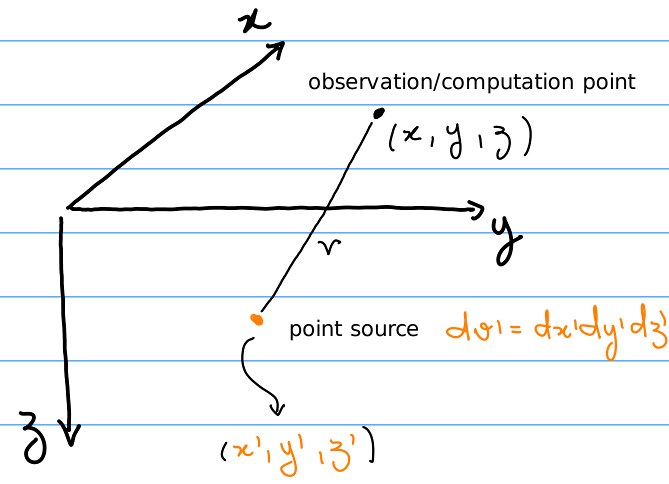
# Point sources




Consider a point source located at $(x', y', z')$, with volume $dv'$.

Consider also that the distante between the point source and an observation/computation point $(x, y, z)$ is defined by $r$.

$$r = \left[(x-x')^2 + (y-y')^2 + (z-z')^2\right]^{1/2}$$

Inverse distance function

$$\frac{1}{r} = \frac{1}{\left[(x-x')^2 + (y-y')^2 + (z-z')^2\right]^{1/2}}$$

first derivative of 1/r with respect to the observation point

$$\partial_\alpha \frac{1}{r} = \left(-\frac{1}{2}\right) \frac{1}{[\cdots]^{3/2}} \, 2(\alpha - \alpha')$$

$$= -\frac{\alpha - \alpha'}{r^3}$$

$$\boxed{\begin{array}{l} \alpha = x, y, z \\ \alpha' = x', y', z' \end{array}}$$

first derivative of 1/r with respect to the source point

$$\partial_{\alpha'} \frac{1}{r} = \left(-\frac{1}{2}\right) \frac{1}{[\cdots]^{3/2}} \, 2(\alpha - \alpha')(-1)$$

$$= \frac{\alpha - \alpha'}{r^3}$$

2nd derivative of 1/r with respect to the observation point

$$\partial_{\alpha\alpha} \frac{1}{r} = \left[\partial_\alpha (\alpha - \alpha')\right]\left(-\frac{1}{r^3}\right) + (\alpha - \alpha')\left[\partial_\alpha \left(-\frac{1}{r^3}\right)\right]$$

$$= -\frac{1}{r^3} + (\alpha - \alpha')\left[-\left(-\frac{3}{2}\right)\frac{1}{[\cdots]^{5/2}} \, 2(\alpha - \alpha')\right]$$

$$= \frac{3(\alpha - \alpha')^2}{r^5} - \frac{1}{r^3}$$

$$\partial_{\beta\alpha} \frac{1}{r} = \frac{3(\beta - \beta')(\alpha - \alpha')}{r^5}$$

$$\boxed{\begin{array}{l} \beta = x, y, z \\ \beta' = x', y', z' \end{array}}$$

2nd derivative of 1/r with respect to the source point

$$\partial_{\alpha'\alpha'}\frac{1}{r} = \left[\partial_{\alpha'}(\alpha-\alpha')\right]\left(\frac{1}{r^3}\right) + (\alpha-\alpha')\left[\partial_{\alpha'}\left(\frac{1}{r^3}\right)\right]$$

$$= \frac{-1}{r^3} + (\alpha-\alpha')\left[\left(-\frac{3}{2}\right)\frac{1}{[\dots]^{5/2}}\,2(\alpha-\alpha')(-1)\right]$$

$$= \frac{3(\alpha-\alpha')^2}{r^5} - \frac{1}{r^3}$$

$$\partial_{\beta'\alpha'}\frac{1}{r} = \frac{3(\beta-\beta')(\alpha-\alpha')}{r^5}$$

$$\boxed{\begin{gathered}\beta = x, y, z \\ \beta' = x', y', z'\end{gathered}}$$

From the derivatives computed above, we obtain

$$\partial_{\alpha}\frac{1}{r} = -\partial_{\alpha'}\frac{1}{r} \qquad \partial_{\alpha\beta}\frac{1}{r} = \partial_{\alpha'\beta'}\frac{1}{r}$$

Laplacian of 1/r

$$\partial_{\alpha\beta}\frac{1}{r} = \partial_{\beta\alpha}\frac{1}{r} \qquad \nabla^2\frac{1}{r} = \partial_{xx}\frac{1}{r} + \partial_{yy}\frac{1}{r} + \partial_{zz}\frac{1}{r} = 0$$

This equation indicates that 1/r is a harmonic function

## Magnetic scalar potential produced by a dipole

$$V'(x,y,z) = -C_m\, \nabla\frac{1}{r}^{T}\, \mathbf{h}'(x',y',z')\, dv'$$

$$\left[\frac{HA}{m}\right] = [T\,m] \qquad [H/m]$$

total magnetization

$$\mathbf{m}'(x',y',z') = \mathbf{h}'(x',y',z')\, dv'$$

magnetic moment = magnetization X volume

$$[A\,m^2] \qquad [A/m] \times [m^3]$$

## Gravitational potential produced by a point mass

$$U'(x,y,z) = G\,\frac{1}{r}\,\rho'(x',y',z')\, dv'$$

$$[m^2/s^2] \qquad \left[\frac{m^3}{Kg\, s^2}\right]$$

density

$$m'(x',y',z') = \rho'(x',y',z')\, dv'$$

mass = density X volume

$$[Kg] \qquad [Kg/m^3] \times [m^3]$$

gradient operator with respect to the coordinates of the observation point

$$\nabla\frac{1}{r} = \begin{bmatrix}\partial_x 1/r \\ \partial_y 1/r \\ \partial_z 1/r\end{bmatrix}$$

Magnetic induction field produced by a dipole

$$\mathbf{B}'(x,y,z) = -\boldsymbol{\nabla} V'(x,y,z)$$

gradient operator with respect to the coordinates of the observation point

$$\boldsymbol{\nabla} V'(x,y,z) = \begin{bmatrix} \partial_x V'(x,y,z) \\ \partial_y V'(x,y,z) \\ \partial_z V'(x,y,z) \end{bmatrix}$$

$\alpha = x, y, z$

$$\partial_\alpha V'(x,y,z) = -c_m \left(\partial_\alpha \boldsymbol{\nabla} \frac{1}{r}\right)^T \mathbf{h}'(x',y',z')\, dv'$$

$$\partial_\alpha \boldsymbol{\nabla} \frac{1}{r} = \begin{bmatrix} \partial_{\alpha x} 1/r \\ \partial_{\alpha y} 1/r \\ \partial_{\alpha z} 1/r \end{bmatrix}$$

Gravitational acceleration field produced by a point mass

$$\boldsymbol{\alpha}'(x,y,z) = \boldsymbol{\nabla} U'(x,y,z)$$

gradient operator with respect to the coordinates of the observation point

$$\boldsymbol{\nabla} U'(x,y,z) = \begin{bmatrix} \partial_x U'(x,y,z) \\ \partial_y U'(x,y,z) \\ \partial_z U'(x,y,z) \end{bmatrix}$$

$\alpha = x, y, z$

$$\partial_\alpha U'(x,y,z) = -G \frac{(\alpha - \alpha')}{r^3} \rho'(x',y',z')\, dv'$$

$$\mathbf{B}'(x,y,z) = -c_m \begin{bmatrix} -\left(\partial_x \boldsymbol{\nabla} \frac{1}{r}\right)^T \mathbf{h}'(x',y',z')\, dv' \\ -\left(\partial_y \boldsymbol{\nabla} \frac{1}{r}\right)^T \mathbf{h}'(x',y',z')\, dv' \\ -\left(\partial_z \boldsymbol{\nabla} \frac{1}{r}\right)^T \mathbf{h}'(x',y',z')\, dv' \end{bmatrix} = c_m \mathbf{H}'(x,y,z,x',y',z')\, \mathbf{h}'(x',y',z')\, dv'$$

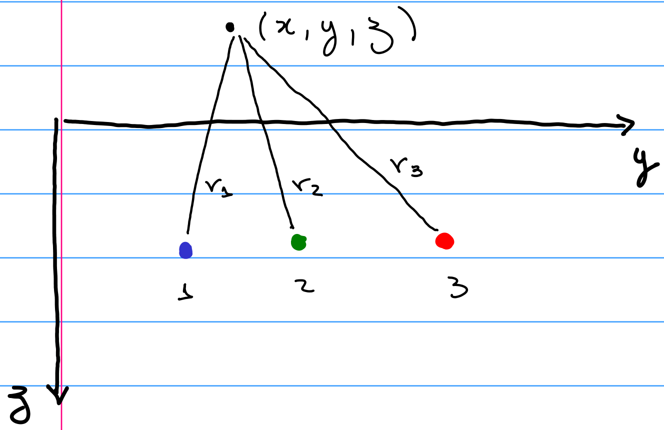
derivatives with respect to the coordinates of the observation point

$$\mathbf{H}'(x,y,z,x',y',z') = \begin{bmatrix} \partial_{xx} 1/r & \partial_{xy} 1/r & \partial_{xz} 1/r \\ \partial_{xy} 1/r & \partial_{yy} 1/r & \partial_{yz} 1/r \\ \partial_{xz} 1/r & \partial_{yz} 1/r & \partial_{zz} 1/r \end{bmatrix} = \boldsymbol{\nabla}^2 \frac{1}{r}$$

gradient tensor of 1/r

Gravitational gradient tensor produced by a point mass

$$\boldsymbol{\Gamma}'(x,y,z) = \boldsymbol{\nabla}^2 U'(x,y,z) = \begin{bmatrix} \partial_{xx} U' & \partial_{xy} U' & \partial_{xz} U' \\ \partial_{xy} U' & \partial_{yy} U' & \partial_{yz} U' \\ \partial_{xz} U' & \partial_{yz} U' & \partial_{zz} U' \end{bmatrix}$$




$$r_j = \left[(x - x'_j)^2 + (y - y'_j)^2 + (z - z'_j)^2\right]^{1/2}$$

$\rho'_1\ ,\ \mathbf{h}'_1$

$\rho'_2\ ,\ \mathbf{h}'_2$

$\rho'_3\ ,\ \mathbf{h}'_3$

$$\rho'_j \equiv \rho'(x'_j, y'_j, z'_j)$$

$$\mathbf{h}'_j \equiv \mathbf{h}'(x'_j, y'_j, z'_j)$$

$$j = 1, 2, 3$$

$$V'(x,y,z) = \sum_{j=1}^{3} V'_j(x,y,z) \quad , \quad V'_j(x,y,z) = -Gm\, \nabla \frac{1}{r_j}^{T} \mathbf{h}'_j\, dv'$$

$$\mathbf{B}'(x,y,z) = \sum_{j=1}^{3} \mathbf{B}'_j(x,y,z)$$

$$U'(x,y,z) = \sum_{j=1}^{3} U'_j(x,y,z)$$

$$\mathbf{a}'(x,y,z) = \sum_{j=1}^{3} \mathbf{a}'_j(x,y,z)$$

$$\mathbf{\Gamma}'(x,y,z) = \sum_{j=1}^{3} \mathbf{\Gamma}'_j(x,y,z)$$