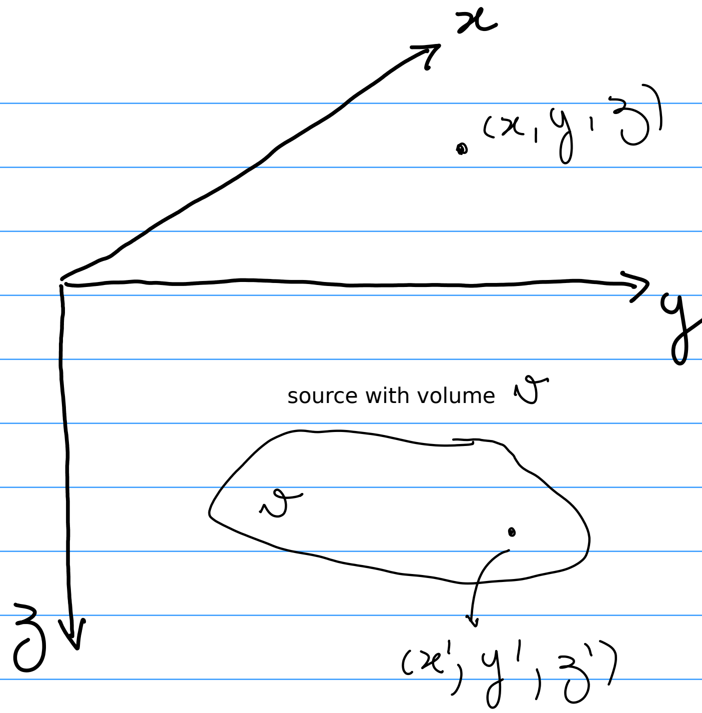
Poisson's relation



scalar magnetic potetial produced by a source
with constant total magnetization

$$V(x,y,z) = -\nabla \Theta(x,y,z)^T \, \hat{\mathbf{h}}$$

$$\Theta(x,y,z) = C_m h \iiint_{\vartheta} \frac{1}{r} \, d\vartheta$$

$$\hat{\mathbf{h}} = \begin{bmatrix} \cos I \cos D \\ \cos I \sin D \\ \sin I \end{bmatrix}$$

$$V(x,y,z) = -\begin{bmatrix} \partial_x \Theta \\ \partial_y \Theta \\ \partial_z \Theta \end{bmatrix}^T \hat{\mathbf{h}} \, , \quad \partial_\alpha \Theta \equiv \frac{\partial \Theta(x,y,z)}{\partial \alpha} = C_m h \iiint_{\vartheta} \frac{\alpha' - \alpha}{r^3} \, d\vartheta$$

$$\alpha = x, y, z$$

gravitational potential produced by a source
with constant density

$$U(x,y,z) = G \rho \iiint_{\vartheta} \frac{1}{r} \, d\vartheta$$

$$\nabla U(x,y,z) = \begin{bmatrix} \partial_x U \\ \partial_y U \\ \partial_z U \end{bmatrix}$$

gradient of the gravitational potential
(gravitational acceleration)

alfa-component of the gravitational acceleration

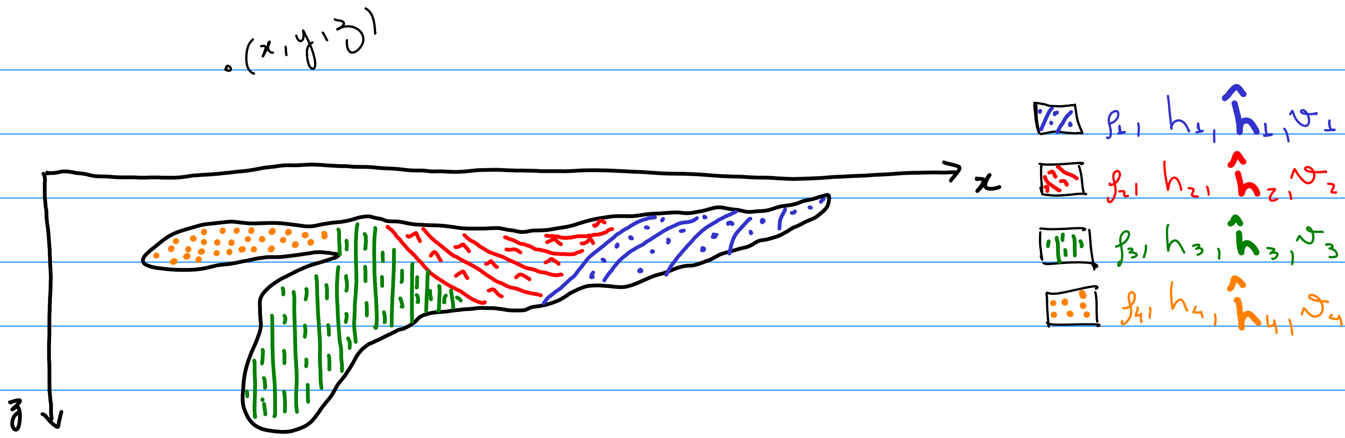
$$\partial_\alpha U \equiv \frac{\partial U(x,y,z)}{\partial \alpha} = G \rho \iiint_{\vartheta} \frac{\alpha' - \alpha}{r^3} \, d\vartheta \, , \quad \alpha = x, y, z$$

$$\partial_\alpha \Theta = \frac{C_m h}{G \rho} \, \partial_\alpha U$$

$$V(x,y,z) = -\frac{C_m h}{G \rho} \overbrace{\nabla U(x,y,z)^T \, \hat{\mathbf{h}}}^{\partial U / \partial h}$$

$$\nabla \Theta(x,y,z) = \frac{C_m h}{G \rho} \nabla U(x,y,z)$$

According to the conditions specified above, the scalar magnetic potential represents the directional derivative of the gravitational potential along the total-magnetization direction of the source. This equation is known as Poisson's relation (Blakely, 1996, p. 91).



$$V(x,y,z) = V_1(x,y,z) + V_2(x,y,z) + V_3(x,y,z) + V_4(x,y,z)$$

$$U(x,y,z) = U_1(x,y,z) + U_2(x,y,z) + U_3(x,y,z) + U_4(x,y,z)$$

$$V_1(x,y,z) = -\frac{C_m h_1}{G\,\rho_1} \nabla U_1(x,y,z)^T \hat{\mathbf{h}}_1 \qquad V_2(x,y,z) = -\frac{C_m h_2}{G\,\rho_2} \nabla U_2(x,y,z)^T \hat{\mathbf{h}}_2$$

$$V_3(x,y,z) = -\frac{C_m h_3}{G\,\rho_3} \nabla U_3(x,y,z)^T \hat{\mathbf{h}}_3 \qquad V_4(x,y,z) = -\frac{C_m h_4}{G\,\rho_4} \nabla U_4(x,y,z)^T \hat{\mathbf{h}}_4$$

Poisson's relation for a heterogeneous body

$$V(x,y,z) = -\left(\frac{C_m h_1}{G\,\rho_1} \nabla U_1(x,y,z)^T \hat{\mathbf{h}}_1 + \frac{C_m h_2}{G\,\rho_2} \nabla U_2(x,y,z)^T \hat{\mathbf{h}}_2 + \right.$$
$$\left. + \frac{C_m h_3}{G\,\rho_3} \nabla U_3(x,y,z)^T \hat{\mathbf{h}}_3 + \frac{C_m h_4}{G\,\rho_4} \nabla U_4(x,y,z)^T \hat{\mathbf{h}}_4 \right)$$

$\mathbf{B}$

Ex: Deduce the relation between the magnetic induction and derivatives of the gravitational potential.

Ex: Deduce the relation between the approximated total-field anomaly and derivatives of the gravotational potential.

$$\hat{\mathbf{F}} = \begin{bmatrix} \hat{F}_x \\ \hat{F}_y \\ \hat{F}_z \end{bmatrix}$$

$$\Delta\tilde{T}(x,y,z) = \hat{\mathbf{F}}^T \mathbf{B}(x,y,z)$$

$$V(x,y,z) = -\frac{C_m h}{G\rho} \nabla U(x,y,z)^T \hat{\mathbf{h}}$$

$$\mathbf{B}(x,y,z) = -\nabla V(x,y,z)$$

$$B_\alpha(x,y,z) = \frac{C_m h}{G\rho} \partial_\alpha \nabla U(x,y,z)^T \hat{\mathbf{h}}$$

$$\partial_\alpha \nabla U(x,y,z) = \begin{bmatrix} \partial_{\alpha x} U(x,y,z) \\ \partial_{\alpha y} U(x,y,z) \\ \partial_{\alpha z} U(x,y,z) \end{bmatrix} \qquad \alpha = x, y, z$$