

|                     |                   |                            |                |
|---------------------|-------------------|----------------------------|----------------|
| <b>G</b> EOCENTRIC  | <b>C</b> ARTESIAN | <b>C</b> OORDINATES/System | <b>(GEC/S)</b> |
|                     | <b>G</b> EODETIC  |                            | <b>(GGC/S)</b> |
|                     | <b>S</b> pherical |                            | <b>(GSC/S)</b> |
| <b>T</b> OPOCENTRIC | <b>C</b> ARTESIAN |                            | <b>(TCC/S)</b> |

### GGC → GEC

$$\begin{aligned} X &= (N+h) \cos \varphi \cos \lambda \\ Y &= (N+h) \cos \varphi \sin \lambda \\ Z &= [N(1-e^2) + h] \sin \varphi \end{aligned}$$

PRIME VERTICAL RADIUS OF CURVATURE

$$N = a / \sqrt{1 - e^2 \sin^2 \varphi}$$

$$e^2 = (a^2 - b^2) / b^2$$

$a$ : major SEMIAXIS

$b$ : minor SEMIAXIS

### GEC → GGC

HILVONEN-MORITZ algorithm

input:  $X, Y, Z, a, b, it_{max}$

$$\lambda = \tan^{-1} \left( \frac{Y}{X} \right)$$

$k=0$

$$\bullet p = \sqrt{X^2 + Y^2}$$

$$\bullet \varphi_0 = \tan^{-1} \left( \frac{Z}{p(1-e^2)} \right)$$

$$\bullet N_0 = a / \sqrt{1 - e^2 \sin^2 \varphi_0}$$

$$\bullet h_0 = \frac{p}{\cos \varphi_0} - N_0$$

for  $k = 1: it_{max}$

$$\bullet \varphi = \tan^{-1} \left[ \frac{Z}{p} \left( 1 - e^2 \frac{N_0}{N_0 + h_0} \right)^{-1} \right]$$

$$\bullet N = a / \sqrt{1 - e^2 \sin^2 \varphi}$$

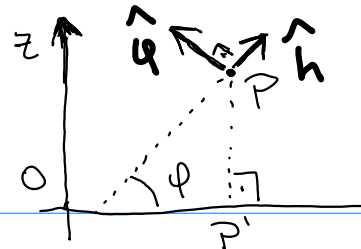
$$\bullet h = \frac{p}{\cos \varphi} - N$$

$$\bullet \varphi_0 \leftarrow \varphi$$

$$\bullet N_0 \leftarrow N$$

$$\bullet h_0 \leftarrow h$$

## Unit vectors relating GGC and GGC



$$\hat{h}(h, \phi, \lambda) = \begin{bmatrix} \cos \phi \cos \lambda \\ \cos \phi \sin \lambda \\ \sin \phi \end{bmatrix}$$

- direction of increasing  $h$
- outward normal of ellipsoid

$$\hat{\phi}(h, \phi, \lambda) = \begin{bmatrix} -\sin \phi \cos \lambda \\ -\sin \phi \sin \lambda \\ \cos \phi \end{bmatrix}$$

- direction of increasing  $\phi$
- tangent to meridian plane
- " " ellipsoid

$$\hat{\lambda}(h, \phi, \lambda) = \begin{bmatrix} -\sin \lambda \\ \cos \lambda \\ 0 \end{bmatrix}$$

- direction of increasing  $\lambda$
- tangent to ellipsoid

### GSC $\rightarrow$ GCC

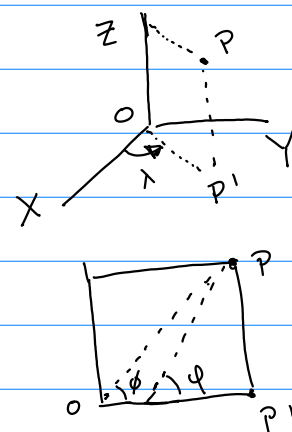
$$\begin{aligned} X &= r \cos \phi \cos \lambda \\ Y &= r \cos \phi \sin \lambda \\ Z &= r \sin \phi \end{aligned}$$

### GCC $\rightarrow$ GSC

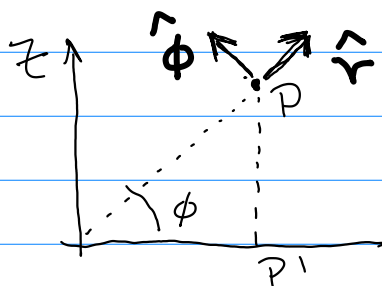
$$r = \sqrt{X^2 + Y^2 + Z^2}$$

$$\phi = \sin^{-1} \left( \frac{Z}{r} \right)$$

$$\lambda = \tan^{-1} (Y/X)$$



Unit vectors  
relating GSC  
and GCC



$$\tan \phi = \frac{a^2}{b^2} \tan \phi$$

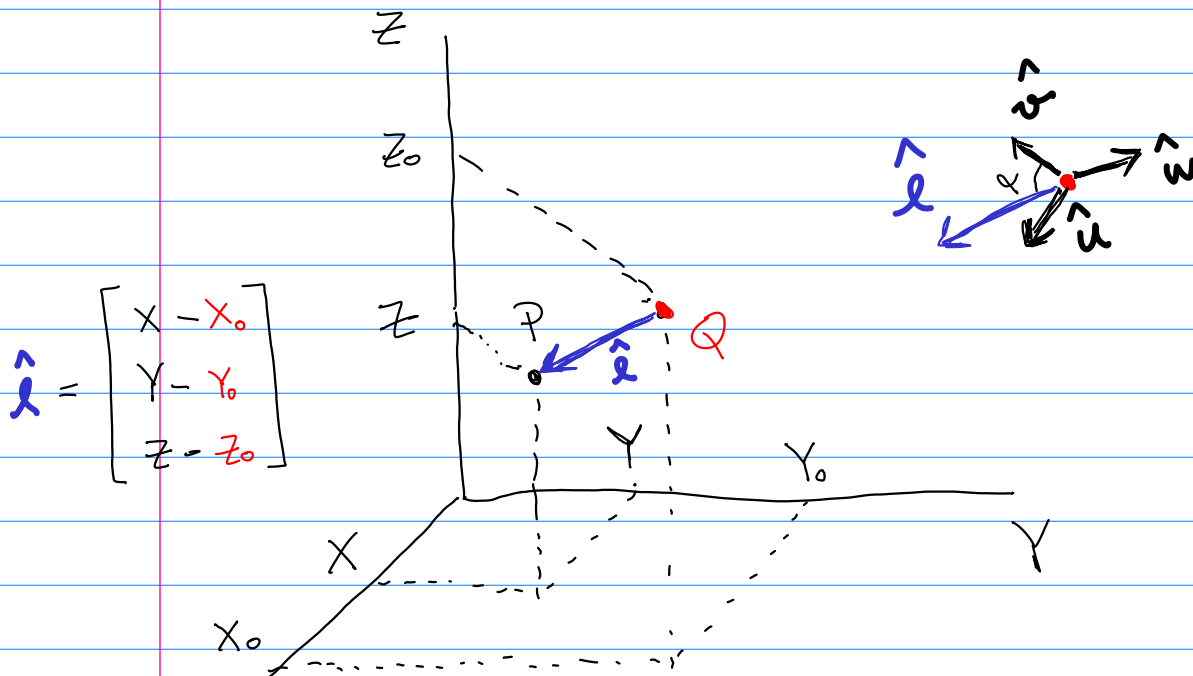
$$\hat{r}(r, \phi, \lambda) = \begin{bmatrix} \cos \phi \cos \lambda \\ \cos \phi \sin \lambda \\ \sin \phi \end{bmatrix} \quad \hat{\phi}(r, \phi, \lambda) = \begin{bmatrix} -\sin \phi \cos \lambda \\ -\sin \phi \sin \lambda \\ \cos \phi \end{bmatrix} \quad \hat{\lambda}(r, \phi, \lambda) = \begin{bmatrix} -\sin \lambda \\ \cos \lambda \\ 0 \end{bmatrix}$$

# TCC

- Origin at a point Q
- Q may be defined with GCC, GGC or GSC

Consider two points defined with GCC

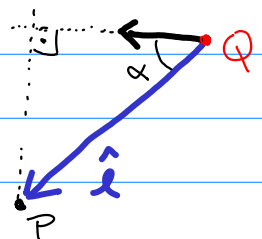
$$Q = (X_0, Y_0, Z_0) \quad P = (X, Y, Z)$$



$$\hat{l} = \begin{bmatrix} x - x_0 \\ y - y_0 \\ z - z_0 \end{bmatrix}$$

$$\begin{aligned} \hat{u} &= \hat{v} \text{ OR } \hat{\phi} \\ \hat{w} &= \hat{v} \text{ OR } \hat{\phi} \\ \hat{u} &= \hat{v} \text{ OR } \hat{w} \end{aligned}$$

$$\|\hat{v}^T \hat{l}\|$$



$$\begin{aligned} x &= \hat{v}^T \hat{l} \\ y &= \hat{w}^T \hat{l} \\ z &= \hat{u}^T \hat{l} \end{aligned} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = R^T \begin{bmatrix} x - x_0 \\ y - y_0 \\ z - z_0 \end{bmatrix}$$

$$R^T = R^{-1} \text{ OK?}$$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix} + R \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$