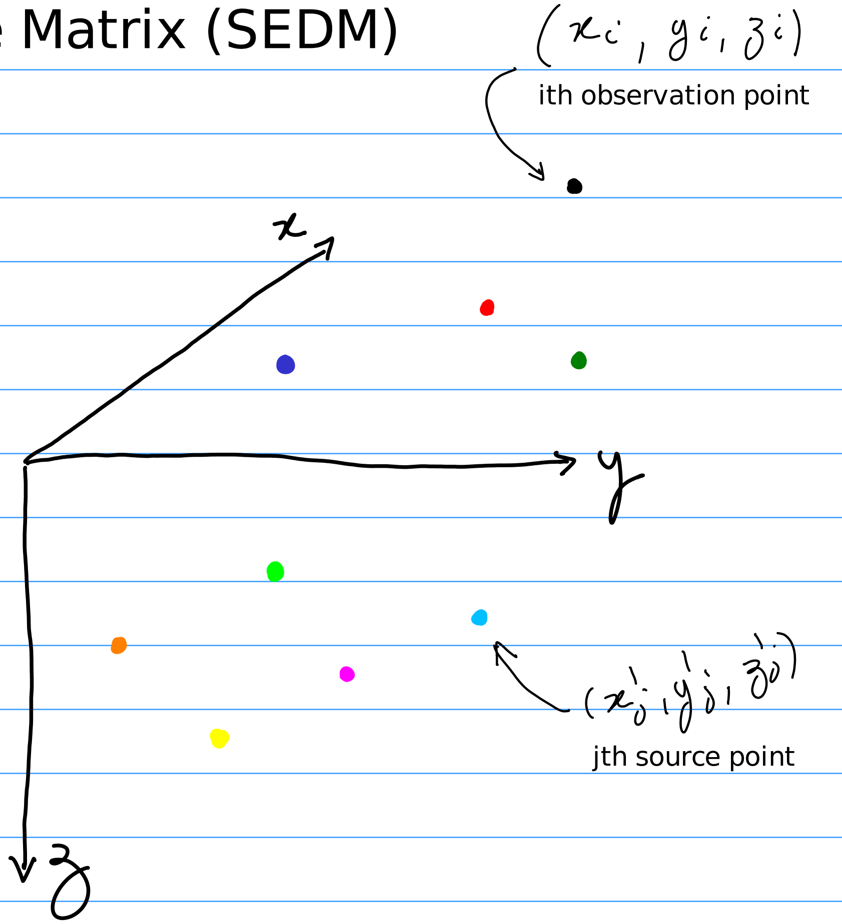
Squared Euclidean Distance Matrix (SEDM)



Observation points matrix

$$
P = \begin{bmatrix} x_0 & x_1 & \cdots & x_{N-1} \\ y_0 & y_1 & \cdots & y_{N-1} \\ z_0 & z_0 & \cdots & z_{N-1} \end{bmatrix}_{3 \times N}
$$

Source points matrix

$$
S = \begin{bmatrix} x'_0 & x'_1 & \cdots & x'_{M-1} \\ y'_0 & y'_1 & \cdots & y'_{M-1} \\ z'_0 & z'_1 & \cdots & z'_{M-1} \end{bmatrix}_{3 \times M}
$$

Squared Euclidean Distance Matrix (SEDM)

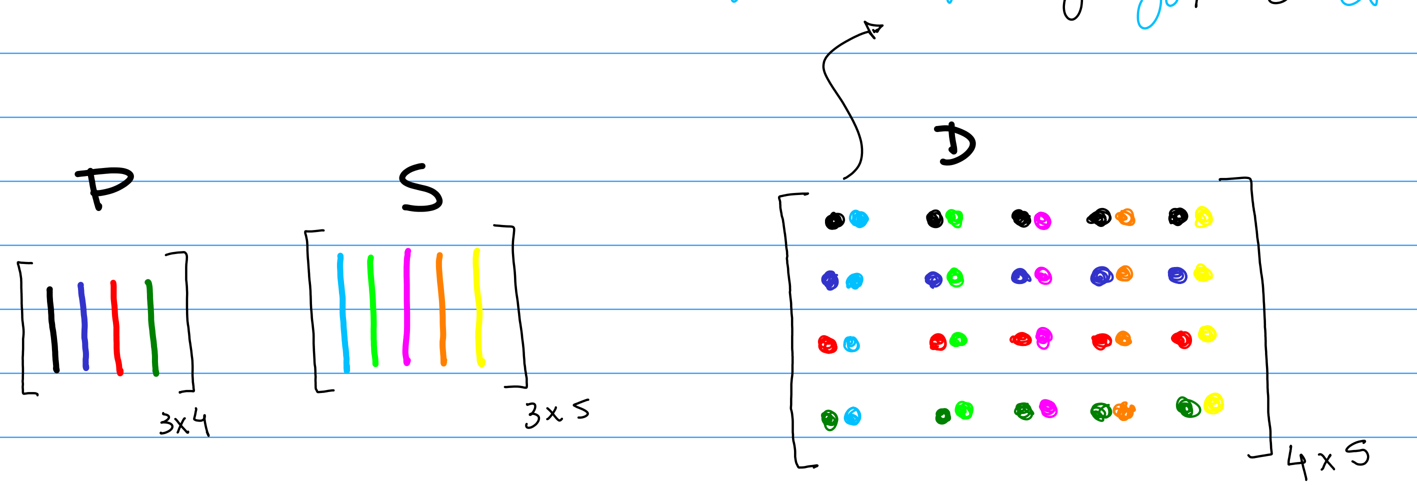
$$
D = \begin{bmatrix} d^2_{00} & \cdots & d^2_{0M-1} \\ \vdots & & \vdots \\ d^2_{N-10} & \cdots & d^2_{N-1M-1} \end{bmatrix}_{N \times M}
$$

Squared Euclidean distance

$$
d^2_{ij} = (x_i - x'_j)^2 + (y_i - y'_j)^2 + (z_i - z'_j)^2
$$

$$
i = 0, \ldots, N-1 \qquad j = 0, \ldots, M-1
$$

$$
d^2_{ij} = (x_i - x'_j)^2 + (y_i - y'_j)^2 + (z_i - z'_j)^2
$$



$$d_{ij}^2 = (x_i - x'_j)^2 + (y_i - y'_j)^2 + (z_i - z'_j)^2$$

$$= (x_i^2 - 2x_i x'_j + x'^2_j) + (y_i^2 - 2y_i y'_j + y'^2_j) + (z_i^2 - 2z_i z'_j + z'^2_j)$$

$$= (x_i^2 + y_i^2 + z_i^2) + (x'^2_j + y'^2_j + z'^2_j) - 2(x_i x'_j + y_i y'_j + z_i z'_j)$$

$$d_{ij}^2 = \mathbf{P}_i^T \mathbf{P}_i + \mathbf{s}_j^T \mathbf{s}_j - 2\mathbf{P}_i^T \mathbf{s}_j$$

$$\mathbf{P}_i = \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix}$$

ith row of matrix $\mathbf{P}$

$$\mathbf{s}_j = \begin{bmatrix} x'_j \\ y'_j \\ z'_j \end{bmatrix}$$

th row of matrix $\mathbf{S}$

constant values along rows

$$\mathbf{D}_1 = \begin{bmatrix} \mathbf{P}_0^T \mathbf{P}_0 & \cdots & \mathbf{P}_0^T \mathbf{P}_0 \\ \mathbf{P}_{N-1}^T \mathbf{P}_{N-1} & \cdots & \mathbf{P}_{N-1}^T \mathbf{P}_{N-1} \end{bmatrix}_{N \times M}$$

$$\mathbf{D}_2 = \begin{bmatrix} \mathbf{s}_0^T \mathbf{s}_0 & \mathbf{s}_{M-1}^T \mathbf{s}_{M-1} \\ \vdots & \vdots \\ \mathbf{s}_0^T \mathbf{s}_0 & \mathbf{s}_{M-1}^T \mathbf{s}_{M-1} \end{bmatrix}_{N \times M}$$

constant values along columns

$$\boxed{\mathbf{D} = \mathbf{D}_1 + \mathbf{D}_2 + \mathbf{D}_3}$$

$$\mathbf{D}_3 = \begin{bmatrix} \mathbf{P}_0^T \mathbf{s}_0 & \mathbf{P}_0^T \mathbf{s}_{M-1} \\ \mathbf{P}_{N-1}^T \mathbf{s}_0 & \mathbf{P}_{N-1}^T \mathbf{s}_{M-1} \end{bmatrix}_{N \times M} = 2\mathbf{P}^T \mathbf{S}$$