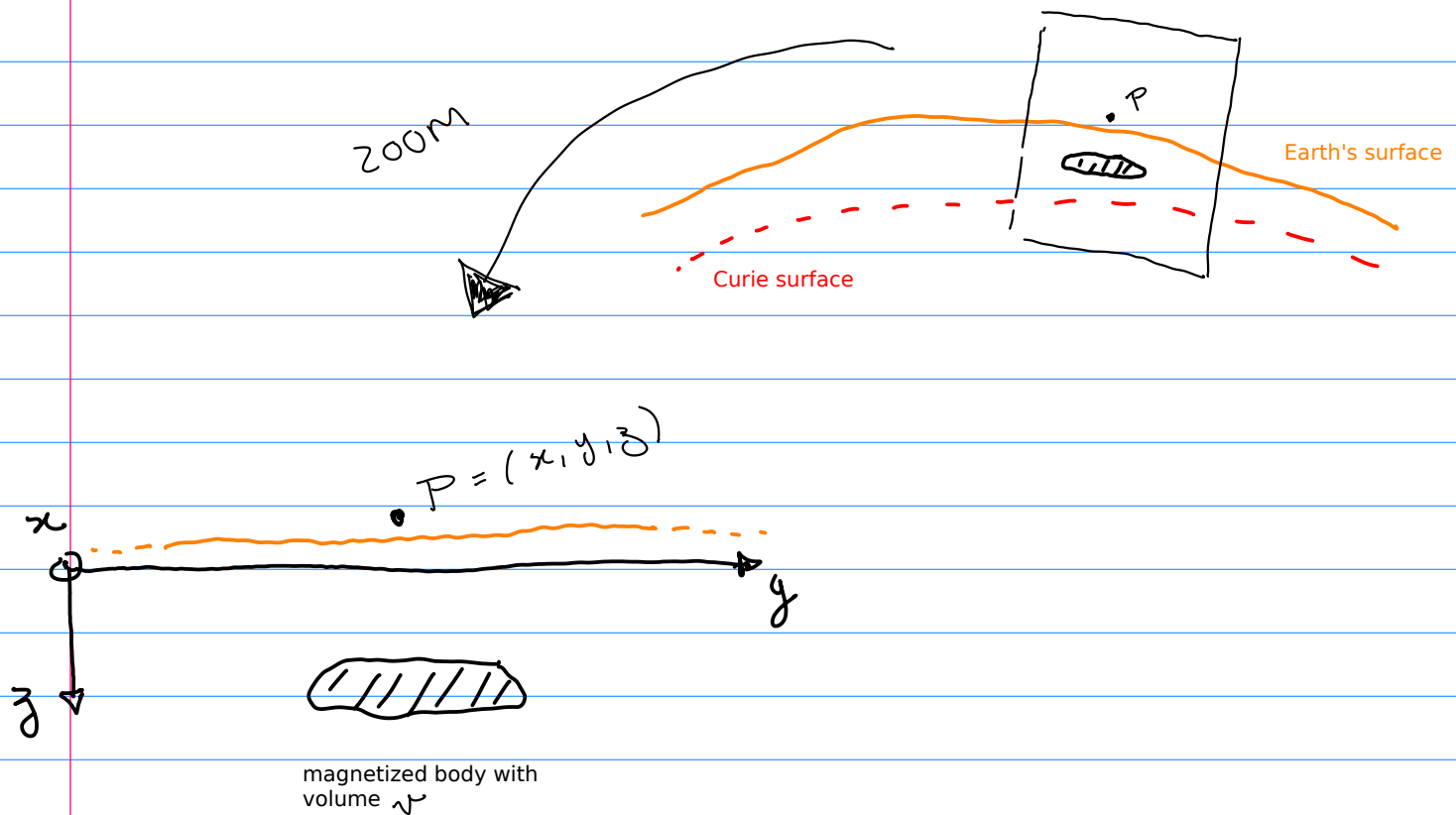


Magnetic modeling



General case (total magnetization)

$$V(x, y, z) = - \mu_0 \iiint_V \left(\nabla \frac{1}{r} \right)^T \mathbf{h}(x', y', z') d\mathbf{v}$$

$$\mathbf{B}(x, y, z) = - \nabla V(x, y, z)$$

$$= \mu_0 \iiint_V \left(\nabla^2 \frac{1}{r} \right) \mathbf{h}(x', y', z') d\mathbf{v}$$

matrix containing second derivatives of $1/r$

$$\mathbf{T}(x, y, z) = \mathbf{F}(x, y, z) + \mathbf{B}(x, y, z)$$

$$\Delta T(x, y, z) = \|\mathbf{T}(x, y, z)\| - \|\mathbf{F}(x, y, z)\|$$

$$\tilde{\Delta T}(x, y, z) = \hat{\mathbf{F}}^T \mathbf{B}(x, y, z) \quad \boxed{\tilde{\Delta T} \approx \Delta T}$$

Constant total-magnetization direction

$$h(x', y', z') = h(x', y', z') \hat{h}, \quad \hat{h} = \begin{bmatrix} \cos I \cos D \\ \cos I \sin D \\ \sin I \end{bmatrix} = \begin{bmatrix} \hat{h}_x \\ \hat{h}_y \\ \hat{h}_z \end{bmatrix}$$

$$V(x, y, z) = -\mu_m \iiint_{\mathcal{V}} h(x', y', z') \left(\nabla \frac{1}{r} \right)^T \hat{h} d\mathcal{V}$$

$$= -\mu_m \left\{ \begin{aligned} &\hat{h}_x \iiint_{\mathcal{V}} h(x', y', z') \partial_x \frac{1}{r} d\mathcal{V} + \\ &+ \hat{h}_y \iiint_{\mathcal{V}} h(x', y', z') \partial_y \frac{1}{r} d\mathcal{V} + \\ &+ \hat{h}_z \iiint_{\mathcal{V}} h(x', y', z') \partial_z \frac{1}{r} d\mathcal{V} \end{aligned} \right\}$$

$$= -\nabla \Phi(x, y, z)^T \hat{h}$$

$$\Phi(x, y, z) = \mu_m \iiint_{\mathcal{V}} h(x', y', z') \frac{1}{r} d\mathcal{V}$$

$$\mathbf{B}(x, y, z) = \nabla^z \Phi(x, y, z) \hat{h}$$

$$\nabla^z \Phi(x, y, z) = \begin{bmatrix} \partial_{xx} \Phi & \partial_{xy} \Phi & \partial_{xz} \Phi \\ \partial_{xy} \Phi & \partial_{yy} \Phi & \partial_{yz} \Phi \\ \partial_{xz} \Phi & \partial_{yz} \Phi & \partial_{zz} \Phi \end{bmatrix}$$

Constant total magnetization

$$\mathbf{h}(x', y', z') = \underset{\substack{\uparrow \\ \text{constant}}}{h} \hat{\mathbf{h}}, \quad \hat{\mathbf{h}} = \begin{bmatrix} \cos I \cos D \\ \cos I \sin D \\ \sin I \end{bmatrix} = \begin{bmatrix} \hat{h}_x \\ \hat{h}_y \\ \hat{h}_z \end{bmatrix}$$

$$V(x, y, z) = -\nabla \cdot (\mathbf{H}(x, y, z))^T \hat{\mathbf{h}} \quad \mathbf{H}(x, y, z) = \mu_0 h \iiint_V \frac{1}{r} d\mathbf{r}$$

$$\mathbf{B}(x, y, z) = \nabla^2 \mathbf{H}(x, y, z) \hat{\mathbf{h}}$$

General case (induced and remanent magnetization components)

total-magnetization vector

$$\mathbf{h}(x', y', z') = \underbrace{\chi(x', y', z') \mathbf{H}(x', y', z')}_{\substack{\uparrow \\ \text{induced magnetization}}} + \underbrace{\mathbf{P}(x', y', z')}_{\substack{\uparrow \\ \text{remanent magnetization}}}$$

$[A/m]$

magnetic intensity field

$$\mathbf{H}(x', y', z') = \underbrace{\mathbf{F}(x', y', z')}_{\substack{\uparrow \\ \text{magnetic induction field}}} / \underbrace{\mu_0}_{\substack{\uparrow \\ \text{magnetic permeability constant (at vacuum)}}}$$

$[A/m]$ $[T]$ $[H/m]$

magnetic susceptibility tensor (dimensionless in the SI)

$$\chi(x', y', z') = \begin{bmatrix} \chi_{11} & \chi_{12} & \chi_{13} \\ \chi_{12} & \chi_{22} & \chi_{23} \\ \chi_{13} & \chi_{23} & \chi_{33} \end{bmatrix}, \quad \chi_{ij} \equiv \chi_{ij}(x', y', z')$$

$i, j = 1, 2, 3$

scalar function

identity matrix

$$\chi(x', y', z') = \chi(x', y', z') \mathbf{I}$$

particular case in which
susceptibility is isotropic

$$V(x, y, z) = -\epsilon_m \iiint_V \left(\nabla \frac{1}{r} \right)^T \mathbf{h}(x', y', z') d\mathbf{r}$$

$$= -\epsilon_m \iiint_V \left(\nabla \frac{1}{r} \right)^T \chi(x', y', z') \mathbf{H}(x', y', z') d\mathbf{r} +$$

$$- \epsilon_m \iiint_V \left(\nabla \frac{1}{r} \right)^T \mathbf{p}(x', y', z') d\mathbf{r}$$