

# Squared Euclidean Distance Matrix (SEDM)

$(x_i, y_i, z_i)$

$i$ th observation point

Observation points matrix

$$P = \begin{bmatrix} x_0 & x_1 & \dots & x_{N-1} \\ y_0 & y_1 & \dots & y_{N-1} \\ z_0 & z_1 & \dots & z_{N-1} \end{bmatrix}_{3 \times N}$$

Source points matrix

$$S = \begin{bmatrix} x'_0 & x'_1 & \dots & x'_{M-1} \\ y'_0 & y'_1 & \dots & y'_{M-1} \\ z'_0 & z'_1 & \dots & z'_{M-1} \end{bmatrix}_{3 \times M}$$

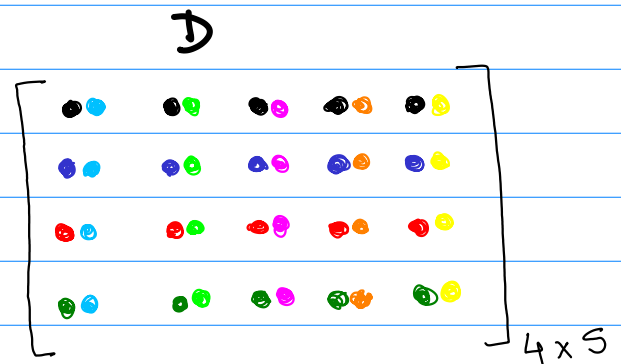
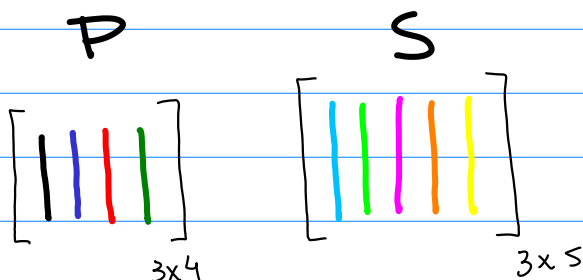
Squared Euclidean distance

$$d_{ij}^z = (x_i - x'_j)^2 + (y_i - y'_j)^2 + (z_i - z'_j)^2$$

$$i = 0, \dots, N-1 \quad j = 0, \dots, M-1$$

Squared Euclidean Distance Matrix (SEDM)

$$D = \begin{bmatrix} d_{00}^z & \dots & d_{0M-1}^z \\ \vdots & & \vdots \\ d_{N-10}^z & \dots & d_{N-1M-1}^z \end{bmatrix}_{N \times M}$$



$$d_{ij}^2 = \mathbf{p}_i^T \mathbf{p}_i + \mathbf{s}_j^T \mathbf{s}_j - 2 \mathbf{p}_i^T \mathbf{s}_j$$

$$\mathbf{p}_i = \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix}$$

$$\mathbf{s}_j = \begin{bmatrix} x'_j \\ y'_j \\ z'_j \end{bmatrix}$$

$$\mathbf{D}_1 = \begin{bmatrix} \mathbf{p}_0^T \mathbf{p}_0 & \dots & \mathbf{p}_0^T \mathbf{p}_N \\ \mathbf{p}_{N-1}^T \mathbf{p}_0 & \dots & \mathbf{p}_{N-1}^T \mathbf{p}_N \end{bmatrix}_{N \times M}$$

$$\mathbf{D}_2 = \begin{bmatrix} \mathbf{s}_0^T \mathbf{s}_0 & \mathbf{s}_{m-1}^T \mathbf{s}_{m-1} \\ \vdots & \vdots \\ \mathbf{s}_0^T \mathbf{s}_0 & \mathbf{s}_{m-1}^T \mathbf{s}_{m-1} \end{bmatrix}_{N \times M}$$

$$\boxed{\mathbf{D} = \mathbf{D}_1 + \mathbf{D}_2 + \mathbf{D}_3}$$

$$\mathbf{D}_3 = \begin{bmatrix} \mathbf{p}_0^T \mathbf{s}_0 & \mathbf{p}_0^T \mathbf{s}_{m-1} \\ \mathbf{p}_{N-1}^T \mathbf{s}_0 & \mathbf{p}_{N-1}^T \mathbf{s}_{m-1} \end{bmatrix} = \mathbf{P}^T \mathbf{S}$$

$N \times M$