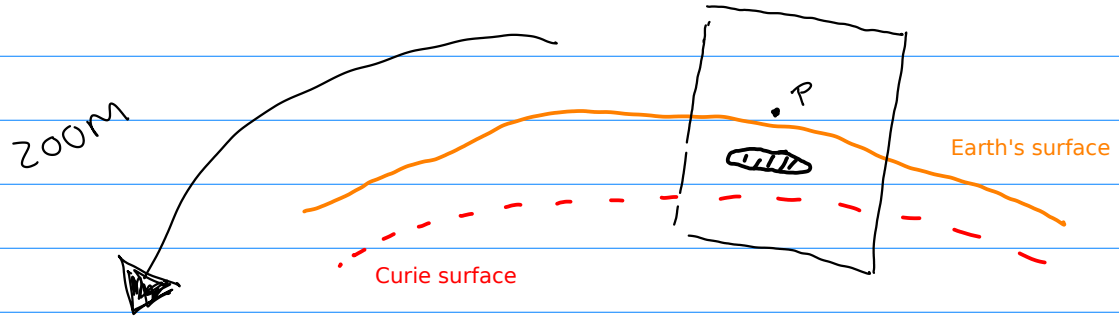
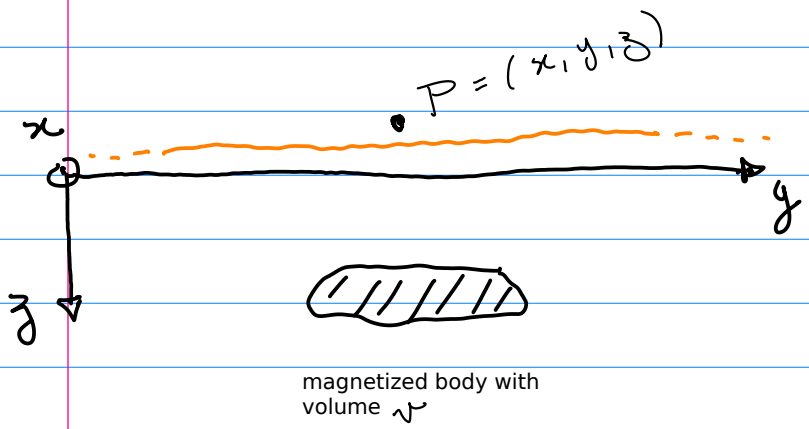
Magnetic modeling





General case (total magnetization)

$$V(x,y,z) = -G\rho \iiint_v \left(\nabla \frac{1}{r}\right)^T \mathbf{h}(x',y',z')\, dv$$

(valid for points outside the source) $$\mathbf{B}(x,y,z) = -\nabla V(x,y,z)$$

$$= G\rho \iiint_v \left(\nabla^2 \frac{1}{r}\right) \mathbf{h}(x',y',z')\, dv$$

matrix containing second derivatives of 1/r

$$\mathbf{T}(x,y,z) = \mathbf{F}(x,y,z) + \mathbf{B}(x,y,z)$$

$$\Delta T(x,y,z) = \|\mathbf{T}(x,y,z)\| - \|\mathbf{F}(x,y,z)\|$$

$$\Delta \tilde{T}(x,y,z) = \hat{\mathbf{F}}^T \mathbf{B}(x,y,z) \qquad \boxed{\Delta \tilde{T} \approx \Delta T}$$

Constant total-magnetization direction

$$\mathbf{h}(x',y',z') = h(x',y',z')\,\hat{\mathbf{h}}, \quad \hat{\mathbf{h}} = \begin{bmatrix} \cos I \cos D \\ \cos I \sin D \\ \sin I \end{bmatrix} = \begin{bmatrix} \hat{h}_x \\ \hat{h}_y \\ \hat{h}_z \end{bmatrix}$$

$$V(x,y,z) = -C_m \iiint_v h(x',y',z') \left(\nabla \frac{1}{r}\right)^T \hat{\mathbf{h}}\, dv$$

$$= -C_m \left\{ \hat{h}_x \iiint_v h(x',y',z')\, \partial_x \frac{1}{r}\, dv + \right.$$

$$+ \hat{h}_y \iiint_v h(x',y',z')\, \partial_y \frac{1}{r}\, dv +$$

$$\left. + \hat{h}_z \iiint_v h(x',y',z')\, \partial_z \frac{1}{r}\, dv \right\}$$

$$= -\nabla \Theta(x,y,z)^T \hat{\mathbf{h}}$$

$$\Theta(x,y,z) = C_m \iiint_v h(x',y',z') \frac{1}{r}\, dv$$

(valid for points outside the source)

$$\mathbf{B}(x,y,z) = \nabla^2 \Theta(x,y,z)\, \hat{\mathbf{h}}$$

$$\nabla^2 \Theta(x,y,z) = \begin{bmatrix} \partial_{xx}\Theta & \partial_{xy}\Theta & \partial_{xz}\Theta \\ \partial_{xy}\Theta & \partial_{yy}\Theta & \partial_{yz}\Theta \\ \partial_{xz}\Theta & \partial_{yz}\Theta & \partial_{zz}\Theta \end{bmatrix}$$

Constant total magnetization

$$\mathbf{h}(x',y',z') = h\,\hat{\mathbf{h}}, \qquad \hat{\mathbf{h}} = \begin{bmatrix} \cos I \cos D \\ \cos I \sin D \\ \sin I \end{bmatrix} = \begin{bmatrix} \hat{h}_x \\ \hat{h}_y \\ \hat{h}_z \end{bmatrix}$$

(h: constant)

$$V(x,y,z) = -\nabla \textcircled{H}(x,y,z)^T \hat{\mathbf{h}} \qquad \textcircled{H}(x,y,z) = G m h \iiint_v \frac{1}{r}\, dv$$

(valid for points outside the source) $$\mathbf{B}(x,y,z) = \nabla^2 \textcircled{H}(x,y,z)\,\hat{\mathbf{h}}$$

General case (induced and remanent magnetization components)

total-magnetization vector $\mathbf{h}(x',y',z')$ [A/m] = induced magnetization $\boldsymbol{\chi}(x',y',z')\mathbf{H}(x',y',z')$ + remanent magnetization $\mathbf{P}(x',y',z')$

magnetic intensity field $\mathbf{H}(x',y',z')$ [A/m] = magnetic induction field $\mathbf{F}(x',y',z')$ [T] / magnetic permeability constant (at vacuum) μ_0 [H/m]

$$\boldsymbol{\chi}(x',y',z') = \begin{bmatrix} \chi_{11} & \chi_{12} & \chi_{13} \\ \chi_{12} & \chi_{22} & \chi_{23} \\ \chi_{13} & \chi_{23} & \chi_{33} \end{bmatrix}, \qquad \chi_{ij} \equiv \chi_{ij}(x',y',z'), \quad i,j = 1,2,3$$

magnetic susceptibility tensor (dimentionless in the SI)

Hrouda, F., 2007. Magnetic susceptibility, anisotropy. In Gubbins, D., and Herrero-Bervera, E. (eds.), Encyclopedia of Geomagnetism and Paleomagnetism, Springer, pp. 546–560. ISBN: 978-1-4020-3992-8

Rochette, P., Jackson, M.J., and Aubourg, C., 1992. Rock magnetism and the interpretation of anisotropy of magnetic susceptibility. Reviews of Geophysics, 30: 209–226. doi: 10.1029/92RG00733

Hrouda, F., 1982. Magnetic anisotropy of rocks and its application in geology and geophysics. Geophysical Surveys, 5, 37–82. doi: 10.1007/BF01450244

AGICO, Manuals and AGICO prints, https://www.agico.com/text/support/agicoprints/agicoprints.php, 21/10/2020

scalar function

identity matrix

$$\boldsymbol{\chi}(x',y',z') = \chi(x',y',z')\,\mathbf{I}$$

particular case in which susceptibility is isotropic

$$V(x,y,z) = -Gm \iiint_v \left(\nabla \frac{1}{r}\right)^T \mathbf{h}(x',y',z')\, dv$$

$$= -Gm \iiint_v \left(\nabla \frac{1}{r}\right)^T \boldsymbol{\chi}(x',y',z')\,\mathbf{H}(x',y',z')\, dv +$$

$$-Gm \iiint_v \left(\nabla \frac{1}{r}\right)^T \mathbf{p}(x',y',z')\, dv$$