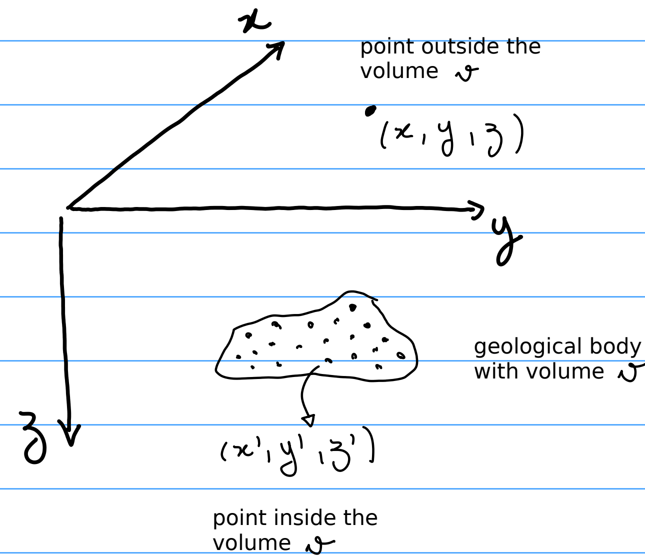
3D sources

To compute the magnetic/gravitational effect produced by the 3D body at the point (x, y, z), consider that it is formed by small volume elements dv = dx'dy'dz' , each one with center at a point (x', y', z').



Magnetic scalar potential produced by the 3D body

$$V(x,y,z) = -C_m \iiint_{v} \nabla \frac{1}{r}^{T} \mathbf{h}(x',y',z')\, dv$$

$\left[\frac{HA}{m}\right] = [T\,m]$ $\quad [H/m]$ $\quad$ total magnetization

$$\mathbf{m}(x',y',z') = \mathbf{h}(x',y',z')\, dv$$

magnetic moment = magnetization X volume

$[A\,m^2] \quad [A/m] \times [m^3]$

Gravitational potential produced by the 3D body

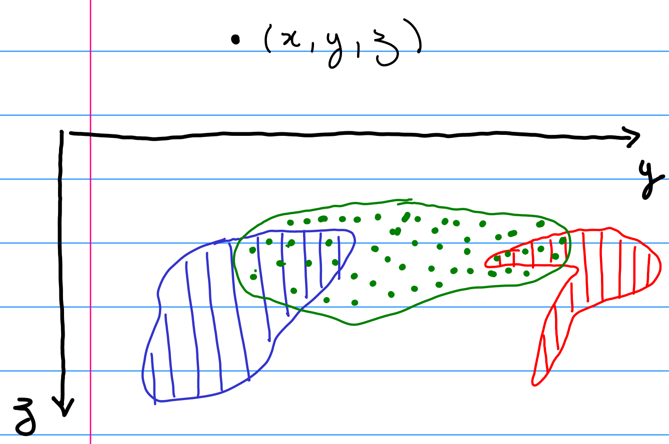
$$U(x,y,z) = G \iiint_{v} \frac{1}{r} \rho(x',y',z')\, dv$$

$[m^2/s^2] \quad \left[\frac{m^3}{Kg\,s^2}\right]$ $\quad$ density

$$m(x',y',z') = \rho(x',y',z')\, dv$$

mass = density X volume

$[Kg] \quad [Kg/m^3] \times [m^3]$



$$v = v_1 \cup v_2 \cup v_3 \cup v_4 \cup v_5$$

$v_1, \rho_1, \mathbf{h}_1$

$v_2, \rho_2 = \rho_1 + \rho_3, \mathbf{h}_2 = \mathbf{h}_1 + \mathbf{h}_3$

$v_3, \rho_3, \mathbf{h}_3$

$v_4, \rho_4 = \rho_3 + \rho_5, \mathbf{h}_4 = \mathbf{h}_3 + \mathbf{h}_5$

$v_5, \rho_5, \mathbf{h}_5$

$$V(x,y,z) = -C_m \iiint_{v} \nabla \frac{1}{r}^{T} \mathbf{h}(x',y',z')\, dv$$

$$= -C_m \sum_{k=1}^{5} \iiint_{v_k} \nabla \frac{1}{r}^{T} \mathbf{h}_k\, dv$$

$$U(x,y,z) = G \iiint_{v} \frac{1}{r} \rho(x',y',z')\, dv$$

$$= G \sum_{k=1}^{5} \iiint_{v_k} \frac{1}{r} \rho_k\, dv$$

$$V(x,y,z) = -G m \Big\{ \iiint_{v_1} \boldsymbol{\nabla}\frac{1}{r}\cdot \mathbf{h}_1 \, dv + \iiint_{v_2} \boldsymbol{\nabla}\frac{1}{r}\cdot \overbrace{\mathbf{h}_2}^{=\mathbf{h}_1+\mathbf{h}_3} \, dv + \iiint_{v_3} \boldsymbol{\nabla}\frac{1}{r}\cdot \mathbf{h}_3 \, dv +$$

$$+ \iiint_{v_4} \boldsymbol{\nabla}\frac{1}{r}\cdot \mathbf{h}_4 \, dv + \iiint_{v_5} \boldsymbol{\nabla}\frac{1}{r}\cdot \mathbf{h}_5 \, dv \Big\}$$

$$= -G m \Big\{ \iiint_{v_1} \boldsymbol{\nabla}\frac{1}{r}\cdot \mathbf{h}_1 \, dv + \Big[\iiint_{v_2} \boldsymbol{\nabla}\frac{1}{r}\cdot \mathbf{h}_1 \, dv + \iiint_{v_2} \boldsymbol{\nabla}\frac{1}{r}\cdot \mathbf{h}_3 \, dv \Big] +$$

$$+ \iiint_{v_3} \boldsymbol{\nabla}\frac{1}{r}\cdot \mathbf{h}_3 \, dv + \Big[\iiint_{v_4} \boldsymbol{\nabla}\frac{1}{r}\cdot \mathbf{h}_3 \, dv + \iiint_{v_4} \boldsymbol{\nabla}\frac{1}{r}\cdot \mathbf{h}_5 \, dv \Big] +$$

$$+ \iiint_{v_5} \boldsymbol{\nabla}\frac{1}{r}\cdot \mathbf{h}_5 \, dv \Big\}$$

$$U(x,y,z) = G \Big\{ \iiint_{v_1} \frac{1}{r} \rho_1 \, dv + \iiint_{v_2} \frac{1}{r} \rho_2 \, dv + \iiint_{v_3} \frac{1}{r} \rho_3 \, dv +$$

$$+ \iiint_{v_4} \frac{1}{r} \rho_4 \, dv + \iiint_{v_5} \frac{1}{r} \rho_5 \, dv \Big\}$$

$$= G \Big\{ \iiint_{v_1} \frac{1}{r} \rho_1 \, dv + \Big[\iiint_{v_2} \frac{1}{r} \rho_1 \, dv + \iiint_{v_2} \frac{1}{r} \rho_3 \, dv \Big] +$$

$$+ \iiint_{v_3} \frac{1}{r} \rho_3 \, dv + \Big[\iiint_{v_4} \frac{1}{r} \rho_3 \, dv + \iiint_{v_4} \frac{1}{r} \rho_5 \, dv \Big] +$$

$$+ \iiint_{v_5} \frac{1}{r} \rho_5 \, dv \Big\}$$

$$\alpha = x, y, z$$

$$\partial_\alpha V(x,y,z) = -G m \iiint_{v} \Big(\partial_\alpha \boldsymbol{\nabla}\frac{1}{r} \Big) \cdot \mathbf{h}(x',y',z') \, dv \qquad \partial_\alpha U(x,y,z) = G \iiint_{v} \partial_\alpha \frac{1}{r} \, \rho(x',y',z') \, dv$$

$$\partial_{\alpha\beta} V(x,y,z) = -G m \iiint_{v} \Big(\partial_{\alpha\beta} \boldsymbol{\nabla}\frac{1}{r} \Big) \cdot \mathbf{h}(x',y',z') \, dv \qquad \partial_{\alpha\beta} U(x,y,z) = G \iiint_{v} \partial_{\alpha\beta} \frac{1}{r} \, \rho(x',y',z') \, dv$$