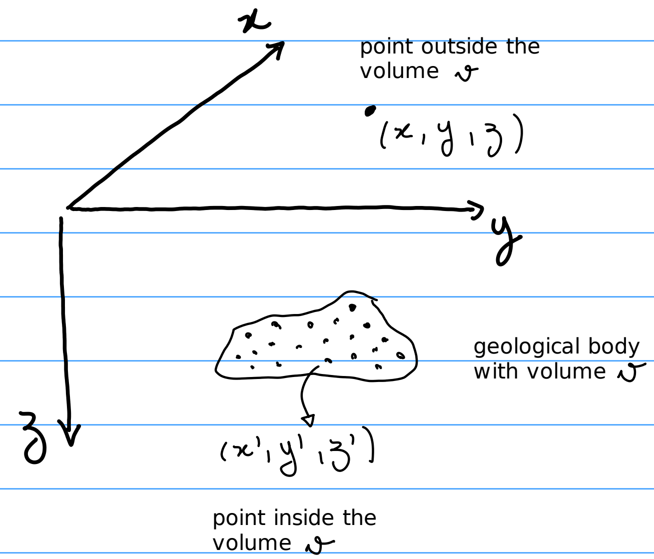
3D sources

To compute the magnetic/gravitational effect produced by a 3D body at the point (x, y, z), consider that it is formed by small volume elements dv' = dx'dy'dz' , each one with center at a point (x', y', z').



Magnetic scalar potential produced by a 3D body

$$V(x,y,z) = -C_m \iiint_{\upsilon} \nabla \frac{1}{r}^{T} \mathbf{h}(x',y',z')\, d\upsilon'$$

$\left[\frac{HA}{m}\right] = [T\,m]$ $\qquad [H/m]$ $\qquad$ total magnetization

$$\mathbf{m}(x',y',z') = \mathbf{h}(x',y',z')\, d\upsilon'$$

magnetic moment = magnetization X volume

$[A\,m^2]$ $\qquad [A/m] \times [m^3]$

Gravitational potential produced by a 3D body

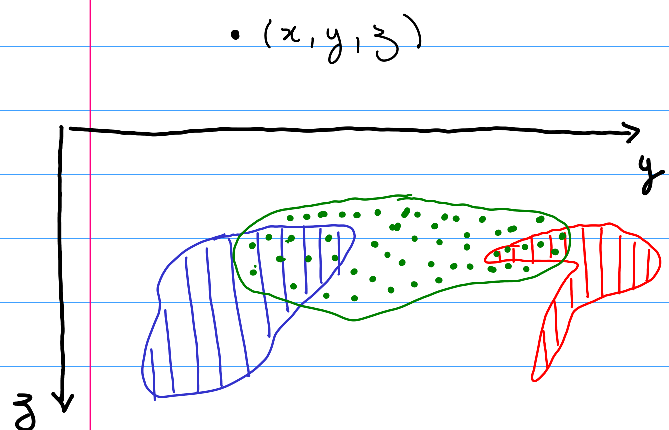
$$U(x,y,z) = G \iiint_{\upsilon} \frac{1}{r}\, \rho(x',y',z')\, d\upsilon'$$

$[m^2/s^2]$ $\qquad \left[\frac{m^3}{Kg\, s^2}\right]$ $\qquad$ density

$$m(x',y',z') = \rho(x',y',z')\, d\upsilon'$$

mass = density X volume

$[Kg]$ $\qquad [Kg/m^3] \times [m^3]$



$$\upsilon = \upsilon_1 \cup \upsilon_2 \cup \upsilon_3 \cup \upsilon_4 \cup \upsilon_5$$

- $\upsilon_1,\ \rho_1,\ \mathbf{h}_1$
- $\upsilon_2,\ \rho_2 = \rho_1 + \rho_3,\ \mathbf{h}_2 = \mathbf{h}_1 + \mathbf{h}_3$
- $\upsilon_3,\ \rho_3,\ \mathbf{h}_3$
- $\upsilon_4,\ \rho_4 = \rho_3 + \rho_5,\ \mathbf{h}_4 = \mathbf{h}_3 + \mathbf{h}_5$
- $\upsilon_5,\ \rho_5,\ \mathbf{h}_5$

$$V(x,y,z) = -C_m \iiint_{\upsilon} \nabla \frac{1}{r}^{T} \mathbf{h}(x',y',z')\, d\upsilon'$$

$$= -C_m \sum_{k=1}^{5} \iiint_{\upsilon_k} \nabla \frac{1}{r}^{T} \mathbf{h}_k\, d\upsilon'$$

$$U(x,y,z) = G \iiint_{\upsilon} \frac{1}{r}\, \rho(x',y',z')\, d\upsilon'$$

$$= G \sum_{k=1}^{5} \iiint_{\upsilon_k} \frac{1}{r}\, \rho_k\, d\upsilon'$$

$$V(x,y,z) = -G_m \Big\{ \iiint_{v_1} \nabla\tfrac{1}{r}\cdot \mathbf{h}_1\, dv' + \iiint_{v_2} \nabla\tfrac{1}{r}\cdot \underbrace{\mathbf{h}_2}_{=\mathbf{h}_1+\mathbf{h}_3}\, dv' + \iiint_{v_3} \nabla\tfrac{1}{r}\cdot \mathbf{h}_3\, dv' +$$

$$+ \iiint_{v_4} \nabla\tfrac{1}{r}\cdot \mathbf{h}_4\, dv' + \iiint_{v_5} \nabla\tfrac{1}{r}\cdot \mathbf{h}_5\, dv' \Big\}$$

$$= -G_m \Big\{ \iiint_{v_1} \nabla\tfrac{1}{r}\cdot \mathbf{h}_1\, dv' + \Big[\iiint_{v_2} \nabla\tfrac{1}{r}\cdot \mathbf{h}_1\, dv' + \iiint_{v_2} \nabla\tfrac{1}{r}\cdot \mathbf{h}_3\, dv' \Big] +$$

$$+ \iiint_{v_3} \nabla\tfrac{1}{r}\cdot \mathbf{h}_3\, dv' + \Big[\iiint_{v_4} \nabla\tfrac{1}{r}\cdot \mathbf{h}_3\, dv' + \iiint_{v_4} \nabla\tfrac{1}{r}\cdot \mathbf{h}_5\, dv' \Big] +$$

$$+ \iiint_{v_5} \nabla\tfrac{1}{r}\cdot \mathbf{h}_5\, dv' \Big\}$$

$$U(x,y,z) = G \Big\{ \iiint_{v_1} \tfrac{1}{r}\,\rho_1\, dv' + \iiint_{v_2} \tfrac{1}{r}\,\rho_2\, dv' + \iiint_{v_3} \tfrac{1}{r}\,\rho_3\, dv' +$$

$$+ \iiint_{v_4} \tfrac{1}{r}\,\rho_4\, dv' + \iiint_{v_5} \tfrac{1}{r}\,\rho_5\, dv' \Big\}$$

$$= G \Big\{ \iiint_{v_1} \tfrac{1}{r}\,\rho_1\, dv' + \Big[\iiint_{v_2} \tfrac{1}{r}\,\rho_1\, dv' + \iiint_{v_2} \tfrac{1}{r}\,\rho_3\, dv' \Big] +$$

$$+ \iiint_{v_3} \tfrac{1}{r}\,\rho_3\, dv' + \Big[\iiint_{v_4} \tfrac{1}{r}\,\rho_3\, dv' + \iiint_{v_4} \tfrac{1}{r}\,\rho_5\, dv' \Big] +$$

$$+ \iiint_{v_5} \tfrac{1}{r}\,\rho_5\, dv' \Big\}$$

$\alpha = x, y, z$

$$\partial_\alpha V(x,y,z) = -G_m \iiint_{v} \left(\partial_\alpha \nabla\tfrac{1}{r}\right)\cdot \mathbf{h}(x',y',z')\, dv' \qquad \partial_\alpha U(x,y,z) = G \iiint_{v} \partial_\alpha \tfrac{1}{r}\, \rho(x',y',z')\, dv'$$

$$\partial_{\alpha\beta} V(x,y,z) = -G_m \iiint_{v} \left(\partial_{\alpha\beta} \nabla\tfrac{1}{r}\right)\cdot \mathbf{h}(x',y',z')\, dv' \qquad \partial_{\alpha\beta} U(x,y,z) = G \iiint_{v} \partial_{\alpha\beta} \tfrac{1}{r}\, \rho(x',y',z')\, dv'$$