A STUDY OF ROCK DENSITIES IN THE ENGLISH MIDLANDS

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Summary

A knowledge of the densities of rock formations is important in the interpretation of gravity anomalies. This paper describes the results of field and laboratory experiments made on the rocks in the Midlands of England. Density measurements were made on nine geological formations from the Silurian to the Cretaceous and also on Metamorphic rocks from the Malvern area. A catalogue of densities has been prepared for general use based on the assumption that rocks in the field are saturated with water. A table of densities and porosities for a large number of rocks is also given. Among the conclusions drawn about the observed sampling variances, the most significant is that rock formations in the West Midlands do not appear to show any "regional variation" of density. The results of the field and laboratory methods are consistent within the experimental errors of each, in marked contrast to the results reported by S. Hammer from observations in some American mines, from somewhat different data.

1. *Introduction*

1.1. The Bouguer gravity anomaly at any place is defined as the observed value of gravity minus the theoretical value as calculated from the following well-known formula:

$$g_{\text{calc.}} = \gamma_0 - k_1 h + k_2 \rho h - T.$$

Here γ_0 is the value of gravity as given by the International Gravity Formula at the latitude of the station, $k_1 h$ is the so-called "free air" correction due to the height (h) of the station above sea-level, and $k_2 \rho h$ is the attraction of an infinite slab of material of density ρ supposed to lie between the station and the sea-level. The assumption that this material is in the form of an infinite slab may not be justified in regions where the topography surrounding the station is not flat and the term ($-T$) is included to allow for the attraction of the topography. T is always negative whether there is a hill near the station or a hollow. The values of the constants k_1 and k_2 when h is expressed in feet (as it is, throughout this paper) are given by:

$$k_1 = 0{\cdot}09406 \text{ mgal/ft.}$$
$$k_2 \rho = 0{\cdot}0128 \rho \text{ mgal/ft.}$$

It is evident that to calculate the Bouguer anomaly it is necessary to know the density of the geological formation underlying a station. An error of 0·1 g/cm^3 in the density corresponds to an error of nearly 0·13 mgal in the Bouguer anomaly for every 100 ft. This in itself is not a very large error but an error of 0·1 g/cm^3 in the density can have a very large effect on the interpretation of the anomalies because we may be concerned with two adjoining rock formations whose densities may differ by 0·3 or 0·4 g/cm³ only. In the case of the "step-structure" not uncommonly encountered, not only will the depth be grossly in error but also the form of the step might be inverted. It is clear, therefore, that density measurements on rocks are of fundamental importance in gravity surveys. In this paper an account is given of an investigation to determine the densities of rocks by two independent methods. In addition, measurements in mines made by Cook and Thirlaway (1951) are discussed.

1.2. *The field method.*—Nettleton (1939) has described a method of density measurement. He calculates gravity profiles for various assumed values of the density of the underlying formation and selects that one which makes the profile least correlated with the topographic section. An advantage of this method is that the regional trend of gravity, if any, needs no separate attention, but the disadvantage is that a rather large number of calculations is required, and at best two values of the density are obtained giving nearly the same result between which the real value is presumed to lie. Further, in this method an estimate of the standard deviations can be made only with a laborious arithmetical procedure.

The author's method is slightly different from Nettleton's method and is described below. Suppose that there are two stations at which the observed values of gravity are g_2 and g_1. Let g_2' and g_1' be the respective calculated values of gravity. Then

$$g_2' = \gamma_{02} - k_1 h_2 + k_2 \rho h_2,$$
$$g_1' = \gamma_{01} - k_1 h_1 + k_2 \rho h_1,$$

where h_2, h_1 are the heights of the stations above sea-level. Now

$$g_2 = g_2' + \text{Bouguer anomaly at station 2},$$
$$g_1 = g_1' + \text{Bouguer anomaly at station 1}.$$

If the Bouguer anomalies at the stations are the same,

$$g_2 - g_1 = g_2' - g_1' = \gamma_{02} - \gamma_{01} - k_1(h_2 - h_1) + k_2(h_2 - h_1)\rho. \tag{1}$$

$g_2 - g_1$ can be observed by a gravimeter and as all other quantities except ρ can be found, we have an equation to determine ρ by the least squares method. In this, of course, it is assumed that the Bouguer anomaly is known at the two stations. But actually it cannot be known without the knowledge of ρ. Hence, in practice we assume the density obtained from measurements on rock samples to find the trend of the anomaly.

If the apparent topographic corrections T_2, T_1 at the two stations are appreciable they can be found by using tables such as those of Hammer (1939) which are usually calculated for an assumed density of ρ_0. Then the corrections for a density ρ are $T_2\rho/\rho_0$ and $T_1\rho/\rho_0$ respectively. If these are included in equation (1), we have

$$\Delta g = g_2 - g_1 = \gamma_{02} - \gamma_{01} - k_1(h_2 - h_1) + [k_2(h_2 - h_1) - (T_2 - T_1)/\rho_0]\rho$$
$$= \Delta\gamma_0 - k_1\Delta h + [k_2\Delta h - (T_2 - T_1)/\rho_0]\rho$$

or

$$\Delta g - \Delta\gamma_0 + k_1\Delta h = [k_2\Delta h - (T_2 - T_1)/\rho_0]\rho. \tag{2}$$

Let us denote the left-hand side by Δy and the right-hand side by $\rho\Delta x$. Now, if a line of stations at different heights be established, then taking one of them as a base, we can calculate Δy and Δx for each of the stations. The graph of Δy against Δx will be a straight line (through the origin) and its slope will be ρ. In the next section such measurements made over a number of formations are described.

The simplest way to apply this method is to make the measurements at various points on the slope of a hill. Special attention must be paid to the following factors:

(1) The hill must be an erosion feature and not an anticline or a faulted structure. River valleys often provide excellent examples of such features.
(2) The surrounding gravity field must be not abnormally disturbed by structures such as intrusions, troughs, faults, etc., otherwise a fictitious density will be obtained.
(3) The hill must be of a sufficient height so that the differences of gravity between consecutive stations are appreciably greater than the standard deviation of a single determination.
(4) The topography of the surrounding country must be allowed for.

1.3. *The laboratory method.*—In this method samples of rocks are collected from outcrops near the station and their density is found from laboratory determinations. This method, however, may have the following drawbacks because the outcrop may not be truly representative of the strata below:

(1) If the strata consist of bands of more than one lithological character (such as alternating bands of shales and limestones) the density from a specimen of one character may be widely different from that found from another. The "effective" density determining the Bouguer anomaly will not simply be a mean of all such densities but will depend on the frequency of each band within the formation.
(2) The rocks may be compacted at great depths. Even with specimens of the same lithological character, the density may be different from different horizons. Thus the density of the Oxford clay (Athleta zone) in the Stewartby brickwork pits was found to vary from 1·85 to 1·88 from the surface to a depth of about 100 ft.
(3) It is usually not known with certainty to what extent the rocks contain water and the best assumption is that the "true" density lies somewhere between the density of a saturated rock specimen and that of a dry one.
(4) The weathering of rocks may affect their density considerably.

The field method is the more basic of the two methods but also the more difficult one. The laboratory method is easy but not necessarily correct owing to the causes just mentioned.

2. *The field measurements of density*

2.1. The measurements were made with an Askania type gravimeter belonging to the Department of Geodesy and Geophysics, Cambridge University. A discussion of the accuracy of measurements with this instrument has been made by Cook (1951). The smallest division on the scale corresponds to 0·06 mgal but the final accuracy of measurement is usually about 0·18 mgal, owing chiefly to the drift of the scale. The calibration has a standard deviation of 0·42 per cent. Some measurements by the field method have been made previously by Cook and Thirlaway (1951) and, using Nettleton's method, by the Anglo-Iranian Oil Company in their gravity surveys (privately circulated work). But both these measurements were restricted in the number of the lithological formations covered. In the present investigation an extended study of the rock densities in the West Midlands has been made. In the next few sections the following measurements are described:

(1) Upper Lias at Upton St. Leonards (Glos.).
(2) Silurian Limestone and Shales at Woolhope Cockshoot (Herefords.).
(3) Old Red Sandstone at Hanley William (Worcs.).
(4) Old Red Sandstone on May Hill (near Monmouth).
(5) Middle Coal Measures Shales and Sandstones—Forest of Wyre (Worcs.).
(6) Carboniferous Limestone—Forest of Dean (Glos.).
(7) Triassic Lower Mottled Sandstone—Bridgnorth Cliffs (Shropshire).

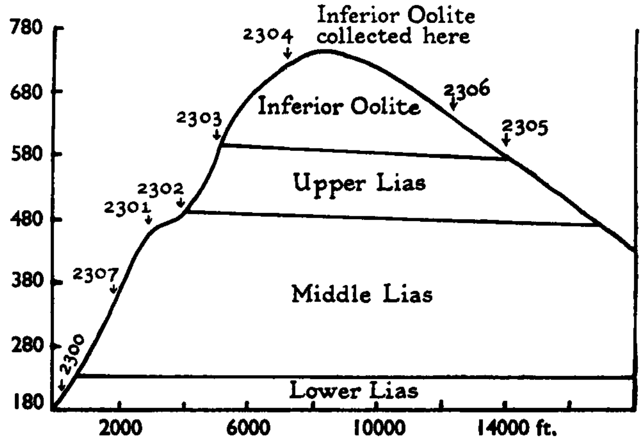

FIG. 1.—*Profile of the hill near Upton St. Leonards with locations of gravimeter stations.* (*Section along line AA′ on Fig.* 4A.) *Ordinates: Height in feet above O.D.*

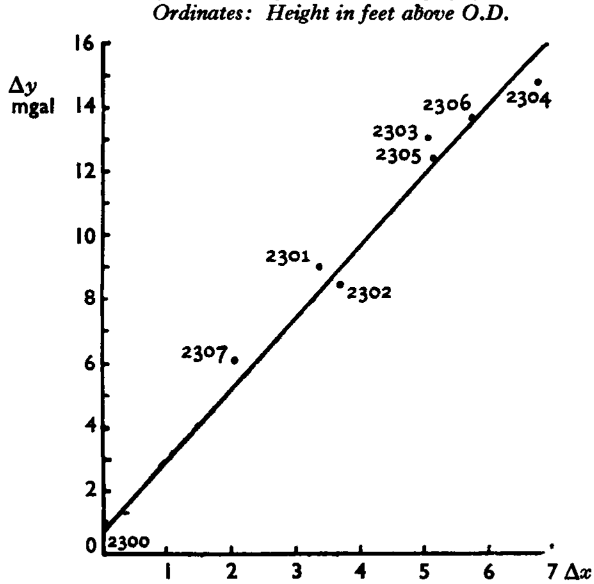

FIG. 2.—*Density line at Upton St. Leonards.*

The observation on Lias is treated first because it illustrates some important points concerning the hill measurements.

2.21. *Upper Lias at Upton St. Leonards* (*Gloucestershire*), Lat. 51° 49′ 51″ N., Long. 2° 12′ 11″ W.* Fig. 1 shows the simplified geological structure of the hill

* The latitude and longitude refer to the lowest station on the particular hill.

which is the erosion work of numerous small streams. The observations, which are reproduced in detail in Table I, were made along the road from Upton to Painswick (B4073). Although the hill is made of three distinct layers of rocks, the observations indicate only one straight line, which means that the densities of these layers are not very different from each other. Fig. 2 is a graph of Δy against Δx.

TABLE I

Observations near Upton St. Leonards

Base 2300. Height of Base above O.D. = 183 ft. $T_1 = 0{\cdot}22$ mgal. $\rho_0 = 2$ g/cm³.

I	II	III	IV	V	VI	VII	VIII	IX	X
Stn. No.	Δg_s mgal	$\Delta \gamma_0$ mgal	Δh ft.	$0{\cdot}09406\Delta h$ mgal	$0{\cdot}0128\Delta h$	T_s mgal	$\frac{T_s - T_1}{2}$	Δx VI–VIII	Δy II–III+V mgal
2307	−10·62	−1·5	+162	+15·24	2·074	0·35	+0·06	2·06	6·12
2301	−18·62	−1·8	+274	25·77	3·51	0·48	+0·13	3·38	8·95
2302	−20·42	−0·9	+296·4	27·88	3·79	0·35	+0·06	3·73	8·36
2303	−26·24	−1·4	+402·2	37·83	5·15	0·40	+0·09	5·06	13·00
2304	−37·38	−1·5	+539	50·70	6·90	0·48	+0·13	6·77	14·82
2305	−28·54	−3·4	+398·5	37·48	5·10	0·23	0·00	5·10	12·34
2306	−32·26	−3·1	+454·7	42·77	5·82	0·38	+0·08	5·74	13·61
2300 (Base)	0	0	0	0	0	0·22	0	0	0

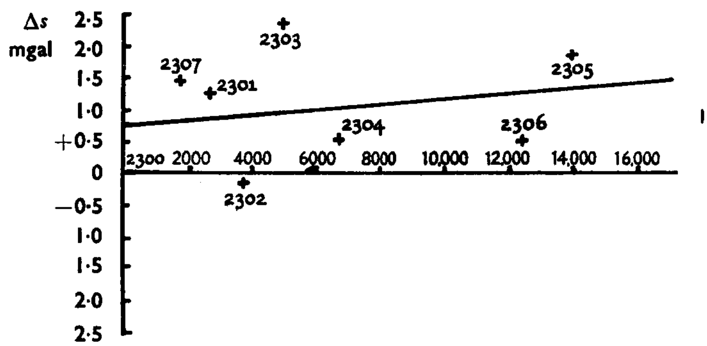

FIG. 3.—*Investigation of the horizontal gradient at Upton St. Leonards. Horizontal line shows distance in feet from Station 2302.*

The best straight line is found to be

$$\Delta y = (2{\cdot}28 \pm 0{\cdot}19)\Delta x + (0{\cdot}574 \pm 0{\cdot}83).$$

The place where samples were collected is shown in Fig. 4A which is a simplified map of the region. The regional trend in this as well as all the other cases was found to be insignificant. The examination is made by plotting the residual $\Delta s = \Delta y_{\text{obs.}} - \rho\Delta x$ (mgal) against D, the distance in feet from the base assuming

$$\Delta s = k'D + b.$$

As an example, the graph of Δs against D in the present case is shown in Fig. 3. The best straight line is found to be

$$\Delta s = (0{\cdot}47 \pm 0{\cdot}80) \times 10^{-4} D + (0{\cdot}73 \pm 0{\cdot}62) \text{ mgal}.$$

This shows that the value of k' is not significantly different from zero.

Result for density:

$$\rho = 2{\cdot}28 \pm 0{\cdot}19 \text{ g/cm}^3.$$

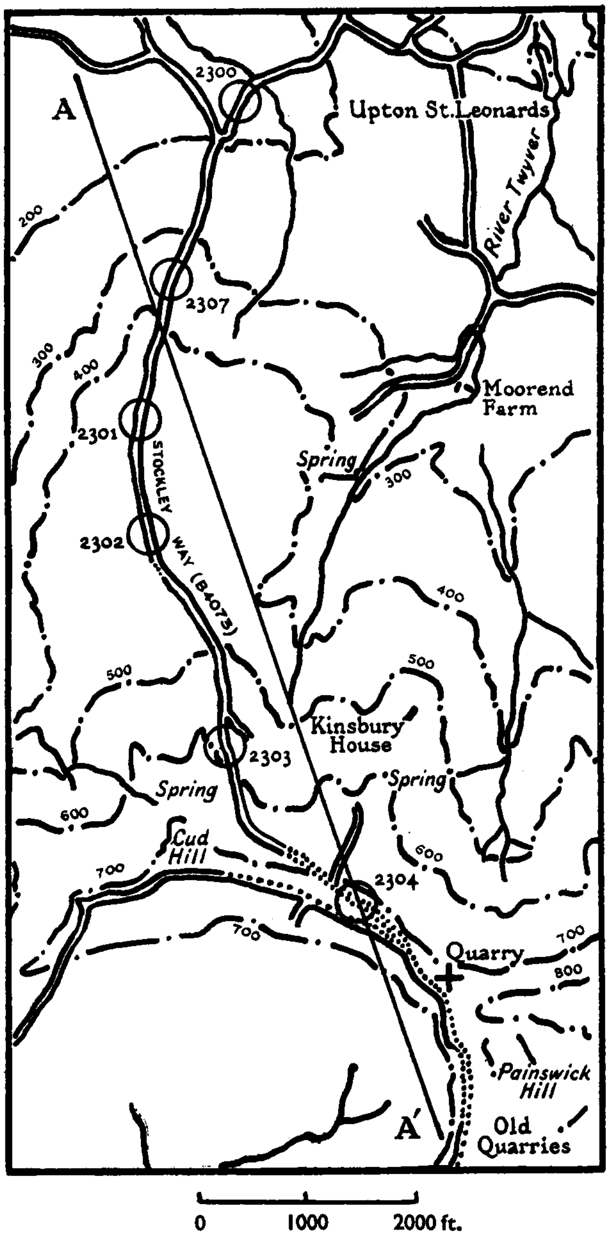

FIG. 4A.—*Area near Upton St. Leonards (Upper Lias). Based on the O.S. Map, with the sanction of the Controller of H.M. Stationery Office.*
Geological section along AA′ is shown in Fig. 1.

Contours of topography at 100 *ft. interval.*		
Exposures from which samples were taken.		*Key for all the maps.*
Gravimeter station.		

2.22. *Silurian limestone and shales: Ludlow beds at Woolhope Cockshoot (Herefordshire).* Lat. 52° 02′ 03″ N., Long. 2° 31′ 19″ W.

Site: The hill has been eroded by the tributaries of the Preston Brook in the Woolhope anticline. It is composed of alternating bands of limestone and shales, the dip being slightly to the south-east.

Samples: The samples were collected in the quarry just above Woolhope Cockshoot. The quarry is marked in Fig. 4B which is a map of the area.

Gravity measurements: Measurements were made at five stations. The height of the lowest one above O.D. is 305 ft. and that of the highest one 734 ft. The best straight line obtained is

$$\Delta y = (2{\cdot}73 \pm 0{\cdot}22)\Delta x + (0{\cdot}109 \pm 0{\cdot}63).$$

The topographic correction was of the order of 0·5 mgal and has been applied.

Result for density:

$$\rho = 2{\cdot}73 \pm 0{\cdot}22 \text{ g/cm}^3.$$

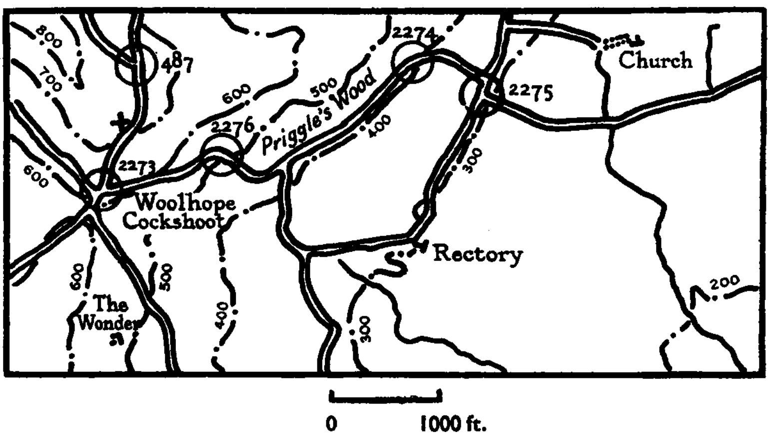

FIG. 4B.—*Area near Woolhope Cockshoot (Silurian limestone and shales). Based on the O.S. Map, with the sanction of the Controller of H.M. Stationery Office.*

2.23. *Old Red Sandstone at Hanley William (Worcestershire).* Lat. 52° 18′ 43″ N., Long. 2° 29′ 43″ W.

Site: This hill, which is eroded by the tributaries of the Teme, is made of Downtonian rocks (Lower Old Red Sandstone). Wickham King (1925) mentions that along the lower Teme valley the strata consist of the Middle and Lower Downtonian. The hill is nearer the lower part of the Teme and it is estimated that it is composed of about 200 ft. of red and green marls underlain by about 300 ft. of calcareous and marly sandstone. Below this is a purple-green fine sandstone 90 ft. in thickness underneath which is a thickness of 400–500 ft. of "deep compact marls devoid of sandstone". This is followed by Lingula marls (about 315 ft.), Ledbury group (400 ft.) and Temeside group (about 150 ft.).

Samples: These were collected from an exposure by the Long Bank that is marked on Fig. 4C.

Gravity measurements: Measurements were made at nine points on the hill. The height above O.D. of the lowest station is 200 ft. and that of the highest one 800 ft.

The best straight line is

$$\Delta y = (2{\cdot}47 \pm 0{\cdot}048)\Delta x - (0{\cdot}18 \pm 0{\cdot}23).$$

The topographic correction was found to be negligible.

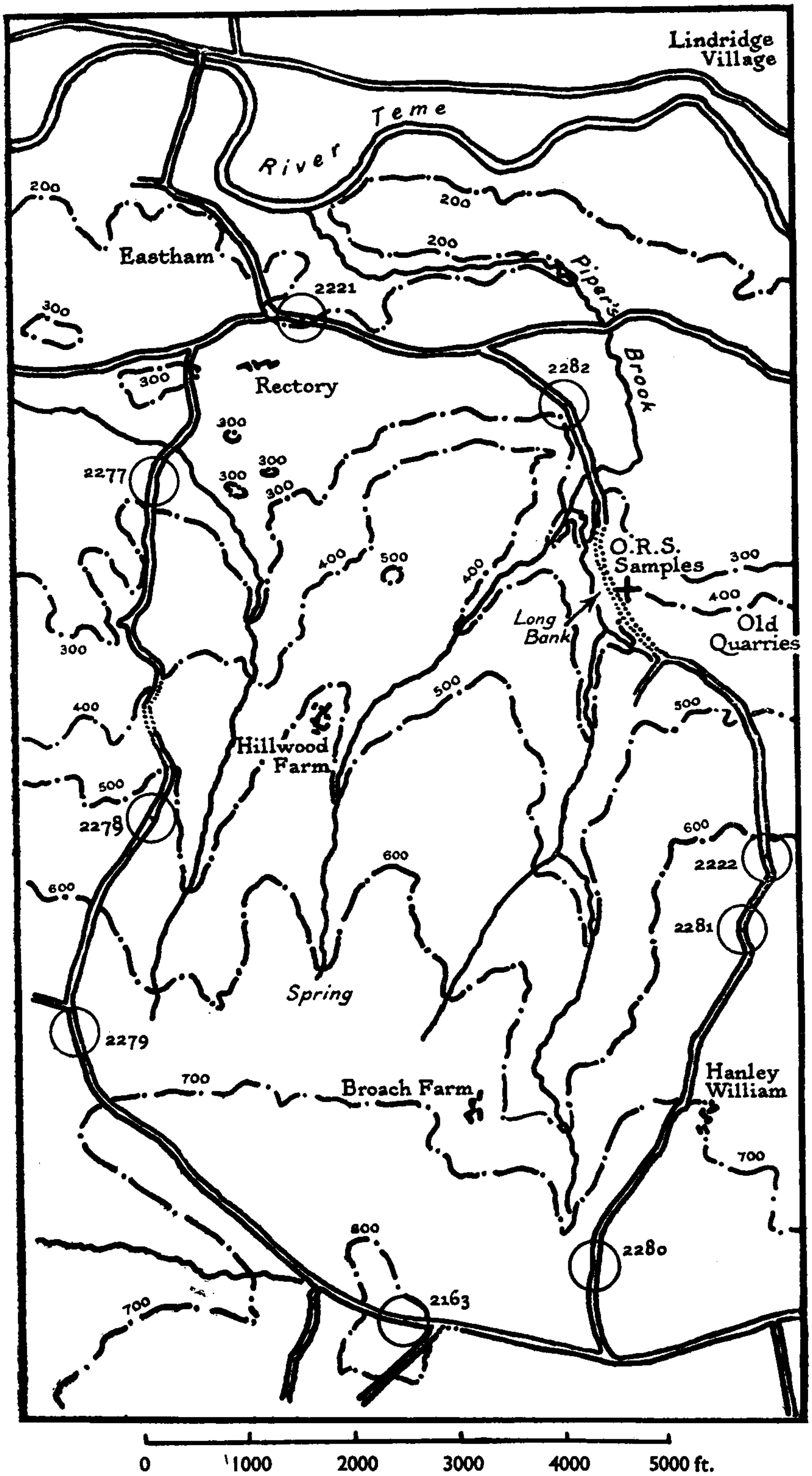

FIG. 4C.—*Map of the region near Hanley William (Old Red Sandstone). Based on the O.S. Map, with the sanction of the Controller of H.M. Stationery Office.*

Result for density:

$$\rho = 2{\cdot}47 \pm 0{\cdot}048 \text{ g/cm}^3.$$

2.24. *Old Red Sandstone at May Hill near Monmouth (Ditton Series).* Lat. 51° 48′ 40″ N., Long. 2° 42′ 02″ W.

Site: The hill has been eroded chiefly by the River Wye and is composed of Lower Old Red Sandstone beds.

Samples: These were collected from an exposure near the "Fiddler's Elbow" marked on Fig. 4D.

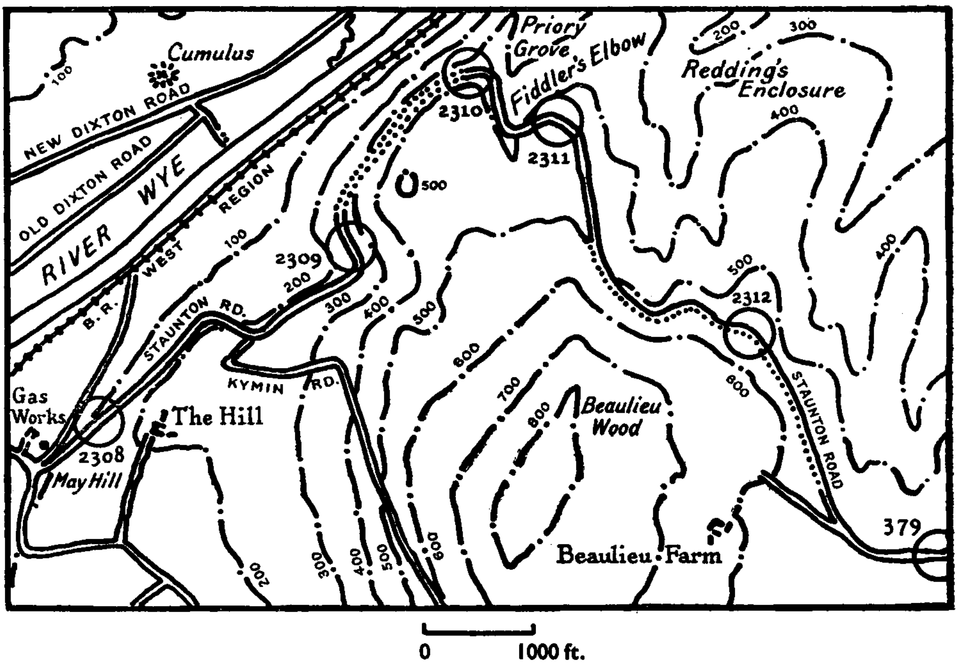

FIG. 4D.—*May Hill near Monmouth (Old Red Sandstone). Based on the O.S. Map, with the sanction of the Controller of H.M. Stationery Office.*

Gravity measurements: The measurements were made at six stations along "Staunton road" which goes over the hill from Monmouth to Staunton. The height above O.D. of the lowest station is 77 ft. and that of the highest one 551 ft. The topographic correction was found to be as high as 1·15 mgal for Station No. 2309; for others it was of the order of 0·50 mgal. The best straight line is

$$\Delta y = (2{\cdot}46 \pm 0{\cdot}08)\Delta x + (0{\cdot}16 \pm 0{\cdot}31).$$

Result for density:

$$\rho = 2{\cdot}46 \pm 0{\cdot}08 \text{ g/cm}^3.$$

This compares well with that at Hanley William in the previous section.

2.25. *Carboniferous Limestone in the Forest of Dean (Gloucestershire).* Lat. 50° 51′ 00″ N., Long. 2° 35′ 40″ W.

Site: The hill is eroded by the River Wye and its tributaries and is situated outside the north-west margin of the Forest of Dean near Lower Lydbrook in the Carboniferous Limestone strip that girdles the coalfield area.

Samples: These were collected from a quarry near Pope's Grove towards the northern part of the hill. Fig. 4E is a map of the region.

Gravity measurements: Eight stations were available. The height above O.D. of the lowest one is 147·1 ft. and that of the highest one 635·5 ft. The best straight line is

$$\Delta y = (2{\cdot}54 \pm 0{\cdot}07)\Delta x - (0{\cdot}25 \pm 0{\cdot}11).$$

The topographic correction is negligible.

Result for density: $\rho = 2{\cdot}54 \pm 0{\cdot}07\ \mathrm{g/cm^3}$.

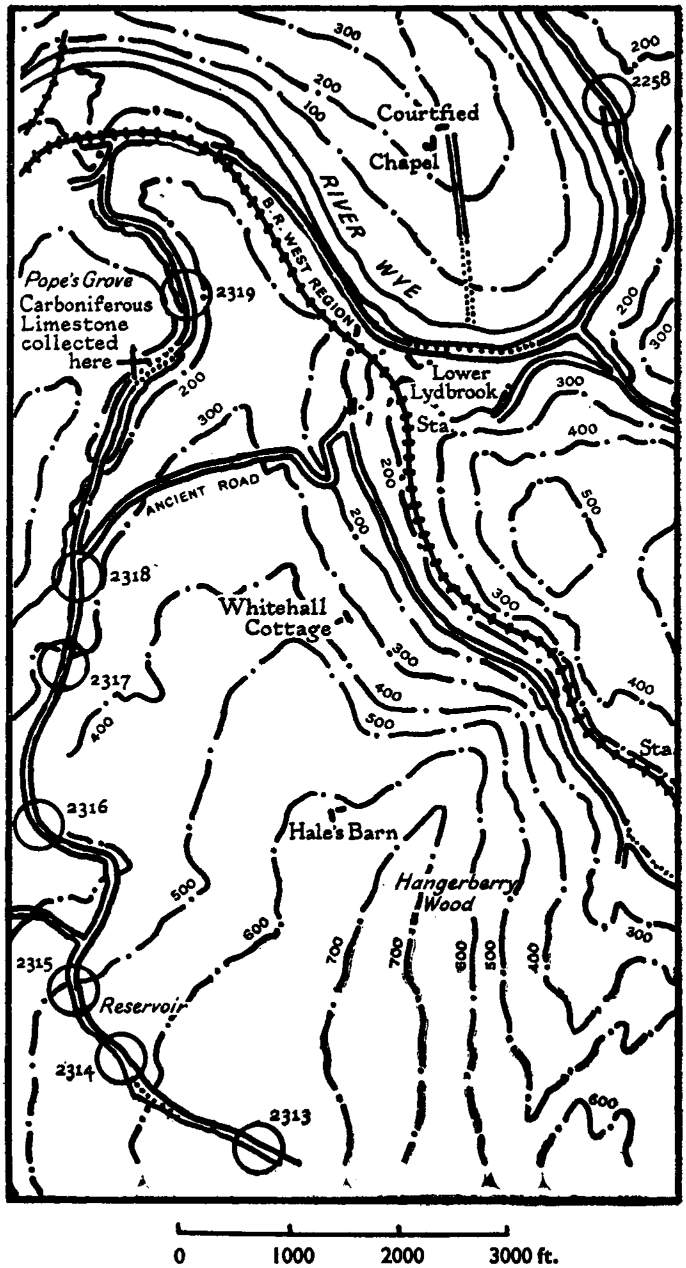

FIG. 4E.—*Forest of Dean near Lower Lydbrook (Carboniferous Limestone). Based on the O.S. Map with the sanction of the Controller of H.M. Stationery Office.*

2.26. *Middle Coal Measures in the Forest of Wyre (Worcestershire).* Lat. 52° 23′ 10″ N., Long. 2° 24′ 40″ W.

Site: The hill is situated near Far Forest and has been eroded by the Dowles, Lem and Baveney brooks. The exposures in the Dowles valley belong to the Middle Coal Measures down to about 880 ft. from the surface (Kidston, Cantrill and Dixon, 1917). They do not here bear any workable coal. A little farther north (at Billingseley), however, it is known that the sweet coal group is present.

Samples: These were collected from an exposure near the confluence of the three brooks north of the Quarry cottages. Fig. 4F is a simplified map of the region.

Gravity measurements: Nine stations were available for these. The height of the lowest one above O.D. is 255 ft. and that of the highest one 468 ft. The best straight line is

$$\Delta y = (2{\cdot}70 \pm 0{\cdot}10)\Delta x - (0{\cdot}039 \pm 0{\cdot}19).$$

The topographic correction is only of the order of 0·1 mgal.

Result for density:

$$\rho = 2{\cdot}70 \pm 0{\cdot}10 \text{ g/cm}^3.$$

2.27. *Triassic Sandstone of Bridgnorth Cliffs (Shropshire).* Lat. 52° 32′ 18″ N., Long. 2° 24′ 42″ W.

Site: Bridgnorth is situated on the Castle Hill (height about 150 ft.) eroded out of Lower Mottled Sandstone by the River Severn. One observation was made at the church on the top of the hill and the other on the bank of the river. It was not found practicable to make observations other than these two.

Result for density:

$$\rho = 2{\cdot}30 \pm 0{\cdot}13 \text{ g/cm}^3.$$

The topographic correction was found to be 0·2 mgal.

Samples: These were collected at various places on the road from Bridgnorth to Quattford. There are several good exposures.

3. *Laboratory measurements of density*

3.1. One object of these measurements was to obtain independent values of rock densities for comparison with the gravimeter values. A second object was to investigate the sampling variances of the densities when several specimens of the same lithological character are obtained from the same exposure and also when they are obtained from different exposures. Samples were collected from the hills on which gravimeter measurements were made and the exact position of the exposure was marked on the 6-inch Ordnance Survey map for future reference. A comparison of the densities obtained by the two methods would show whether a random sampling is sufficient to determine the density to be used in the calculation of anomalies.

3.2. *The measurement of density*.—It has been pointed out that the "field" density of a rock must lie between the saturated and the dry density. The following method was used to saturate the rock specimen:

The specimen was placed under water in a small vessel connected to a vacuum pump. The vessel was evacuated down to about 10 mm of mercury and the specimen kept under vacuum for about 30 minutes after air bubbles had ceased to come out of it. It was then left under atmospheric pressure for about 16 hours.

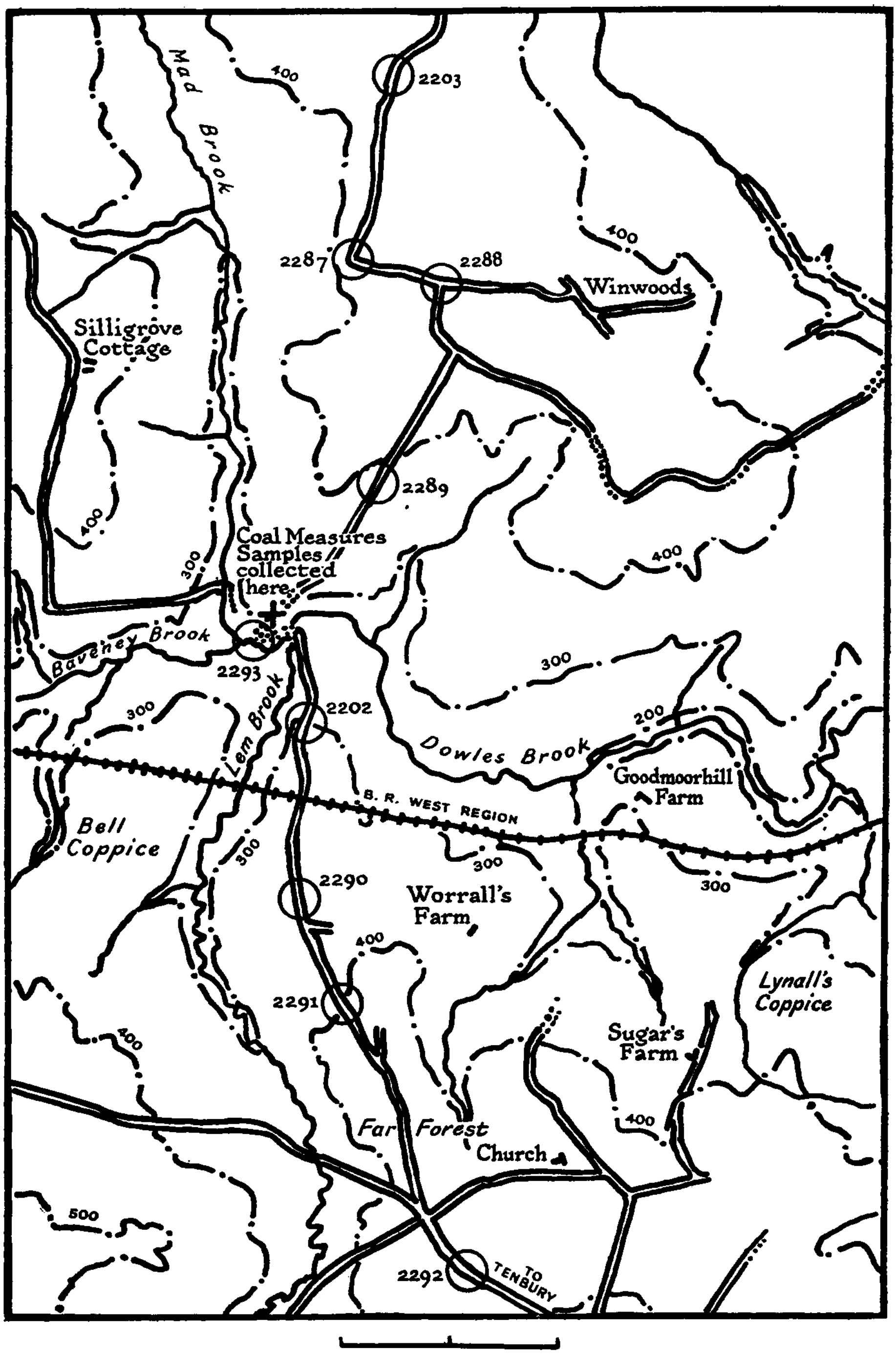

FIG. 4F.—*Part of the Forest of Wyre (Middle Coal Measures). Based on the O.S. Map, with the sanction of the Controller of H.M. Stationery Office.*

This method is slightly different from that used by Thirlaway (see Cook and Thirlaway, 1951). The following example shows the difference in weights when the specimen was weighed at various instants during the process of saturation.

Specimen—Chalk

Weight immediately after evacuation for 30 minutes ...	34·27 g.
Weight after 6 hours' soaking	34·38 g.
Weight after 16 hours' soaking	34·51 g.
Weight after 30 hours' soaking	34·51 g.

It was found convenient to leave the specimen soaking overnight and this usually meant 16 hours' soaking.

The specimen was dried by heating in an oven at 110 deg. C for about three hours. This time was found sufficient to dry it completely. It was cooled in a desiccator.

The following procedure of measurements was adopted:

Filter paper was used to remove the surface moisture from the saturated specimen which was then weighed in air and water. It was next dried by heating and finally weighed again in air.

Let V be the volume occupied by the grains in a specimen and let v be the total volume of the pores. Let ρ_g be the grain density, ρ_s the density when saturated and ρ_d that when dry.

If w_1 = weight of the saturated specimen in water,

w_2 = weight of the dry specimen in air,

w_3 = weight of the saturated specimen in air,

it follows that

$$w_1 = V(\rho_g - 1) \ldots (a); \quad w_2 = V\rho_g \ldots (b); \quad \text{and} \quad w_3 = V\rho_g + v \ldots (c).$$

Further, $\rho_s = w_3/(V+v)$ and $\rho_d = w_2/(V+v)$,

because the total volume is $V+v$.

From (a) and (b) we obtain

$$\rho_g = w_2/(w_2 - w_1),$$

and hence also

$$\rho_g = \rho_d \div \{\rho_d - w_1/(V+v)\} \ldots (d),$$

ρ_s can be expressed as $1 + w_1(V+v)$.

Therefore (d) becomes

$$\rho_g = \rho_d/(\rho_d - \rho_s + 1) = \rho_d/(1-\Delta), \quad \text{where} \quad \Delta = \rho_s - \rho_d.$$

Also, v/V which is called the void ratio and is denoted by ϵ, can be seen to be equal to

$$\Delta/(1-\Delta).$$

We can define the porosity of a specimen as $100\Delta/(1-\Delta)$.

Ordinary tap water was used instead of distilled water in all the measurements but no detectable difference was found in the weights, as the following example shows:

Specimen—Granite (Dartmoor)

Weight in air	88·073 g.
Weight in tap water	54·060 g.
Weight in distilled water	54·060 g.
Density of specimen	2·58 g/cm³.

Allowance must be made for the temperature of water. This is done by dividing the density obtained by the density of water at the temperature concerned. This allowance amounts to 1 part in 1000 at temperatures of 15–20 deg. C and hence is quite negligible. Where this was not the case in the following measurements the necessary correction has been made.

3.3. As data on density measurements of rocks are not very plentiful it is thought desirable to reproduce the results in detail. The existing information in Great Britain is more or less confined to a paper by Moore (1902), another one by Holmes (1921) and the work of Cook and Thirlaway (1951). Table II shows the results and the columns are self-explanatory. The standard deviation of a single measurement of density is $\pm 0{\cdot}01$ g/cm³, whence it follows that the standard deviation σ_Δ of Δ is $\pm 0{\cdot}014$ g/cm³. These were not found to vary with lithology. The standard deviation σ_ϵ of ϵ is equal to $[1/(1-\Delta)^2]\sigma_\Delta$ and Table III gives some values of $1-\Delta$ and σ_ϵ.

The maximum error in ϵ in the observed values of Table II is of the order of $\pm 0{\cdot}06$ per cent. The standard error in the value of ρ_g is $\pm 0{\cdot}05$ g/cm³ for $\rho_g = 3$ and $1-\Delta = 0{\cdot}9$. The error, which is inversely proportional to $(1-\Delta)^2$, increases with ρ_g and as a value of $\rho_g = 3{\cdot}00$ g/cm³ is rather a high one, $\pm 0{\cdot}05$ g/cm³ may be taken to be the maximum standard error of ρ_g.

3.4. The total variance in the measurement of rock density is the combined effect of σ_e^2, σ_l^2 and σ_r^2 where these are respectively the contributions of the errors of measurement, the variation within one exposure and the regional variation of a certain type of rock. The important question is whether the variance of density within one exposure can also account for the variance among different exposures of the same rock. If there are k samples in each exposure, then the variance of a single measurement is $\sigma_e^2 + \sigma_l^2$, while the variance of the mean of these k samples will be $(\sigma_e^2 + \sigma_l^2)/k$. As there are $(k-1)$ degrees of freedom in each exposure, this variance will have $m(k-1)$ degrees of freedom. If for each one of the m exposures a mean density is found, this will be distributed with a variance of $\sigma_r^2 + (\sigma_e^2 + \sigma_l^2)/k$ which will have $(m-1)$ degrees of freedom. The total number of samples is $mk = n$. We can then construct the following table:

	Degrees of freedom	Value of variance
Between exposures	$m-1$	$\frac{\sigma_e^2}{k} + \frac{\sigma_l^2}{k} + \sigma_r^2$
Within exposures	$m(k-1)$	$\sigma_e^2 + \sigma_l^2$
Total	$mk-1$	$\left(\frac{1+k}{k}\right)(\sigma_e^2 + \sigma_l^2) + \sigma_r^2$

Whether the two estimates of the variance differ significantly can be tested by calculating Fisher's parameter z (= half the difference of the natural logarithms of the two estimates) and then finding the probability of exceeding this value by chance on the given degrees of freedom from tables given by Fisher (1946). Table IV gives the values of the variances, the respective degrees of freedom, the parameter z and the probability P in the case of Chalk, Silurian limestone, Old Red Sandstone and Lower Mottled Sandstone.

The values of P clearly show that there is no difference among the several exposures in one rock which is not accounted for as a random sampling effect of the differences within each exposure. In other words, there is no "regional variation of density" for the rocks in Table IV.

TABLE II

Densities and Porosities of Rocks

Specimen No.	Approximate lithology	Locality	Saturation density ρ_s g/cm³	Dry density ρ_d g/cm³	Δ $\rho_s-\rho_d$ g/cm³	Grain density ρ_g g/cm³	ϵ per cent
1	Limestone	Woolhope Cockshoot	2·67	2·64	0·03	2·72	3·1
2	Keuper s.s.		2·13	1·81	0·32	2·67	47·0
3·1	Sandy l.s.	Hanley William	2·56	2·47	0·09	2·72	9·9
3·2	Shaly s.s.	Hanley William	2·56	2·47	0·09	2·72	9·9
3·3	Marly shales	Hanley William	2·53	2·41	0·12	2·74	14·0
3·4	Calcareous s.s.	Hanley William	2·62	2·59	0·03	2·67	3·1
4·1	Calcareous shales	Nr. Woolhope	2·55	2·45	0·10	2·72	12·2
4·2	Shale	Nr. Woolhope	2·51	2·43	0·08	2·64	8·7
5·1	Calcareous shales	Outside Woolhope	2·58	2·50	0·08	2·72	8·7
5·2	Limestone	Outside Woolhope	2·68	2·65	0·03	2·73	3·1
6·4	Siltstone	Little Malvern	2·62	2·56	0·06	2·72	6·4
7·1	Carbonaceous s.s.	Wyre Forest	2·50	2·35	0·15	2·76	17·2
8·1	Shale	Nr. Highley	2·49	2·32	0·17	2·80	20·5
8·2	Sandstone	Nr. Highley	2·41	2·22	0·19	2·74	23·5
8·3	Sandstone	Nr. Highley	2·43	2·27	0·16	2·70	19·0
9	Calcareous s.s. (Keele beds)	Nr. Chelmarsh	2·46	2·31	0·15	2·72	17·2
10	Lower Mottled s.s.	Nr. Quattford	2·26	2·01	0·25	2·68	33·3
11	Calcareous s.s. (Enville beds)	Nr. Tuckhill	2·33	2·11	0·22	2·71	28·2
12	Mottled s.s.		2·22	1·95	0·27	2·68	37·0
13	Oolitic l.s.	Nr. Upton St. Leonards	2·36	2·20	0·16	2·62	19·0
14·1	Liassic l.s.	Aust Cliff	2·56	2·46	0·10	2·74	11·1
14·2	Liassic l.s.	Aust Cliff	2·55	2·44	0·11	2·74	12·4
14·3	Green Marl	Aust Cliff	2·38	2·13	0·25	2·84	33·3
14·4	Green Marl	Aust Cliff	2·40	2·13	0·27	2·92	37·0
14·5	Blue Paper shales	Aust Cliff	1·90	1·63	0·27	2·24	37·1
14·61	Red Marl	Aust Cliff	2·44	2·26	0·18	2·76	22·0
14·62	Red Marl	Aust Cliff	2·42	2·23	0·19	2·76	23·5
14·63	Red Marl	Aust Cliff	2·44	2·26	0·18	2·76	22·0
14·64	Red Marl	Aust Cliff	2·44	2·25	0·19	2·78	23·5
15	Keuper s.s.	Nr. Newent	2·26	2·04	0·22	2·62	28·2
16·1	Metamorphic (gneiss)	Malverns	2·62	2·59	0·03	2·67	3·1
16·2	Metamorphic (gneiss)	Malverns	2·73	2·72	0·01	2·75	1·01
16·3	Metamorphic (gneiss)	Malverns	2·64	2·60	0·04	2·71	4·2
16·6	Metamorphic (gneiss)	Malverns	2·64	2·60	0·04	2·71	4·2
16·7	Metamorphic (gneiss)	Malverns	2·75	2·72	0·03	2·80	3·1
17·1	O.R.S.	May Hill (nr. Monmouth)	2·61	2·56	0·05	2·70	5·27
17·2	Sandy Marl	May Hill (nr. Monmouth)	2·60	2·53	0·07	2·72	7·52
18·11	Carbonaceous l.s.	Nr. Lydbrook (Forest of Dean)	2·66	2·64	0·02	2·70	2·02
18·12	Carbonaceous l.s.	Nr. Lydbrook (Forest of Dean)	2·71	2·69	0·02	2·74	2·02
18·2	Carbonaceous l.s.	Nr. Lydbrook (Forest of Dean)	2·64	2·60	0·04	2·71	4·15
18·31	Carbonaceous l.s.	Nr. Lydbrook (Forest of Dean)	2·64	2·62	0·02	2·68	2·02
18·32	Carbonaceous l.s.	Nr. Lydbrook (Forest of Dean)	2·67	2·65	0·02	2·71	2·02
19	Silurian l.s.	Wenlock Edge	2·65	2·63	0·02	2·69	2·02
20·1	Calcite	Vale of Severn (nr. Presthope)	2·70	2·69	0·01	2·72	1·01
20·2	Limestone	Vale of Severn (nr. Presthope)	2·65	2·61	0·04	2·72	4·15
20·3	Limestone with calcite	Vale of Severn (nr. Presthope)	2·69	2·68	0·01	2·71	1·01

TABLE II—*continued*

21	Limestone	West of Much Wenlock	2·61	2·54	0·07	2·73	7·5
A1	Chalk	Nr. Cambridge	1·93	1·55	0·38	2·50	61·4
A2	Chalk	Nr. Cambridge	1·97	1·60	0·37	2·54	58·8
A3	Chalk	Nr. Cambridge	1·94	1·52	0·42	2·62	72·5
B1	Chalk	Nr. Cambridge	1·94	1·59	0·35	2·45	54·0
B2	Chalk	Nr. Cambridge	1·96	1·60	0·36	2·50	56·4
C1	Chalk	Nr. Cambridge	1·96	1·54	0·42	2·66	72·5
C2	Chalk	Nr. Cambridge	1·95	1·52	0·43	2·67	75·5
C3	Chalk	Nr. Cambridge	1·95	1·54	0·41	2·61	69·5

Notes: Specimen numbers have no textual significance.
s.s. = sandstone ; l.s. = limestone.
O.R.S. = Old Red Sandstone.
Letters A, B, C denote pits from which the Chalk samples were obtained.

TABLE III

The Errors in Porosity

$1-\Delta$ g/cm^3	σ_E per cent
1·00	0·014
0·75	0·025
0·50	0·056
0·25	0·224
0·10	1·40
0·05	5·60
0·00	∞

TABLE IV

The Analysis of Variance

Rock formation		ν	Variance (g/cm^3)^2	z	P
Chalk	Between exposures	2	0·0001	1·040	Slightly
	Within exposures	6	0·0008		>5 per cent
Silurian limestone	Between exposures	4	0·0007	0·178	≫5 per cent
	Within exposures	5	0·0010		
O.R.S.	Between exposures	1	0·0005	1·131	>5 per cent
	Within exposures	3	0·0013		
Lower Mottled s.s.	Between exposures	1	0·0008	1·040	≫5 per cent
	Within exposures	1	0·0001		

3.5. *Discussion of sampling variances.*—The specimens in Table II naturally fall into the following five groups: (1) sandstones with four sub-groups, (2) limestones with three sub-groups, (3) marls with two sub-groups, (4) shales, and (5) metamorphic rocks. Table V gives the mean saturated density of each group, the observed standard deviation and the number of degrees of freedom on which the variance has been calculated. The corresponding quantities for the dry density could also be given but it is more useful to consider the porosity, and the table also contains the necessary entries in this respect.

TABLE V

Sampling Variance of Density and Porosity

No.	Lithological group	Mean $\bar{\rho}_g$ g/cm³	Standard deviation σ_ϱ g/cm³	Degrees of freedom ν	Mean $\bar{\epsilon}$	Standard deviation σ_ϵ	Degrees of freedom ν
1	Sandstones						
1*a*	Old Red	2·58	0·035	5	7·1	2·46	4
1*b*	Carboniferous	2·42	0·061	5	21·95	4·84	5
1*c*	New Red (all)	2·22	0·061	3	36·4	7·95	3
1*d*	Lower Mottled	2·247	0·023	2	...	...	...
2	Limestones						
2*a*	Silurian	2·65	0·034	8	4·31	1·88	9
2*b*	Carboniferous	2·66	0·029	4	2·44	0·95	4
2*c*	Chalk	1·95	0·0126	8	65·13	8·57	8
3	Marls	2·42	0·025	5	...	...	...
3*a*	Red Marls (Aust Cliff)	...	...	...	22·75	0·87	3
3*b*	Green (Aust Cliff)	...	...	...	35·15	2·57	1
4	Shales (all)	2·56	0·055	6	...	...	...
5	Metamorphic rocks from Malvern Hills	2·69	0·056	4	3·55	1·96	4

Several interesting conclusions can be drawn from this analysis. In the first place, Silurian and Carboniferous limestones seem to form a remarkably uniform deposit despite their different ages. Sandstones, on the other hand, are more variable, but sandstones of the same age group show a small variance of density. The porosity of sandstones even of the same age group has a rather large variance. In the case of New Red Sandstone, this is not unexpected judging from the conditions of its deposition. Chalk appears to be the most uniform deposit so far as density is concerned and also the most variable in porosity. As chalk is supposed to be of shallow-water origin it might be argued that a full differentiation of the sediments could not have taken place and consequently chalk would be very far from being a homogeneous deposit and then the first fact would seem very surprising. However, a glance at Table II will show that the grain density of chalk is very variable. The outstanding fact is its low value, although pure calcite has a density of 2·70 g/cm³. It was suggested by Dr Cook that as chalk grains may be shells of animals, they might have cavities so minute that complete drying of a specimen cannot be achieved. Therefore, a sample of chalk was pulverized and dried when its grain density was found to be 2·64 g/cm³ (obtained by a pycnometer), which seems to support the above suggestion. This value is still too low and the difference is probably due to a number of shells not being crushed at all or due to an adsorbed film of water on each grain which cannot be driven out except at about 300 deg. C. or beyond. The second reason seems to be the more likely one from a number of similar experiments made by the author on granular materials. As chalk contains a considerable amount of soluble material the porosity would depend very much on the water table, the dip of the strata, etc. and would show the large variance that has been found. The variances of density and porosity of the metamorphic rocks are rather surprisingly small. But the low values of porosity are not unexpected from the nature of the rocks. The slightly higher grain density of green marls may be due to the presence of glauconite. The difference of density between Silurian limestone and Carboniferous limestone (2*a*, 2*b* in Table V) is not significant, but that between the Old Red Sandstone and the Carboniferous Sandstone is.

4. *Comparison of the field and laboratory methods*

4.1. In order to test whether the density values found by the two methods refer to the same population of rock formations in each case, Table VI was constructed. In this table the following details are given: the locality, the type of rock, the values of density obtained by the field method (H), its standard deviation, the laboratory measurement (L), its standard deviation, the difference between the two density values, the standard deviation of the difference (σ), and the ratio $(H-L)^2/\sigma^2$. χ^2 $(=\Sigma[(H-L)^2/\sigma^2])$ is calculated and the probability is found of χ^2 exceeding a certain value on eight degrees of freedom (the actual number) from Fisher's tables (Fisher, 1946). It is found that the probability is 0·2, which means that H and L are not significantly different from one another. No correlation can be found between $(H-L)^2/\sigma^2$ and either lithology or porosity.

TABLE VI

Comparison of the Two Methods

Densities in g/cm^3

Locality	Rock type	Field value H	Std. dev. H	Lab. value L	Std. dev. L	$H-L$	σ	$(H-L)^2/\sigma^2$	ϵ (per cent)
Woolhope Cockshoot	Silurian l.s.	2·73	0·25	2·67	0·034	+0·06	0·25	0·058	4·31 ± 1·88
Hanley William	O.R.S.	2·47	0·048	2·58	0·035	−0·11	0·06	3·38	7·10 ± 2·46
May Hill (Monmouth)	O.R.S.	2·46	0·08	2·58	0·035	−0·12	0·09	1·78	7·10 ± 2·46
Wyre Forest	Coal Measures	2·70	0·10	2·42	0·061	+0·28	0·12	4·71	21·95 + 4·84
Forest of Dean	Carb. l.s.	2·54	0·073	2·66	0·029	−0·12	0·14	0·74	2·44 ± 0·95
Bridgnorth Cliffs	Triassic s.s.	2·30	0·13	2·25	0·023	+0·05	0·15	0·11	36·4 ± 7·95
Upton St. Leonards	Lias	2·28	0·19	2·40	0·05	−0·12	0·20	0·36	19·0 ± 1·0
Gog Magog Hills (Cambridge)	Chalk	1·92*	0·20	1·95	0·012	−0·03	0·20	0·02	65·1 ± 8·6

Degrees of freedom 8 $\chi^2 = 11·16$

* From measurements made by Cook and Thirlaway (unpublished).

4.2. *Measurements in a mine.*—The value of the density of a formation can also be found by measuring the difference of gravity at two points in a mine shaft. Such measurements have been made by Cook and Thirlaway (1951) and Table VII gives the analysis of these. They are grouped separately because they are not directly comparable with the hill measurements. Mine measurements are expected to be more accurate because there is no error arising from a horizontal gradient of gravity, and a comparison of Tables VI and VII shows that this is so.

The conclusion is that M and L do not differ significantly.

5. *Conclusions and catalogue of densities*

The study of sampling variances described in this paper shows that in gravity surveys in the Western Midlands, laboratory measurements on rock densities will be sufficiently accurate even if hill measurements are dispensed with. This conclusion is completely different from that reached by Hammer (1950) from density measurements in America. However, his results comprise (repeated) gravimeter measurements in a mine shaft and determinations on several samples from the shaft and a borehole in the vicinity and do not include any density profile determinations.

TABLE VII

Density Measurements in Mine Shafts

(Observers : Cook and Thirlaway)

Locality of mine	Formation	Field value M	Standard deviation σ_M	Laboratory value L	σ_L	$M-L$	σ	$\frac{(M-L)^2}{\sigma^2}$
S. Wales and Forest of Dean (Mean)	Pennant grit	2·65	0·05	2·67	0·01	−0·02	0·05	0·16
S. Wales (Britannia Pit, Pengam)	Deri Beds	2·77	0·02	2·73	0·06	+0·04	0·064	0·39
Alveley Pit	Coal Measures	2·52	0·01	2·42	0·06	+0·10	0·06	2·78
Degrees of freedom 3.								$\chi^2=3{\cdot}33$
P (from tables) $>0{\cdot}3$.								

In the sampling of rocks, attention must be paid to lithology as variances tend to vary with it. Table IV is based on the values of density when the specimens are saturated with water and the results show that this is the most likely condition in nature. As no regional variation in density was found, a sample collection from one exposure should be sufficient for laboratory measurements. It must be pointed out that this conclusion is only true for the particular district studied in this paper, and it is not proved that there may not be considerable variation in density in the same formation in other districts. Table VIII has been formed for general use in gravity surveys of the West Midlands. The adopted values are the mean of the field value and the saturated density in laboratory measurements. Chalk has also been included from the point of view of the completeness of the paper.

TABLE VIII

Catalogue of Densities

No.	Formation	Adopted density (g/cm^3)
1	Silurian limestones and shales	2·70
2	Old Red Sandstone	2·52
3	Carboniferous limestone	2·60
4	Middle Coal Measures Shales and Sandstones	2·48
5	Keuper Sandstone	2·27
6	Keuper Marl	2·42
7	Upper Lias	2·34
8	Chalk	1·94
9	Malvernian Gneiss	2·69

Notes on variability:

(1) Silurian rocks and Carboniferous limestone do not show wide variation in sampling.

(2) Sandstones show a wider variation than limestones.

(3) Chalk is very uniform in density.

(4) The density of metamorphic rocks is applicable to the Malvern area only.

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