yz}\Theta \\ \partial_{xz}\Theta & \partial_{yz}\Theta & \partial_{zz}\Theta \end{bmatrix}$$

Constant total magnetization

$$\mathbf{h}(x', y', z') = \underbrace{h}_{\text{constant}} \hat{\mathbf{h}} \quad , \quad \hat{\mathbf{h}} = \begin{bmatrix} \cos I \cos D \\ \cos I \sin D \\ \sin I \end{bmatrix} = \begin{bmatrix} \hat{h}_x \\ \hat{h}_y \\ \hat{h}_z \end{bmatrix}$$

$$V(x, y, z) = -\nabla \Theta(x, y, z)^{T} \hat{\mathbf{h}} \qquad \Theta(x, y, z) = C_m h \iiint_{v} \frac{1}{r} \, dv$$

$$\mathbf{B}(x, y, z) = \nabla^{2} \Theta(x, y, z) \hat{\mathbf{h}}$$

General case (induced and remanent magnetization components)

total-magnetization vector

$$\mathbf{h}(x', y', z') = \overbrace{\boldsymbol{\chi}(x', y', z') \mathbf{H}(x', y', z')}^{\text{induced magnetization}} + \underbrace{\mathbf{p}(x', y', z')}_{\text{remanent magnetization}}$$

$[A/m]$

magnetic intensity field

$$\mathbf{H}(x', y', z') = \mathbf{F}(x', y', z') / \mu_0$$

$\mathbf{H}$: $[A/m]$ — magnetic induction field $\mathbf{F}$: $[T]$ — magnetic permeability constant (at vacuum) μ_0: $[H/m]$

$$\boldsymbol{\chi}(x', y', z') = \begin{bmatrix} \chi_{11} & \chi_{12} & \chi_{13} \\ \chi_{12} & \chi_{22} & \chi_{23} \\ \chi_{13} & \chi_{23} & \chi_{33} \end{bmatrix} , \quad \chi_{ij} \equiv \chi_{ij}(x', y', z') \quad i, j = 1, 2, 3$$

magnetic susceptibility tensor
(dimentionless in the SI)

Hrouda, F., 2007. Magnetic susceptibility, anisotropy. In Gubbins, D., and Herrero-Bervera, E. (eds.), Encyclopedia of Geomagnetism and Paleomagnetism, Springer, pp. 546–560. ISBN: 978-1-4020-3992-8

Rochette, P., Jackson, M.J., and Aubourg, C., 1992. Rock magnetism and the interpretation of anisotropy of magnetic susceptibility. Reviews of Geophysics, 30: 209–226. doi: 10.1029/92RG00733

Hrouda, F., 1982. Magnetic anisotropy of rocks and its application in geology and geophysics. Geophysical Surveys, 5, 37–82. doi: 10.1007/BF01450244

AGICO, Manuals and AGICO prints, https://www.agico.com/text/support/agicoprints/agicoprints.php, 21/10/2020

scalar function

identity matrix

$$\boldsymbol{\chi}(x', y', z') = \chi(x', y', z')\,\mathbf{I}$$

particular case in which
susceptibility is isotropic

$$V(x, y, z) = -C_m \iiint_v \left(\nabla \frac{1}{r}\right)^T \mathbf{h}(x', y', z')\, dv$$

$$= -C_m \iiint_v \left(\nabla \frac{1}{r}\right)^T \boldsymbol{\chi}(x', y', z')\,\mathbf{H}(x', y', z')\, dv \; +$$

$$- C_m \iiint_v \left(\nabla \frac{1}{r}\right)^T \mathbf{p}(x', y', z')\, dv$$