Heights

Geoidal undulation

$$N \equiv N(\varphi, \lambda)$$

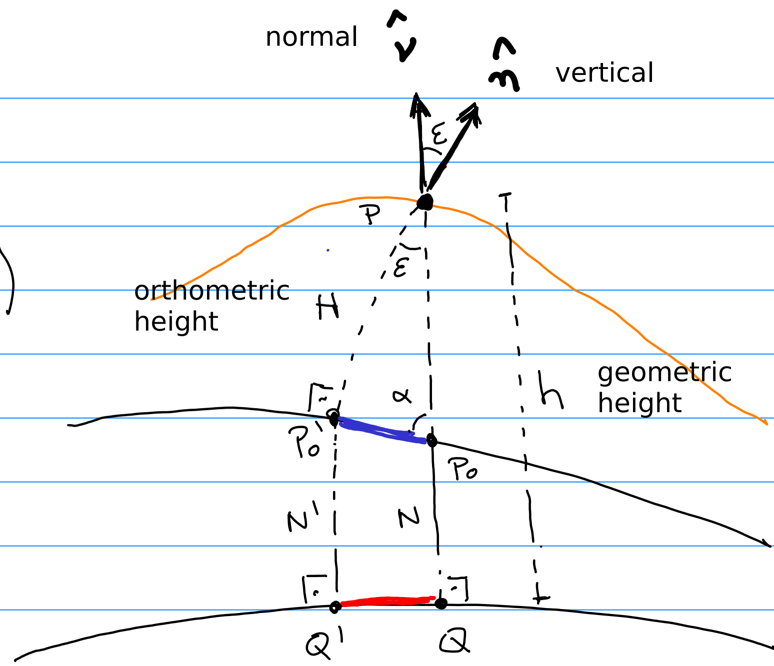
$$|N| \leq 120\ m \ (\text{world})$$

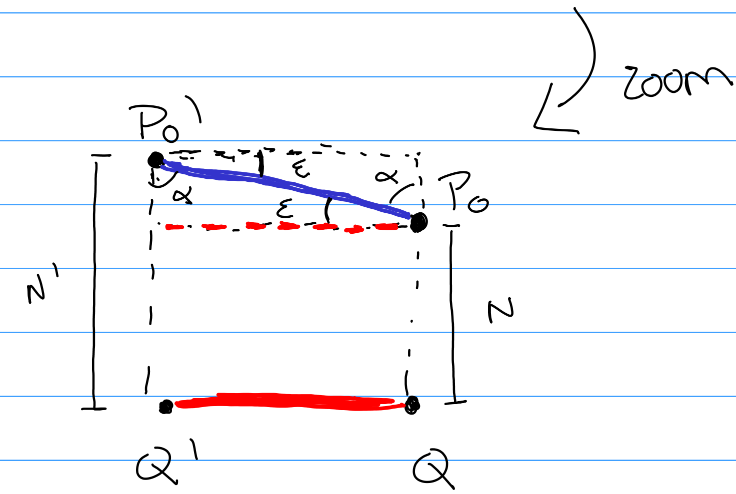
IBGE - Modelo de ondulação geoidal (MAPGEO2015)

$$|N| \leq 30\ m \ (\text{Brazil})$$

Deflection of vertical

$$\varepsilon \leq \left(1/60\right)^\circ \approx 3 \times 10^{-4}$$





Approximations (Sansò and Sideris, 2013, p 46):

$$\alpha \approx 90^\circ - \varepsilon$$

Law of sines

$$\frac{H}{\sin\alpha} \approx \frac{P_0'P_0}{\sin\varepsilon}$$

$$H \sin\varepsilon \approx P_0'P_0 \sin\alpha$$

Small-angle approximation

$$\sin\varepsilon \approx \varepsilon$$

$$P_0'P_0 \sin\alpha \approx Q'Q$$

$$Q'Q \approx H\varepsilon$$

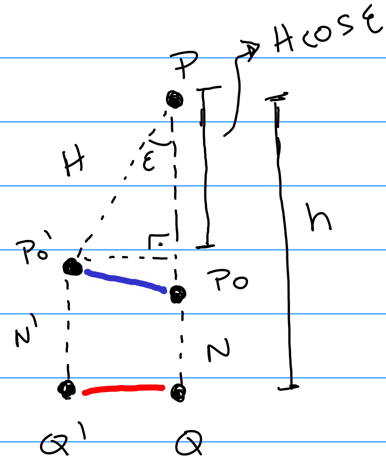
Ex.: consider $H = 9000\ m$ (Everest)

$$Q'Q \approx 2{,}7\ m$$

$$\frac{N' - N}{Q'Q} \approx tg\,\varepsilon \approx \varepsilon$$

Small-angle approximation

$$N' - N \approx (H\varepsilon)\,\varepsilon = H\varepsilon^2$$



$$h \approx H\cos\varepsilon + N'$$

$$\cos\varepsilon \approx 1 - \frac{1}{2}\varepsilon^2$$

Small-angle approximation

$$h \approx H - \frac{1}{2}\varepsilon^2 H + N'$$

$$h \approx H - \frac{1}{2}\varepsilon^2 H + N + H\varepsilon^2$$

$$h \approx H + N + \frac{1}{2}\varepsilon^2 H$$

$$\frac{1}{2}\varepsilon^2 H \approx 0{,}4\ mm$$