

# Heights

Geoidal undulation

$$N \equiv N(\varphi, \lambda)$$

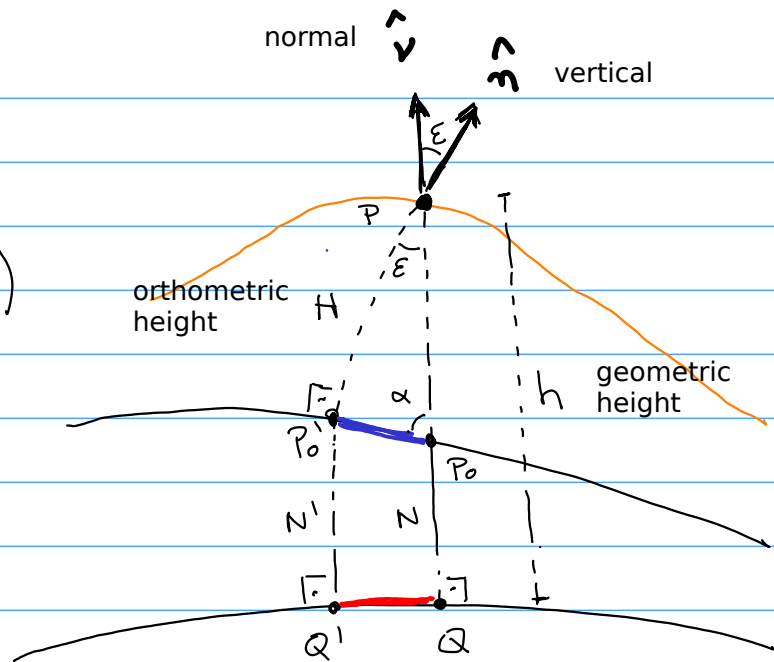
$$|N| \leq 120 \text{ m (world)}$$

IBGE - Modelo de  
ondulação geoidal  
(MAPGEO2015)

$$|N| \leq 30 \text{ m (Brazil)}$$

Deflection of vertical

$$\epsilon \leq \left(\frac{1}{60}\right)^\circ \approx 3 \times 10^{-4}$$



Approximations (Sansò and Sideris, 2013, p 46):

$$\alpha \approx 90^\circ - \epsilon$$

Law of  
sines

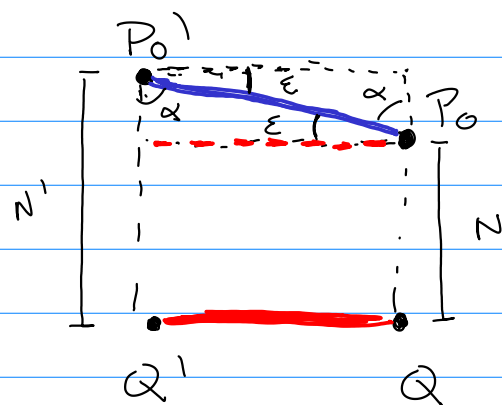
$$\frac{H}{\sin \alpha} \approx \frac{P_0'P_0}{\sin \epsilon}$$

$$H \sin \epsilon \approx P_0'P_0 \sin \alpha$$

$$\sin \epsilon \approx \epsilon$$

$$P_0'P_0 \sin \alpha \approx Q'Q$$

$$Q'Q \approx H \epsilon$$

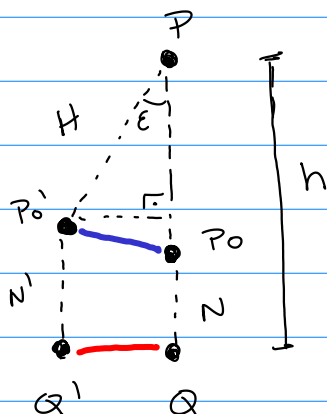


Ex.: consider  $H = 9000 \text{ m (EVEREST)}$   
 $Q'Q \approx 2,7 \text{ m}$

$$\frac{N' - N}{Q'Q} \approx \tan \epsilon \approx \epsilon$$

Small-angle  
approximation

$$N' - N \approx (H \epsilon) \epsilon = H \epsilon^2$$



$$h \approx H \cos \epsilon + N'$$

$$\cos \epsilon \approx 1 - \frac{1}{2} \epsilon^2 \quad \text{Small-angle approximation}$$

$$h \approx H - \frac{1}{2} \epsilon^2 H + N'$$

$$h \approx H - \frac{1}{2} \epsilon^2 H + N + H \epsilon^2$$

$$h \approx H + N + \left(\frac{1}{2} \epsilon^2 H\right)$$

$$\frac{1}{2} \epsilon^2 H \approx 0,4 \text{ mm}$$