

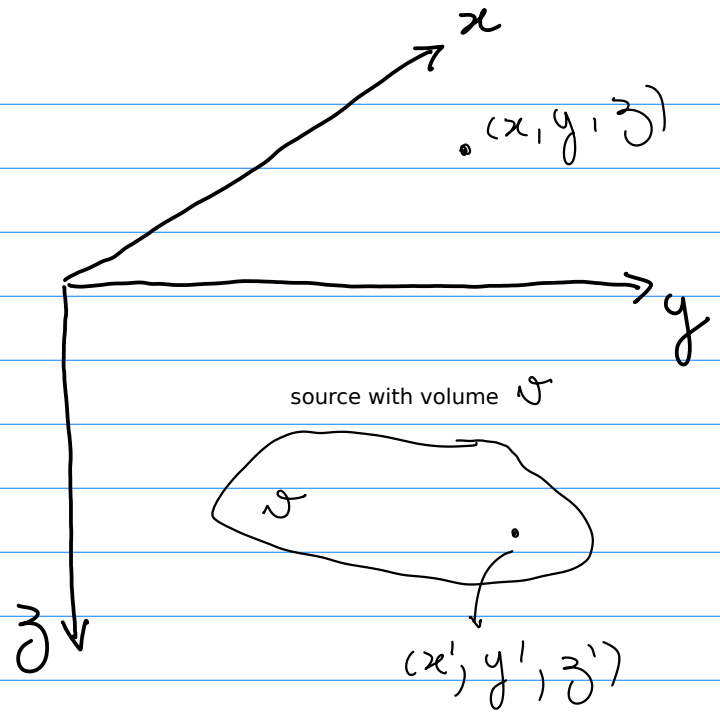
Poisson's relation

scalar magnetic potential produced by a source with constant total magnetization

$$V(x, y, z) = - \nabla \mathcal{H}(x, y, z)^T \hat{h}$$

$$\mathcal{H}(x, y, z) = G_m h \iiint_V \frac{1}{r} d\tau$$

$$\hat{h} = \begin{bmatrix} \cos I \cos D \\ \cos I \sin D \\ \sin I \end{bmatrix}$$



$$V(x, y, z) = - \begin{bmatrix} \frac{\partial \mathcal{H}}{\partial x} \\ \frac{\partial \mathcal{H}}{\partial y} \\ \frac{\partial \mathcal{H}}{\partial z} \end{bmatrix}^T \hat{h}, \quad \frac{\partial \mathcal{H}}{\partial \alpha} \equiv \frac{\partial \mathcal{H}(x, y, z)}{\partial \alpha} = G_m h \iiint_V \frac{\alpha' - \alpha}{r^3} d\tau$$

$\alpha = x, y, z$

gravitational potential produced by a source with constant density

$$U(x, y, z) = G \rho \iiint_V \frac{1}{r} d\tau$$

$$\nabla U(x, y, z) = \begin{bmatrix} \frac{\partial U}{\partial x} \\ \frac{\partial U}{\partial y} \\ \frac{\partial U}{\partial z} \end{bmatrix}$$

gradient of the gravitational potential
(gravitational acceleration)

alpha-component of the gravitational acceleration

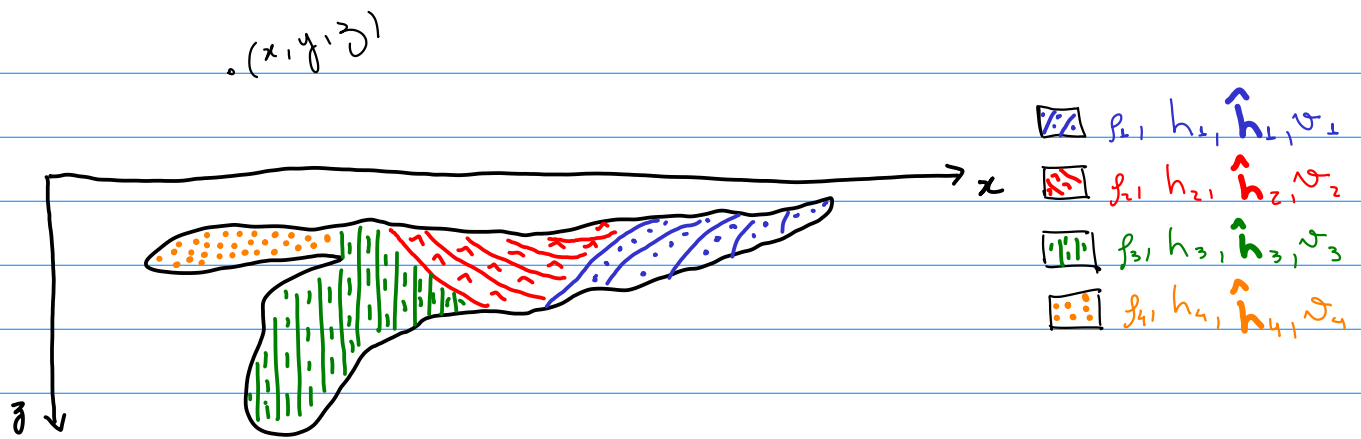
$$\frac{\partial U}{\partial \alpha} \equiv \frac{\partial U(x, y, z)}{\partial \alpha} = G \rho \iiint_V \frac{\alpha' - \alpha}{r^3} d\tau, \quad \alpha = x, y, z$$

$$\frac{\partial \mathcal{H}}{\partial \alpha} = \frac{G_m h}{G \rho} \frac{\partial U}{\partial \alpha}$$

$$V(x, y, z) = - \frac{G_m h}{G \rho} \underbrace{\nabla U(x, y, z)^T}_{\frac{\partial U}{\partial h}} \hat{h}$$

$$\nabla \mathcal{H}(x, y, z) = \frac{G_m h}{G \rho} \nabla U(x, y, z)$$

According to the conditions specified above, the scalar magnetic potential represents the directional derivative of the gravitational potential along the total-magnetization direction of the source. This equation is known as Poisson's relation (Blakely, 1996, p. 91).



$$V(x, y, z) = V_1(x, y, z) + V_2(x, y, z) + V_3(x, y, z) + V_4(x, y, z)$$

$$U(x, y, z) = U_1(x, y, z) + U_2(x, y, z) + U_3(x, y, z) + U_4(x, y, z)$$

$$V_1(x, y, z) = -\frac{G \rho_1 h_1}{g_1} \nabla U_1(x, y, z)^T \hat{h}_1 \quad V_2(x, y, z) = -\frac{G \rho_2 h_2}{g_2} \nabla U_2(x, y, z)^T \hat{h}_2$$

$$V_3(x, y, z) = -\frac{G \rho_3 h_3}{g_3} \nabla U_3(x, y, z)^T \hat{h}_3 \quad V_4(x, y, z) = -\frac{G \rho_4 h_4}{g_4} \nabla U_4(x, y, z)^T \hat{h}_4$$

Poisson's relation for a heterogeneous body

$$V(x, y, z) = -\left(\frac{G \rho_1 h_1}{g_1} \nabla U_1(x, y, z)^T \hat{h}_1 + \frac{G \rho_2 h_2}{g_2} \nabla U_2(x, y, z)^T \hat{h}_2 + \frac{G \rho_3 h_3}{g_3} \nabla U_3(x, y, z)^T \hat{h}_3 + \frac{G \rho_4 h_4}{g_4} \nabla U_4(x, y, z)^T \hat{h}_4 \right)$$

B

Ex: Deduce the relation between the magnetic induction and derivatives of the gravitational potential.

Ex: Deduce the relation between the approximated total-field anomaly and derivatives of the gravitational potential.

$$\hat{\mathbf{F}} = \begin{bmatrix} \hat{F}_x \\ \hat{F}_y \\ \hat{F}_z \end{bmatrix}$$

$$\Delta T(x, y, z) = \hat{\mathbf{F}}^T \mathbf{B}(x, y, z)$$

$$V(x, y, z) = - \frac{cm h}{G \rho} \nabla U(x, y, z)^T \hat{h}$$

$$\mathbf{B}(x, y, z) = - \nabla V(x, y, z)$$

$$\mathbf{B}_\alpha(x, y, z) = \frac{cm h}{G \rho} \partial_\alpha \nabla U(x, y, z)^T \hat{h}$$

$$\partial_\alpha \nabla U(x, y, z) = \begin{bmatrix} \partial_\alpha x U(x, y, z) \\ \partial_\alpha y U(x, y, z) \\ \partial_\alpha z U(x, y, z) \end{bmatrix} \quad \alpha = x, y, z$$