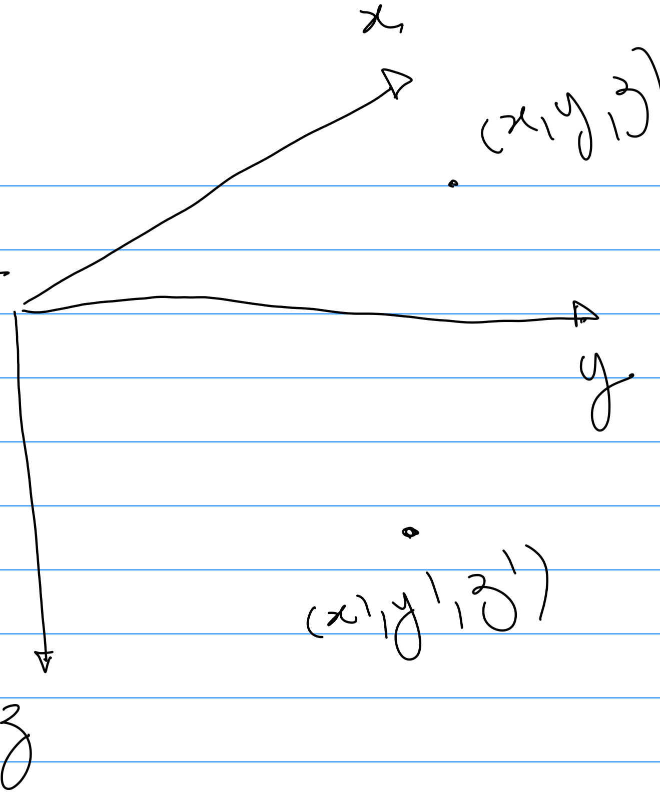
Inverse distance function

$$r = \left[(x-x')^2 + (y-y')^2 + (z-z')^2\right]^{1/2}$$



$$\cancel{\frac{1}{r}}$$

$$\partial_x \frac{1}{r} = -\frac{1}{\cancel{2}}\left[\quad\right]^{-3/2} \cancel{2}(x-x')$$

$$= -\frac{(x-x')}{r^3}$$

$$\partial_y \frac{1}{r} = -\frac{(y-y')}{r^3} \qquad \partial_z \frac{1}{r} = -\frac{(z-z')}{r^3}$$

$$\partial_{xx} \frac{1}{r} = \partial_x \left[\partial_x \frac{1}{r}\right] = \partial_x \left[-\frac{x-x'}{r^3}\right]$$

$$= [-1]\cdot\left(\frac{1}{r^3}\right) + \left[-(x-x')\right]\left(-\frac{3}{2}\right)\left[\quad\right]^{-5/2} 2(x-x')$$

$$= \frac{3(x-x')(x-x')}{r^5} - \frac{1}{r^3}$$

$$\partial_{yx}\frac{1}{r} = \partial_y\left[\partial_x \frac{1}{r}\right] = \partial_y\left[-\frac{(x-x')}{r^3}\right]$$

$$= -(x-x')\left(-\frac{3}{2}\right)\left[\quad\right]^{-5/2} 2(y-y')$$

$$= \underline{\frac{3(x-x')(y-y')}{r^5}} \,//$$

$$\underline{\partial_{yx}\frac{1}{r} = \partial_{xy}\frac{1}{r}} \,//$$

$$\nabla^2\frac{1}{r} = \partial_{xx}\frac{1}{r} + \partial_{yy}\frac{1}{r} + \partial_{zz}\frac{1}{r} =$$

$$= \frac{3(x-x')^2}{r^5} + \frac{3(y-y')^2}{r^5} + \frac{3(z-z')^2}{r^5} - \frac{3}{r^3} =$$

$$= 3\underbrace{\left[(x-x')^2 + (y-y')^2 + (z-z')^2\right]}_{r^2}\frac{1}{r^5} - \frac{3}{r^3}$$

$$= \frac{3}{r^3} - \frac{3}{r^3} = \underline{0} \,//$$

1/r is harmonic!