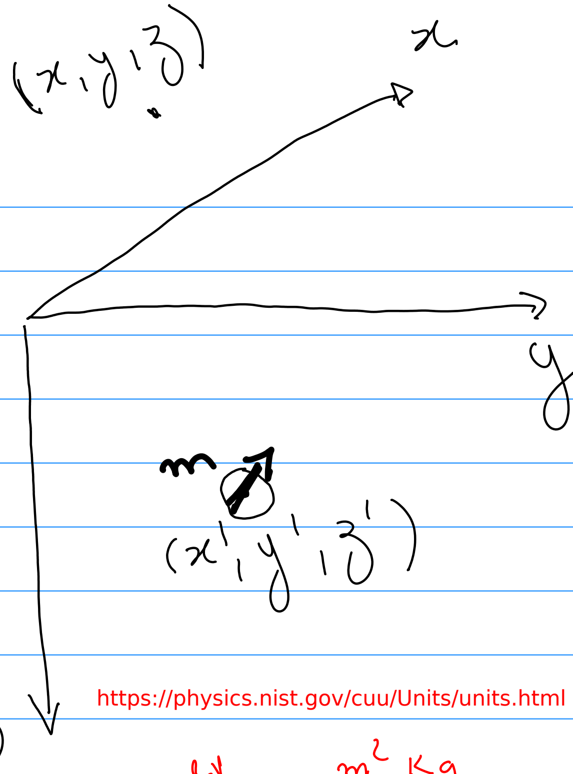
Dipolo



$$\mathbf{B}(x,y,z)\ ?$$

condições magnetostáticas
(ausência de corrente elétrica)

https://physics.nist.gov/cuu/Units/units.html

HENRY $\quad H = \dfrac{m^2\,Kg}{s^2\,A^2}$

TESLA (T)

potencial magnético escalar

$$V(x,y,z) = -C_m\, \nabla \frac{1}{r}^{T}\, \mathbf{m} \qquad \left[\frac{HA}{m}\right] = \left[\frac{m\,Kg}{s^2\,A}\right] = [Tm]$$

$$\mathbf{r} = \begin{bmatrix} x-x' \\ y-y' \\ z-z' \end{bmatrix} \qquad r = \left[(x-x')^2 + (y-y')^2 + (z-z')^2\right]^{1/2}$$

$$\nabla \frac{1}{r} = \begin{bmatrix} \partial_x \frac{1}{r} \\ \partial_y \frac{1}{r} \\ \partial_z \frac{1}{r} \end{bmatrix} \qquad \left[\frac{1}{m^2}\right]$$

$$\mathbf{m} = m\,\hat{\mathbf{m}}$$

$$\hat{\mathbf{m}} = \begin{bmatrix} \cos I' \cos D' \\ \cos I' \sin D' \\ \sin I' \end{bmatrix}$$

$$C_m = 10^9 \frac{\mu_0}{4\pi} \quad \frac{H}{m}$$

m = volume × intensidade de magnetização total

$[m^3]$ $\qquad$ $[A/m]$

$$\nabla \frac{1}{r} = -\frac{1}{r^3}\,\mathbf{r}$$

indução magnética

$$\mathbf{B}(x,y,z) = -\nabla V(x,y,z) = -\begin{bmatrix} \partial_x V \\ \partial_y V \\ \partial_z V \end{bmatrix}$$

$$-\partial_x V = -\partial_x \left[-c m \left(\partial_x \frac{1}{r} m_x + \partial_y \frac{1}{r} m_y + \partial_z \frac{1}{r} m_z \right) \right]$$

$$\partial_x \frac{1}{r} = -\frac{1}{\cancel{2}} \frac{\cancel{2}(x-x')}{r^3} = -\frac{(x-x')}{r^3}$$

$$\partial_y \frac{1}{r} = -\frac{y-y'}{r^3} \qquad \partial_z \frac{1}{r} = -\frac{z-z'}{r^3}$$

$$\nabla \frac{1}{r} = -\frac{1}{r^3} \mathbf{r}$$

$$\partial_{xx} \frac{1}{r} = \left(-1 \cdot \frac{1}{r^3} \right) + -(x-x')\left(-\frac{3}{2}\right) \frac{1}{r^5} 2(x-x')$$

$$= \frac{3(x-x')^2}{r^5} - \frac{1}{r^3}$$

$$\partial_{xy} \frac{1}{r} = -(y-y')\left(-\frac{3}{2}\right) \frac{1}{r^5} 2(x-x')$$

$$= \frac{3(x-x')(y-y')}{r^5}$$

$$-\partial_x V = C_m \left[\partial_{xx}\frac{1}{r} \quad \partial_{xy}\frac{1}{r} \quad \partial_{xz}\frac{1}{r} \right] \mathbf{m}$$

$$-\partial_y V = C_m \left[\partial_{xy}\frac{1}{r} \quad \partial_{yy}\frac{1}{r} \quad \partial_{yz}\frac{1}{r} \right] \mathbf{m}$$

$$-\partial_z V = C_m \left[\partial_{xz}\frac{1}{r} \quad \partial_{yz}\frac{1}{r} \quad \partial_{zz}\frac{1}{r} \right] \mathbf{m}$$

Ex $\partial_{xy}\frac{1}{r} = \partial_{yx}\frac{1}{r}$

Ex $\partial_{xx}\frac{1}{r} + \partial_{yy}\frac{1}{r} + \partial_{zz}\frac{1}{r}$

Ex $\partial_x \frac{1}{r} = -\partial_{x'}\frac{1}{r}$

$$\mathbf{B}(x,y,z) = C_m \begin{bmatrix} \partial_{xx}\frac{1}{r} & \partial_{xy}\frac{1}{r} & \partial_{xz}\frac{1}{r} \\ \partial_{xy}\frac{1}{r} & \partial_{yy}\frac{1}{r} & \partial_{yz}\frac{1}{r} \\ \partial_{xz}\frac{1}{r} & \partial_{yz}\frac{1}{r} & \partial_{zz}\frac{1}{r} \end{bmatrix} \mathbf{m}$$

$$\mathbf{B}(x,y,z) = C_m \, \mathbf{H}(x,y,z,x',y',z')\, \mathbf{m}$$

$$[T] \qquad \left[\frac{H}{m}\right] \qquad \left[\frac{1}{m^3}\right] \qquad [A \cdot m^2]$$

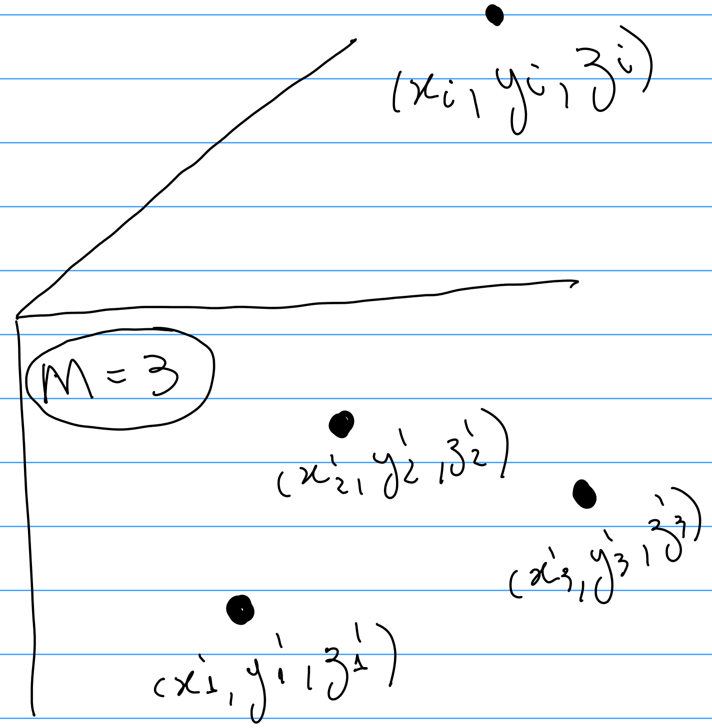
$$\frac{kg\, m^2}{s^2 A^2}\ \frac{1}{m}\ \frac{1}{m^3}\ A\ m^2 = \frac{kg}{A\, s^2} = T$$

$$\mathbf{B}_{ij} \equiv \mathbf{B}(x_i, y_i, z_i) = C_m \mathbf{H}(x_i, y_i, z_i, x'_j, y'_j, z'_j)\, \mathbf{m}_j$$

$$\mathbf{B}_{ij} = C_m \mathbf{H}_{ij} \mathbf{m}_j$$

$$\mathbf{B}_i = \sum_{j=0}^{M-1} \mathbf{B}_{ij}$$

$$V_i = \sum_{j=0}^{M-1} V_{ij}$$



ptos. observação — dipolos

```
def dipolo_V (x, y, z, x', y', z', m, I', D'):
    V = 0 (vetor de zeros)
    for (xj, yj, zj, mj, Ij, Dj) in (x', y', z', m, I', D'):
        mx, my, mz = moment(mj, Ij, Dj)
        for (xi, yi, zi) in (x, y, z):
            dx, dy, dz = deriv 1/r (xi, yi, zi, xj, yj, zj)
            V[i] += dx mx + dy my + dz mz
    V *= Cm
```

$$x = \begin{bmatrix} x[0] \\ x[1] \\ \vdots \\ x[N-1] \end{bmatrix} \qquad x' = \begin{bmatrix} x'[0] \\ x'[1] \\ x'[2] \end{bmatrix}$$

```
def magnetic_scalar_potential(x, y, z, x', y', z', m, I', D'):
    V = 0 #array with null elements
    for (xj, yj, zj, mj, Ij, Dj) in (x', y', z', m, I', D'):
        mx, my, mz = moment(mj, Ij, Dj)
        for (xi, yi, zi) in (x, y, z):
            dx, dy, dz = first_deriv_1_r(xi, yi, zi, xj, yj, zj)
            V[i] += dx*mx + dy*my + dz*mz
    V *= Cm
    return V
```

```
def moment(mj, Ij, Dj):

    # convert angle from degree to radian
    inc = deg2rad(Ij)
    dec = deg2rad(Dj)

    # compute cosine and sine
    cos_inc = cos(inc)
    sin_inc = sin(inc)
    cos_dec = cos(dec)
    sin_dec = sin(dec)

    # components of moment vector
    mx = mj*cos_inc*cos_dec
    my = mj*cos_inc*sin_dec
    mz = mj*sin_inc

    return mx, my, mz
```

```
def first_deriv_1_r(xi, yi, zi, xj, yj, zj):

    # define new variables
    X = xi - xj
    Y = yi - yj
    Z = zi - zj

    # compute distance r
    r = sqrt(X**2 + Y**2 + Z**2)

    # first derivatives
    dx = - X / r**3
    dy = - Y / r**3
    dz = - Z / r**3

    return dx, dy, dz
```