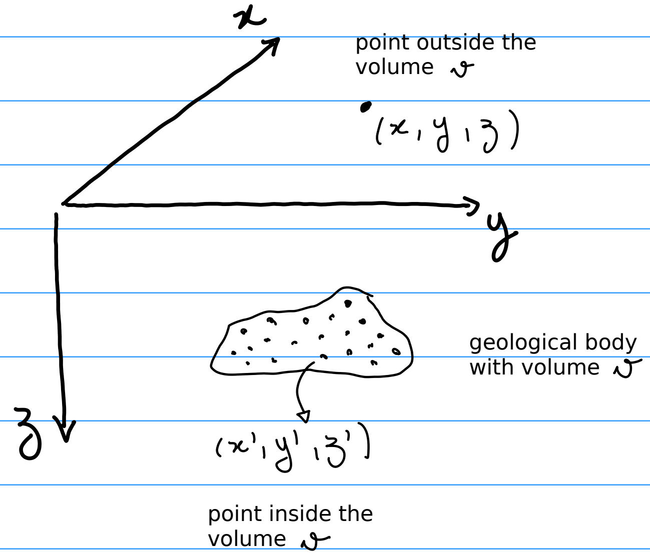
3D sources

To compute the magnetic/gravitational effect produced by the 3D body at the point (x, y, z), consider that it is formed by small volume elements dv = dx'dy'dz' , each one with center at a point (x', y', z').



Magnetic scalar potential produced by the 3D body

$$\underset{[\frac{HA}{m}] = [T\,m]}{V(x,y,z)} = -\underset{[H/m]}{C_m} \iiint_v \nabla \frac{1}{r} \cdot \underset{\text{total magnetization}}{\mathbf{h}(x',y',z')}\, dv$$

$$\mathbf{m}(x',y',z') = \mathbf{h}(x',y',z')\, dv$$

magnetic moment = magnetization X volume

$[A\,m^2]$ $[A/m] \times [m^3]$

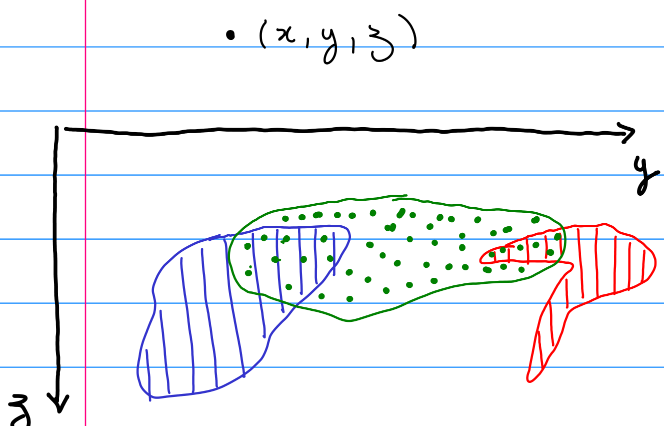
Gravitational potential produced by the 3D body

$$\underset{[m^2/s^2]}{U(x,y,z)} = \underset{[\frac{m^3}{Kg\,s^2}]}{G} \iiint_v \frac{1}{r}\, \underset{\text{density}}{\rho(x',y',z')}\, dv$$

$$m(x',y',z') = \rho(x',y',z')\, dv$$

mass = density X volume

$[Kg]$ $[Kg/m^3] \times [m^3]$



$$v = v_1 \cup v_2 \cup v_3 \cup v_4 \cup v_5$$

$v_1, \rho_1, \mathbf{h}_1$

$v_2, \rho_2 = \rho_1 + \rho_3, \mathbf{h}_2 = \mathbf{h}_1 + \mathbf{h}_3$

$v_3, \rho_3, \mathbf{h}_3$

$v_4, \rho_4 = \rho_3 + \rho_5, \mathbf{h}_4 = \mathbf{h}_3 + \mathbf{h}_5$

$v_5, \rho_5, \mathbf{h}_5$

$$V(x,y,z) = -C_m \iiint_v \nabla \frac{1}{r} \cdot \mathbf{h}(x',y',z')\, dv$$

$$= -C_m \sum_{K=1}^{5} \iiint_{v_K} \nabla \frac{1}{r} \cdot \mathbf{h}_K\, dv$$

$$U(x,y,z) = G \iiint_v \frac{1}{r}\, \rho(x',y',z')\, dv$$

$$= G \sum_{K=1}^{5} \iiint_{v_K} \frac{1}{r}\, \rho_K\, dv$$

$$V(x,y,z) = -Gm\left\{ \iiint_{v_1} \nabla\frac{1}{r}\cdot \mathbf{h}_1\, dv + \iiint_{v_2} \nabla\frac{1}{r}\cdot \mathbf{h}_2\, dv + \iiint_{v_3} \nabla\frac{1}{r}\cdot \mathbf{h}_3\, dv + \right.$$

$$\left. + \iiint_{v_4} \nabla\frac{1}{r}\cdot \mathbf{h}_4\, dv + \iiint_{v_5} \nabla\frac{1}{r}\cdot \mathbf{h}_5\, dv \right\}$$

$$= -Gm\left\{ \iiint_{v_1} \nabla\frac{1}{r}\cdot \mathbf{h}_1\, dv + \left[\iiint_{v_2} \nabla\frac{1}{r}\cdot \mathbf{h}_1\, dv + \iiint_{v_2} \nabla\frac{1}{r}\cdot \mathbf{h}_3\, dv \right] + \right.$$

$$+ \iiint_{v_3} \nabla\frac{1}{r}\cdot \mathbf{h}_3\, dv + \left[\iiint_{v_4} \nabla\frac{1}{r}\cdot \mathbf{h}_3\, dv + \iiint_{v_4} \nabla\frac{1}{r}\cdot \mathbf{h}_5\, dv \right] +$$

$$\left. + \iiint_{v_5} \nabla\frac{1}{r}\cdot \mathbf{h}_5\, dv \right\}$$

$$U(x,y,z) = G\left\{ \iiint_{v_1} \frac{1}{r}\rho_1\, dv + \iiint_{v_2} \frac{1}{r}\rho_2\, dv + \iiint_{v_3} \frac{1}{r}\rho_3\, dv + \right.$$

$$\left. + \iiint_{v_4} \frac{1}{r}\rho_4\, dv + \iiint_{v_5} \frac{1}{r}\rho_5\, dv \right\}$$

$$= G\left\{ \iiint_{v_1} \frac{1}{r}\rho_1\, dv + \left[\iiint_{v_2} \frac{1}{r}\rho_1\, dv + \iiint_{v_2} \frac{1}{r}\rho_3\, dv \right] + \right.$$

$$+ \iiint_{v_3} \frac{1}{r}\rho_3\, dv + \left[\iiint_{v_4} \frac{1}{r}\rho_3\, dv + \iiint_{v_4} \frac{1}{r}\rho_5\, dv \right] +$$

$$\left. + \iiint_{v_5} \frac{1}{r}\rho_5\, dv \right\}$$

$$\partial_\alpha V(x,y,z) = -Gm \iiint_{v} \left(\partial_\alpha \nabla\frac{1}{r}\right)\cdot \mathbf{h}(x',y',z')\, dv \qquad \partial_\alpha U(x,y,z) = G\iiint_{v} \partial_\alpha \frac{1}{r}\, \rho(x',y',z')\, dv$$

$$\partial_{\alpha\beta} V(x,y,z) = -Gm \iiint_{v} \left(\partial_{\alpha\beta} \nabla\frac{1}{r}\right)\cdot \mathbf{h}(x',y',z')\, dv \qquad \partial_{\alpha\beta} U(x,y,z) = G\iiint_{v} \partial_{\alpha\beta} \frac{1}{r}\, \rho(x',y',z')\, dv$$